# Comparison of Dynamic Mode Decompositon and Iterative Matrix Pencil Method for Power System Modal Analysis

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***Abstract*—This paper compares two modal analysis techniques developed for handling large sets of data, Dynamic Mode Decomposition and the Iterative Matrix Pencil method. The two methods are compared by analyzing large-scale synthetic grids and using metrics such as the average cost function and computation time. Dynamic Mode Decomposition is shown to be able to quickly identify modes, but suffering inaccuracies in reproduced data. These difficulties are explored in various case studies and are compared to the Iterative Matrix Pencil method. Sampling frequency is also considered in order to examine the impact of larger data sets when comparing the two methodologies.**

## I. Introduction

With the existence of consistent load changes due to a variety of causes, the operating point of a power system primarily exists in what is known as a pseudo-steady state, where the operating point fluctuates within a small range [1]. However, this pseudo-steady state is also vulnerable to perturbations due to any number of causes, ranging from natural causes such as tornadoes and storms, to technical problems at various generators or lines in the system. Similarly, advancements in the grid such as the inclusion of more and more renewable energy sources has resulted in more oscillations being existent in the system [2]. Furthermore, the introduction of new technologies such as synchrophasors has allowed for more data to be widely available for analysis when considering the power grid [3]. As a result, modal analysis techniques exist in order to help gauge the small signal stability of systems, which measures how a system handles minor disturbances [4]. Proper mode identification allows for the characterization of the dynamics of the system, which in turn may lead to better control methods to maintain the oscillations within a reasonable range [4].



Signal-based modal analysis techniques seek to approximate a signal, $y(t)$, where:

$$y(t) \approx \sum_{i=1}^{M} R_i e^{z_i t} + n(t), \qquad 0 \leq t \leq T \tag{1}$$

where $M$ is the total number of modes that are used to represent the signal. Each mode $z_i$ is defined by a damping $b_i$ and angular frequency $\omega_i$. Each signal $y(t)$ also has a magnitude $|R_i|$ and phase $\theta_i$, which we refer to collectively as the mode shape. Finally, $n(t)$ represents some amount of noise that is associated with each signal.

There are a number of signal-based modal analysis techniques, and each one has its own pros and cons associated with them. One of the first techniques developed was Prony Analysis [5], and has been widely used since its inception. However, Prony Analysis was limited due to its inability to accurately determine modes under noise, and as such, newer modal analysis techniques were developed [6]. The Matrix Pencil Method was developed to estimate the modes of a system using eigenvalues and matrices [7], all of which were built from measurements taken from the system [6].

Despite being more robust relative to Prony Analysis, the Matrix Pencil Method determines the mode shapes of the system using a linear-least squares fit [8], which does not allow us to account for the non-linear nature of power systems. As such, modal analysis techniques such as the Variable Projection Method (VPM) have been used to project linear mode shapes into non-linear modes, and then solve the resulting non-linear optimization problem [9], [10].

The aforementioned techniques, however, do not quickly and efficiently handle large scale systems. In an era where quick and accurate modal analysis techniques have risen in demand [1], there have been several developments. Dynamic Mode Decomposition (DMD) has been established as a modal analysis technique whose origins lie primarily in the understanding of fluid flows [11]. Applications of DMD to large scale power grids have arisen in recent years, seeking to utilize the methodology in order to better understand the non-linearities of systems, especially those occurring after contingencies [1], [12].

Another technique that has been developed in handling the modal analysis of large scale systems is the Iterative Matrix Pencil (IMP) method [13]. The IMP method involves using the Matrix Pencil method for determining the modes, and using the cost function as a metric in order to determine the accuracy of the reproduced data [13]. The primary advantage of the method is that it can accurately replicate a given measured data set by only using a smaller, more impactful, set of data to determine the modes [13].

The rest of the paper will go as follows. Section II will discuss the IMP method and how the various values of interest will be calculated. Section III explains the basis for DMD and the underlying method behind the technique. Section IV will apply both the DMD and IMP method to large-scale synthetic grids, in an attempt to understand how each technique performs. Section V will consider how the sampling frequency of the data impacts the results of each method. Finally, Section VI will conclude the paper.

## II. Iterative Matrix Pencil Method

The IMP method utilize an iterative approach to reproducing a large set of signals $y(t)$, represented in the form seen in (1). Beginning with one signal, the IMP method seeks to approximate (1) with an equation of the form

$$\hat{y}(t) = d(t) + \sum_{i=1}^{M} |R_i| e^{b_i t} \cos(\omega_i t + \theta_i) \tag{2}$$

The primary difference between (1) and (2) is the existence of a detrend term, $d(t)$. Data detrending is an important aspect of modal analysis; without it, data may look as though there are fewer oscillations after a contingency, due to the scale of the oscillation relative to the scale of the data of interest [9]. As a result of this, data detrending will be considered on a case-by-case basis, in order to avoid instances where modes that are identified are associated with noise, rather than the oscillations.

After data detrending, the IMP method begins by selecting a single signal within all the other given signals. For example, if we are considering the frequencies of all the buses in a 2000 bus case, we would begin by selecting the signal from one bus. This signal is used to find the modes of the system through the Matrix Pencil method, described in Section II-A.

From the modes, the cost functions, which are described in Section II-B, are then calculated. These cost functions are a metric of fit; the higher the cost function, the worse the fit. Thus, the IMP method looks for the signal with the highest cost function, which indicates that the signal has the worst fit given the current modes. The signal with the highest cost function has information that is important to determining the behavior of the oscillations, but is not being properly represented in the current set of modes. As a result of this, the IMP method takes the the signal with the highest cost function, and uses it in conjunction to the prior signals in order to calculate a new set of modes.

This procedure is done iteratively; for any user-defined $m$ amount of iterations, the IMP will continuously follow through this procedure, including more and more signals. This iterative approach helps to minimize the amount of computation time needed to determine the modes, as we are only using a small subset of all the signals in the system.

### A. Matrix Pencil Method

The IMP method is dependent on the Matrix Pencil method, which begins with creating a Hankel Matrix, denoted by $[Y]$, which is constructed using measured data $y(t)$, as seen in (3). The Hankel matirx is an $(N-L) \times L$ matrix, where N is the number of sampled data points, and L is the pencil parameter. By setting the pencil parameter $L = N/2$, we note that our results approach the Cramer-Rao bound, which is a bound indicating that our result is the most accurate given a noisy environment [14].

$$[Y] = \begin{bmatrix} y(0) & y(1) & \dots & y(L) \\ y(1) & y(2) & \dots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & \dots & y(N-1) \end{bmatrix} \tag{3}$$

For multiple signals $y_k(t)$, a Hankel Matrix $[Y_k']$ is made for each signal, and then vertically concatenated into one large Hankel Matrix [9], which is denoted as $[Y']$, which can be seen in (4).

$$[Y'] = \begin{bmatrix} [Y_1'] \\ [Y_2'] \\ \vdots \\ [Y_n'] \end{bmatrix} \tag{4}$$

The Hankel Matrix may be broken up into two separate matrices, $[Y_2]$ and $[Y_1]$. These matrices are important, as the eigenvalues of the pencil, defined in (5), help to define our modes $z_i$, which in turn give the frequencies and dampings used in (2).

$$[Y_2] - \lambda [Y_1] \tag{5}$$

In order to get $[Y_1]$ and $[Y_2]$, the IMP method utilizes the singular value decomposition (SVD) of $[Y]$, in order to reduce the dimension of the data set. While larger power systems result in larger dimensional data sets, SVD helps to reduce the computational load that comes as a result of these larger data sets, while also maintaining patterns of interest. SVD results in three matrices; $[U]$, a unitary matrix containing the eigenvectors of $[Y][Y]^T$, $[S]$, a diagonal matrix containing the square roots of the eigenvalues of $[U]$ or $[V]$ in descending order, and $[V]$, another unitary matrix, but this time containing eigenvectors corresponding to $[Y]^T[Y]$.

The right singular matrix, $[V]$, is used to form two matrices, $[V_1]$ and $[V_2]$, which are then used to define $[Y_1]$ and $[Y_2]$. To form $[V_1]$ and $[V_2]$, we define a SVD threshold, $q$, which acts as a metric to define which singular values are associated with noise, versus being associated with the actual oscillations of the signal. Beginning with $[S]$, we consider the ratio of all singular values relative to the largest singular value, and compare that to $q$. If the ratio falls below $q$, we remove the singular value from consideration. Thus, we generate two separate matrices, $[V_1]$ and $[V_2]$, defined in (6).

$$[V_1] = \begin{bmatrix} v_1 & v_2 & v_3 & \dots & v_{q-1} \end{bmatrix}$$
$$[V_2] = \begin{bmatrix} v_2 & v_3 & v_4 & \dots & v_q \end{bmatrix} \quad (6)$$

We may use the definitions in (6) in order to define our matrices $[Y_1]$ and $[Y_2]$ as the following.

$$[Y_1] = [V_1]^T[V_1] \qquad [Y_2] = [V_2]^T[V_1] \quad (7)$$

The eigenvalues of the matrix pair $\{[Y_2], [Y_1]\}$ are the poles $z_i$ of the system, which may be further used to calculate the damping and angular frequency associated with each mode. This is done by finding the eigenvalues of the system, $\lambda_i$, by converting $z_i$ to $\lambda_i$ through (8), where $\Delta t$ is the spacing between each time point in the input data set.

$$z_i = e^{\lambda_i \Delta t}, \quad \text{Where: } \lambda_i = \frac{\ln z_i}{\Delta t} \quad (8)$$

Finally, in order to calculate the mode shapes, which give us the amplitude $|R_i|$ and phase $\theta_i$, we utilize the matrix equation $Z = RY$, which can be expanded in (9).

$$\begin{bmatrix} z_1^0 & z_2^0 & \dots & z_M^0 \\ z_1^1 & z_2^1 & \dots & z_M^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} \quad (9)$$

### B. Cost Function

The cost function is a metric that is used to directly measure the quality of the reproduced data relative to the actual data. Defined in (10), we measure the difference between the actual data $y(t)$ and the reproduced data $\hat{y}(t)$, and utilize a Euclidean normalization of the residuals.

$$c(t) = \frac{1}{2}||r(t)||_2^2 = \frac{1}{2}||y(t) - \hat{y}(t)||_2^2 \quad (10)$$

The primary benefit of the cost function is its computation time; by calculating all of the time cost functions for each signal, the IMP method is able to identify which signals to add in each iteration, resulting in a smaller subset of signals to analyze relative to the larger system.

## III. Dynamic Mode Decomposition

DMD was designed with the intent to quickly extract modes from large sets of data, and reproduces it in the form seen in (11).

$$\hat{y}_m[n] = u_m^T \sum_{i=1}^{M} e_i \alpha_i \mu_i^n \quad (11)$$

Where $m$ is the signal of interest, $n$ is the current time point, and $M$ is the number of modes. We also note that $u_m$ is the associated row of the left singular matrix $[U]$ that comes as a result of SVD of $[Y_0]$, $e_i$ is the eigenvector associated with the current mode, $\alpha_i$ is the DMD amplitude, and $\mu_i$ is the eigenvalue associated with the current mode.

One of the underlying assumptions of DMD is an approximate linear mapping $A$ between two consecutive points for the entire time window [11]. This means that for any signal $Y_k$ in a time window, we may express the measurements in the form seen in (12).

$$y_{k,n} \approx A y_{k,n-1} \implies Y_k = \{y_1, Ay_1, A^2 y_1, \dots, A^{N-1} y_1\} \quad (12)$$

Thus, we may understand the dynamics of any system by looking at $A$. In order to approximate $A$, we split the data into two separate matrices, $[Y_0]$ and $[Y_1]$, which are defined in (13). These matrices may be assumed to have an approximate linear mapping $A$ between the two matrices, with a residual error. However, much like in the IMP method, we utilize a SVD of $[Y_0]$ in order to both project the matrix into a lower dimension that is computationally simpler to handle and filter the singular values to avoid modes that are associated with noise. The filtering occurs in the same manner as the IMP method, where given a user-determined SVD threshold $q$, we only select singular values whose ratios relative to the largest singular value are greater than the threshold.

$$[Y_0] = \begin{bmatrix} y_0 & y_1 & y_2 & \dots & y_{N-1} \end{bmatrix}$$
$$[Y_1] = \begin{bmatrix} y_1 & y_2 & y_3 & \dots & y_N \end{bmatrix} \quad (13)$$

The resulting $[U]$, $[S]$, and $[V]$ matrices from SVD are used in conjunction with $[Y_1]$ to define an approximation of A, $\tilde{A}$, as seen in (14). This is done to increase the robustness of the methodology for handling data with noise and other contaminants [11].

$$\tilde{A} = [U]^*[Y_1][V][S]^{-1} \quad (14)$$

The eigenvalues $\mu_i$ and eigenvectors $e_i$ of $\tilde{A}$ are found via an eigendecomposition of $\tilde{A}$, and are used in (11) to reproduce data. However, in order to fully reconstruct the data from the modes found via DMD, we must also determine the amplitude, $\alpha_i$. This amplitude may be calculated through (15). DMD does utilize a least-squares optimization approach, so the values for both the eigenvalues $\mu_i$ and the amplitudes $\alpha_i$ are considered optimal through the minimization of the residue between the measured data and the reconstructed data [15].

$$\alpha = E^{-1}[S][V]^H \quad (15)$$

It is important to note that DMD reconstructs the signal $y(t)$ in terms of discrete-time eigenvalues, whereas the IMP method reconstructs the signal $y(t)$ using a continuous time format. However, we may convert these values into the same continuous time eigenvalues used in the IMP method through (16), where $\Delta t$ is the time separation between any two points, where all the data for DMD is uniformly sampled.

$$\lambda_i = \frac{\ln(\mu_i)}{\Delta t} = \frac{\ln|\mu_i| + j\angle\mu_i}{\Delta t} \quad (16)$$

In order to provide a more direct comparison between DMD and the IMP method, we will consider how DMD calculates

TABLE I
2000 BUS CASE COST FUNCTION

| | DMD | IMP # of Iterations | | |
|---|---|---|---|---|
| | | 1 | 10 | 20 |
| Avg $c(t)$ | 0.0166 | 0.0068 | 0.0023 | 0.0022 |
| Max $c(t)$ | 0.0207 | 0.0138 | 0.0042 | 0.0048 |
| Min $c(t)$ | 0.0131 | 0.0026 | 0.0011 | 0.001 |
| t (s) | 8.3 | 4.58 | 12.75 | 24.56 |

TABLE II
10,000 BUS CASE COST FUNCTION

| | DMD | IMP # of Iterations | | |
|---|---|---|---|---|
| | | 1 | 10 | 20 |
| Average $c(t)$ | 0.0256 | 0.0119 | 0.0028 | 0.0024 |
| Max $c(t)$ | 0.0410 | 0.0324 | 0.0049 | 0.0079 |
| Min $c(t)$ | 0.0214 | 0.0012 | 0.0018 | 0.0010 |
| t (s) | 36.68 | 19.92 | 58.50 | 89.39 |

the continuous time eigenvalues $\lambda_i$, and how effectively it recreates the data using these modes. The mode shapes may be calculated using the same method as (9).

## IV. CASE STUDIES

We apply both DMD and the IMP method to two different case studies. These case studies are synthetic power networks, which represent properties of actual large-scale power systems, but do not reveal any information that is considered confidential [16]–[19]. We begin with a 2000 bus case, which is based on the footprint of Texas, and transition to a 10,000 bus case, which is representative of the western United States. Nonlinearity in the system is already considered when generating data from these synthetic grids.

We compare DMD and IMP based on several statistics. The average cost function is a measure of the overall fit of any reproduced data, and gives a gauge on how well the two methods reproduce all the signals in a large system. The maximum and minimum cost function indicate the worst and best fits using the methods, allowing us to not only see how well the overall system is reproduced, but how strong or how poor the reproduced data is for the best and worst cases, respectively. Finally, because DMD has been considered a modal analysis technique which quickly extracts information from a set of input data, we also look at how fast each method is at computing the modes.

### *A. 2000 Bus Case*

In the 2000 bus simulation, the contingency that we use is a generator opening after one second. The frequency at each bus is measured for a total of 20s, at a sampling rate of 5 Hz. Each signal is linearly detrended, and scaled by a factor of its standard deviation. For consistency, the SVD threshold being considered for all cases is 0.025.

The results in Table I show a few trends of interest. First and foremost, we note that DMD does not outperform the IMP method in regards to accuracy, across any of the three metrics considered. Even at one iteration, the average cost function of the signals after one iteration in the IMP method is 59.03% better than that of DMD. This discrepancy in accuracy is exacerbated further when considering comparisons to the 10 and 20 iteration cases, with differences in the average cost function being 86.14% and 86.75%, respectively.

When considering the best and worst cases for the techniques, DMD again does not show any improvements over the IMP method. At 1 iteration, the IMP method has a 33.33% better max cost function when compared to the max cost function of DMD, and this percentage increases to 79.71% at 10 iterations.

However, while DMD suffers in accuracy, it still maintains higher speeds when compared to the IMP method at larger iterations. DMD is slower than one iteration of the IMP method by 44.82%, but is faster than the IMP method at 10 iterations by 53.61%, and 20 iterations by a much larger 195.90%.

### *B. 10,000 Bus Case*

Similar to the 2000 bus system, the 10,000 bus system involves a generator being opened at 1s as the contingency, while measuring the frequency at every bus for a duration of 20s. There is a linear detrend in this case, with the standard deviation being used to scale the data. The SVD threshold is again set to be 0.025.

Table II shows many of the same traits as seen in the 2000 bus case. When considering the average cost function of each modal analysis technique, we see that DMD again does not have a lower average cost function when compared to the IMP method at 1, 10, or 20 iterations. There is a difference of 53.52% for the average cost functions between DMD and the IMP method at 1 iteration, while there is an 89.06% difference when we increase the number of iterations in the IMP method to 10.

We may also visualize the results of the modal analysis techniques by plotting the reproduced data, and comparing it to the original data. Figures 1 and 2 show the minimum cost function for the IMP method and DMD, respectively. When looking at the results of the IMP method, we note there are almost no discrepancies between the original data and the reproduced data. However, when considering the results of DMD, we see that the reproduced data initially matches the original data, but then begins to diverge at about 5s.

Furthermore, contours may be used to visualize how the modal analysis techniques represent the entire grid [20]. Figure 3 shows a contour of the 10 iteration IMP method, with a color scale of [0,0.005]. The overall cohesive nature of the contour shows the efficacy of the IMP method when it comes to matching the original signals of the system. However, when considering DMD, we note that even the minimum cost function is larger than the maximum of the IMP method, meaning the entire contour would show nothing but red, based on the color scale.

While DMD may be able to determine a set of modes quickly from a large set of data, the assumption of a linear mapping between consecutive points is shown to break down

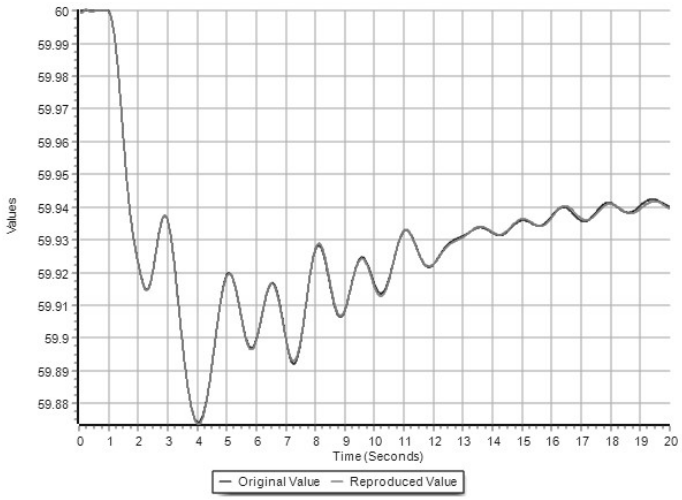


Fig. 1. Minimum $c(t)$ using IMP for 10k Case

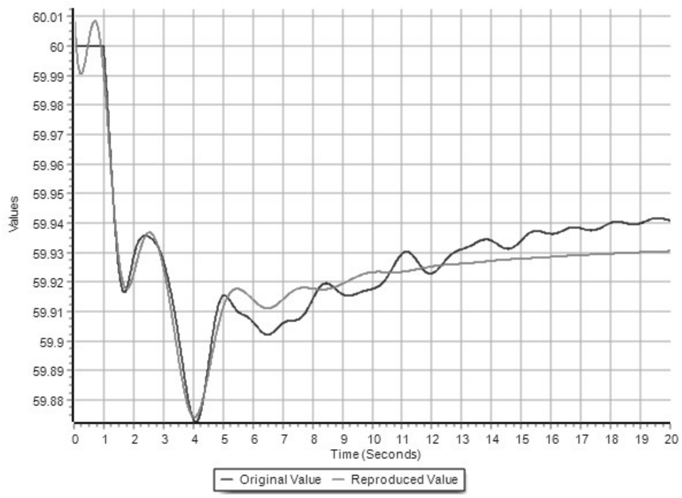


Fig. 2. Minimum $c(t)$ using DMD for 10k Case

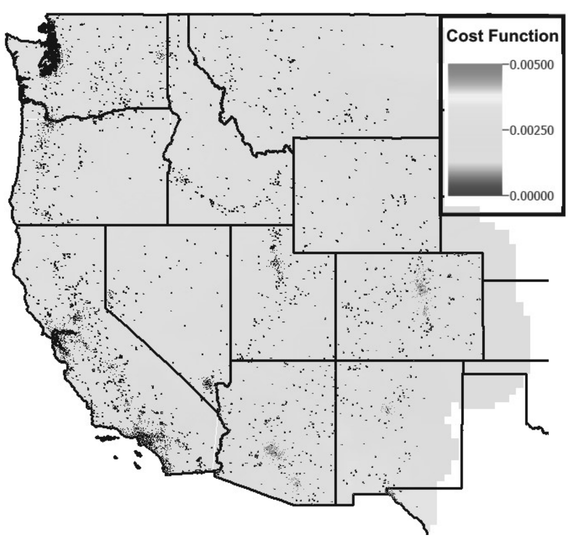


Fig. 3. Contour of $c(t)$ using IMP for 10k Case

TABLE III
RESULTS FOR DMD AND IMP AT DIFFERENT SAMPLING FREQUENCIES

| | | Sampling Frequency | | |
|---|---|---|---|---|
| | | 5 Hz | 10 Hz | 15 Hz |
| DMD | Avg $c(t)$ | 0.0256 | 0.0165 | 0.0129 |
| | Max $c(t)$ | 0.041 | 0.0281 | 0.0233 |
| | Min $c(t)$ | 0.0214 | 0.0124 | 0.0085 |
| | t (s) | 36.68 | 112.89 | 245.64 |
| IMP | Avg $c(t)$ | 0.0028 | 0.0024 | 0.0019 |
| | Max $c(t)$ | 0.0049 | 0.0038 | 0.0033 |
| | Min $c(t)$ | 0.0018 | 0.0016 | 0.0014 |
| | t (s) | 58.5 | 111.34 | 215.74 |

as larger networks with more non-linearities are considered. The IMP method manages to compensate for any linear assumptions by continuously optimizing the guess of the modes through the inclusion of other signals per each iteration. This result shows an important trade-off when considering modal analysis techniques; being able to identify modes from large sets of data faster will result in less accurate results.

## V. SAMPLING FREQUENCY

While our case studies have shown that the IMP method generally outperforms DMD as a modal analysis technique in terms of accuracy, it is important to note that DMD still does maintain quick mode identification in comparison. In order to compare the two techniques further, we test the sensitivity of the two approaches to two separate factors; the SVD threshold, which we used to determine how many singular values to keep, and the sampling frequency, which determines the number of points that are being measured.

Table III shows the results of running DMD and the IMP method, at 10 iterations, at different sampling frequencies for the 10,000 bus case. These results show similar trends to the case studies, but also provides some contrasting data. In particular, as we increase the sampling frequency from 5 Hz to 10 Hz, there is a decrease in the average cost function for DMD of approximately 31.46%, while for the IMP method, we see a decrease of only 22.45%. However, while DMD does improve its accuracy by a larger percentage when compared to the IMP method, we see that there is an increase in computation time of 76.21s, while the IMP method only takes 52.84s longer at 10 Hz when compared to 5 Hz.

Furthermore, if we increase the sampling frequency to 15 Hz, we see that the average cost function for DMD and the IMP method decrease by 21.82% and 20.83%, respectively. This indicates that even at higher frequencies, the rate at which the two techniques improve their reproduce approaches the same value. At 15 Hz, the IMP method also has a faster computation time, calculating the modes 12.17% faster than DMD does.

While we have visualized the differences in the best fits between the two techniques in Figures 1 and 2, we may also use the worst fits as a metric of comparison at a higher sampling frequency. Figures 4 and 5 show the maximum cost function at 15 Hz for the IMP method and DMD, respectively. The IMP method shows very little deviation between the

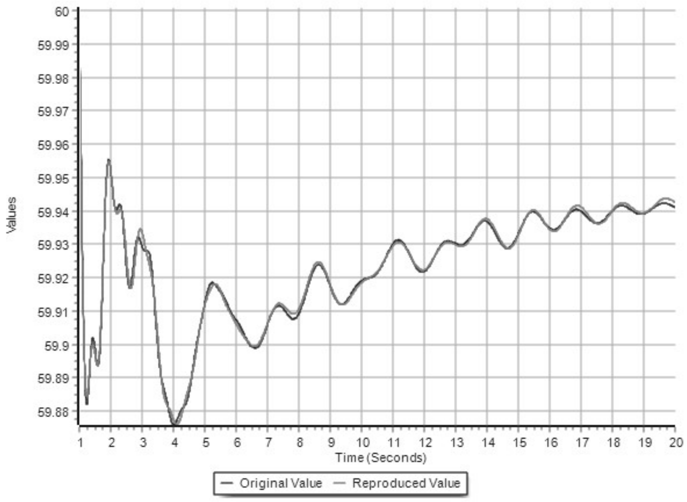


Fig. 4. Maximum $c(t)$ using IMP at 15 Hz

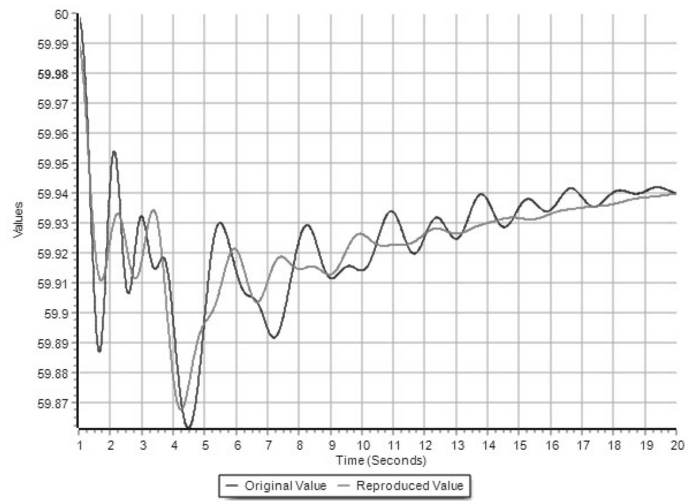


Fig. 5. Maximum $c(t)$ using DMD at 15 Hz

original and reproduced data, while the reproduced data from DMD shows a similar trend in oscillation, but with varying magnitudes when compared to the original data.

## VI. Conclusion

One of the first techniques introduced to handle modal analysis for large power systems was Dynamic Mode Decomposition, which specialized in the quick calculation of modes, but at the cost of the accuracy of the reproduced data. The Iterative Matrix Pencil method, while only being recently applied to large power systems, has shown considerable accuracy and speed when handling large-scale synthetic grids.

The primary advantages of the Iterative Matrix Pencil method may be split into two main concepts; the number of signals required to accurately calculate the modes, and the speed at which the modes are calculated. Because the IMP method begins with one signal, and is constrained by a user-determined number of iterations, the number of signals required to calculate the modes is low, relative to the size of the system. Results found here and previous research [13] show that accurate reproduction of data may be performed with the use of only 10-15 signals. As an extension of this, because the IMP method requires so few signals, the time needed to calculate the modes decreases as a direct result.

Further work may seek to compare the IMP method and DMD in situations that do not involve synthetic grids; applying both techniques to data that is of lesser quality due to missing information or more noise may be tested. Applications of these results include using modal analysis techniques to help identify and characterize the oscillations that occur as a result of natural disasters, along with allowing for adjustments of remedial controls in large power systems.

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