# ECEN 667
# Power System Stability

## Lecture 7: Stability Overview, Synchronous Machine Modeling

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# Announcements

- Homework 2 is due on Thursday September 23
- Read Chapter 5
- The EPG dinner will again take place this semester, hosted by Dr. Begovic and his wife on Saturday September 25th from 5 to 7:30pm. This is for all EPG Faculty, Staff and Students including families (and anyone in 667 is eligible). The meal will be catered. However you must RSVP by today at https://forms.gle/XyN3hc6Md1Mi3YUv9

# Kersting Example 4.1

- For this example the full **Z** matrix is

$$\mathbf{Z} = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix}$$

- Partition the matrix and solve $\mathbf{Z}_p = \left[ \mathbf{Z}_A - \mathbf{Z}_B \mathbf{Z}_D^{-1} \mathbf{Z}_C \right]$
- The result in Ω/mile is

$$\mathbf{Z}_p = \begin{bmatrix} 0.4576 + 1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix}$$

# Kersting Example 4.1, cont.

- Then to convert to the sequence matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \text{ with } \alpha = 1\angle 120°$$

Then

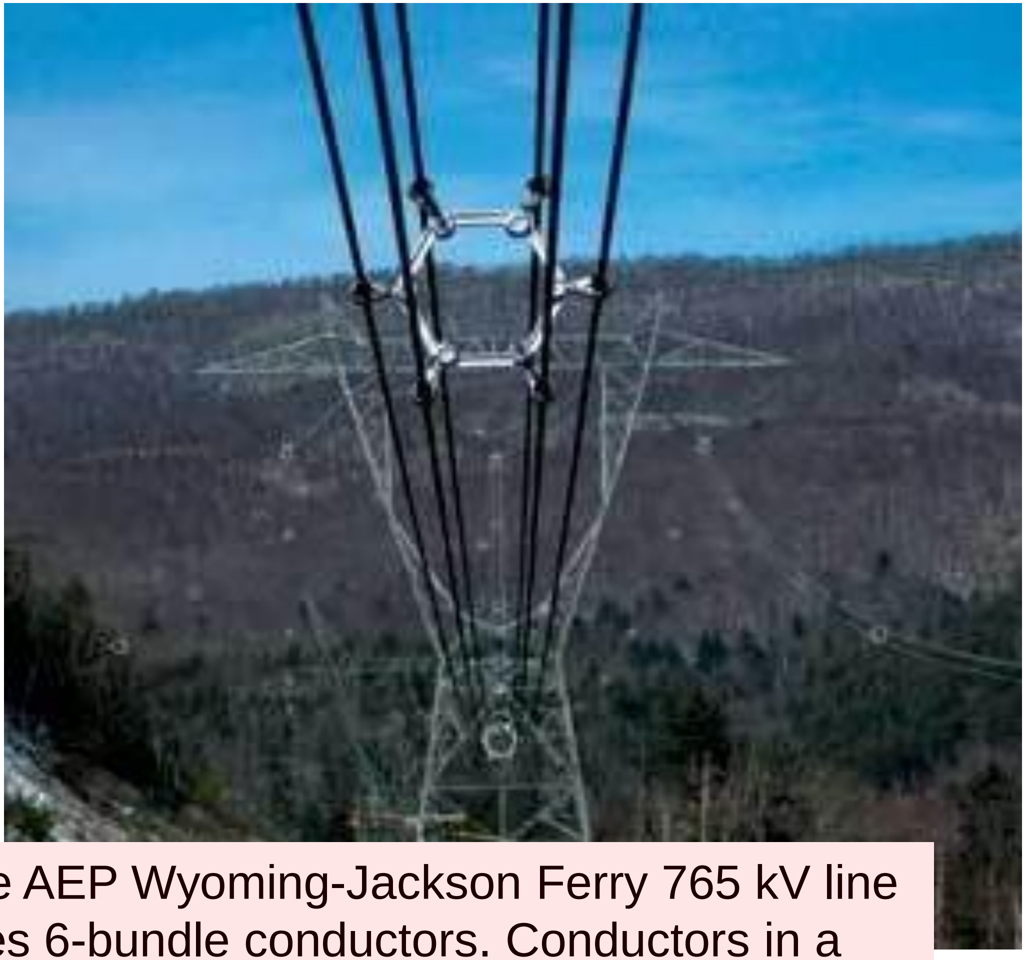
$$Z_s = \mathbf{A}^{-1}\mathbf{Z}_p\mathbf{A} = \begin{bmatrix} 0.7735 + j1.9536 & 0.0256 + j0.0115 & -0.321 + j0.0159 \\ -0.0321 + j0.0159 & 0.3061 + j0.6270 & -0.0723 - j0.0060 \\ 0.0256 + j0.0115 & 0.0723 - j0.0059 & 0.3061 + j0.6270 \end{bmatrix}$$

The diagonal elements are the sequence values, with the positive and negative sequence values equal, and the zero sequence about three times their value. The non-zero off-diagonals indicates that there is mutual coupling between the phases.

# Substation Bus

# Symmetric Line Spacing – 69 kV

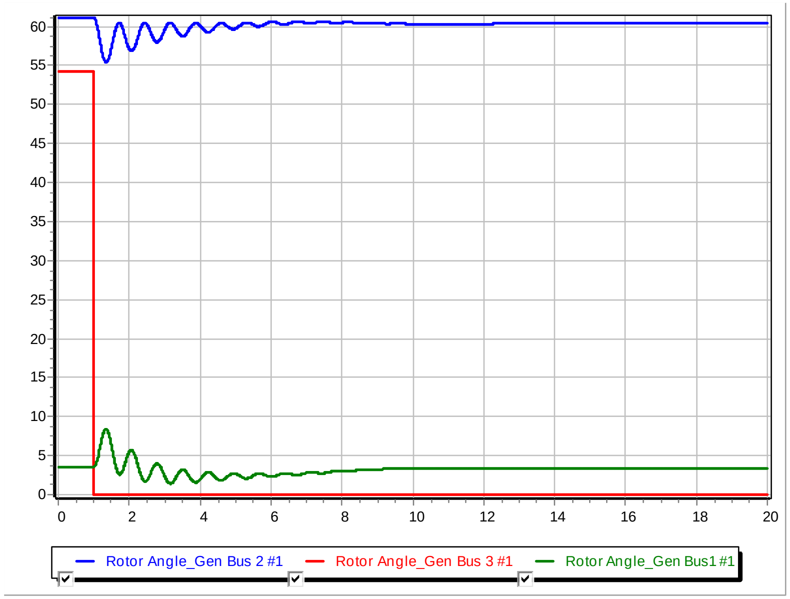
# Bundled Conductor Pictures

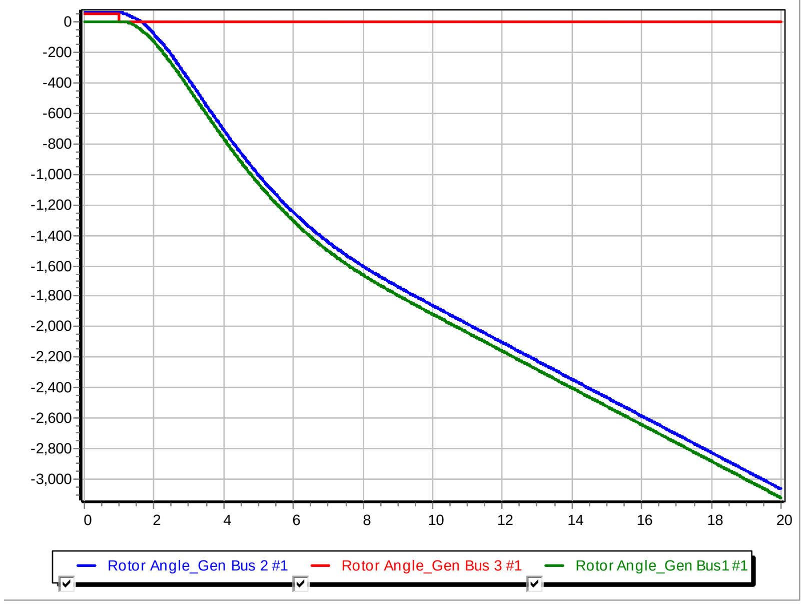


The AEP Wyoming-Jackson Ferry 765 kV line uses 6-bundle conductors. Conductors in a bundle are at the same voltage!

Photo Source: BPA and American Electric Power

# Returning to the Simulation: Generator Angles on Different Reference Frames



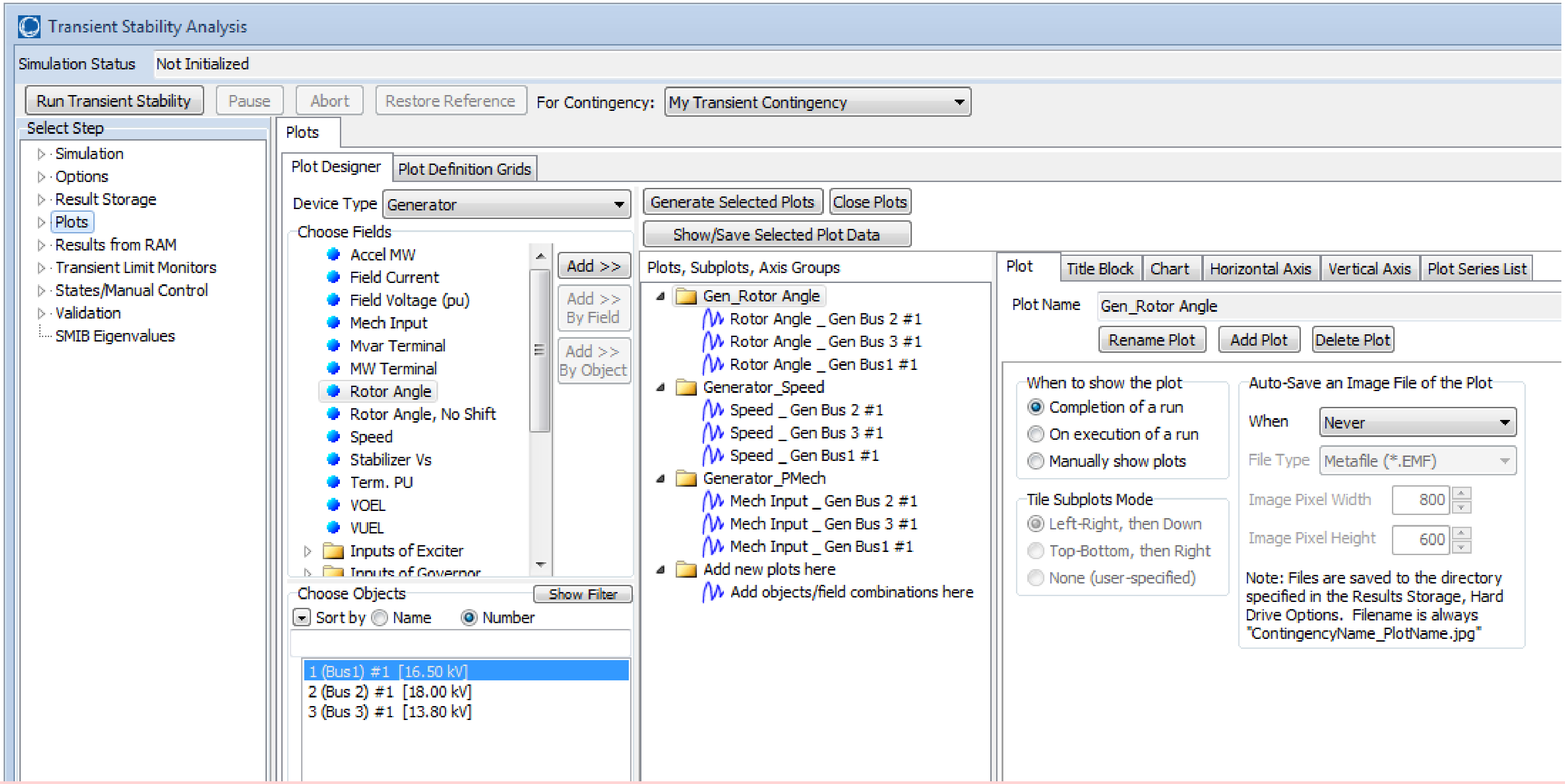




Average of Generator Angles Reference Frame

Synchronous Reference Frame

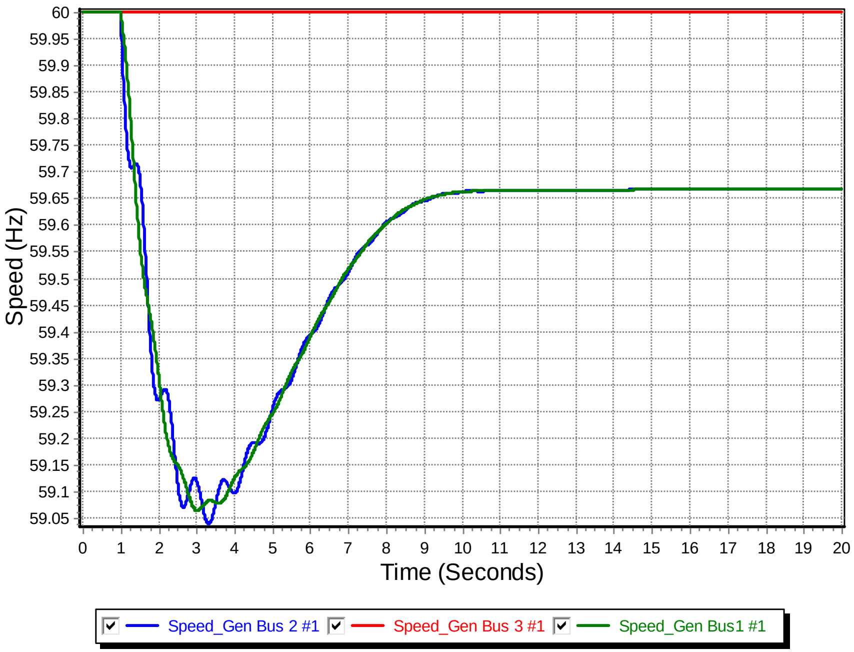
Both are equally "correct", but it is much easier to see the rotor angle variation when using the average of generator angles reference frame

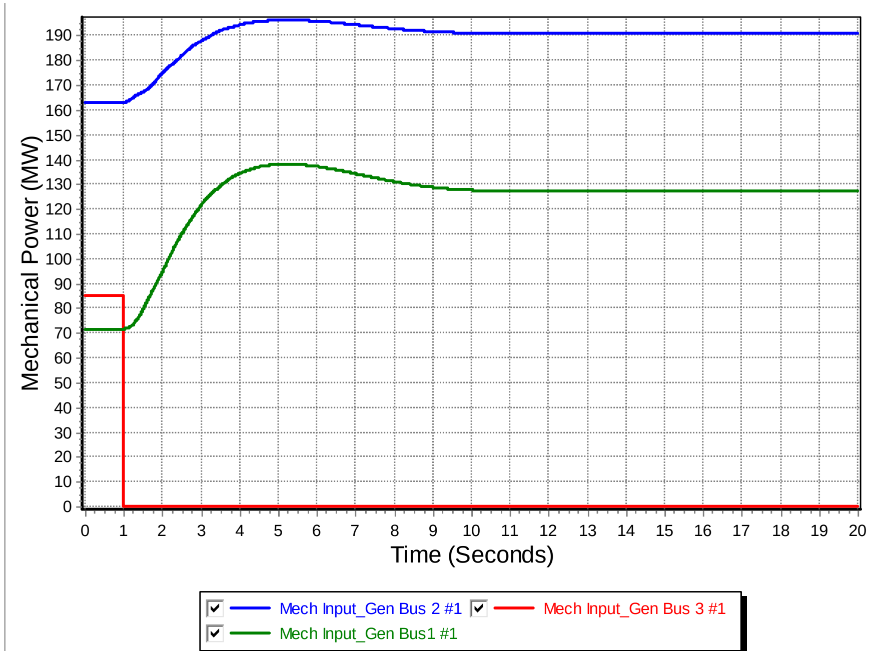
# Plot Designer with New Plots with the WSCC Nine Bus Case




Note that when new plots are added using "Add Plot", new Folders appear in the plot list. This will result in separate plots for each group

# Gen 3 Open Contingency Results

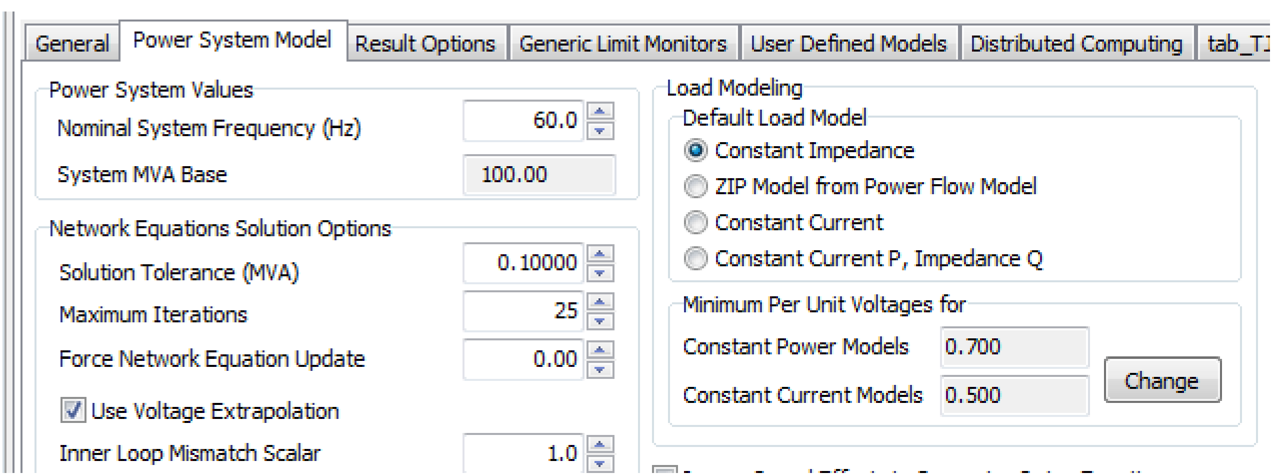



The left figure shows the generator speed, while the right figure shows the generator mechanical power inputs for the loss of generator 3. This is a severe contingency since more than 25% of the system generation is lost, resulting in a frequency dip of almost one Hz. Notice frequency does not return to 60 Hz.

# Load Modeling

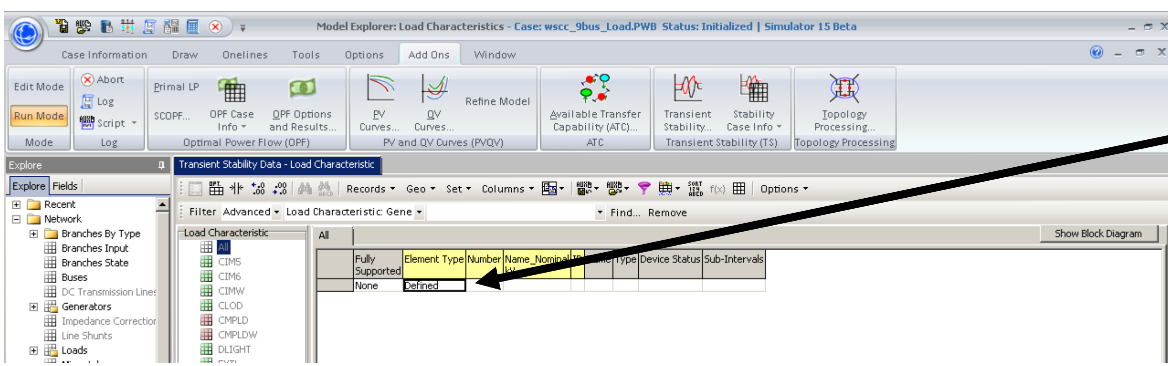
- The load model used in transient stability can have a significant impact on the results
- By default PowerWorld uses constant impedance models but makes it very easy to add more complex loads.
- The default (global) models are specified on the Options, Power System Model page.




These models are used only when no other models are specified.

# Load Modeling

- More detailed models are added by selecting **Case Information, Model Explorer, Transient Stability, Load Characteristics Models**.
- Models can be specified for the entire case (system), or individual areas, zones, owners, buses or loads.
- To insert a load model click right click and select insert to display the Load Characteristic Information dialog.



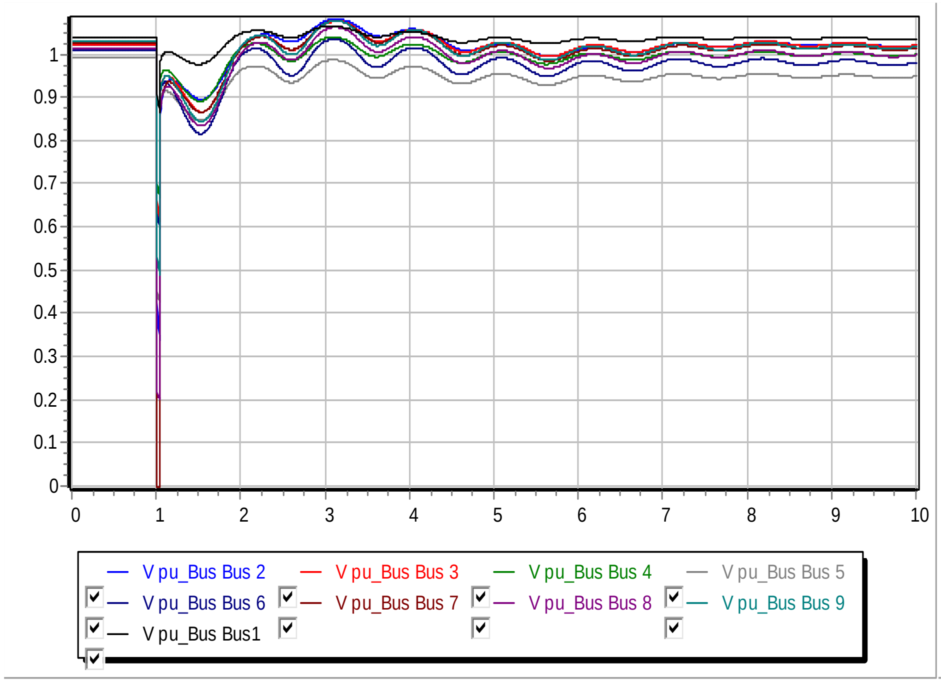

Right click here to get local menu and select insert.

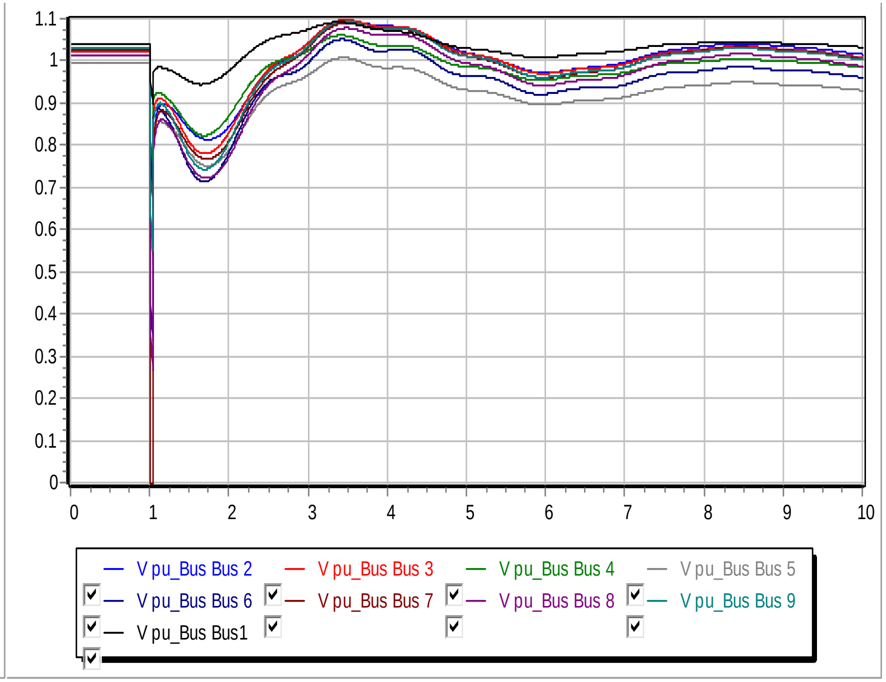
# Dynamic Load Models

- Loads can either be static or dynamic, with dynamic models often used to represent induction motors
- Some load models include a mixture of different types of loads; one example is the CLOD model represents a mixture of static and dynamic models
- Loads models/changed in PowerWorld using the Load Characteristic Information Dialog
- Next slide shows voltage results for static versus dynamic load models
- Case Name: **WSCC_9Bus_Load**

# WSCC Case Without/With Complex Load Models

- Below graphs compare the voltage response following a fault with a static impedance load (left) and the CLOD model, which includes induction motors (right)



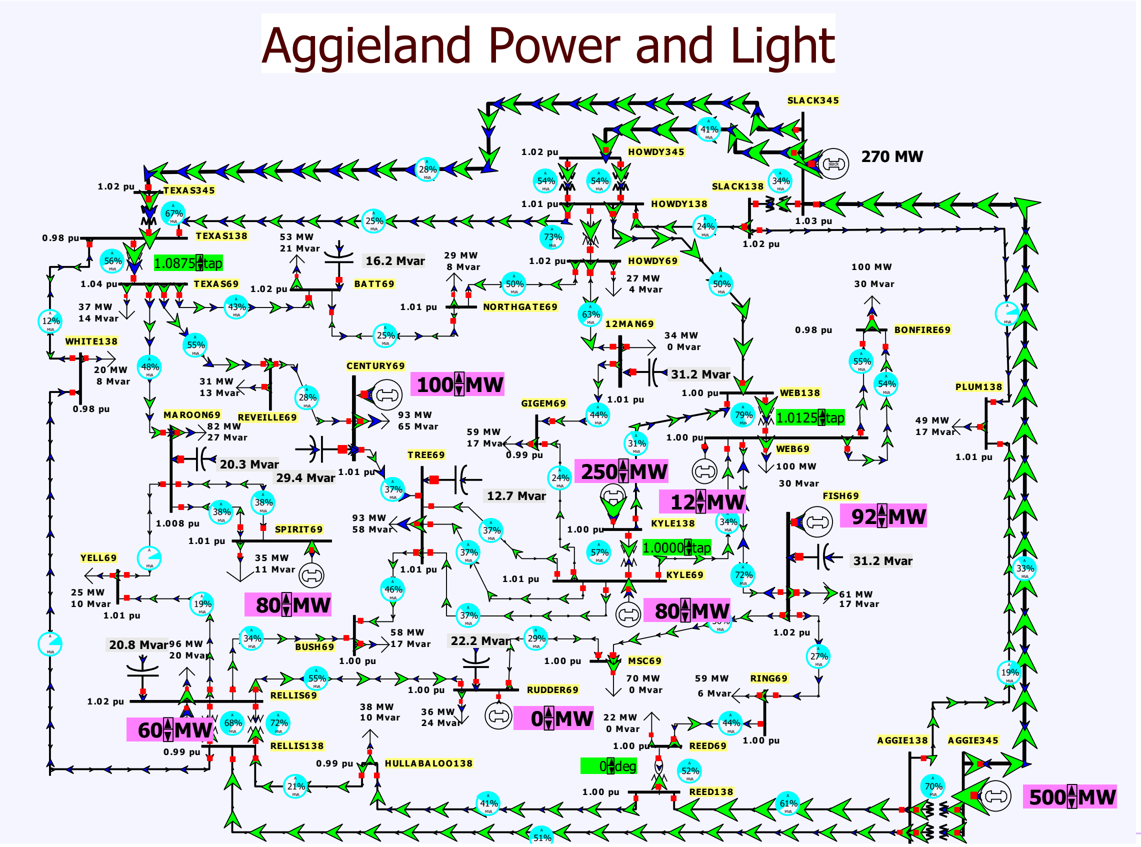



# Under-Voltage Motor Tripping

- In the PowerWorld CLOD model, under-voltage motor tripping may be set by the following parameters
  - Vi = voltage at which trip will occur (default = 0.75 pu)
  - Ti (cycles) = length of time voltage needs to be below Vi before trip will occur (default = 60 cycles, or 1 second)
- In this example change the tripping values to 0.8 pu and 30 cycles and you will see the motors tripping out on buses 5, 6, and 8 (the load buses) – this is especially visible on the bus voltages plot. These trips allow the clearing time to be a bit longer than would otherwise be the case.
- Set Vi = 0 in this model to turn off motor tripping.

# 37 Bus System

- Next we consider a slightly larger, ten generator, 37 bus system. To view this system open case **AGL37_TS**. The system one-line is shown below.




To see summary listings of the transient stability models in this case select “Stability Case Info” from the ribbon, and then either “TS Generator Summary” or “TS Case Summary”

# Transient Stability Case and Model Summary Displays

Models in Use | Generators | Load Characteristics | Load Summary

Filter: Advanced | TSModelSummaryObject

| | Model Class | Object Type | Active and Online Count | Active Count | Inactive Count | Fully Supported |
|---|---|---|---|---|---|---|
| 1 | Machine Model | GENSAL | 1 | 1 | 0 | YES |
| 2 | Machine Model | GENROU | 9 | 9 | 0 | YES |
| 3 | Exciter | IEEET1 | 10 | 10 | 0 | YES |
| 4 | Governor | TGOV1 | 10 | 10 | 0 | YES |

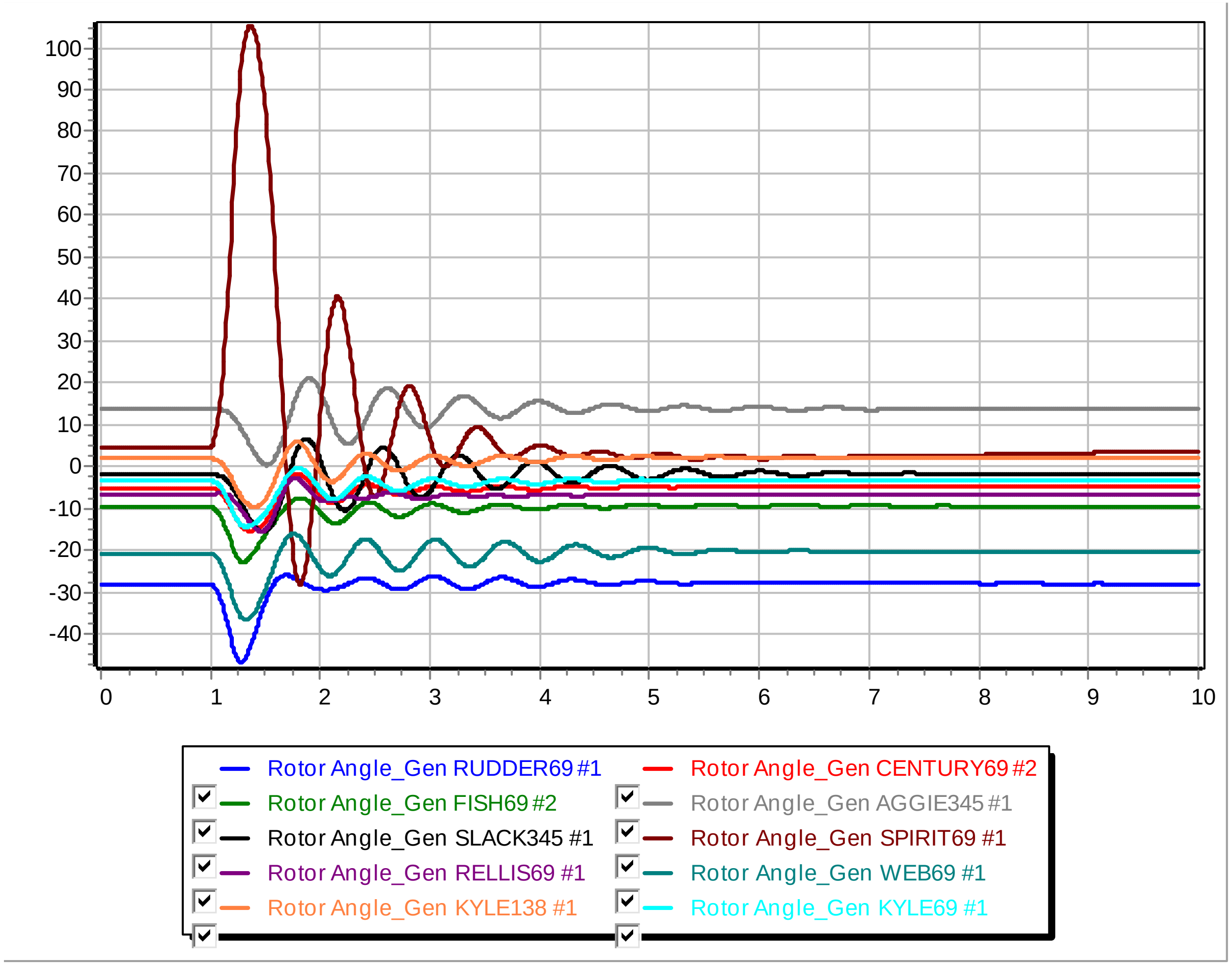
Right click on a line and select "Show Dialog" for more information.

Generator Model Use | Model Summary | Generators | Load Characteristics | Load Summary

Filter: Advanced | Generator

| | Number of Bus | Name of Bus | ID | Status | Gen MW | MVA Base | Machine | Exciter | Governor | Stabilizer | Other Model | Governor Response Limits | H (system base) | TS Rcom (system b |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 14 | RUDDER69 | 1 | Closed | 0.00 | 50.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 1.50000 | 0.0 |
| 2 | 16 | CENTURY69 | 2 | Closed | 100.00 | 120.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 3.60000 | 0.0 |
| 3 | 20 | FISH69 | 2 | Closed | 91.75 | 130.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 3.90000 | 0.0 |
| 4 | 28 | AGGIE345 | 1 | Closed | 500.00 | 600.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 36.00000 | 0.0 |
| 5 | 31 | SLACK345 | 1 | Closed | 270.20 | 600.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 36.00000 | 0.0 |
| 6 | 37 | SPIRIT69 | 1 | Closed | 80.00 | 90.00 | GENSAL | IEEET1 | TGOV1 | | | Normal | 2.70000 | 0.0 |
| 7 | 44 | RELLIS69 | 1 | Closed | 60.00 | 80.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 2.40000 | 0.0 |
| 8 | 48 | WEB69 | 1 | Closed | 12.30 | 80.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 2.40000 | 0.0 |
| 9 | 53 | KYLE138 | 1 | Closed | 250.00 | 300.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 9.00000 | 0.0 |
| 10 | 54 | KYLE69 | 1 | Closed | 80.00 | 100.00 | GENROU | IEEET1 | TGOV1 | | | Normal | 3.00000 | 0.0 |

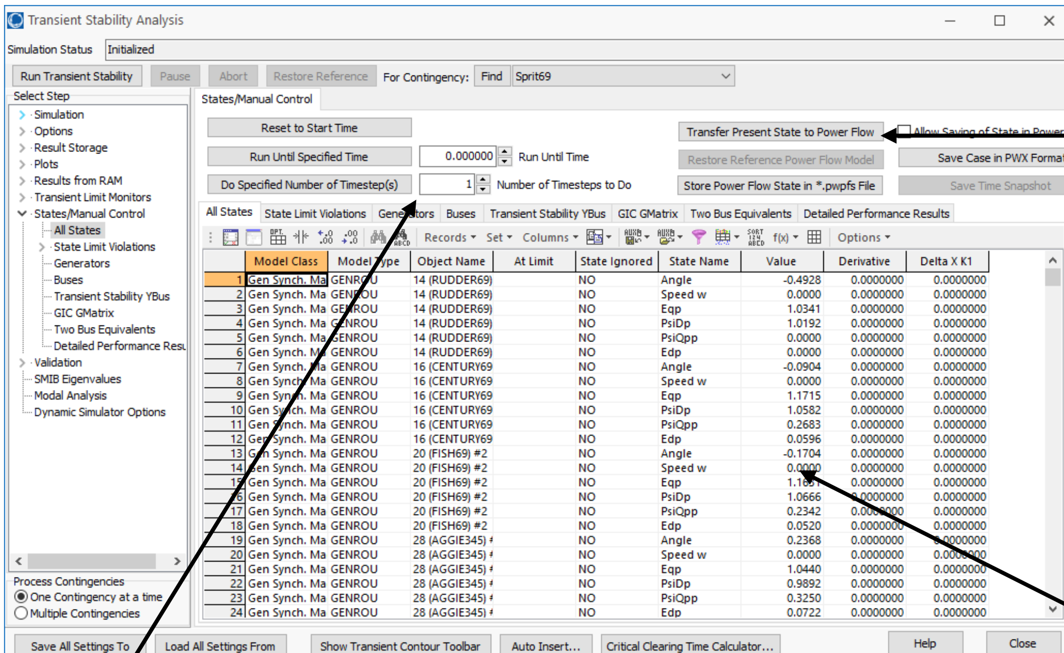
# 37 Bus Case Solution




Graph shows the rotor angles following a line fault

# Stepping Through a Solution

- Simulator provides functionality to make it easy to see what is occurring during a solution.  This functionality is accessed on the States/Manual Control Page

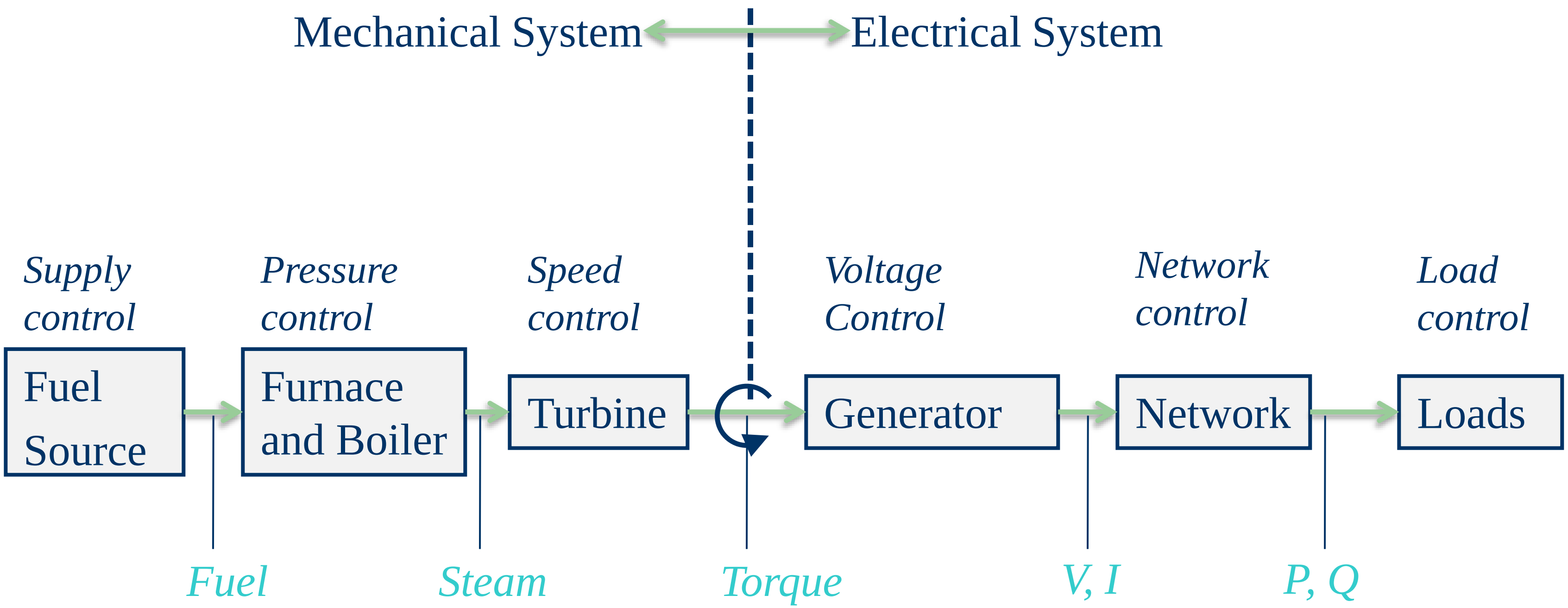



Transfer results to Power Flow to view using standard PowerWorld displays and one-lines

Run a Specified Number of  Timesteps or Run Until a Specified Time, then Pause.

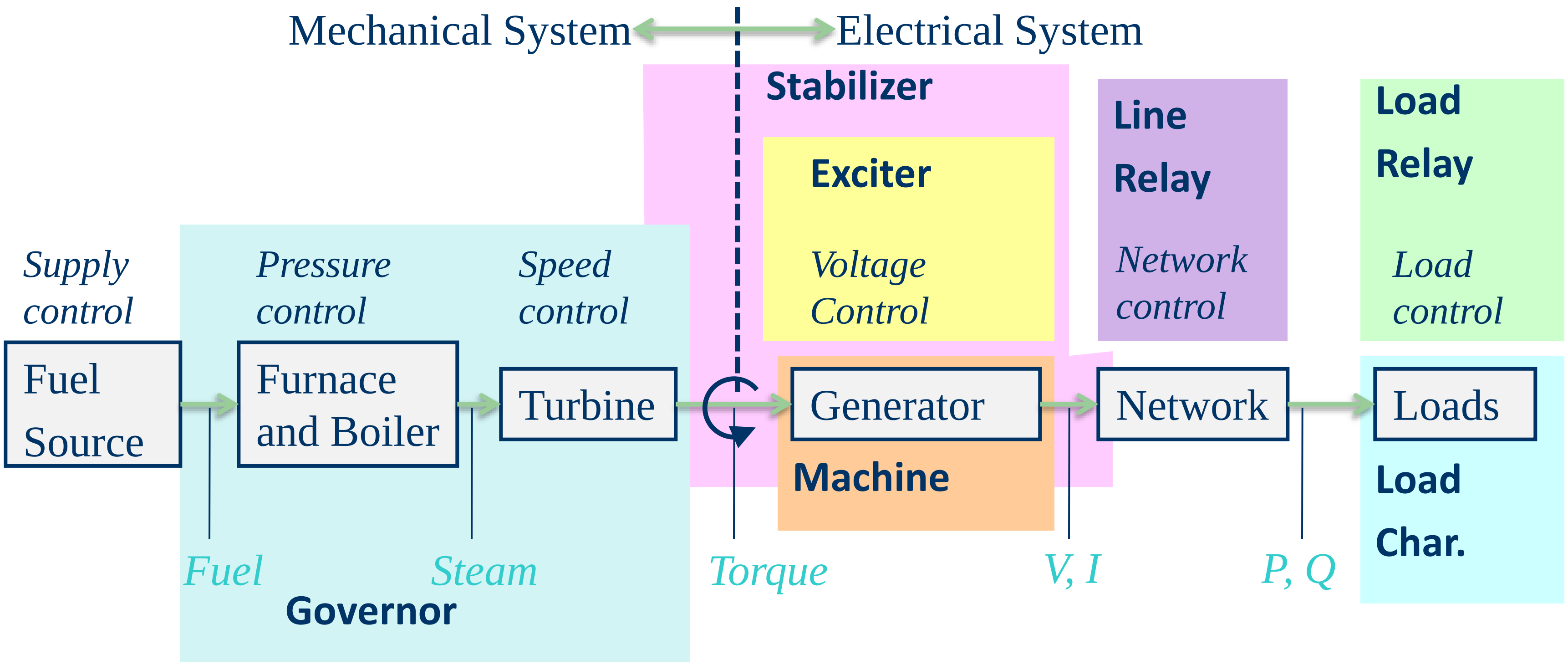
See detailed results at the Paused Time

# Physical Structure
# Power System Components




P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

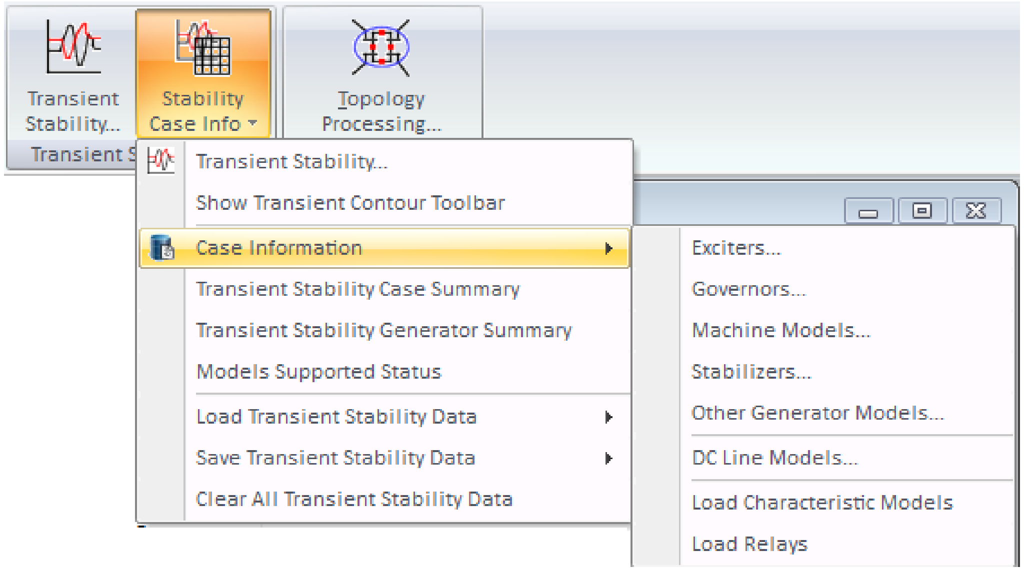
# Dynamic Models in the Physical Structure



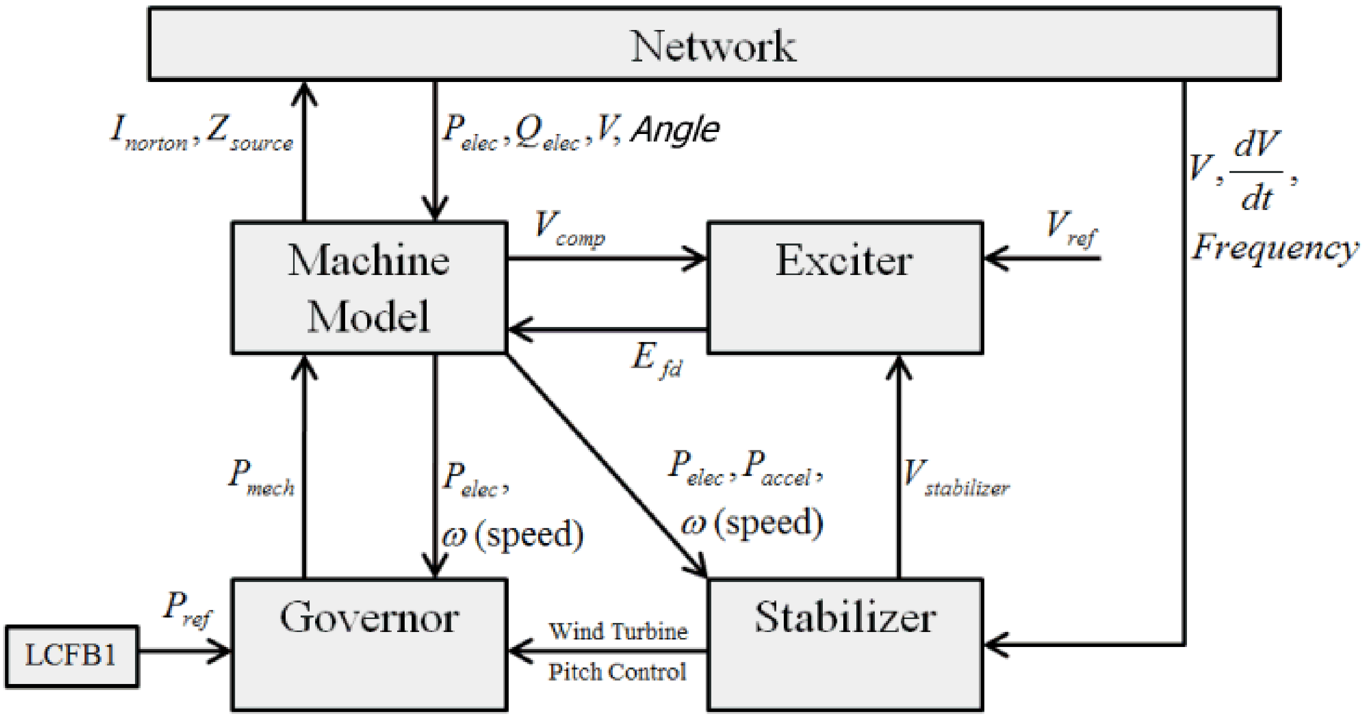

P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

# Generator Models

- Generators can have several classes of models assigned to them
  - Machine Models
  - Exciter
  - Governors
  - Stabilizers
- Others also available
  - Excitation limiters, voltage compensation, turbine load controllers, and generator relay model



# Generator Models




$P_{elec}$ = Electrical Power

$Q_{elec}$ = Electrical Reactive Power

$V$ = Voltage at Terminal Bus

$\frac{dV}{dt}$ = Derivate of Voltage

$V_{comp}$ = Compensated Voltage

$P_{mech}$ = Mechanical Power

$\omega$(speed) = Rotor Speed (often it's deviation from nominal speed)

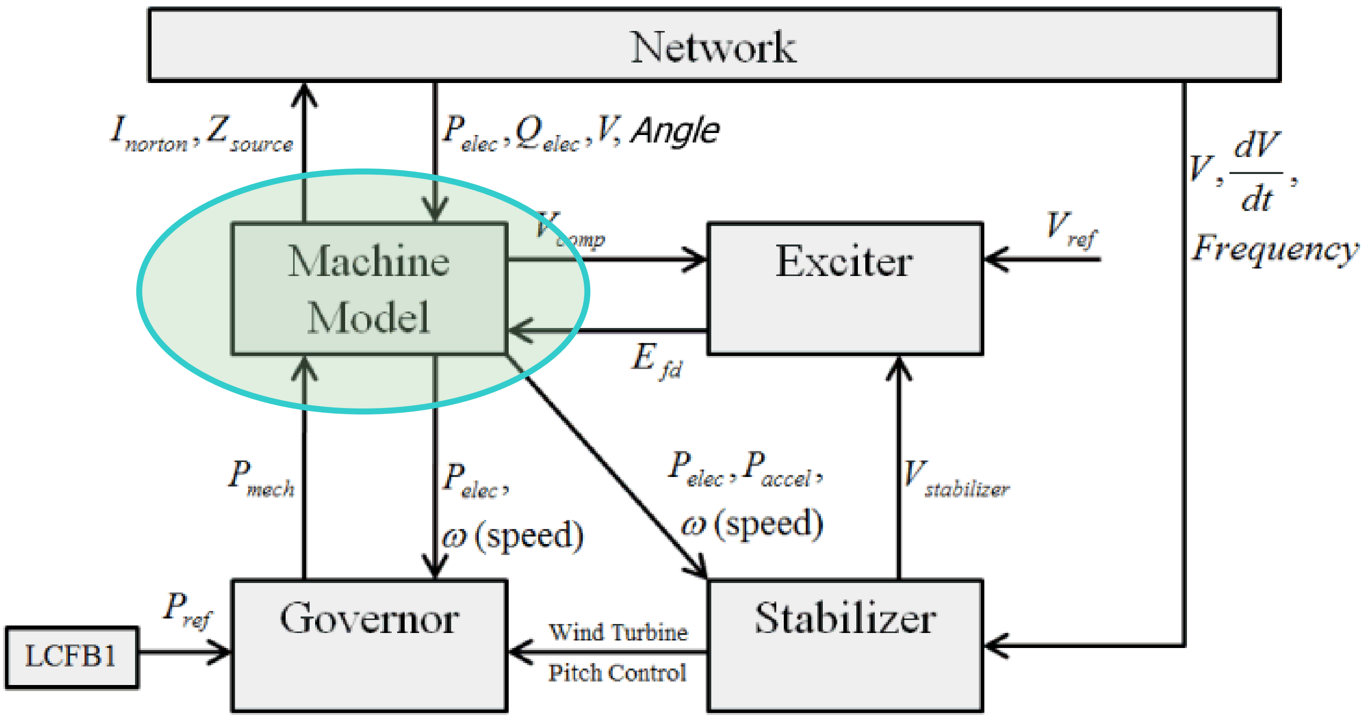
$P_{accel}$ = Accelerating Power

$V_{stabilizer}$ = Output of Stabilizer

$V_{ref}$ = Exciter Control Setpoint (determined during initialization)

$P_{ref}$ = Governor Control Setpoint (determined during initialization)

# Machine Models

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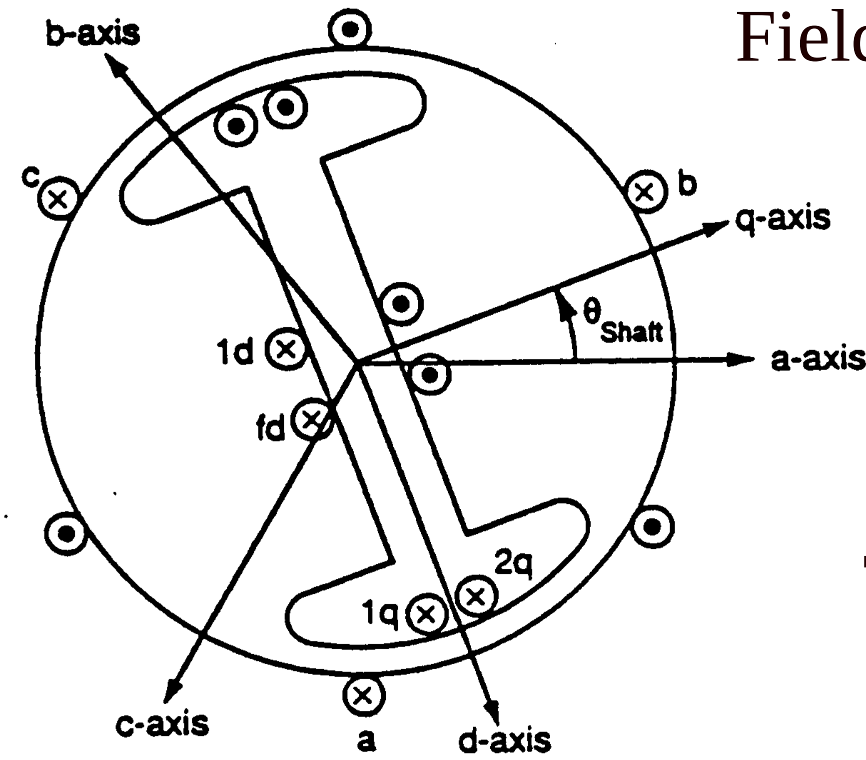
# Synchronous Machine Modeling

- Electric machines are used to convert mechanical energy into electrical energy (generators) and from electrical energy into mechanical energy (motors)
  - Many devices can operate in either mode, but are usually customized for one or the other
- Vast majority of electricity is generated using synchronous generators and some is consumed using synchronous motors, so we'll start there
- There is much literature on subject, and sometimes it is overly confusing with the use of different conventions and nomenclature

# Synchronous Machine Modeling

3ϕ bal. windings (a,b,c) – stator




Field winding (fd) on rotor

Damper in "d" axis (1d) on rotor

Two dampers in "q" axis (1q, 2q) on rotor

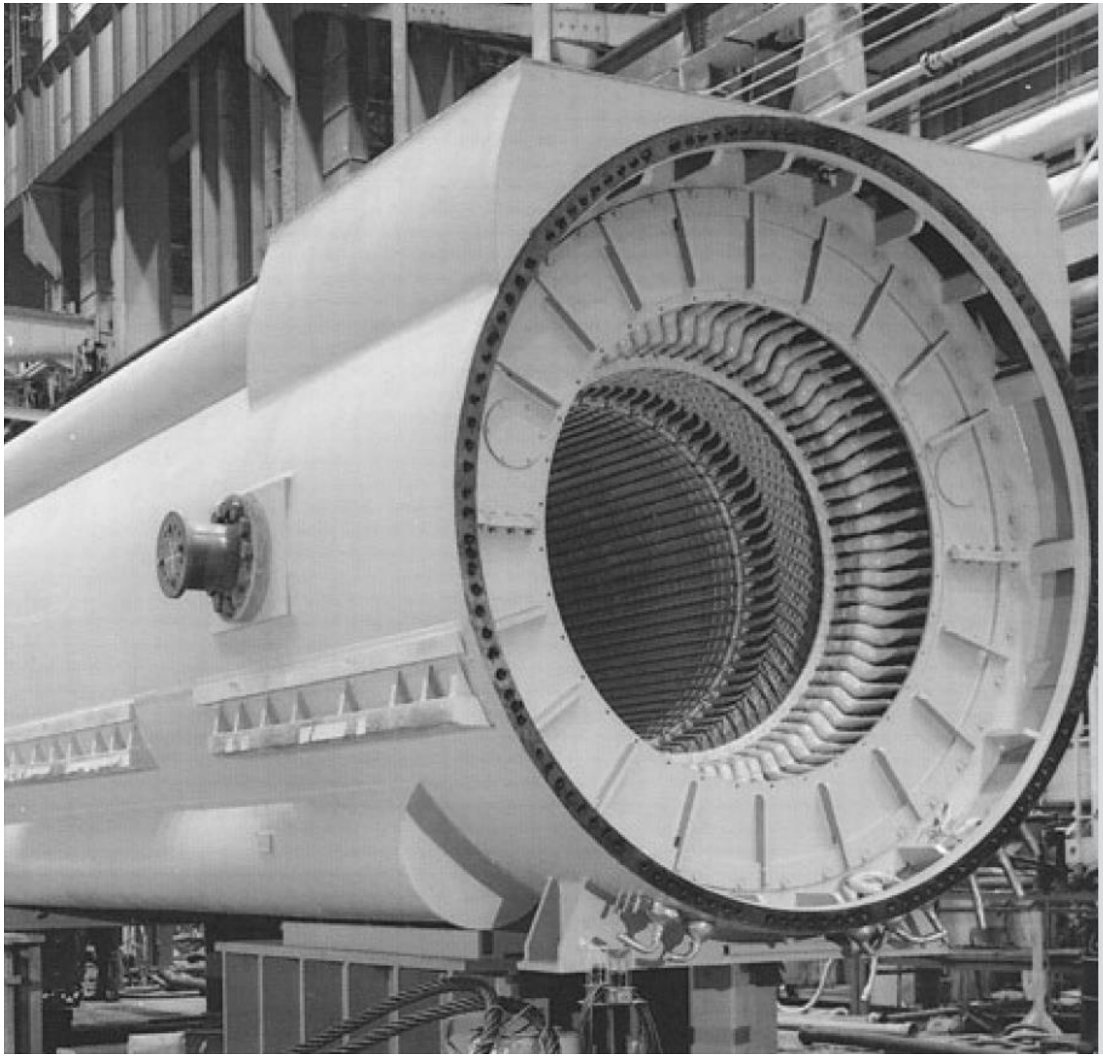
# Two Main Types of Synchronous Machines

- Round Rotor
  - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
  - Air-gap varies circumferentially
  - Used with many pole, slower machines such as hydro
  - Narrowest part of gap in the d-axis and the widest along the q-axis

# Dq0 Reference Frame

- Stator is stationary, rotor is rotating at synchronous speed
- Rotor values need to be transformed to fixed reference frame for analysis
- Done using Park's transformation into what is known as the dq0 reference frame (direct, quadrature, zero)
  - Parks' 1929 paper voted 2nd most important power paper of 20th century at the 2000 NAPS Meeting (1st was Fortescue's sym. components)
- Convention used here is the q-axis leads the d-axis (which is the IEEE standard)

# Synchronous Machine Stator

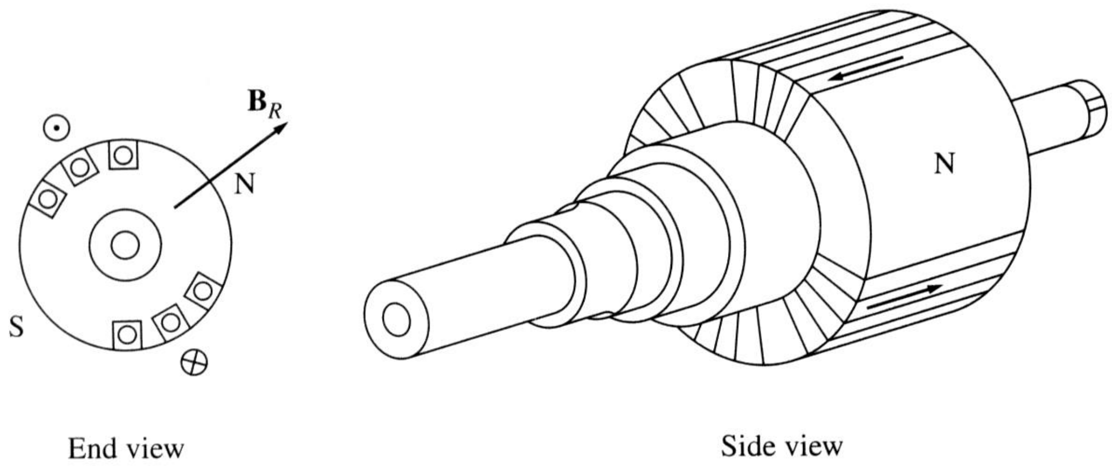


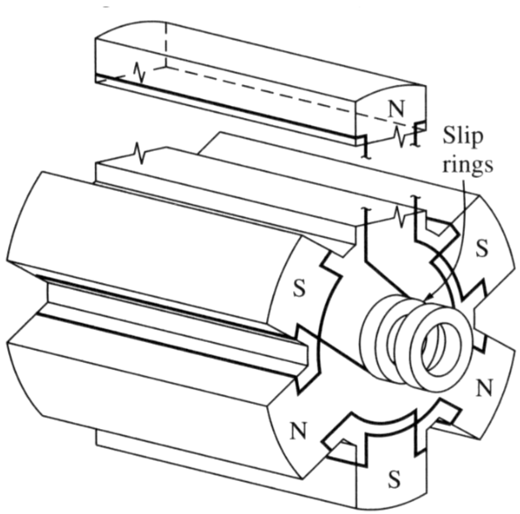
Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

# Synchronous Machine Rotors

- Rotors are essentially electromagnets




Two pole (P) round rotor

Six pole salient rotor

Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

# Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

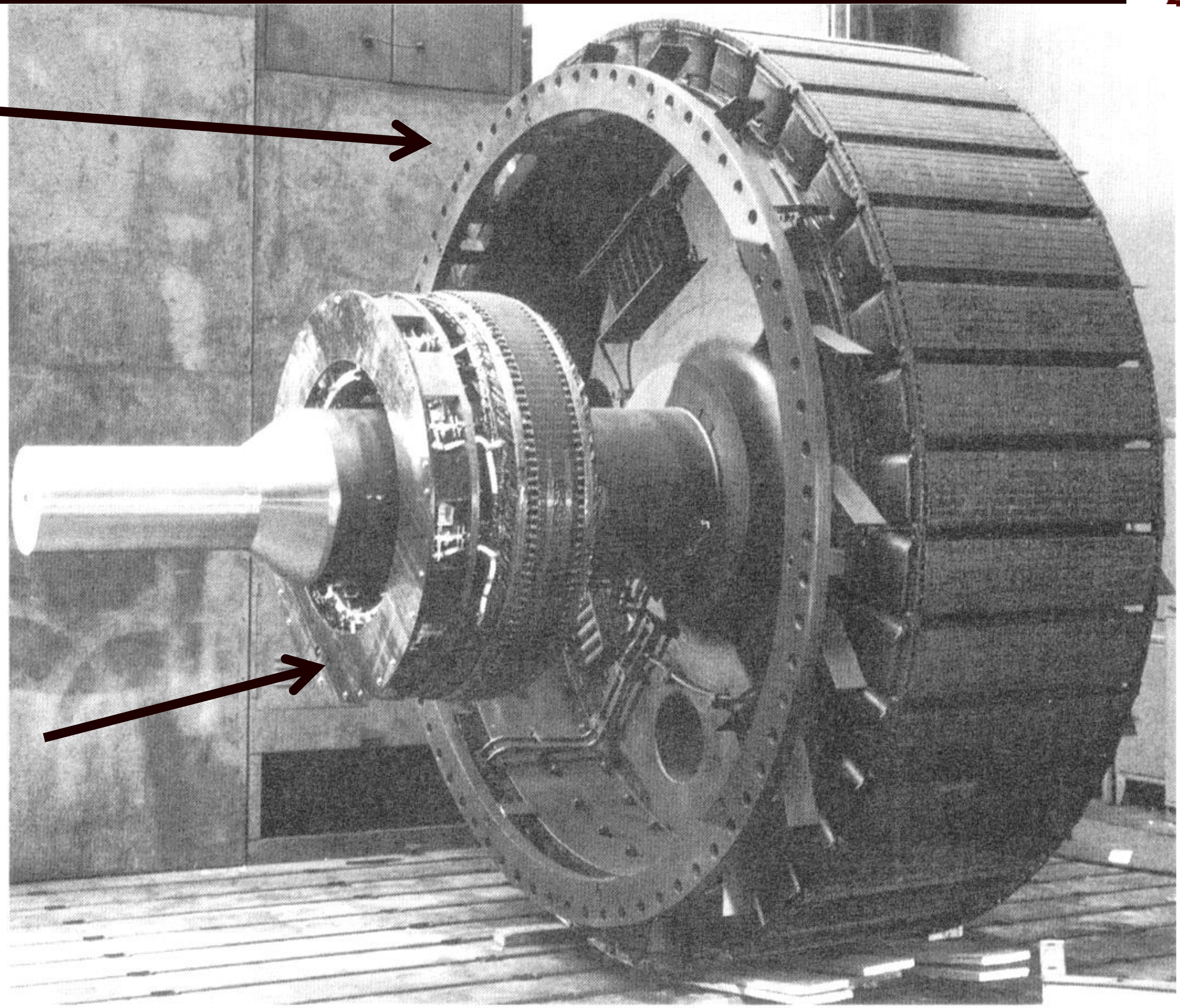


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

# Fundamental Laws

- Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

**Stator**

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

**Rotor**

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

**Shaft**

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

The rotor winds are the field winding and then three damper windings (added to provide damping)

# Dq0 Transformations

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or} \quad i, \lambda$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

In the next few slides we'll quickly go through how these basic equations are transformed into the standard machine models. The point is to show the physical basis for the models.

# Dq0 Transformations

$$T_{dqo} \triangleq \frac{2}{3}\begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \cos\frac{P}{2}\theta_{shaft} & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

with the inverse,

$$T_{dqo}^{-1} = \begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \cos\frac{P}{2}\theta_{shaft} & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

# Transformed System

### Stator

$$v_d = r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

### Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

### Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

We are now in the dq0 space

# Electrical & Mechanical Relationships

Electrical system:

$$v = iR + \frac{d\lambda}{dt} \quad \text{(voltage)}$$

$$vi = i^2 R + i\frac{d\lambda}{dt} \quad \text{(power)}$$

P is the number of poles (e.g., 2,4,6); Tfw is the friction and windage torque

Mechanical system:

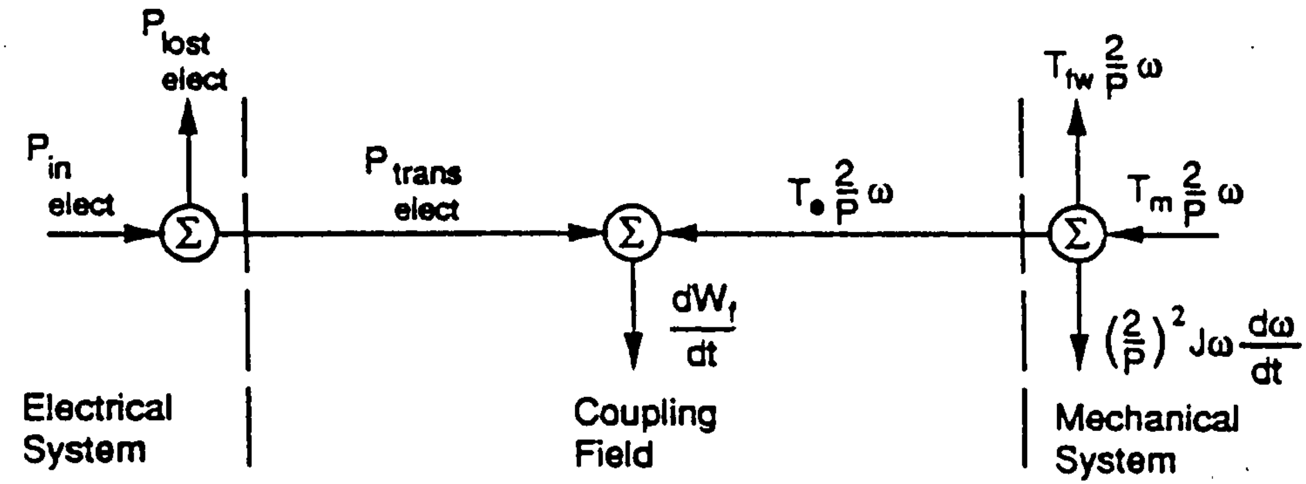
$$J\left(\frac{2}{P}\right)\frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad \text{(torque)}$$

$$J\left(\frac{2}{P}\right)^2 \omega\frac{d\omega}{dt} = \frac{2}{P}\omega T_m - \frac{2}{P}\omega T_e - \frac{2}{P}\omega T_{fw} \quad \text{(power)}$$

# Torque Derivation

- Torque is derived by looking at the overall energy balance in the system
- Three systems: electrical, mechanical and the coupling magnetic field
  - Electrical system losses are in the form of resistance
  - Mechanical system losses are in the form of friction
- Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

# Energy Conversion




The coupling field stores and discharges energy but has no losses

Look at the instantaneous power:

$$v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o$$

# Change to Conservation of Power

$$P_{\substack{in \\ elect}} = v_a i_a + v_b i_b + v_c i_c + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q}$$

$$+ v_{2q} i_{2q}$$

$$P_{\substack{lost \\ elect}} = r_s \left( i_a^2 + i_b^2 + i_c^2 \right) + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{\substack{trans \\ elect}} = i_a \frac{d\lambda_a}{dt} + i_b \frac{d\lambda_b}{dt} + i_c \frac{d\lambda_c}{dt} + i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt}$$

$$+ i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

We are using
$v = d\lambda/dt$

# With the Transformed Variables

$$P_{in_{elect}} = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{lost_{elect}} = \frac{3}{2} r_s i_d^2 + \frac{3}{2} r_s i_q^2 + 3 r_s i_o^2 + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

# With the Transformed Variables

$$P_{\substack{trans\\elect}} = -\frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_q i_d + \frac{3}{2} i_d \frac{d\lambda_d}{dt} + \frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_d i_q$$

$$+\frac{3}{2} i_q \frac{d\lambda_q}{dt} + 3 i_o \frac{d\lambda_o}{dt} + i_{fd}\frac{d\lambda_{fd}}{dt} + i_{1d}\frac{d\lambda_{1d}}{dt}$$

$$+ i_{1q}\frac{d\lambda_{1q}}{dt} + i_{2q}\frac{d\lambda_{2q}}{dt}$$

# Change in Coupling Field Energy

$$\frac{dW_f}{dt} = \boxed{T_e \frac{2}{P}} \frac{d\theta}{dt} + \boxed{i_a} \frac{d\lambda_a}{dt} + \boxed{i_b} \frac{d\lambda_b}{dt}$$

$$+ \boxed{i_c} \frac{d\lambda_c}{dt} + \boxed{i_{fd}} \frac{d\lambda_{fd}}{dt} + \boxed{i_{1d}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{i_{1q}} \frac{d\lambda_{1q}}{dt} + \boxed{i_{2q}} \frac{d\lambda_{2q}}{dt}$$

First term on right is what is going on mechanically, other terms are what is going on electrically

This requires the lossless coupling field assumption

# Change in Coupling Field Energy

For independent states $\theta, \lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q}$

$$\frac{dW_f}{dt} = \boxed{\frac{\partial W_f}{\partial \theta}} \frac{d\theta}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_a}} \frac{d\lambda_a}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_b}} \frac{d\lambda_b}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_c}} \frac{d\lambda_c}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{fd}}} \frac{d\lambda_{fd}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{1d}}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_{1q}}} \frac{d\lambda_{1q}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{2q}}} \frac{d\lambda_{2q}}{dt}$$

# Equate the Coefficients

$$T_e \frac{2}{P} = \frac{\partial W_f}{\partial \theta} \qquad i_a = \frac{\partial W_f}{\partial \lambda_a} \qquad \text{etc.}$$

There are eight such "reciprocity conditions for this model.

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

# Equate the Coefficients

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e$$

$$\frac{\partial W_f}{\partial \lambda_d} = \frac{3}{2} i_d\,, \qquad \frac{\partial W_f}{\partial \lambda_q} = \frac{3}{2} i_q, \qquad \frac{\partial W_f}{\partial \lambda_o} = 3 i_o$$

$$\frac{\partial W_f}{\partial \lambda_{fd}} = i_{fd}\,, \qquad \frac{\partial W_f}{\partial \lambda_{1d}} = i_{1d}\,, \qquad \frac{\partial W_f}{\partial \lambda_{1q}} = i_{1q}, \qquad \frac{\partial W_f}{\partial \lambda_{2q}} = i_{2q}$$

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

# Coupling Field Energy

- The coupling field energy is calculated using a path independent integration
  - For integral to be path independent, the partial derivatives of all integrands with respect to the other states must be equal

For example, $\frac{3}{2}\frac{\partial i_d}{\partial \lambda_{fd}} = \frac{\partial i_{fd}}{\partial \lambda_d}$

- Since integration is path independent, choose a convenient path
  - Start with a de-energized system so variables are zero
  - Integrate shaft position while other variables are zero
  - Integrate sources in sequence with shaft at final value

# Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

$\omega_s$ is the rated synchronous speed
$\delta$ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

# Synchronous Machine Equations in Per Unit

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I 2 + V_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

Units of $H$ are seconds

The $\psi$ variables are in the $\lambda$ variables in per unit (see book 3.50 to 3.52)

# Sinusoidal Steady-State

$$V_a = \sqrt{2}V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2}I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application to balanced, sinusoidal conditions

# Simplifying Using δ

- Define $\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$
- Hence
$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$
- These algebraic equations can be written as complex equations
$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

# Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of $\theta_{shaft}$ when expressed in the dq0 coordinate system