ECEN 615
Methods of Electric Power Systems Analysis

Lecture 16: State Estimation, EMS

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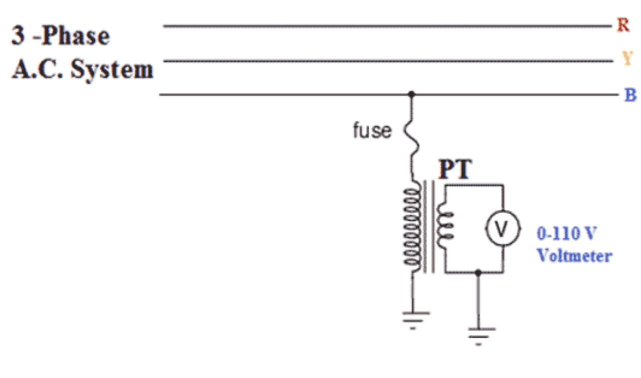
Announcements

- Read Chapter 9
- Homework 4 is due today
  - In problem 5 the LODF is a vector
  - In problem 6 find the OTDF on the line between buses 1 and 2

Electric Grid Measurements

- The two major types of measurements are voltages and currents
  - The challenge for both types are doing these measurements at the high electric grid voltage levels
- Potential transformers (PTs) are used to measure voltage, using a transformer sometimes with a set of series capacitors to drop the voltage

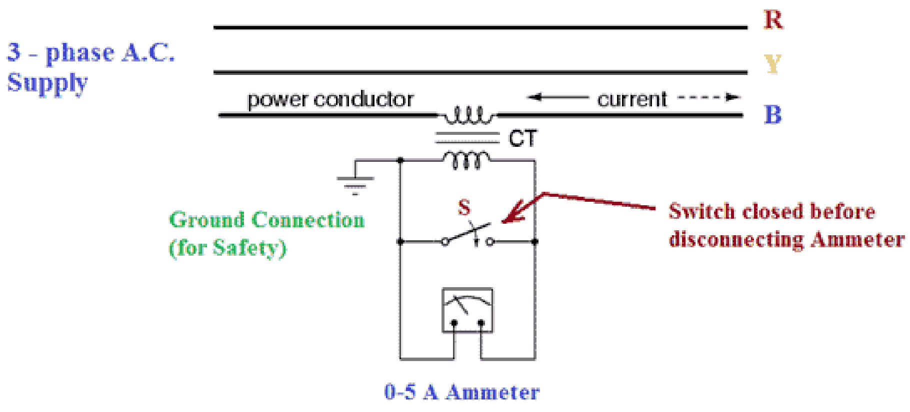


Potential Transformer (P.T.)

Image source: www.electrical4u.com/instrument-transformers/

Electric Grid Measurements

- Current transformers (CTs) are used to measure current, with the primary often consisting of the transmission wire itself; the secondary then has its number of turns set to give a specified current (say 5A) at a specified line current
  - Many CTs are used in the protection system so these need to be calibrated to correctly measure fault current; others are used to give more accurate load current values
- All meters have errors



Current Transformer (C.T.)

Image source: www.electrical4u.com/instrument-transformers/

Phasor Measurement Units (PMUs)

- All AC signals have a magnitude and phase. It is very easy to measure the phase angle differences between local signals (e.g., at an electrical substation)
  - These differences are used to calculate power values
- However, it had been challenging to measure phase angle differences between signals at different locations
  - This requires access to a precise time source
  - At 60 Hz one cycle takes 16.67 ms, which means one degree takes 46 µs.

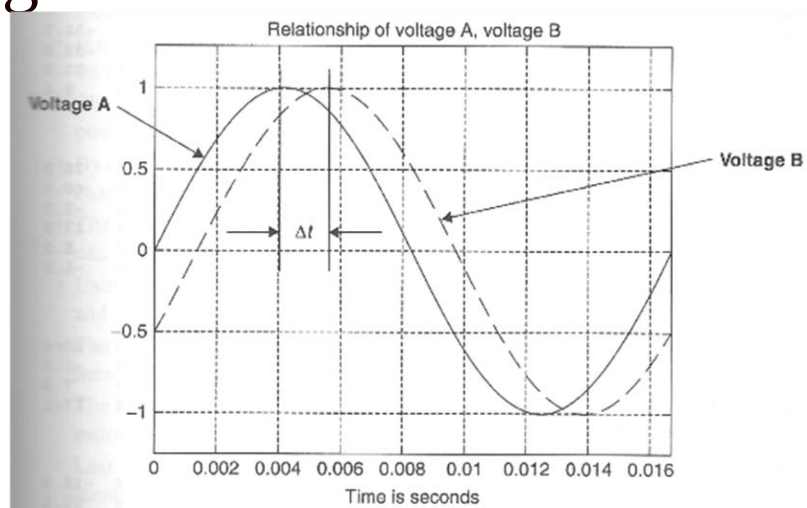


FIGURE 9.19 Phase difference as a function of time difference.

Phasor Measurement Units (PMUs)

- Widespread access to precise time became available in the 1980's when civilian use of the GPS was allowed
- PMUs use the GPS signals to determine the phase angles of voltages and currents (relative to some global reference)
  - The inputs to PMUs come from the CTs and PTs
- PMUs sample the system at rates on the order of 30 times per second
- PMU values are being used in SE algorithms

SE Example: Two Bus Case

- Assume a two bus case with a generator supplying a load through a single line with x=0.1 pu. Assume measurements of the p/q flow on both ends of the line (into line positive), and the voltage magnitude at both the generator and the load end. So B_{12} = B_{21}=10.0

$$P_{ij}^{meas} - \left[V_i V_j \left(B_{ij} \sin\left(\theta_i - \theta_j \right) \right) \right]$$

$$Q_{ij}^{meas} - \left[V_i^2 B_{ij} + V_i V_j \left(-B_{ij} \cos\left(\theta_i - \theta_j \right) \right) \right]$$

$$V_i^{meas} - V_i = 0$$

We need to assume a reference angle unless we're directly measuring phase angles

Example: Two Bus Case

- Let $$\mathbf{z}^{meas} = \begin{bmatrix} P_{12} \\ Q_{12} \\ P_{21} \\ Q_{21} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 1.01 \\ 0.87 \end{bmatrix} \qquad x^0 = \begin{bmatrix} V_1 \\ \theta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \sigma_i = 0.01$$

We assume an angle reference of $\theta_1 = 0$

$$H(\mathbf{x}) = \begin{bmatrix} V_2 10 \sin(-\theta_2) & -V_1 V_2 10 \cos(-\theta_2) & V_1 10 \sin(-\theta_2) \\ 20V_1 - V_2 10 \cos(-\theta_2) & -V_1 V_2 10 \sin(-\theta_2) & -V_1 10 \cos(-\theta_2) \\ V_2 10 \sin(\theta_2) & V_1 V_2 10 \cos(\theta_2) & V_1 10 \sin(\theta_2) \\ -V_2 10 \cos(\theta_2) & V_1 V_2 10 \sin(\theta_2) & 20V_2 - V_1 10 \cos(\theta_2) \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Two Bus Case

- With a flat start guess we get

$$H(\mathbf{x}^0) = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{z} - \mathbf{f}(\mathbf{x}^0) = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

Example: Two Bus Case

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = 1e^6 \times \begin{bmatrix} 2.01 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 2.01 \end{bmatrix}$$

$$\mathbf{x}^1 = \mathbf{x}^0 + \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix} = \begin{bmatrix} 1.003 \\ -0.2 \\ 0.8775 \end{bmatrix}$$

Assumed SE Measurement Accuracy

- The assumed measurement standard deviations can have a significant impact on the resultant solution, or even whether the SE converges
- The assumption is a Gaussian (normal) distribution of the error with no bias

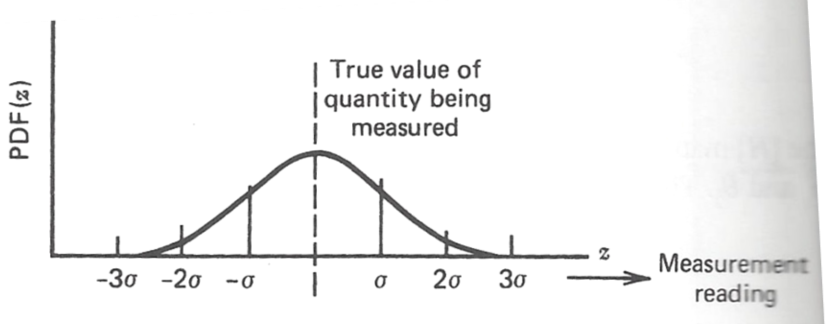


FIGURE 9.8 Normal distribution of meter errors.

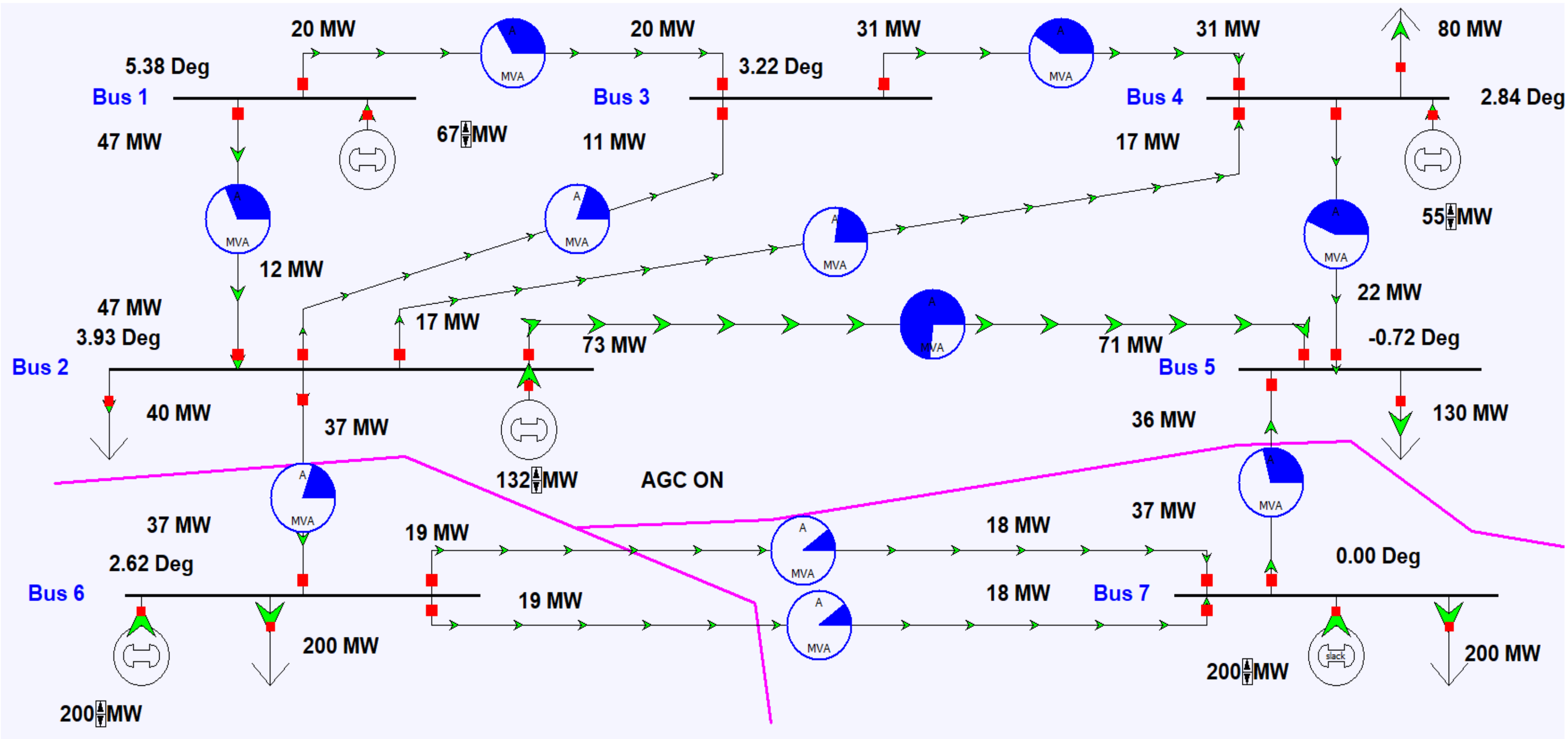
SE Observability

- In order to estimate all n states we need at least n measurements. However, where the measurements are located is also important, a topic known as observability
  - In order for a power system to be fully observable usually we need to have a measurement available no more than one bus away
  - At buses we need to have at least measurements on all the injections into the bus except one (including loads and gens)
  - Loads are usually flows on feeders, or the flow into a transmission to distribution transformer
  - Generators are usually just injections from the GSU

Pseudo Measurements

- Pseudo measurements are used at buses in which there is no load or generation; that is, the net injection into the bus is know with high accuracy to be zero
  - In order to enforce the net power balance at a bus we need to include an explicit net injection measurement
- To increase observability sometimes estimated values are used for loads, shunts and generator outputs
  - These "measurements" are represented as having a higher much standard deviation

SE Observability Example



SE Bad Data Detection

- The quality of the measurements available to an SE can vary widely, and sometimes the SE model itself is wrong. Causes include
  - Modeling Errors: perhaps the assumed system topology is incorrect, or the assumed parameters for a transmission line or transformer could be wrong
  - Data Errors: measurements may be incorrect because of in correct data specifications, like the CT ratios or even flipped positive and negative directions
  - Transducer Errors: the transductors may be failing or may have bias errors
  - Sampling Errors: SCADA does not read all values simultaneously and power systems are dynamic

SE Bad Data Detection

- The challenge for SE is to determine when there is likely a bad measurement (or multiple ones), and then to determine the particular bad measurements
- J(**x**) is random number, with a probability density function (PDF) known as a chi-squared distribution, $\chi^2(K)$, where K is the degrees of freedom, K=m-n
- It can be shown the expected mean for J(**x**) is K, with a standard deviation of $\sqrt{2K}$
  - Values of J(**x**) outside of several standard deviations indicate possible bad measurements, with the measurement residuals used to track down the likely bad measurements
- SE can be re-run without the bad measurements

QR Factorization

- Used in SE since it handles ill-conditioned m by n matrices (with m >= n)
- Can be used with sparse matrices
- We will first split the $\mathbf{R}^{-1}$ matrix

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = \mathbf{H}^T\mathbf{R}^{-1/2}\mathbf{R}^{-1/2}\mathbf{H} = \mathbf{H}'^T\mathbf{H}'$$

- QR factorization represents the m by n **H'** matrix as

$$\mathbf{H}' = \mathbf{Q}\,\mathbf{U}$$

with **Q** an m by m orthonormal matrix and **U** an upper triangular matrix (most books use **Q R** but we use **U** to avoid confusion with the previous **R**)

Orthonormal Matrices

- The term orthogonal is used with vectors to indicate their dot product is zero (i.e., they are perpendicular to each other)
- Orthonormal is used to indicate they are orthogonal and each has unit length (magnitude of 1)
- The definition of an orthogonal matrix is $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
  - This implies its inverse always exists
- Its determinant is 1
- They can be used for transformations such as an angular rotation

QR Factorization

- We then have $\mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\mathbf{Q}^T\mathbf{Q}\,\mathbf{U}$
- But since $\mathbf{Q}$ is an orthonormal matrix, $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
- Hence we have $\mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\,\mathbf{U}$

Originally $\Delta\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

With $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = \mathbf{H}'^T\mathbf{H}' = \mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\mathbf{U}$

Q is an m by m matrix

Let $\hat{\mathbf{z}} = \mathbf{Q}^T\mathbf{R}^{-\frac{1}{2}}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

$$\Delta\mathbf{x} = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-\frac{1}{2}}\mathbf{R}^{-\frac{1}{2}}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right] = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{U}^T\hat{\mathbf{z}}$$

$$\mathbf{U}^T\mathbf{U}\Delta\mathbf{x} = \mathbf{U}^T\hat{\mathbf{z}} \rightarrow \Delta\mathbf{x} = \mathbf{U}^{-1}\hat{\mathbf{z}}$$

QR Factorization

- Next we'll briefly discuss the QR factorization algorithm
- When factored the **U** matrix (i.e., what most call the **R** matrix) will be an m by n upper triangular matrix
- Several methods are available including the Householder method and the Givens method
- Givens is preferred when dealing with sparse matrices
- A good reference is Gene H. Golub and Charles F. Van Loan, "Matrix Computations," second edition, Johns Hopkins University Press, 1989.

Givens Algorithm for Factoring a Matrix A

- The Givens algorithm works by pre-multiplying the initial matrix, **A**, by a series of matrices and their transposes, starting with $\mathbf{G}_1\mathbf{G}_1^{\mathrm{T}}$
  - If **A** is m by n, then each **G** is an m by m matrix
- The algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of **A**, starting at the bottom of each column

$$\mathbf{G}_1 \ldots \mathbf{G}_p \mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{Q}\mathbf{U}$$

$$\mathbf{G}_1 \ldots \mathbf{G}_p = \mathbf{Q}$$

$$\mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{U}$$

If **A** is sparse, then we can take advantage of sparsity going up the column

Givens Algorithm

- To zero out element **A**[i,j], with i > j we first solve with a=A[k,j], b= A[i,j]

$$\begin{bmatrix} c & -s \\ s & c \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

$$r = \sqrt{a^2 + b^2}$$

To zero out an element we need a non-zero pivot element in column j; assume this row as k.

- A numerically safe algorithm is

If b=0 then c=1, s=0 // i.e, no rotation is needed

Else If $|b| > |a|$ then $\tau = -a/b; \quad s = 1/\sqrt{1+\tau^2}; c = s\tau$

Else $\tau = -b/a; \quad c = 1/\sqrt{1+\tau^2}; s = c\tau$

Givens G Matrix

- The orthogonal **G**(i,k,θ) matrix is then

$$\mathbf{G}(i,\mathrm{k},\theta)=\begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ 0 & \cdots & c & \cdots & s & \cdots & 0 \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ 0 & \cdots & -s & \cdots & c & \cdots & 0 \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & 0 & \cdots & 1 \end{bmatrix}$$

As noted, to zero out an element we need a non-zero pivot element in column j; assume this row as k. Row k here is the first non-zero above row i.

- Premultiplication by **G**(i,k,θ)^T is a rotation by θ radians in the (i,k) coordinate plane

Small Givens Example

- Let

$$\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix}$$

First start in column j=1; we will zero out A[4,1] with i=4, k=2

- First we zero out **A**[4,1], a=1, b=2 giving s= 0.8944, c=**-**0.4472

$$\mathbf{G}_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.4472 & 0 & 0.8944 \\ 0 & 0 & 1 & 0 \\ 0 & -0.8944 & 0 & -0.4472 \end{bmatrix} \qquad \mathbf{G}_1^T\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -2.236 & -0.8944 \\ 0 & 5 \\ 0 & -0.4472 \end{bmatrix}$$

Small Givens Example

- Next zero out A[2,1] with a=4, b=-2.236, giving c= -0.8729, s=0.4880

k=1 with A[k,j]=4

$$\mathbf{G}_2 = \begin{bmatrix} 0.873 & 0.488 & 0 & 0 \\ -0.488 & 0.873 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5 \\ 0 & -0.447 \end{bmatrix}$$

j=2, k=3 with A[k,j]=5

- Next zero out A[4,2] with a=5, b=-0.447, c=0.996, s=0.089

$$\mathbf{G}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.996 & 0.089 \\ 0 & 0 & -0.089 & 0.996 \end{bmatrix} \quad \mathbf{G}_3^T \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5.020 \\ 0 & 0 \end{bmatrix}$$

Small Givens Example

- Next zero out A[3,2] with a=0.195, b=5.02, c=-0.039, s=0.999

j=2, k=2 with A[k,j]=0.195

$$\mathbf{G}_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.039 & 0.999 & 0 \\ 0 & -0.999 & -0.039 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{G}_4^T \mathbf{G}_3^T \mathbf{G}_2^T \mathbf{G}_1^T \mathbf{A} = \mathbf{U} = \begin{bmatrix} 4.58 & 2.18 \\ 0 & -5.023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Also we have

$$\mathbf{Q} = \mathbf{G}_1 \mathbf{G}_2 \mathbf{G}_3 \mathbf{G}_4 = \begin{bmatrix} 0.872 & -0.019 & 0.487 & 0 \\ 0.218 & 0.094 & -0.387 & 0.891 \\ 0 & -0.995 & -0.039 & 0.089 \\ 0.436 & -0.009 & -0.782 & -0.445 \end{bmatrix}$$

Start of Givens for SE Example

- Starting with the **H** matrix we get

$$\mathbf{H}' = \mathbf{R}^{-1/2}\mathbf{H} = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- To zero out **H**'[5,1]=1 we have b=100, a=-1000, giving c=0.995, s=0.0995

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.995 & 0.0995 & 0 \\ 0 & 0 & 0 & -0.0995 & 0.995 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Here the column (j) is 1, while i is 5 and k is 4.

Start of Givens for SE Example

- Which gives

$$\mathbf{G}_1^T \mathbf{H}' = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ 10.049 & 0 & -9.95 \\ 0 & 0 & 0.995 \\ 0 & 0 & 1 \end{bmatrix}$$

- The next rotation would be to zero out element [4,1], continuing until all the elements in the lower triangle have been reduced

Givens Comments

- For a full matrix, Givens is $O(mn^2)$ since each element in the lower triangle needs to be zeroed $O(nm)$, and each operation is $O(n)$
- Computation can be drastically reduced for a sparse matrix since we only need to zero out the elements that are initially non-zero, and any that become non-zero (i.e., the fills)
  - Also, for each multiply we only need to deal with the nonzeros in the impacted row
- Givens rotation is commonly used to solve the SE

Example SE Application: PJM and MISO

- PJM provides information about their EMS model in
  - www.pjm.com/-/media/documents/manuals/m03a.ashx

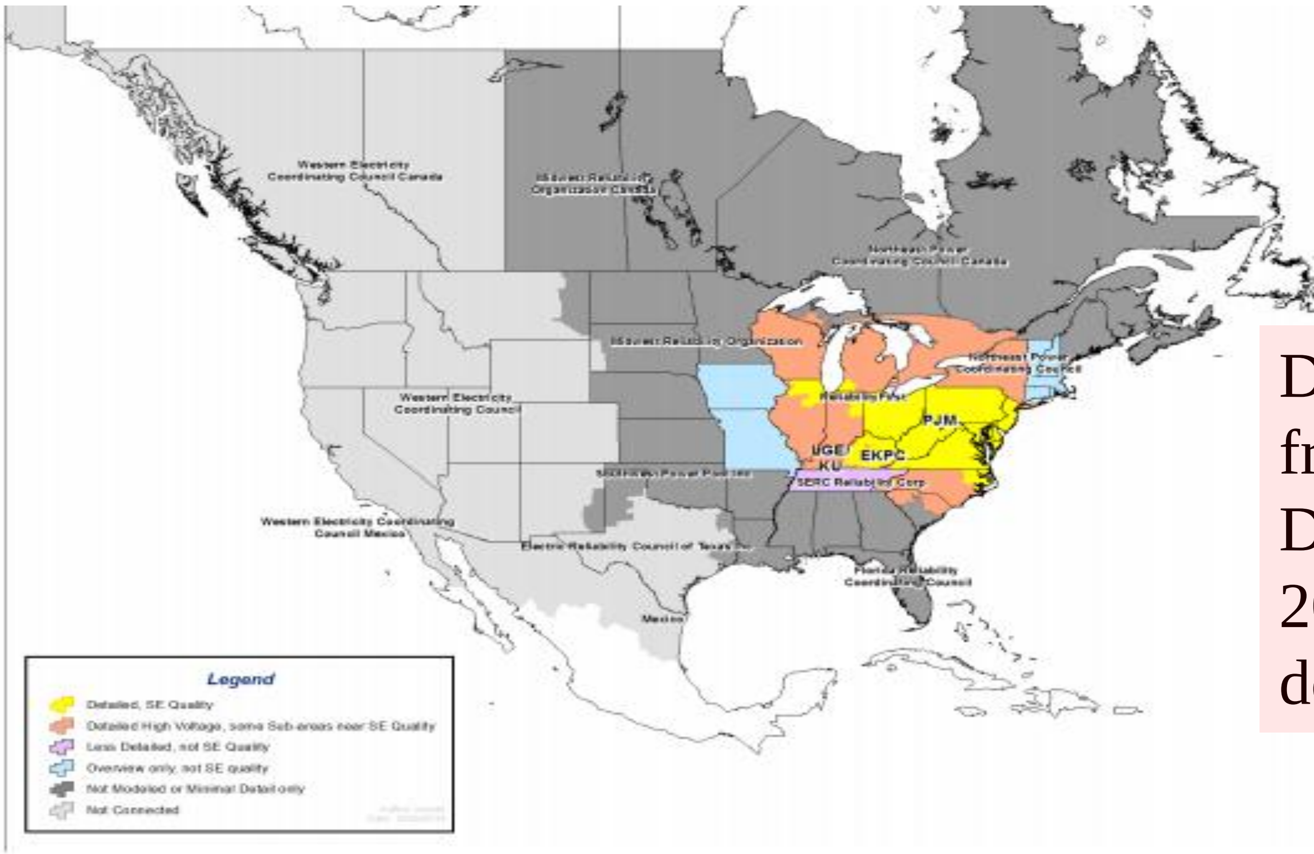


Exhibit 4: PJM EMS Model Details

Data here is from the December 2019 (Rev 18) document

Example SE Application: PJM and MISO

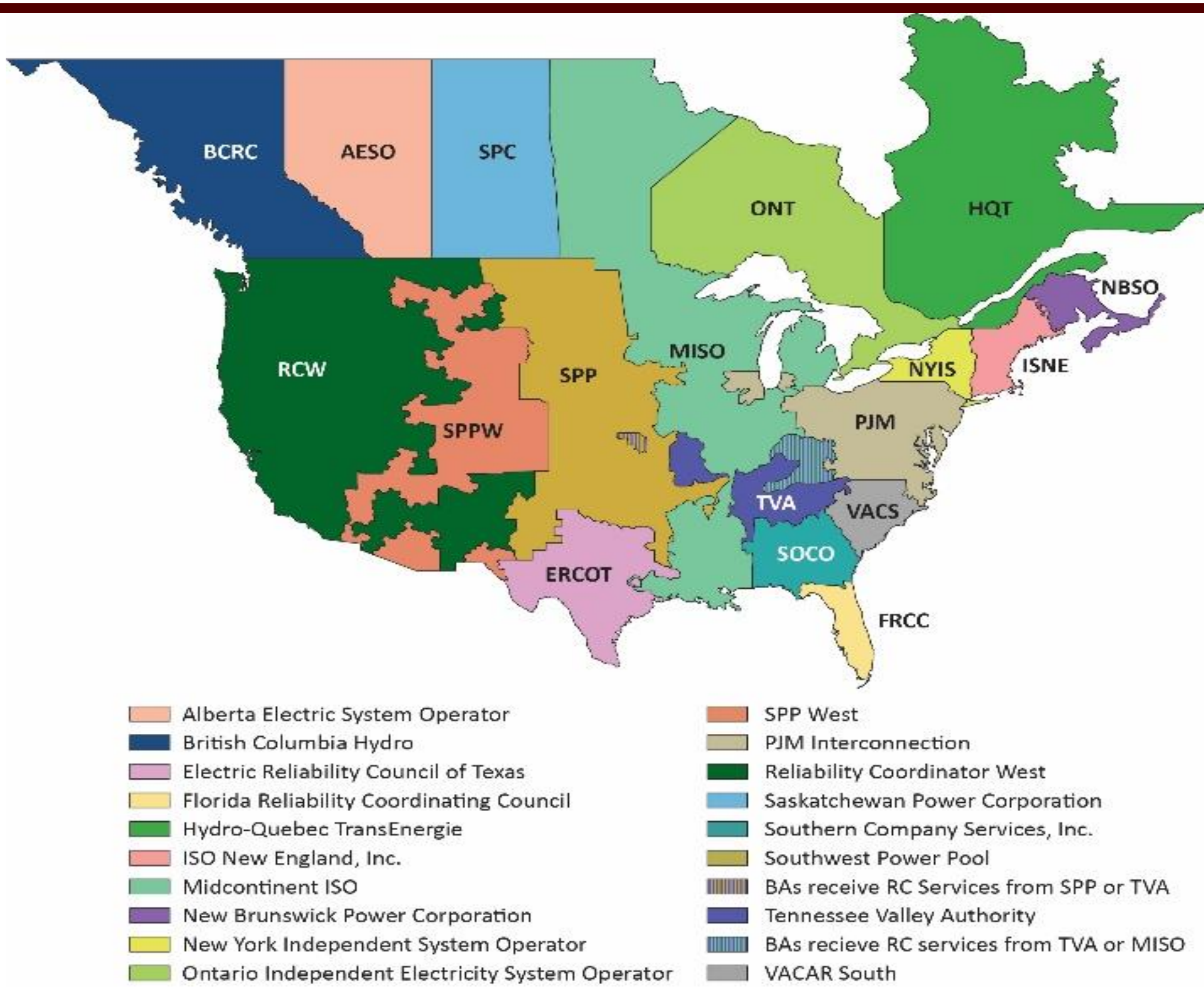
- PJM measurements are required for 69 kV and up
- PJM SE is triggered to execute every minute
- PJM SE solves well over 98% of the time
- Below reference provides info on MISO SE from March 2015
  - 54,433 buses
  - 54,415 network branches
  - 6332 generating units
  - 228,673 circuit breakers
  - 289,491 mapped points

https://www.naspi.org/sites/default/files/2017-05/3a%20MISO-NASPIWokshop-Synchrophasor%20Data%20and%20State%20Estimation.pdf

Energy Management Systems (EMSs)

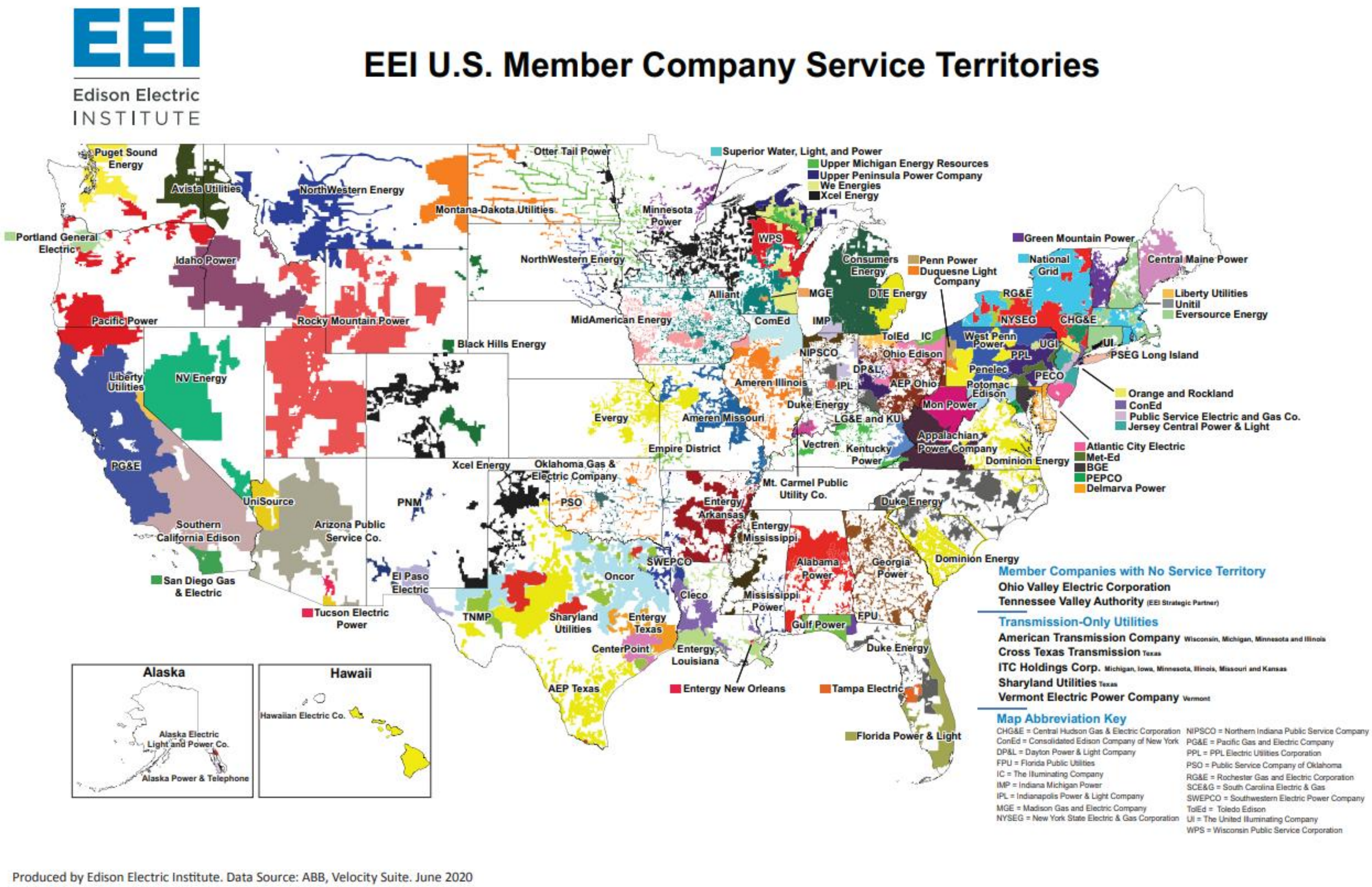
- EMSs are now used to control most large scale electric grids
- EMSs developed in the 1970's and 1980's out of SCADA systems
  - An EMS usually includes a SCADA system; sometimes called a SCADA/EMS
- Having a SE is almost the definition of an EMS. The SE then feeds data to the more advanced functions
- EMSs have evolved as the industry as evolved as the industry has evolved, with functionality customized for the application (e.g., a reliability coordinator or a vertically integrated utility)

NERC Reliability Coordinators

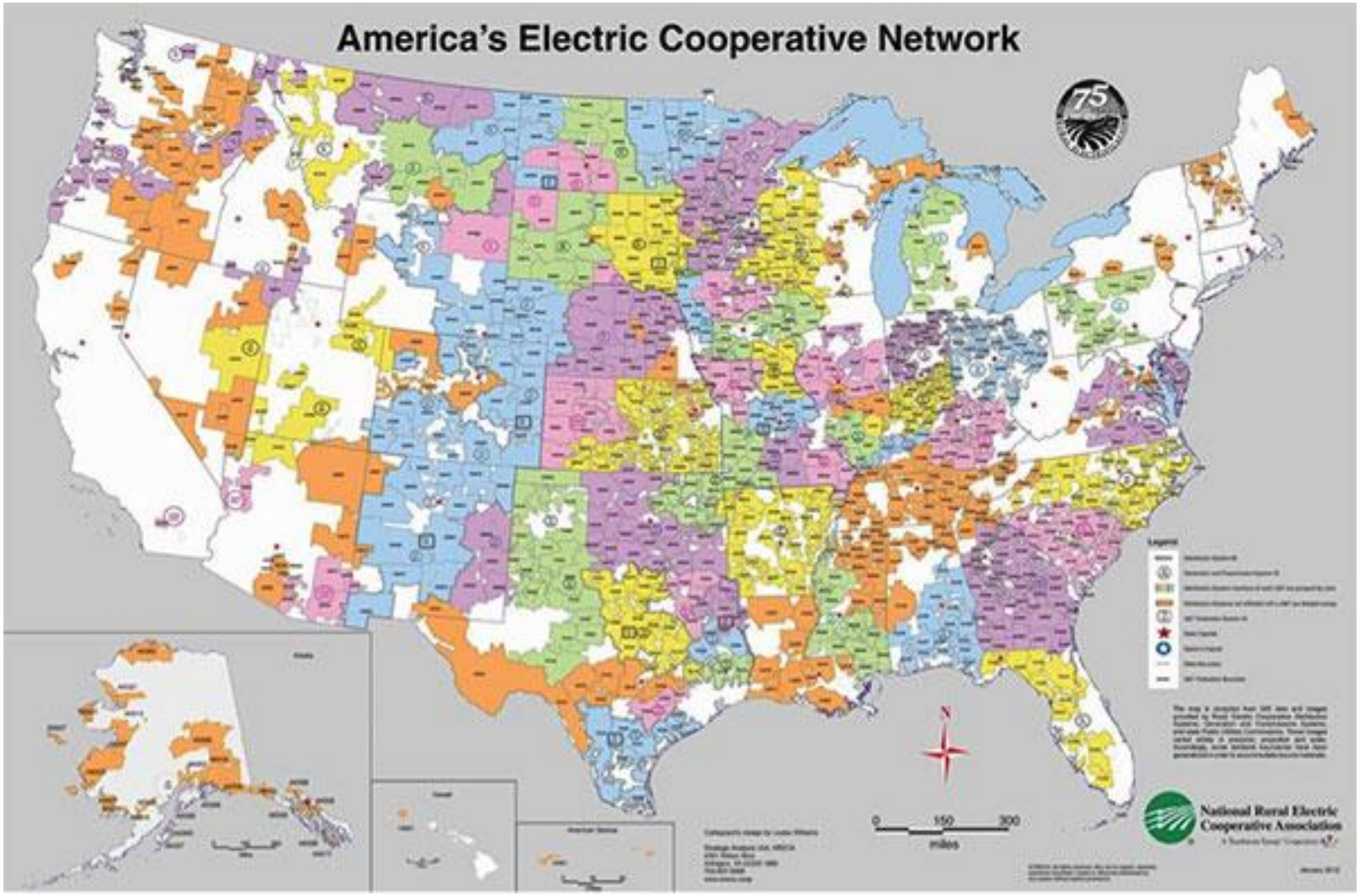


Source: www.nerc.com/pa/rrm/TLR/Pages/Reliability-Coordinators.aspx

EEI Member Companies

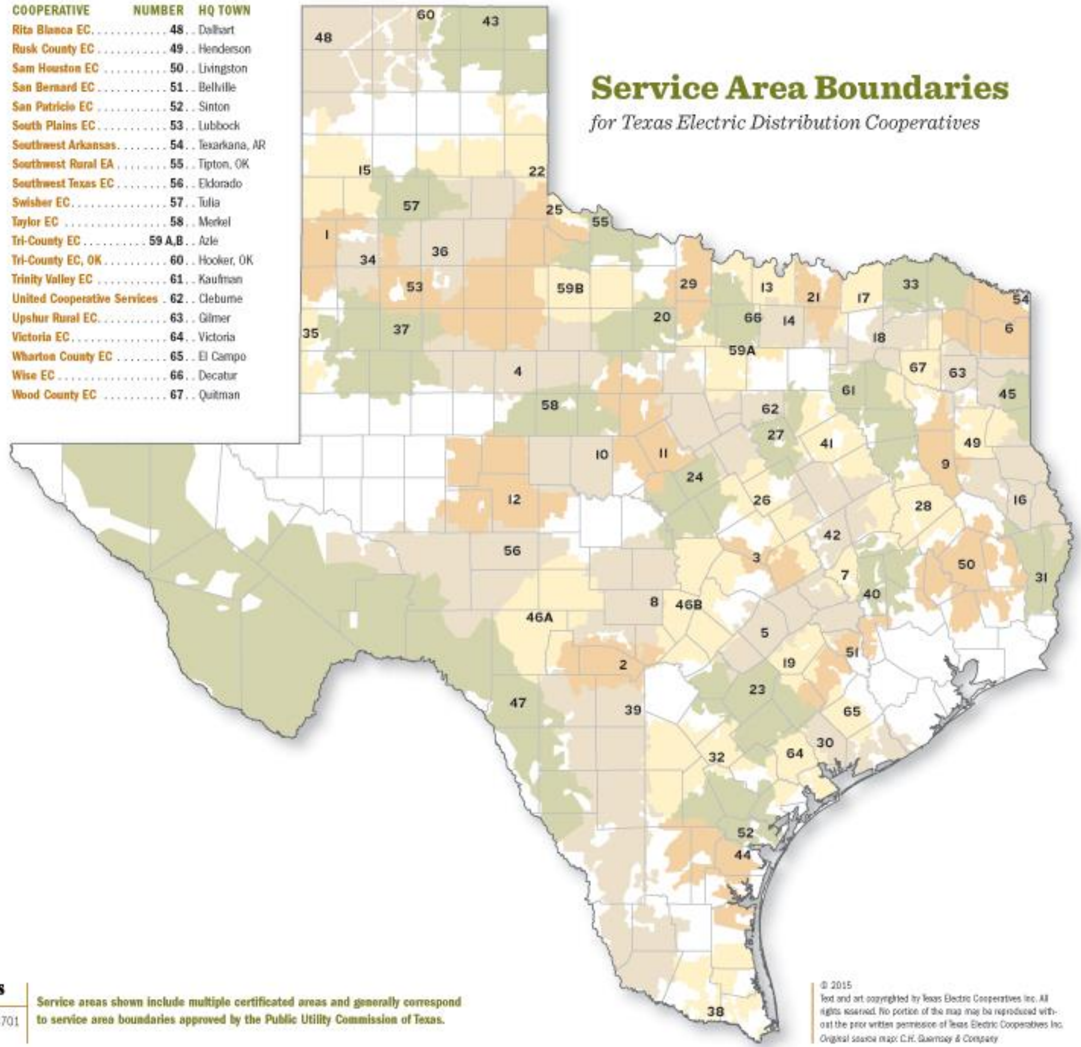


Electric Coops



Texas Electric Coops

COOPERATIVE	NUMBER	HQ TOWN
Bailey County ECA	1	Muleshoe
Bandera EC	2	Bandera
Bartlett EC	3	Bartlett
Big Country EC	4	Roby
Bluebonnet EC	5	Bastrop
Bowie-Cass EC	6	Douglassville
Bryan Texas Utilities	7	Bryan
Central Texas EC	8	Fredericksburg
Cherokee County ECA	9	Rusk
Coleman County EC	10	Coleman
Comanche EC	11	Comanche
Concho Valley EC	12	San Angelo
Cooke County ECA	13	Muenster
CoServ Electric	14	Corinth
Deaf Smith EC	15	Hereford
Deep East Texas EC	16	San Augustine
Fannin County EC	17	Bonham
Farmers EC	18	Greenville
Fayette EC	19	La Grange
Fort Belknap EC	20	Olney
Grayson-Collin EC	21	Van Alstyne
Greenbelt EC	22	Wellington
Guadalupe Valley EC	23	Gonzales
Hamilton County ECA	24	Hamilton
Harmon EA	25	Hollis, OK
Heart of Texas EC	26	McGregor
HILCO EC	27	Itasca
Houston County EC	28	Crockett
J-A-C EC	29	Bluegrove
Jackson EC	30	Edna
Jasper-Newton EC	31	Kirbyville
Karnes EC	32	Karnes City
Lamar County ECA	33	Paris
Lamb County EC	34	Littlefield
Lea County EC	35	Lovington, NM
Lighthouse EC	36	Floydada
Lyntegar EC	37	Tahoka
Magic Valley EC	38	Mercedes
Medina EC	39	Hondo
Mid-South Synergy	40	Navasota
Navarro County EC	41	Corsicana
Navasota Valley EC	42	Franklin
North Plains EC	43	Perryton
Nueces EC	44	Corpus Christi
Panola-Harrison EC	45	Marshall
Pedernales EC	46 A, B	Johnson City
Rio Grande EC	47	Brackettville
Rita Blanca EC	48	Dalhart
Rusk County EC	49	Henderson
Sam Houston EC	50	Livingston
San Bernard EC	51	Bellville
San Patricio EC	52	Sinton
South Plains EC	53	Lubbock
Southwest Arkansas	54	Texarkana, AR
Southwest Rural EA	55	Tipton, OK
Southwest Texas EC	56	Eldorado
Swisher EC	57	Tulia
Taylor EC	58	Merkel
Tri-County EC	59 A,B	Azle
Tri-County EC, OK	60	Hooker, OK
Trinity Valley EC	61	Kaufman
United Cooperative Services	62	Cleburne
Upshur Rural EC	63	Gilmer
Victoria EC	64	Victoria
Wharton County EC	65	El Campo
Wise EC	66	Decatur
Wood County EC	67	Quitman



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Service areas shown include multiple certificated areas and generally correspond to service area boundaries approved by the Public Utility Commission of Texas.