ECEN 667
Power System Stability

Lecture 16: Load Modeling

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Announcements

- Read Chapter 7
- Homework 5 is due on Oct 28.

Done with Transient Stability Solutions: On to Load Modeling

- Load modeling is certainly challenging!
- For large system models an aggregate load can consist of many thousands of individual devices
- The load is constantly changing, with key diurnal and temperature variations
  - For example, a higher percentage of lighting load at night, more air conditioner load on hot days
- Load model behavior can be quite complex during the low voltages that may occur in transient stability
- Testing aggregate load models for extreme conditions is not feasible – we need to wait for disturbances!

Load Modeling

- Traditionally load models have been divided into two groups
  - Static: load is a algebraic function of bus voltage and sometimes frequency
  - Dynamic: load is represented with a dynamic model, with induction motor models the most common
- The simplest load model is a static constant impedance
  - Has been widely used
  - Allowed the $\mathbf{Y}_{bus}$ to be reduced, eliminating essentially all non-generator buses
  - Presents no issues as voltage falls to zero
  - No longer commonly used

Load Modeling References

- Many papers and reports are available!
- A classic reference on load modeling is by the IEEE Task Force on Load Representation for Dynamic Performance, "Load Representation for Dynamic Performance Analysis," IEEE Trans. on Power Systems, May 1993, pp. 472-48
- NERC 2016, "Dynamic Load Modeling"; available at https://www.nerc.com/comm/PC/LoadModelingTaskForceDL/Dynamic%20Load%20Modeling%20Tech%20Ref%202016-11-14%20-%20FINAL.PDF
- EPRI Technical Guide to Composite Load Modeling, August 2020 (with a draft at the below link) https://www.wecc.org/Administrative/Mitra%20-%20Technical%20Guide%20on%20Composite%20Load%20Modeling.pdf

ZIP Load Model

- Another common static load model is the ZIP, in which the load is represented as

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k\right|^2 + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k\right|^2 + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)$$

- Some models allow more general voltage dependence

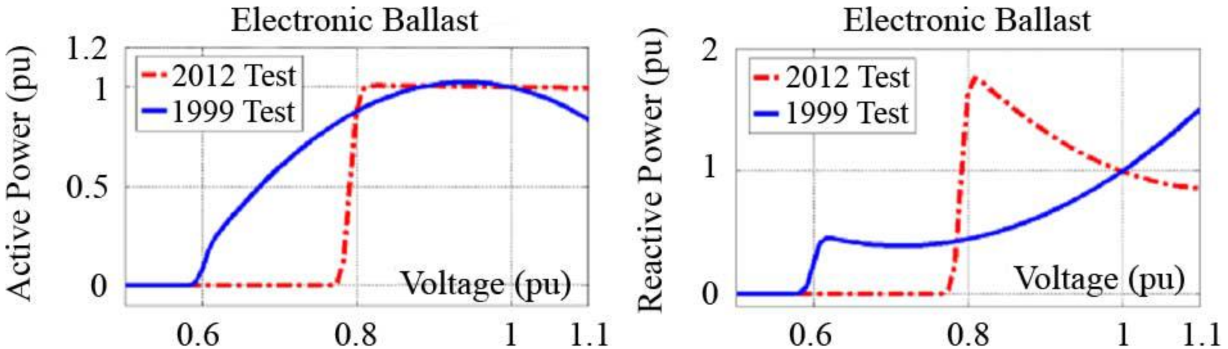
$$P_{Load,k} = P_{BaseLoad,k}\left(a_{1,k}\left|\bar{V}_k\right|^{n1} + a_{2,k}\left|\bar{V}_k\right|^{n2} + a_{3,k}\left|\bar{V}_k\right|^{n3}\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(a_{4,k}\left|\bar{V}_k\right|^{n4} + a_{5,k}\left|\bar{V}_k\right|^{n5} + a_{6,k}\left|\bar{V}_k\right|^{n6}\right)$$

The voltage exponent for reactive power is often > 2

ZIP Model Coefficients

- An interesting paper on the experimental determination of the ZIP parameters is A. Bokhari, et. al., "Experimental Determination of the ZIP Coefficients for Modern Residential and Commercial Loads, and Industrial Loads," IEEE Trans. Power Delivery, 2014
  - Presents test results for loads as voltage is varied; also highlights that load behavior changes with newer technologies
    - Below figure (part of fig 4 of paper), compares real and reactive behavior of light ballast



ZIP Model Coefficients

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13
LCD Television	1	100	120	208.03	-20.58	0.11	-0.17	1.06	1.58	-1.72	1.14
LED light	1	100	120	3.38	5.85	0.58	1.13	-0.71	1.78	-0.8	0.02
Magnetic ballast	1	100	120	81.23	8.2	-1.58	3.79	-1.21	36.18	-67.78	32.6
Mercury vapor HID light	2	100	120	268.27	77.66	0.52	1.02	-0.54	-1.33	2.4	-0.07
Metal halide HID electronic ballast	2	100	120	113.7	26.37	1	-2.02	2.02	8.8	-18.64	10.84
Metal halide HID magnetic ballast	2	100	120	450	102.94	0.86	-0.66	0.8	32.54	-59.83	28.29
Microwave	2	100	120	1365.53	451.02	1.39	-1.96	1.57	50.07	-93.55	44.48
Minibar	1	100	120	90.65	126.94	2.5	-4.1	2.6	2.56	-2.76	1.2
PC (Monitor & CPU)	1	100	120	118.9	172.79	0.2	-0.3	1.1	0	0.6	0.4

The Z,I,P coefficients sum to zero; note that for some models the absolute values of the parameters are quite large, indicating a difficult fit

A portion of Table VII from Bokhari 2014 paper

Discharge Lighting Models

- Discharge lighting (such as fluorescent lamps) is a major portion of the load (10-15%)
- Discharge lighting has been modeled for sufficiently high voltage with a real power as constant current and reactive power with a high voltage dependence
  - Linear reduction for voltage between 0.65 and 0.75 pu
  - Extinguished (i.e., no load) for voltages below

$$P_{DischargeLighting} = P_{Base}\left(\left|\bar{V}_k\right|\right)$$

$$Q_{DischargeLighting} = Q_{Base}\left(\left|\bar{V}_k\right|^{4.5}\right)$$

May need to change with newer electronic ballasts – e.g., reactive power increasing as the voltage drops!

Static Load Model Frequency Dependence

- Frequency dependence is sometimes included, to recognize that the load could change with the frequency

$$P_{Load,k} = P_{BaseLoad,k}\left(P_{z,k}\left|\bar{V}_k^2\right| + P_{i,k}\left|\bar{V}_k\right| + P_{p,k}\right)\left(1 + P_{f,k}\left(f_k - 1\right)\right)$$

$$Q_{Load,k} = Q_{BaseLoad,k}\left(Q_{z,k}\left|\bar{V}_k^2\right| + Q_{i,k}\left|\bar{V}_k\right| + Q_{p,k}\right)\left(1 + Q_{f,k}\left(f_k - 1\right)\right)$$

- Here f_k is the per unit bus frequency, which is calculated as

$$\theta_k \rightarrow \boxed{\frac{s}{1+sT}} \rightarrow f_k$$

A typical value for T is about 0.02 seconds. Some models just have frequency dependence on the constant power load

- Typical values for P_f and Q_f are 1 and -1 respectively

Induction Machines

- Term induction machine is used to indicate either generator or motor; most uses are as motors
- Induction machines have two major components
  - A stationary stator, which is supplied with an ac voltage; windings in stator create a rotating magnetic field
  - A rotating rotor, in which an ac current is induced (hence the name)
- Two basic design types based on rotor design
  - Squirrel-cage: rotor consists of shorted conducting bars laid into magnetic material in a cage structure
  - Wound-rotor: rotor has windings similar to stator, with slip rings used to provide external access to the rotor windings

Squirrel Cage Rotor Picture

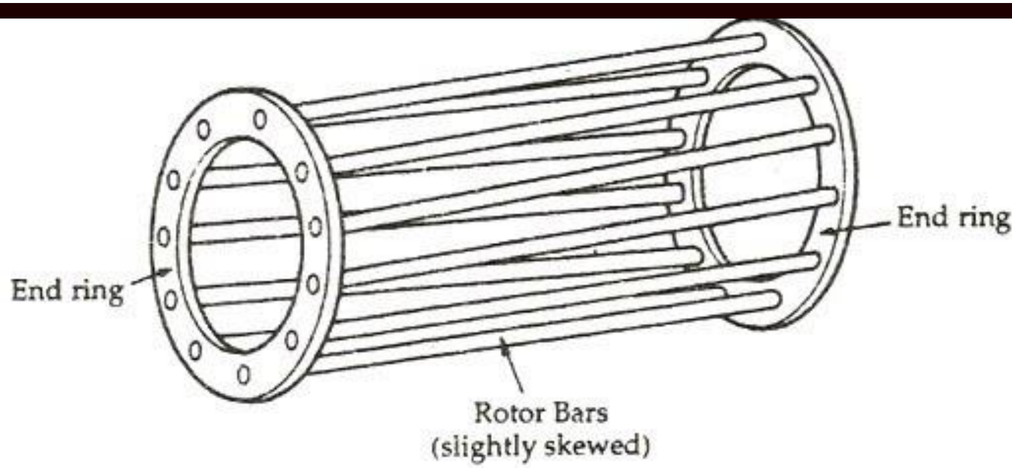
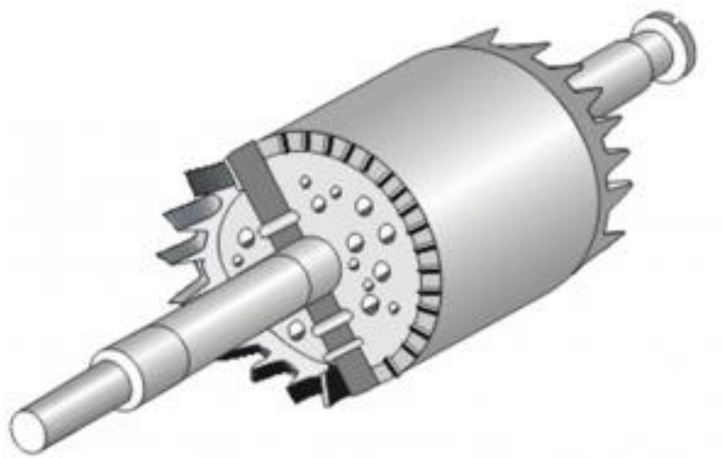


Image 1 Source: www.quora.com/What-will-happen-If-the-Squirrel-cage-motor-rotor-conductors-are-not-skewed



Embedded in laminated magnetic material

Image 2 Source: www.polytechnichub.com/squirrel-cage-rotor/

Induction Machine Overview

- Speed of rotating magnetic field (synchronous speed) depends on number of poles

$N_s = f_s \dfrac{120}{p}$ where N_s is the synchronous speed in RPM, f_s is the stator electrical frequency (e.g., 60 or 50Hz) and p is the number of poles

- Frequency of induced currents in rotor depends on frequency difference between the rotating magnetic field and the rotor

$$\omega_r = \omega_s - \left(\frac{p}{2}\right)\omega_m$$

where ω_s is the stator electrical frequency, ω_m is mechanical speed, and ω_r is the rotor electrical frequency

Induction Machine Slip

- Key value is slip, s, defined as

$$s = \frac{N_s - N_{act}}{N_s}$$

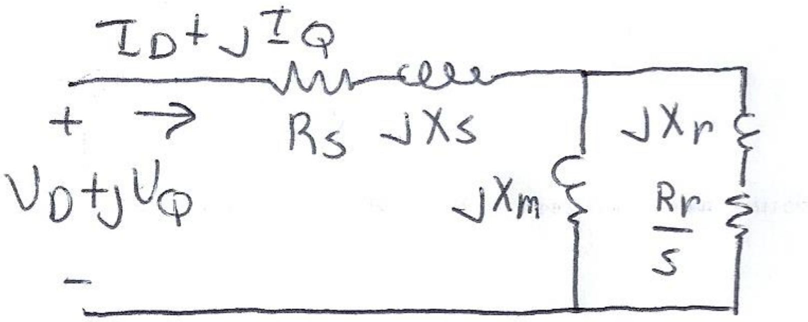
where N_s is the synchronous speed, and

N_{act} is the actual speed (in RPM)

- As defined, when operating as a motor an induction machine will have a positive slip, slip is negative when operating as a generator
  - Slip is zero at synchronous speed, a speed at which no rotor current is induced; s=1 at stand still

Basic Induction Machine Model

- A basic (single cage) induction machine circuit model is given below
  - Model is derived in an undergraduate machines class



$$\frac{R_r}{s} = R_r + \frac{(1-s)}{s}R_r$$

- Circuit is useful for understanding the static behavior of the machine
- Effective rotor resistance (R_r/s) models the rotor electrical losses (R_r) and the mechanical power R_r(1-s)/s

Induction Machine Dynamics

- Expressing all values in per unit (with the base covered later), the mechanical equation for a machine is

$$\frac{ds}{dt} = \frac{1}{2H}\left(T_M - T_E\right)$$

where H is the inertia constant, T_M is the mechanical torque and T_E is the electrical torque (to be defined)

- Similar to what was done for a synchronous machine, the induction machine can be modeled as an equivalent voltage behind a stator resistance and transient reactance (later we'll introduce, but not derive, the subtransient model)

Induction Machine Dynamics

- Define

$$X' = X_s + \frac{X_r X_m}{X_r + X_m}$$

$$X = X_s + X_m$$

where X' is the apparent reactance seen when the rotor is locked (s=1) and X is the synchronous reactance

- Also define the open circuit time constant

$$T_o' = \frac{\left(X_r + X_m\right)}{\omega_s R_r}$$

Induction Machine Dynamics

- Electrically the induction machine is modeled similar to the classical generator model, except here we use the "motor convention" in which I_D+jI_Q is assumed positive into the machine

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = \omega_s s E'_Q - \frac{1}{T'_o}\left(E'_D + \left(X - X'\right) I_Q\right)$$

$$\frac{dE'_Q}{dt} = -\omega_s s E'_D - \frac{1}{T'_o}\left(E'_Q - \left(X - X'\right) I_D\right)$$

All calculations are done on the network reference frame

Induction Machine Dynamics

- The induction machine electrical torque, T_E, and terminal electrical load, P_E, are then

$$T_E = \frac{\left(E'_D I_D + E'_Q I_Q\right)}{\omega_s}$$

$$P_E = V_D I_D + V_Q I_Q$$

Recall we are using the motor convention so positive P_E represents load

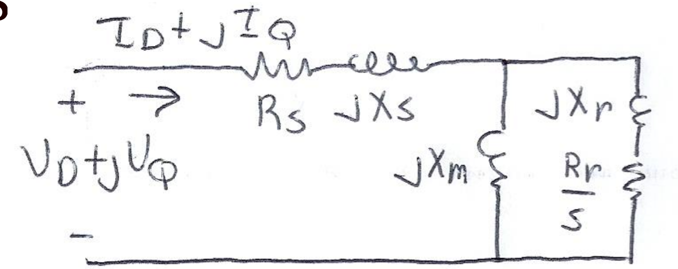
- Similar to a synchronous machine, once the initial values are determined the differential equations are fairly easy to simulate
  - Key initial value needed is the slip

Specifying Induction Machine Parameters

- In transient stability packages induction machine parameters are specified in per unit
  - If unit is modeled as a generator in the power flow (such as CIMTR1 or GENWRI) then use the generator's MVA base (as with synchronous machines)
  - With loads it is more complicated.
    - Sometimes an explicit MVA base is specified. If so, then use this value. But this can be cumbersome since often the same per unit machine values are used for many loads
    - The default is to use the MW value for the load, often scaled by a multiplier (say 1.25)

Determining the Initial Values

- To determine the initial values, it is important to recognize that for a fixed terminal voltage there is only one independent value: the slip, s
  - For a fixed slip, the model is just a simple circuit with resistances and reactances



- The initial slip is chosen to match the power flow real power value. Then to match the reactive power value (for either a load or a generator), the approach is to add a shunt capacitor in parallel with the induction machine
- We'll first consider torque-speed curves, then return to determining the initial slip

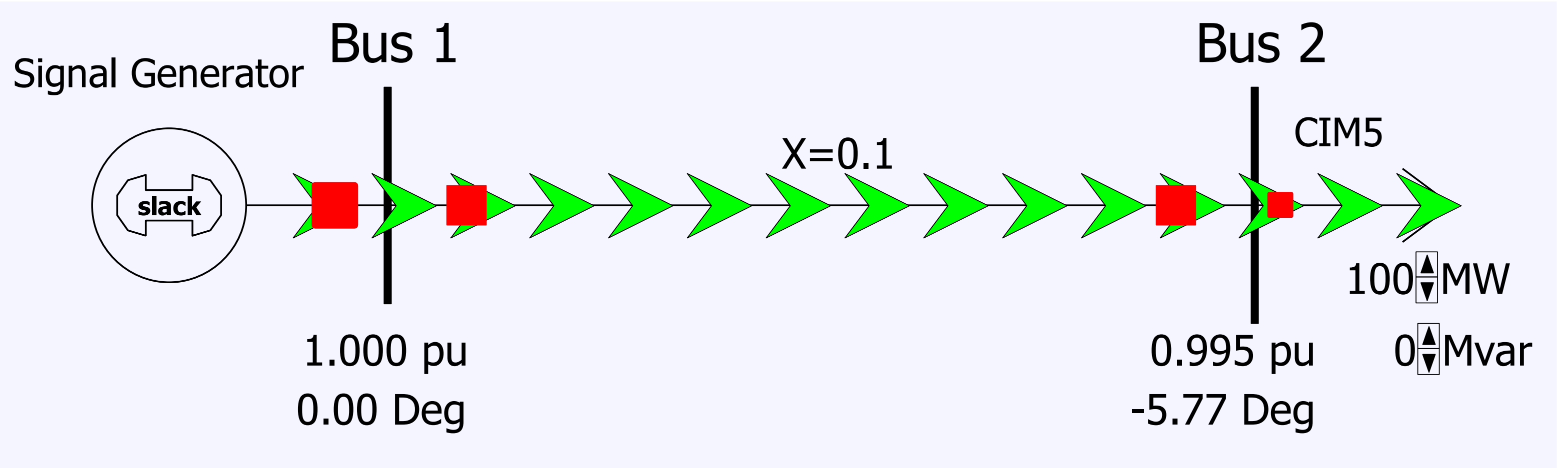
Torque-Speed Curves

- To help understand the behavior of an induction machine it is useful to plot various values as a function of speed (or equivalently, slip)
  - Solve the equivalent circuit for a specified terminal voltage, and varying values of slip
  - Plot results
  - Recall torque times speed = power
    - Here speed is the rotor speed
  - When using per unit, the per unit speed is just 1-s

$$P_E = T_E\left(1 - s\right)$$

Induction Motor Example

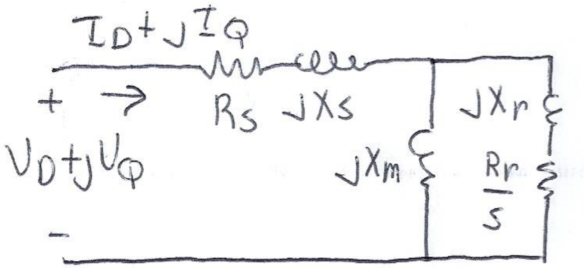
- Assume the below 60 Hz system, with the entire load modeled as a single cage induction motor with per unit values on a 125 MVA base of H=1.0, R_s=0.01, X_s=0.06, X_m=4.0, R_r=0.03, X_r=0.04
  - In the CIM5 model R_1=R_r and X_1=X_r



PowerWorld case **B2_IndMotor**

Induction Motor Example

- With a terminal voltage of 0.995∠0° we can solve the circuit for specified values of s



- The input impedance and current are

$$Z_{in} = (R_s + jX_s) + \frac{jX_m\left(\frac{R_r}{s} + jX_r\right)}{\frac{R_r}{s} + j\left(X_r + X_m\right)}, \qquad \bar{I} = \frac{\bar{V}}{Z_{in}} = \frac{0.995\angle 0°}{Z_{in}}$$

- Then with s=1 we get

Note, values are per unit on a 125 MVA base

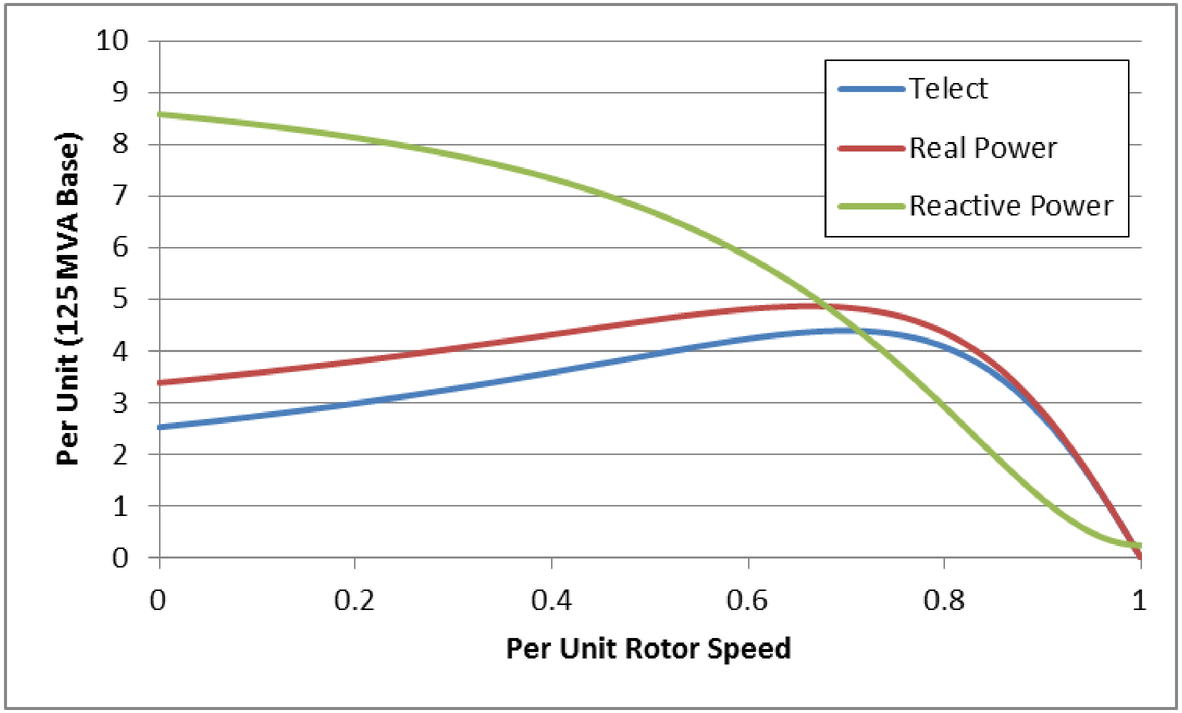
$$\bar{I} = \frac{0.995}{0.0394 + j0.0998} = 3.404 - j8.624 \rightarrow S = 3.387 + j8.581$$

Induction Motor Example

- PowerWorld allows for display of the variation in various induction machine values with respect to speed
  - Right click on load, select Load Information Dialog, Stability
  - On bottom of display click Show Torque Speed Dialog
  - Adjust the terminal voltage and pu scalar as desired; set v=0.995 and the pu scalar to 1.0 to show values on the 125 MVA base used in the previous solution
  - Right click on column and select Set/Toggle/Columns, Plot Column to plot the column

Induction Motor Example Torque-Speed Curves

- The below graph shows the torque-speed curve for this induction machine; note the high reactive power consumption on starting (which is why the lights may dim when starting a cloth dryer!)



From the graph you can see with a 100 MW load (0.8 pu on the 125 MW base), the slip is about 0.025

Calculating the Initial Slip

- One way to calculate the initial slip is to just solve the below five equations for five unknowns (s, I_D, I_Q, E'_D, E'_Q) with P_E, V_D and V_Q inputs

$$P_E = V_D I_D + V_Q I_Q$$

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = 0 = \omega_s s E'_Q - \frac{1}{T'_o}\left(E'_D + \left(X - X'\right) I_Q\right)$$

$$\frac{dE'_Q}{dt} = 0 = \omega_s s E'_Q - \frac{1}{T'_o}\left(E'_Q - \left(X - X'\right) I_D\right)$$

These are nonlinear equations that can have multiple solutions so use Newton's method, with an initial guess of s small (say 0.01)

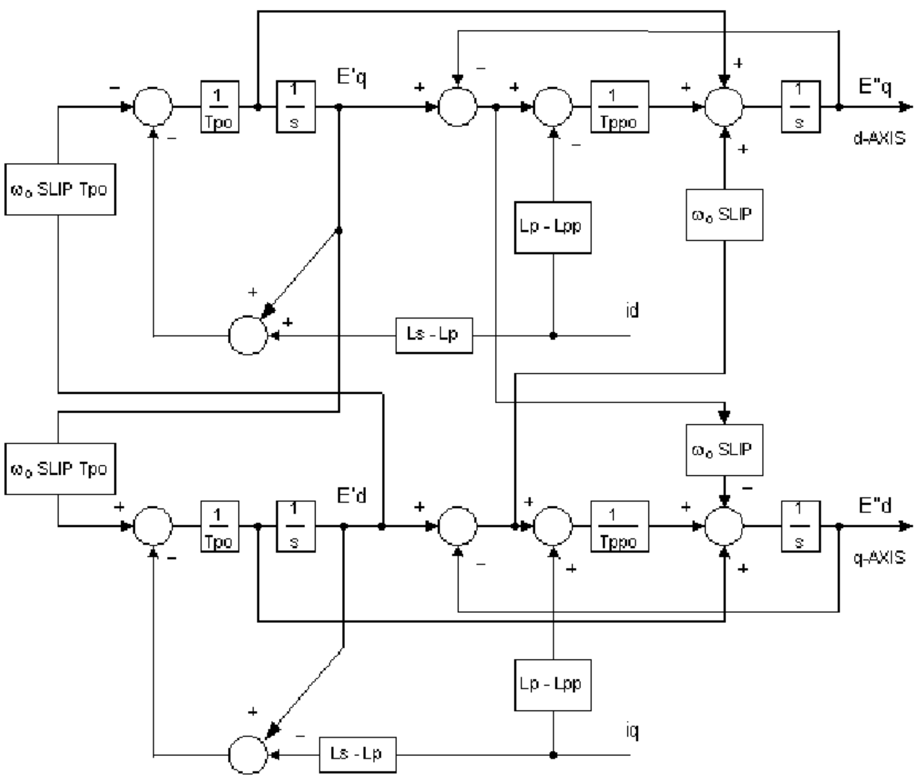
Initial slip in example is 0.0251

Double Cage Induction Machines

- In the design of induction machines, there are various tradeoffs, such as between starting torque (obviously one needs enough to start) and operating efficiency
  - The highest efficiency possible is 1-slip, so operating at low slip is desirable
- A common way to achieve high starting torque with good operating efficiency is to use a double cage design
  - E.g., the rotor has two embedded squirrel cages, one with a high R and lower X for starting, and one with lower R and higher X for running
  - Modeled by extending our model by having two rotor circuits in parallel; add subtransient values X'' and T''_o

Example Double Cage Model

- Double cage rotors are modeled by adding two additional differential equations



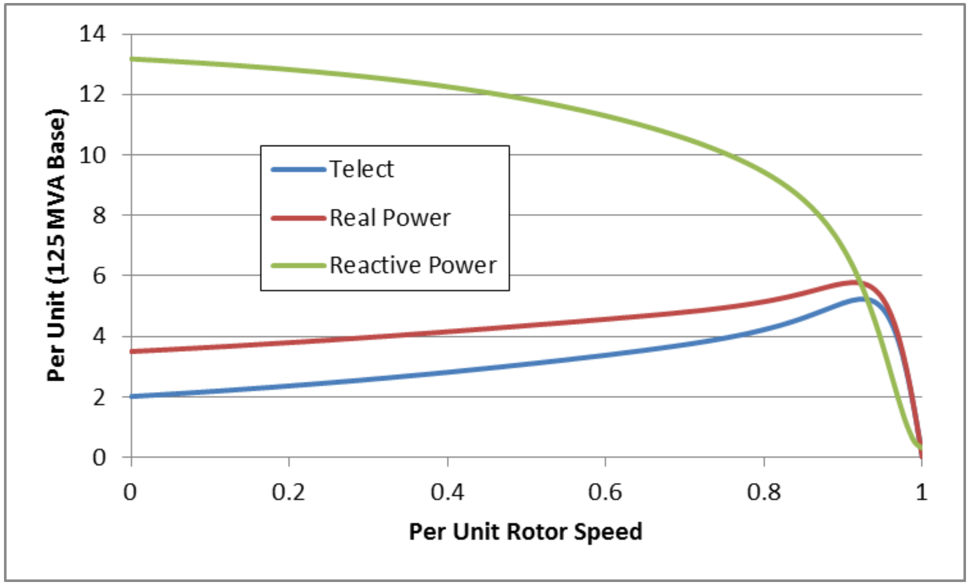
Some models also include saturation, a topic that we will skip

PowerWorld case **B2_IndMotor_DoubleCage**

Image source: PSLF Manual, version 18.1_02; MotorW

Double Cage Induction Motor Model

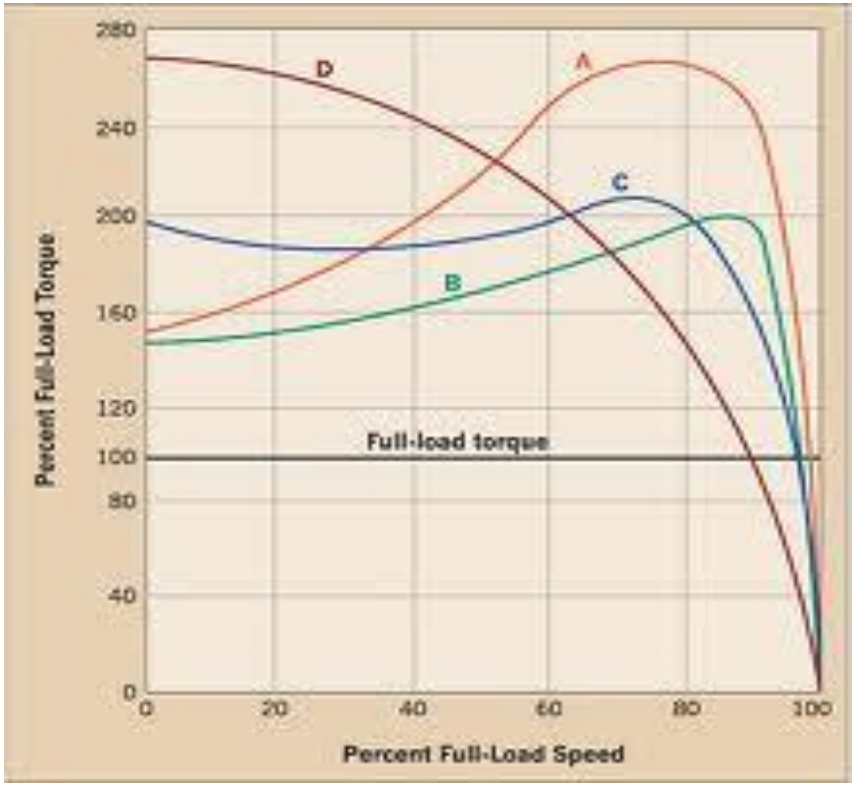
- The previous example can be extended to model a double cage rotor by setting R_2=0.01, X_2=0.08
  - The below graph shows the modified curves, notice the increase in the slope by s=0, meaning it is operating with higher efficiency (s=0.0063 now!)



The additional winding does result in lower initial impedance and hence a higher starting reactive power

Induction Motor Classes

- Four major classes of induction motors, based on application. Key values are starting torque, pull-out torque, full-load torque, and starting current



In steady-state the motor will operate on the right side of the curve at the point at which the electrical torque matches the mechanical torque

A: Fans, pumps machine tools

B: Similar to A

C: Compressors, conveyors

D: High inertia such as hoists

Image source: ecmweb.com/motors/understanding-induction-motor-nameplate-information

Induction Motor Stalling

- Height of the torque-speed curve varies with the square of the terminal voltage
- When the terminal voltage decreases, such as during a fault, the mechanical torque can exceed the electrical torque
  - This causes the motor to decelerate, perhaps quite quickly, with the rate proportional to its inertia
  - This deceleration causing the slip to increase, perhaps causing the motor to stall with s=1, resulting in a high reactive current draw
  - Too many stalled motors can prevent the voltage from recovering