Homework #2, Problem #1

Tuesday, September 21, 2021 11:34 AM

Parrot Conductor : GMR = 0.0507 ft

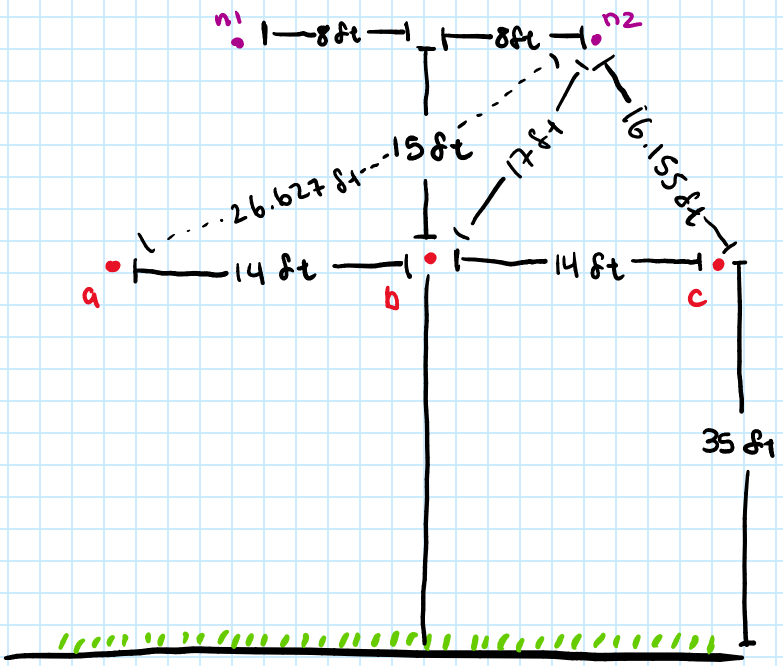
R @ 60 Hz = 0.0622 Ω/mi

Partridge Conductor : GMR = 0.0217 ft

R @ 60 Hz = 0.35 Ω/mi

$\rho = 60\ \Omega \cdot m \qquad f = 60\ Hz$

$R_k' = 9.869 \times 10^{-7} \times f\ \Omega/m \approx 0.0953\ \Omega/mi$



Approach #1 : $Z_P = [Z_A - Z_B Z_D^{-1} Z_C]$

$D_{kk}' = 658.5 \sqrt{\rho / f} \qquad L_{km} \approx 2 \times 10^{-7} \ln\left(\frac{D_{kk}'}{D_{km}}\right)$

$\Rightarrow D_{kk}' = 658.5 \sqrt{60/60} = 658.5\ m \approx 2160.433\ ft$

Note: 1 mi = 1609.34 m , 1 m = 3.28084 ft

For $Z = R + j\omega L$,
- All diagonal resistances are R_k' plus the conductor resistance
- All off-diagonal resistances are R_k'
- Inductances are calculated using L_{km}

- For the phase values, $R = 0.0622 + 0.0953 = 0.1575\ \Omega/mi$
- For the neutral, $R = 0.35 + 0.0953 = 0.4453\ \Omega/mi$
- Because there are 5 points (3 conductors, 2 neutrals), the resultant Z matrix is 5×5

$$\Rightarrow Z = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an1} & Z_{an2} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn1} & Z_{bn2} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn1} & Z_{cn2} \\ Z_{n1a} & Z_{n1b} & Z_{n1c} & Z_{n1n1} & Z_{n1n2} \\ Z_{n2a} & Z_{n2b} & Z_{n2c} & Z_{n2n1} & Z_{n2n2} \end{bmatrix}$$

Red Partition: Z_A
Green Partition: Z_B
Blue Partition: Z_C
Purple Partition: Z_D

For Z_A:

$$Z_{aa} = Z_{bb} = Z_{cc} = \overset{(\Omega/mi)}{0.1575} + j\omega \overset{(H/m)}{\left[2\times10^{-7} \ln\left(\frac{2160.433}{0.0507}\right)\right]}$$

$$= \overset{(\Omega/mi)}{0.1575} + j\omega \overset{(H/m)}{(2.132\times10^{-6})}$$

$$= 0.1575 + j\,1.293\ \Omega/mi$$

$Z_{ab} = Z_{ba} = Z_{bc} = Z_{cb}$ (Because a/c are equidistant from b)

$$= \overset{(\Omega/mi)}{0.0953} + j\omega \overset{(H/m)}{\left[2\times10^{-7} \ln\left(\frac{2160.433}{14}\right)\right]}$$

$$= \overset{(\Omega/mi)}{0.0953} + j\omega \overset{(H/m)}{(1.0078\times10^{-6})}$$

$$= 0.0953 + j\,0.611\ \Omega/mi$$

$$Z_{ac} = Z_{ca} = \overset{(\Omega/mi)}{0.0953} + j\omega \overset{(H/m)}{\left[2\times10^{-7} \ln\left(\frac{2160.433}{28}\right)\right]}$$

$$= \overset{(\Omega/mi)}{0.0953} + j\omega \overset{(H/m)}{(8.691\times10^{-7})}$$

$= 0.0953 + j\,0.527\ \Omega/mi$

For Z_B:

$Z_{an1} = Z_{cn2}$ (Symmetric system)

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\left[\overset{(H/m)}{2\times10^{-7}\ln\left(\frac{2160.433}{16.155}\right)}\right]$$

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\overset{(H/m)}{\left(9.792\times10^{-7}\right)}$$

$$= 0.0953 + j\,0.594\ \Omega/mi$$

$Z_{bn1} = Z_{bn2}$ (Symmetric system)

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\left[\overset{(H/m)}{2\times10^{-7}\ln\left(\frac{2160.433}{17}\right)}\right]$$

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\overset{(H/m)}{\left(9.689\times10^{-7}\right)}$$

$$= 0.0953 + j\,0.587\ \Omega/mi$$

$Z_{an2} = Z_{cn1}$ (Symmetric system)

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\left[\overset{(H/m)}{2\times10^{-7}\ln\left(\frac{2160.433}{26.627}\right)}\right]$$

$$= \underset{(\Omega/mi)}{0.0953} + j\omega\overset{(H/m)}{\left(8.792\times10^{-7}\right)}$$

$$= 0.0953 + j\,0.533\ \Omega/mi$$

For Z_c:

$$Z_{n1a} = Z_{n2c} = Z_{an_1} = 0.0953 + j0.594 \ \Omega/mi$$

$$Z_{n1b} = Z_{n2b} = Z_{bn_1} = 0.0953 + j0.587 \ \Omega/mi$$

$$Z_{n1c} = Z_{n2a} = Z_{cn_1} = 0.0953 + j0.533 \ \Omega/mi$$

For Z_D:

$$Z_{n_1n_1} = Z_{n_2n_2} = \overset{(\Omega/mi)}{0.4453} + j\omega\left[\overset{(H/m)}{2\times10^{-7} \ln\left(\frac{2160.433}{0.0217}\right)}\right]$$

$$= \overset{(\Omega/mi)}{0.4453} + j\omega\,\overset{(H/m)}{(2.3017\times10^{-6})}$$

$$= 0.4453 + j1.396 \ \Omega/mi$$

$$Z_{n_1n_2} = Z_{n_2n_1} = \overset{(\Omega/mi)}{0.0953} + j\omega\left[\overset{(H/m)}{2\times10^{-7} \ln\left(\frac{2160.433}{16}\right)}\right]$$

$$= \overset{(\Omega/mi)}{0.0953} + j\omega\,\overset{(H/m)}{(9.811\times10^{-7})}$$

$$= 0.0953 + j0.595 \ \Omega/mi$$

An alternative approach to filling out Z is to use the modified Carson's Equations (from the Kersting textbook)

$$(4.37) \quad z_{ii} = r_i + \pi^2 f G + j4\pi f G\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right)$$

$$(4.38) \quad z_{ij} = \pi^2 f G + j4\pi f G\left(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right)$$

where $G = 0.1609347\times10^{-3} \ \Omega/mi$

Using those two equations to construct Z results in the same matrix and partitions.

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a) $Z_p = \left[Z_A - Z_B Z_D^{-1} Z_C \right]$

$$= \begin{bmatrix} 0.136 + j0.976 & 0.0745 + j0.282 & 0.0723 + j0.213 \\ 0.0745 + j0.282 & 0.138 + j0.950 & 0.0745 + j0.282 \\ 0.0723 + j0.213 & 0.0745 + j0.282 & 0.136 + j0.976 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$ where $\alpha = 1\angle 120^\circ$

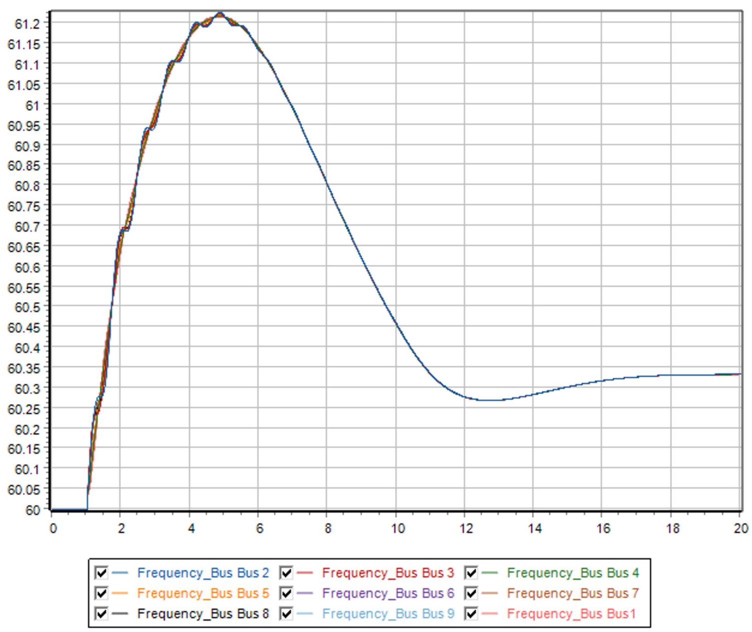
$\Rightarrow Z_S = A^{-1} Z_p A$

$$= \begin{bmatrix} 0.284 + j1.485 & 0.012 - j0.008 & -0.0133 - j0.006 \\ -0.0133 - j0.0061 & 0.0631 + j0.708 & -0.0467 + j0.0278 \\ 0.012 - j0.008 & 0.0474 + j0.0265 & 0.0631 + j0.708 \end{bmatrix}$$

Homework #2, Problems 2-4

Tuesday, September 21, 2021 7:00 PM

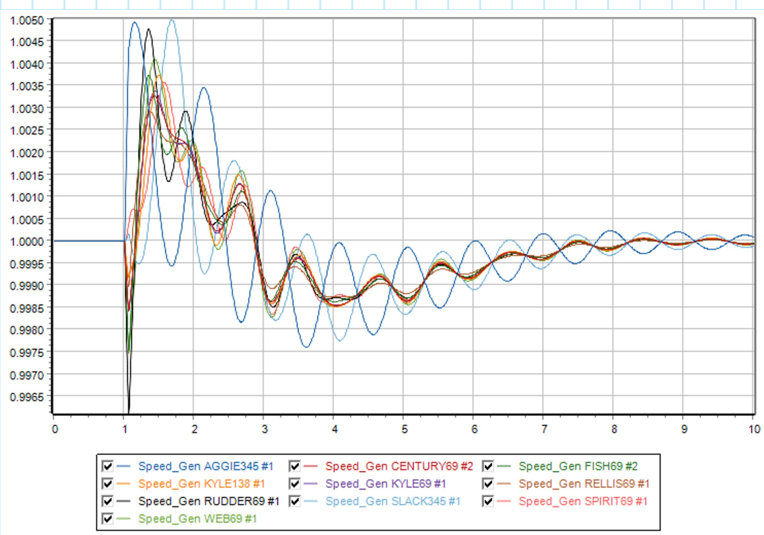
Problem # 2



Highest Bus Frequency : 61.2281 Hz

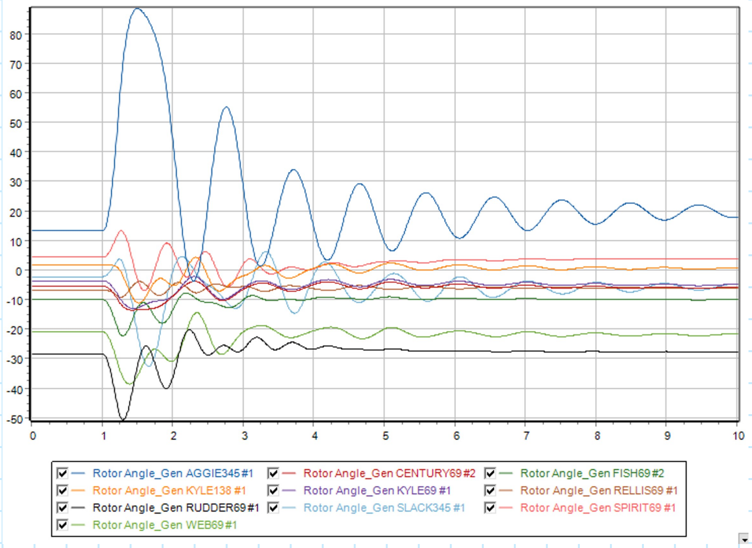
Final Bus Frequency : 60.338 Hz

Problem # 3



Maximum Speed = 60.3 Hz = 1.005 pu

⇒ Fault duration such that you do not exceed 1.005 pu generator speed is 0.063s



The system loses synchronism when the rotor angles diverge.

⇒ Maximum Fault Duration such that you do not lose Synchronism is 0.209s

- If you wanted to maintain both conditions, the maximum fault duration would be 0.063s

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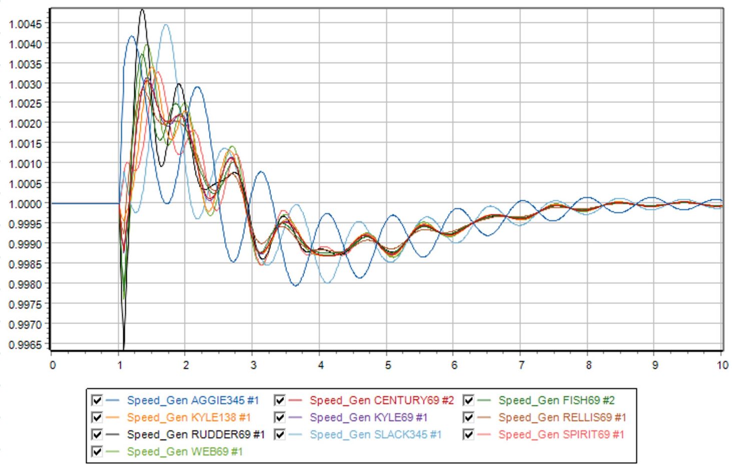
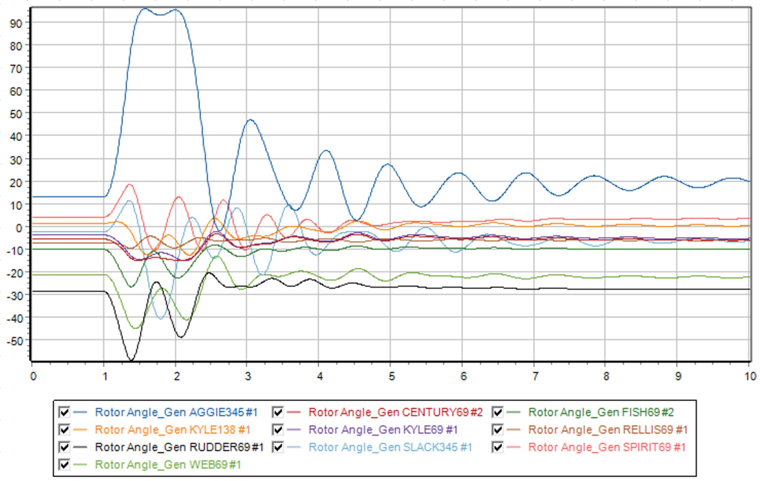
Problem #4



Maximum Speed = 60.3 Hz = 1.005 pu

⇒ Fault duration such that you do not exceed 1.005 pu generator speed is 0.072s



The system loses synchronism when the rotor angles diverge.

⇒ Maximum Fault Duration such that you do not lose synchronism is 0.322s

- If you wanted to maintain both conditions, the maximum fault duration would be 0.072s