ECEN 615
Methods of Electric Power Systems Analysis

Lecture 19: Voltage Stability, Economic Dispatch, Optimal Power Flow

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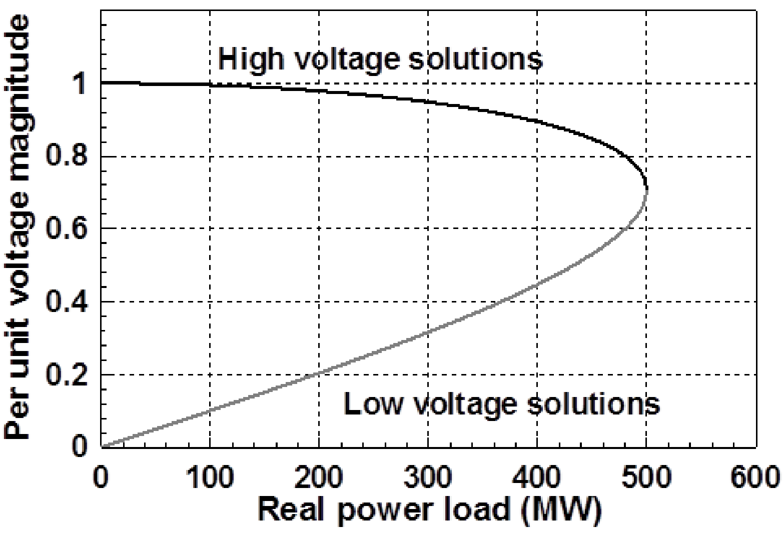
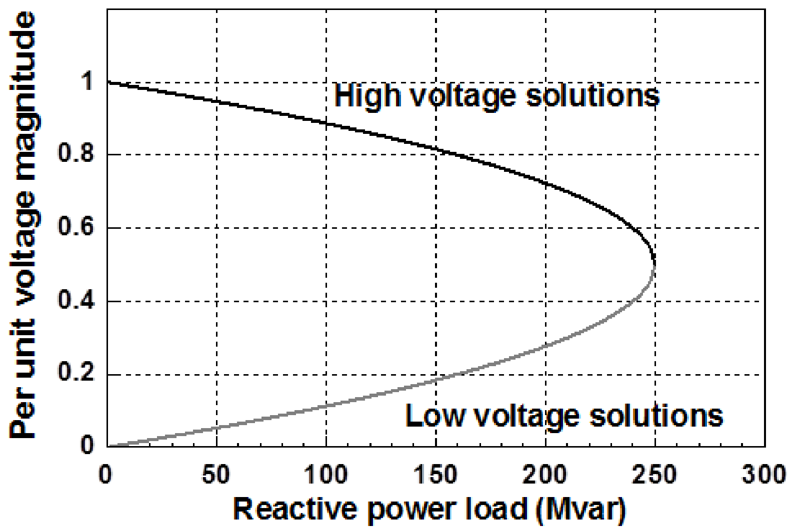
Announcements

- Read Chapter 8
- Homework 5 is due on Thursday Oct 29

PV and QV Curves

- PV curves can be traced by plotting the voltage as the real power is increased; QV curves as reactive power is increased
  - At least for the upper portion of the curve
- Two bus example PV and QV curves

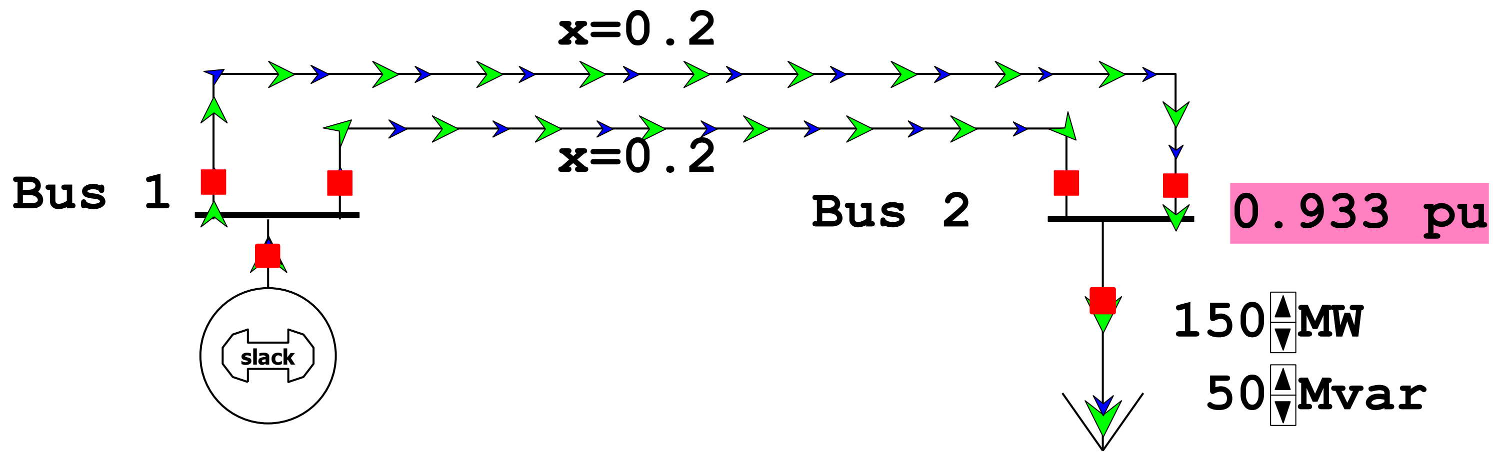


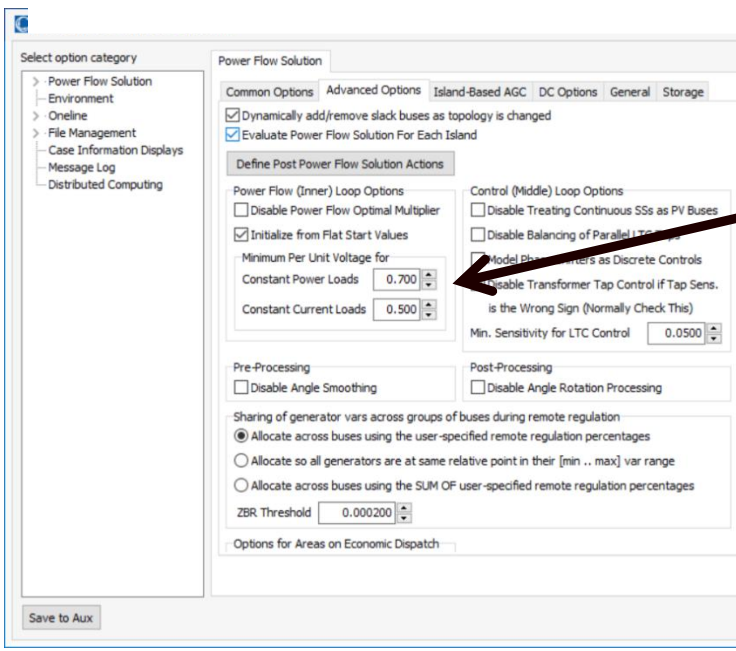


Small Disturbance Voltage Collapse

- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
  - V is phasor voltage, I is phasor current
  - This is the reason for using a high voltage grid
- Line real power losses are given by RI^2 and reactive power losses by XI^2
  - R is the line's resistance, and X its reactance; for a high voltage line X >> R
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses

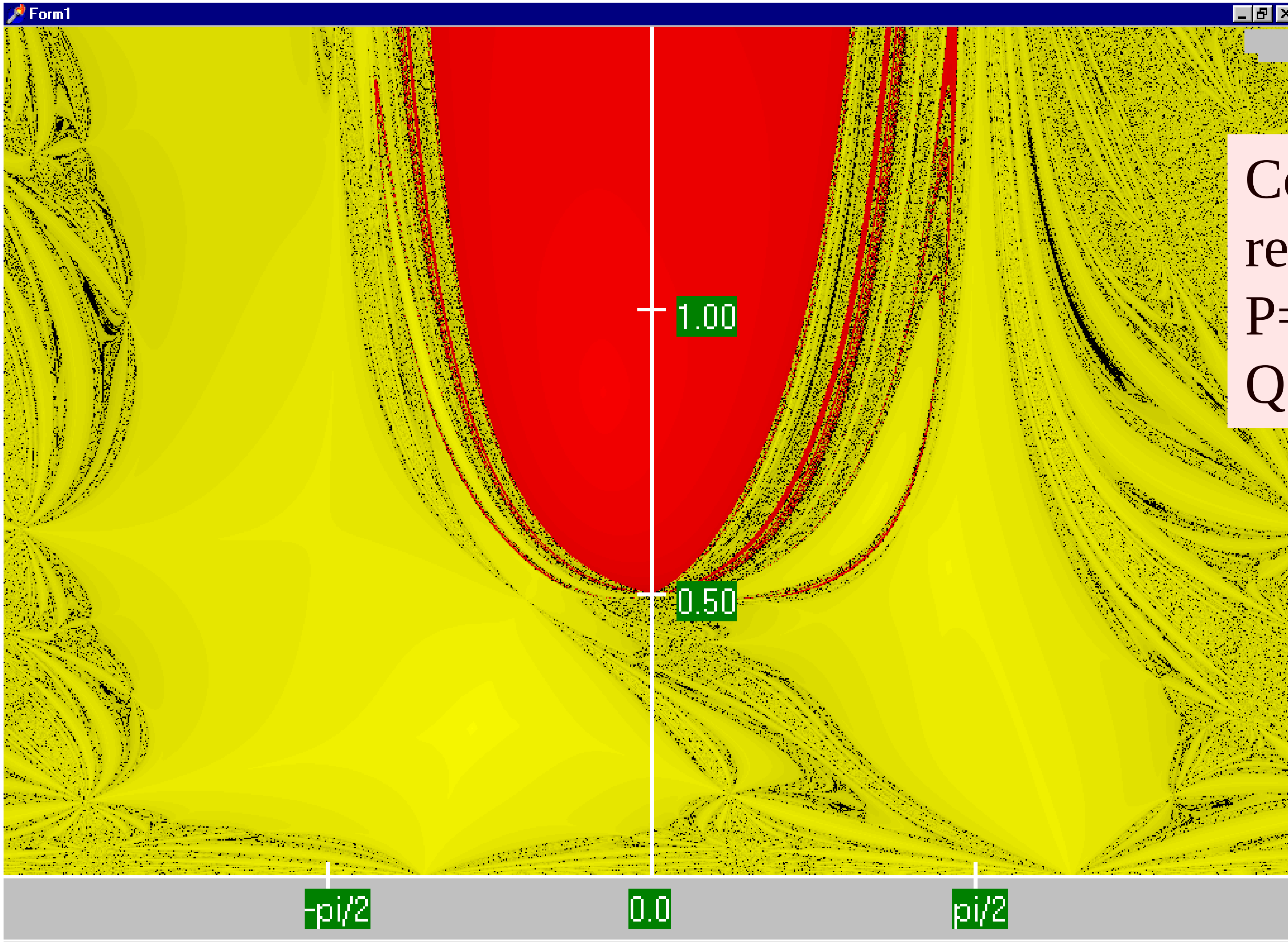
PowerWorld Two Bus Example





Commercial power flow software usually auto converts constant power loads at low voltages; set these fields to zero to disable this conversion

Power Flow Region of Convergence

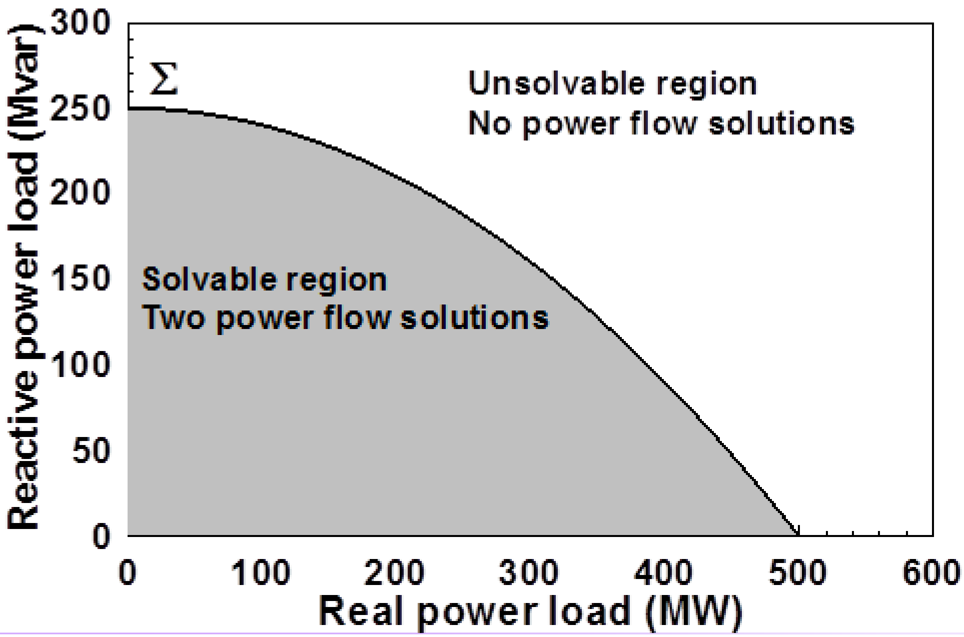


Convergence regions with P=100 MW, Q=0 Mvar

Load Parameter Space Representation

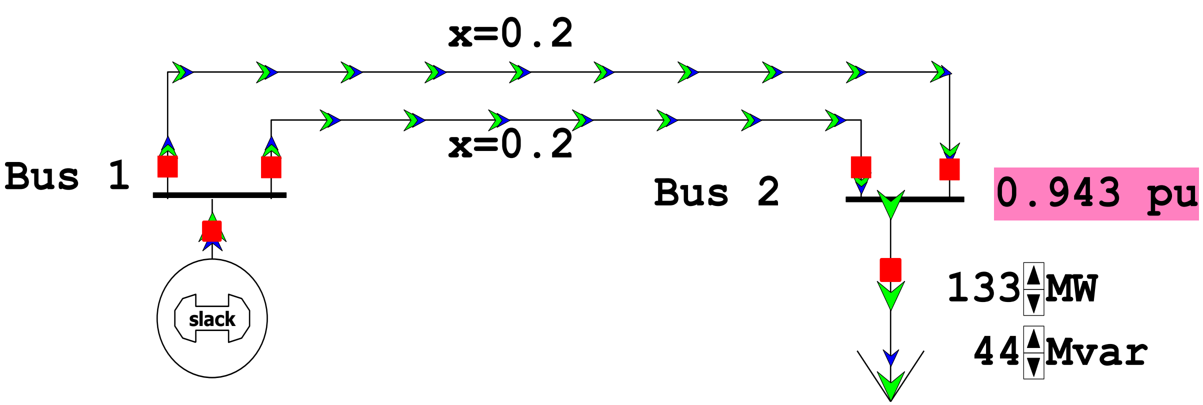
- With a constant power model there is a maximum loadability surface, Σ
  - Defined as point in which the power flow Jacobian is singular
  - For the lossless two bus system it can be determined as

$$-\frac{P_L^2}{B} + Q_L + \frac{1}{4}B = 0$$



Load Model Impact

- With a static load model regardless of the voltage dependency the same PV curve is traced
  - But whether a point of maximum loadability exists depends on the assumed load model
    - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai "Sufficient conditions on static load models for network solvability,"NAPS 1992, pp. 262-271)



Change load to constant impedance; hence it becomes a linear model

ZIP Model Coefficients

- One popular static load model is the ZIP; lots of papers on the “correct” amount of each type

TABLE I
ZIP COEFFICIENTS FOR EACH CUSTOMER CLASS

Class	Z_p	I_p	P_p	Z_q	I_q	P_q
Large commercial	0.47	-0.53	1.06	5.30	-8.73	4.43
Small commercial	0.43	-0.06	0.63	4.06	-6.65	3.59
Residential	0.85	-1.12	1.27	10.96	-18.73	8.77
Industrial	0	0	1	0	0	1

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13

Table 1 from M. Diaz-Aguilo, et. al., “Field-Validated Load Model for the Analysis of CVR in Distribution Secondary Networks: Energy Conservation,” IEEE Trans. Power Delivery, Oct. 2013

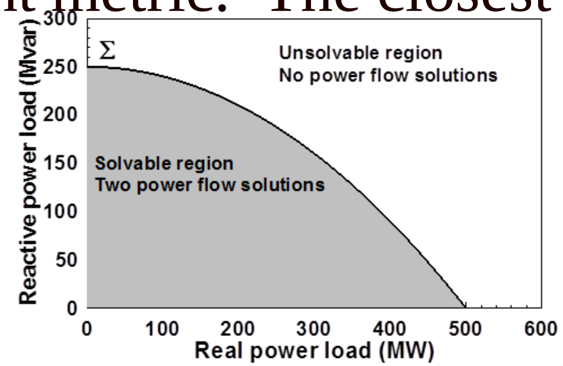
Table 7 from A, Bokhari, et. al., “Experimental Determination of the ZIP Coefficients for Modern Residential, Commercial, and Industrial Loads,” IEEE Trans. Power Delivery, June. 2014

Application: Conservation Voltage Reduction (CVR)

- If the "steady-state" load has a true dependence on voltage, then a change (usually a reduction) in the voltage should result in a total decrease in energy consumption
- If an "optimal" voltage could be determined, then this could result in a net energy savings
- Some challenges are 1) the voltage profile across a feeder is not constant, 2) the load composition is constantly changing, 3) a decrease in power consumption might result in a decrease in useable output from the load, and 4) loads are dynamic and an initial decrease might be balanced by a later increase

Determining a Metric to Voltage Collapse

- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
  - PV and QV curves (or some combination) can determine such a metric along a particular path
  - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction



  - Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits

Determining a Metric to Voltage Collapse

- A paper by Dobson in 1992 (see below) noted that at a saddle node bifurcation, in which the power flow Jacobian is singular, that
  – The right eigenvector associated with the Jacobian zero eigenvalue tells the direction in state space of the voltage collapse
  – The left eigenvector associated with the Jacobian zero eigenvalue gives the normal in parameter space to the boundary Σ. This can then be used to estimate the minimum distance in parameter space to bifurcation.

I. Dobson, "Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems," IEEE Trans. Circuits and Systems, March 1992

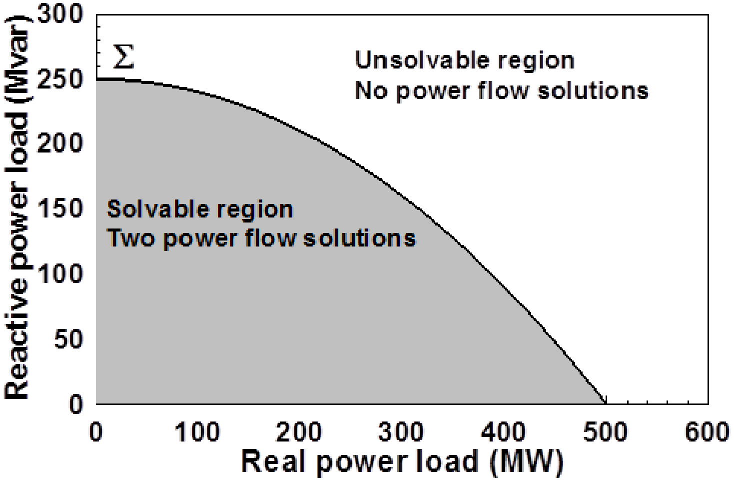
Determining a Metric to Voltage Collapse Example

- For the previous two bus example we had

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

$$\mathbf{J}(\theta,V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$



Singular when $\left(2V\cos\theta - 1\right) = 0$

So consider $B = -10,\ V = 0.6,\ \theta = -33.56°$, then $P_L = 3.317$, $Q_L = 1.400$

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

Determining a Metric to Voltage Collapse Example

- Calculating the right and left eigenvectors associated with the zero eigenvalue we get

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 0.742 \\ 0.671 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0.553 \\ 0.833 \end{bmatrix}$$

Quantifying Power Flow Unsolvability

- Since lack of power flow convergence can be a major problem, it would be nice to have a measure to quantify the degree of unsolvability of a power flow
  - And then figure out the best way to restore solvabiblity
- T.J. Overbye, "A Power Flow Measure for Unsolvable Cases," IEEE Trans. Power Systems, August 1994

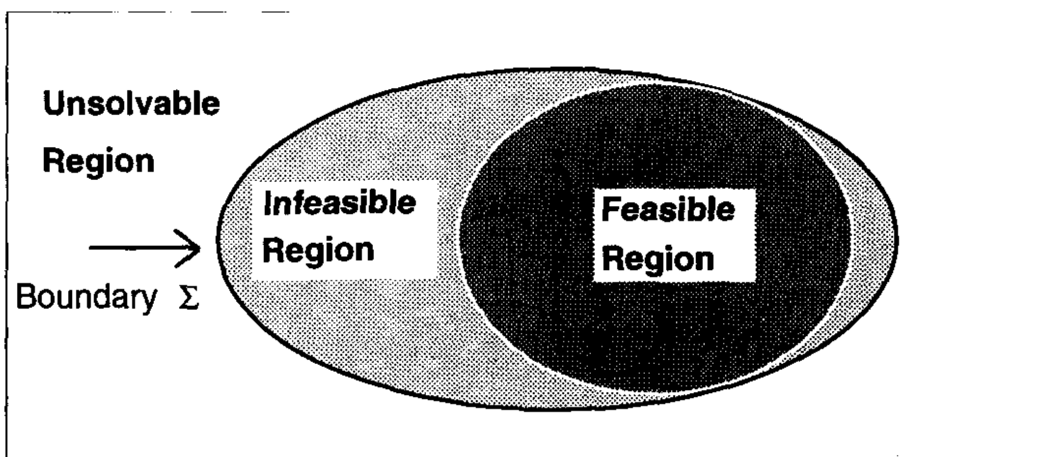


Figure 1 : Power Flow Security Regions

Quantifying Power Flow Unsolvability

- To setup the problem, first consider the power flow iteration without and with the optimal multiplier

$$\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}^k$$

$$\Delta\mathbf{x}^k = -\mathbf{J}(\mathbf{x}^k)^{-1}\left(\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right)$$

With the optimal multiplier we are minimizing

$$\mathrm{F}(\mathbf{x}^{k+1}) = \frac{1}{2}\left(\mathbf{f}(\mathbf{x}^k) + \mu\Delta\mathbf{x}^k - \mathbf{S}\right)^T\left(\mathbf{f}(\mathbf{x}^k) + \mu\Delta\mathbf{x}^k - \mathbf{S}\right)$$

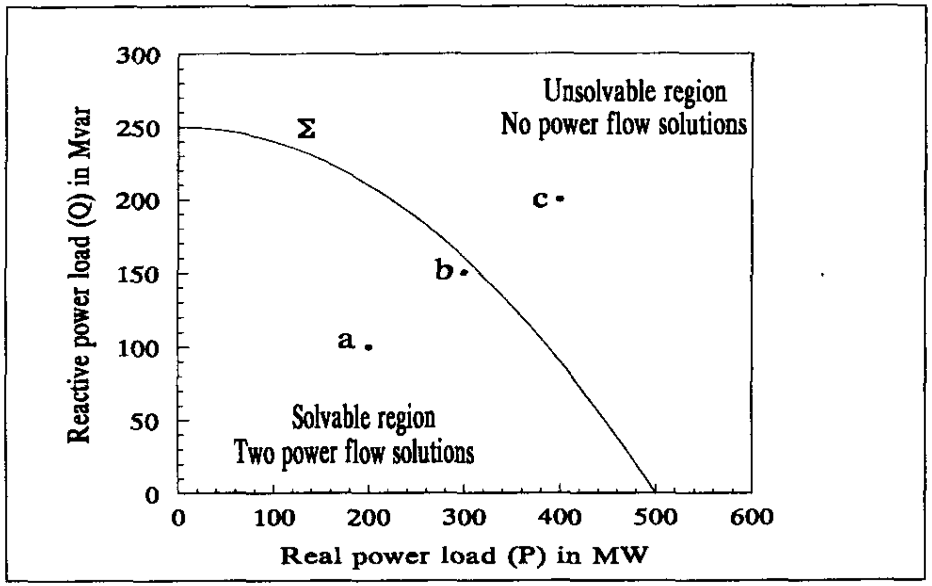
When there is a solution $\mu \rightarrow 1$ and the cost function goes to zero

Quantifying Power Flow Unsolvability

$$\det(\mathbf{J}) \quad = \quad B_{12}(B_{12} + 2eB_{22}) \quad = \quad 0 \qquad (12)$$

Here, where $B_{12} = -B_{22}$, the solution of (12) is $e = 0.5$. Substituting this solution for e into (10b) and using (10a) to solve for the f component of the bus 2 voltage, one gets Σ to be the set of all points where

$$\frac{P^2}{B_{12}} + Q - \frac{1}{4}B_{12} = 0 \qquad (13)$$



• Figure 2 : Solvable and Unsolvable Regions in Parameter Space

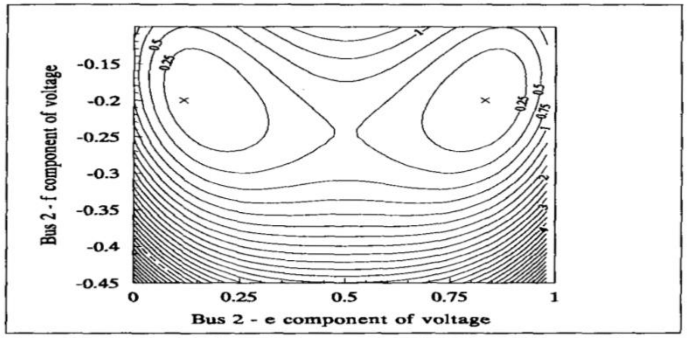


Figure 3a : Two Bus Cost Contours - Load of 200 MW and 100 Mvar

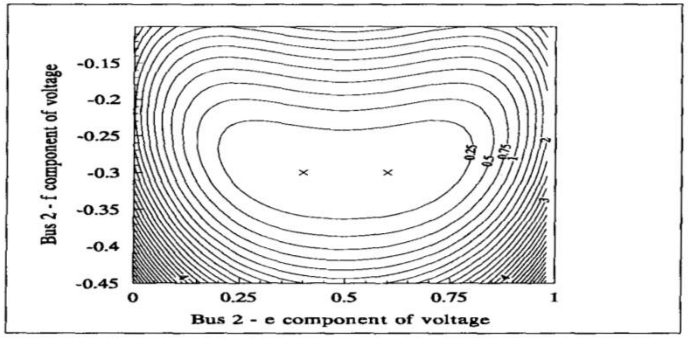


Figure 3b : Two Bus Cost Contours - Load of 300 MW and 150 Mvar

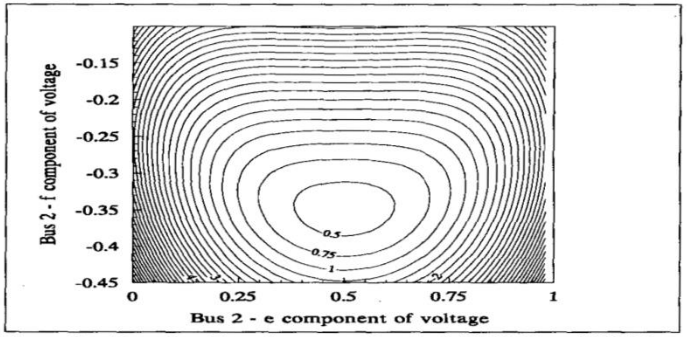


Figure 3c : Two Bus Cost Contours - Load of 400 MW and 200 Mvar

Quantifying Power Flow Unsolvability

- However, when there is no solution the standard power flow would diverge. But the approach with the optimal multiplier tends to point in the direction of minimizing $F(x^{k+1})$. That is,

$$\nabla F(\mathbf{x}^k) = \left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^k)$$

Also

$$\Delta \mathbf{x}^k = -\mathbf{J}(\mathbf{x}^k)^{-1}\left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]$$

where how far to move in this direction is limited by μ.

Quantifying Power Flow Unsolvability

- The only way we cannot reduce the cost function some would be if the two directions were perpendicular, hence with a zero dot product. So

$$\frac{\nabla F\left(\mathbf{x}^k\right)\cdot\Delta\mathbf{x}^k}{\left\|\mathbf{x}^k\right\|} = \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^{\mathrm{k}})\mathbf{J}(\mathbf{x}^{\mathrm{k}})^{-1}\left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

$$= \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

(provided the Jacobian is not singular). As we approach singularity this goes to zero. Hence we converge to a point on the boundary Σ, but not necessarilty at the closest boundary point.

Quantifying Power Flow Unsolvability

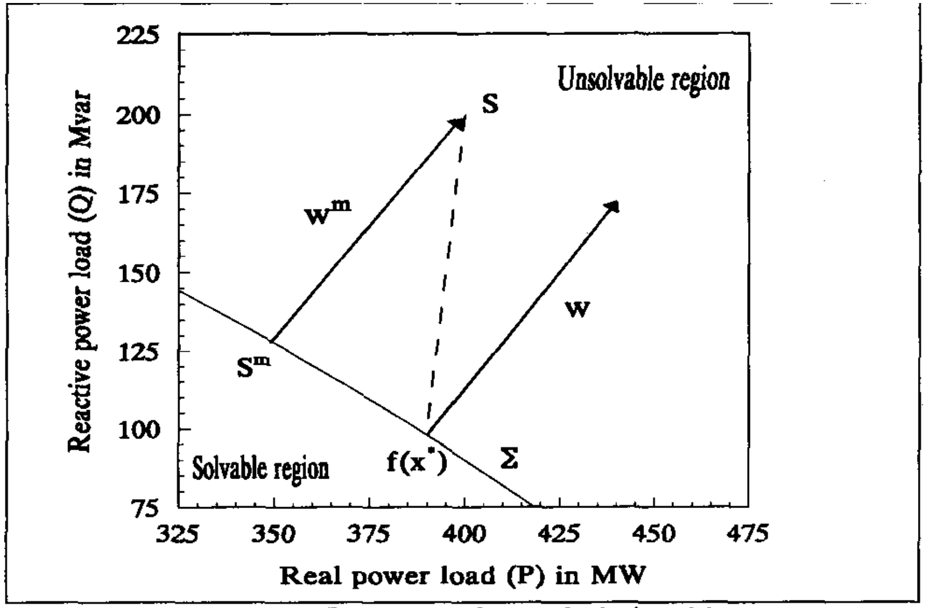


Figure 5 : Parameter Space Relationships

If Σ were flat then **w** is parallel to **w**^{m}

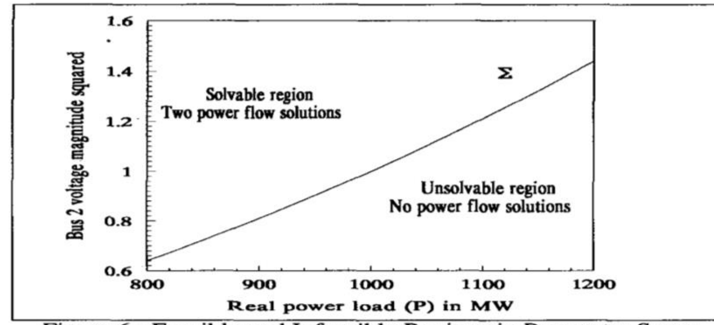


Figure 6 : Feasible and Infeasible Regions in Parameter Space

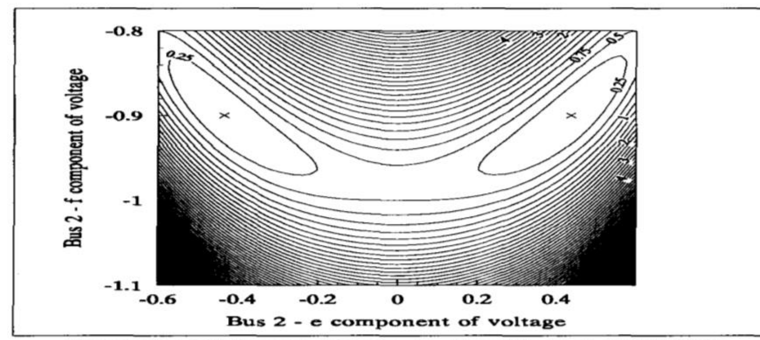


Figure 7a : PV Bus Cost Contours - Feasible load of 900 MW

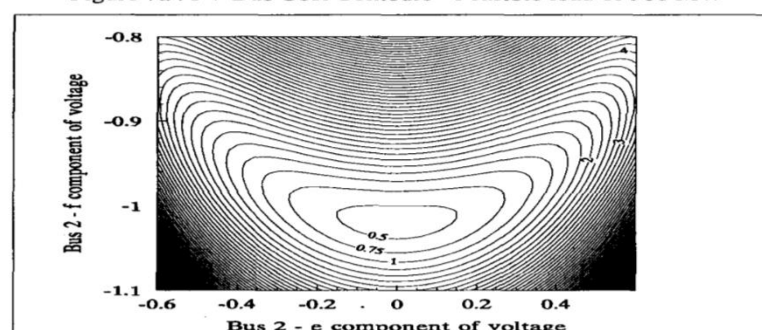


Figure 7b : PV Bus Cost Contours - Infeasible load of 1100 MW

Quantifying Power Flow Unsolvability

- The left eigenvector associated with the zero eigenvalue of the Jacobian (defined as $\mathbf{w}^{i*}$) is perpendicular to Σ (as noted in the early 1992 Dobson paper)
- We can get the closest point on the Σ just by iterating, updating the **S** Vector as

$$\mathbf{S}^{i+1} = \mathbf{S} + [(\mathbf{f}(\mathbf{x}^{i*}) - \mathbf{S}) \cdot \mathbf{w}^{i*}]\, \mathbf{w}^{i*}$$

(here **S** is the initial power injection, $\mathbf{x}^{i*}$ a boundary solution)

- Converges when $\left\|(\mathbf{f}(\mathbf{x}^{i*}) - \mathbf{S}^{i})\right\| < \varepsilon$

Challenges

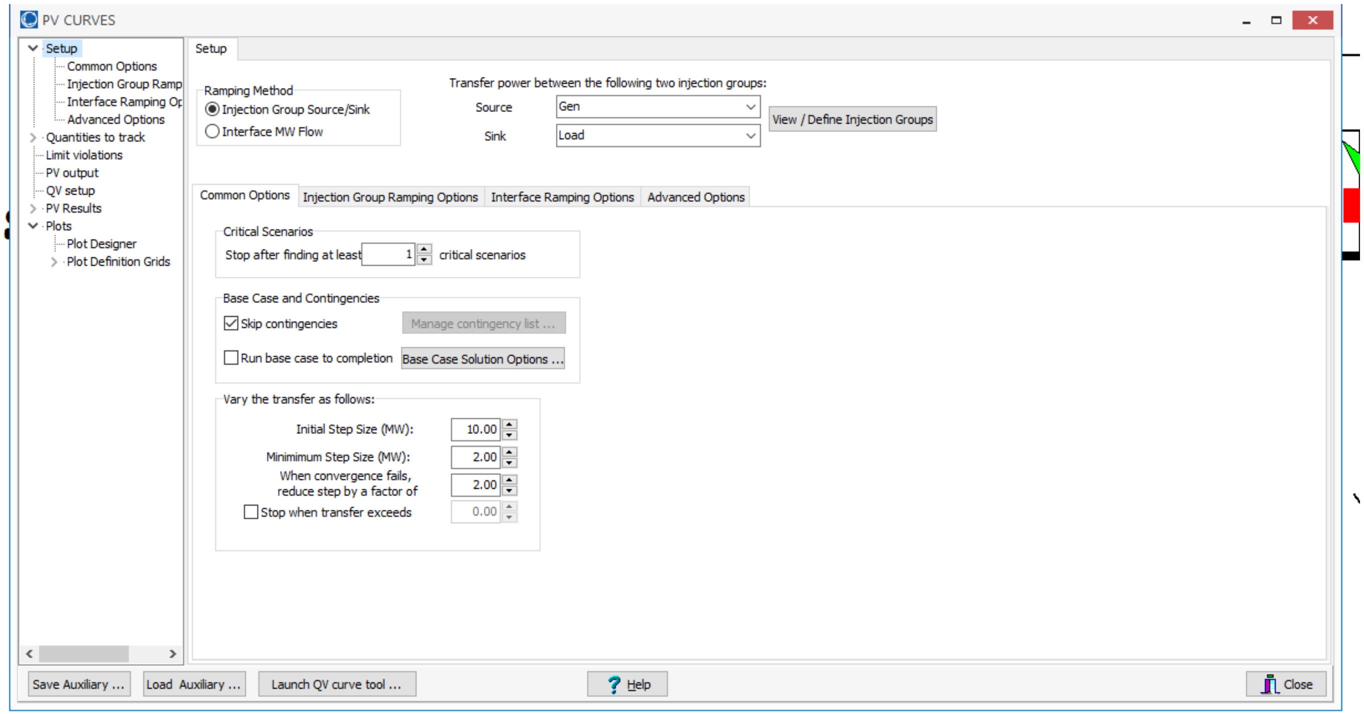
- The key issues is actual power systems are quite complex, with many nonlinearities. For example, generators hitting reactive power limits, switched shunts, LTCs, phase shifters, etc.
- Practically people would like to know how far some system parameters can be changed before running into some sort of limit violation, or maximum loadability.
  - The system is changing in a particular direction, such as a power transfer; this often includes contingency analysis
- Line limits and voltage magnitudes are considered
  - Lower voltage lines tend to be thermally constrained
- Solution is to just to trace out the PV or QV curves

PV and QV Analysis in PowerWorld

- Requires setting up what is known in PowerWorld as an injection group
  - An injection group specifies a set of objects, such as generators and loads, that can inject or absorb power
  - Injection groups can be defined by selecting **Case Information, Aggregation, Injection Groups**
- The PV and/or QV analysis then varies the injections in the injection group, tracing out the PV curve
- This allows optional consideration of contingencies
- The PV tool can be displayed by selecting **Add-Ons, PV**

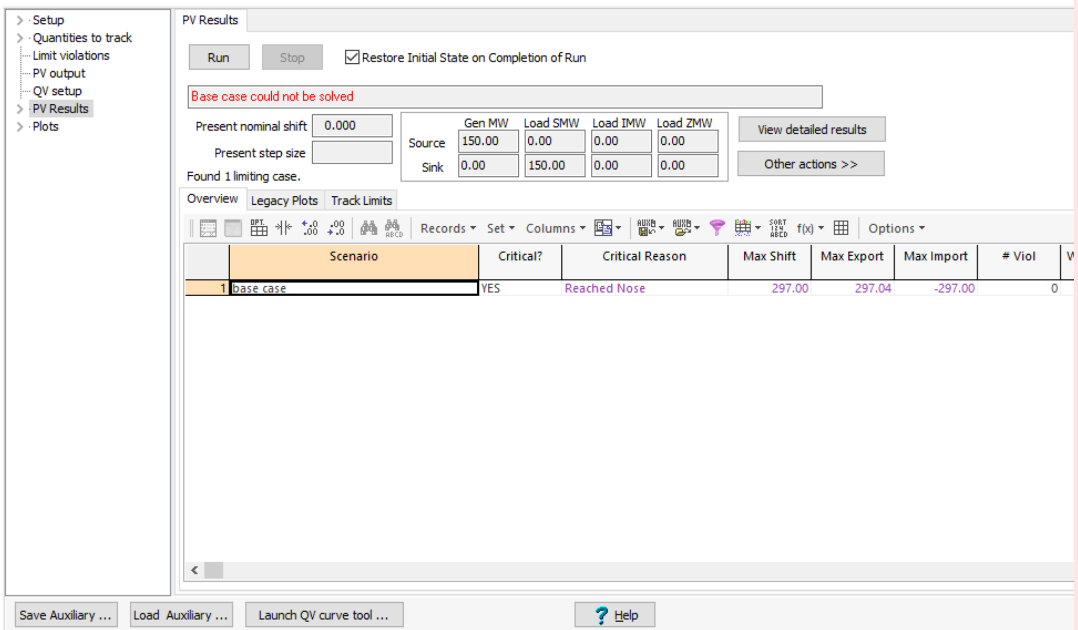
PV and QV Analysis in PowerWorld: Two Bus Example

- Setup page defines the source and sink and step size



PV and QV Analysis in PowerWorld: Two Bus Example

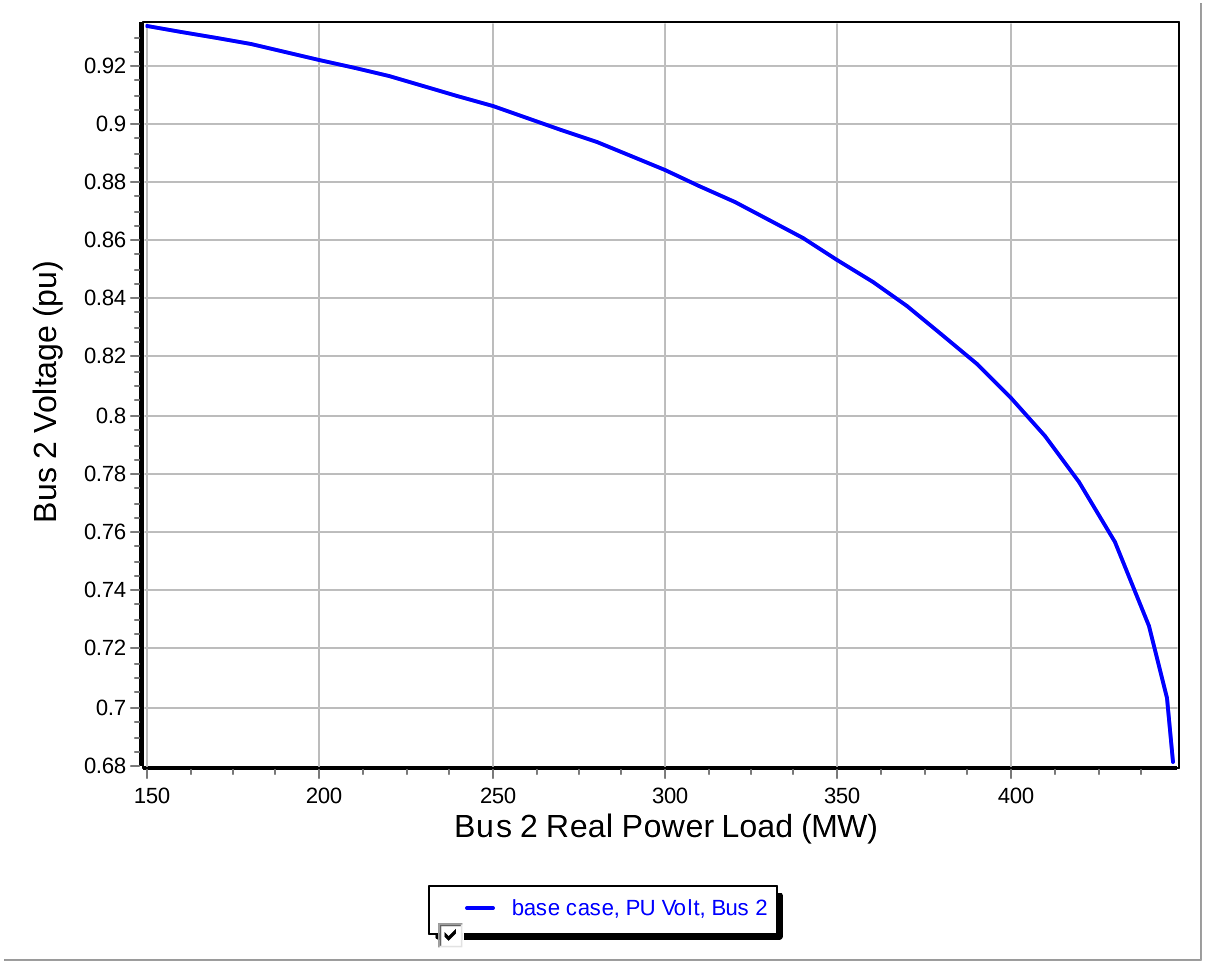
- The PV Results Page does the actual solution
  - Plots can be defined to show the results
  - **Other Actions, Restore initial state** restores the pre-study state



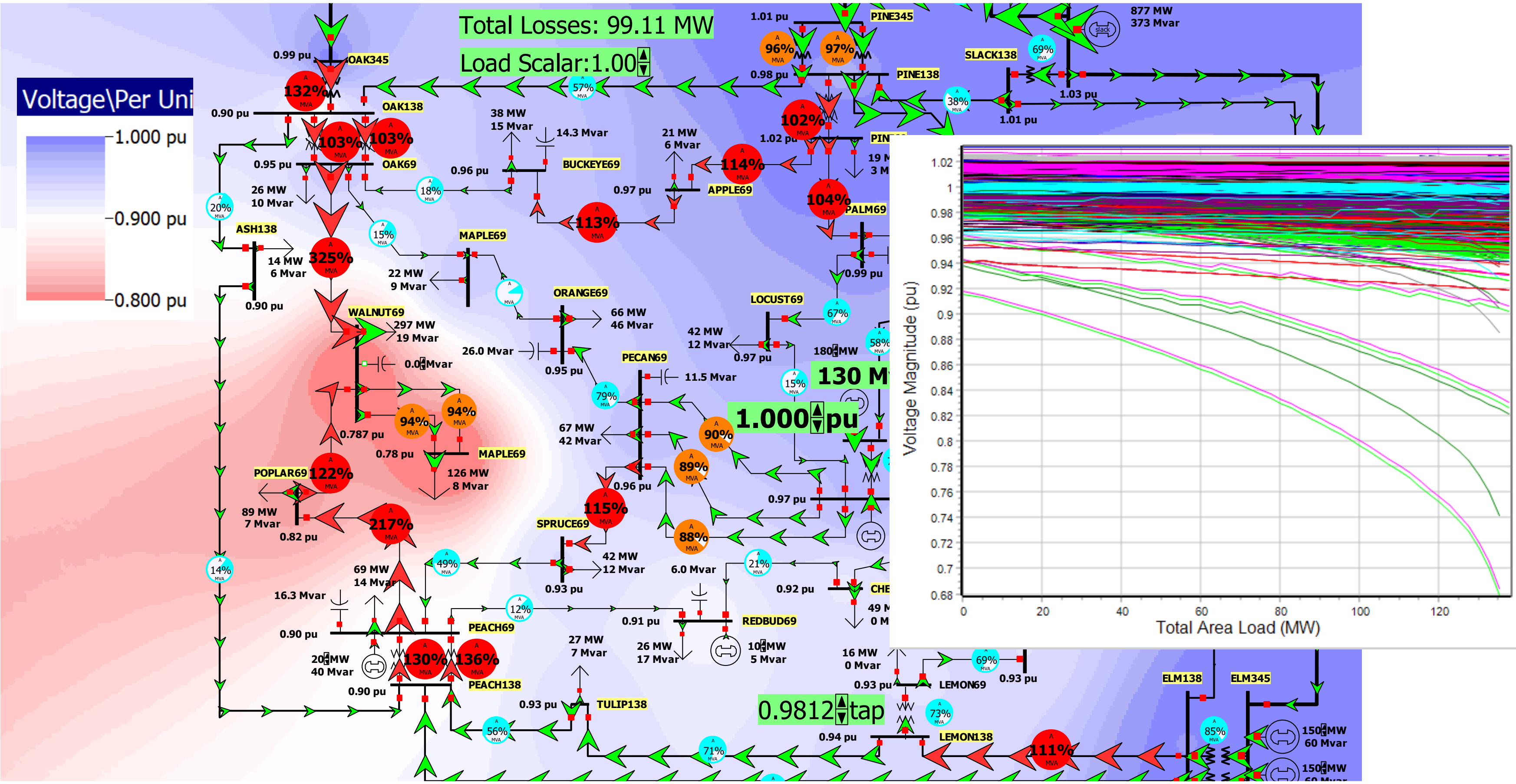
Click the Run button to run the PV analysis;

Check the **Restore Initial State on Completion of Run** to restore the pre-PV state (by default it is not restored)

PV and QV Analysis in PowerWorld: Two Bus Example



PV and QV Analysis in PowerWorld: 37 Bus Example



Usually other limits also need to be considered in doing a realistic PV analysis

Power System Economic Dispatch

- Generators can have vastly different incremental operational costs
  - Some are essentially free or low cost (wind, solar, hydro, nuclear)
  - Because of the large amount of natural gas generation, electricity prices are very dependent on natural gas prices
- Economic dispatch is concerned with determining the best dispatch for generators without changing their commitment
- Unit commitment focuses on optimization over several days. It is discussed in Chapter 4 of the book, but will not be not covered here in-depth

Power System Economic Dispatch

- Economic dispatch is formulated as a constrained minimization
  - The cost function is often total generation cost in an area
  - Single equality constraint is the real power balance equation
- Solved by setting up the Lagrangian (with P_D the load and P_L the losses, which are a function the generation)

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D + P_L(\mathbf{P}_G) - \sum_{i=1}^{m} P_{Gi}\right)$$

- A necessary condition for a minimum is that the gradient is zero. Without losses this occurs when all generators are dispatched at the same marginal cost (except when they hit a limit)

Power System Economic Dispatch

$$\mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

$$\frac{\partial \mathrm{L}(\mathbf{P}_{\mathrm{G}},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

- If losses are neglected then there is a single marginal cost (lambda); if losses are included then each bus could have a different marginal cost

Economic Dispatch Penalty Factors

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}} = 0$$

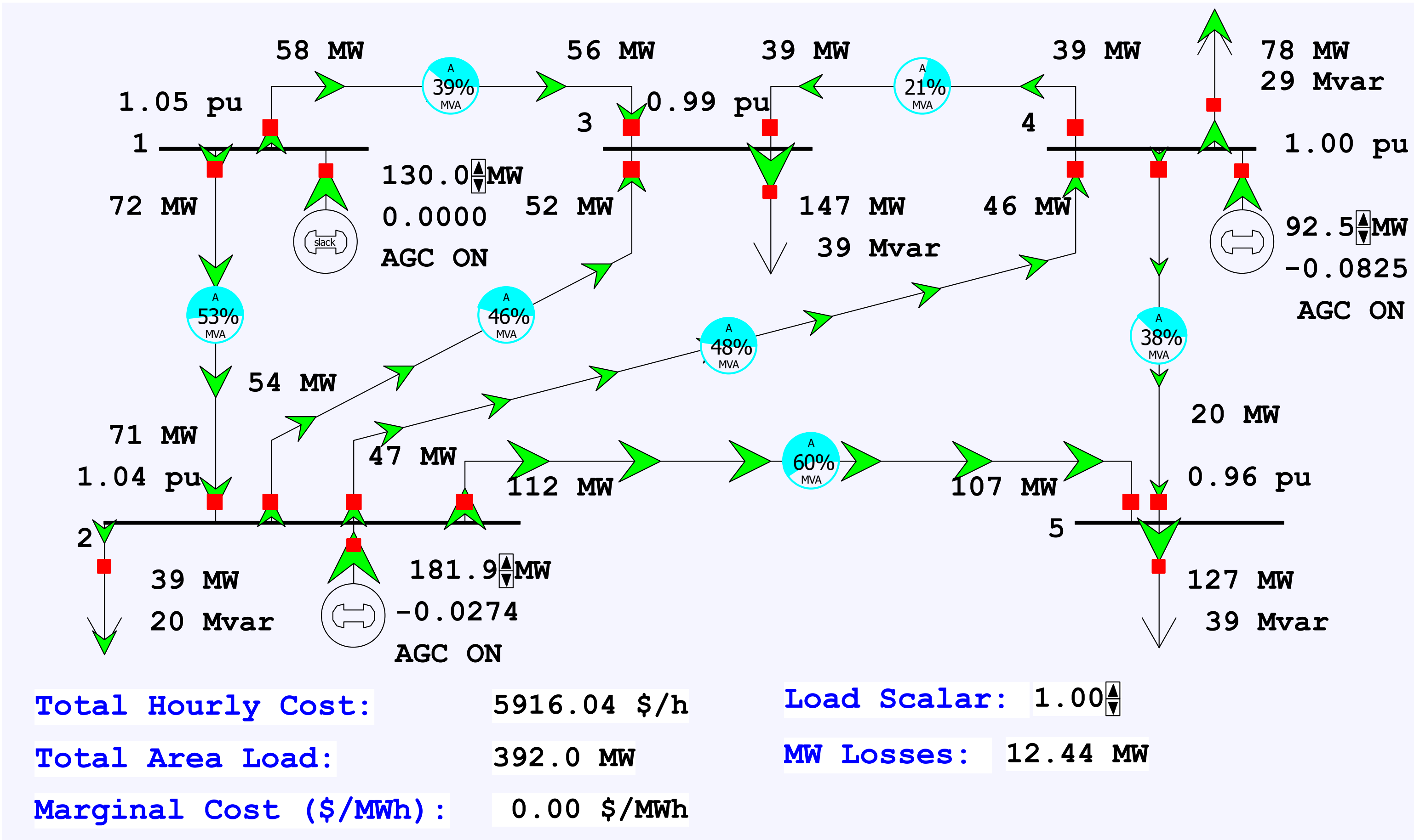
$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

Define the penalty factor L_i for the i^{th} generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

The penalty factor at the slack bus is always unity!

Economic Dispatch Example



Case is GOS_Example6_22; use **Power Flow Solution Options, Advanced Options** to set Penalty Factors

Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch
- SCOPF adds in contingency analysis
- Goal of OPF and SCOPF is to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

OPF, cont.

- Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls
  - generator MW outputs
  - transformer taps and phase angles
  - reactive power controls

Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
  - Generation costs (and other costs) represented by quadratic or cubic functions
- Linear Programming
  - fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - used in PowerWorld Simulator
  - generation costs (and other costs) represented by piecewise linear functions
- Both can be implemented using an ac or dc power flow

OPF and SCOPF Current Status

- OPF (really SCOPF) is currently an area of active research, with ARPA-E having an SCOPF competition and recently awarding about $5 million for improved algorithms (see gocompetition.energy.gov)
- A 2016 National Academies Press report, titled "Analytic Research Founds for the Next-Generation Electric Grid," recommended improved AC OPF models
  - I would recommend reading this report; it provides good background on power systems include OPF
  - It is available for free at www.nap.edu/catalog/21919/analytic-research-foundations-for-the-next-generation-electric-grid

OPF and SCOPF History

- A nice OPF history from Dec 2012 is provided by the below link, and briefly summarized here
- Prior to digital computers economic dispatch was solved by hand and the power flow with network analyzers
- Digital power flow developed in late 50's to early 60's
- First OPF formulations in the 1960's
  - J. Carpienterm, "Contribution e l'étude do Dispatching Economique," Bulletin Society Francaise Electriciens, 1962
  - H.W. Dommel, W.F. Tinney, "Optimal power flow solutions," *IEEE Trans. Power App. and Sys*tems, Oct. 1968
    - "Only a small extension of the power flow program is required"

www.ferc.gov/industries/electric/indus-act/market-planning/opf-papers/acopf-1-history-formulation-testing.pdf
(by M Cain, R. O'Neill, A. Castillo)

OPF and SCOPF History

- A linear programming (LP) approach was presented by Stott and Hobson in 1978
  - B. Stott, E. Hobson, “Power System Security Control Calculations using Linear Programming,” (Parts 1 and 2) *IEEE Trans. Power App and Syst*., Sept/Oct 1978
- Optimal Power Flow By Newton’s Method
  - D.I. Sun, B. Ashley, B. Brewer, B.A. Hughes, and W.F. Tinney, "Optimal Power Flow by Newton Approach", *IEEE Trans. Power App and Syst*., October 1984
- Follow-up LP OPF paper in 1990
  - O. Alsac, J. Bright, M. Prais, B. Stott, “Further Developments in LP-based Optimal Power Flow,” *IEEE Trans. Power Systems*, August 1990

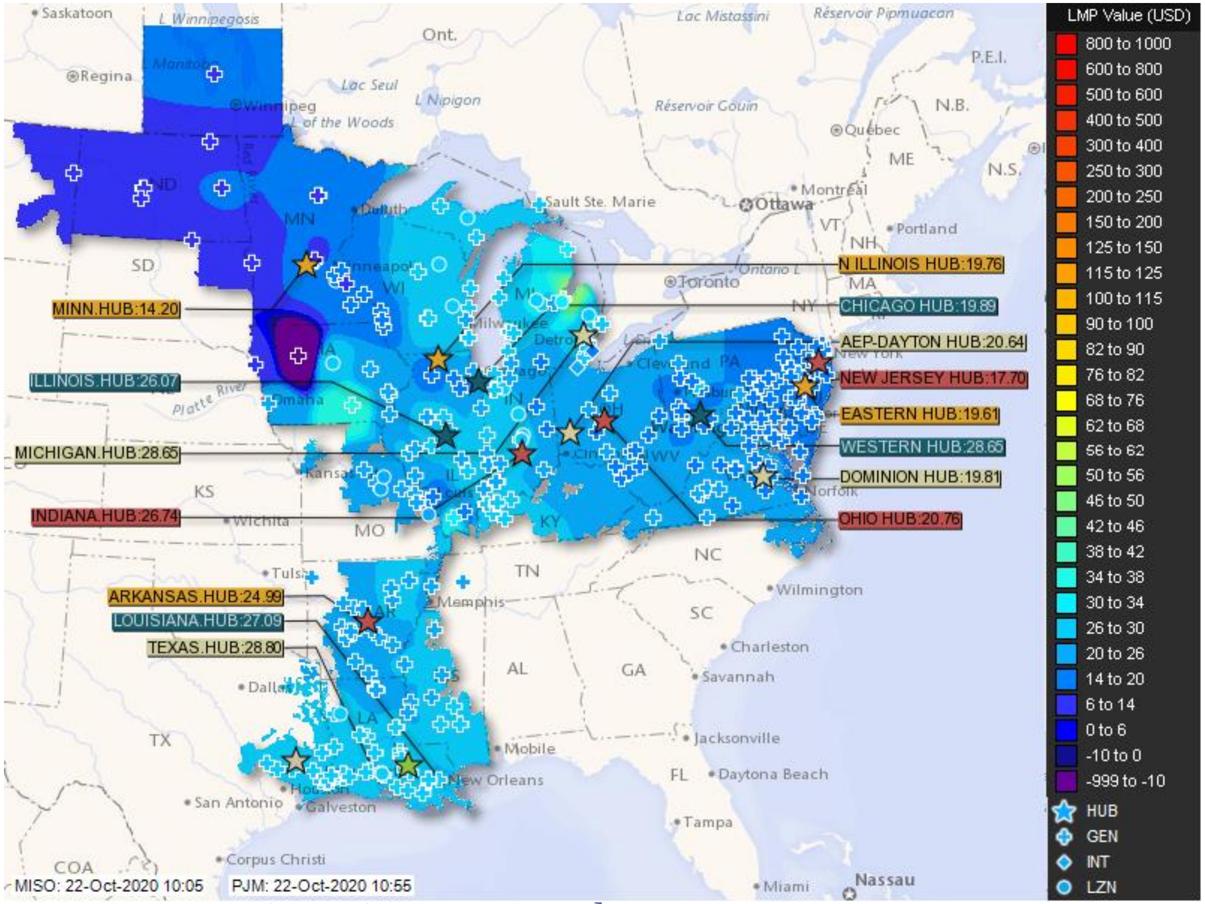
OPF and SCOPF History

- Critique of OPF Algorithms
  - W.F. Tinney, J.M. Bright, K.D. Demaree, B.A. Hughes, "Some Deficiencies in Optimal Power Flow," *IEEE Trans. Power Systems*, May 1988
- Hundreds of other papers on OPF
- Comparison of ac and dc optimal power flow methods
  - T.J. Overbye, X. Cheng, Y. San, "A Comparison of the AC and DC Power Flow Models for LMP Calculations," Proc. 37th Hawaii International Conf. on System Sciences, 2004

Key SCOPF Application: Locational Marginal Prices (LMPs)

- The locational marginal price (LMP) tells the cost of providing electricity to a given location (bus) in the system
- Concept introduced by Schweppe in 1985
  - F.C. Schweppe, M. Caramanis, R. Tabors, "Evaluation of Spot Price Based Electricity Rates," *IEEE Trans. Power App and Syst.*, July 1985
- LMPs are a direct result of an SCOPF, and are widely used in many electricity markets worldwide
  –

Example LMP Contour, 10/22/2020



LMPs are now widely visualized using color contours; the first use of LMP color contours was presented in [1]

https://www.miso-pjm.com/markets/contour-map.aspx

[1] T.J. Overbye, R.P. Klump, J.D. Weber, "A Virtual Environment for Interactive Visualization of Power System Economic and Security Information," IEEE PES 1999 Summer Meeting, Edmonton, AB, Canada, July 1999

OPF Problem Formulation

- The OPF is usually formulated as a minimization with equality and inequality constraints

$$\text{Minimize } F(\mathbf{x},\mathbf{u})$$

$$\mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

$$\mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where $\mathbf{x}$ is a vector of dependent variables (such as the bus voltage magnitudes and angles), $\mathbf{u}$ is a vector of the control variables, $F(\mathbf{x},\mathbf{u})$ is the scalar objective function, $\mathbf{g}$ is a set of equality constraints (e.g., the power balance equations) and $\mathbf{h}$ is a set of inequality constraints (such as line flows)

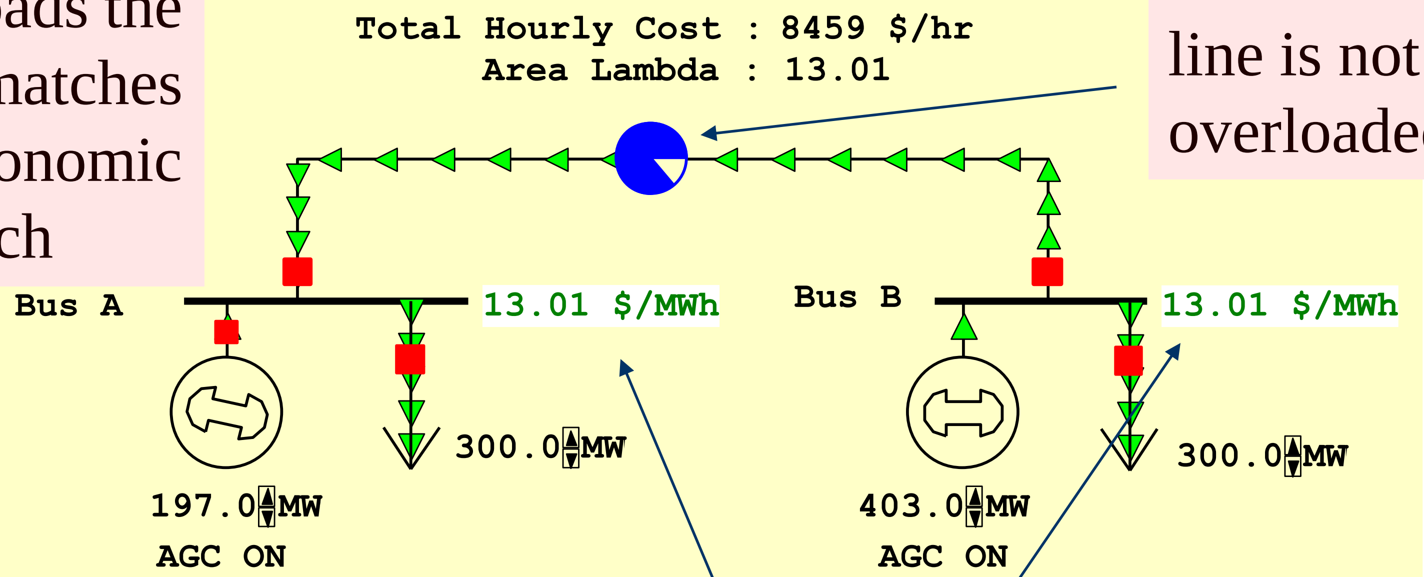
LP OPF Solution Method

- Solution iterates between
  - solving a full ac or dc power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

Two Bus with Unconstrained Line

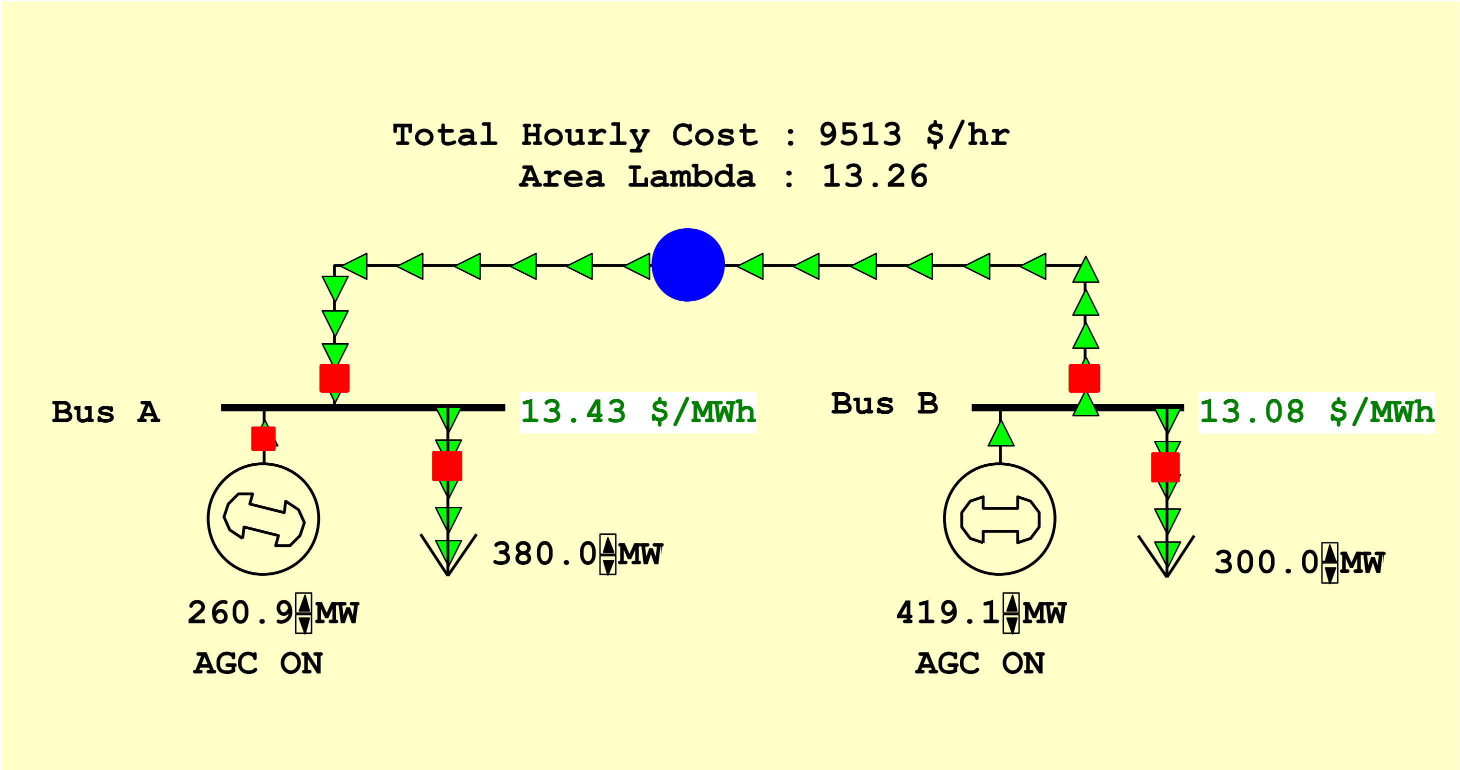
With no overloads the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

Two Bus with Constrained Line

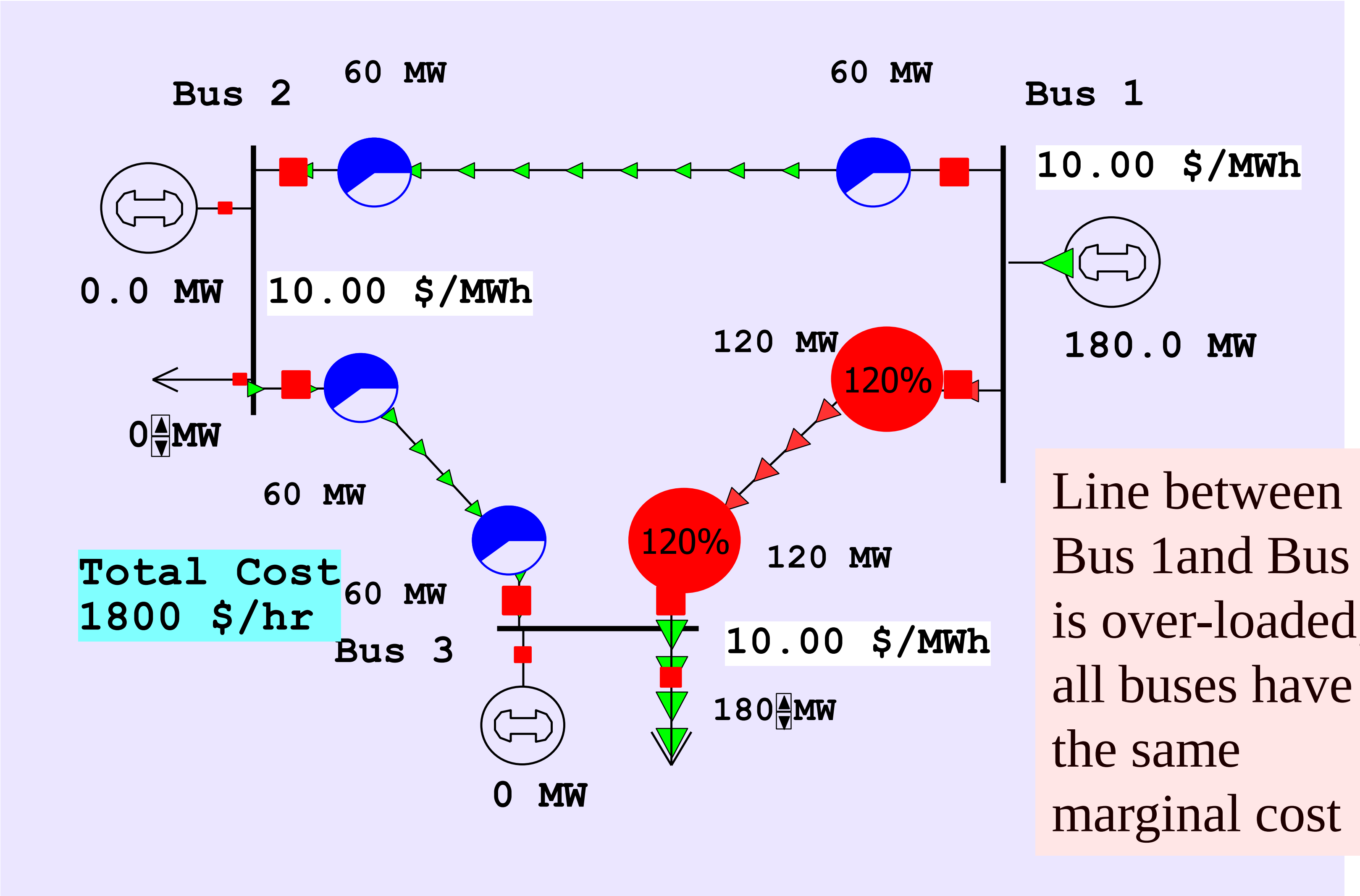


With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

Three Bus (B3) Example

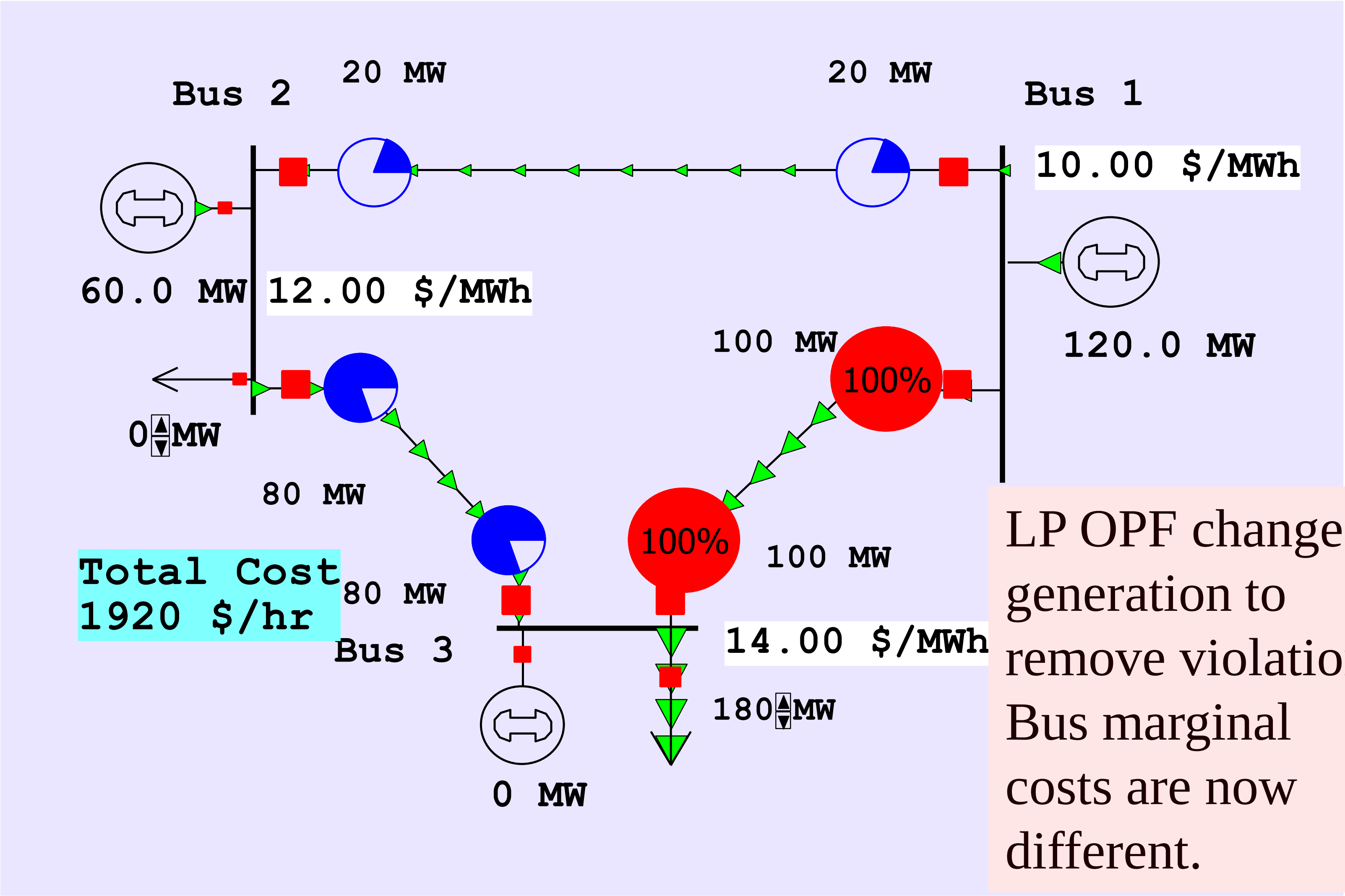
- Consider a three bus case (Bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

B3 with Line Limits NOT Enforced



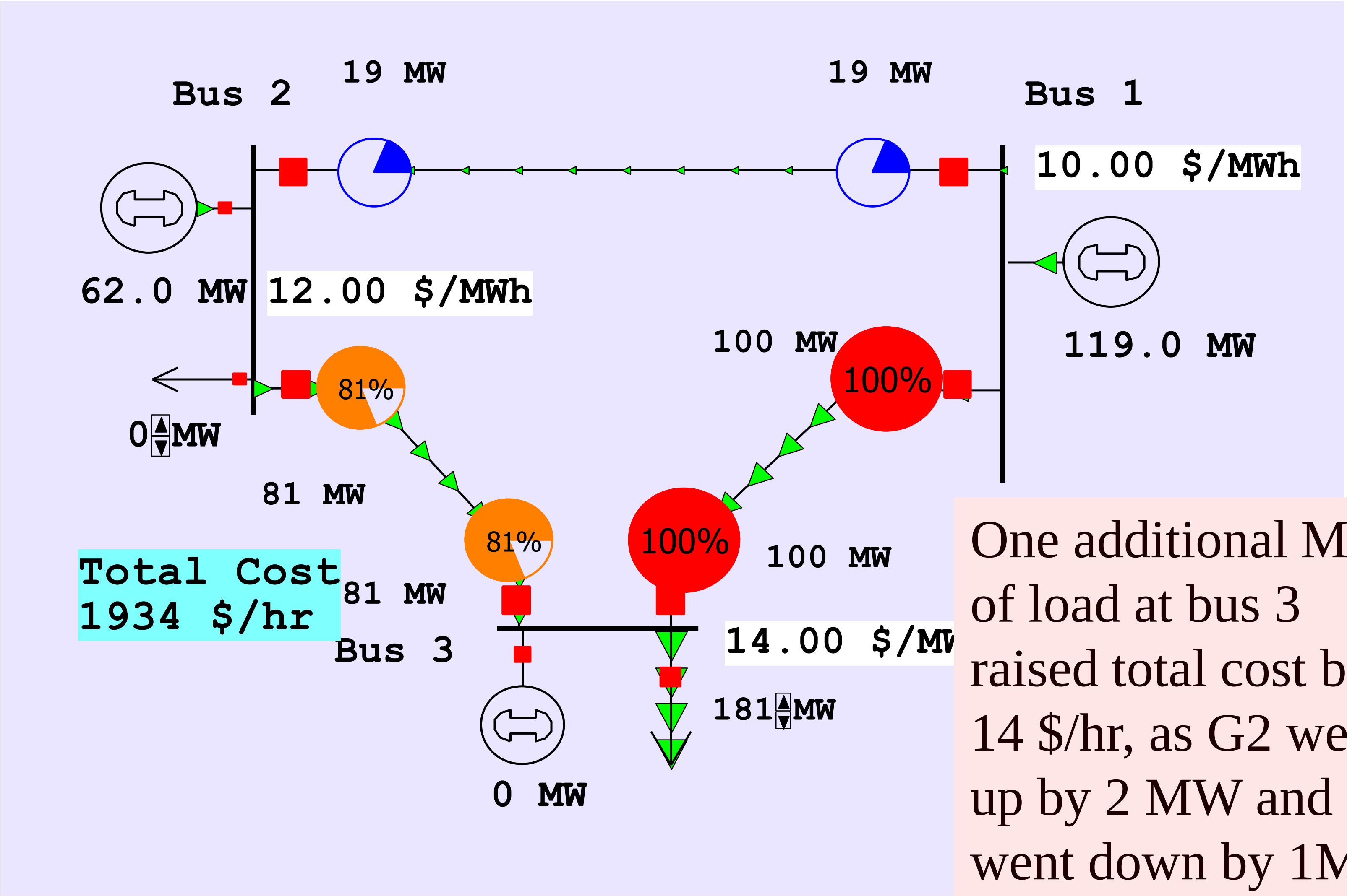
Line between Bus 1and Bus 3 is over-loaded; all buses have the same marginal cost

B3 with Line Limits Enforced



LP OPF changes generation to remove violation. Bus marginal costs are now different.

Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

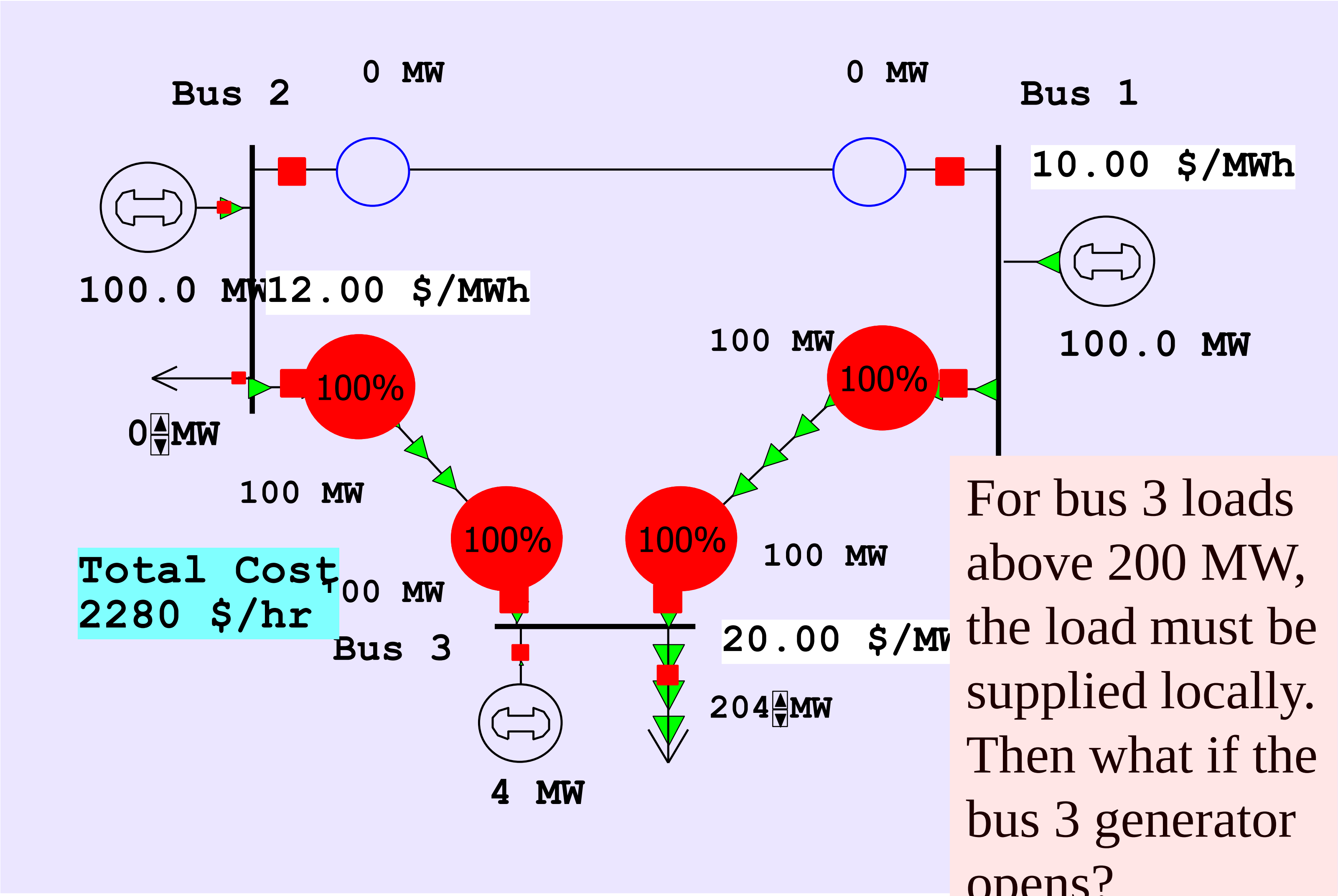
Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would “loop around” from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

Why is bus 3 LMP $ 14 / MWh, cont'd

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - $\Delta P_{G1} + \Delta P_{G2} = 1$ MW
  - $2/3\ \Delta\ P_{G1} + 1/3\ \Delta\ P_{G2} = 0$; (no more flow on 1-3)
- Solving requires we up P_{G2} by 2 MW and drop P_{G1} by 1 MW -- a net increase of $24 – $10 = $14.

Both lines into Bus 3 Congested



For bus 3 loads above 200 MW, the load must be supplied locally. Then what if the bus 3 generator opens?

Both lines into Bus 3 Congested