Sensitivity of Modes from Modal Analysis of Electric Grids

Wei Trinh, Thomas J Overbye
Department of Electrical and Computer Engineering
Texas A& M University, College Station, TX, USA
{weit1, overbye}@ tamu.edu

***Abstract*—A sensitivity analysis is applied to the mode and mode shape calculations of the Matrix Pencil method, in order to measure the robustness of the results. The methodology has been proven to work quickly and accurately when identifying modes, but there has not been testing regarding the ability of the MP method to operate under different time windows and sampling frequencies. A variety of signals are used for testing, including both signals with known and unknown modes in conjunction with the moving time windows and sampling frequencies.**

I. Introduction

Modal analysis techniques exist as a tool used to analyze signals from systems, and break them down into their constituent components for further analysis, such as examining the small signal stability of systems [1]. In the North American power grid, the frequency is intended to stay close to 60 Hz, but due to constantly fluctuating loads, it maintains a pseudo-steady state, where the operating point fluctuates [2]. However, it is also vulnerable to perturbations from larger sources of oscillations, such as generators tripping and demand spiking.

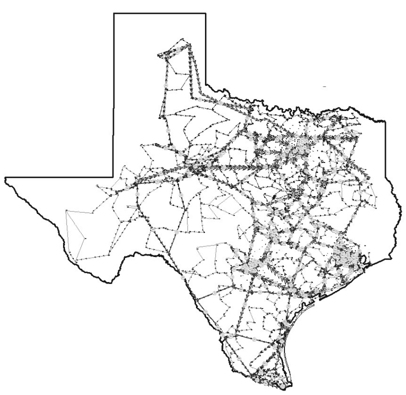

Fig. 1. Synthetic Texas Grid [3]

Large, interconnected grids, like the one seen in Figure 1, have many avenues in which instability may be introduced. In both post-event analysis and real time operation, consistency in results is key. Thus, it is important to have accurate and robust analysis techniques, especially in situations where they are used to design grids that are secure against voltage stability constraints [4] or when developing control methods for maintaining stable grid operation [1].

While modal analysis techniques may be split into either model-based or signal-based approaches, with the advent of PMUs and large amounts of data being produced for large systems, model-based modal analysis is difficult to implement, given the amount of components necessary to consider. As such, this work will focus on signal-based modal analysis, looking at the Matrix Pencil (MP) method. As the name entails, the MP method utilizes eigenvalues and matrix analysis techniques to estimate the modes of a system [5].

More generally, the MP method looks to approximate a signal, denoted by $y(t)$, where:

$$y(t) \approx \sum_{i=1}^{M} R_i e^{z_i t} + n(t) \qquad 0 \leq t \leq T \tag{1}$$

Here, z_i is a mode of the system, associated with a damping σ_i and angular frequency ω_i. There are M modes in the system, and each signal has a corresponding mode shape, which consists of a magnitude $|R_i|$ and phase angle θ_i. Finally, there is some noise associated with each signal, which is denoted by $n(t)$. In order to approximate $y(t)$, the MP method generates a similar looking function, defined as $\hat{y}(t)$, where:

$$\hat{y}(t) = d(t) + \sum_{i=1}^{M} |R_i| e^{\sigma_i t} \cos(\omega_i t + \theta_i) \tag{2}$$

Here, the primary differentiation made between Equations (1) and (2) is the $d(t)$ term, which refers to the detrending term.

Sensitivity of modal analysis results can be found in a variety of work; [6] looks at the sensitivity of resonant modes with respect to different network components, which are then used to identify and efficiently mitigate problems that arise in the grid. [4] uses the sensitivity of voltage to reactive power injections as a measure of voltage stability or instability.

The MP method and its applications in power systems has been discussed in previous works [7], [8], and has been proven to be better than existing techniques like Prony analysis at handling data with more noise [9]. It has also been integrated into new power system modal analysis tools [10], along with being extended into techniques developed to handle large-scale synthetic networks [11].

However, these applications of the MP method all operate under a similar assumption that the input data to the method is the result of a transient stability study, or a full signal over an entire intended time frame. In real time grid operation, this is a type of luxury that is not always feasible, and as such, it is important to test the MP method against smaller and potentially more dynamic time windows and sampling frequencies to ensure there are consistent results.

The rest of the paper will go as follows; Section II discusses the approach and types of data used to test the MP method. Section III shows the results of the analysis, along with a discussion regarding the implication of said results. Section IV will conclude the paper, and discuss further work.

II. Methodology

A. The Matrix Pencil Method

In order to understand what exactly is being tested, an understanding of the MP method must be established. A full mathematical description of the technique can be found in [5], but a brief description of the technique is as follows.

As mentioned in Section I, given an input signal of form $y(t)$, the MP method looks to make a reconstruction of the signal, $\hat{y}(t)$, whose form is given in (2). This begins with the construction of what is known as a Hankel matrix. Denoted as $[Y]$, the Hankel matrix is defined in (3), and is constructed using the input data $y(t)$. The matrix has two variables of note; N,which represents the number of sampled data points, and L, which is known as the pencil parameter. The pencil parameter is a tune-able parameter which is utilized to mitigate the effects of noise, but results in [8] indicate that the most optimal value for L is $N/2$, which results in the performance of the technique approaching the Cramer-Rao bound. It should also be noted that for multiple signals, a Hankel matrix is generated for each one, and vertically concatenated.

$$[Y] = \begin{bmatrix} y(0) & y(1) & \dots & y(L) \\ y(1) & y(2) & \dots & y(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(N-L-1) & y(N-L) & \dots & y(N-1) \end{bmatrix} \quad (3)$$

The term ”pencil” in the MP method refers to the matrix pair $([Y_1], [Y_2])$, whose eigenvalues correspond to the modes of the system, z_i. The definitions for $[Y_1]$ and $[Y_2]$ are generated using Singular Value Decomposition, or SVD for short. SVD as a technique exists primarily to reduce the dimension of the a data set while maintaining the behaviors and trends of the higher dimensional data set. This helps to alleviate computational burden while preserving the accuracy of analysis applied to the resultant matrix.

Mathematically, SVD results in three matrices; $[U]$, $[S]$, and $[V]$. The matrices $[U]$ and $[V]$ contain the eigenvectors for $[Y][Y]^T$ and $[Y]^T[Y]$, respectively. On the other hand, $[S]$ is a diagonal matrix comprised of the square roots of the eigenvalues of $[Y]$, which are known as singular values. In order to mitigate the effects of noise, a SVD threshold, denoted as q, is used. Because $[S]$ orders the singular values in descending order, the ratio of each singular value with respect to the largest one is considered. If any ratio falls beneath q, the corresponding singular value is removed from consideration. This helps to filter out singular values associated with noise rather than actual oscillations of concern. The results of SVD after the threshold filtering are used to generate two matrices, $[V_1]$ and $[V_2]$, whose definitions are seen in (4).

$$[V_1] = \begin{bmatrix} v_1 & v_2 & v_3 & \dots & v_{q-1} \end{bmatrix}$$
$$[V_2] = \begin{bmatrix} v_2 & v_3 & v_4 & \dots & v_q \end{bmatrix} \quad (4)$$

The matrices $[V_1]$ and $[V_2]$ are then used to construct $[Y_1]$ and $[Y_2]$, defined in (5).

$$[Y_1] = [V_1]^T[V_1] \qquad [Y_2] = [V_2]^T[V_1] \quad (5)$$

The eigenvalues of the matrix pair $([Y_1], [Y_2])$ correspond to the modes of the system, and are denoted as z_i. In order to get the damping and frequency of the mode, z_i is converted to λ_i using (6), where $\lambda_i = \sigma_i + j\omega_i$. Here, Δt refers to the spacing between the data points.

$$z_i = e^{\lambda_i \Delta t} \implies \lambda_i = \frac{\ln(z_i)}{\Delta t} \quad (6)$$

The final component that needs to be considered are the mode shapes. These can be quickly solved for using a simple matrix equation $ZR = Y$, which is expanded upon in (7).

$$\begin{bmatrix} z_1^0 & z_2^0 & \dots & z_M^0 \\ z_1^1 & z_2^1 & \dots & z_M^1 \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix} = \begin{bmatrix} y(0) \\ y(1) \\ \vdots \\ y(N-1) \end{bmatrix} \quad (7)$$

B. Robust Result Testing

As mentioned in Section I, the input data, which we will denote as Y, in all prior work, has been the full results from a transient stability study, or is a signal over an entire intended time frame. In order to test the consistency of the MP method against different time frames, the first data set used will be the output from a single signal with known frequencies and dampings. The MP method will then be applied to this signal over varying time frames and sampling frequencies, in order to measure the consistency of the results. These are the types of signals that are more common in the grid, as the oscillations seen in standard operation and extreme events are not known to be characterized by singular sine or cosine functions. It is important to note here that the frequencies and dampings of the signal are flexible, and will be tested with various combinations of high and low frequencies, along with different dampings that result in slower or quicker oscillation decaying.

This approach is then extended to two signals with relatively close frequencies and dampings, in order to see whether or not the same behaviors seen with one signal are consistent with multiple signals. Finally, we apply the MP method with the same varying time frames and sampling frequencies to a simplified power system network, again in an attempt to observe if there are similar trends as before.

In order to measure consistency, there are a few metrics that can be used. First, the cost functions of the signals may be used, in order to see whether or not our variations in input data will result in more or less accurate results. The cost function $c(t)$ is a direct measure of accuracy of the reproduced signal $\hat{y}(t)$ against the original signal $y(t)$, and is defined in Equation 8.

$$c(t) = \frac{1}{2}||r(t)||_2^2 = \frac{1}{2}||y(t) - \hat{y}(t)||_2^2 \quad (8)$$

In the single and double signal cases, since the modes are known, we may also gauge the accuracy simply by observing the modes that are a result from the MP method, and comparing them against what the expected values are. For moving time windows, the evolution of the modes will be observed. In particular, since the dampings are fixed, as the window moves further along in the signal, modes with higher damping should tend to 0 faster than modes with lower damping.

Finally, this methodology will be extended to example systems; while variations were built on the MP method to handle large systems [11], this work will be applying the MP method on smaller grids, in order to more effectively measure the ability of the MP method to accurately reproduce signals without concerns of overloading the technique with large quantities of data it was not originally developed to manage. More specifically, work will be performed on the WSCC 9-Bus System, an equivalent system that approximates the Western System Coordinating Council (WSCC) with nine buses and 3 generators [12].

III. Results

A. Sampling Frequency

Sampling frequency refers to how often the input data to the MP method is sampled, which affects the resolution of the data. Higher sampling frequencies indicate higher amounts of data being taken in. While this would lead to more accurate results, there are also trade-offs in computational speed. As such, testing the MP method under differing sampling frequencies allows a glimpse into whether or not the technique is capable of producing consistent results despite differences in the input data.

1) 1 Signal Example: We start by generating a signal with four known frequencies and different dampings. The set modes are given in Table I.

TABLE I
Fixed Modes for 1 Signal

Mode	Frequency (Hz)	σ
1	0.25	0.5
2	0.4	0.5
3	0.6	0.05
4	0.8	0.05

Creating a signal with a frequency of 30 Hz over a 30 second simulation results in the signal seen in Figure 2. The goal is to compare results from different sampling frequencies, and testing to see if there are any significant differences (either improvements or failures), between the different sampling frequencies. The two resampling frequencies tested will be

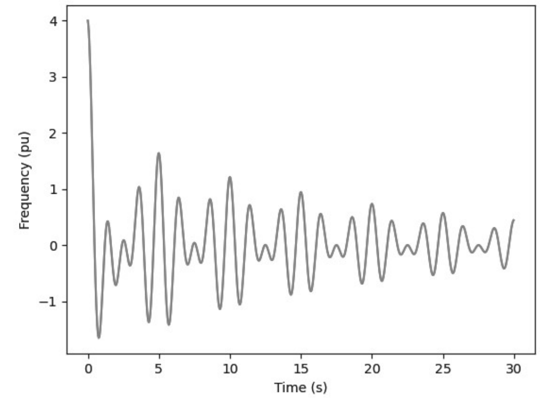

Fig. 2. Combined Signal with 4 Different Modes

at 20 Hz and 10 Hz. The results of the MP method applied at these frequencies, compared to the original data, can be seen in Table II.

From a preliminary glance, it's seen that the three different sampling frequencies all produce similar results, although the speed of computation for higher sampling frequencies is obviously higher due to there being more data to process. Furthermore, there are slight differences in the cost functions across the different sampling frequencies. This can be attributed again to the larger quantity of data being processed at 30 Hz when compared to 20 or 10 Hz, but differences in results are minor.

This can be further seen by looking at the reproduced results in graphical format; Figure 3 shows the different reproductions for each sampling frequency. As can be seen, for all cases, most of the signal is being very well matched, with some minor deviations towards the end when using the 30 Hz data, which is reflected in the cost functions shown in Table II.

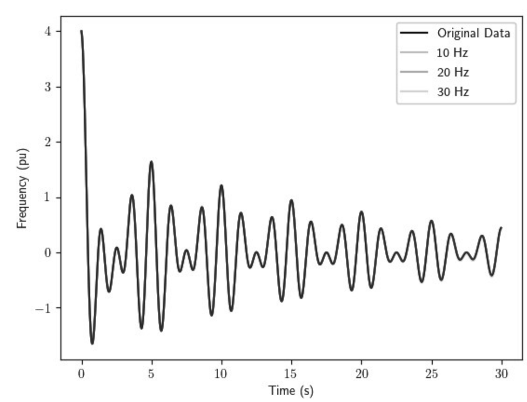

Fig. 3. Reproduced Data vs Original Data for Different Sampling Frequencies in One Signal Approach

2) 2 Signal Example: Now, the focus shifts to examining multiple signals. In this case, two signals are generated, with very similar frequencies. The goal here is to check whether or not the consistencies observed in the 1 signal example still hold with two signals of relatively close frequencies.

TABLE II
SAMPLING FREQUENCY RESULTS FOR ONE SIGNAL

	30 Hz					20 Hz					10 Hz				
Modes (Hz)	0.8	0.599	0.396	0.25	0	0.8	0.6	0.4	0.25	0	0.8	0.6	0.4	0.25	0
Damping %	0.997	1.329	20.948	27.956	100	0.995	1.326	19.512	30.331	100	0.995	1.326	19.512	30.331	100
Cost Function	0.0059					0					0				
Time Taken (s)	0.165					0.067					0.028				

TABLE III
FIXED MODES FOR 2 SIGNALS

	Signal 1		Signal 2	
Mode	Frequency (Hz)	σ	Frequency (Hz)	σ
1	0.25	0.5	0.25	0.5
2	0.4	0.5	0.3	0.5
3	0.6	0.05	0.6	0.05
4	0.8	0.05	0.75	0.05

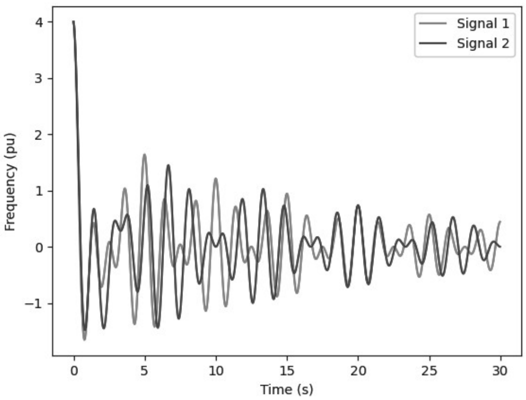

Fig. 4. Two Signal Example

Table III has the fixed modes for the two signals that are generated. The associated plot is seen in Figure 4. Here, the second signal introduced has equivalent damping to the first signal, but the values for two of the frequencies are slightly lower than the first signal. The interest here is in whether or not the MP method manages to pick up these unique modes, and whether or not the introduction of more signals with close frequencies impacts the quality of the results.

Following the one signal example, Table IV shows the results of the MP method applied to both signals. Here we see that the technique picks up almost every unique mode, with the exception of the higher damped modes, of which it only makes rough approximations that are close to the expected values. Looking at the cost functions, the cost functions for each signal are still low, even if they are not 0 as was the case with the single signal approach. The results in Table IV indicate that the MP method still performs accurately in the presence of multiple signals, but does make some approximations in regards to higher damped modes, as opposed to the single signal approach which accurately identified each mode.

3) WSCC 9 Bus System: The final case study will be the WSCC 9 Bus System [12], to test the MP method on signals whose modes are not known exactly. The case is seen in Figure 5and the contingency of interest will be a 3 phase fault on the transmission line between buses 5 and 7 at 1s, which will be cleared at 1.05s. Here, instead of looking at bus frequency, we will instead look at bus voltage magnitude for each bus.

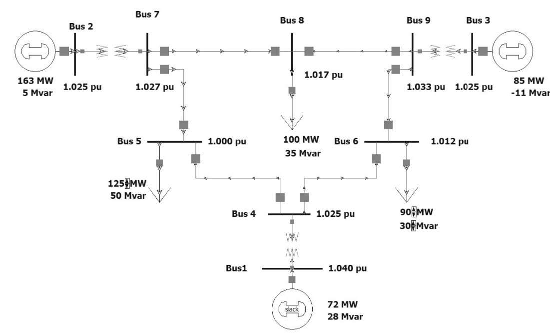

Fig. 5. WSCC 9 Bus System

Applying the aforementioned contingency results in the voltage magnitudes seen in Figure 6. The behavior of the voltages are all relatively similar, and it is very apparent when the contingency occurs, as the voltages drop to 0. Applying the MP method to this signal at varying sampling frequencies gives the results seen in Table V. Initial looks at this table indicate something much different from prior examples, showing an excessively large amount of modes with cost functions that are orders of magnitude larger than prior examples.

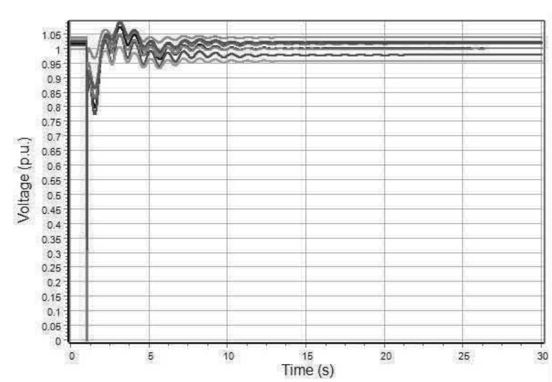

Fig. 6. WSCC 9 Bus System Bus Voltage

Looking at the lowest cost function signal from the 30 Hz resampling, we note that the extreme nature of the voltage drop, going from a constant value to less than half of it's original value instantaneously at the time of the event, coupled

TABLE IV
SAMPLING FREQUENCY RESULTS FOR TWO SIGNALS

	30 Hz				20 Hz				10 Hz			
Modes (Hz)	0.8	0.75	0.6	0.389	0.8	0.75	0.6	0.389	0.8	0.75	0.6	0.39
	0.267	0	0		0.267	0	0		0.267	0	0	
Damping %	0.986	1.059	1.318	16.328	0.986	1.059	1.318	16.330	0.986	1.059	1.319	16.339
	31.726	100	-100		31.705	100	-100		31.653	100	-100	
Cost Function	Signal 1: 0.000363				Signal 1: 0.000443				Signal 1: 0.000623			
	Signal 2: 0.000221				Signal 2: 0.000271				Signal 2: 0.000385			
Time Taken	0.249				0.129				0.0625			

TABLE V
FULL MP RESULTS FOR 9 BUS CASE

	30 Hz	20 Hz	10 Hz
# of Modes	21	17	11
Cost Function	Highest: 0.100 Lowest: 0.0673	Highest: 0.0021 Lowest: 0.0013	Highest: 0.0030 Lowest: 0.0018
Time Taken (s)	2.602	1.18	0.27

with the fact that the MP method is also trying to accurately match the oscillations afterwards, results in larger magnitude cost functions than those in Tables II and IV.

As such, the focus of the analysis is shifted to when the fault is cleared, at 1.05s, where the oscillatory behavior while the system tries to re-stabilize begins. Applying the MP method to this adjusted time window gives the results in Table VI. These number of modes has stabilized, and the associated cost functions are lower than the full counterparts. The exception to this is the 10 Hz resampling, which has the appearance of a 5 Hz mode along with much higher cost functions in this adjusted time frame. This would imply that lower sampling frequencies can result in quite a bit of lost information, as the other modes are similar to the 20 Hz and 30 Hz resampling.

B. Moving Time Window

As mentioned before, one of the assumptions so far in the MP method is the notion that all the data that is to be processed is already available. While this is convenient and mostly holds true for post event analysis, if modal analysis was to be used in real-time power system operation, say, during an event, then not all the data is readily available. Thus, the goal in this section is to test the ability of the MP method to accurately observe the behaviors of a signal as it moves over the signal, which is similar to processing data live.

1) 1 Signal Example: Using the same one signal example as when testing the sampling frequency, two examples of window lengths can be seen in Figure 7. We apply the MP method with a time window of 10 seconds, which moves 1 second over every instance, ie: the second time window will be from 1 second to 11 seconds, the third from 2 to 12, etc. The goal here it to observe whether or not the quality of the results from the MP method suffers as it moves along the signal, and to observe the evolution of the modes. In order to show this, a plot of the modes over each time window can be seen in Figure 8. What is particularly interesting about these results is the fact that the damping of the modes can be observed over

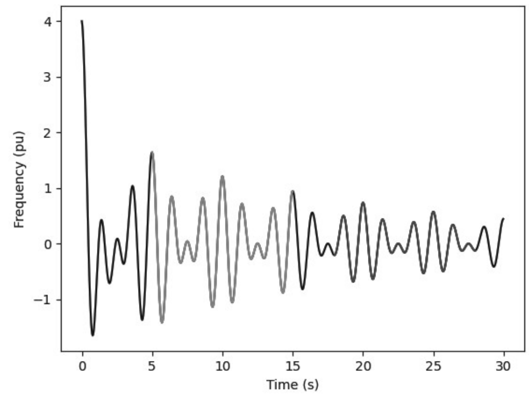

Fig. 7. Different Time Windows in One Signal Approach

time. The 0.25 and 0.4 modes, which have a σ value of 0.5, visibly damp out faster than the 0.6 and 0.8 modes, which have a σ value of 0.05, which proves that a larger σ value indicates that the signal will damp out faster.

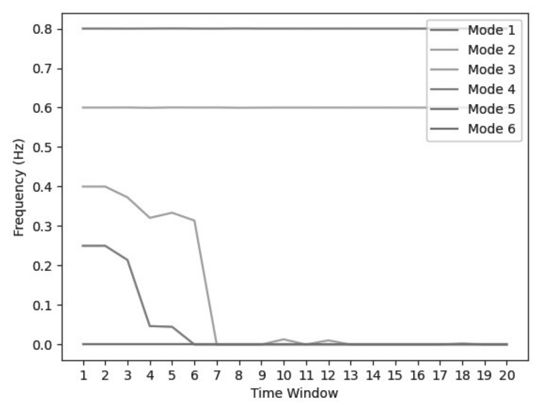

Fig. 8. Modes for Each Time Window in One Signal Approach

2) 2 Signal Example: Using the signals from Table III, the property of interest is again the evolution of the modes over the time windows, and whether or not the MP method properly tracks the damping of the modes over time. The resulting plot can be seen in Figure 9. Similar to Figure 8, we see that the larger modes that had lower damping are still maintained throughout each time window, but the higher damped modes decay quickly and go to 0, which indicates that the results are still consistent from the MP method.

TABLE VI
ADJUSTED MP RESULTS FOR 9 BUS CASE

	30 Hz				20 Hz				10 Hz			
Modes (Hz)	1.992	0.995	0.904	0.650	1.979	0.984	0.886	0.652	1.949	0.978	0.751	0.197
	0.205	0.011	0		0.202	0.010	0		0.00265	5	0	
Damping %	3.288	5.141	7.281	42.532	3.459	5.161	8.595	48.394	2.779	4.339	20.334	35.942
	35.674	-8.098	100		35.204	-9.296	100		-24.443	77.612	100	
Cost Function	Highest: 0.00310				Highest: 0.00168				Highest: 0.00109			
	Lowest: 0.00182				Lowest: 0.00085				Lowest: 0.00259			
Time Taken (s)	2.464				1.006				0.24			

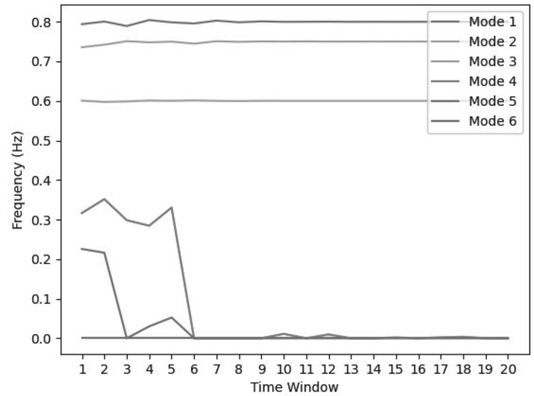

Fig. 9. Modes for Each Time Window in Two Signal Approach

3) WSCC 9 Bus System: Applying the same methodology to the adjusted WSCC 9 Bus System, we get the results seen in Figure 10. Here, we see similar behavior to both prior examples, where some modes damp out quicker than others. However, it should be noted that in this case the modes are not exactly similar to the modes found in Table VI, as was the case for the 1 and 2 signal cases. This is primarily due to the fact that the signals in this case are much more complex, including non-zero phase shifts and non-unitary magnitudes, so the modes are not explicitly defined as in synthetically generated data, but there are still matching modes that are clearly defined, such as the approximately 1 Hz mode and the 0.2 Hz mode.

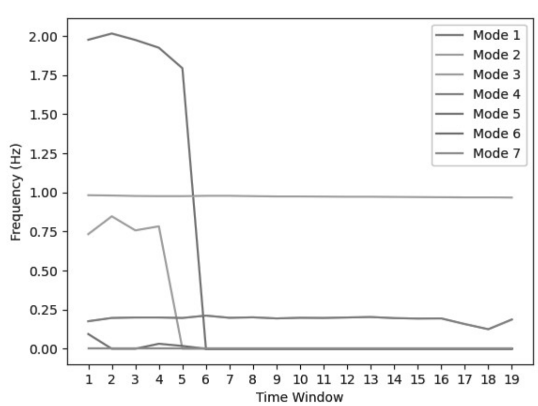

Fig. 10. Modes for Each Time Window in WSCC 9 Bus Case

C. Conclusion

The goal of this work was to look at the performance of the MP method under varying sampling frequencies and it's ability to track modes when applying the method to a moving time window. The MP method is shown to accurately identify modes over different signals and different sampling frequencies, including synthetically generated data where the modes are known, and simpler cases, such as the WSCC 9-Bus system. It is also shown that as the MP method is applied over a sliding time window, it also observes the behavior of modes of the system that decay over time. A base assumption of prior works involving the MP method assumed that all of the data was available to process, but this paper also shows that the MP method allows for accurate tracking of modes over time, without need of all the data from an event. Future work will involve utilizing this technique in further complex cases in other implementations, looking to observe the nature of system modes and their behavior, along with possible implementations in real-time system monitoring.

REFERENCES

[1] M. Klein, G. J. Rogers, and P. Kundur, "A fundamental study of inter-area oscillations in power systems," *IEEE Transactions on Power Systems*, vol. 6, no. 3, pp. 914–921, Aug 1991.

[2] S. Mohapatra and T. J. Overbye, "Fast modal identification, monitoring, and visualization for large-scale power systems using dynamic mode decomposition," in *2016 Power Systems Computation Conference (PSCC)*, June 2016, pp. 1–7.

[3] A. B. Birchfield and T. J. Overbye, "Convergence characteristics of the variable projection method for mode extraction," in *2017 IEEE Texas Power and Energy Conference (TPEC)*, Feb 2017, pp. 1–6.

[4] B. Gao, G. K. Morison, and P. Kundur, "Voltage stability evaluation using modal analysis," *IEEE Transactions on Power Systems*, vol. 7, no. 4, pp. 1529–1542, 1992.

[5] Y. Hua and T. Sarkar, "Matrix pencil method for estimating parameters of exponentially damped/undamped sinusoids in noise," *IEEE Transactions on Signal Processing*, vol. 38, no. 5, pp. 814–824, 5 1990.

[6] Z. Huang, Y. Cui, and W. Xu, "Application of modal sensitivity for power system harmonic resonance analysis," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 222–231, 2007.

[7] Y. Hua and T. K. Sarkar, "Generalized pencil-of-function method for extracting poles of an em system from its transient response," *IEEE Transactions on Antennas and Propagation*, vol. 37, no. 2, pp. 229–234, Feb 1989.

[8] M. L. Crow and A. Singh, "The matrix pencil for power system modal extraction," *IEEE Transactions on Power Systems*, vol. 20, no. 1, pp. 501–502, Feb 2005.

[9] L. L. Grant and M. L. Crow, "Comparison of matrix pencil and prony methods for power system modal analysis of noisy signals," in *2011 North American Power Symposium*, Aug 2011, pp. 1–7.

[10] A. R. Borden, B. C. Lesieutre, and J. Gronquist, "Power system modal analysis tool developed for industry use," in *2013 North American Power Symposium (NAPS)*, Sept 2013, pp. 1–6.

[11] W. C. Trinh, K. S. Shetye, I. Idehen, and T. J. Overbye, "Iterative matrix pencil method for power system modal analysis," in *52nd Hawaii International Conference on System Sciences (HICSS-52)*, Jan 2019.
[12] A. S. Al-Hinai, "Voltage collapse prediction for interconnected power systems," Master's thesis, West Virginia University, 2000.