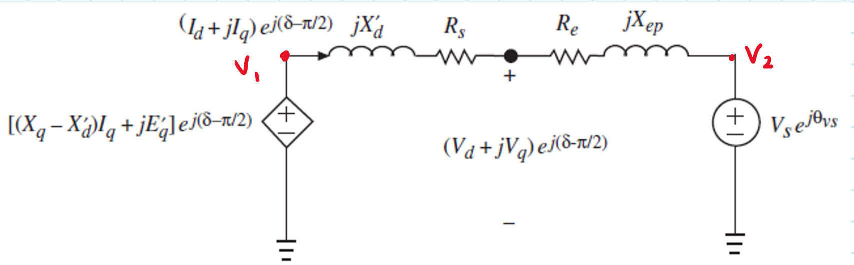
Homework #3, Problem #1

Wednesday, September 29, 2021 1:45 PM

5.2 Given the one-axis dynamic circuit of Figure 5.3, solve for I_d and I_q in terms of the circuit parameters plus δ, E'_q and the source V_s, θ_{vs}.



By Ohm's Law:

$$(I_d + jI_q)e^{j(\delta-\pi/2)} = \frac{V_1 - V_2}{(R_s + jX'_d) + (R_e + jX_{ep})}$$

where: $V_1 = [(X_q - X'_d)I_q + jE'_q]e^{j(\delta-\pi/2)}$

$V_2 = V_s e^{j\theta_{vs}}$

eliminating from both ↓ sides

$$\Rightarrow (I_d + jI_q)\cancel{e^{j(\delta-\pi/2)}} = \frac{\left([(X_q - X'_d)I_q + jE'_q] - V_s e^{j(\theta_{vs}-\delta+\pi/2)}\right)\cancel{e^{j(\delta-\pi/2)}}}{(R_s + R_e) + j(X'_d + X_{ep})}$$

$$\Rightarrow I_d + jI_q = \frac{[(X_q - X'_d)I_q + jE'_q] - V_s e^{j(\theta_{vs}-\delta+\pi/2)}}{(R_s + R_e) + j(X'_d + X_{ep})}$$ ← Converting to rectangular form

$$= \frac{[(X_q - X'_d)I_q + jE'_q] - \left(V_s\cos(\theta_{vs} - \delta + \pi/2) + jV_s\sin(\theta_{vs} - \delta + \pi/2)\right)}{(R_s + R_e) + j(X'_d + X_{ep})}$$

$\cos(\theta + \pi/2) = -\sin\theta$ $\sin(\theta + \pi/2) = \cos(\theta)$

$$= \frac{[(X_q - X'_d)I_q + jE'_q] + V_s\sin(\theta_{vs} - \delta) - jV_s\cos(\theta_{vs} - \delta)}{(R_s + R_e) + j(X'_d + X_{ep})}$$

$$= \frac{[(X_q - X'_d)I_q + V_s\sin(\theta_{vs} - \delta)] + j[E'_q - V_s\cos(\theta_{vs} - \delta)]}{(R_s + R_e) + j(X'_d + X_{ep})}$$

↑ Multiply numerator / denominator by the complex conjugate.

$$= \frac{\left[(X_q - X_d')I_q + V_s \sin(\theta_{vs} - \delta)\right] + j\left[E_q' - V_s \cos(\theta_{vs} - \delta)\right]}{(R_s + R_e) + j(X_d' + X_{ep})} \times \frac{(R_s + R_e) - j(X_d' + X_{ep})}{(R_s + R_e) - j(X_d' + X_{ep})}$$

For the denominator:

$$\left[(R_s + R_e) + j(X_d' + X_{ep})\right] \times \left[(R_s + R_e) - j(X_d' + X_{ep})\right]$$

$$= (R_s + R_e)^2 + (X_d' + X_{ep})^2$$

For the numerator:

$$= \left[(X_q - X_d') I_q + V_s \sin(\theta_{vs} - \delta)\right](R_s + R_e) -$$

$$-j\left[(X_q - X_d') I_q + V_s \sin(\theta_{vs} - \delta)\right](X_d' + X_{ep}) +$$

$$+j\left[E_q' - V_s \cos(\theta_{vs} - \delta)\right](R_s + R_e) +$$

$$+\left[E_q' - V_s \cos(\theta_{vs} - \delta)\right](X_d' + X_{ep})$$

- Green: Real
- Red: Complex

For the real component:

$$I_d = \frac{\left[(X_q - X_d') I_q + V_s \sin(\theta_{vs} - \delta)\right](R_s + R_e) + \left[E_q' - V_s \cos(\theta_{vs} - \delta)\right](X_d' + X_{ep})}{(R_s + R_e)^2 + (X_d' + X_{ep})^2}$$

For the imaginary component:

$$I_q = \frac{\left[E_q' - V_s \cos(\theta_{vs} - \delta)\right](R_s + R_e) - \left[(X_q - X_d')I_q + V_s \sin(\theta_{vs} - \delta)\right](X_d' + X_{ep})}{(R_s + R_e)^2 + (X_d' + X_{ep})^2}$$

Thus we have two equations with two unknowns. Plugging this into an equation solver, we get:

$$I_q = \frac{(R_s + R_e)\left[E_q' - V_s \cos(\delta - \theta_{vs})\right] + (X_d' + X_{ep}) V_s \sin(\delta - \theta_{vs})}{(R_s + R_e)^2 + (X_d' + X_{ep})(X_{ep} + X_q)}$$

$$I_q = \frac{(R_s + R_e)\left[E'_q - V_s \cos(\delta - \theta_{vs})\right] + (X'_d + X_{ep})\, V_s \sin(\delta - \theta_{vs})}{(R_s + R_e)^2 + (X'_d + X_{ep})(X_{ep} + X_q)}$$

$$I_d = \frac{(X_{ep} + X_q)\left[E'_q - V_s \cos(\delta - \theta_{vs})\right] - (R_e + R_s)\, V_s \sin(\delta - \theta_{vs})}{(R_s + R_e)^2 + (X'_d + X_{ep})(X_{ep} + X_q)}$$

Homework #3, Problem #2

Wednesday, September 29, 2021 5:48 PM

Given a synchronous generator with a two-axis model:

$$(V_d + jV_q)e^{j(\delta-\pi/2)} = 1\angle 0 \text{ pu}$$

$$(I_d + jI_q)e^{j(\delta-\pi/2)} = 0.5\angle 30^o \text{ pu}$$

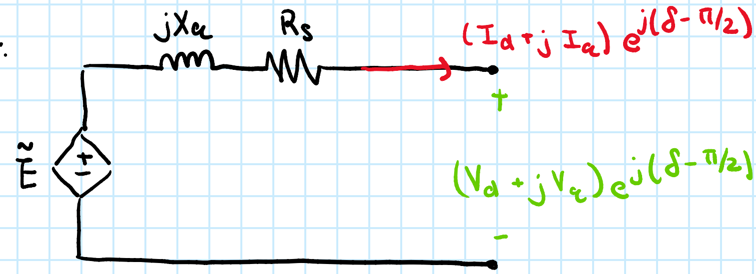
$R_s = 0$ pu, $X_d =$ pu, $X_q =$ pu, $X_d' = 0.2$ pu, $X_q' = 0.2$ pu

(a) Find the steady-state values of δ, E_q', E_d'.

(b) Find the inputs E_{fd} and T_m.

(c) Find E'^o and δ'^o for the classical model.

(a) At steady state:



$$\Rightarrow |E|\angle\delta = (R_s + jX_q)(I_d + jI_q)e^{j(\delta-\pi/2)} + (V_d + jV_q)e^{j(\delta-\pi/2)}$$

$$= j1.1(0.5\angle 30^\circ) + (1\angle 0)$$

$$= j1.1\left[0.5\cos(30) + j0.5\sin(30)\right] + 1$$

$$= 0.867\angle 33.043^\circ \text{ pu}$$

$$\Rightarrow \boxed{\delta = 33.043^\circ}$$

$$\Rightarrow \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix}\begin{bmatrix} V_r \\ V_i \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.549 \\ 0.836 \end{bmatrix} \text{pu}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix}\begin{bmatrix} I_r \\ I_i \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix}\begin{bmatrix} 0.433 \\ 0.25 \end{bmatrix} = \begin{bmatrix} 0.029 \\ 0.499 \end{bmatrix} \text{pu}$$

$$\Rightarrow E_q' = V_q + R_s I_q + X_d' I_d = 0.449 \text{ pu}$$

$$E_d' = V_d - R_s I_d - X_q' I_q = 0.841 \text{ pu}$$

$$\Rightarrow \boxed{\begin{aligned} E_d' &= 0.449 \text{ pu} \\ E_q' &= 0.841 \text{ pu} \end{aligned}}$$

(b) $E_{fd} = E_q' + (X_d - X_d') I_d = 0.876\,pu$ From Chapter 3

$T_m = \psi_d I_q - \psi_q I_d + T_{FW}$, where: $\psi_d = E_q' - X_d' I_d$

$\psi_q = -X_q' I_q - E_d'$

$T_{FW} = 0$

$= E_q' I_q - X_d' I_d I_q + X_q' I_d I_q + E_d' I_d$

$= E_q' I_q + E_d' I_d + (X_q' - X_d') I_d I_q = 0.433\,pu$

$\Rightarrow \boxed{E_{fd} = 0.876\,pu \quad T_m = 0.433\,pu}$

(c)

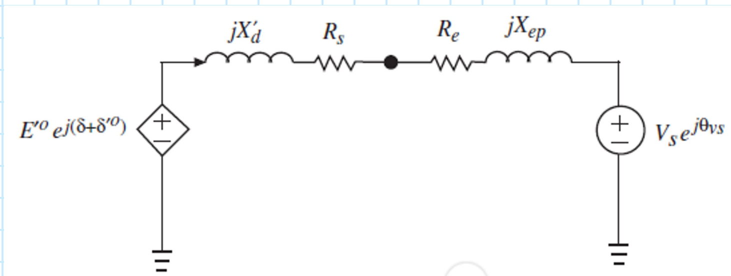


Figure 5.4 of a classical model from textbook

$\Rightarrow E'^0 e^{j(\delta + \delta'^0)} = jX_d'(0.5\angle 30^\circ) + (1\angle 0^\circ)$

current from circuit

voltage at the other end

$= 0.954 \angle 5.209^\circ\ pu$

$\Rightarrow \delta + \delta'^0 = 5.209^\circ \Rightarrow \boxed{\begin{array}{l}\delta'^0 = -28.096^\circ \\ E'^0 = 0.954\,pu\end{array}}$

Alternatively, use 5.150 and 5.151 from the textbook:

$$E'^0 = \sqrt{(E_d'^0 + (X_q' - X_d') I_q^0)^2 + (E_q'^0)^2}$$

$$\delta'^0 = \tan^{-1}\left(\frac{E_q'^0}{E_d'^0 + (X_q' - X_d') I_q^0}\right) - \frac{\pi}{2}$$

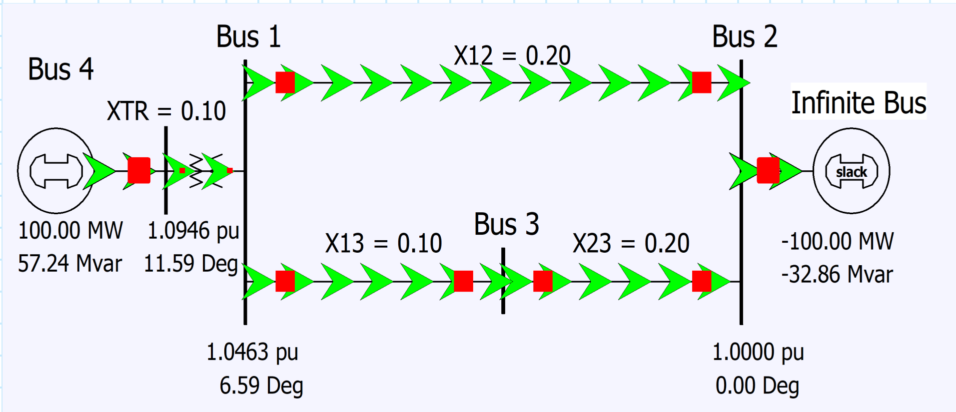
$\Rightarrow \boxed{\begin{array}{l}\delta'^0 = -28.096^\circ \\ E'^0 = 0.954\,pu\end{array}}$

These give equivalent results as above.

Homework #3, Problem #3

Thursday, September 30, 2021 5:47 PM

From Lecture 8, Slide 46



- Generator supplies 1pu real power at 0.95pf lagging
- Current is $1\angle 0^\circ$ pu into the bus.

$\Rightarrow \tilde{I} = 1\angle 0^\circ$ pu

current flowing through the line

$$\tilde{V}_s = (1\angle 0^\circ) + j\,0.22(1\angle 0^\circ)$$

equivalent impedance of line

voltage at infinite bus

$$= 1 + j0.22 \text{ pu}$$

$$\Rightarrow |E|\angle\delta' = \tilde{V}_s + (\cancel{R_s}^{\,0} + jX_q)\tilde{I}$$

$$= (1 + j0.22) + j2(1\angle 0)$$

$$= 1 + j2.22 = 2.435\angle 65.751^\circ \text{ pu}$$

$$\Rightarrow \boxed{\delta = 65.751^\circ}$$

$$\Rightarrow \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

$$= \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 1 \\ 0.22 \end{bmatrix}$$

$$= \begin{bmatrix} 0.821 \\ 0.611 \end{bmatrix} pu \Rightarrow \boxed{\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.821 \\ 0.611 \end{bmatrix} pu}$$

$$\Rightarrow \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

$$= \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.912 \\ 0.411 \end{bmatrix} pu \Rightarrow \boxed{\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.912 \\ 0.411 \end{bmatrix} pu}$$

$$E_q' = V_q + R_s I_q + X_d' I_d \Rightarrow \boxed{E_q' = 0.884 \, pu}$$

$$E_d' = V_d - R_s I_d - X_q' I_q \Rightarrow \boxed{E_d' = 0.616 \, pu}$$

Homework #3, Problem #4

Wednesday, October 6, 2021 11:15 AM

From the B4_GENSAL_SAT system:

$H = 3.0$ $\quad X_d = 2.1$ $\quad X_d'' = X_q'' = 0.2$ $\quad T_{do}'' = 0.07$

$D = 0$ $\quad X_q = 2.0$ $\quad X_\ell = 0.13$ $\quad T_{qo}'' = 0.07$

$R_a = 0$ $\quad X_d' = 0.3$ $\quad T_{do}' = 7.0$ $\quad S(1.0) = 0.02$

$S(1.2) = 0.1$

Given that the power delivered is $2 + j\,0.2$ pu

$$S = V I^* \Rightarrow 2 + j2 = (1\angle 0) I^* \Rightarrow \tilde{I} = 2 - j2 \text{ pu}$$

$$\Rightarrow \tilde{V} = (1\angle 0^\circ) + j\,0.22\,(2 - j2)$$

$$= 1.044 + j\,0.44 \text{ pu}$$

$$\Rightarrow |E|\angle\delta = \tilde{V}_s + (\cancel{R_s}^{\,0} + j X_q)\,\tilde{I}$$

$$= 4.669 \angle 71.9842^\circ \text{ pu}$$

$$\Rightarrow \boxed{\delta = 71.9842^\circ}$$

$$\Rightarrow \begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

$$= \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 1.044 \\ 0.44 \end{bmatrix}$$

$$= \begin{bmatrix} 0.857 \\ 0.741 \end{bmatrix} \text{pu} \Rightarrow \boxed{\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.857 \\ 0.741 \end{bmatrix} \text{pu}}$$

$$\Rightarrow \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_r \\ I_i \end{bmatrix}$$

$$= \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.964 \\ 0.428 \end{bmatrix} pu \quad \Rightarrow \boxed{\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 1.964 \\ 0.428 \end{bmatrix} pu}$$

$$\Rightarrow E'' = \tilde{V} + (\cancelto{0}{R_s} + jX'')\tilde{I}, \text{ where } X'' = 0.2$$

$$= (1.044 + j0.44) + j0.2(2 - j2)$$

$$= 1.084 + j0.84 \ pu$$

$\Rightarrow$ Assuming $\omega = 1$ pu, we get:

$$\begin{bmatrix} -\psi_q'' \\ \psi_d'' \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} E_r'' \\ E_i'' \end{bmatrix}$$

$$= \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 1.084 \\ 0.84 \end{bmatrix}$$

$$= \begin{bmatrix} 0.771 \\ 1.134 \end{bmatrix} \Rightarrow \boxed{\begin{bmatrix} \psi_q'' \\ \psi_d'' \end{bmatrix} = \begin{bmatrix} -0.771 \\ 1.134 \end{bmatrix}}$$

$\Rightarrow$ At steady state: $E_q' = V_q + R_s I_q + X_d' I_d$

$$\psi_d' = E_q' - (X_d' - X_\ell) I_d$$

$$\Rightarrow E_q' = 0.741 + 0.3(1.964)$$

$$= 1.3304 pu$$

$$\psi_d' = 1.3304 - (0.3 - 0.13)(1.964)$$

$$= 0.997 pu$$

$$\Rightarrow \boxed{\begin{aligned} E_q' &= 1.3304 \ pu \\ \psi_d' &= 0.997 \ pu \end{aligned}}$$

- However, when calculating E_{fd}, we need to consider saturation.

Given: $S(1) = 0.02$ $\quad S = B(E_q' - A)^2$

$S(1.2) = 0.1$

$$\Rightarrow B = \frac{S}{(E_q' - A)^2} \Rightarrow B = \frac{S(1)}{(1-A)^2} = \frac{S(1.2)}{(1.2-A)^2}$$

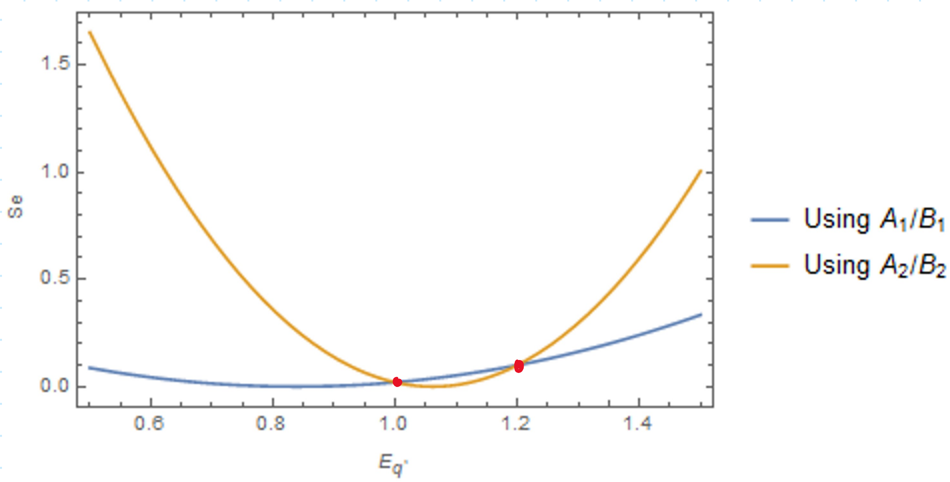
$$\Rightarrow S(1) = \frac{S(1.2)}{(1.2-A)^2} \cdot (1-A)^2$$

$$\Rightarrow (1.2-A)^2 S(1) = (1-A)^2 S(1.2)$$

$\Rightarrow$ Either: $A_1 = 0.838 \Rightarrow B_1 = 0.763$

OR

$A_2 = 1.0618 \Rightarrow B_2 = 5.236$



Since we don't want the minimum to be between the marked points, we use $A_1 = 0.838$

$B_1 = 0.763$

$$\Rightarrow E_{fd} = E_q'(1 + S(E_q')) + (X_d - X_d') I_d$$

$$\Rightarrow \boxed{E_{fd} = 5.112 \text{ pu}}$$

Homework #3, Problem #5

Wednesday, October 6, 2021 2:23 PM

Parameter	Initial Value	Final Value	Critical Clearing Time	1st Swing Peak	4th Swing Peak	"Damping"	CCT Delta	Damping Delta
Inertia (H)	3	4	0.18	131.977	70.774	0.261011833	0.01	-0.013834096
Damping Factor (D)	0	1	0.17	151.524	73.93	0.242001817	0	-0.032844112
Stator Resistance (Ra)	0	1	0.04	51.765	47.166	-0.763420245	-0.13	-1.038266175
D-Axis Synchronous Reactance (Xd)	2.1	3.1	0.16	142.129	75.071	0.27872908	-0.01	0.003883151
Q-Axis Synchronous Reactance (Xq)	2	3	0.16	145.981	78.391	0.301929274	-0.01	0.027083345
D-Axis Transient Reactance (Xdp)	0.3	1.3	0.1	113.976	55.142	0.092334038	-0.07	-0.182511891
Q-Axis Transient Reactance (Xqp)	0.5	1.5	0.17	145.399	75.743	0.276241142	0	0.001395213
D-Axis Subtransient Reactance (Xdpp)	0.28	0.29	0.17	159.146	80.899	0.288592496	0	0.013746566
Stator Leakage Reactance (Xl)	0.13	0.23	0.17	154.801	80.56	0.297253039	0	0.022407109
Open Circuit D-Axis Transient Time Constant (Tdop)	7	8	0.17	155.975	78.155	0.2714711	0	-0.003374829
Q-Axis Transient Time Constant (Tqop)	0.75	0.85	0.17	157.013	78.079	0.268153835	0	-0.006692094
Open Circuit D-Axis Subtransient Time Constant (Tdopp)	0.073	0.173	0.17	155.444	78.409	0.275217101	0	0.000371172
Q-Axis Subtransient Time Constant (Tqopp)	0.07	0.17	0.17	162.847	86.579	0.329158237	0	0.054312308
Saturation @ 1 flux (S1)	0.05	0.15	0.16	137.917	74.748	0.288316809	-0.01	0.01347088
Saturation @ 1.2 flux (S12)	0.2	0.3	0.17	152.545	73.968	0.239979495	0	-0.034866434
Compensating Resistance (Rcomp)	0	1	0.17	158.36	79.171	0.274845929	0	0
Compensating Reactance (Xcomp)	0	1	0.17	158.36	79.171	0.274845929	0	0

From the above data, we note that the most sensitive parameters are the inertia (H), damping factor (D), stator resistance (Ra), and D-axis transient reactance X'_d.

Thus:

Parameter	CCT Delta	Damping Delta
Inertia (H)	0.01	-0.013834096
Damping Factor (D)	0	-0.032844112
Stator Resistance (Ra)	-0.13	-1.038266175
D-Axis Transient Reactance (Xdp)	-0.07	-0.182511891

Shows how the critical clearing time (CCT) and the damping change as we change the parameters by 1.