ECEN 667
Power System Stability

Lecture 14: Time-Domain Simulation Solutions (Transient Stability)

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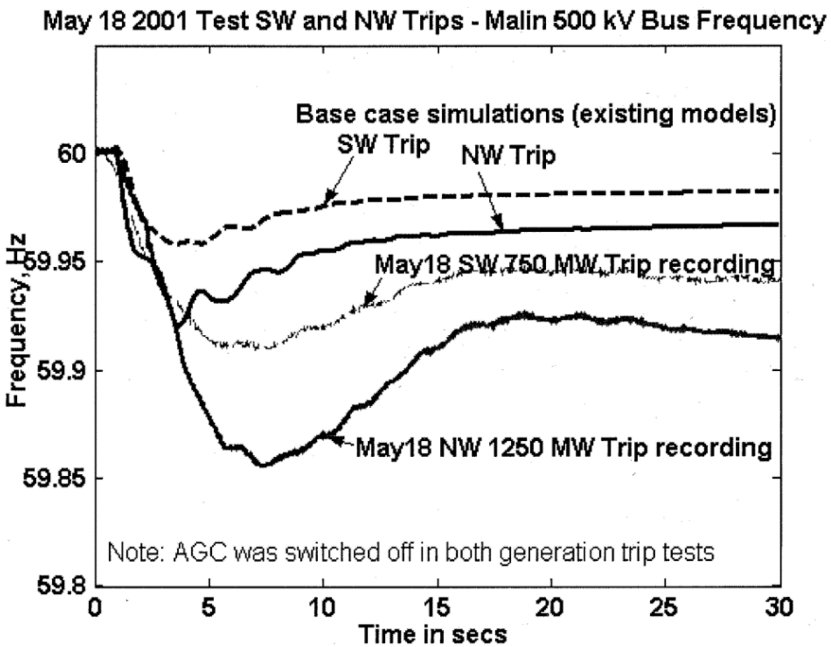
Announcements

- Read Chapter 7
- Exam 1 average was 78
- Homework 5 is assigned today; it will be due on Oct 28.
- A classic paper in this area is B. Stott, “Power System Dynamic Response Calculations,” *Proc. IEEE*, February 1979, pp. 219-241

GGOV1

- GGOV1 is a relatively newer governor model introduced in early 2000's by WECC for modeling thermal plants
  - Existing models greatly under-estimated the frequency drop
  - GGOV1 is now the most common WECC governor, used with about 40% of the units
- A useful reference is L. Pereira, J. Undrill, D. Kosterev, D. Davies, and S. Patterson, "A New Thermal Governor Modeling Approach in the WECC," *IEEE Transactions on Power Systems*, May 2003, pp. 819-829

GGOV1: Selected Figures from 2003 Paper



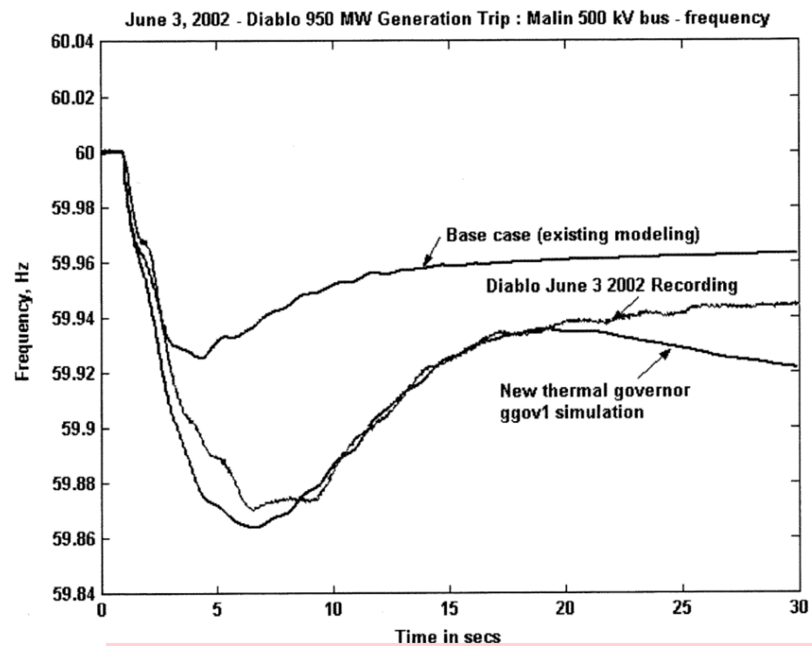
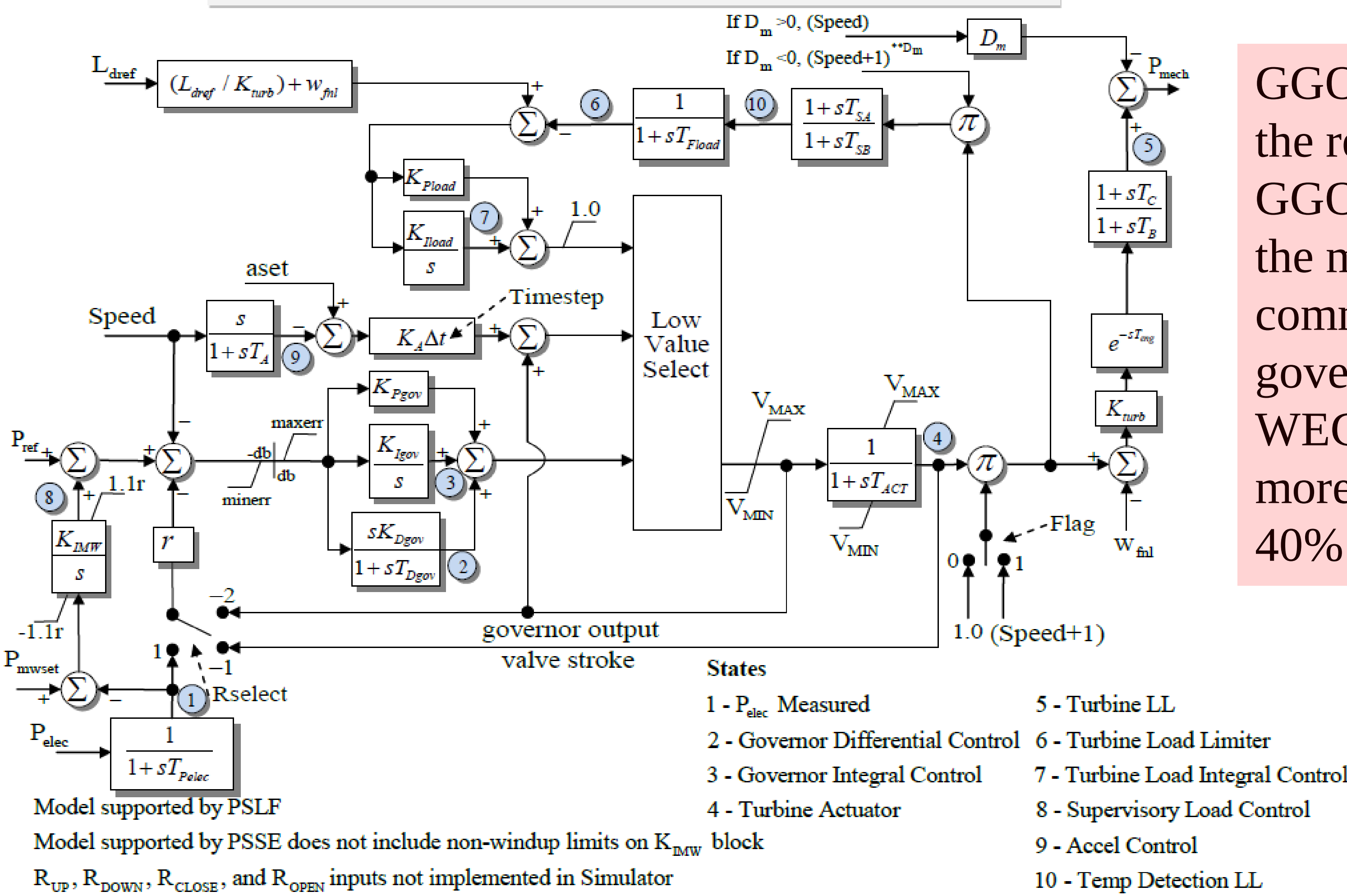


Fig. 1. Frequency recordings of the SW and NW trips on May 18, 2001. Also shown are simulations with existing modeling (base case).

Governor model verification—950-MW Diablo generation trip on June 3, 2002.

Diablo Canyon is California’s last nuclear plant, with Unit 1 now scheduled to shutdown in 2024 and Unit 2 in 2025 (though there has been recent controversy about this)

GGOV1 Block Diagram



GGOV1 and the related GGOV3 are the most common governors in WECC, with more than 40% in 2019

Power System Stability Assessment

- As given in [1] the formal definition of power system stability is
  - "Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact"
- The previously developed models will help in power system stability assessment
- Different techniques are used including time-domain simulations, eigenvalue analysis and power flow

1] IEEE/PES Power System Dynamic Performance Committee, "Stability definitions and characterization of dynamic behavior in systems with high penetration of power electronic interfaced technologies", PES-TR77, April 2020

Power System Stability Terms

- The below image (from Figure 4 of [1]), and also shown lecture 2, helps define the terms

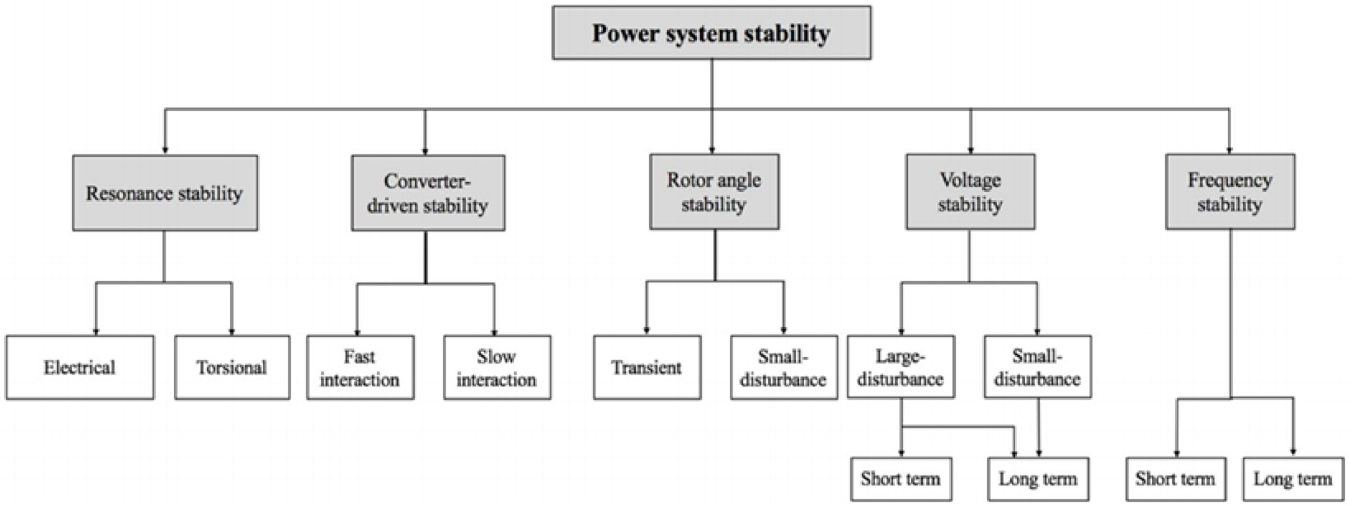


Fig. 4. Classification of power system stability

The two main time scales are the electromagnetic (left two branches) and the electromechanical (right three branches). The focus in 667 is mostly on the electromechanical time scale with ECEN 616 focusing on the electromagnetic.

1] IEEE/PES Power System Dynamic Performance Committee, "Stability definitions and characterization of dynamic behavior in systems with high penetration of power electronic interfaced technologies", PES-TR77, April 2020

Transient Stability Multimachine Simulations

- Next, we'll be putting the models we've covered so far together
- Later we'll add in new model types such as stabilizers, loads and wind turbines
- By way of history, prior to digital computers, network analyzers were used for system stability studies as far back as the 1930's (perhaps earlier)
  - For example see, J.D. Holm, "Stability Study of A-C Power Transmission Systems," AIEE Transactions, vol. 61, 1942, pp. 893-905
- Digital approaches started appearing in the late 1950's

A Little History

- A nice early reference is
  - Dyrkacz, Young, Maginniss, “A Digital Transient Stability Program Including the Effects of Regular, Exciter and Governor Response,” Proc. AIEE, Part 3, February 1961, pp. 1245-1254

This 1961 demonstrates results on a 96 bus system, shown below; note that the simulation is quite short, less than one second

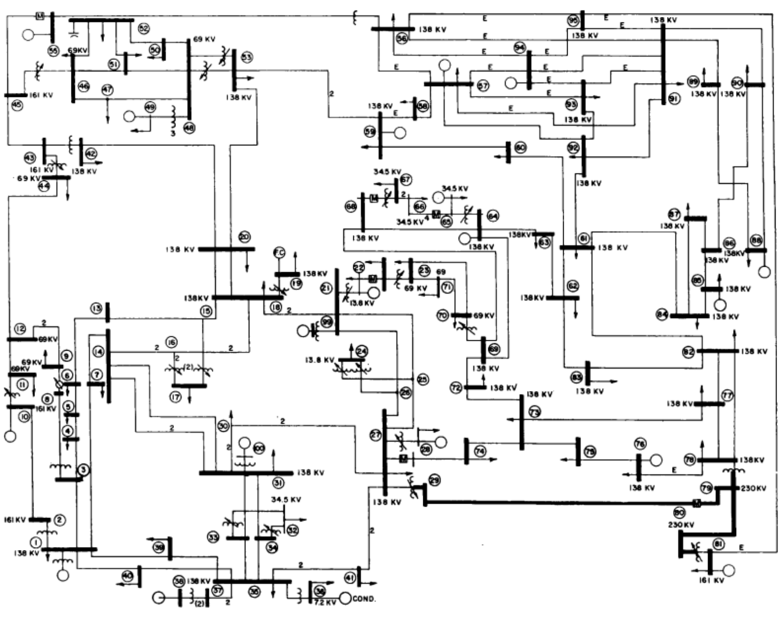
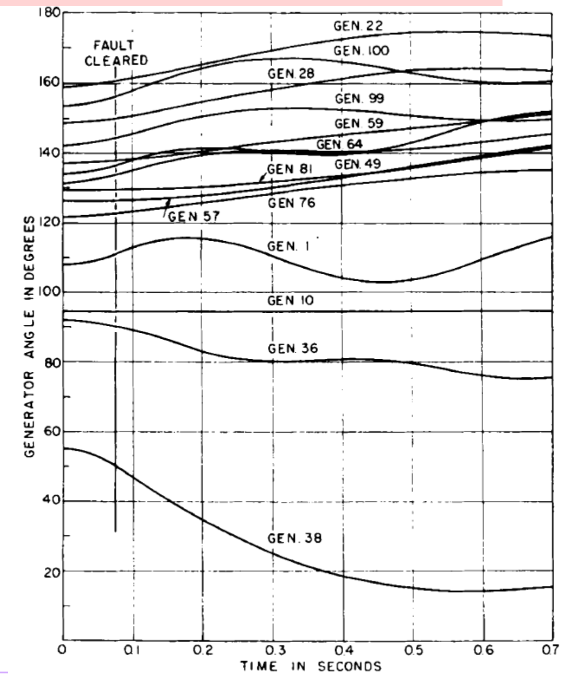


Fig. 4. One-line diagram of Union Electric Company system



Power System Multimachine Simulations

- The general structure is as a set of differential-algebraic equations
  - Differential equations describe the behavior of the machines (and the loads and other dynamic devices)
  - Algebraic equations representing the network constraints

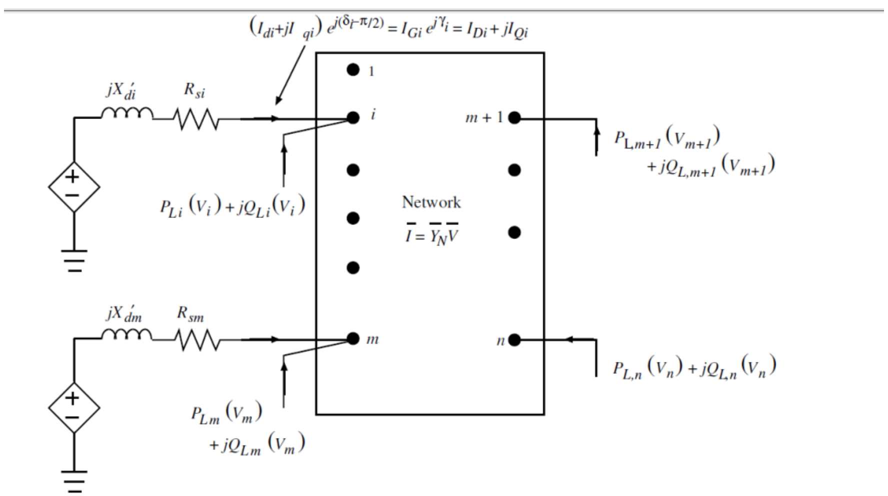


Figure 7.2: *Interconnection of synchronous machine dynamic circuit and the rest of the network*

In EMTP applications the transmission line delays decouple the machines; here they are assumed to be coupled together by the algebraic network equations

General Form

- The general form of the problem is solving

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$
$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

where $\mathbf{x}$ is the vector of the state variables (such as the generator δ's), $\mathbf{y}$ is the vector of the algebraic variables (primarily the bus complex voltages), and $\mathbf{u}$ is the vector of controls (such as the exciter voltage setpoints)

Stability Simulations General Solution

- General solution approach is
  - Solve power flow to determine initial conditions
  - Back solve to get initial states, starting with machine models, then exciters, governors, stabilizers, loads, etc
  - Set $t = t_{start}$, time step = Δt, abort = false
  - While (t <= tend) and (not abort) Do Begin
    - Apply any contingency event
    - Solve differential and algebraic equations
    - If desired store time step results and check other conditions (that might cause the simulation to abort)
    - $t = t + \Delta t$
  - End while

Algebraic Constraints

- The **g** vector of algebraic constraints is similar to the power flow equations, but usually rather than formulating the problem like in the power flow as real and reactive power balance equations, it is formulated in the current balance form

$$\mathbf{I}(\mathbf{x},\mathbf{V}) = \mathbf{Y}\,\mathbf{V} \quad \text{or} \quad \mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{V}) = \mathbf{0}$$

where $\mathbf{Y}$ is the n $\times$ n bus admittance matrix $(\mathbf{Y} = \mathbf{G} + j\mathbf{B}), \mathbf{V}$ is the complex vector of the bus voltages, and $\mathbf{I}$ is the complex vector of the bus current injections

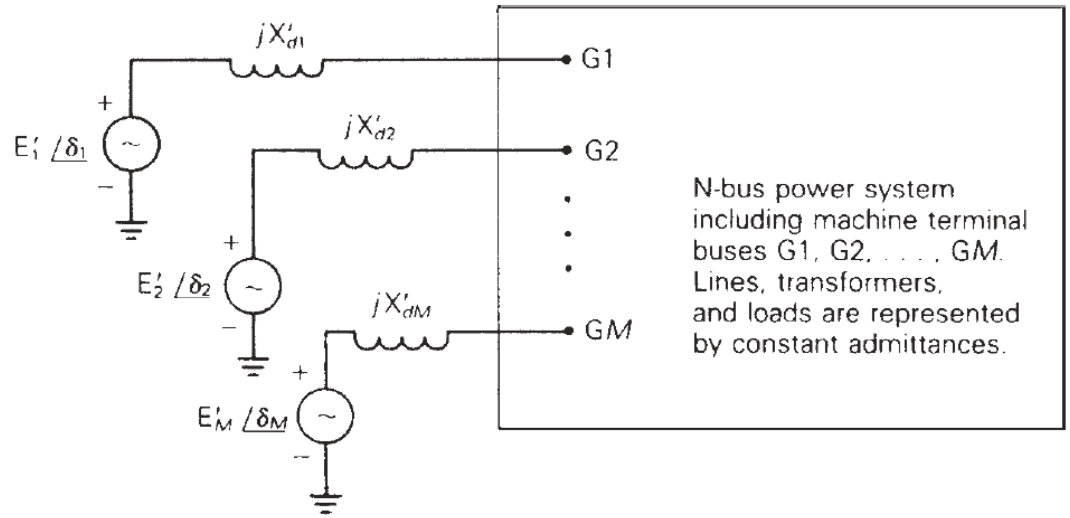
Simplest cases can have I independent of **x** and **V**, allowing a direct solution; otherwise we need to iterate

Why Not Use the Power Flow Equations?

- The power flow equations were ultimately derived from
  $\mathbf{I}(\mathbf{x}, \mathbf{V}) = \mathbf{Y}\ \mathbf{V}$
- However, the power form was used in the power flow primarily because
  - For the generators the real power output is known and either the voltage setpoint (i.e., if a PV bus) or the reactive power output
  - In the quasi-steady state power flow time frame the loads can often be well approximated as constant power
  - The constant frequency assumption requires a slack bus
- These assumptions do not hold for transient stability

Algebraic Equations for Classical Model

- To introduce the coupling between the machine models and the network constraints, consider a system modeled with just classical generators and impedance loads



In this example because we are using the classical model all values are on the network reference frame

We'll extend the figure slightly to include stator resistances, $R_{s,i}$

Image Source: Fig 11.15, Glover, Sarma, Overbye, *Power System Analysis and Design*, 5th Edition, Cengage Learning, 2011

Algebraic Equations for Classical Model

- Replace the internal voltages and their impedances by their Norton Equivalent

Review Norton and Thevenin equivalents if you are rusty on them

$$\bar{I}_i = \frac{E_i'\angle\delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$

- Current injections at the non-generator buses are zero since the constant impedance loads are included in **Y**
  - We'll modify this later when we talk about dynamic loads
- The algebraic constraints are then **I** – **Y V** = **0**

Swing Equation

- The first two differential equations for any synchronous machine correspond to the swing equation

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $T_{Ei} = \psi_{de,i} i_{qi} - \psi_{qe,i} i_{di}$

Swing Equation Speed Effects

- There is often confusion about these equations because of whether speed effects are included
  - Recognizing that often $\omega \approx \omega_s$ (which is one per unit), some stability books have neglected speed effects
- For a rotating machine with a radial torque, power = torque times speed
- For a subtransient model

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}, \quad E''_d + jE''_q = \left(-\psi''_q + j\psi''_d\right)\omega$$

$$T_E = \psi''_d I_q - \psi''_q I \quad and$$

$$P_E = T_E \omega = \left(E''_d + jE''_q\right)\left(I_d - jI_q\right) = E''_d I_d + E''_q I_q$$

Classical Swing Equation

- Often in an introductory coverage of transient stability with the classical model the assumption is $\omega \approx \omega_s$ so the swing equation for the classical model is given as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_i\right)$$

$$\text{with } \mathrm{P}_{Ei} = \left(E_i'\angle\delta_i\right)\left(E_i'\angle\delta_i - \bar{V}_i\right)Y_i$$

- We'll use this simplification for our initial example

As an example of this initial approach see Anderson and Fouad, *Power System Control and Stability*, 2nd Edition, Chapter 2 (with a newer version third edition of this book now available adding Vijay Vittal and Jim McCalley as authors).

Numerical Solution

- There are two main approaches for solving

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

  - Partitioned-explicit: Solve the differential and algebraic equations separately (alternating between the two) using an explicit integration approach
  - Simultaneous-implicit: Solve the differential and algebraic equations together using an implicit integration approach

Outline of the Solution Process

- The next group of slides will provide basic coverage of the solution process, partitioned explicit, then the simultaneous-implicit approach
- We'll start out with a classical model supplying an infinite bus, which can be solved by embedded the algebraic constraint into the differential equations

We'll start out just solving $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$

and then will extend to solving the full problem of

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

Classical Swing Equation with Embedded Power Balance

- With a classical generator at bus i supplying an infinite bus with voltage magnitude V_{inf}, we can write the problem without algebraic constraints as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left(P_{Mi} - \frac{E_i' V_{\inf}}{X_{th}}\sin\delta_i - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

$$\text{with } P_{Ei} = \frac{E_i' V_{\inf}}{X_{th}}\sin\delta_i$$

Note we are using the per unit speed approach

Explicit Integration Methods

- As covered on the first day of class, there are a wide variety of explicit integration methods
  - We considered Forward Euler, Runge-Kutta, Adams-Bashforth
- Here we will just consider Euler's, which is easy to explain but not too useful, and a second order Runge-Kutta, which is commonly used

Forward Euler

- Recall the Forward Euler approach is approximate

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \frac{\mathrm{d}\mathbf{x}}{\mathrm{dt}} \text{ as } \frac{\Delta\mathbf{x}}{\Delta\mathrm{t}}$$

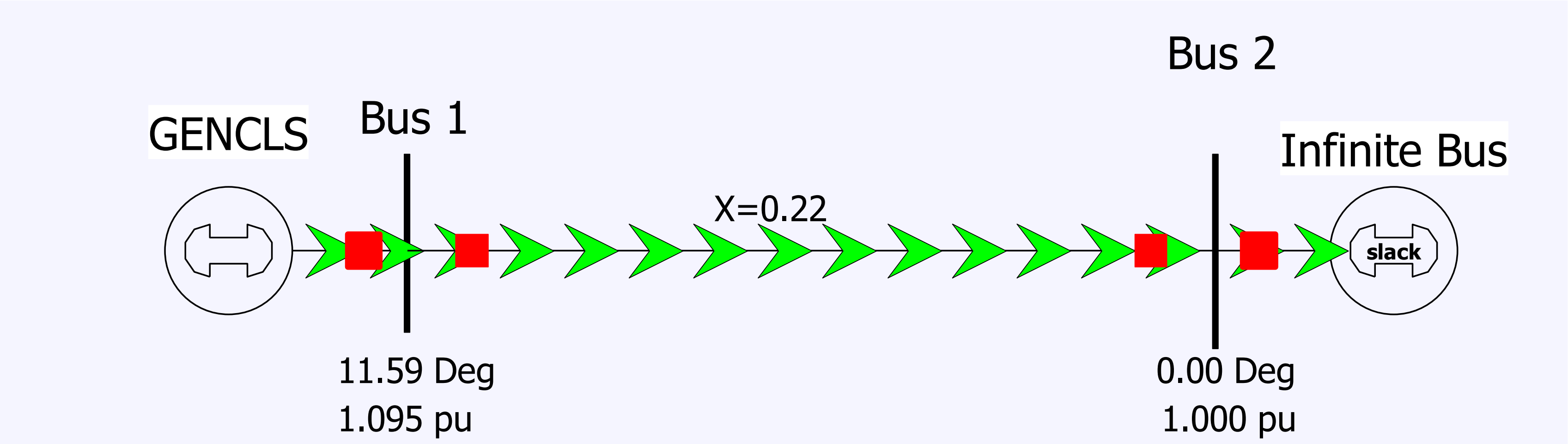
Then

$$\mathbf{x}(t+\Delta t) \approx \mathbf{x}(t) + \Delta t\,\mathbf{f}(\mathbf{x}(t))$$

- Error with Euler's varies with the square of the time step

Infinite Bus GENCLS Example using the Forward Euler's Method

- Use the four bus system from before, except now gen 4 is modeled with a classical model with $X_d'=0.3$, $H=3$ and $D=0$; also we'll reduce to two buses with equivalent



In this example $X_{th} = (0.22 + 0.3)$, with the internal voltage $\bar{E}'_1 = 1.281\angle 23.95°$ giving $E'_1=1.281$ and $\delta_1= 23.95°$

Infinite Bus GENCLS Example

- The associated differential equations for the bus 1 generator are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}\left(1 - \frac{1.281}{0.52}\sin\delta_1\right)$$

- The value of $P_{M1} = 1$ is determined from the initial conditions, and would stay constant in this simple example without a governor
- The value $\delta_1 = 23.95°$ is readily verified as an equilibrium point (which is 0.418 radians)

Infinite Bus GENCLS Example

- Assume a solid three phase fault is applied at the generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear}, resulting in the post-fault system being identical to the pre-fault system
  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}\left(1-0\right)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Euler's Solution

- The initial value of **x** is

$$\mathbf{x}(0) = \begin{bmatrix} \delta_1(0) \\ \Delta\omega_{1,pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Assuming a time step Δt = 0.02 seconds, and a T^{clear} of 0.1 seconds, then using Euler's

$$\mathbf{x}(0.02) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

- Iteration continues until t = T^{clear}

Note Euler's assumes δ stays constant during the first time step

Euler's Solution

- At $t = T^{clear}$ the fault is self-cleared, with the equations changing to

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

- The integration continues using the new equations

Euler's Solution Results (Δt=0.02)

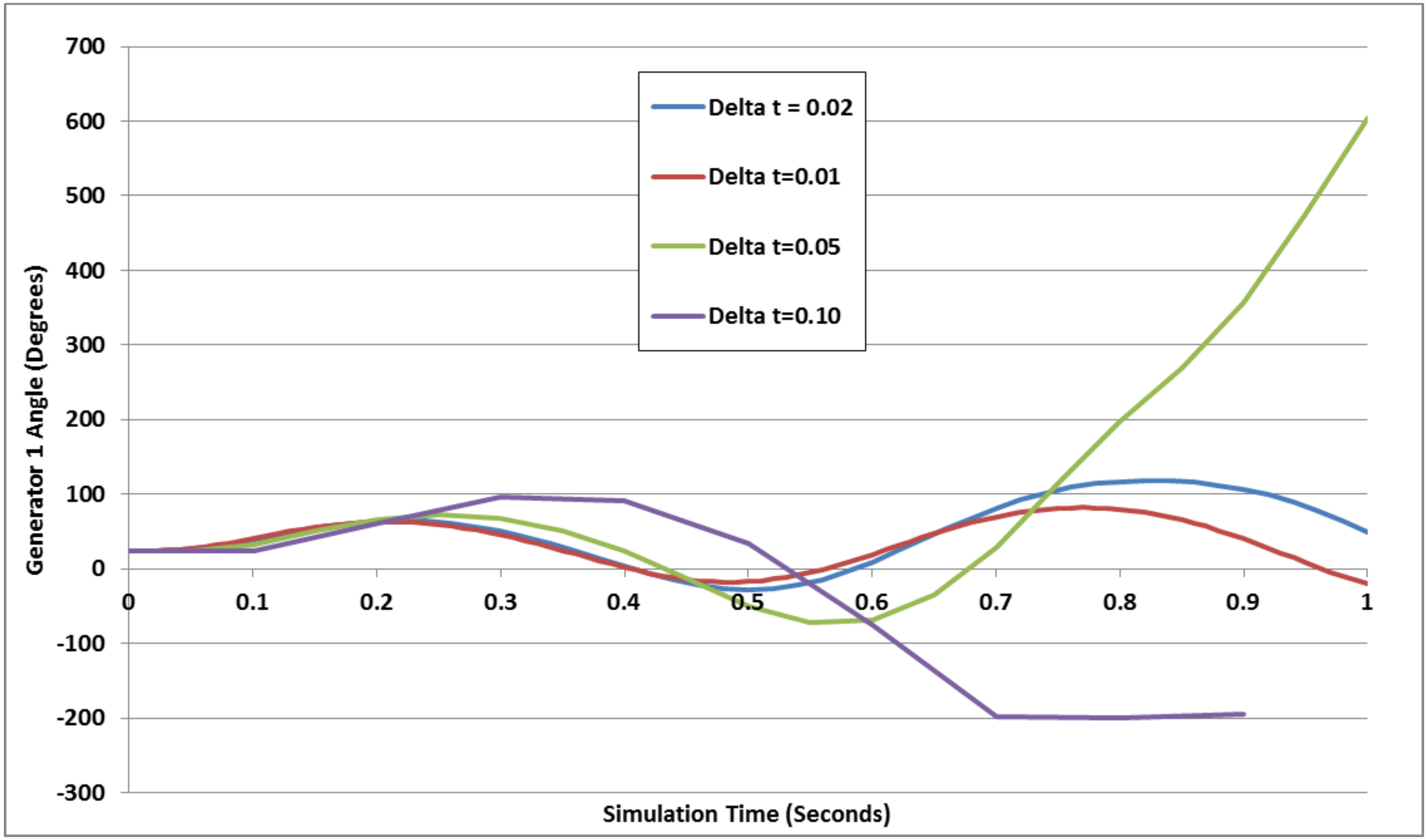
- The below table gives the results using Δt = 0.02 for the beginning time steps

Time	Gen 1 Rotor Angle, Degrees	Gen 1 Speed (Hz)
0	23.9462	60
0.02	23.9462	60.2
0.04	25.3862	60.4
0.06	28.2662	60.6
0.08	32.5862	60.8
0.1	38.3462	61
0.1	38.3462	61
0.12	45.5462	60.8943
0.14	51.9851	60.7425
0.16	57.3314	60.5543
0.18	61.3226	60.3395
0.2	63.7672	60.1072
0.22	64.5391	59.8652
0.24	63.5686	59.6203
0.26	60.8348	59.3791
0.28	56.3641	59.1488

This is saved as PowerWorld case **B2_CLS_Infinite**. The integration method is set to Euler's on the Transient Stability, Options, Power System Model page

Generator 1 Delta: Euler's

- The below graph shows the generator angle for varying values of Δt; numerical instability is clearly seen



Second Order Runge-Kutta

- Runge-Kutta methods improve on Euler's method by evaluating **f**(**x**) at selected points over the time step
- One approach is a second order method (RK2) in which

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$$

where

$$\mathbf{k}_1 = \Delta t\ \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}(\mathbf{x}(t) + \mathbf{k}_1)$$

This is also known as Heun's method or as the Improved Euler's or Modified Euler's Method

- That is, $\mathbf{k}_1$ is what we get from Euler's; $\mathbf{k}_2$ improves on this by reevaluating at the estimated end of the time step
- Error varies with the cubic of the time step

Second Order Runge-Kutta (RK2)

- Again assuming a time step $\Delta t = 0.02$ seconds, and a T^{clear} of 0.1 seconds, then using Heun's approach

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \Delta\omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

$$\mathbf{k}_1 = 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.00333 \end{bmatrix}, \quad \mathbf{x}(0) + \mathbf{k}_1 = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{k}_2 = 0.02 \begin{bmatrix} 1.257 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{x}(0.020) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{1}{2}\left(\mathbf{k}_1 + \mathbf{k}_2\right) = \begin{bmatrix} 0.431 \\ 0.00333 \end{bmatrix}$$

RK2 Solution Results (Δt=0.02)

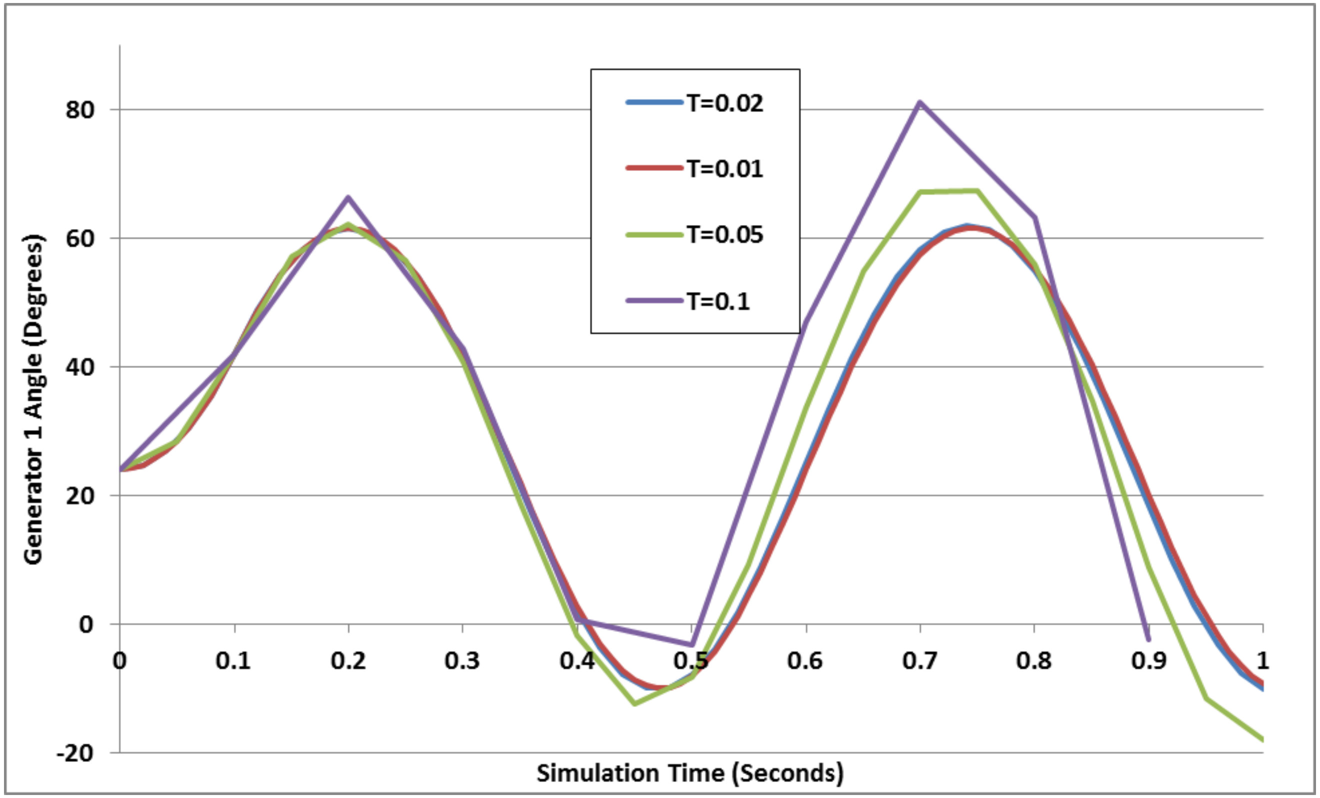
- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time	Gen 1 Rotor Angle, Degrees	Gen 1 Speed (Hz)
0	23.9462	60
0.02	24.6662	60.2
0.04	26.8262	60.4
0.06	30.4262	60.6
0.08	35.4662	60.8
0.1	41.9462	61
0.1	41.9462	61
0.12	48.6805	60.849
0.14	54.1807	60.6626
0.16	58.233	60.4517
0.18	60.6974	60.2258
0.2	61.4961	59.9927
0.22	60.605	59.7598
0.24	58.0502	59.5343
0.26	53.9116	59.3241
0.28	48.3318	59.139

This is saved as PowerWorld case B2_CLS_Infinite. The integration method should be changed to Second Order Runge-Kutta on the Transient Stability, Options, Power System Model page

Generator 1 Delta: RK2

- The below graph shows the generator angle for varying values of Δt; much better than Euler's but still the beginning of numerical instability with larger values of Δt



Adding Network Equations

- Previous slides with the network equations embedded in the differential equations were a special case
- In general with the explicit approach we'll be alternating between solving the differential equations and solving the algebraic equations
- Voltages and currents in the network reference frame can be expressed using either polar or rectangular coordinates
- In rectangular with the book's notation we have

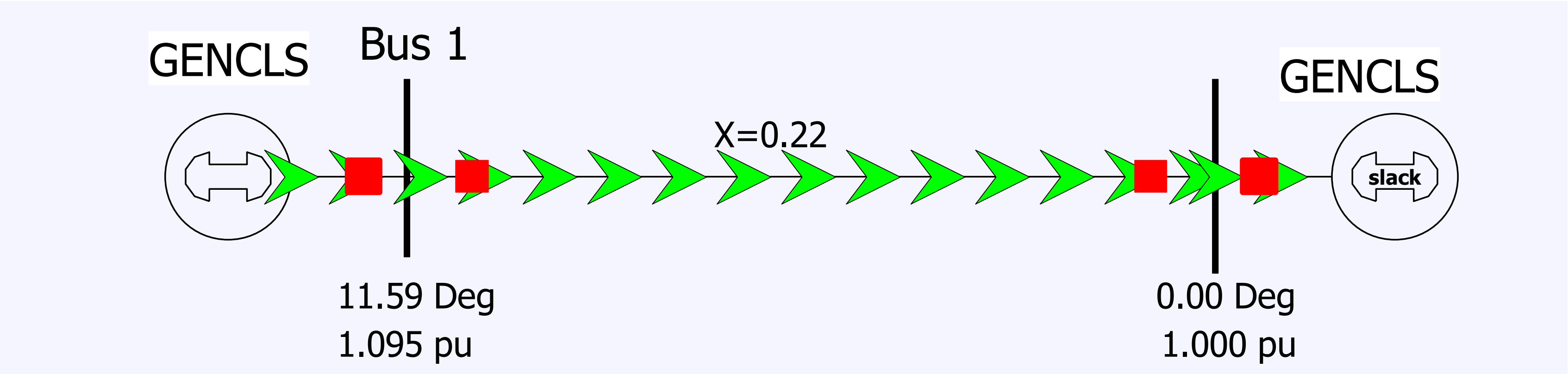
$$\bar{V}_i = V_{Di} + jV_{Qi}, \quad \bar{I}_i = I_{Di} + jI_{Qi}$$

Adding Network Equations

- Network equations will be written as **Y V- I(x,V) = 0**
  - Here **Y** is as from the power flow, except augmented to include the impact of the generator's internal impedance
  - Constant impedance loads are also embedded in **Y**; non-constant impedance loads are included in **I(x,V)**
- If **I** is independent of **V** then this can be solved directly: $\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I}(\mathbf{x})$
- In general an iterative solution is required, which we'll cover shortly, but initially we'll go with just the direct solution

Two Bus Example, Except with No Infinite Bus

- To introduce the inclusion of the network equations, the previous example is extended by replacing the infinite bus at bus 2 with a classical model with $X_{d2}'=0.2$, $H_2=6.0$



PowerWorld Case is **B2_CLS_2Gen**

Bus Admittance Matrix

- The network admittance matrix is

$$\mathbf{Y}_N = \begin{bmatrix} -j4.545 & j4.545 \\ j4.545 & -j4.545 \end{bmatrix}$$

- This is augmented to represent the Norton admittances associated with the generator models (X_{d1}'=0.3, X_{d2}'=0.2)

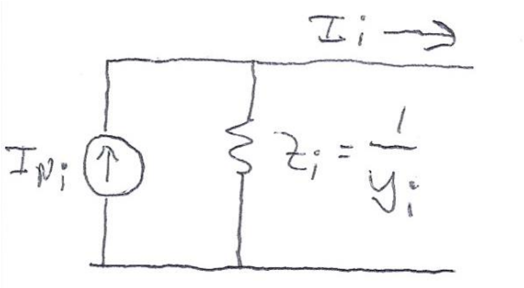
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.3} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

In PowerWorld you can see this matrix by selecting **Transient Stability, States/Manual Control, Transient Stability Ybus**

Current Vector

- For the classical model the Norton currents are given by

$$\bar{I}_{Ni} = \frac{E_i' \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$



- The initial values of the currents come from the power flow solution
- As the states change (δ_i for the classical model), the Norton current injections also change

B2_CLS_Gen Initial Values

- The internal voltage for generator 1 is as before

$$\bar{I} = 1 - j0.3286$$

0.4179 radians

$$\bar{E}_1 = 1.0 + (j0.22 + j0.3)\bar{I} = 1.1709 + j0.52 = 1.281\angle 23.95°$$

- We likewise solve for the generator 2 internal voltage

$$\bar{E}_2 = 1.0 - (j0.2)\bar{I} = 0.9343 - j0.2 = 0.9554\angle -12.08$$

- The Norton current injections are then

0.2108 radians

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

Keep in mind the Norton current injections are not the current out of the generator

B2_CLS_Gen Initial Values

- To check the values, solve for the voltages, with the values matching the power flow values

$$\mathbf{V} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Swing Equations

- With the network constraints modeled, the swing equations are modified to represent the electrical power in terms of the generator's state and current values

$$\mathrm{P}_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$I_{Di}+jI_{Qi}$ is the current being injected into the network by the generator

- Then swing equation is then

$$\frac{d\delta_i}{dt} = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left(P_{Mi} - \left(E_{Di} I_{Di} + E_{Qi} I_{Qi}\right) - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

Two Bus, Two Generator Differential Equations

- The differential equations for the two generators are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

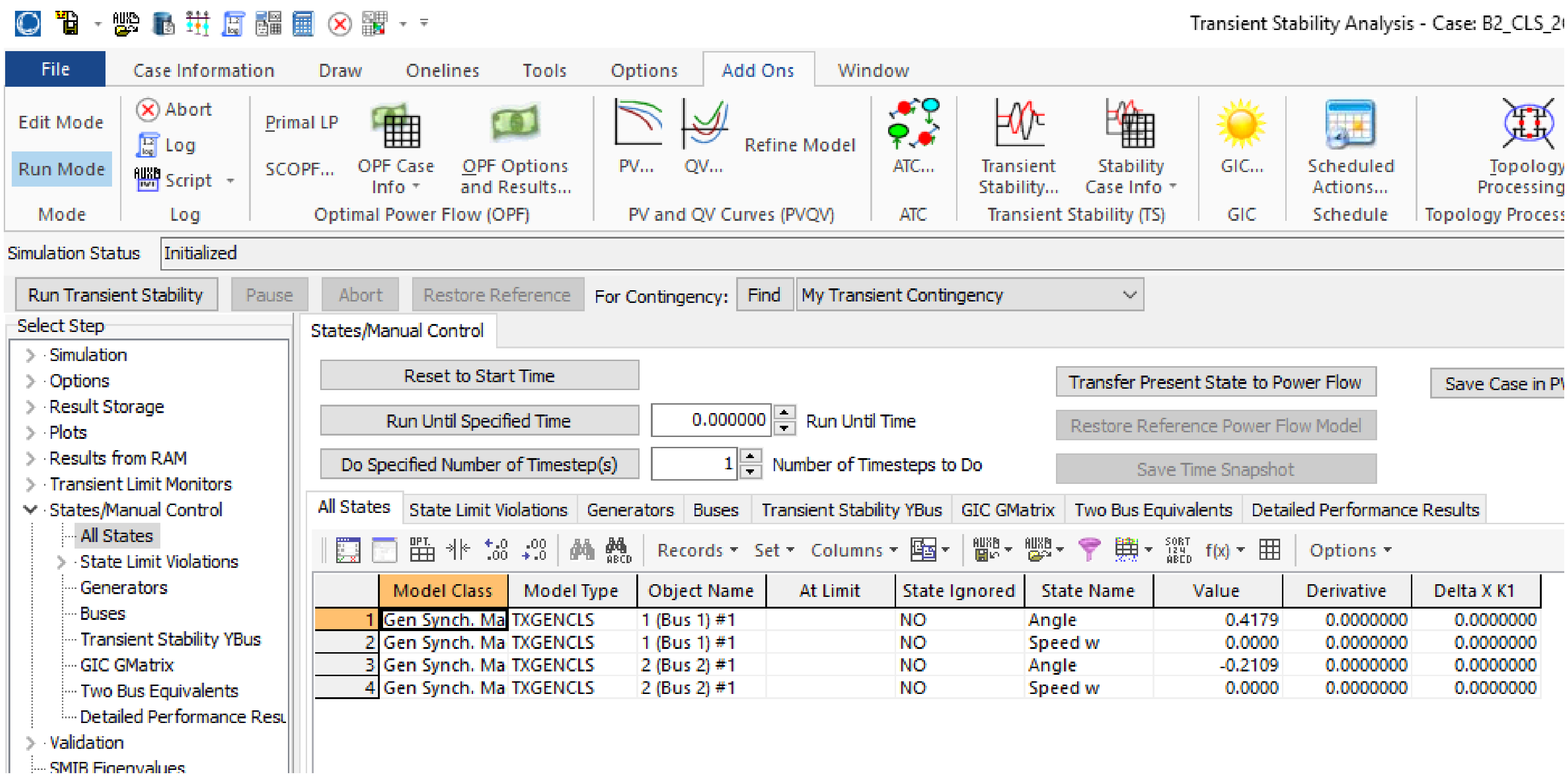
$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - \left(E_{D1}I_{D1} + E_{Q1}I_{Q1}\right)\right)$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - \left(E_{D2}I_{D2} + E_{Q2}I_{Q2}\right)\right)$$

In this example
P_{M1} = 1 and
P_{M2} = -1

PowerWorld GENCLS Initial States



Solution at t=0.02

- Usually a time step begins by solving the differential equations. However, in the case of an event, such as the solid fault at the terminal of bus 1, the network equations need to be first solved
- Solid faults can be simulated by adding a large shunt at the fault location
  - Amount is somewhat arbitrary, it just needs to be large enough to drive the faulted bus voltage to zero
- With Euler's the solution after the first time step is found by first solving the differential equations, then resolving the network equations

Solution at t=0.02

- Using Y_{fault} = -j1000, the fault-on conditions become

$$\mathbf{V}=\begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1}\begin{bmatrix} 1.733-j3.903 \\ -1-j4.671 \end{bmatrix}$$

$$=\begin{bmatrix} -0.006-j0.001 \\ 0.486-j0.1053 \end{bmatrix}$$

Solving for the currents into the network

$$I_1=\frac{(1.1702+j0.52)-V_1}{j0.3}=1.733-j3.900$$

$$I_2=\frac{(0.9343-j0.2)-(0.486-j0.1053)}{j0.2}=-0.473-j2.240$$

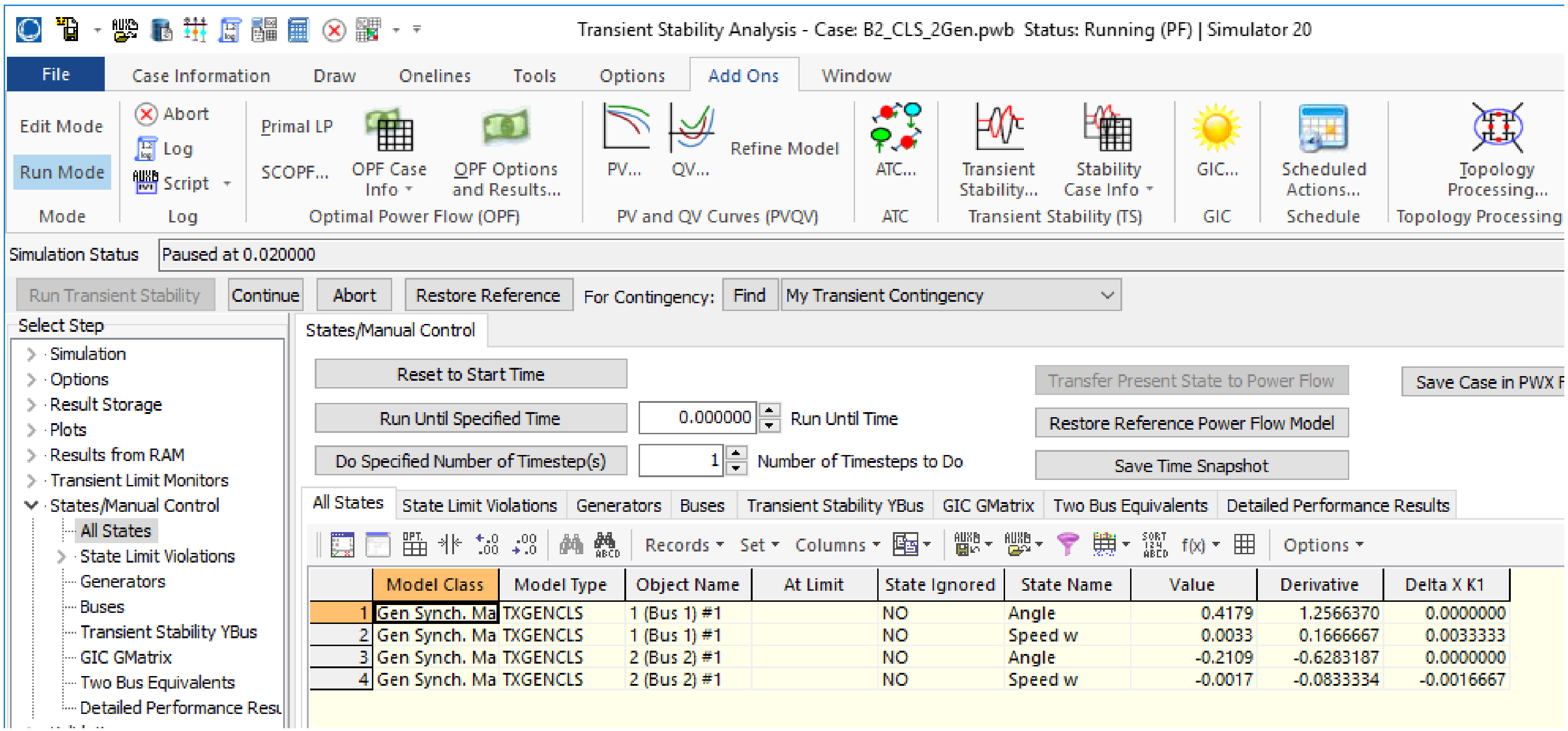
Solution at t=0.02

- Then the differential equations are evaluated, using the new voltages and currents
  - These impact the calculation of P_{Ei} with P_{E1}=0, P_{E2}=0

$$\begin{bmatrix} \delta_1(0.02) \\ \Delta\omega_1(0.02) \\ \delta_2(0.02) \\ \Delta\omega_1(0.02) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.0 \\ -0.211 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ \frac{1}{6}(1-0) \\ 0 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- If solving with Euler's this is the final state value; using these state values the network equations are resolved, with the solution the same here since the δ's didn't vary

PowerWorld GENCLS at t=0.02



Solution Values Using Euler's

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	23.9462	60.2	-12.0829	59.9
0.04	25.3862	60.4	-12.8029	59.8
0.06	28.2662	60.6	-14.2429	59.7
0.08	32.5862	60.8	-16.4029	59.6
0.1	38.3462	61	-19.2829	59.5
0.1	38.3462	61	-19.2829	59.5
0.12	45.5462	60.9128	-22.8829	59.5436
0.14	52.1185	60.7966	-26.169	59.6017
0.16	57.8541	60.6637	-29.0368	59.6682
0.18	62.6325	60.5241	-31.426	59.7379
0.2	66.4064	60.385	-33.3129	59.8075
0.22	69.1782	60.2498	-34.6988	59.8751
0.24	70.9771	60.1197	-35.5982	59.9401
0.26	71.8392	59.9938	-36.0292	60.0031
0.28	71.7949	59.8702	-36.0071	60.0649

Solution at t=0.02 with RK2

- With RK2 the first part of the time step is the same as Euler's, that is solving the network equations with

$$\mathbf{x}(\mathrm{t}+\Delta \mathrm{t})^{(1)} = \mathbf{x}(\mathrm{t}) + \mathbf{k}_1 = \mathbf{x}(\mathrm{t}) + \Delta \mathrm{T}\mathbf{f}(\mathbf{x}(\mathrm{t}))$$

- Then calculate k2 and get a final value for **x**(t+Δt)

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}\left(\mathbf{x}(t) + \mathbf{k}_1\right)$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{2}\left(\mathbf{k}_1 + \mathbf{k}_2\right)$$

- Finally solve the network equations using the final value for **x**(t+Δt)

Solution at t=0.02 with RK2

- From the first half of the time step

$$x(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- Then

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}\left(\mathbf{x}(t)+\mathbf{k}_1\right) = 0.02\begin{bmatrix} 1.256 \\ \frac{1}{6}(1-0) \\ -0.628 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \\ -0.0126 \\ -0.00167 \end{bmatrix}$$

Solution at t=0.02 with RK2

- The new values for the Norton currents are

$$\bar{I}_{N1} = \frac{1.281\angle 24.69^\circ}{j0.3} = 1.851 - j3.880$$

$$\bar{I}_{N2} = \frac{0.9554\angle -12.43^\circ}{j0.2} = -1.028 - j4.665$$

$$\mathbf{V}(0.02) = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.851 - j3.880 \\ -1.028 - j4.665 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.108 \end{bmatrix}$$

Solution Values Using RK2

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	24.6662	60.2	-12.4429	59.9
0.04	26.8262	60.4	-13.5229	59.8
0.06	30.4262	60.6	-15.3175	59.7008
0.08	35.4662	60.8	-17.8321	59.6008
0.1	41.9462	61	-21.0667	59.5008
0.1	41.9462	61	-21.0667	59.5008
0.12	48.7754	60.8852	-24.4759	59.5581
0.14	54.697	60.7538	-27.4312	59.6239
0.16	59.6315	60.6153	-29.8931	59.6931
0.18	63.558	60.4763	-31.8509	59.7626
0.2	66.4888	60.3399	-33.3109	59.8308
0.22	68.4501	60.2071	-34.286	59.8972
0.24	69.4669	60.077	-34.789	59.9623
0.26	69.5548	59.9481	-34.8275	60.0267
0.28	68.7151	59.8183	-34.4022	60.0916

Angle Reference

- The initial angles are given by the angles from the power flow, which are based on the slack bus's angle
- As presented the transient stability angles are with respect to a synchronous reference frame
  - Sometimes this is fine, such as for either shorter studies, or ones in which there is little speed variation
  - Oftentimes this is not best since the when the frequencies are not nominal, the angles shift from the reference frame
- Other reference frames can be used, such as with respect to a particular generator's value, which mimics the power flow approach; the selected reference has no impact on the solution