ECEN 667
Power System Stability

Lecture 3: Numeric Solution of Differential Equations, Electromagnetic Transients

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Announcements

- Start reading Chapters 1 and 2 from the book (Chapter 1 is Introduction, Chapter 2 is Electromagnetic Transients)
- Homework 1 is assigned today. It is due on Thursday September 16
- Classic reference paper on EMTP is H.W. Dommel, "Digital Computer Solution of Electromagnetic Transients in Single- and Multiphase Networks," *IEEE Trans. Power App. and Syst.*, vol. PAS-88, pp. 388-399, April 1969

Putting ECEN 667 in Context

- Research can be broken into two broad categories
  - Those with problems looking for solutions
  - Those with solutions looking for problems
- Research is needed in both areas!
- Power systems is more in the first category: we've got problems associated with designing and operating large-scale electric grids, and often look to other domains for solutions
  - More a focus on domain knowledge; we know the domain and then have a working knowledge of many solution methods
  - This is the approach used in ECEN 667

Sources of Technical Information

- There are many sources of technical information, with many students are most familiar with textbooks and courses
- Journals, conferences, reports, and presentations (e.g., webinars) are extremely useful additional sources
- In rough order of the newest of the information (from oldest to newest)
  - Books including textbooks; often lectures (depending on the instructor); reports
  - Journals, Conferences; both can have peer review
  - Webinars, and industry periodicals
- Many are available open access; someone needs to pay

Research (Graduate School) Success

- The goal of a PhD program, and to some extent the MS programs with a thesis, is to teach students to do independent research
  - Much of this is learned from being in a research group
- If you want to become a researcher you need to get familiar with journals and conferences in your area
  - For ECEN 667 material much of it is in IEEE Explorer, with key journals being *IEEE Transactions on Power Systems*, *IEEE Transactions on Smart Grid*, *IEEE Open Access Journal of Power and Energy* (there are others)
- TAMU students can access IEEE Xplore at ieeexplore.ieee.org/Xplore/home.jsp

Research (Graduate School) Success

- Read all your advisor's papers
- References (endnotes) matter. The endnotes in a paper can provide useful background information
- With tools such as Google Scholar they can also give an indication of the most important papers in a field
  - Those that are commonly referenced
  - Not all scholars have google scholar profiles
- IEEE now keeps track of paper citations, for example the 1969 by Dommel has 1071 paper citations (in IEEE so its total number is likely higher); this is a high number for electric power!

Off Campus IEEE Xplore Access

- As TAMU students, faculty and staff we can get IEEE Explorer Access off campus by going to

  tamu.libguides.com/c.php?g=607356&p=4211300

  (then click on the IEEE Xplore link; you'll then login with your TAMU credentials)
  - I'm sure there are other ways, this is just how I access it
- More general library access is available at
  - library.tamu.edu
- Authors often post prepublication copies of papers on their websites
- On campus you can also just google papers for direct access

RK2 Versus Euler's

- RK2 requires twice the function evaluations per iteration, but gives much better results
- With RK2 the error tends to vary with the cube of the step size, compared with the square of the step size for Euler's
- The smaller error allows for larger step sizes compared to Euler’s
- There are other RK2 implementations
  - What was presented here is known as Heun’s method
  - Other approaches include the Explicit midpoint method (evaluate the function at the midpoint) and Ralston’s method (evaluate the function 2/3 of the way to the Euler value)

Fourth Order Runge-Kutta

- Other Runge-Kutta algorithms are possible, including the fourth order

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{6}\left(\mathbf{k}_1 + 2\mathbf{k}_2 + 2\mathbf{k}_3 + \mathbf{k}_4 \right)$$

where

$$\mathbf{k}_1 = \Delta t \; \mathbf{f}\left(\mathbf{x}(t)\right)$$

$$\mathbf{k}_2 = \Delta t \; \mathbf{f}\left(\mathbf{x}(t) + \frac{1}{2}\mathbf{k}_1 \right)$$

$$\mathbf{k}_3 = \Delta t \; \mathbf{f}\left(\mathbf{x}(t) + \frac{1}{2}\mathbf{k}_2 \right)$$

$$\mathbf{k}_4 = \Delta t \; \mathbf{f}\left(\mathbf{x}(t) + \mathbf{k}_2 \right)$$

RK4 Oscillating Cart Example

- RK4 gives much better results, with error varying with the time step to the fifth power

time	actual $x_1(t)$	$x_1(t)$ with RK4 $\Delta t=0.25$
0	1	1
0.25	0.9689	0.9689
0.50	0.8776	0.8776
0.75	0.7317	0.7317
1.00	0.5403	0.5403
10.0	-0.8391	-0.8392
100.0	0.8623	0.8601

Multistep Methods

- Euler's and Runge-Kutta methods are single step approaches, in that they only use information at $\mathbf{x}(t)$ to determine its value at the next time step
- Multistep methods take advantage of the fact that using we have information about previous time steps $\mathbf{x}(t-\Delta t)$, $\mathbf{x}(t-2\Delta t)$, etc
- These methods can be explicit or implicit (dependent on $\mathbf{x}(t+\Delta t)$ values; we'll just consider the explicit Adams-Bashforth approach

Multistep Motivation

- In determining $\mathbf{x}$(t+Δt) we could use a Taylor series expansion about $\mathbf{x}$(t)

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\dot{\mathbf{x}}(t) + \frac{\Delta t^2}{2}\ddot{\mathbf{x}}(t) + O(\Delta t^3)$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\mathbf{f}(t) + \frac{\Delta t^2}{2}\left(\frac{\mathbf{f}(\mathbf{x}(t)) - \mathbf{f}(\mathbf{x}(t-\Delta t))}{\Delta t} + O(\Delta t)\right)$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\left(\frac{3}{2}\mathbf{f}\left(\mathbf{x}(t)\right) - \frac{1}{2}\mathbf{f}\left(\mathbf{x}(t-\Delta t)\right)\right) + O(\Delta t^3)$$

(note Euler's is just the first two terms on the right-hand side)

Adams-Bashforth

- What we derived is the second order Adams-Bashforth approach. Higher order methods are also possible, by approximating subsequent derivatives. Here we also present the third order Adams-Bashforth

Second Order

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\big(3\mathbf{f}(\mathrm{x}(t)) - \mathbf{f}(x(t-\Delta t))\big) + O(\Delta t^3)$$

Third Order

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{12}\big(23\mathbf{f}(\mathrm{x}(t)) - 16\mathbf{f}(x(t-\Delta t)) + 5\mathbf{f}(x(t-2\Delta t))\big) + O(\Delta t^4)$$

Adams-Bashforth Versus Runge-Kutta

- The key Adams-Bashforth advantage is the approach only requires one function evaluation per time step while the RK methods require multiple evaluations
- A key disadvantage is when discontinuities are encountered, such as with limit violations
  - In some simulations limits can be hit often
  - Another method needs to be used until there are sufficient past solutions
- They also have difficulties if variable time steps are used

Numerical Instability

- All explicit methods can suffer from numerical instability if the time step is not correctly chosen for the problem eigenvalues

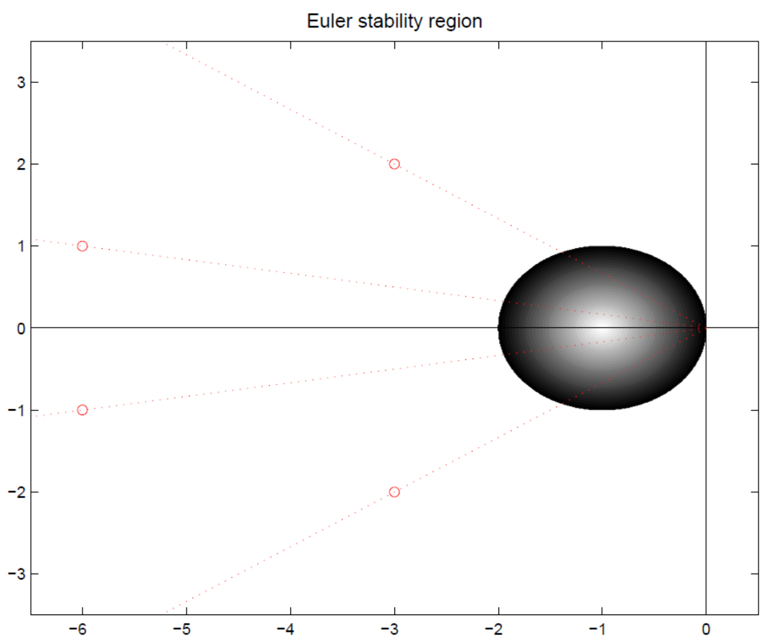


Figure 10.2: The spectrum of A is scaled by h. Stability of the origin is recovered if $h\lambda$ is in t region of absolute stability $|1 + z| < 1$ in the complex plane.

Values are scaled by the time step; the shape for RK2 has similar dimensions but is closer to a square. Key point is to make sure the time step is small enough relative to the eigenvalues

Image source: http://www.staff.science.uu.nl/~frank011/Classes/numwisk/ch10.pdf

Stiff Differential Equations

- Stiff differential equations are ones in which the desired solution has components the vary quite rapidly relative to the solution
- Stiffness is associated with solution efficiency: in order to account for these fast dynamics we need to take quite small time steps

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -1000x_1 - 1001x_2$$
$$\dot{\mathbf{x}} \rightarrow = \begin{bmatrix} 0 & 1 \\ -1000 & -1000 \end{bmatrix} \mathbf{x}$$
$$x_1(t) = Ae^{-t} + Be^{-1000t}$$

Stiff differential equations are common in power systems, but there are efficient techniques for handling them

Implicit Methods

- Implicit solution methods have the advantage of being numerically stable over the entire left half plane
- Only methods considered here are the is the Backward Euler and Trapezoidal

Initially we'll assume linear equations

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \mathbf{A}\mathbf{x}(t))$$

Then using backward Euler

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\mathbf{A}(\mathbf{x}(t+\Delta t))$$

$$\left[I - \Delta t\,\mathbf{A}\right]\mathbf{x}(t+\Delta t) = \mathbf{x}(t)$$

$$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1}\mathbf{x}(t)$$

Backward Euler Cart Example

- Returning to the cart example

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}(t))$$

Then using backward Euler with $\Delta t = 0.25$

$$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1} \mathbf{x}(t) = \begin{bmatrix} 1 & -0.25 \\ 0.25 & 1 \end{bmatrix}^{-1} \mathbf{x}(t)$$

Backward Euler Cart Example

- Results with $\Delta t = 0.25$ and 0.05

time	actual $x_1(t)$	$x_1(t)$ with Δt=0.25	$x_1(t)$ with Δt=0.05
0	1	1	1
0.25	0.9689	0.9411	0.9629
0.50	0.8776	0.8304	0.8700
0.75	0.7317	0.6774	0.7185
1.00	0.5403	0.4935	0.5277
2.00	-0.416	-0.298	-0.3944

Note: Just because the method is numerically stable doesn't mean it is error free! RK2 is more accurate than backward Euler.

Trapezoidal Linear Case

- For the trapezoidal with a linear system we have

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \mathbf{A}\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[\mathbf{A}(\mathbf{x}(t)) + \mathbf{A}(\mathbf{x}(t+\Delta t))\right]$$

$$\left[I - \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t+\Delta t) = \left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

$$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1}\left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

Trapezoidal Cart Example

- Results with $\Delta t = 0.25$, comparing between backward Euler and trapezoidal

time	actual $x_1(t)$	Backward Euler	Trapezoidal
0	1	1	1
0.25	0.9689	0.9411	0.9692
0.50	0.8776	0.8304	0.8788
0.75	0.7317	0.6774	0.7343
1.00	0.5403	0.4935	0.5446
2.00	-0.416	-0.298	-0.4067

Electromagnetic Transients

- The modeling of very fast power system dynamics (much less than one cycle) is known as electromagnetics transients program (EMTP) analysis
  - Covers issues such as lightning propagation and switching surges; they can also be used with inverter-based controls
- Concept originally developed by Prof. Hermann Dommel for his PhD in the 1960's (now emeritus at Univ. British Columbia)
  - After his PhD work Dr. Dommel worked at BPA where he was joined by Scott Meyer in the early 1970's
  - Alternative Transients Program (ATP) developed in response to commercialization of the BPA code

Power System Time Frames

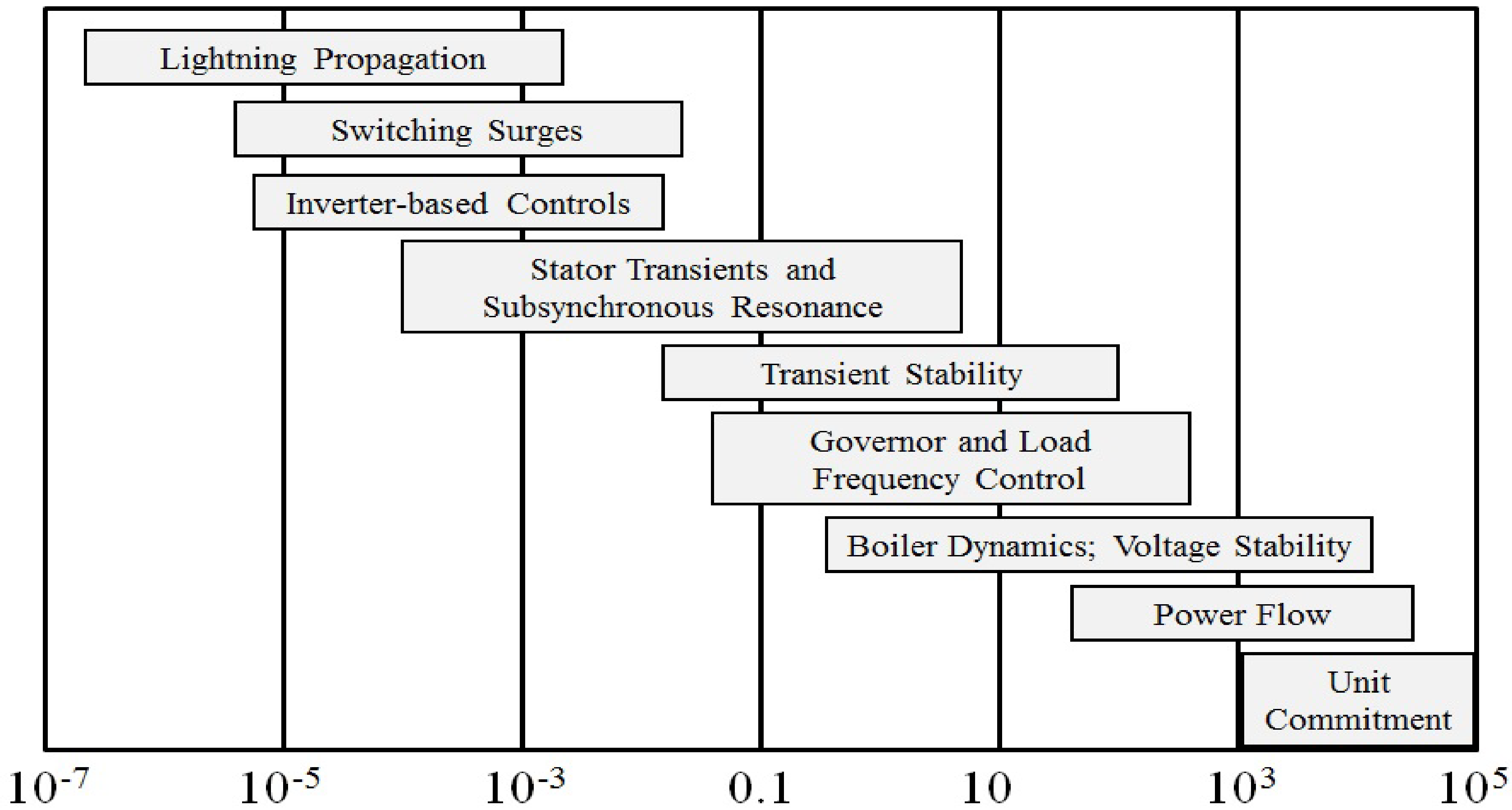


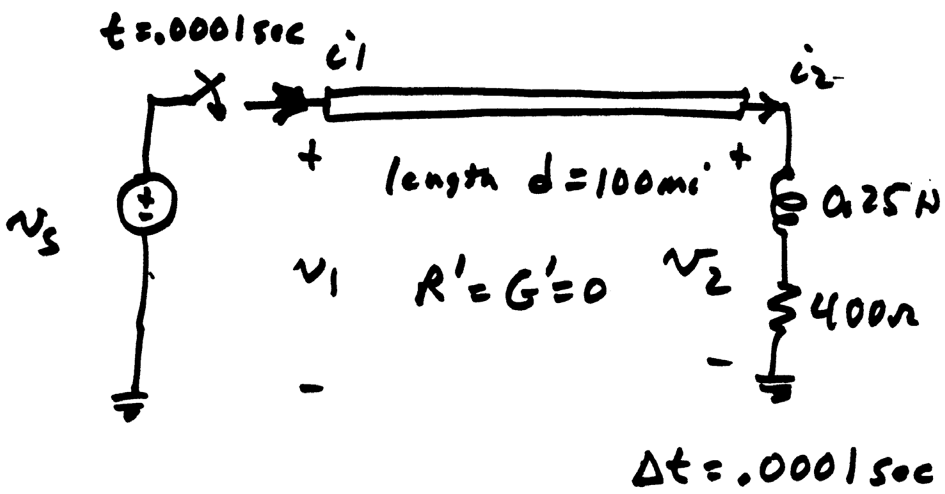
Image source: P.W. Sauer, M.A. Pai, Power System Dynamics and Stability, 1997, Fig 1.2, modified

Transmission Line Modeling

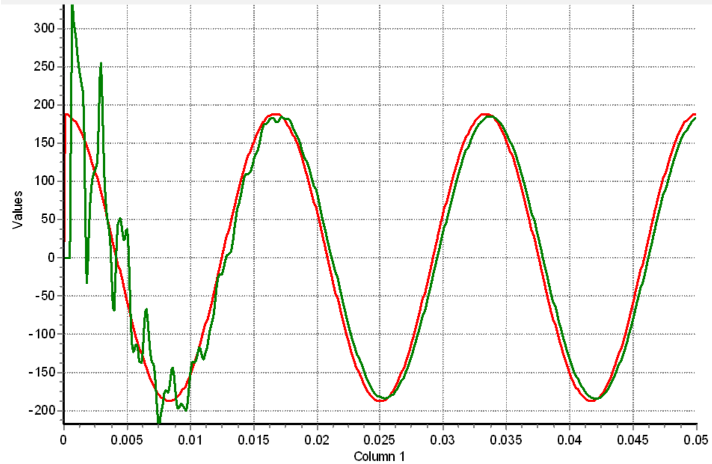
- In power flow and transient stability transmission lines are modeled using a lumped parameter approach
  - Changes in voltages and current in the line are assumed to occur instantaneously
  - Transient stability time steps are usually a few ms (1/4 cycle is common, equal to 4.167ms for 60Hz)
- In EMTP time-frame this is no longer the case; speed of light is 300,000km/sec or 300km/ms or 300m/μs
  - Change in voltage and/or current at one end of a transmission cannot instantaneously affect the other end

Need for EMTP

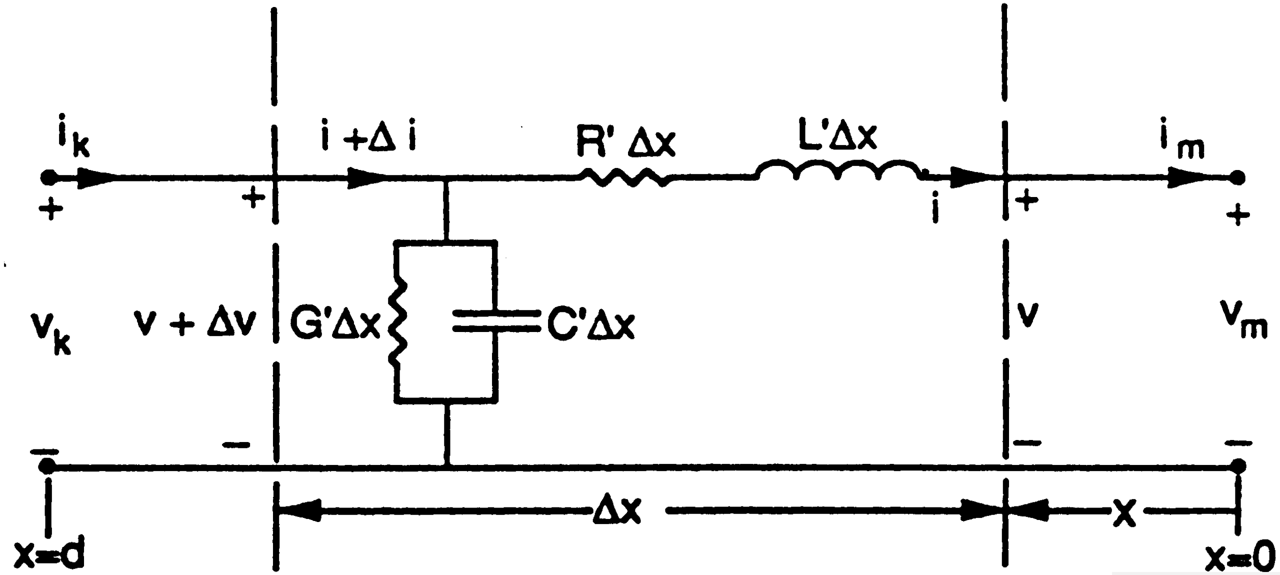
- The change isn't instantaneous because of propagation delays, which are near the speed of light; there also wave reflection issues



Red is the v_s end, green the v_2 end



Incremental Transmission Line Modeling



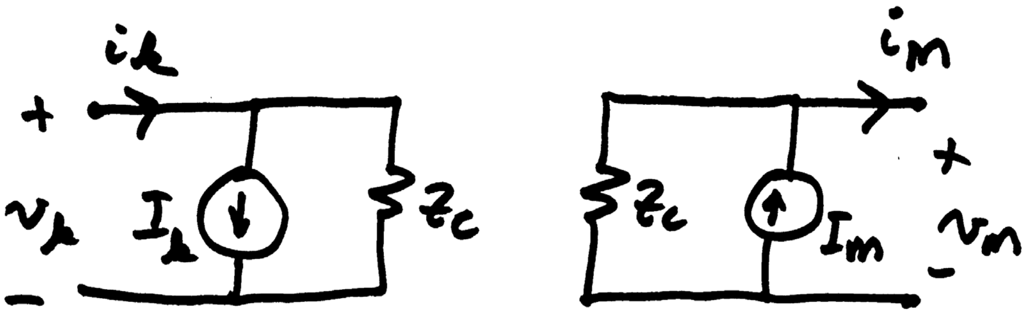
$$\Delta v = R' \Delta x i + L' \Delta x \frac{\partial i}{\partial t}$$

$$\Delta i = G' \Delta x \left(v + \Delta v \right) + C' \Delta x \frac{\partial}{\partial t} \left(v + \Delta v \right)$$

Define the receiving end as bus m (x=0) and the sending end as bus k (x=d)

Where We Will End Up

- Goal is to come up with model of transmission line suitable for numeric studies on this time frame



$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Both ends of the line are represented by Norton equivalents

Assumption is we don’t care about what occurs along the line

Incremental Transmission Line Modeling

We are looking to determine v(x,t) and i(x,t)

Recall $\Delta i = G'\Delta x\left(v+\Delta v\right) + C'\Delta x \frac{\partial}{\partial t}\left(v+\Delta v\right)$

Substitute $\Delta v = \Delta x\left(R'i + L'\frac{\partial i}{\partial t}\right)$

Into the equation for Δi and divide both by Δx

$$\frac{\Delta i}{\Delta x} = G'v + G'\left(R'\Delta x i + L'\Delta x\frac{\partial i}{\partial t}\right) + C'\frac{\partial v}{\partial t}$$

$$+C'\left[R'\Delta x\frac{\partial i}{\partial t} + L'\Delta x\frac{\partial^2 i}{\partial t^2}\right]$$

Incremental Transmission Line Modeling

Taking the limit we get

$$\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = \boxed{\frac{\partial v}{\partial x} = R' i + L' \frac{\partial i}{\partial t}}$$

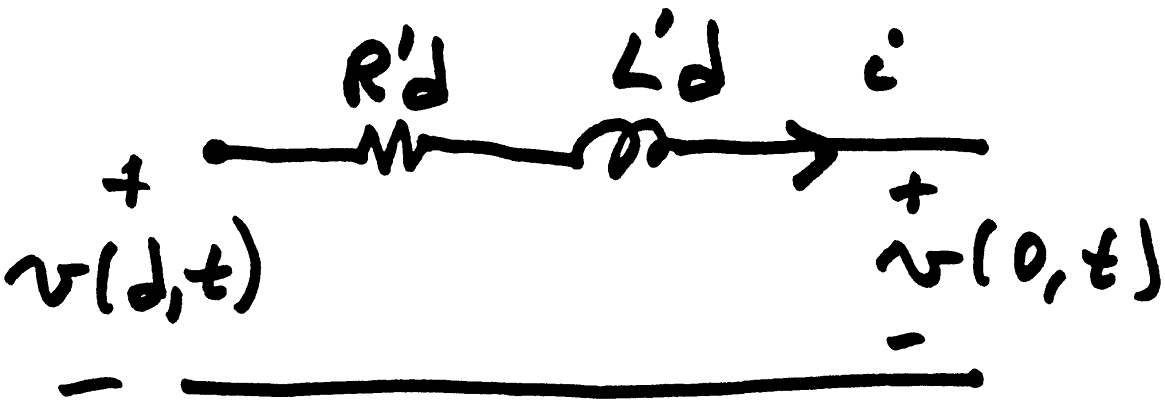
$$\lim_{\Delta x \to 0} \frac{\Delta i}{\Delta x} = \boxed{\frac{\partial i}{\partial x} = G' v + C' \frac{\partial v}{\partial t}}$$

Some authors have a negative sign with these equations; it just depends on the direction of increasing x; note that the values are function of both x and t

Special Case 1

C' = G' = 0 (neglect shunts)

$$v(x,t) = v(0,t) + R'x_i + L'x\frac{di}{dt}$$



This just gives a lumped parameter model, with all electric field effects neglected

Special Case 2: Wave Equation

The lossless line (R'=0, G'=0), which gives

$$\frac{\partial v}{\partial x} = L'\frac{\partial i}{\partial t}, \quad \frac{\partial i}{\partial x} = C'\frac{\partial v}{\partial t}$$

This is the wave equation with a general solution of

$$i(x,t) = -f_1\left(x - v_p t\right) - f_2\left(x + v_p t\right)$$

$$v(x,t) = z_c f_1\left(x - v_p t\right) - z_c f_2\left(x + v_p t\right)$$

$$z_c = \sqrt{L'/C'}\ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Z_c is the characteristic impedance and v_p is the velocity of propagation

Special Case 2: Wave Equation

- This can be thought of as two waves, one traveling in the positive x direction with velocity v_p, and one in the opposite direction
- The values of f_1 and f_2 depend upon the boundary (terminal) conditions

$$i(x,t) = -f_1\left(x - v_p t\right) - f_2\left(x + v_p t\right)$$

$$v(x,t) = z_c f_1\left(x - v_p t\right) - z_c f_2\left(x + v_p t\right)$$

$$z_c = \sqrt{L' / C'}\ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Boundaries are receiving end with x=0 and the sending end with x=d