Graph Crossings in Electric Transmission Grids

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***Abstract*—The number of times that transmission lines in a power system intersect one another without connecting electrically, that is, the number of graph crossings, gives insight into the level of non-planarity for a power grid network. This paper describes the importance of these crossings metrics and discusses challenges associated with calculating them. Example results are given for the geographic spatial embedding of actual United States power grid networks, based on public data from the U. S. Energy Information Administration. These results show that within voltage classes and removing main sources of data errors, there are up to 15% as many edge crossings as the number of lines, with fewer crossings both in the very highest voltage levels and in those closer to sub-transmission. A comparison is also made between using actual right-of-way routing and straight-line routing, with the latter circumventing more data errors and producing smaller and more consistent crossing counts.**

***Keywords*—power grids, crossing numbers, complex networks, intersections**

I. INTRODUCTION

In recent years, significant insights have been drawn from analyzing the electric grid as a complex network, that is, applying graph theoretic analytical approaches [1]-[2]. One important application is vulnerability assessment, such as for understanding the risk of cascading outages [3] and targeted cyber or physical threats [4]. Recent work has leveraged structural analysis for building synthetic networks that mimic actual grid properties, first with only a synthetic graph topology [5], then with full electrical analysis as well [6]-[7]. Visualization applications also often make important assumptions about electric grid structure [8]-[9]. Topological analysis is further enhanced when metrics are added that consider spatial embedding of circuit nodes: the geographic context of electric grid networks. Such a paradigm is particularly relevant when it comes to the subject of planning for a robust network, as potential new transmission lines are, by nature, geographically constrained [3].

Power grids fall into a specialized category among real-world networks in that they are highly geographically constrained but decidedly non-planar. Unlike professional collaboration graphs and the World Wide Web, there are strong spatial constraints that prevent any two arbitrary vertices from being connected. However, unlike waterways and biking trails, the fact that two lines intersect does not mean there is an electric connection there. By analyzing these non-connecting intersections or *crossings*, this paper aims to quantify where along the spectrum of non-planarity power grids lie. (An analysis of road networks is given in [10] which shows these are nonplanar as well, though these systems have far fewer crossings than electric grids do, as this paper will show.) Hence this paper applies the concept of *graph crossing number* as a topological-geographic metric for characterizing electric grid network structure in a real or synthetic grid planning context.

Understanding graph crossing number properties of large, high-voltage electric grid networks is of interest for several reasons. First, they are a metric for system planning and design. Intuitively, transmission line crossings are spatially inefficient, difficult to build, and would tend to be avoided; hence in comparing grids for validation and design evaluation, this number shows where historical, engineering, or economic factors constrained the idealistic system layout. Second, crossing number properties have implications for grid visualization and diagram drawing. Planar graphs have many drawing techniques, but power grids are non-planar, and the crossings form challenges in showing a system for maximum human comprehension [8]. Third, there are implications for energy resilience analysis in the study of reliability and cascading failures: grids with high crossing numbers have more points at which multiple transmission lines are jointly vulnerable.

Previous work has looked at many properties of power grid networks in their topological structure, such as degree distribution, clustering, and shortest-path diameter [1]-[3], [5]-[6]. But prior work has not emphasized consideration of the spatial embedding, partially due to limited availability of geographic information for power systems. Hence the authors have not found prior analysis of power system crossing number metrics. The contribution of this paper is to apply graph crossing numbers as a design metric for electric grids, present a modeling framework for accurately calculating it, and discuss its potential applications for grid planning and validation. Example results are also given for public U.S. datasets.

II. GRAPH CROSSING NUMBER

Given a graph $\boldsymbol{G}(V, E)$, with vertices V and edges E, a drawing $\boldsymbol{D}$ of that graph is a mapping of the edges and vertices to a surface, such as a finite 2D plane. One way to do this is to assign each vertex a spatial coordinate (x_i, y_i) and each edge a path consisting of a sequence of of linear segments, with waypoints marked out by coordinates (x_j, y_j). A *crossing* is a place where the paths of two edges $e_1 \in E$ and $e_2 \in E$ geometrically intersect other than at a vertex. Calculating the classical graph crossing number $\mathrm{gc}(\boldsymbol{G})$ involves finding the drawing $\boldsymbol{D}$ among all possible drawings of $\boldsymbol{G}$ for which the number of crossings is at a minimum, then reporting the number of crossings for that drawing [11].

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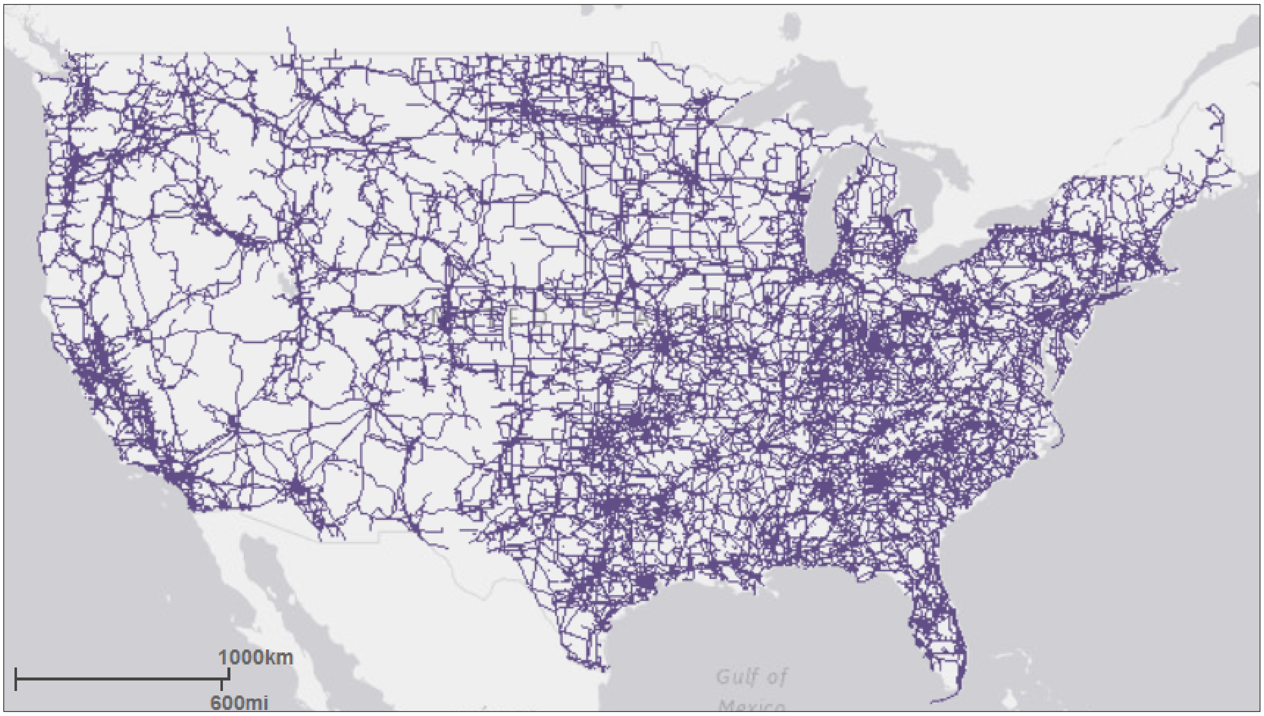

Fig. 1. Transmission line geographic embedding map, from EIA dataset [13].

While this property has the advantage that it is purely reflective of the network topology and does not depend on how one draws the graph, calculating this number is a proven NP-hard problem and is primarily practical only for very small graphs, unlike large power systems [12]. In addition, for most purposes related to this work, only one spatial embedding is relevant: the geographic embedding of the network on a two-dimensional map.

III. Crossing Number Analysis on Electric Grids

A. *Representing the Electric Grid as a Graph*

Electric grids topological analysis is highly sensitive to how a grid is modeled or represented as a graph [7]. Highly-detailed models are available, where each breaker and circuit node is included. Or, more typically, bus-branch modeling can be used, with only lines and transformers as edges. Here, any edge internal to a substation is irrelevant, as we are only interested in crossings external to substations. Each substation is considered to be a single vertex and transmission lines only are edges. Fig. 2 illustrates this difference.

B. *Dividing by Voltage Class*

Modeling every substation and transmission line in this way can lead to an absurdly large number of crossings. For an example U.S. dataset, over 44,000 crossings can be identified for 74,000 transmission lines—about 60%. The main reason this simplified framework leads to such a large percentage of crossings is that crossings between transmission lines of different voltage classes are distinct from same-voltage crossings. Different-voltage pairs of lines could not connect anyway without a substation transformer; extra-high-voltage lines are designed to carry long distances and must therefore pass by many lower-voltage substations that are not configured to connect to that voltage level. While this is a true aspect of the combined grid's non-planarity, a better approach is to recognize that the full power grid is actually the union of several graphs, one at each nominal voltage class. It would be somewhat unintuitive to have two same-voltage transmission lines pass one another without connecting electrically, but this certainly can occur due to engineering constraints, or economic or physical barriers. Therefore, the approach here, which seems most suitable as a design and validation metric, is to examine each voltage class independently as a graph and report its own count of line crossings.

C. *Straight Line Routing*

The methodology this paper proposes offers two variations for the exact path to use for transmission line edges in a given geographic drawing ***D***. For some datasets, the actual right-of-way path of a given line is available, although issues in data quality could lead to superfluous reported crossings (which can be somewhat mitigated, see Appendix). An alternative to using actual right-of-way for transmission lines is to approximate each

TABLE I
GEOGRAPHIC CROSSINGS FOR EIA DATASET VOLTAGE NETWORKS

Voltage class	Number of substations	Number of lines	Number of lines, no parallel	Crossings, straight-line		Crossings, right-of-way		Crossings, no parallel	
				Number	% of lines	Number	% of lines	Number	% of lines
765 kV	40	42	42	1	2.4	1	2.4	1	2.4
500 kV	529	732	596	67	9.2	219	29.9	162	27.2
345 kV	1526	2171	1778	297	13.7	628	28.9	563	31.7
230 kV	4648	6233	5109	935	15.1	2266	36.4	1977	38.7
161 kV	2633	3172	2858	405	13.0	952	30.0	845	29.6
138 kV	8611	10684	9129	1617	15.3	2951	27.6	2658	29.1
115 kV	12826	15031	13161	1485	10.2	3137	20.9	2734	20.8
100 kV	894	1595	1002	118	7.5	282	17.7	215	21.5
69 kV	8022	8022	7271	289	3.7	618	7.7	582	8.0

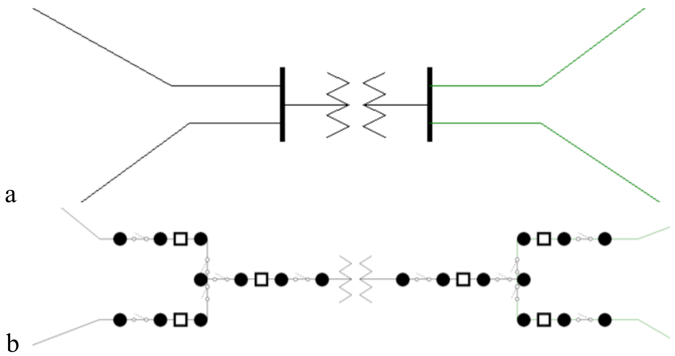

Fig 2. Effect of modeling detail on network metrics (from [7]). This substation can be modeled with two buses (a) or with a full topology of 20 nodes (b). With all switches and breakers closed, these two models are electrically identical, but expanding to a full topology representation can affect multiple metrics—in this case, reducing the number of intersections as a percentage of total nodes or branches.

line as a single straight segment between its two endpoints, grouping endpoints into substation locations. In principle, this could introduce new crossings, as some lines might be routed non-straight to avoid intersecting another line. But in practice, as example results below will show, the number of crossings is significantly reduced. The major advantage of this approach is that it circumvents many cases of data errors. In most cases, the substation geographic location is quite accurate, and many lines which cross each other in practice will still do so with straight-line approximations.

IV. EXAMPLE RESULTS FOR PUBLIC U.S. GRID DATA

As an example application of this metric, this section of the paper analyzes public data for the United States electric transmission network, with geographic coordinates available from the U. S. Energy Information Administration (EIA) [13]. Fig. 1 shows the diagram of this dataset. This dataset gives actual paths for each high-voltage transmission line in the U. S., plus other information such as the nominal voltage class.

Intersections are found by comparing every pair of poly-segmented paths to find geometric points of intersection. A direct calculation for the 74,553 transmission line routes in the EIA dataset yields 44,732 crossings, which is 60.0% of the number of lines. (All numbers here exclude intersections very close to either line's endpoint, which are assumed to actually be a vertex incidence rather than a meaningful crossing. In addition, between any given pair of branches only the first intersection is counted.)

When the graph is divided by voltage class as explained in Section II.B, the results in the seventh and eighth column of Table I appear. As the header indicates, this is again using the full right-of-way paths as specified in the dataset and comparing each set of transmission lines pairwise. The number of lines in column three does not sum to the total given above because there are other voltage levels that are too small to be looked at separately, and some lines were not labeled for voltage class in the dataset. When examining the percent crossing data in column eight of Table I, several observations can be made. It seems that the very highest-voltage level, 765 kV, has only one crossing. The lower voltage levels close to subtransmission (69 kV to 115 kV) also have fewer crossings than those in the middle, up to about 20%. The rest (138 kV to 500 kV) have crossings around 30% of the number of lines. This trend, that planarity increases both near the upper and lower ends of the transmission voltage class spectrum, persists through the analysis here.

Results for straight-line routing are given in the fifth and sixth columns of Table I. Many of the same general properties are present as in the right-of-way routing. The 765 kV grid is still nearly-planar with only one crossing, and the lower voltages (69 kV to 115 kV) also have lower crossing counts. Besides the 765 kV grid, every grid has half or fewer of the number of crossings as before, with an average of 10% instead of 23%. The highest-crossing graphs, 230 kV and 138 kV, have about 15% crossings.

V. CONCLUSIONS

This paper has shown how the concept of graph crossing number can be applied as a structural metric of electric power system networks. Transmission line right-of-way data from a public EIA information source [13] was used as an example to examine graph crossings in power systems. Studying each voltage level network independently, the results indicate that using the full right-of-way subjects the analysis to a number of

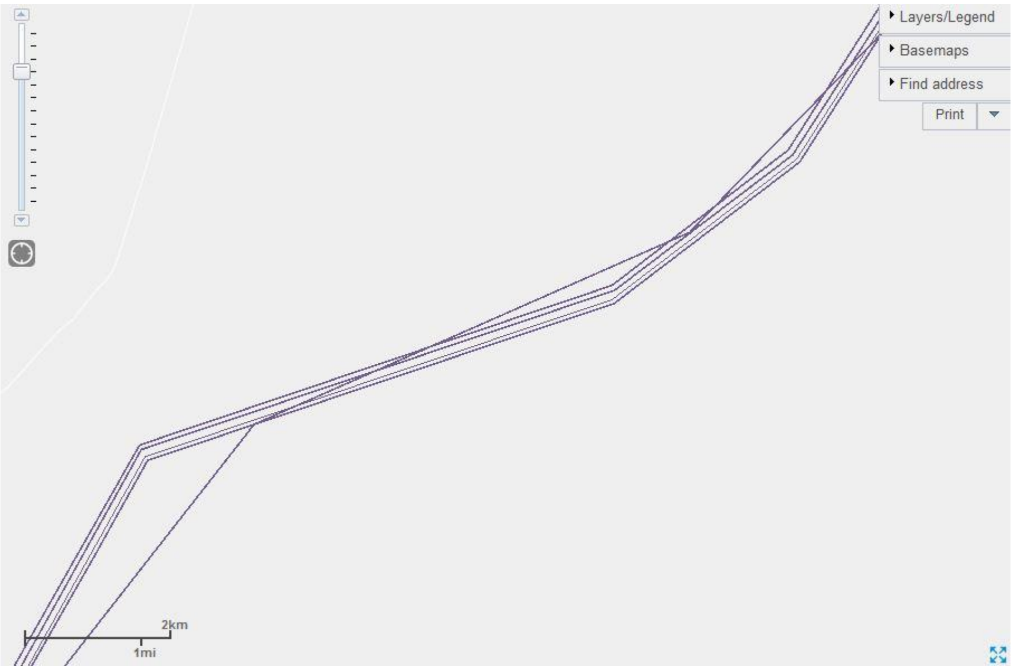

Fig. 3. Example of a double-reported line causing multiple apparent crossings due to data quality limitation, from EIA dataset [7]. There are only two lines here in reality, a single double-circuit tower. But five are reported, one with a different resolution and slightly different geographic coordinates.

errors due to limitations of the data quality for the full right-of-way coordinates. When straight-line routing is used, although some of the realism in detail is lost, the results are more robust to data errors and hence show more consistently that voltage networks tend to have up to 15% crossings of the number of lines, and fewer both closer to the sub-transmission and at the very highest voltages. These observations contextualize the power grid in the complex network terms and underscore the importance of geography and voltage class for understanding, planning, and visualizing transmission grids.

APPENDIX: ADDRESSING DATA QUALITY LIMITATIONS

Using a right-of-way based crossings analysis depends heavily on the accuracy of the data given. While the accuracy of the dataset used here appears to be high in general, small errors can introduce new crossings that inflate the apparent numbers.

Frequently, multiple transmission lines will share a transmission tower or common right-of-way for some distance. If the lines are very close together, or even vertically stacked on a tower, their reported paths may geometrically intersect though the lines do not actually cross physically. This could potentially happen multiple times along a shared path, which is one reason only the first intersection is counted in this letter's analysis. However, two lines may also share a right-of-way for a distance, then legitimately cross and continue in a opposite directions. These two situations cannot be readily distinguished.

Another example of a data quality limitation that was observed with this dataset is double-reporting of lines. Double, triple, and higher-order circuit transmission lines are certainly common. However, this dataset is a collection of data elements from various sources, and on occasion the same physical lines appear twice with slightly different approximations of the path. Such pairs of lines will potentially cross one another multiple times along their nearly-shared route. This phenomenon is illustrated in Fig. 3.

Data quality varies significantly by geographic area, which is reasonable considering the heterogeneous sources of the data. The 732 lines in the 500 kV grid, for example, cross 759 times when multiple crossings are allowed per pair of lines, with most of these additional crossings from a subset of lines in one portion of the network. A supplementary analysis was done that excluded approximately 10% of the land area of the U. S. and about half of the 500 kV lines. This area had been manually determined to have a large number of data errors of the types discussed above. The remaining 257 lines outside this area had only 69 crossings, down from 103.7% to 26.8%. This shows the significant impact data errors can have on the right-of-way crossing counts.

One approach to examine the multiple-circuit and shared right-of-way issues, in addition to restricting the crossings to one per pair, can be observed in columns nine and ten of Table I. In this analysis, parallel circuits (real or artefacts of data errors) are removed by checking whether for each pair of lines they share starting and ending points within a small neighborhood. This removes some of the lines, as noted in column four of Table I, but only affects branches that are truly parallel throughout their route. The effect on most of the networks was not significant, even detrimental to the number of crossings, if there were lots of parallel lines without crossings. Hence the parallel branches do not have a major impact on the crossing counts.

Other data quality limitations for which multiple examples were found include mis-labeled voltage classes, different levels of detail in the right-of-way coordinates, and double-reported lines which did not match the original topology. An example of the last would be the same line being reported as a single line from one data source and a sequence of two lines in the second data source. These cases would not be filtered out by removing parallel lines.

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