ECEN 667
Power System Stability

Lecture 4: Electromagnetic Transients

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Announcements

- Homework 1 is due on Thursday September 16
- Starting read Chapter 3
- Reference for modeling three-phase lines is W. Kersting, *Distribution System Modeling and Analysis*, 4th Edition, CRC Press, 2018

Special Case 2: Wave Equation

The lossless line (R'=0, G'=0), which gives

$$\frac{\partial v}{\partial x} = L'\frac{\partial i}{\partial t}, \quad \frac{\partial i}{\partial x} = C'\frac{\partial v}{\partial t}$$

This is the wave equation with a general solution of

$$i(x,t) = -f_1\left(x - v_p t\right) - f_2\left(x + v_p t\right)$$

$$v(x,t) = z_c f_1\left(x - v_p t\right) - z_c f_2\left(x + v_p t\right)$$

$$z_c = \sqrt{L'/C'}\,, \quad v_p = \frac{1}{\sqrt{L'C'}}$$

z_c is the characteristic impedance and v_p is the velocity of propagation

Special Case 2: Wave Equation

- This can be thought of as two waves, one traveling in the positive x direction with velocity v_p, and one in the opposite direction
- The values of f_1 and f_2 depend upon the boundary (terminal) conditions

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$z_c = \sqrt{L' / C'} \ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Boundaries are receiving end with x=0 and the sending end with x=d

Calculating v_p

- To calculate v_p for a line in air we go back to the definition of L' and C'

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right), \quad C' = \frac{2\pi\varepsilon_0}{\ln D/r}$$

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu_0\varepsilon_0 \dfrac{\ln D/r'}{\ln D/r}}} = c\frac{1}{\sqrt{\dfrac{\ln D/r'}{\ln D/r}}}$$

With r'=0.78r this is very close to the speed of light

Important Insight

- The amount of time for the wave to go between the terminals is $d/v_p = \tau$ seconds
  - To an observer traveling along the line with the wave, $x+v_p t$, will appear constant
- What appears at one end of the line impacts the other end τ seconds later

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$v(x,t) + z_c i(x,t) = -2 z_c f_2(x + v_p t)$$

Both sides of the bottom equation are constant when $x+v_p t$ is constant

Determining the Constants

- If just the terminal characteristics are desired, then an approach known as Bergeron's method can be used.
- Knowing the values at the receiving end m (x=0) we get

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$i_m(t) = i(0,t) = -f_1(-v_p\, t) - f_2(v_p\, t)$$

$$v_m(t) = z_c f_1(-v_p\, t) - z_c f_2(v_p\, t)$$

This can be used to eliminate f_1

Determining the Constants

- Eliminating f_1 we get

$$v_m(t) = z_c f_1\left(-v_p\, t\right) - z_c f_2\left(v_p\, t\right)$$

$$f_1\left(-v_p\, t\right) = \frac{v_{m(t)}}{z_c} + f_2\left(v_p\, t\right)$$

$$i_m(t) = -\frac{v_m}{z_c} - 2 f_2\left(v_p\, t\right)$$

Solve for f_1 and replace it in the equation from the previous slide

Determining the Constants

- To solve for f_2 we need to look at what is going on at the sending end (i.e., k at which x=d) $\tau = d/v_p$ seconds in the past

$$i_k\left(t-\frac{d}{v_p}\right)=-f_1\left(d-v_p\left(t-\frac{d}{v_p}\right)\right)-f_2\left(d+v_p\left(t-\frac{d}{v_p}\right)\right)$$

$$i_k\left(t-\frac{d}{v_p}\right)=-f_1\left(2d-v_pt\right)-f_2\left(v_pt\right)$$

$$v_k\left(t-\frac{d}{v_p}\right)=z_cf_1\left(2d-v_pt\right)-z_cf_2\left(v_pt\right)$$

Determining the Constants

- Dividing v_k by z_c, and then adding it with i_k gives

$$i_k\left(t-\frac{d}{v_p}\right)+\frac{v_k}{z_c}\left(t-\frac{d}{v_p}\right)=-2f_2\left(v_p t\right)$$

- Then substituting for f_2 in i_m gives

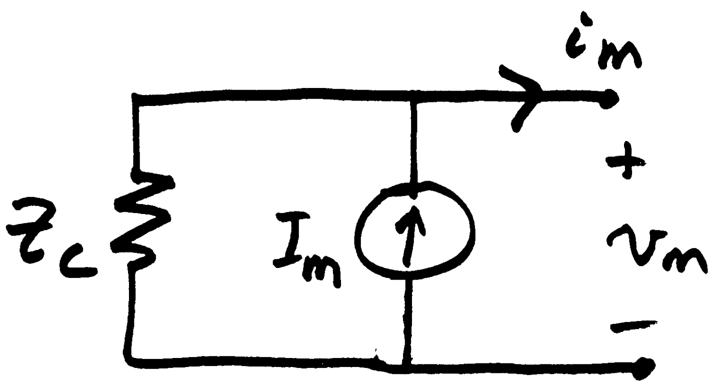
$$i_m\left(t\right)=-\frac{v_m\left(t\right)}{z_c}+i_k\left(t-\frac{d}{v_p}\right)+\frac{1}{z_c}v_k\left(t-\frac{d}{v_p}\right)$$

Hence $i_m(t)$ depends on current conditions at m and past conditions at k

Equivalent Circuit Representation

- The receiving end can be represented in circuit form as

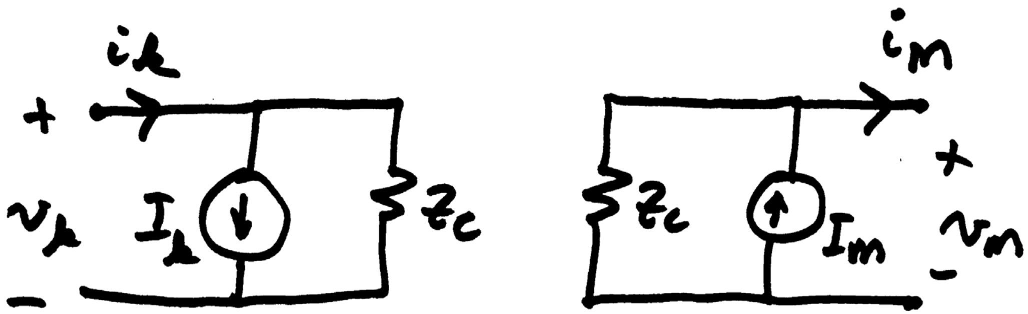
$$i_m(t) = -\frac{v_m(t)}{Z_c} + \boxed{i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{Z_c} v_k\left(t - \frac{d}{v_p}\right)} \longrightarrow I_m$$



Since $\tau = d/v_p$, I_m just depends on the voltage and current at the other end of the line from τ seconds in the past. Since these are known values, it looks like a time-varying current source

Repeating for the Sending End

- The sending end has a similar representation



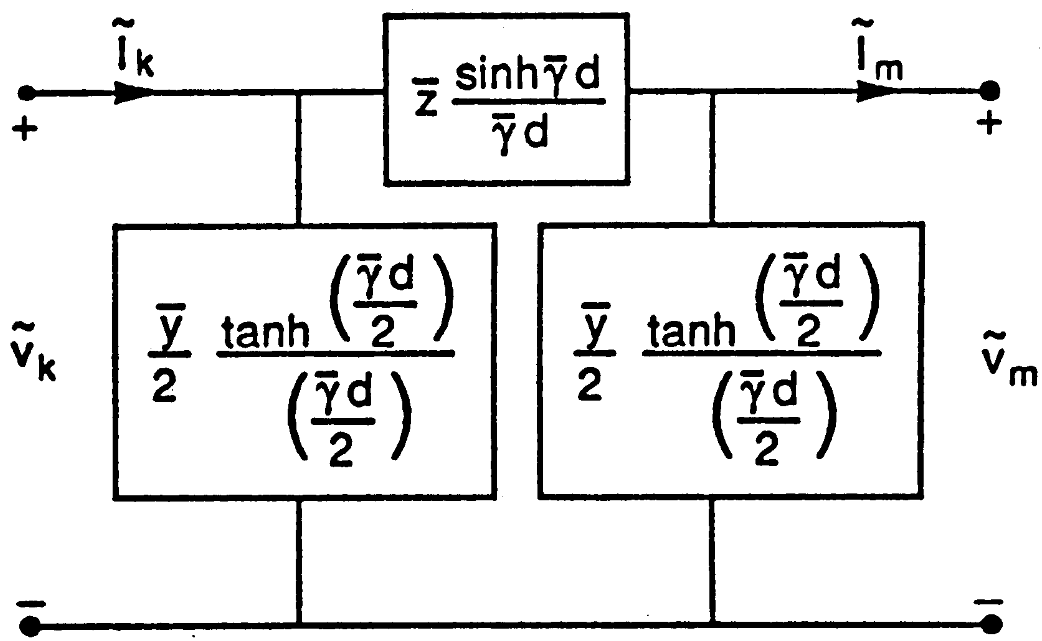
$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Both ends of the line are represented by Norton equivalents

Lumped Parameter Model

- In the special case of constant frequency, book shows the derivation of the common lumped parameter model



This is used in power flow and transient stability; in EMTP the frequency is not constant

Including Line Resistance

- An approach for adding line resistance, while keeping the simplicity of the lossless line model, is to just to place ½ of the resistance at each end of the line
  - Another, more accurate approach, is to place ¼ at each end, and ½ in the middle
- Standalone resistance, such as modeling the resistance of a switch, is just represented as an algebraic equation

$$i_{k,m} = \frac{1}{R}\left(v_k - v_m\right)$$

Numerical Integration with Trapezoidal Method

- Numerical integration is often done using the trapezoidal method discussed last time
  - Here we show how it can be applied to inductors and capacitors
- For a general function the trapezoidal approach is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[f(\mathbf{x}(t)) + f(\mathbf{x}(t+\Delta t)) \right]$$

- Trapezoidal integration introduces error on the order of Δt^3, but it is numerically stable

Trapezoidal Applied to Inductor with Resistance

- For a lossless inductor,

$$v = L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{v}{L} \qquad i(0) = i^0$$

$$i(t+\Delta t) = i(t) + \frac{\Delta t}{2L}\left(v(t) + v\left(t+\Delta t\right)\right)$$

This is a linear equation

- This can be represented as a Norton equivalent with current into the equivalent defined as positive (the last two terms are the current source)

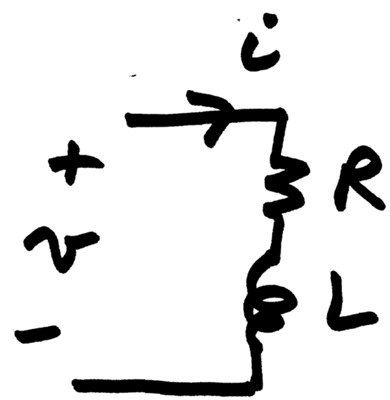
$$i(t+\Delta t) = \frac{v\left(t+\Delta t\right)}{2L/\Delta t} + i(t) + \frac{v(t)}{2L/\Delta t}$$

Trapezoidal Applied to Inductor with Resistance

- For an inductor in series with a resistance we have

$$v = iR + L\frac{di}{dt}$$

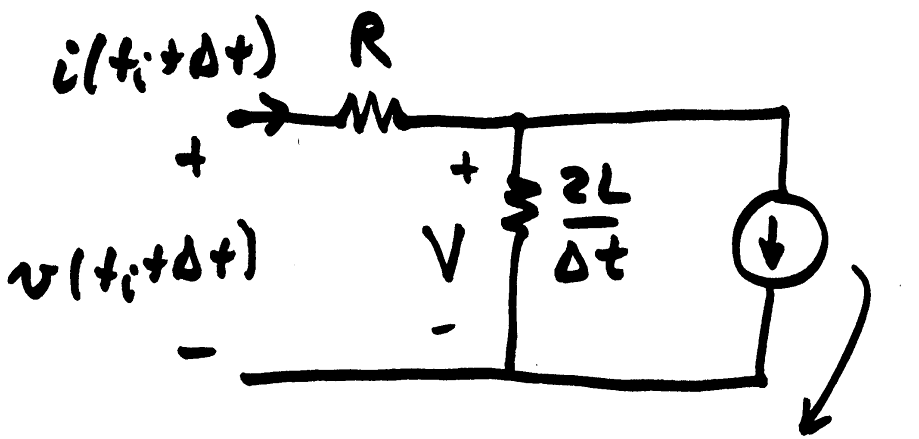
$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$



Trapezoidal Applied to Inductor with Resistance

$$i(t_i+\Delta t) \approx i(t_i) + \frac{\Delta t}{2}\left[-\frac{R}{L}i(t_i) + \frac{1}{L}v(t_i) - \frac{R}{L}i(t_i+\Delta t) + \frac{1}{L}v(t_i+\Delta t)\right]$$

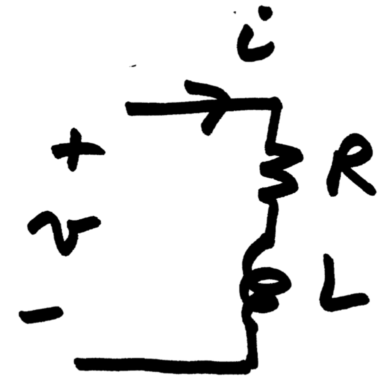
This also becomes a Norton equivalent. A similar expression will be developed for capacitors



$$i(t_i) + \frac{v(t_i) - R\,i(t_i)}{2L/\Delta t}$$

RL Example

- Assume a series RL circuit with an open switch with R= 200Ω and L = 0.3H, connected to a voltage source with $v = 133{,}000\sqrt{2}\cos\left(2\pi 60 t\right)$
- Assume the switch is closed at t=0
- The exact solution is



$$i = -712.4 e^{-667t} + 578.8\sqrt{2}\cos\left(2\pi 60t - 29.5^{0}\right)$$

$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L} i + \frac{1}{L} v \qquad i(0) = i^{0}$$

R/L=667, so the dc offset decays quickly

RL Example Trapezoidal Solution

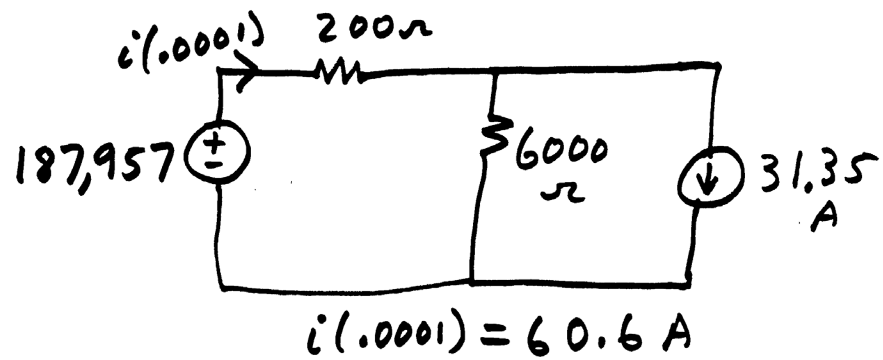
$$\frac{2L}{\Delta t} = \frac{2*0.3}{0.0001} = 6000$$

$$\Delta t = 0.0001\,\text{sec}$$

$$t = 0 \quad i(0) = 0$$

$$t = 0.0001$$

$$i(0) + \frac{v(0) - Ri(0)}{6000} = 31.35\text{A}$$



Numeric solution: $i(0.0001) = \dfrac{187{,}957}{6200} + \dfrac{31.35 \times 6000}{6200} = 60.65\text{A}$

Exact solution:

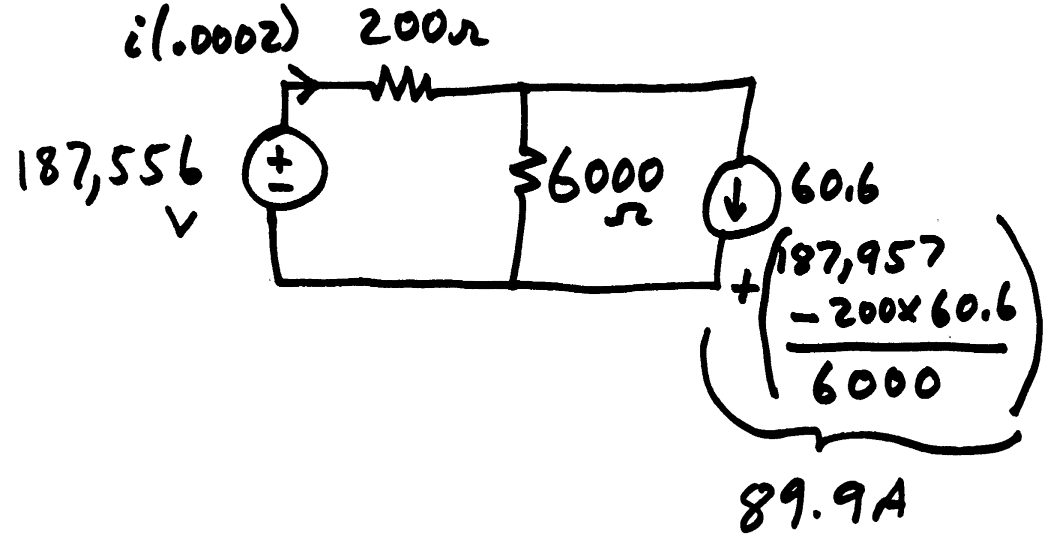
$$i(0.0001) = -712.4e^{-.0677} + 578.8\sqrt{2}\cos\left(2\pi 60 \times .0001 - 29.5\frac{\pi}{180}\right)$$

$$= -666.4 + 727.0$$

$$= 60.6A$$

RL Example Trapezoidal Solution

$t = 0.0002$



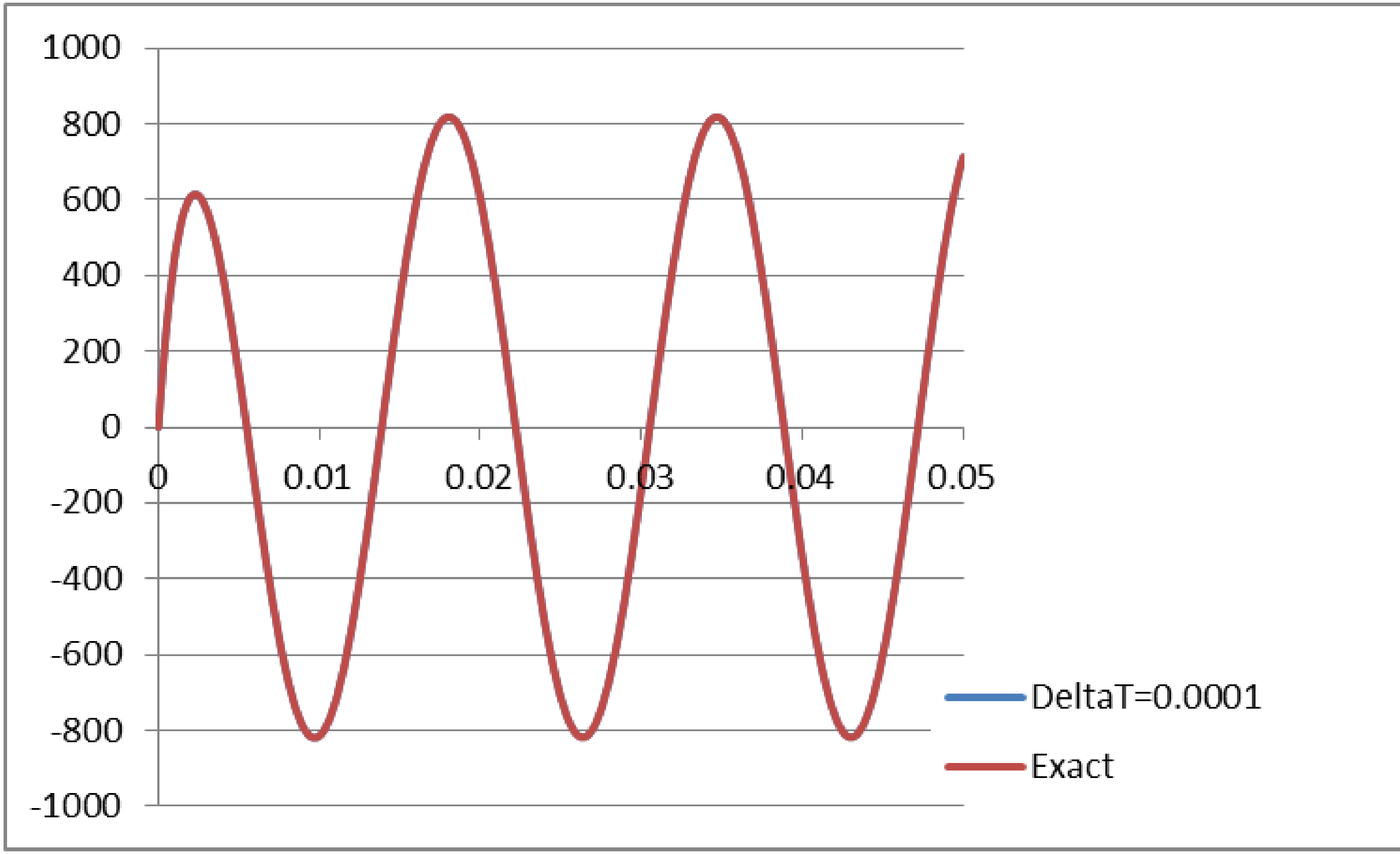
Solving for i(0.0002)

$i(0.0002) = 117.3\text{A}$

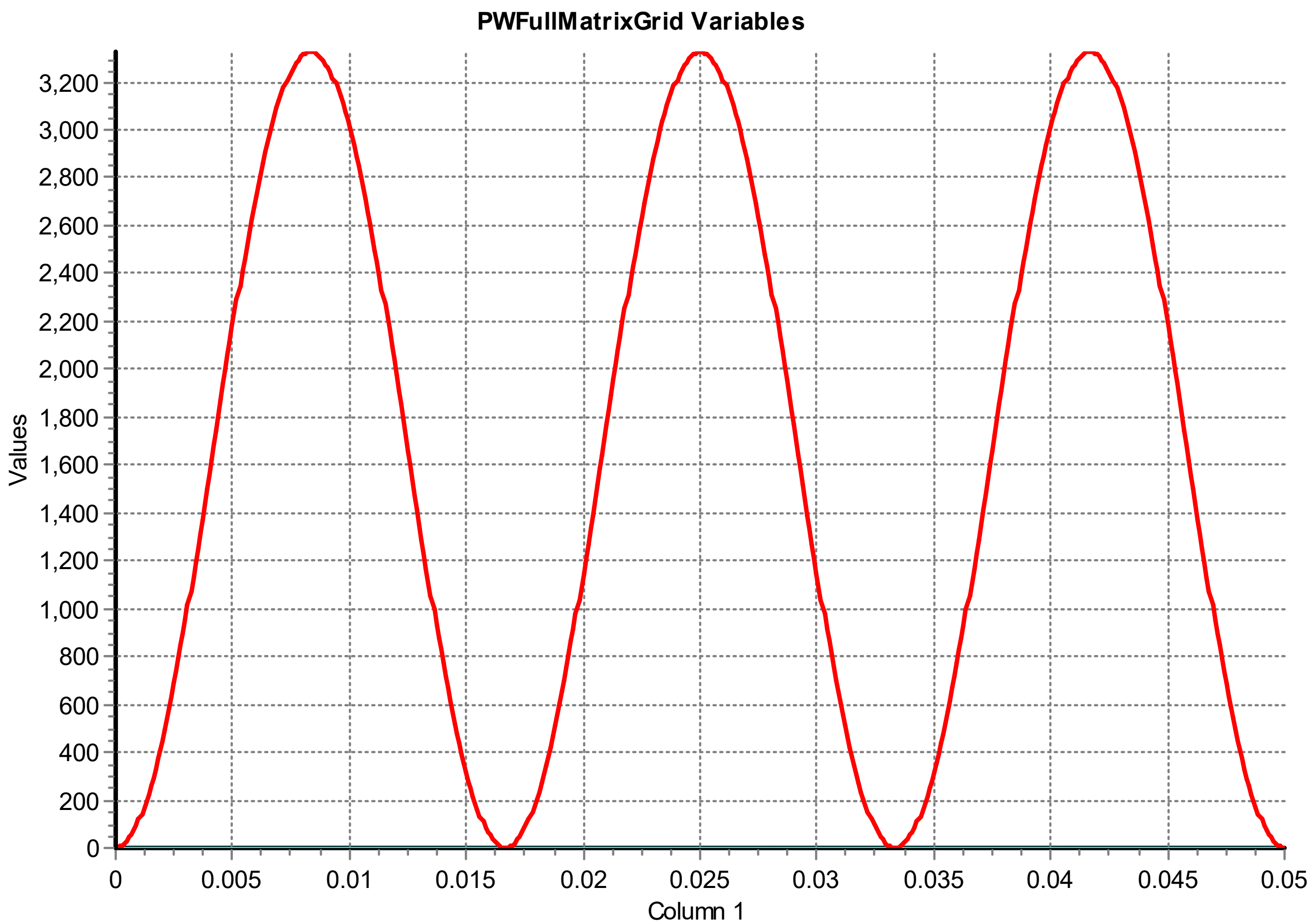
Compare to the exact solution

$i(0.0002) = 117.3\text{A}$

Full Solution Over Three Cycles



A Favorite Problem: R=0 Case, with v(t) = Sin(2*pi*60)



Note that the current is never negative!

Lumped Capacitance Model

- The trapezoidal approach can also be applied to model lumped capacitors

$$i(t) = C\frac{dv(t)}{dt}$$

- Integrating over a time step gives

$$v(t+\Delta t) = v(t) + \frac{1}{C}\int_{t}^{t+\Delta t} i(t)$$

- Which can be approximated by the trapezoidal as

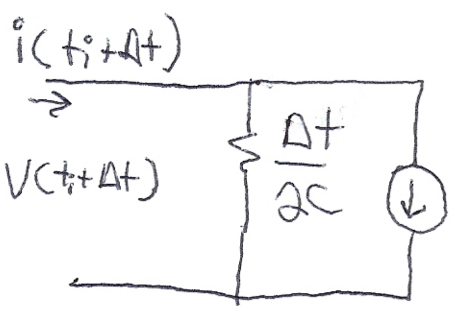
$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

Lumped Capacitance Model

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\big(i(t+\Delta t) + i(t)\big)$$

$$i(t+\Delta t) = \frac{v(t+\Delta t)}{\Delta t/2C} - \frac{v(t)}{\Delta t/2C} - i(t)$$

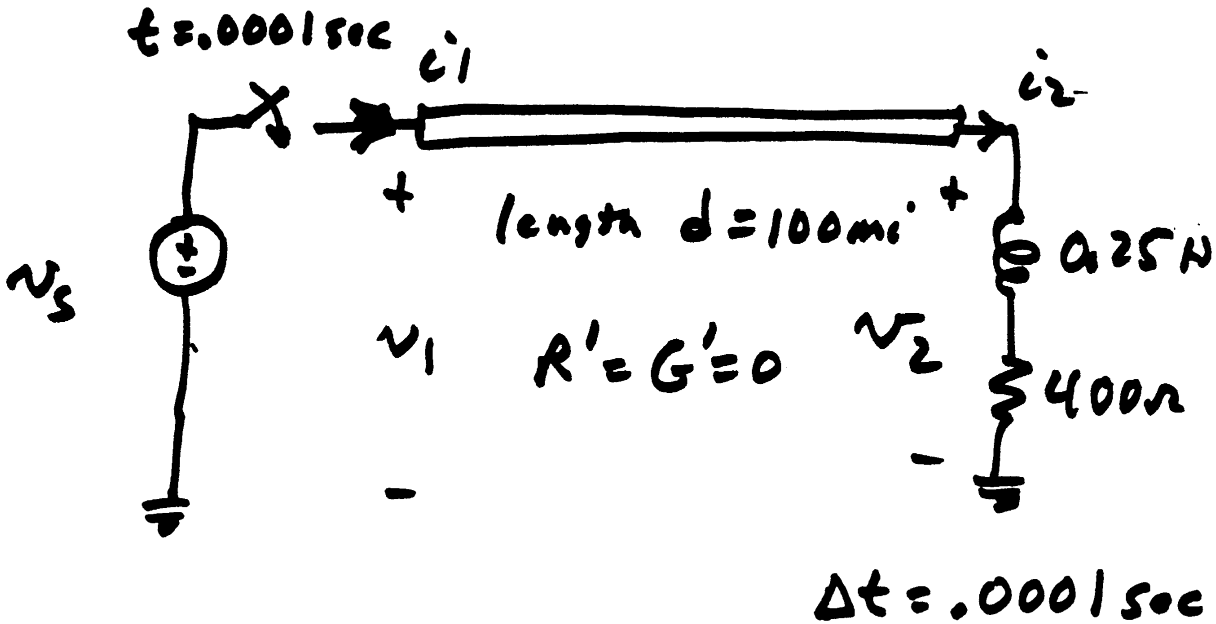
- Hence we can derive a circuit model similar to what was done for the inductor



$$-\frac{v(t)}{\Delta t/2C} - i(t)$$

This is a current source that depends on the past values

Example 2.1: Line Closing



$$L' = 1.5 \times 10^{-3} H / mi$$

$$C' = 0.02 \times 10^{-6} F / mi$$

Switch is closed at time t = 0.0001 sec

Example 2.1: Line Closing

Initial conditions: $i_1 = i_2 = \mathrm{v}_1 = \mathrm{v}_2 = 0$

for $t < 0.0001$ sec

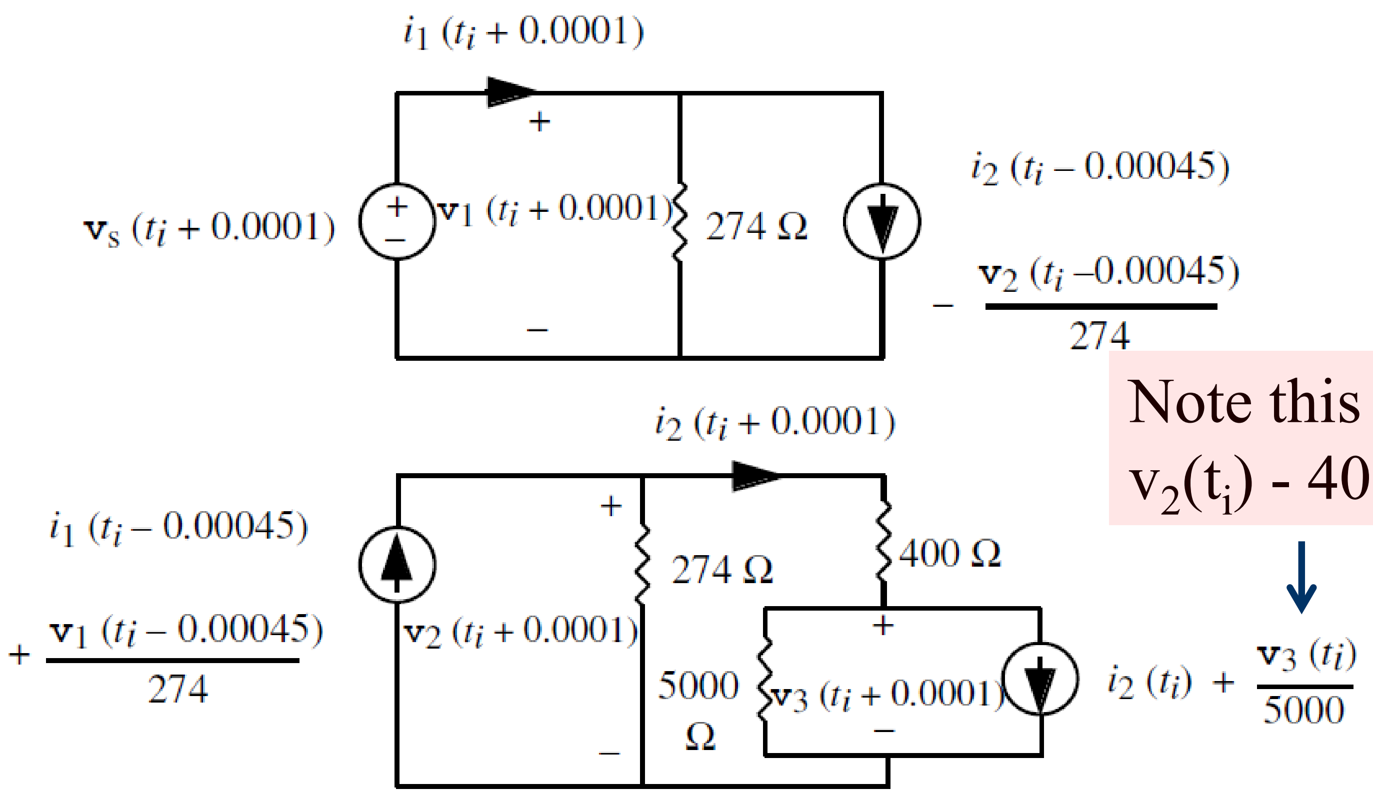
$$z_c = \sqrt{\frac{L'}{C'}} = 274\Omega \qquad v_p = \frac{1}{\sqrt{L'C'}} = 182{,}574\, mi/\sec$$

$$\frac{d}{v_p} = 0.00055\sec$$

$$\frac{2L}{\Delta t} = 5000\Omega$$

Because of finite propagation speed, the receiving end of the line will not respond to energizing the sending end for at least 0.00055 seconds

Example 2.1: Line Closing



Note this is $v_3(t_i) = v_2(t_i) - 400*i_2(t_i)$

Figure 2.8: *Single line and R-L load circuit at* $t = t_i + 0.0001$

Note we have two separate circuits, coupled together only by past values

Example 2.1: t=0.0001

Need: $i_1(-0.00045),\ \ v_1(-0.00045),\ \ i_2(-0.00045),$
$v_2(-0.00045),\ \ i_2(0),\ \ v_3(0),\ \ v_s(0.0001)$

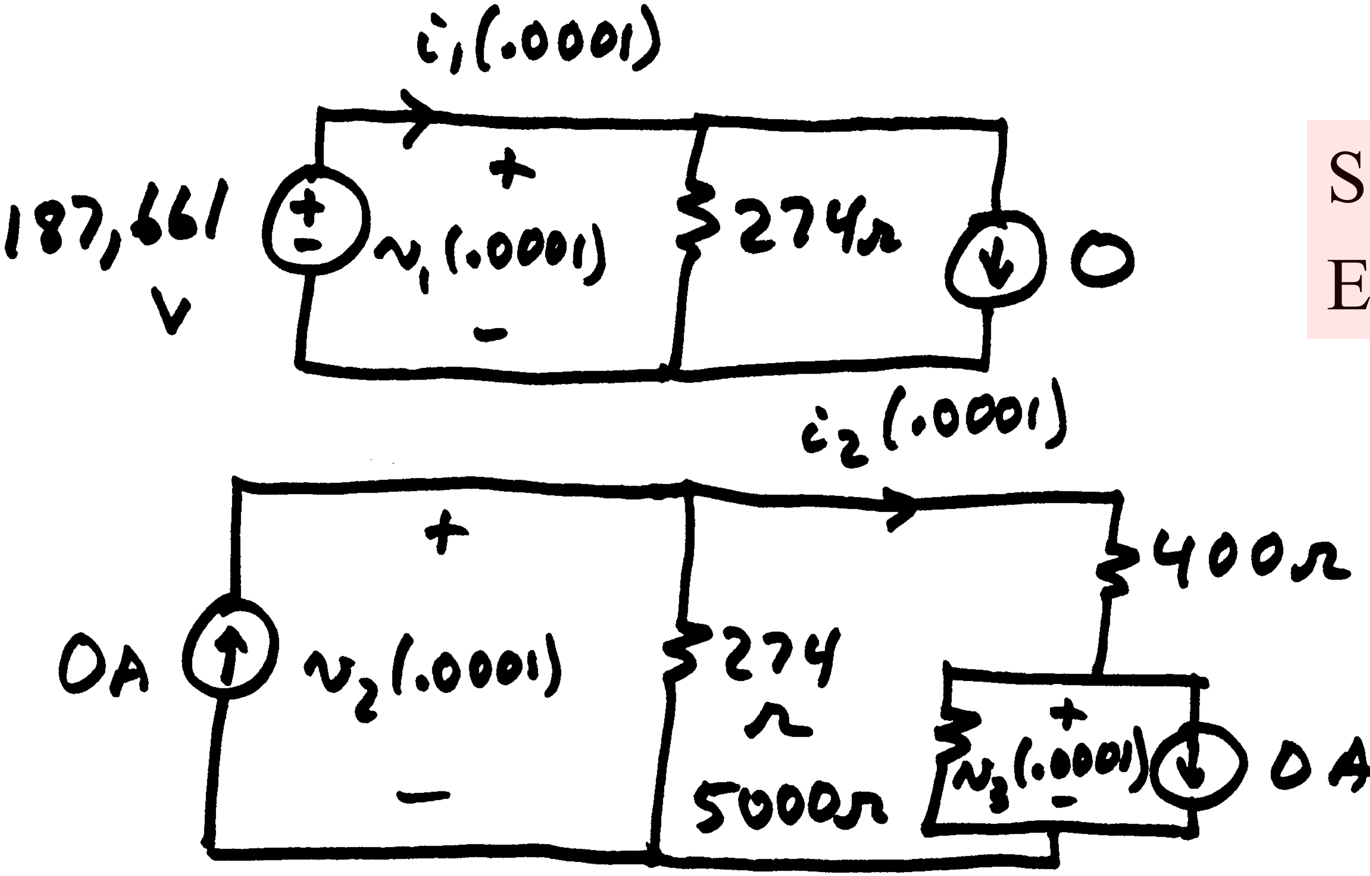
$$i_1(-0.00045)=0 \quad i_2(0)=0$$

$$v_1(-0.00045)=0 \quad v_3(0)=0$$

$$i_2(-0.00045)=0 \quad v_2(-0.00045)=0$$

$$v_s(0.0001)=230{,}000\sqrt{\frac{2}{3}}\cos(2\pi 60\times 0.0001)=187{,}661\,V$$

Example 2.1: t=0.0001



Sending End

Receiving End

Example 2.1: t=0.0001

$$i_1(0.0001) = 685A$$

$$v_1(0.0001) = 187{,}661V$$

Instantaneously changed from zero at $t = 0.0001$ sec.

$$i_2(0.0001) = 0$$

$$v_2(0.0001) = 0$$

$$v_3(0.0001) = 0$$

Example 2.1: t=0.0002

Need:

$$i_1(-0.00035) = 0$$

$$v_1(-0.00035) = 0$$

$$i_2(-0.00035) = 0$$

$$v_2(-0.00035) = 0$$

$$i_2(0.0001) = 0$$

$$v_3(0.0001) = 0$$

$$v_s(0.0002) = 187,261V$$

Circuit is essentially the same

$$i_1(0.0002) = 683A$$

$$v_1(0.0002) = 187,261V$$

$$i_2(0.0002) = 0.$$

$$v_2(0.0002) = 0.$$

$$v_3(0.0002) = 0.$$

Wave is traveling down the line

Example 2.1: t=0.0002 to 0.006

$$\frac{d}{v_p} = 0.00055 \qquad \Delta t = 0.0001$$

$$t_i = 0 \qquad\qquad t = 0.0001 \leftarrow$$

switch closed

$$t_i = 0.0001 \qquad\qquad t = 0.0002$$

$$= 0.0002 \qquad\qquad = 0.0003$$

$$= 0.0003 \qquad\qquad = 0.0004$$

$$= 0.0004 \qquad\qquad = 0.0005$$

$$= 0.0005 \qquad\qquad = 0.0006 \leftarrow$$

$$= 0.0006 \qquad\qquad = 0.0007 \leftarrow$$

With interpolation receiving end will see wave

Example 2.1: t=0.0007

Need: $i_1(.00015)$

$v_1(.00015),\ v_2(.00015)$

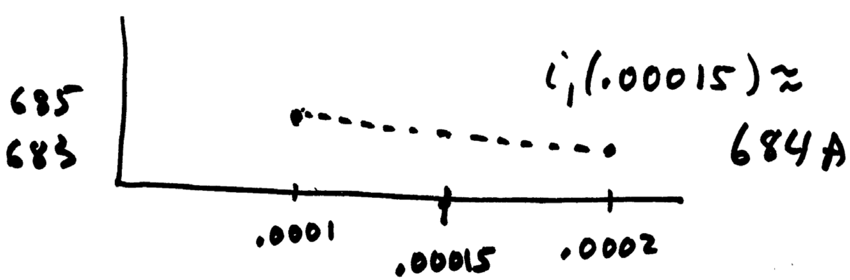
$i_2(.0006),\ v_3(.0006),\ v_s(.0007)$

$i_1(.0001) = 685A$

$i_1(.0002) = 683A$

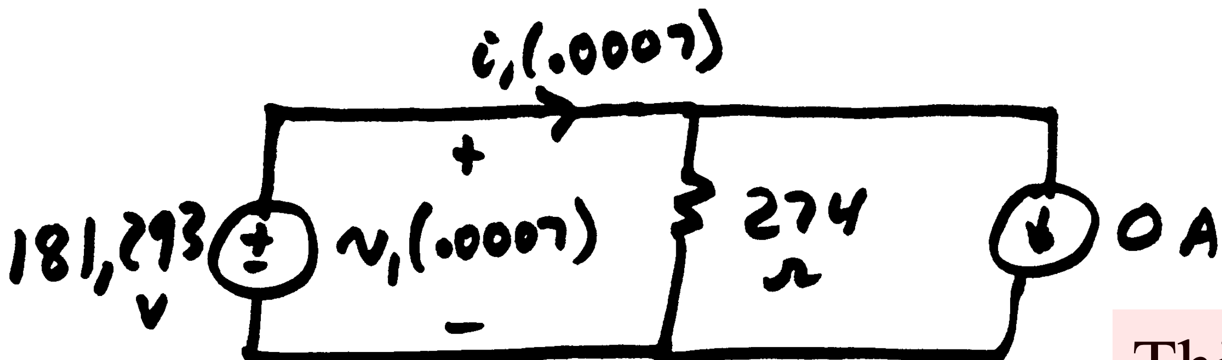
(linear interpolation)

$$i_1(.00015) \approx i_1(.0001) + \frac{.00015 - .0001}{.0002 - .0001} \times \left(i_1(.0002) - i_1(.0001)\right)$$



Example 2.1: t=0.0007

For $t_i = .0006$ ($t = .0007$ sec) at the sending end

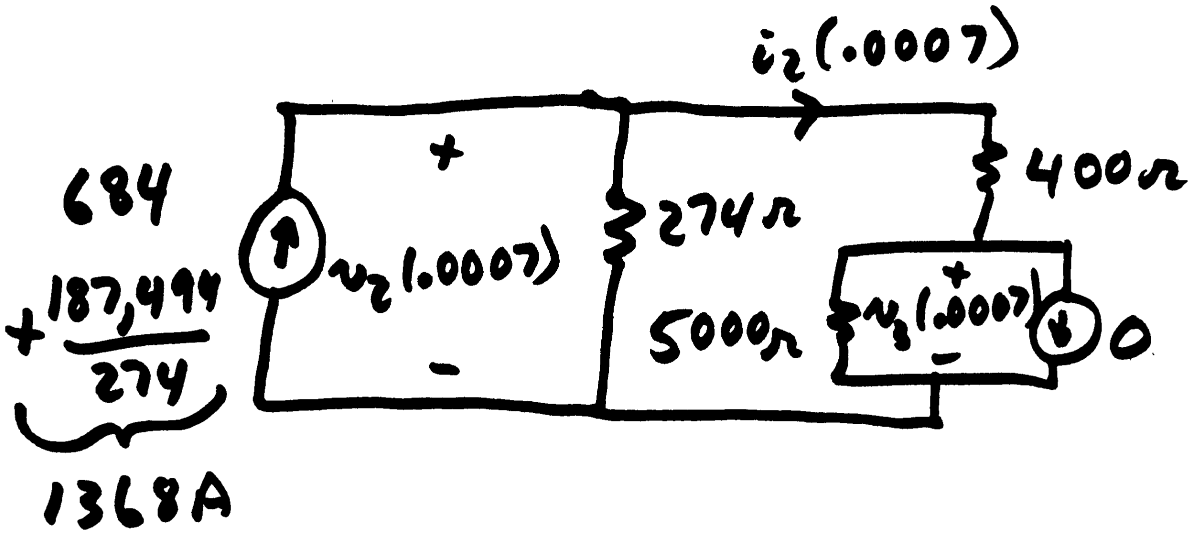


$$i_1(.0007) = 662A$$
$$v_1(.0007) = 181{,}293V$$

This current source will stay zero until we get a response from the receiving end, at about 2τ seconds

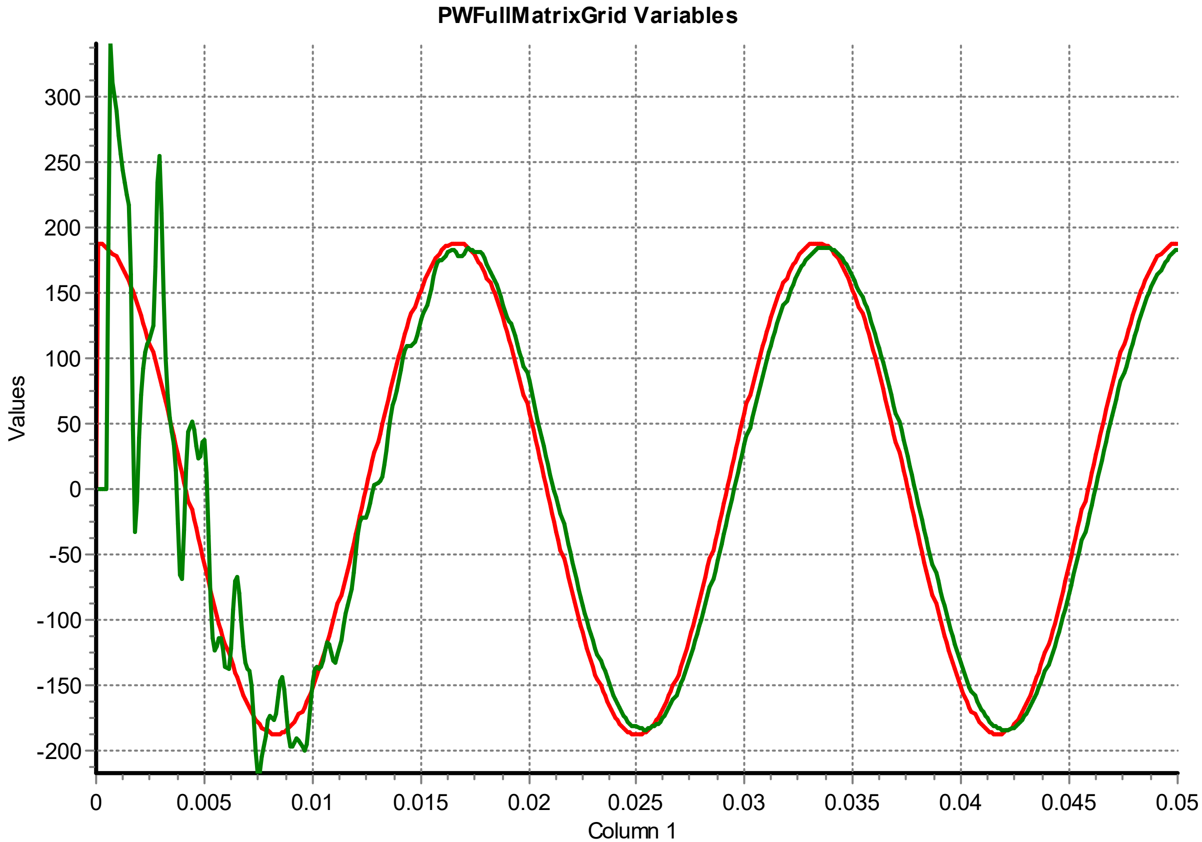
Example 2.1: t=0.0007

For t_i = .0006 (t = .0007 sec) at the receiving end



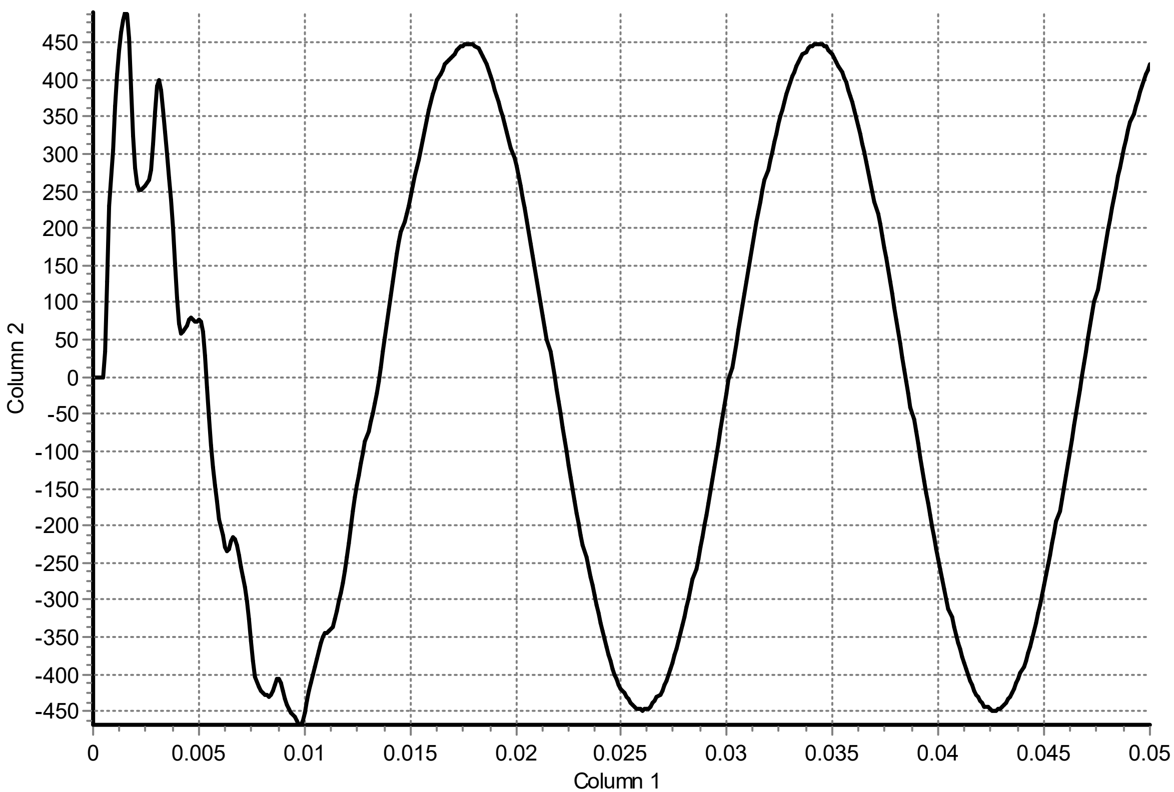
$$v_2(.0007) = 356{,}731V$$
$$i_2(.0007) = 66A$$

Example 2.1: First Three Cycles



Red is the sending end voltage (in kv), while green is the receiving end voltage. Note the approximate voltage doubling at the receiving end

Example 2.1: First Three Cycles

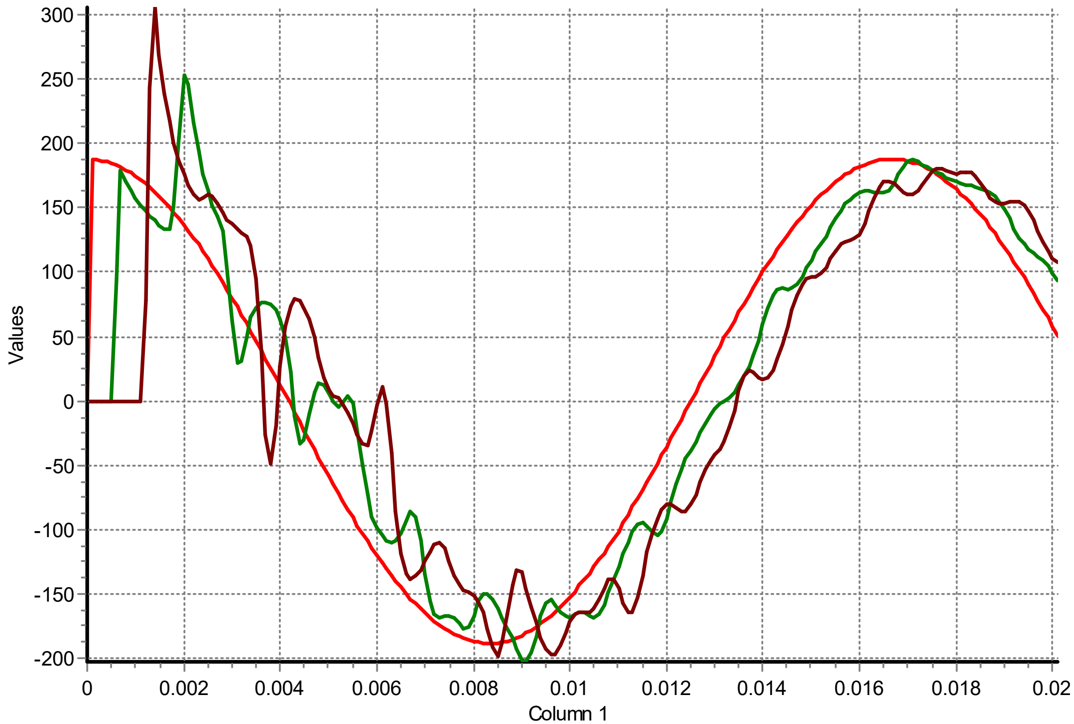


Graph shows the current (in amps) into the RL load over the first three cycles.

To get a **ballpark** value on the expected current, solve the simple circuit assuming the transmission line is just an inductor

$$I_{load,rms} = \frac{230,000/\sqrt{3}}{400 + j94.2 + j56.5} = 311\angle -20.6°,$$ hence a peak value of 439 amps

Three Node, Two Line Example

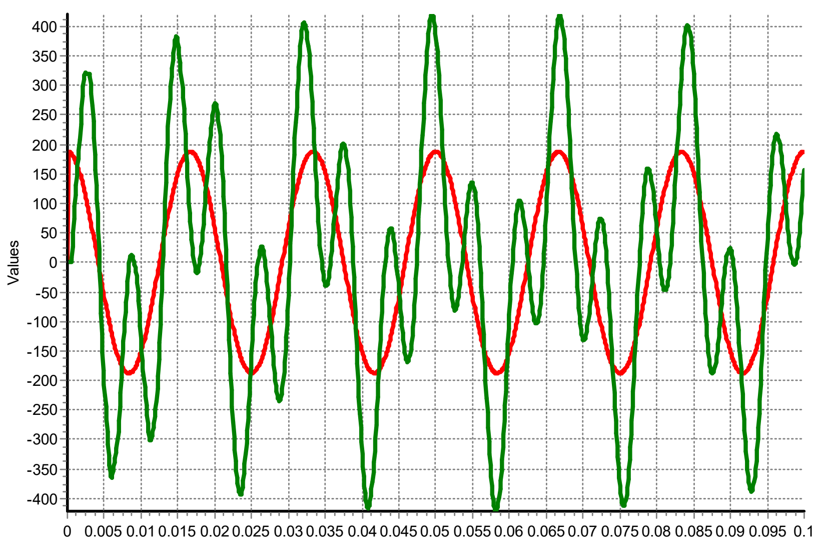


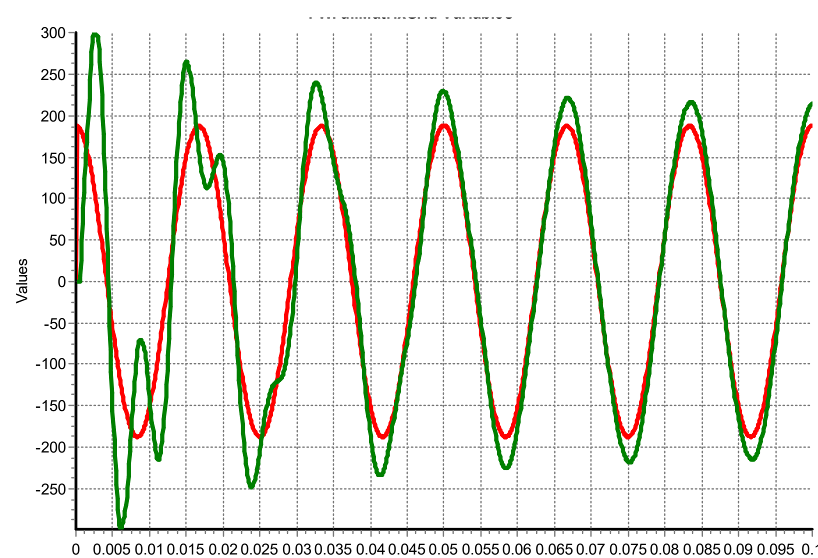
Graph shows the voltages for a total of 0.02 seconds for the Example 2.1 case extended to connect another 120 mile line to the receiving end with an identical load

Note that there is no longer an initial overshoot for the receiving (green) end since wave continues into the second line

Example 2.1 with Capacitance

- Below graph shows example 2.1 except the RL load is replaced by a 5 μF capacitor (about 100 Mvar)
- Graph on left is unrealistic case of no resistance in line
  - Since there is no resistance, there is no damp (dissipation)
- Graph on right has R=0.1 Ω/mile





EMTP Network Solution

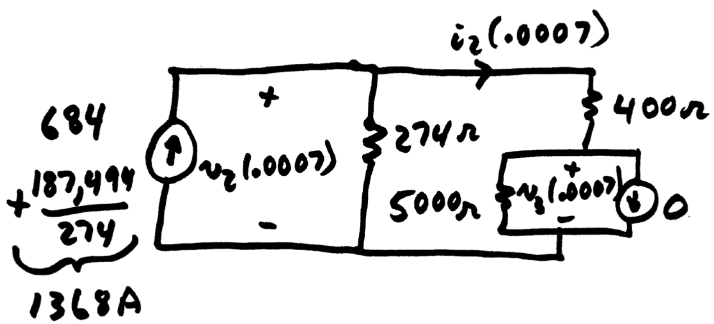
- The EMTP network is represented in a manner quite similar to what is done in the dc power flow or the transient stability network power balance equations or geomagnetic disturbance modeling (GMD)
- Solving set of dc equations for the nodal voltage vector **V** with

  $$\mathbf{V} = \mathbf{G}^{-1}\mathbf{I}$$

  where **G** is the bus conductance matrix and **I** is a vector of the Norton current injections

EMTP Network Solution

- Fixed voltage nodes can be handled in a manner analogous to what is done for the slack bus: just change the equation for node i to $V_i = V_{i,fixed}$
- Because of the time delays associated with the transmission line models **G** is often quite sparse, and can often be decoupled
- Once all the nodal voltages are determined, the internal device currents can be set
  - E.g., in example 2.1 one we know v_2 we can determine v_3



Three-Phase EMTP

- What we just solved was either just for a single phase system, or for a balanced three-phase system
  - That is, per phase analysis (positive sequence)
- EMTP type studies are often done on either balanced systems operating under unbalanced conditions (i.e., during a fault) or on unbalanced systems operating under unbalanced conditions
  - Lightning strike studies
- In this introduction to EMTP will just covered the balanced system case (but with unbalanced conditions)
  - Solved with symmetrical components

Modeling Transmission Lines

- Undergraduate power classes usually derive a per phase model for a uniformly transposed transmission line

$$L = \frac{\mu_0}{2\pi}\ln\frac{D_m}{R_b} = 2\times10^{-7}\ln\frac{D_m}{R_b} \quad \text{H/m}$$

$$C = \frac{2\pi\varepsilon}{\ln \frac{D_m}{R_b^c}}$$

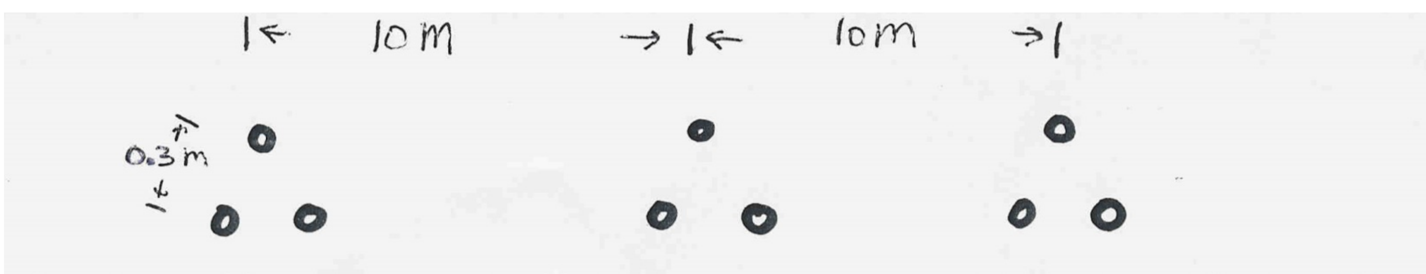
$$\text{D}_\text{m} = \left[d_{ab}d_{ac}d_{bc}\right]^{1/3} \qquad R_b = (\text{r}' d_{12}\cdots d_{1n})^{1/n}$$

$$\text{R}_\text{b}^\text{c} = (r d_{12}\cdots d_{1n})^{1/n} \quad \text{(note r NOT r')}$$

$$\varepsilon \text{ in air} = \varepsilon_\text{o} = 8.854\times10^{-12} \text{ F/m}$$

Modeling Transmission Lines

- Resistance is just the Ω per unit length times the length
- Calculate the per phase inductance and capacitance per km of a balanced 3ϕ, 60 Hz, line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1.5 cm radius and resistance = 0.06 Ω/km



Modeling Transmission Lines

$$D_m = (10\times10\times20)^{1/3} = 12.6\text{m}$$

$$R_b = (0.78\times0.015\times0.3\times0.3)^{1/3} = 0.102\text{m}$$

$$L = 2\times10^{-7}\ln\frac{12.6}{0.102} = 9.63\times10^{-7}\text{H/m} = 9.63\times10^{-4}\text{H/km}$$

$$R_b^c = (0.015\times0.3\times0.3)^{1/3} = 0.1105\text{m}$$

$$C = \frac{2\pi\times8.854\times10^{-12}}{\ln{12.6}/{0{,}1105}} = 1.17\times10^{-11}\text{F/m} = 1.17\times10^{-8}\text{F/km}$$

- Resistance is 0.06/3=0.02Ω/km
  - Divide by three because three conductors per bundle