Announcements

• Read Chapter 9
• Homework 4 is due today
  • In problem 5 the LODF is a vector
  • In problem 6 find the OTDF on the line between buses 1 and 2
Electric Grid Measurements

• The two major types of measurements are voltages and currents
  – The challenge for both types are doing these measurements at the high electric grid voltage levels

• Potential transformers (PTs) are used to measure voltage, using a transformer sometimes with a set of series capacitors to drop the voltage

Image source: www.electrical4u.com/instrument-transformers/
Electric Grid Measurements

- Current transformers (CTs) are used to measure current, with the primary often consisting of the transmission wire itself; the secondary then has its number of turns set to give a specified current (say 5A) at a specified line current.
  - Many CTs are used in the protection system so these need to be calibrated to correctly measure fault current; others are used to give more accurate load current values.
- All meters have errors.

Image source: www.electrical4u.com/instrument-transformers/
Phasor Measurement Units (PMUs)

• All AC signals have a magnitude and phase. It is very easy to measure the phase angle differences between local signals (e.g., at an electrical substation)
  – These differences are used to calculate power values

• However, it had been challenging to measure phase angle differences between signals at different locations
  – This requires access to a precise time source
  – At 60 Hz one cycle takes 16.67 ms, which means one degree takes 46 μs.
Phasor Measurement Units (PMUs)

• Widespread access to precise time became available in the 1980’s when civilian use of the GPS was allowed

• PMUs use the GPS signals to determine the phase angles of voltages and currents (relative to some global reference)
  – The inputs to PMUs come from the CTs and PTs

• PMUs sample the system at rates on the order of 30 times per second

• PMU values are being used in SE algorithms
SE Example: Two Bus Case

- Assume a two bus case with a generator supplying a load through a single line with \( x = 0.1 \) pu. Assume measurements of the \( p/q \) flow on both ends of the line (into line positive), and the voltage magnitude at both the generator and the load end. So \( B_{12} = B_{21} = 10.0 \)

\[
P_{ij}^{\text{meas}} = - \left[ V_i V_j \left( B_{ij} \sin(\theta_i - \theta_j) \right) \right]
\]

\[
Q_{ij}^{\text{meas}} = - \left[ V_i^2 B_{ij} + V_i V_j \left( -B_{ij} \cos(\theta_i - \theta_j) \right) \right]
\]

\[
V_i^{\text{meas}} - V_i = 0
\]

We need to assume a reference angle unless we’re directly measuring phase angles
Example: Two Bus Case

- Let

\[ \mathbf{Z}^{\text{meas}} = \begin{bmatrix} P_{12} \\ Q_{12} \\ P_{21} \\ Q_{21} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 1.01 \\ 0.87 \end{bmatrix} \]

\[ \mathbf{x}^0 = \begin{bmatrix} V_1 \\ \theta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \sigma_i = 0.01 \]

We assume an angle reference of \( \theta_1 = 0 \)

\[ \mathbf{H}(\mathbf{x}) = \begin{bmatrix} V_2 10 \sin(-\theta_2) & -V_1 V_2 10 \cos(-\theta_2) & V_1 10 \sin(-\theta_2) \\ 20V_1 - V_2 10 \cos(-\theta_2) & -V_1 V_2 10 \sin(-\theta_2) & -V_1 10 \cos(-\theta_2) \\ V_2 10 \sin(\theta_2) & -V_1 V_2 10 \cos(\theta_2) & V_1 10 \sin(\theta_2) \\ -V_2 10 \cos(\theta_2) & V_1 V_2 10 \sin(\theta_2) & 20V_2 - V_1 10 \cos(\theta_2) \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \]
Example: Two Bus Case

- With a flat start guess we get

\[ H(x^0) = \begin{bmatrix}
0 & -10 & 0 \\
10 & 0 & -10 \\
0 & 10 & 0 \\
-10 & 0 & 10 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}, \quad z - f(x^0) = \begin{bmatrix}
2.02 \\
1.5 \\
-1.98 \\
-1 \\
0.01 \\
-0.13
\end{bmatrix} \]

\[ R = \begin{bmatrix}
0.0001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.0001 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.0001 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.0001 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.0001 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.0001 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0001
\end{bmatrix} \]
Example: Two Bus Case

\[ H^T R^{-1} H = 1e^6 \times \begin{bmatrix} 2.01 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 2.01 \end{bmatrix} \]

\[ x^1 = x^0 + \left[ H^T R^{-1} H \right]^{-1} H^T R^{-1} \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix} = \begin{bmatrix} 1.003 \\ -0.2 \\ 0.8775 \end{bmatrix} \]
Assumed SE Measurement Accuracy

- The assumed measurement standard deviations can have a significant impact on the resultant solution, or even whether the SE converges.
- The assumption is a Gaussian (normal) distribution of the error with no bias.

![Diagram of normal distribution of meter errors]

**FIGURE 9.8** Normal distribution of meter errors.
In order to estimate all \( n \) states we need at least \( n \) measurements. However, where the measurements are located is also important, a topic known as observability.

- In order for a power system to be fully observable usually we need to have a measurement available no more than one bus away.
- At buses we need to have at least measurements on all the injections into the bus except one (including loads and gens).
- Loads are usually flows on feeders, or the flow into a transmission to distribution transformer.
- Generators are usually just injections from the GSU.
Pseudo Measurements

- Pseudo measurements are used at buses in which there is no load or generation; that is, the net injection into the bus is known with high accuracy to be zero.
  - In order to enforce the net power balance at a bus we need to include an explicit net injection measurement.

- To increase observability sometimes estimated values are used for loads, shunts and generator outputs.
  - These “measurements” are represented as having a higher much standard deviation.
SE Observability Example
SE Bad Data Detection

- The quality of the measurements available to an SE can vary widely, and sometimes the SE model itself is wrong. Causes include
  - Modeling Errors: perhaps the assumed system topology is incorrect, or the assumed parameters for a transmission line or transformer could be wrong
  - Data Errors: measurements may be incorrect because of incorrect data specifications, like the CT ratios or even flipped positive and negative directions
  - Transducer Errors: the transducers may be failing or may have bias errors
  - Sampling Errors: SCADA does not read all values simultaneously and power systems are dynamic
SE Bad Data Detection

- The challenge for SE is to determine when there is likely a bad measurement (or multiple ones), and then to determine the particular bad measurements.
- $J(x)$ is a random number, with a probability density function (PDF) known as a chi-squared distribution, $\chi^2(K)$, where $K$ is the degrees of freedom, $K=m-n$.
- It can be shown the expected mean for $J(x)$ is $K$, with a standard deviation of $\sqrt{2K}$.
  - Values of $J(x)$ outside of several standard deviations indicate possible bad measurements, with the measurement residuals used to track down the likely bad measurements.
- SE can be re-run without the bad measurements.
QR Factorization

- Used in SE since it handles ill-conditioned m by n matrices (with m >= n)
- Can be used with sparse matrices
- We will first split the $R^{-1}$ matrix

$$H^T R^{-1} H = H^T R^{-1/2} R^{-1/2} H = H' H'$$

- QR factorization represents the m by n $H'$ matrix as

$$H' = Q U$$

with $Q$ an m by m orthonormal matrix and $U$ an upper triangular matrix (most books use $Q R$ but we use $U$ to avoid confusion with the previous $R$)
Orthonormal Matrices

• The term orthogonal is used with vectors to indicate their dot product is zero (i.e., they are perpendicular to each other)

• Orthonormal is used to indicate they are orthogonal and each has unit length (magnitude of 1)

• The definition of an orthogonal matrix is $Q^TQ = I$
  – This implies its inverse always exists

• Its determinant is 1

• They can be used for transformations such as an angular rotation
QR Factorization

• We then have $H' H' = U^T Q^T Q U$
• But since $Q$ is an orthonormal matrix, $Q^T Q = I$
• Hence we have $H' H' = U^T U$

Originally $\Delta x = \left[ H^T R^{-1} H \right]^{-1} H^T R^{-1} \left[ z^{meas} - f(x) \right]$

With $H^T R^{-1} H = H' H' = H'^T H' = U^T U$

Let $\hat{z} = Q^T R^{-1/2} \left[ z^{meas} - f(x) \right]$

$\Delta x = \left[ U^T U \right]^{-1} H^T R^{-1/2} R^{-1/2} \left[ z^{meas} - f(x) \right] = \left[ U^T U \right]^{-1} U^T \hat{z}$

$U^T U \Delta x = U^T \hat{z} \rightarrow \Delta x = U^{-1} \hat{z}$

Q is an $m$ by $m$ matrix
Next we’ll briefly discuss the QR factorization algorithm.

When factored the $U$ matrix (i.e., what most call the $R$ matrix) will be an $m$ by $n$ upper triangular matrix.

Several methods are available including the Householder method and the Givens method.

Givens is preferred when dealing with sparse matrices.

Givens Algorithm for Factoring a Matrix A

- The Givens algorithm works by pre-multiplying the initial matrix, \( A \), by a series of matrices and their transposes, starting with \( G_1 G_1^T \)
  - If \( A \) is \( m \) by \( n \), then each \( G \) is an \( m \) by \( m \) matrix
- The algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of \( A \), starting at the bottom of each column

\[
G_1 \ldots G_p G_p^T \ldots G_1^T A = QU
\]

\[
G_1 \ldots G_p = Q
\]

\[
G_p^T \ldots G_1^T A = U
\]

If \( A \) is sparse, then we can take advantage of sparsity going up the column
Givens Algorithm

• To zero out element $A[i,j]$, with $i > j$ we first solve with $a = A[k,j]$, $b = A[i,j]$

\[
\begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}
\]

\[r = \sqrt{a^2 + b^2}\]

• A numerically safe algorithm is

If $b=0$ then $c=1$, $s=0$ // i.e, no rotation is needed

Else If $|b| > |a|$ then $\tau = -a / b$; $s = 1 / \sqrt{1 + \tau^2}$; $c = s\tau$

Else $\tau = -b / a$; $c = 1 / \sqrt{1 + \tau^2}$; $s = c\tau$
Givens G Matrix

• The orthogonal $G(i,k,\theta)$ matrix is then

$$G(i,k,\theta) = \begin{bmatrix}
1 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & c & \cdots & s & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & -s & \cdots & c & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \cdots & 1
\end{bmatrix}$$

As noted, to zero out an element we need a non-zero pivot element in column $j$; assume this row as $k$. Row $k$ here is the first non-zero above row $i$.

• Premultiplication by $G(i,k,\theta)^T$ is a rotation by $\theta$ radians in the $(i,k)$ coordinate plane
Small Givens Example

- Let \( A = \begin{bmatrix} 4 & 2 \\ 1 & 0 \\ 0 & 5 \\ 2 & 1 \end{bmatrix} \)

- First start in column \( j=1 \); we will zero out \( A[4,1] \) with \( i=4, k=2 \)

- First we zero out \( A[4,1] \), \( a=1, b=2 \) giving \( s = 0.8944, c = -0.4472 \)

\[
G_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & -0.4472 & 0 & 0.8944 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -0.8944 & 0 & -0.4472 & 0 \\
0 & -0.8944 & 0 & -0.4472 & 0
\end{bmatrix}
\]

\[
G_1^T A = \begin{bmatrix}
4 & 2 \\
-2.236 & -0.8944 \\
0 & 5 \\
0 & -0.4472
\end{bmatrix}
\]
Small Givens Example

- Next zero out $A[2,1]$ with $a=4$, $b=-2.236$, giving $c=-0.8729$, $s=0.4880$
  
  $$G_2 = \begin{bmatrix} 0.873 & 0.488 & 0 & 0 \\ -0.488 & 0.873 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

  $$G_2^T G_1^T A = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5 \\ 0 & -0.447 \end{bmatrix}$$

  $k=1$ with $A[k,j]=4$

- $j=2$, $k=3$ with $A[k,j]=5$

- Next zero out $A[4,2]$ with $a=5$, $b=-0.447$, $c=0.996$, $s=0.089$
  
  $$G_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0.996 & 0.089 & 0 \\ 0 & 0 & -0.089 & 0.996 & 0 \end{bmatrix}$$

  $$G_3^T G_2^T G_1^T A = \begin{bmatrix} 4.58 & 2.18 \\ 0 & 0.195 \\ 0 & 5.020 \\ 0 & 0 \end{bmatrix}$$
Small Givens Example

- Next zero out $A[3,2]$ with $a=0.195$, $b=5.02$, $c=-0.039$, $s=0.999$ for $j=2$, $k=2$ with $A[k,j]=0.195$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -0.039 & 0.999 & 0 \\ 0 & -0.999 & -0.039 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} G_4 = \begin{bmatrix} 4.58 & 2.18 \\ 0 & -5.023 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Also we have

$$Q = G_1G_2G_3G_4 = \begin{bmatrix} 0.872 & -0.019 & 0.487 & 0 \\ 0.218 & 0.094 & -0.387 & 0.891 \\ 0 & -0.995 & -0.039 & 0.089 \\ 0.436 & -0.009 & -0.782 & -0.445 \end{bmatrix}$$
Start of Givens for SE Example

- Starting with the $H$ matrix we get

$$H' = R^{-\frac{1}{2}}H = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- To zero out $H'[5,1]=1$ we have $b=100$, $a=-1000$, giving $c=0.995$, $s=0.0995$

Here the column (j) is 1, while i is 5 and k is 4.
Start of Givens for SE Example

• Which gives

\[ G_1^T H' = 100 \times \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ 10.049 & 0 & -9.95 \\ 0 & 0 & 0.995 \\ 0 & 0 & 1 \end{bmatrix} \]

• The next rotation would be to zero out element [4,1], continuing until all the elements in the lower triangle have been reduced
For a full matrix, Givens is $O(mn^2)$ since each element in the lower triangle needs to be zeroed $O(nm)$, and each operation is $O(n)$.

Computation can be drastically reduced for a sparse matrix since we only need to zero out the elements that are initially non-zero, and any that become non-zero (i.e., the fills).

Also, for each multiply we only need to deal with the nonzeros in the impacted row.

Givens rotation is commonly used to solve the SE.
Example SE Application: PJM and MISO

- PJM provides information about their EMS model in
  - [www.pjm.com/-/media/documents/manuals/m03a.ashx](http://www.pjm.com/-/media/documents/manuals/m03a.ashx)

Data here is from the December 2019 (Rev 18) document
Example SE Application: PJM and MISO

- PJM measurements are required for 69 kV and up
- PJM SE is triggered to execute every minute
- PJM SE solves well over 98% of the time
- Below reference provides info on MISO SE from March 2015
  - 54,433 buses
  - 54,415 network branches
  - 6332 generating units
  - 228,673 circuit breakers
  - 289,491 mapped points

Energy Management Systems (EMSs)

- EMSs are now used to control most large scale electric grids
- EMSs developed in the 1970’s and 1980’s out of SCADA systems
  - An EMS usually includes a SCADA system; sometimes called a SCADA/EMS
- Having a SE is almost the definition of an EMS. The SE then feeds data to the more advanced functions
- EMSs have evolved as the industry as evolved as the industry has evolved, with functionality customized for the application (e.g., a reliability coordinator or a vertically integrated utility)
NERC Reliability Coordinators

Source: www.nerc.com/pa/rrm/TLR/Pages/Reliability-Coordinators.aspx
EEI Member Companies

EEI U.S. Member Company Service Territories

Electric Coops
Texas Electric Coops

Service Area Boundaries
for Texas Electric Distribution Cooperatives

© 2015. All rights reserved. Texas Electric Cooperatives. 10 April. No part of this material may be reproduced without the prior written permission of Texas Electric Cooperatives, Inc.