ECEN 615
Methods of Electric Power Systems Analysis

Lecture 18: Synthetic Electric Grids, Voltage Stability

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Announcements

- Skim Chapters 3, 4 and 5
- Starting reading Chapter 8
- Homework 5 is due on Thursday Oct 29

Synthetic Electric Grids

- Synthetic electric grids are models of electric grids that were not created to represent any actual electric grid
- The below image shows the five bus synthetic grid I used as an undergraduate

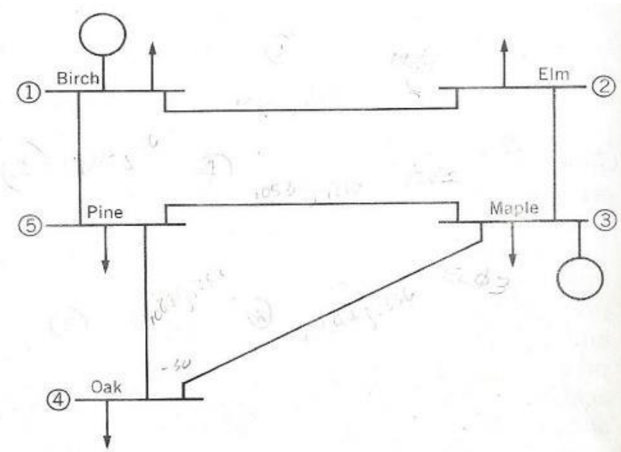


Figure 8.1 One-line diagram for Example 8.1.

Image Source: W.D. Stevenson, *Elements of Power Systems*, Fourth Edition, McGraw-Hill Book Company New York, 1982 (the first edition was in 1955)

Geographically-Based Synthetic Electric Grids

- Synthetic electric grids can be created with or without reference to actual geography
- The image shows an early geographically-based synthetic electric grid
- This grid was designed to show concepts to regulators

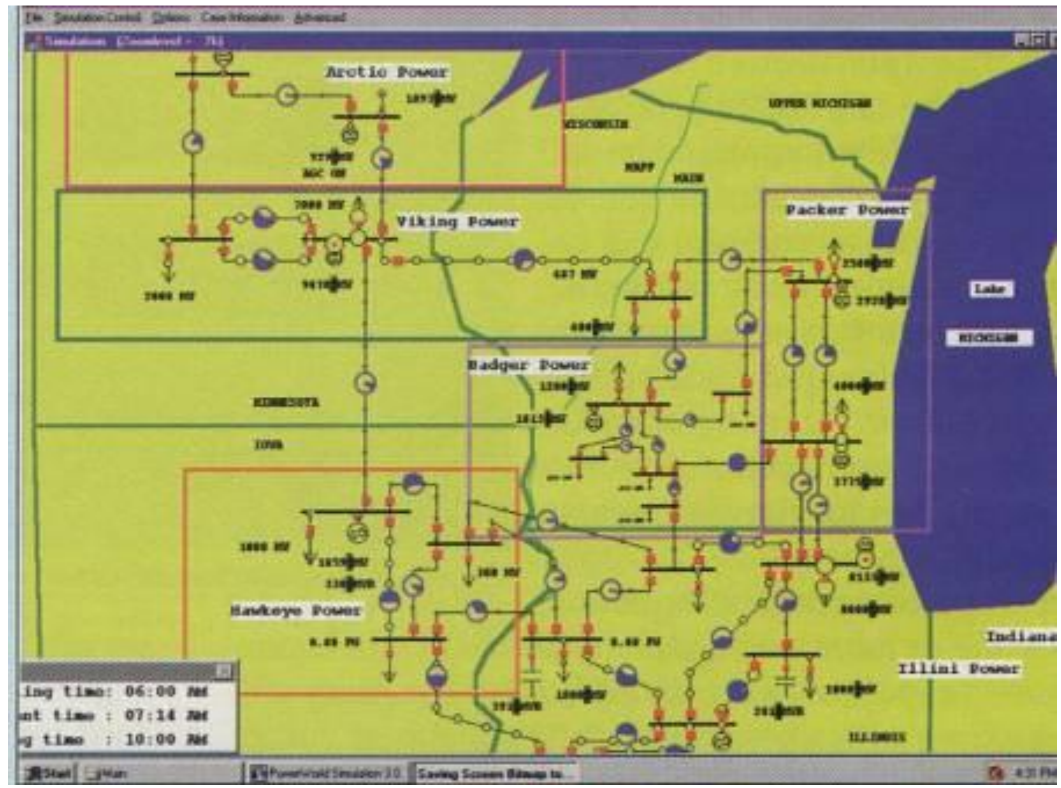


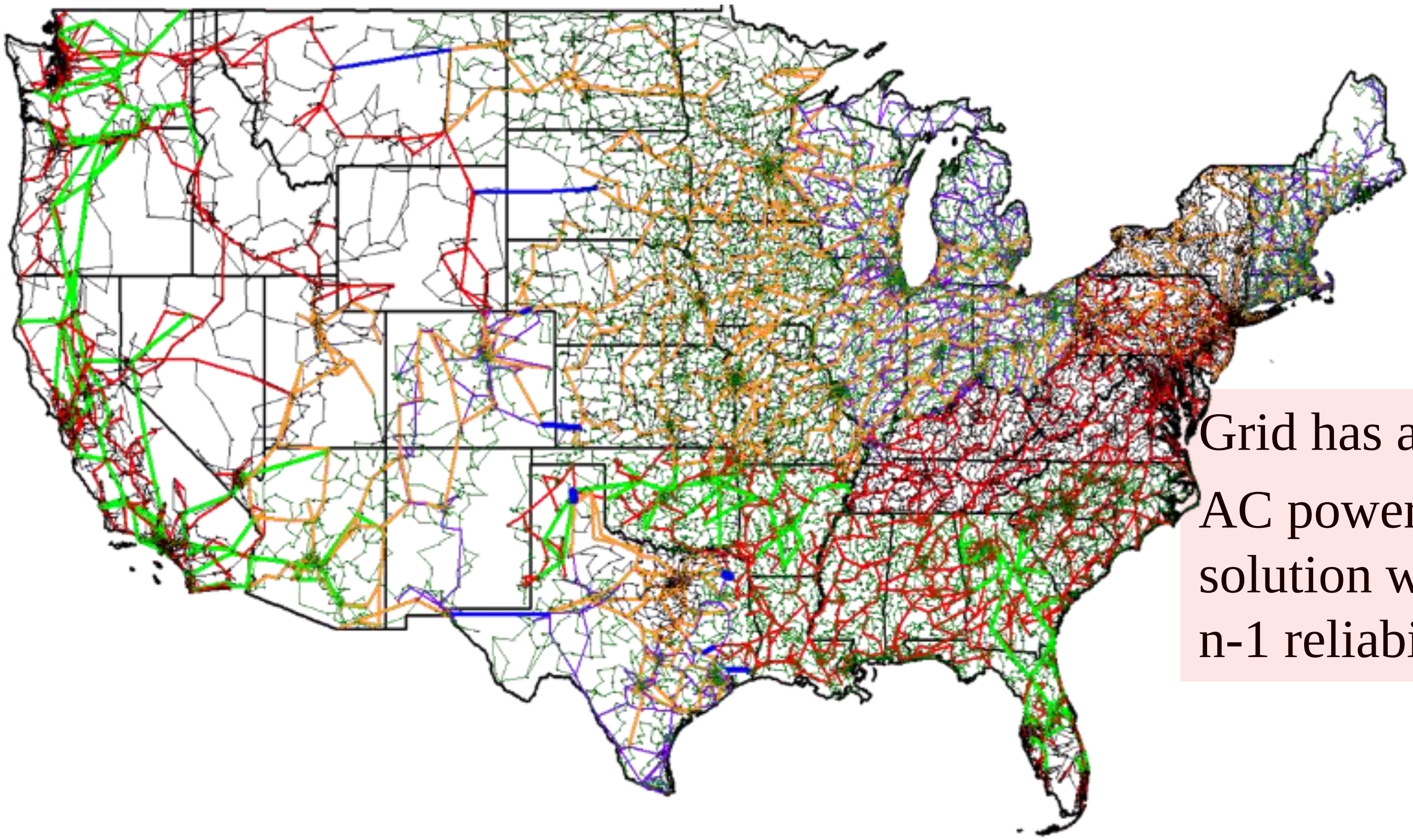
Image Source: PowerWorld Corporation, 1995

High-Quality, Geographically-Based Synthetic Electric Grids

- High-quality synthetic electric grids are designed to have a wide range of characteristics that are similar to those found in actual electric grids
  - “Realistic but not real” to quote Wisconsin colleagues
  - Fictional, but hopefully good fiction
  - **Developed techniques can be applied to real grids**
- However, importantly these grids are not designed to try to duplicate any actual grid
- Over the last five years tremendous progress has been made through ARPA-E at both the transmission and distribution levels

Current Status: Large-Scale Grids are Now Available

This is an 82,000 bus synthetic model that we publicly released in summer 2018 at **electricgrids.engr.tamu.edu**



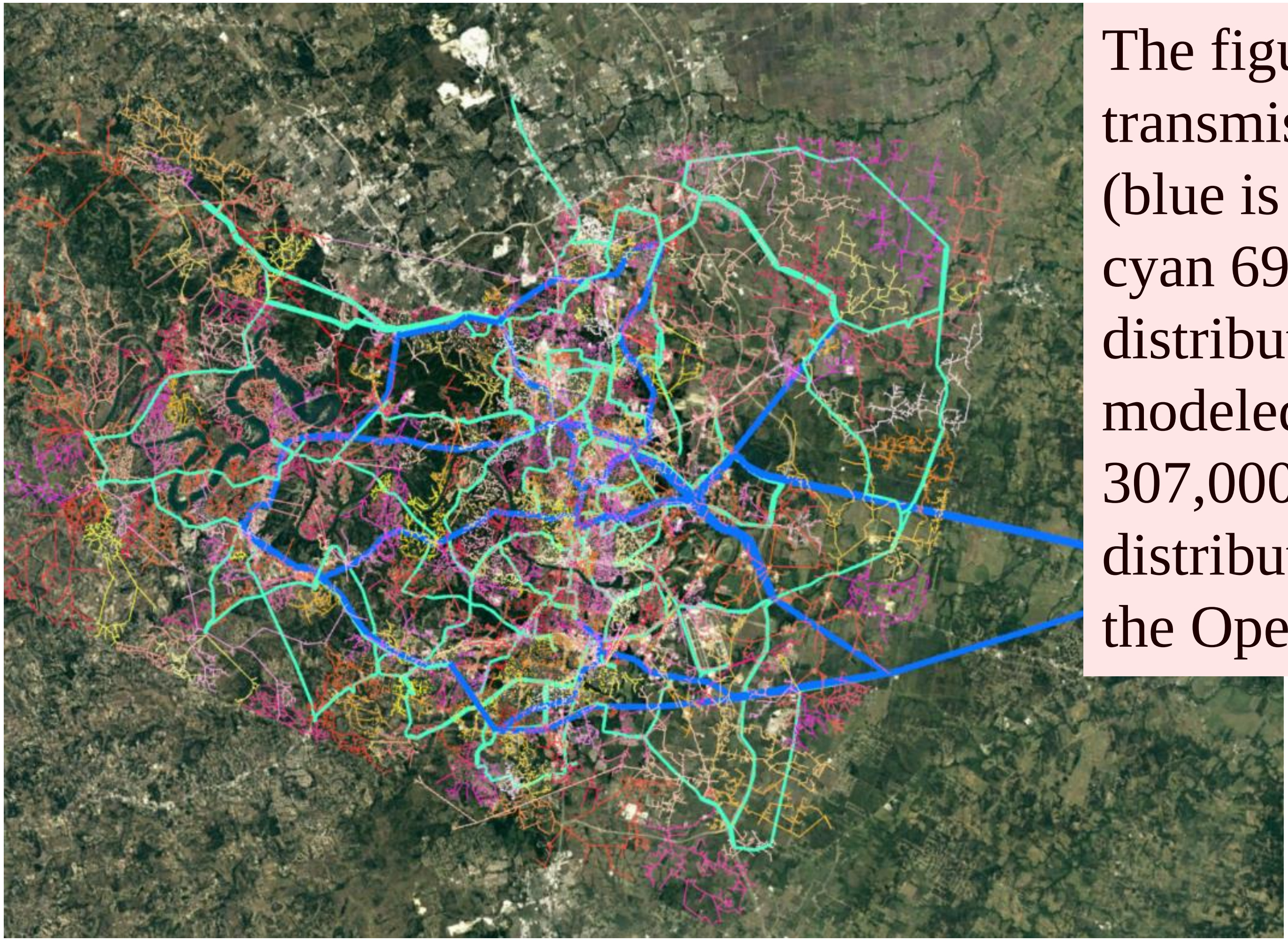
Grid has an

AC power flow solution with n-1 reliability

Highly Detailed Combined Transmission and Distribution Grids

- Previous transmission grids were geographic to the zip code level
- On current ARPA-E project we (with NREL, MIT and Comillas-IIT University) are developing "down to the meter" synthetic grids
- Actual parcel data is used to determine location of the electric meters. The parcels are connected by a distribution system, and the distribution system by a transmission grid
- Currently we have almost all of the load in Texas done!

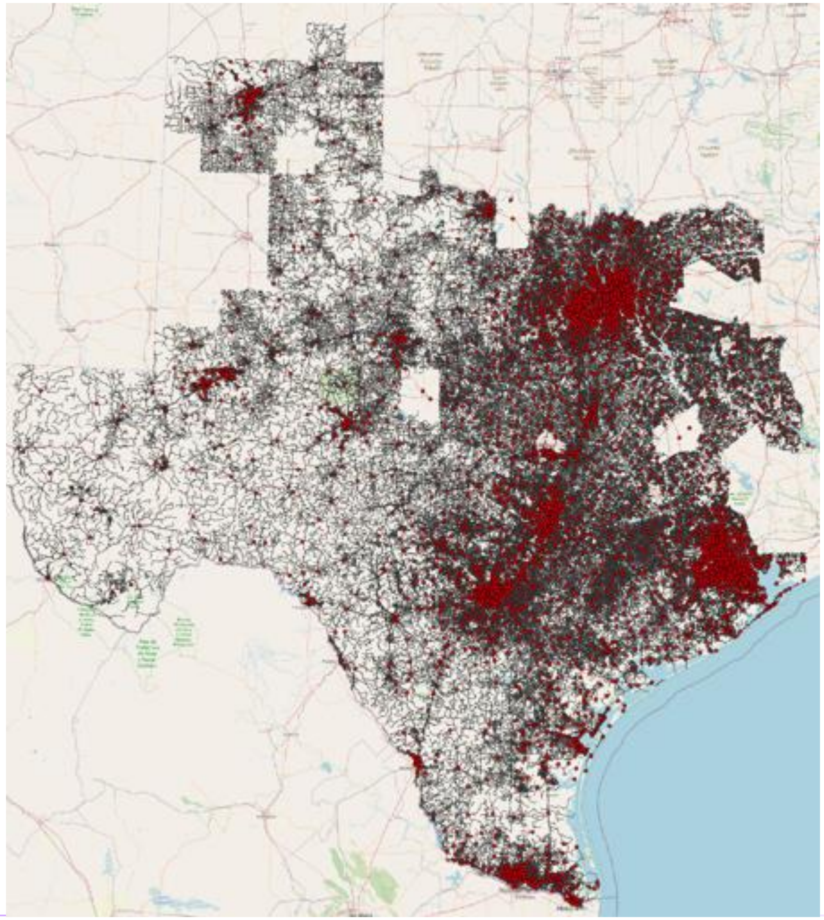
Initial Work: Travis County, Texas (location of Austin, TX)



The figure shows the transmission system (blue is 230 kV and cyan 69 kV) and the distribution system modeled down to 307,000 meters. The distribution data is in the OpenDSS format.

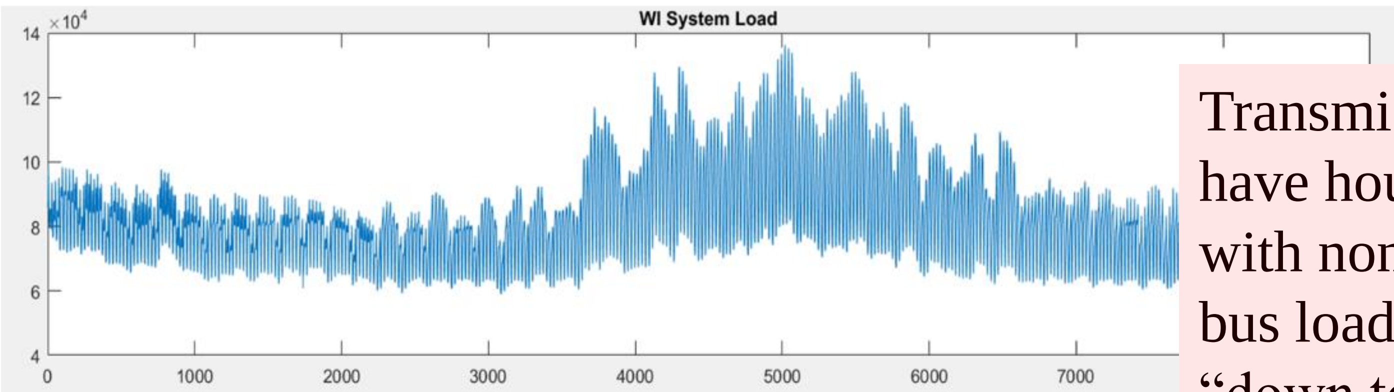
Full ERCOT Footprint Model

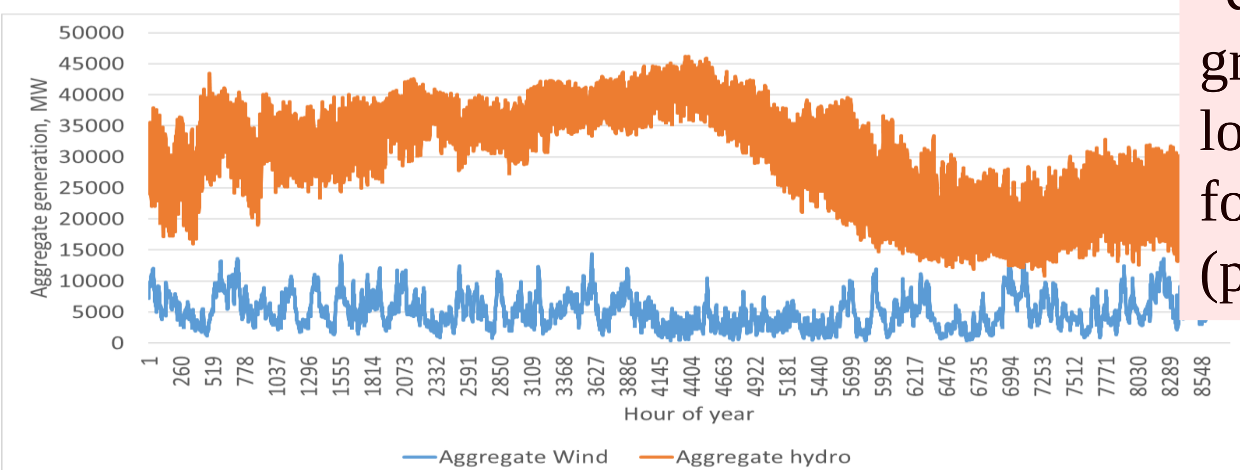
- Working with NREL and some other universities for ARPA-E we're creating combined T&D synthetic grids covering Texas down to the electric meter
- The finished grid has 15 million customers and about 50 million electric nodes



Synthetic Electric Grid Scenarios

- We have developed detailed, yearly scenarios with variation of both bus level load and gen.

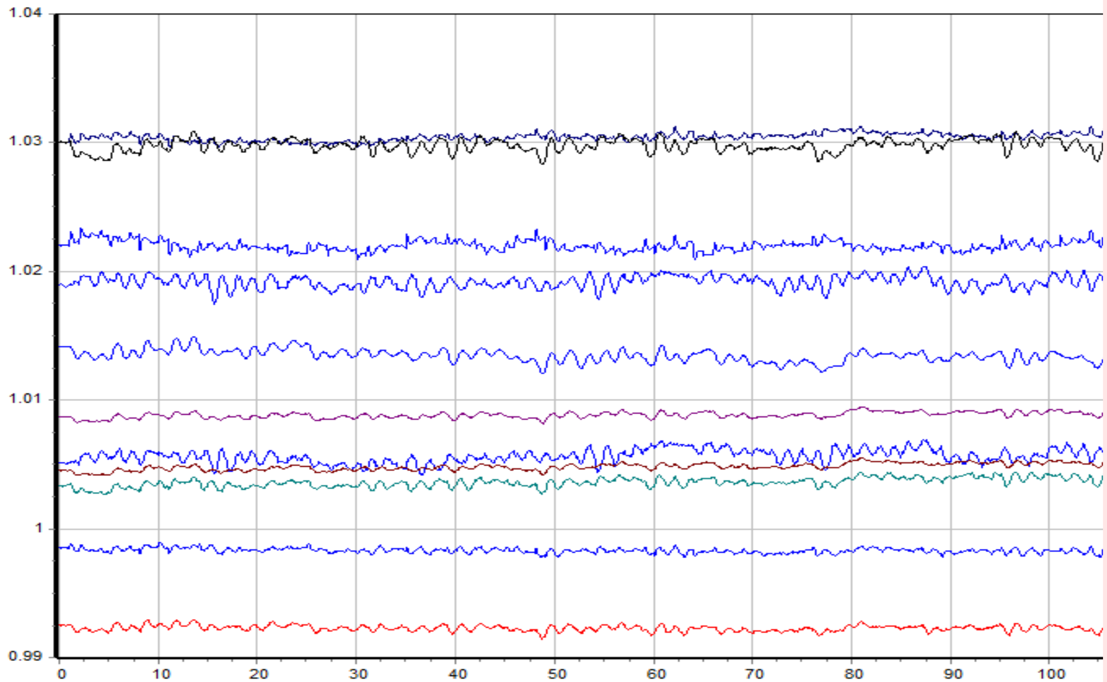




Transmission models have hourly snapshots, with nonconforming bus loads modeled. The "down to the meter" grids have synthetic loads every 15 minutes for each meter (provided by NREL).

Synthetic PMU Data

- To create synthetic PMU data we use dynamic simulations with realistic variation in the load and generation, and appropriate PMU errors.



On our 2000 bus Texas synthetic grid was can provide more then 10,000 signals at PMU data rates (30 times per second) in real-time. This is data creation at about 4GB per hour.

The Need for Synthetic Grids

- Prior to 9/11/01, a lot of grid information was publically available
- Now access to data and models about the actual power grid in the US is quite restricted (e.g., critical energy/electricity infrastructure [CEII])
  - What is available is often partial, and can't be shared
- To do effective research, and to drive innovation, researchers need access to common, realistic grid models and data sets
  - Scientific principle of reproducibility of results

The Need for Synthetic Grids, cont.

- Synthetic grids and datasets are, of course, designed to augment, not replace actual grids
- But the synthetic grids offer some significant advantages, both to industry and researchers
  - Since there are no CEII or privacy concerns, full models and their associated datasets can be freely shared; this is particularly helpful for interdisciplinary research
  - Synthetic grids can allow future grid scenarios to be considered in-depth (i.e., high renewables or high impact, low frequency events) yet still be potentially public

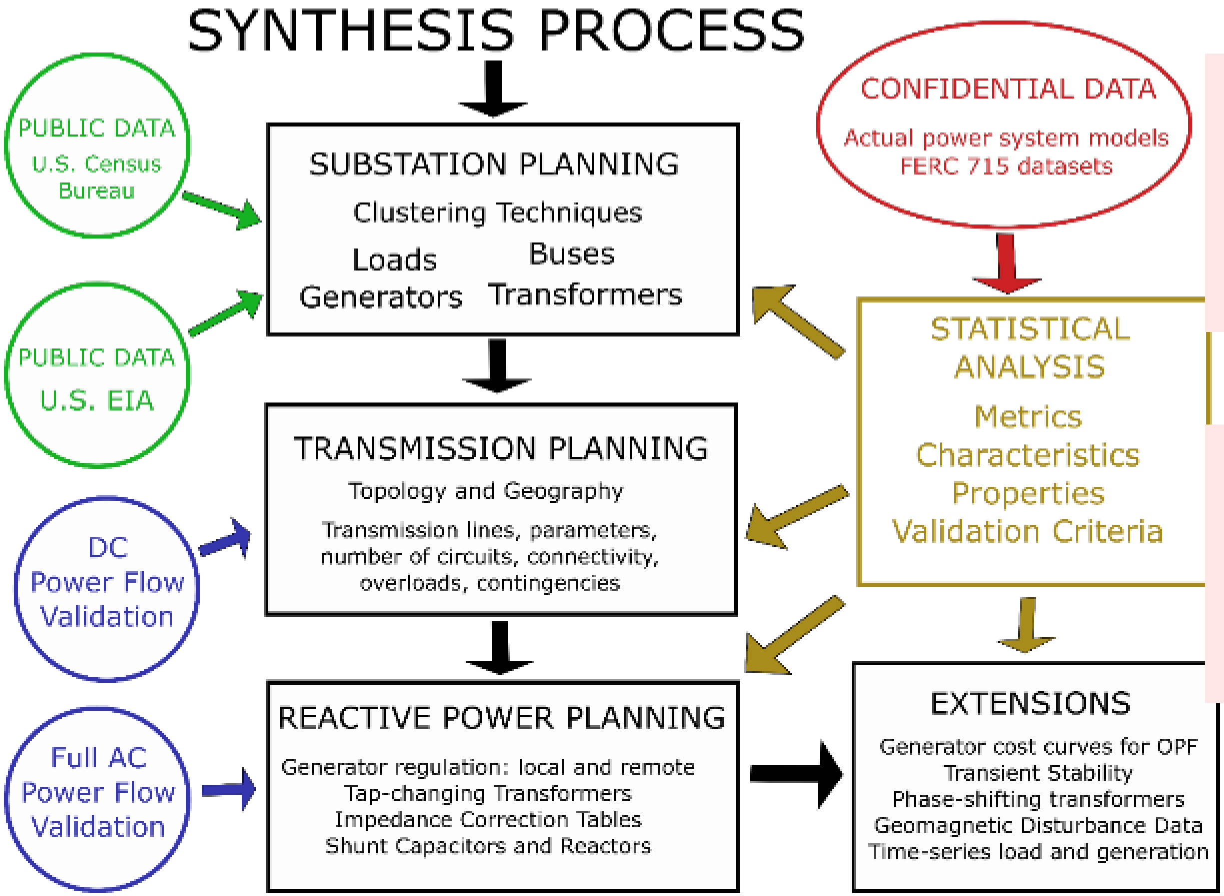
The Need for Synthetic Grids, cont.

- Advantages, cont.
  - Synthetic grids can be customized to represent particular grid idiosyncrasies; utilities can provide this to researchers or potential vendors
    - We've deliberately designed ours using different voltage levels than those used in the actual grid (e.g., 500/230 versus 345/138 in Texas) to emphasis they are synthetic
  - The highly detailed (down to the meter grid) allow coupling with real infrastructures
    - We're working with the Texas Transportation Institute to study electric grid/transportation couplings
  - Synthetic grids can be used for education, including vendor training and short courses

Our Synthetic Grid Approach

- Make grids that look real and familiar by siting them geographically (North America for us) and serving a population density the mimics actual
- Goal is to leverage widely available public data
  - Geography
  - Population density (easily available by post office)
  - Load by utility (US FERC 714), state-wide averages
  - Existing and planned generation (Form US EIA-860, which contains lots of generator information)
- Substation locations and transmission system is entirely fictional (but hopefully good fiction!)

Synthetic Model Design Process



The assumed peak load is based on population, scaled by geographic values

Much of this is automated, but there is still some manual adjustment

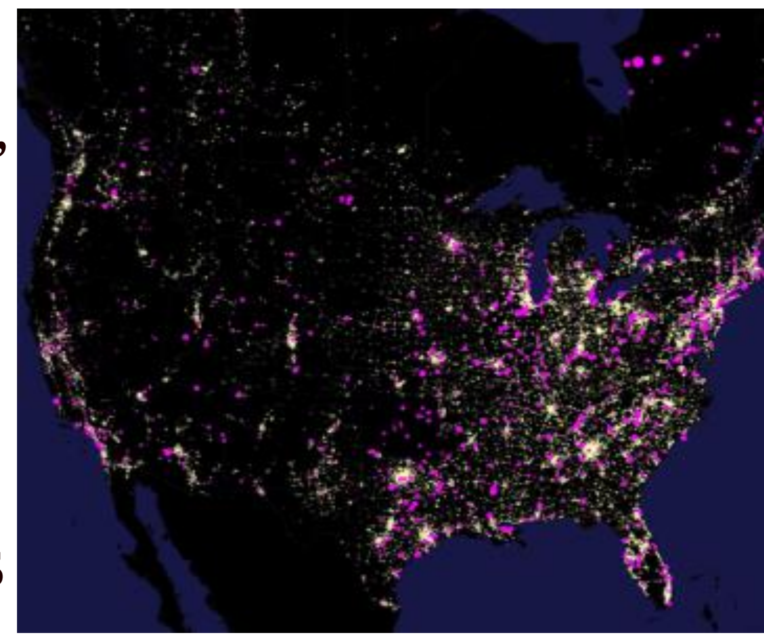
This process can be augmented to couple with detailed distribution grids

More Details on Design Process

- Substation planning: cluster actual population and energy data into correctly-sized substations and assign load, generation, bus voltage levels, and internal branches, along with parameters.
- Transmission planning: use iterative penalty-based dc power flow algorithm to place transmission lines, with the Delaunay triangulation and neighborhood as base
- Reactive power planning: iterative ac power flow starting from known solution to place capacitors and adjust generator set points.

Understanding Actual Electric Grid Topology

- To make realistic synthetic grids it is important to understand the geographic of actual grids
- Actual power grids are geographically consistent
  - This is an inherent characteristic that has profound modeling implications
  - Examples include line impedance, and constraints such as lakes, rivers, and mountains



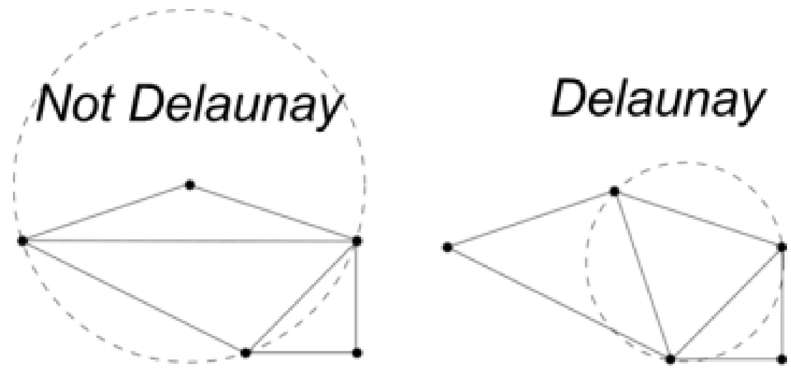
- Traditionally power system planning models did not include geographic coordinates but this has now changed

Understanding Actual Electric Grid Topology

- Assuming the substation geographic locations are known, the topology of the grid can be discovered, voltage level by voltage level
  - Transformers are within substations
- At a particular voltage level think of each substation as a node; the minimum spanning tree (MST) is the shortest set of lines to connect all the nodes
- The Delaunay triangulation is a triangulation of these points such that no point is inside the circumcircle of any triangle.
- The Delaunay triangulation contains the MST

Use of Delaunay Triangulation

- Helps to capture geometric constraints of grids, i.e. substations generally connect to nearby neighbors.
- Considering only Delaunay triangulation and 2-3 neighbors cuts potential lines from n^2 to $21n$, but captures about 98% of actual lines

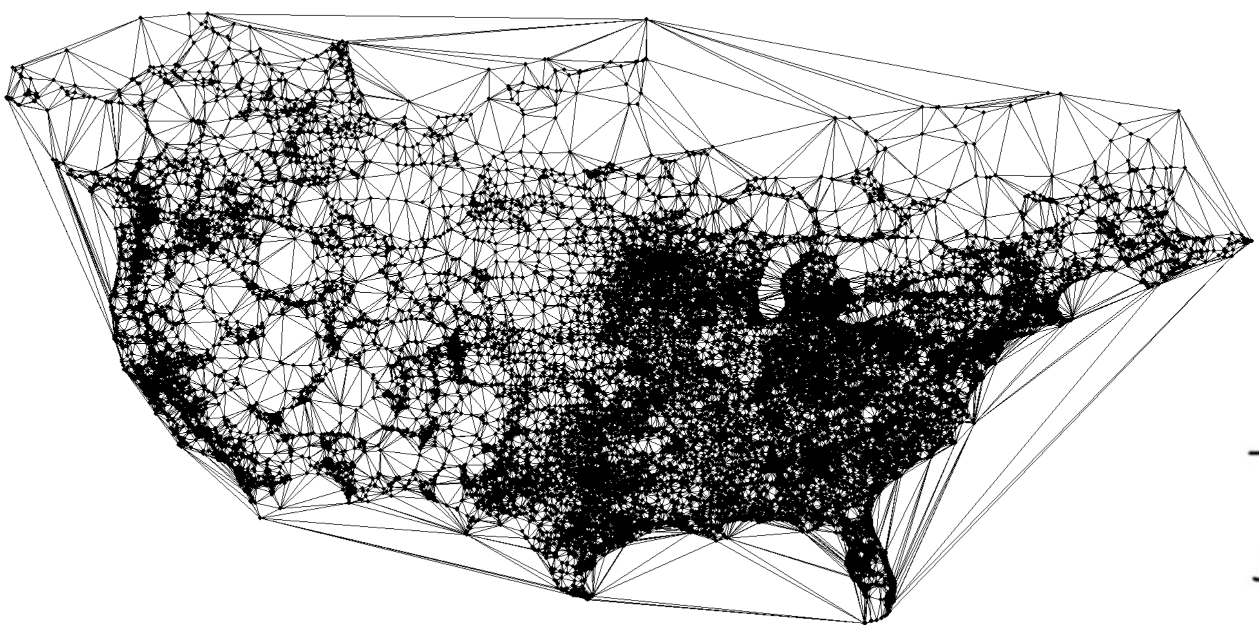


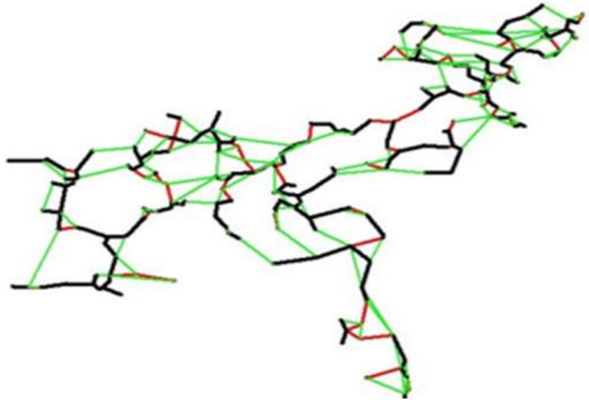
Transmission Line Category	Average percentage for Eastern Interconnect	Average percentage for Western Interconnect
Min. Span. Tree	47.8%	44.3%
Delaunay	75.6%	71.1%
2 neighbor	18.3%	21.5%
3 neighbor	4.6%	5.3%
4 neighbor	1.1%	1.4%
5+ neighbor	0.4%	0.7%

Actual Grid Delaunay Triangulation and Minimum Spanning Tree

Below image shows Delaunay triangulation of 42,000 North America substations; statistics only consider single voltage levels; this is computationally fast (order n ln(n))

MST for the EI 500 kV grid black is actual on MST; red is MST without a line; green is other





Outlier Characteristics are Key

- A 76,000 bus North American power flow model has 27,622 transformers with 98 phase shifters
  - Impedance correction tables are used for 351, including about 2/3 of the phase shifters; tables can change the impedance by more than two times
- The voltage magnitude is controlled at about 19,000 buses (by Gens, LTCs, switched shunts)
  - 94% regulate their own terminals with about 1100 doing remote regulation. Of this 277 are regulated by three or more devices
- Reactive power control interaction is a issue

Validation: Insuring High Quality

- Based upon data from actual grids we’ve developed a large number of metrics that cover many aspects of both transmission and distribution grids
- For example:
  - Buses/substation, Voltage levels, Load at each bus
  - Generator commitment, dispatch
  - Transformer reactance, MVA limit, X/R ratio
  - Percent of lines on minimum spanning tree and various neighbors of the Delaunay triangulation
  - Bus phase angle differences, flow distribution

Example Validation Metrics

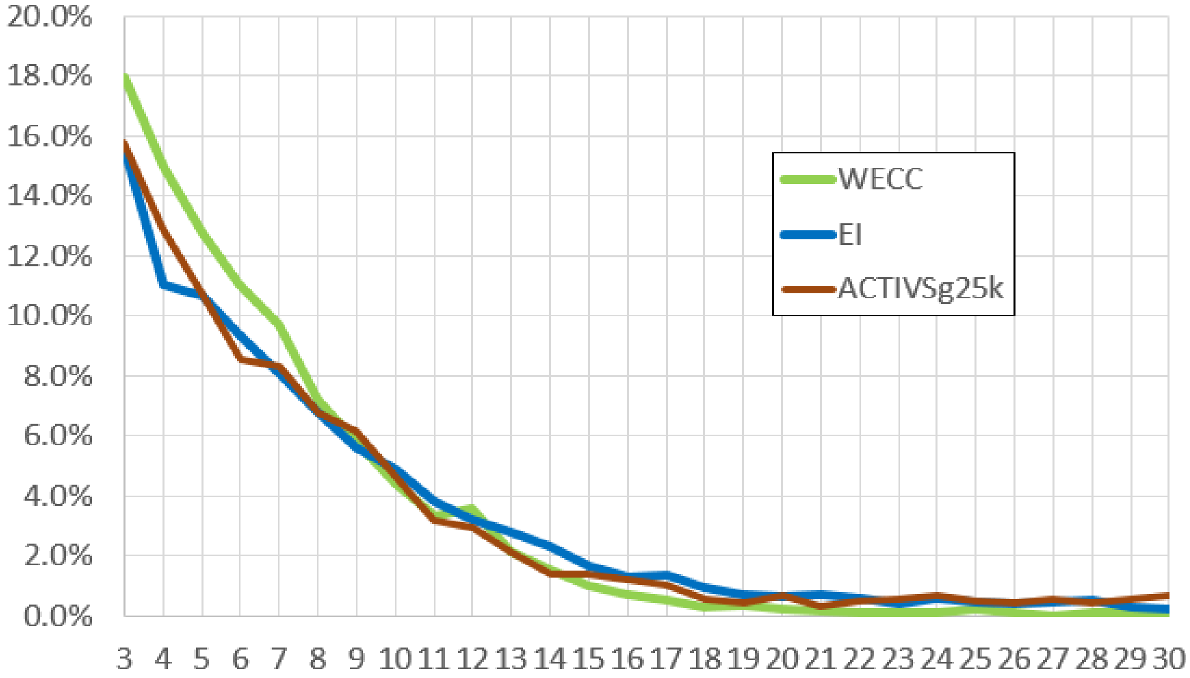
Validation Metric	Criteria	ACTIVSg25k
Voltage control and reactive power		
Shunt capacitors and reactors	10-25% of subs shunts 30-50% above 200 kV	9.1% 35.1%
Off-nominal network transformer taps	60-75% off-nominal 30-50% control voltage	67.2% 44.1%
Grid proportions, generators, load, and substations		
Buses per substation	Mean 1.7-3.5 Exponential decay	2.7 Next slide
Substations containing buses in kV range	<200 kV, 85-100% >201 kV, 7-25%	99.5% 12.9%
Substations with load	75-90%	85.0%
Load per bus	Mean 6-18 MW Exponential decay	9.4 MW Next slide
Load power factor	Mean 0.93-0.95 Decreasing distribution	0.959 Next slide
Generation capacity / load	1.2-1.6	1.30
Substations with generators	5-25%	17.6%
Generator MW maximum capacities	25-200 MW, 40+% 200+ MW, 2-20%	24.7% 9.2%
Generator MW minimum	Coal average 0.37-0.47 Gas average 0.27-0.37	0.3 0.31
Committed Generators	60-80%	78%
Generators dispatched > 80%	50+%	76%
Cycle distribution	Decreasing distribution	Next slide

Example Validation Metrics, cont.

Network topology / branch params.	765	500	345	230	161	138	115	100	69
XF p.u. X, own base	98	98	98	98	95	98	96	98	98
XF X/R ratio and MVA lim	51/58 49/42 100/98	56/53 44/47 100/99	49/47 51/53 99/91	48/37 52/62 100/99	48/50 52/50 99/89	51/34 50/66 100/99	54/62 46/38 100/88	47/66 54/35 100/90	47/66 54/35 100/90
Line X	79	94	99	98	97	100	98	98	98
Line X/R and MVA	100/99	100/99	100/96	100/99	100/ 100	100/98	100/ 100	100/ 100	100/ 100
Lines/subs	1.19	1.20	1.33	1.15	1.16	1.24	1.16	1.21	1.21
Lines MST	53.5	53.5	51.7	53.0	50.8	53.3	54.8	53.7	53.7
Delaunay distance	80.6 15.9 3.5	80.8 15.9 3.3	79.8 16.4 3.7	80.2 16.6 3.2	75.9 20.0 4.1	79.6 16.6 3.8	80.3 16.0 3.7	80.5 16.1 3.4	80.5 16.1 3.4
length / MST	1.38	1.55	1.74	1.46	1.46	1.64	1.57	1.42	1.42

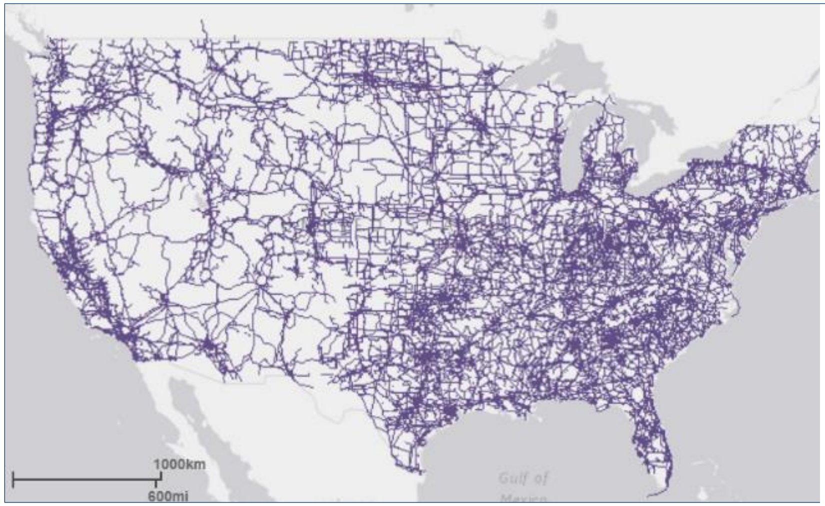
Validation Example: Cycle Sizes

- Cycles are use loops of connected buses. Power systems have many cycles and an important metric to match is to have a similar cycle basis



Intersections (Transmission Line Crossings)

- We've found it is important to match properties of electric grids that depend on their physical layout; for example transmission line crossings
  - Transmission line right-of-ways are available in EIA datasets; hence line crossings can be analyzed

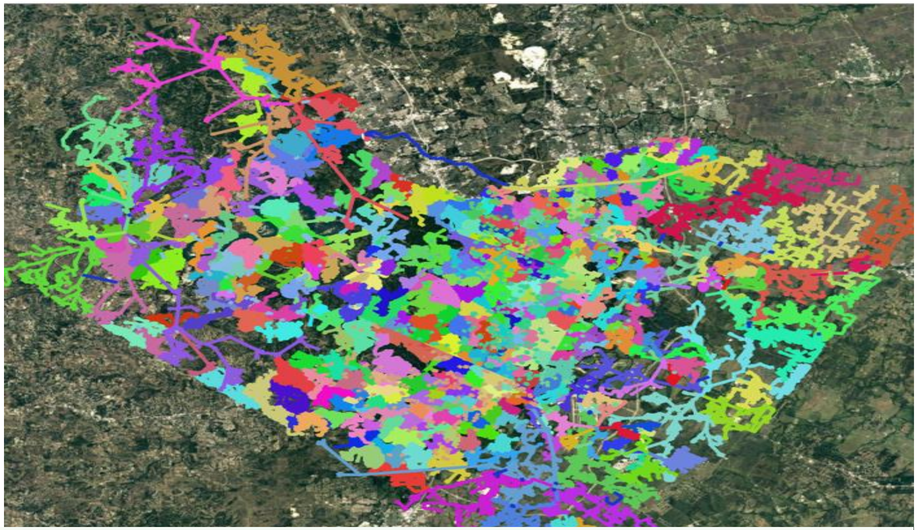


GEOGRAPHIC CROSSINGS FOR EIA DATASET VOLTAGE NETWORKS

Voltage class	Number of substations	Number of lines	Crossings, straight-line Number	Crossings, straight-line % of lines
765 kV	40	42	1	2.4
500 kV	529	732	67	9.2
345 kV	1526	2171	297	13.7
230 kV	4648	6233	935	15.1
161 kV	2633	3172	405	13.0
138 kV	8611	10684	1617	15.3
115 kV	12826	15031	1485	10.2
100 kV	894	1595	118	7.5
69 kV	8022	8022	289	3.7

Detailed Transmission and Distribution Testing

- Full transmission and distribution system studies are being done using a co-simulation framework
  - PowerWorld Simulator is used to solve the transmission system and OpenDSS is used to solve each of the distribution circuits
  - The simulations are coupled together using the national lab developed HELICS package; one year of simulations took about 3 hours



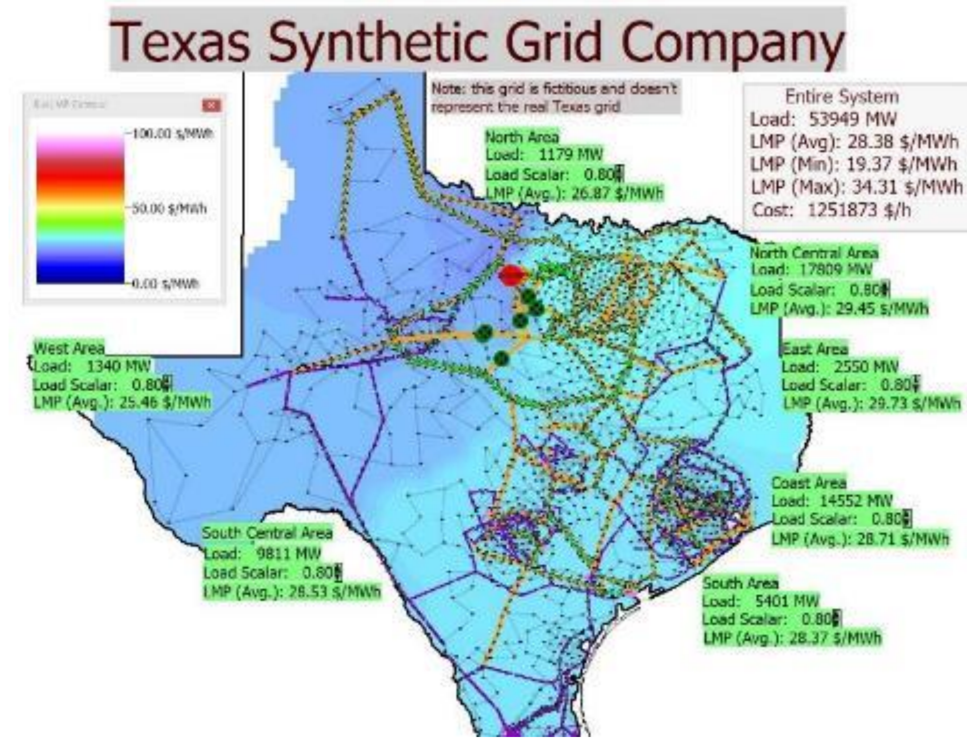
For Travis County (population 1.2 million) 307,236 meters are served by 488 distribution circuits

Different Levels of Modeling

- Just because we have detailed grids, doesn't mean we always simulate the coupled transmission and distribution models. Other options are
  - Transmission only
  - Distribution only
  - Full transmission with distribution topology; this can be quite useful for doing multi-infrastructure simulation in which we just need to know what parts of the distribution system are out-of-service

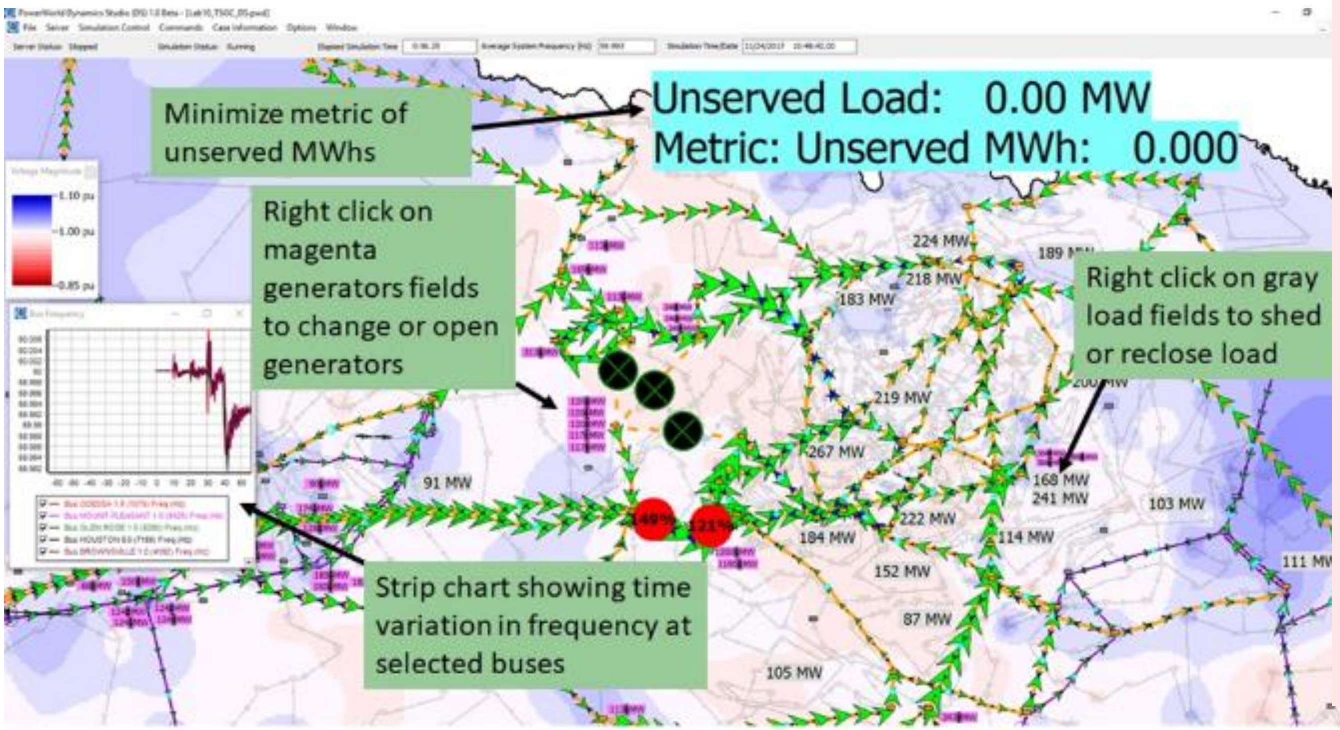
Synthetic Grid Applications: Innovative Electric Power Education

- Lab assignments involving a 2000 bus case have been integrated into Texas A&M's power classes
- Class includes large-system exercises for power flow, economic dispatch, contingency analysis, SCOPF, and transient stability



Innovative Electric Power Education

- One lab challenges students to save the synthetic Texas grid from voltage collapse following a simulated tornado in real-time!



The lab was introduced in Fall 2017; in Fall 2018 it was expanded to be a multi-user simulation. However it did not involve integrated analysis.

Giving Students (and Others) Experience In Grid Operations

- Most electric power students have little or no experience in actually operating an electric grid (real or simulated)
- One of our goals is to provide such an experience both in an individually and as part of a team
- Developing this involves a combination of the electric grid, the scenario, and the associated simulation environment, and the path to give the users experience with the environment
- This also generates data

The Simulation Environment

Applications: Coupled Infrastructures and Extreme Events

- Since the highly detail synthetic grids are linked to actual parcels, they can be used in coupled infrastructure simulation
  - Metadata is used to indicate the number of people at a meter and other attributions
- By partnering with the Texas Transportation Institute (TTI) we're moving forward with coupled electric grid/transportation studies
  - Blackouts affect people, and large blackouts can affect a lot of people, causing transportation impacts
  - Studies can be done looking at different rates of transportation electrification

Power System Voltage Stability

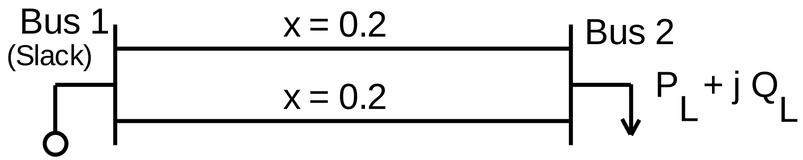
- **Voltage Stability**: The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability**: Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse**: Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system. Typically results in loss of system load.

Voltage Stability

- Two good references are
  - P. Kundur, et. al., “Definitions and Classification of Power System Stability,” *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
  - T. Van Cutsem, “Voltage Instability: Phenomena, Countermeasures, and Analysis Methods,” *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
  - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
  - Short-term (several seconds) or long-term (many seconds to minutes) (covered in ECEN 667)

Small Disturbance Voltage Stability

- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)
- Depending on the assumed load model, the power flow can have multiple (or no) solutions
- PV curve is created by plotting power versus voltage



Assume V_{slack}=1.0

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

Where B is the line susceptance =-10,

V∠θ is the load voltage

Small Disturbance Voltage Stability

- Question: how do the power flow solutions vary as the load is changed?
- A Solution: Calculate a series of power flow solutions for various load levels and see how they change
- Power flow Jacobian

$$\mathbf{J}(\theta,V)=\begin{bmatrix}-BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta-2BV\end{bmatrix}$$

$$\det\mathbf{J}(\theta,V)=VB^2\left(2V\cos\theta-\cos^2\theta-\sin^2\theta\right)$$

Singular when $\left(2V\cos\theta-1\right)=0$

Maximum Loadability When Power Flow Jacobian is Singular

- An important paper considering this was by Sauer and Pai from IEEE Trans. Power Systems in Nov 1990, “Power system steady-state stability and the load-flow Jacobian”
- Other earlier papers were looking at the characteristics of multiple power flow solutions
- The power flow Jacobian depends on the assumed load model (we’ll see the impact in a few slides)

Relationship Between Stability and Power Flow Jacobian

- The Sauer/Pai paper related system stability to the power flow Jacobian by noting the system dynamics could be written as a set of differential algebraic equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y}, \mathbf{p})$$

Linearing about and equilibrium gives

$$\begin{bmatrix} \Delta\dot{\mathbf{x}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \mathbf{f}}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{f}}{\partial \mathbf{y}} \\ \dfrac{\partial \mathbf{g}}{\partial \mathbf{x}} & \dfrac{\partial \mathbf{g}}{\partial \mathbf{y}} \end{bmatrix} \begin{bmatrix} \Delta\mathbf{x} \\ \Delta\mathbf{y} \end{bmatrix}$$

Relationship Between Stability and Power Flow Jacobian

- Then

Assuming $\dfrac{\partial \mathbf{g}}{\partial \mathbf{y}}$ is nonsingular then

$$\Delta\dot{\mathbf{x}} = \left[\frac{\partial \mathbf{f}}{\partial \mathbf{x}} - \frac{\partial \mathbf{f}}{\partial \mathbf{y}}\left[\frac{\partial \mathbf{g}}{\partial \mathbf{y}}\right]^{-1}\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right]\Delta\mathbf{x}$$

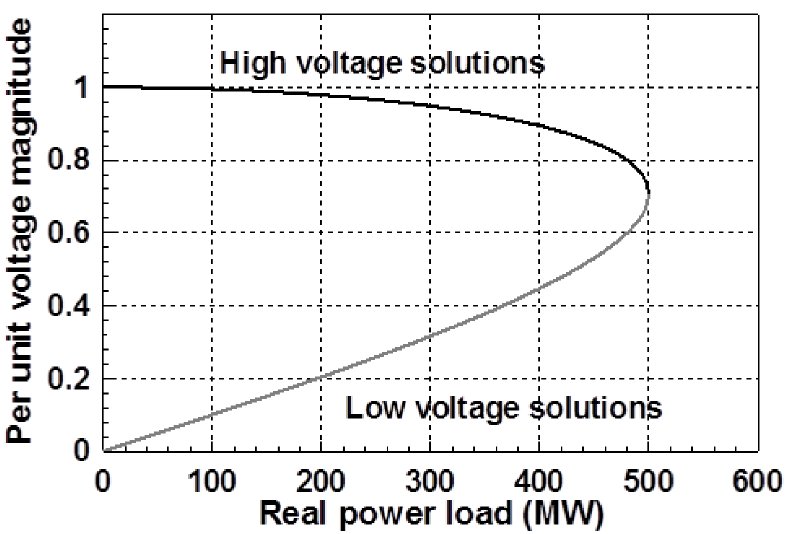
- What Sauer and Pai show is if $\partial \mathbf{g}/\partial \mathbf{y}$ is singular then the system is unstable; if $\partial \mathbf{g}/\partial \mathbf{y}$ is nonsingular then the system may or may not be stable
- Hence it provides an upper bound on stability

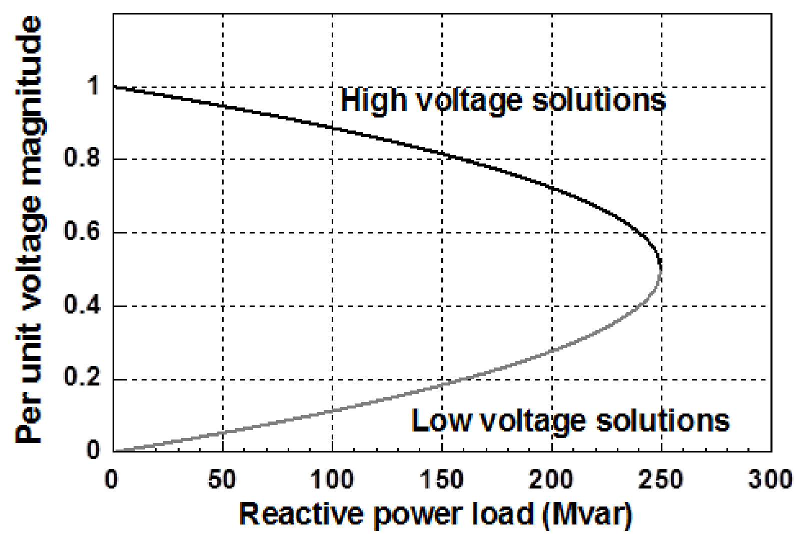
Bifurcations

- In general, bifurcation is the division of something into two branches or parts
- For a dynamic system, a bifurcation occurs when small changes in a parameter cause a new quality of motion of the dynamic system
- Two types of bifurcation are considered for voltage stability
  - Saddle node bifurcation is the disappearance of an equilibrium point for parameter variation; for voltage stability it is two power flow solutions coalescing with parameter variation
  - Hopf bifurcation is cause by two eigenvalues crossing into the right-half plane

PV and QV Curves

- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
  - At least for the upper portion of the curve
- Two bus example PV and QV curves

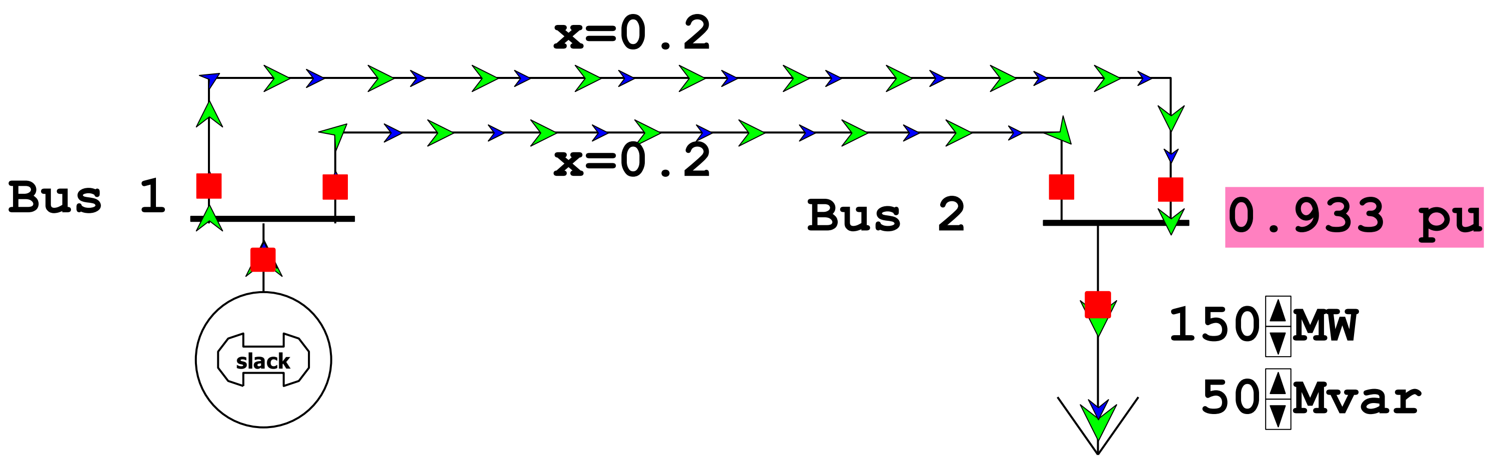


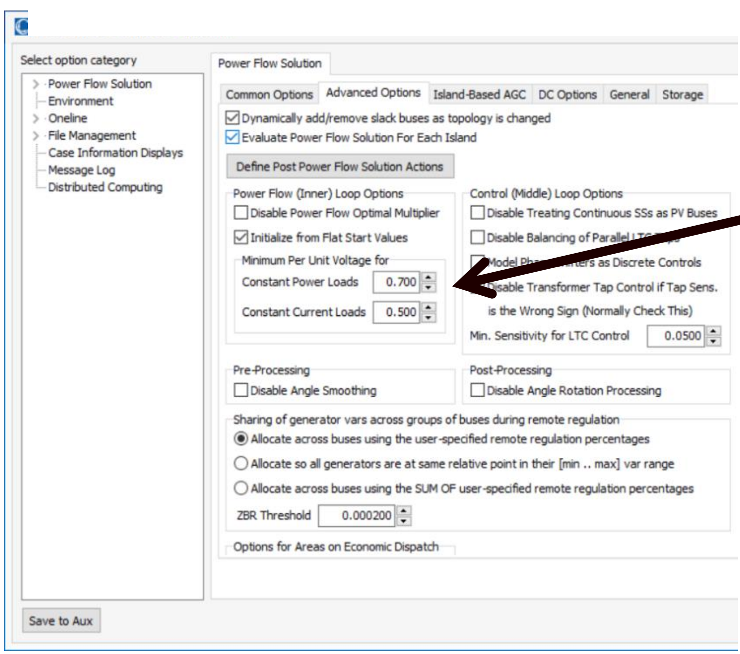


Small Disturbance Voltage Collapse

- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
  - V is phasor voltage, I is phasor current
  - This is the reason for using a high voltage grid
- Line real power losses are given by RI^2 and reactive power losses by XI^2
  - R is the line's resistance, and X its reactance; for a high voltage line X >> R
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses

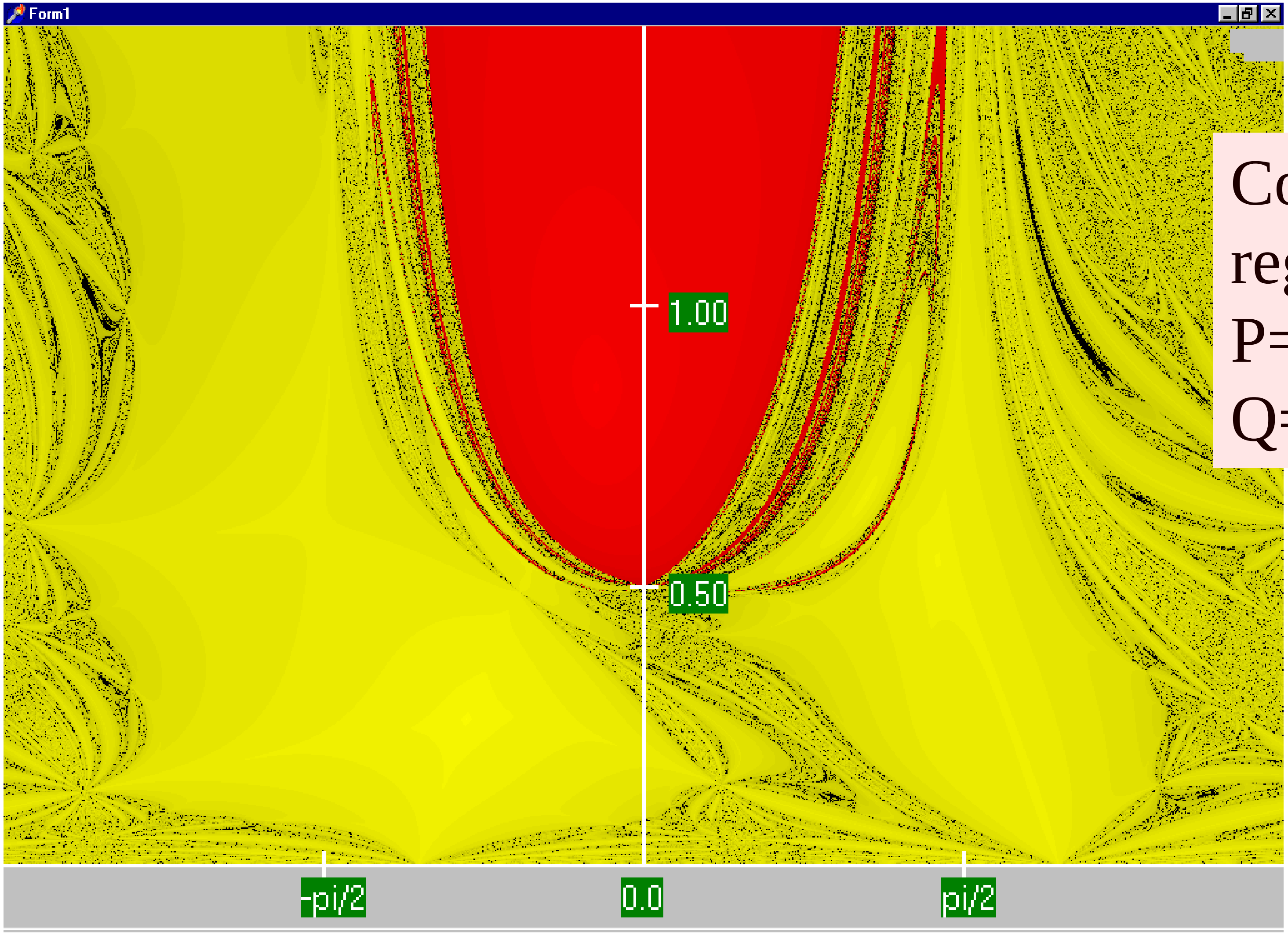
PowerWorld Two Bus Example





Commercial power flow software usually auto converts constant power loads at low voltages; set these fields to zero to disable this conversion

Power Flow Region of Convergence

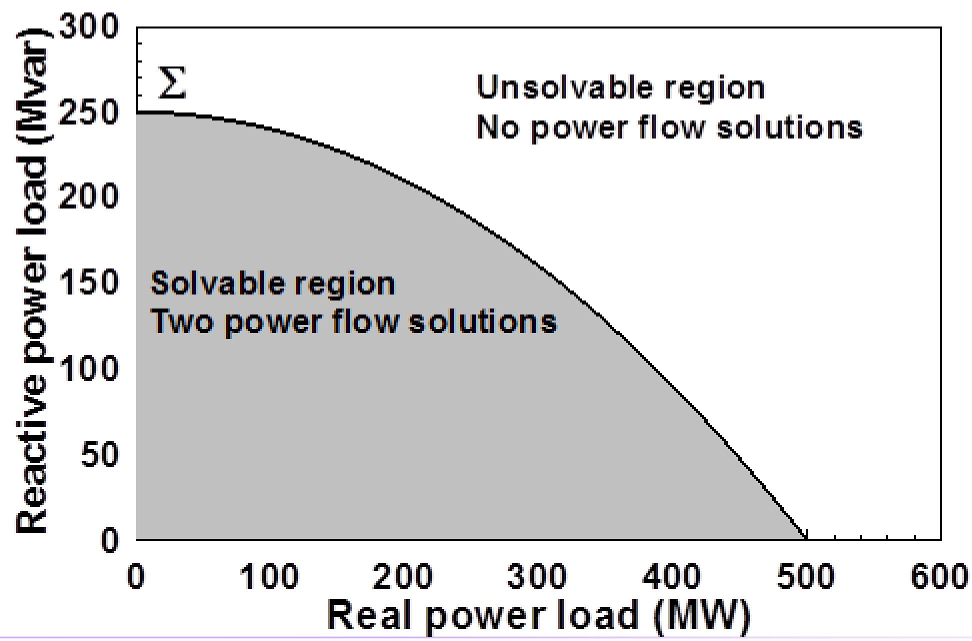


Convergence regions with P=100 MW, Q=0 Mvar

Load Parameter Space Representation

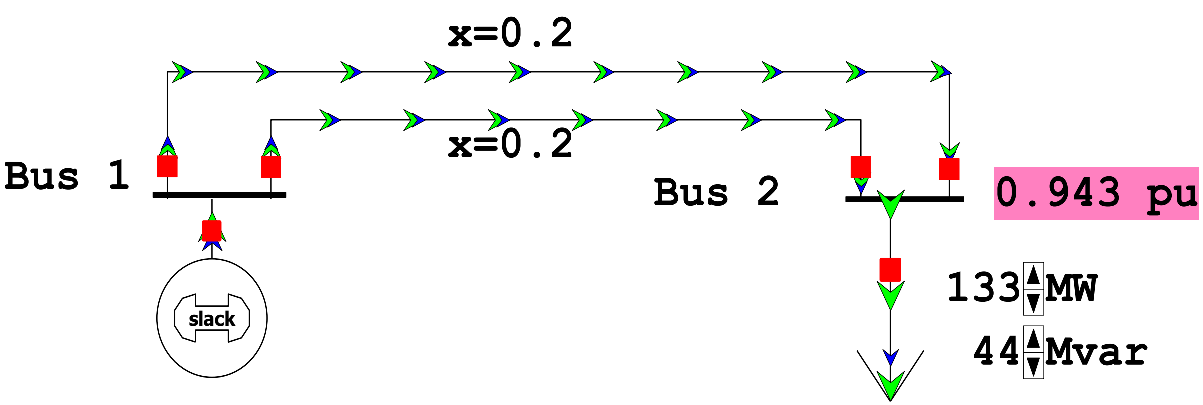
- With a constant power model there is a maximum loadability surface, Σ
  - Defined as point in which the power flow Jacobian is singular
  - For the lossless two bus system it can be determined as

$$-\frac{P_L^2}{B}+Q_L+\frac{1}{4}B=0$$



Load Model Impact

- With a static load model regardless of the voltage dependency the same PV curve is traced
  - But whether a point of maximum loadability exists depends on the assumed load model
    - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai "Sufficient conditions on static load models for network solvability,"NAPS 1992, pp. 262-271)



Change load to constant impedance; hence it becomes a linear model

ZIP Model Coefficients

- One popular static load model is the ZIP; lots of papers on the "correct" amount of each type

TABLE I
ZIP COEFFICIENTS FOR EACH CUSTOMER CLASS

Class	Z_p	I_p	P_p	Z_q	I_q	P_q
Large commercial	0.47	-0.53	1.06	5.30	-8.73	4.43
Small commercial	0.43	-0.06	0.63	4.06	-6.65	3.59
Residential	0.85	-1.12	1.27	10.96	-18.73	8.77
Industrial	0	0	1	0	0	1

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13

Table 1 from M. Diaz-Aguilo, et. al., "Field-Validated Load Model for the Analysis of CVR in Distribution Secondary Networks: Energy Conservation," IEEE Trans. Power Delivery, Oct. 2013

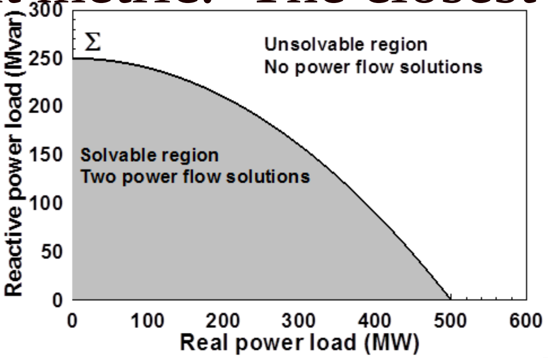
Table 7 from A, Bokhari, et. al., "Experimental Determination of the ZIP Coefficients for Modern Residential, Commercial, and Industrial Loads," IEEE Trans. Power Delivery, June. 2014

Application: Conservation Voltage Reduction (CVR)

- If the "steady-state" load has a true dependence on voltage, then a change (usually a reduction) in the voltage should result in a total decrease in energy consumption
- If an "optimal" voltage could be determined, then this could result in a net energy savings
- Some challenges are 1) the voltage profile across a feeder is not constant, 2) the load composition is constantly changing, 3) a decrease in power consumption might result in a decrease in useable output from the load, and 4) loads are dynamic and an initial decrease might be balanced by a later increase

Determining a Metric to Voltage Collapse

- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
  - PV and QV curves (or some combination) can determine such a metric along a particular path
  - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction



  - Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits

Determining a Metric to Voltage Collapse

- A paper by Dobson in 1992 (see below) noted that at a saddle node bifurcation, in which the power flow Jacobian is singular, that
  - The right eigenvector associated with the Jacobian zero eigenvalue tells the direction in state space of the voltage collapse
  - The left eigenvector associated with the Jacobian zero eigenvalue gives the normal in parameter space to the boundary Σ. This can then be used to estimate the minimum distance in parameter space to bifurcation.

I. Dobson, "Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems," IEEE Trans. Circuits and Systems, March 1992

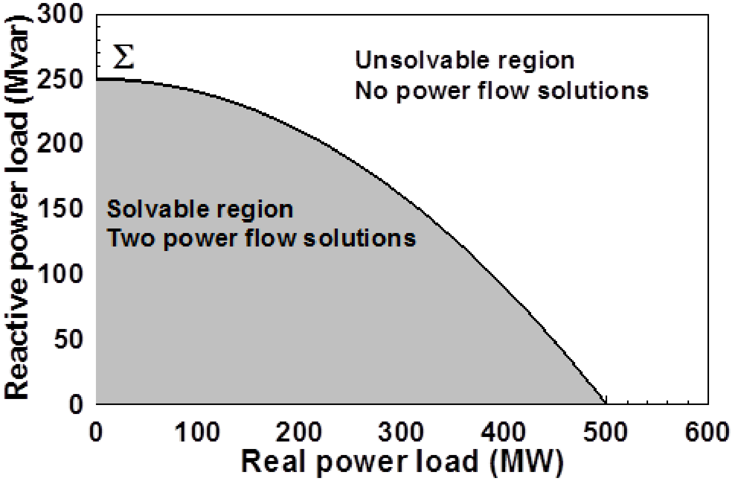
Determining a Metric to Voltage Collapse Example

- For the previous two bus example we had

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

$$\mathbf{J}(\theta,V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$



Singular when $\left(2V\cos\theta - 1\right) = 0$

So consider $B = -10,\ V = 0.6,\ \theta = -33.56°$, then $P_L = 3.317$, $Q_L = 1.400$

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

Determining a Metric to Voltage Collapse Example

- Calculating the right and left eigenvectors associated with the zero eigenvalue we get

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 0.742 \\ 0.671 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0.553 \\ 0.833 \end{bmatrix}$$