ECEN 667
Power System Stability

Lecture 19: Oscillations, Modal Analysis

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Announcements

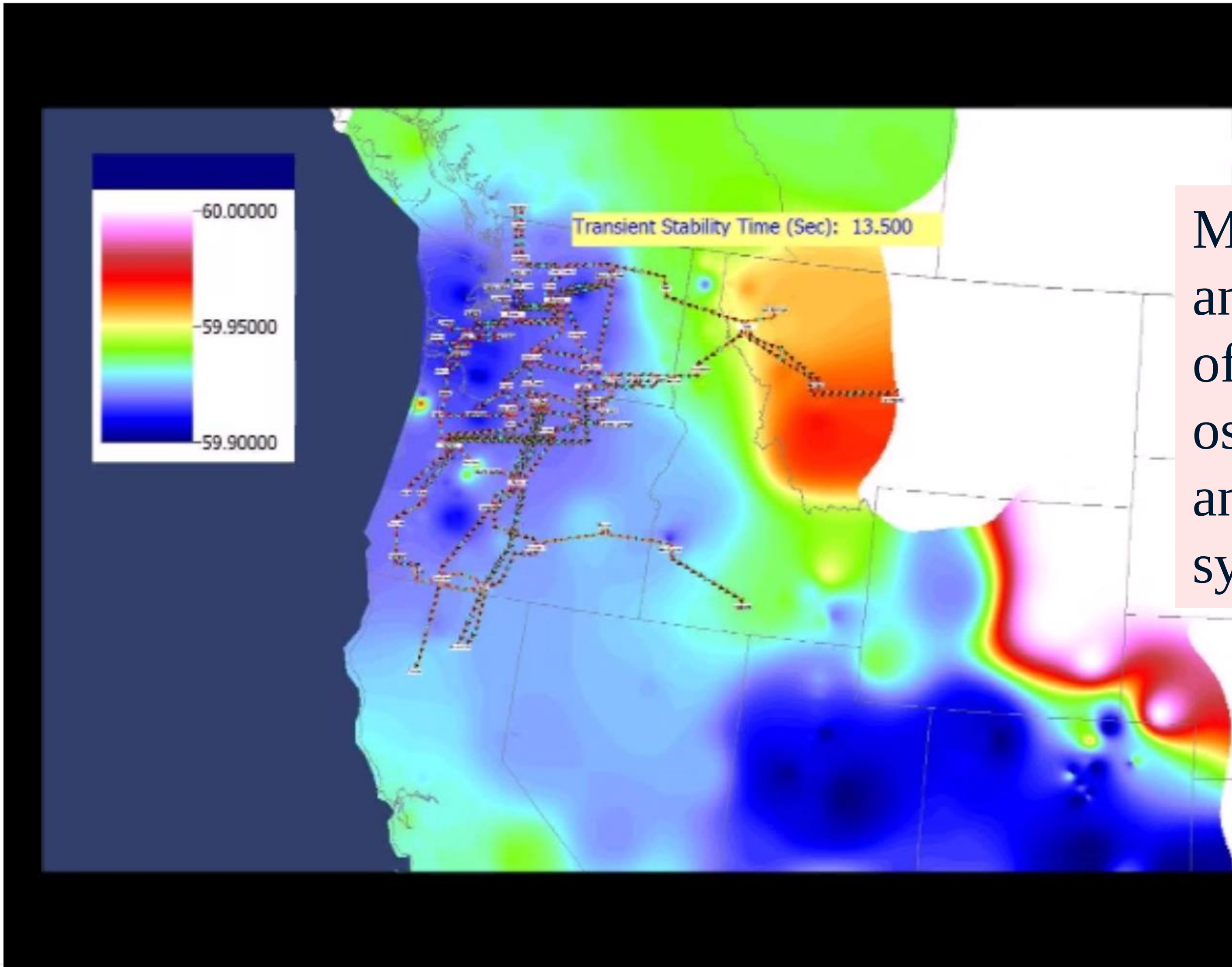
- Read Chapter 8
- Homework 6 is due on November 11
- With respect to the 1996 WECC blackout, more information is available at
  - www.nerc.com/pa/rrm/ea/System%20Disturbance%20Reports%20DL/1996SystemDisturbance.pdf
  - The July 2, 1996 event was caused by a tree contact
- There is a 2019 NERC document on oscillations at www.nerc.com/comm/PC/SMSResourcesDocuments/Interconnection_Oscillation_Analysis.pdf

Modes

- A mode is a concept from linear system analysis
  - Electric grids certainly are not linear, but usually their response to small disturbances is approximated as linear
- A mode corresponds to one of the eigenvalues of the response or, for oscillations, a complex pair of eigenvalues
- A mode has a frequency and damping; all parts of the system oscillate with this pattern
- The mode shape tells how parts of the system participate in the mode
- There can be multiple modes in a system; power systems can have many modes

Fictitious System Oscillation

Movies & TV



Movie shows an example of sustained oscillations in an equivalent system

Causes of Power System Oscillations

- The response of a simple system can be divided into its natural response versus its forced response
  - The natural response tells how the system will response to an initial disturbance without any additional (external) influences; this response shows the system's modes
  - A forced response is associated with an external disturbance; if the external disturbance is periodic then the system will oscillate at least partially at this frequency
  - Often forced oscillations are due to control failures
- Resonance occurs when a forced response is at a similar frequency to one of the system's modes
- An power system can experience both types of oscillations

Forced Oscillations in WECC (from [1])

- Summer 2013 24 hour data: 0.37 Hz oscillations observed for several hours. Confirmed to be forced oscillations at a hydro plant from vortex effect.
- 2014 data: Another 0.5 Hz oscillation also observed. Source points to hydro unit as well. And 0.7 Hz. And 1.12 Hz. And 2 Hz.
- Resonance possible when system modes are poorly damped and close to the forcing function. Resonance can be observed in model simulations.

1. M. Venkatasubramanian, "Oscillation Monitoring System", June 2015
http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

Observing Modes and Damping

- With the advent of wide-scale PMU deployments, the modes and damping can be observed two ways
  - Event (ringdown) analysis – this requires an event
  - Ambient noise analysis – always available, but not as distinct

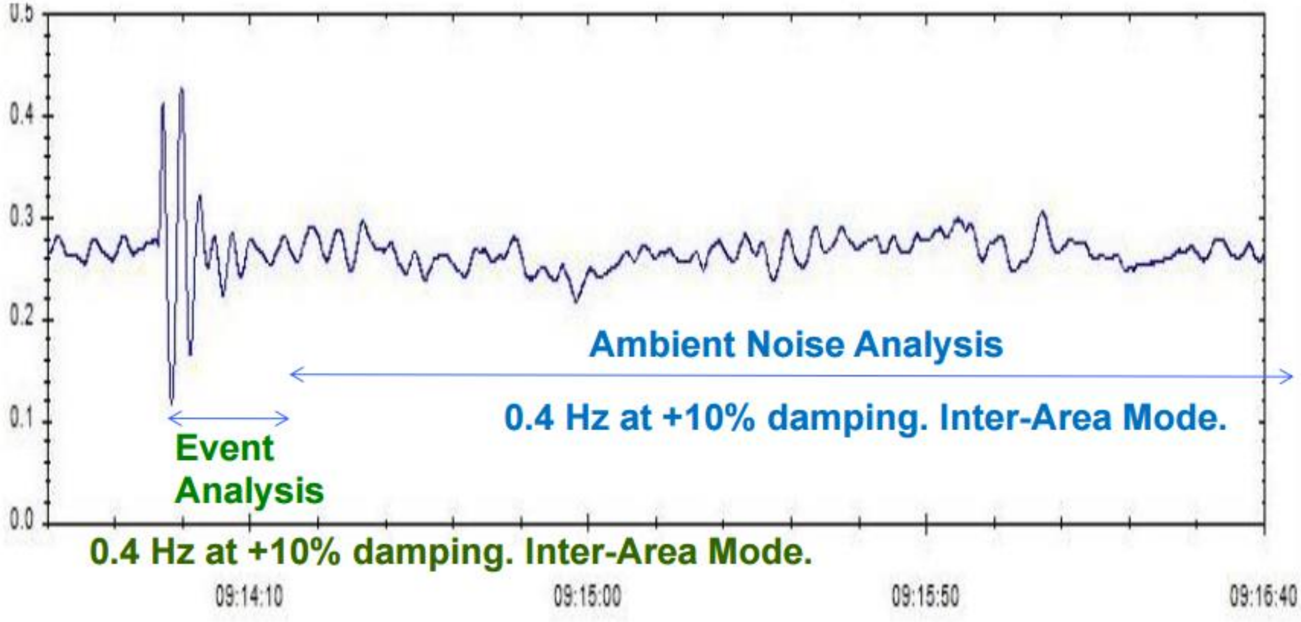


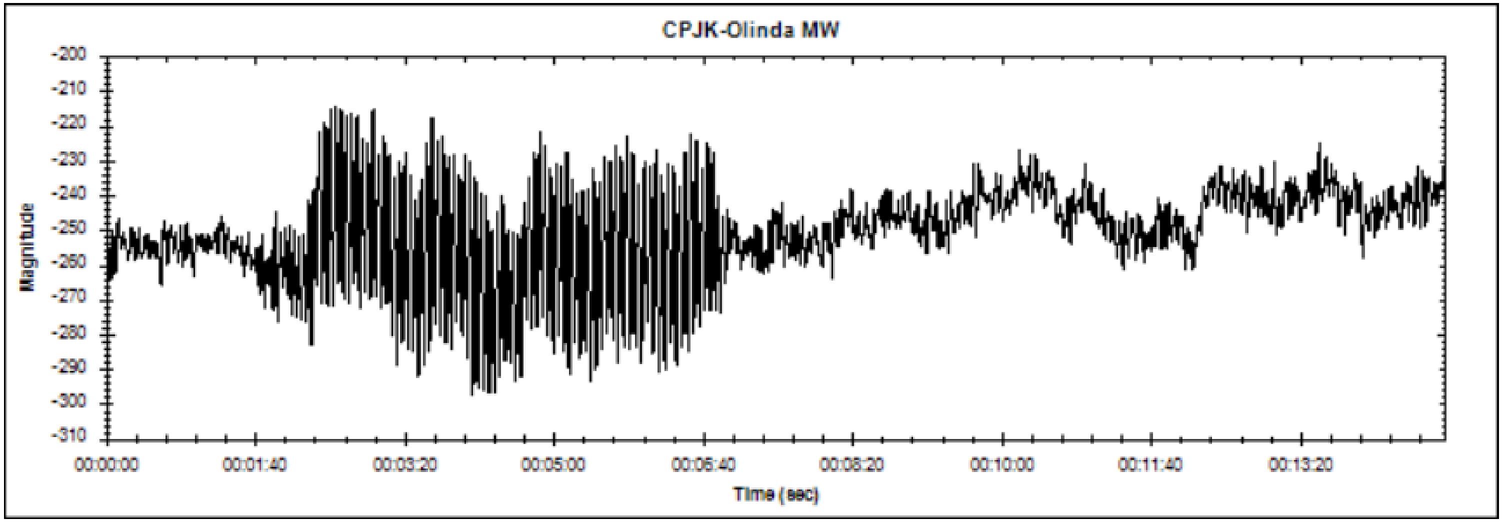
Image Source: M. Venkatasubramanian, "Oscillation Monitoring System", June 2015
http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

Resonance with Interarea Mode [1]

- Resonance effect high when:
  - Forced oscillation frequency near system mode frequency
  - System mode poorly damped
  - Forced oscillation location near the two distant ends of mode
- Resonance effect medium when
  - Some conditions hold
- Resonance effect small when
  - None of the conditions holds

1. M. Venkatasubramanian, "Oscillation Monitoring System", June 2015
http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

Medium Resonance on 11/29/2005



- 20 MW 0.26 Hz Forced Oscillation in Alberta Canada
- 200 MW Oscillations on California-Oregon Inter-tie
- System mode 0.27 Hz at 8% damping
- Two out of the three conditions were true.

1. M. Venkatasubramanian, “Oscillation Monitoring System”, June 2015
http://www.energy.gov/sites/prod/files/2015/07/f24/3.%20Mani%20Oscillation%20Monitoring.pdf

An On-line Oscillation Detection Tool

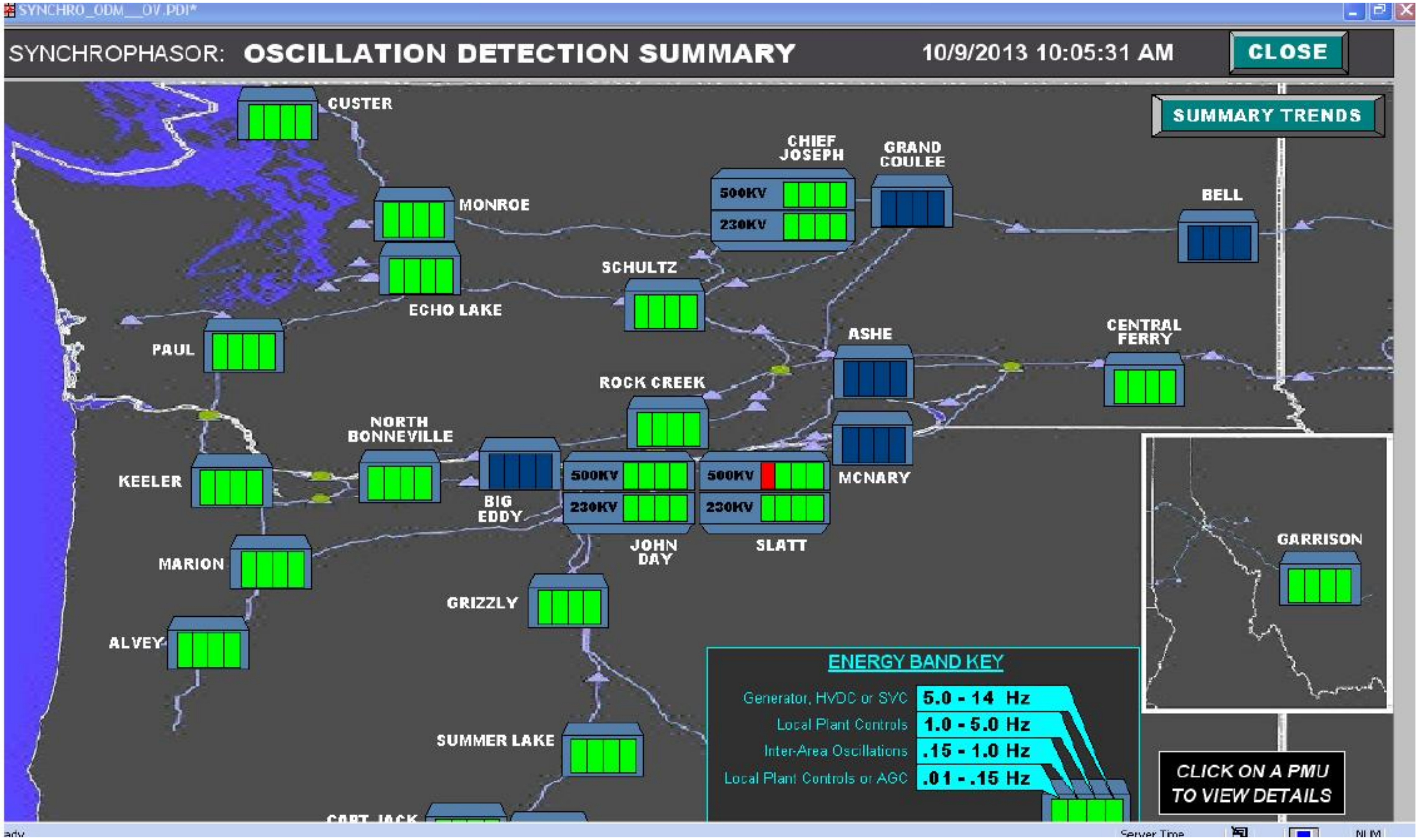


Image source: WECC Joint Synchronized Information Subcommittee Report, October 2013

Small Signal Stability Analysis

- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
  - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) obviously must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
  - There must be positive damping

Model Based SSA

- Assume the power system is modeled in our standard form as

$$\dot{\mathbf{x}} = \mathbf{f}\left(\mathbf{x}, \mathbf{y}\right)$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

- The system can be linearized about an equilibrium point

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{y}$$

$$\mathbf{0} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{y}$$

If there are just classical generator models then **D** is the power flow Jacobian; otherwise it also includes the stator algebraic equations

- Eliminating **Δy** gives

$$\Delta\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\Delta\mathbf{x} = \mathbf{A}_{sys}\Delta\mathbf{x}$$

Model Based SSA

- The matrix $\mathbf{A}_{sys}$ can be calculated doing a partial factorization, just like what is done with Kron reduction in creating power system equivalents
- SSA is done by looking at the eigenvalues (and other properties) of $\mathbf{A}_{sys}$

Modal Analysis - Comments

- Modal analysis (analysis of small signal stability through eigenvalue analysis) is at the core of SSA software
- In Modal Analysis one looks at:
  - Eigenvalues
  - Eigenvectors (left or right)
  - Participation factors
  - Mode shape

Goal is to determine how the various parameters affect the response of the system

- Power System Stabilizer (PSS) design in a multi-machine context can be done using modal analysis method.

Eigenvalues, Right Eigenvectors

- For an n by n matrix **A** the eigenvalues of **A** are the roots of the characteristic equation:

$$\det[\mathbf{A} - \lambda \mathbf{I}] = \left|\mathbf{A} - \lambda \mathbf{I}\right| = 0$$

- Assume $\lambda_1 \ldots \lambda_n$ as distinct (no repeated eigenvalues).
- For each eigenvalue λ_i there exists an eigenvector such that:

$$\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$$

- $\mathbf{v}_i$ is called a right eigenvector
- If λ_i is complex, then $\mathbf{v}_i$ has complex entries

Left Eigenvectors

- For each eigenvalue λ_i there exists a left eigenvector $\mathbf{w}_i$ such that:

$$\mathbf{w}_i^t \mathbf{A} = \mathbf{w}_i^t \lambda_i$$

- Equivalently, the left eigenvector is the right eigenvector of $\mathbf{A}^T$; that is,

$$\mathbf{A}^t \mathbf{w}_i = \lambda_i \mathbf{w}_i$$

Eigenvector Properties

- The right and left eigenvectors are orthogonal i.e.

$$\mathbf{w}_i^t \mathbf{v}_i \neq 0\ ,\ \mathbf{w}_i^t \mathbf{v}_j = 0 \quad (i \neq j)$$

- We can normalize the eigenvectors so that:

$$\mathbf{w}_i^t \mathbf{v}_i = 1\ ,\ \mathbf{w}_i^t \mathbf{v}_j = 0 \quad (i \neq j)$$

Eigenvector Example

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \ |\mathbf{A} - \lambda \mathbf{I}| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 4 \\ 3 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 10 = 0 \Rightarrow \lambda_{1,2} = \frac{3 \pm \sqrt{(3)^2 + 4(10)}}{2} = \frac{3 \pm \sqrt{49}}{2} = 5, -2$$

Right Eigenvectors $\lambda_1 = 5$

$$\mathbf{A}\mathbf{v}_1 = 5\mathbf{v}_1 \Rightarrow \mathbf{v}_1 = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \Rightarrow \begin{matrix} v_{11} + 4v_{21} = 5v_{11} \\ 3v_{11} + 2v_{21} = 5v_{21} \end{matrix}$$

choose $\mathbf{v}_{21} = 1 \Rightarrow \mathbf{v}_{11} = 1$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Similarly,

$$\lambda_2 = -2 \Rightarrow \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

Eigenvector Example

- Left eigenvectors

$$\lambda_1 = 5 \;\; \mathbf{w}_1^t \mathbf{A} = \mathbf{w}_1^t 5 \Rightarrow [w_{11} \;\; w_{21}] \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = 5[w_{11} \;\; w_{21}]$$

$$\begin{aligned} w_{11} + 3w_{21} &= 5w_{11} \\ 4w_{11} + 2w_{21} &= 5w_{21} \end{aligned} \Rightarrow \textit{Let } \; w_{21} = 4, \textit{ then } \; w_{11} = 3$$

$$\mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \lambda_2 = -2 \Rightarrow \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \;\; \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \;\; \mathbf{w}_1 = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \;\; \mathbf{w}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\textit{Verify} \;\; \mathbf{w}_1^t \mathbf{v}_1 = 7 \;,\; \mathbf{w}_2^t \mathbf{v}_2 = 7 \;,\; \mathbf{w}_2^t \mathbf{v}_1 = 0 \;,\; \mathbf{w}_1^t \mathbf{v}_2 = 0$$

We would like to make $w_i^t v_i = 1.$

This can be done in many ways.

Eigenvector Example

$$Let \quad \mathbf{W} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

$$Then \quad \mathbf{W}^T\mathbf{V} = \mathbf{I}$$

$$Verify \quad \frac{1}{7}\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- It can be verified that $\mathbf{W}^T = \mathbf{V}^{-1}$.
- The left and right eigenvectors are used in computing the participation factor matrix.

Modal Matrices

- The deviation away from an equilibrium point can be defined as

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x}$$

- From this equation it is difficult to determine how parameters in **A** affect a particular **x** because of the variable coupling
- To decouple the problem first define the matrices of the right and left eigenvectors (the modal matrices)

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2 ..... \mathbf{v}_n] \ \ \& \ \ \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, ... \mathbf{w}_n]$$

$$\mathbf{AV} = \mathbf{V\Lambda} \quad \text{when} \quad \mathbf{\Lambda} = Diag(\lambda_i)$$

V is for the right eigenvectors

Modal Matrices

- It follows that

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \mathbf{\Lambda}$$

- To decouple the variables define **z** so

$$\Delta\mathbf{x} = \mathbf{V}\mathbf{z} \rightarrow \Delta\dot{\mathbf{x}} = \mathbf{V}\dot{\mathbf{z}} = \mathbf{A}\Delta\mathbf{x} = \mathbf{A}\mathbf{V}\mathbf{z}$$

- Then

$$\dot{\mathbf{z}} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{W}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{\Lambda}\mathbf{z}$$

- Since $\mathbf{\Lambda}$ is diagonal, the equations are now uncoupled with $\dot{z}_i = \lambda_i z_i$

- So $\Delta\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$

Example

- Assume the previous system with

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

$$\mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$$

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}$$

Modal Matrices

- Thus the response can be written in terms of the individual eigenvalues and right eigenvectors as

$$\Delta \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Note, we are requiring that the eigenvalues be distinct!

- Furthermore with

$$\Delta \mathbf{x} = \mathbf{V}\mathbf{Z} \Rightarrow \quad \mathbf{z} = \mathbf{V}^{-1}\mathbf{x} = \mathbf{W}^{T}\mathbf{x}$$

- So z(t) can be written as using the left eigenvectors as

$$\mathbf{z}(t) = \mathbf{W}^{t}\mathbf{x}(t) = [\mathbf{w}_1 \;\; \mathbf{w}_2 \ldots . \mathbf{w}_n]^{t} \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}$$

Modal Matrices

- We can then write the response x(t) in terms of the modes of the system

$$z_i(t) = w_i^t x(t)$$

$$z_i(0) = w_i^t x(0) \triangleq c_i$$

$$\text{so} \quad \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_i c_i e^{\lambda_i t}$$

$$\text{Expanding} \quad \Delta x_i(t) = v_{i1} c_1 e^{\lambda_1 t} + v_{i2} c_2 e^{\lambda_2 t} + \ldots v_{in} c_n e^{\lambda_n t}$$

- So c_i is a scalar that represents the magnitude of excitation of the i^{th} mode from the initial conditions

Numerical example

$$\begin{bmatrix} \Delta\dot{x}_1 \\ \Delta\dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix}, \; \Delta\mathbf{x}(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Eigenvalues are $\lambda_1 = -4$, $\lambda_2 = 2$

Eigenvectors are $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Modal matrix $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$

Normalize so $\mathbf{V} = \begin{bmatrix} 0.2425 & 0.4472 \\ -0.9701 & 0.8944 \end{bmatrix}$

Numerical example (contd)

Left eigenvector matrix is:

$$\mathbf{W}^{\mathrm{T}} = \mathbf{V}^{-1} = \begin{bmatrix} 1.3745 & -0.6872 \\ 1.4908 & 0.3727 \end{bmatrix}$$

$$\dot{\mathbf{z}} = \mathbf{W}^{\mathrm{T}}\mathbf{A}\mathbf{V}\mathbf{z}$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

Numerical example (contd)

$$\dot{z}_1 = -4z_1 \;,\; \mathbf{z}(0) = V^{-1}\mathbf{x}(0)$$

$$\dot{z}_2 = 2z_2 \;,\; \begin{bmatrix} z_1(0) \\ z_2(0) \end{bmatrix} = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$z_1(t) = z_1(0)e^{-4t} \;;\; z_2(t) = z_2(0)e^{2t}, \mathbf{C} = \mathbf{W}^T\mathbf{x}(0) = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{V}\mathbf{z}$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix}$$

$$= c_1 \begin{bmatrix} 0.2425 \\ -0.9701 \end{bmatrix} z_1(t) + c_2 \begin{bmatrix} 0.4472 \\ 0.8944 \end{bmatrix} z_2(t) = \sum_{i=1}^{2} c_i \mathbf{v}_i z_i(0) e^{\lambda_i t}$$

Because of the initial condition, the 2nd mode does not get excited

Mode Shape, Sensitivity and Participation Factors

- So we have

$$\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t), \quad \mathbf{z}(t) = \mathbf{W}^t\mathbf{x}(t)$$

- **x**(t) are the original state variables, **z**(t) are the transformed variables so that each variable is associated with only <u>one</u> mode.
- From the first equation the right eigenvector gives the "mode shape" i.e. relative activity of state variables when a particular mode is excited.
- For example the degree of activity of state variable x_k in $\mathbf{v}_i$ mode is given by the element V_{ki} of the right eigenvector matrix **V**

Mode Shape, Sensitivity and Participation Factors

- The magnitude of elements of $\mathbf{v}_i$ give the extent of activities of *n* state variables in the i^{th} mode and angles of elements (if complex) give phase displacements of the state variables with regard to the mode.
- The left eigenvector $\mathbf{w}_i$ identifies which combination of original state variables display only the i^{th} mode.

Eigenvalue Parameter Sensitivity

- To derive the sensitivity of the eigenvalues to the parameters recall $\mathbf{Av}_i = \lambda_i \mathbf{v}_i$; take the partial derivative with respect to A_{kj} by using the chain rule

$$\frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}} \mathbf{v}_i + \mathbf{A} \frac{\partial \mathbf{v_i}}{\partial A_{kj}} = \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i + \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

Multiply by $\mathbf{w}_i^t$

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}} \mathbf{v}_i + \mathbf{w}_i^t \mathbf{A} \frac{\partial \mathbf{v_i}}{\partial A_{kj}} = \mathbf{w}_i^t \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i + \mathbf{w}_i^t \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i + \mathbf{w}_i^t [\mathbf{A} - \lambda_i \mathbf{I}] \frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \mathbf{w}_i^t \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i$$

Eigenvalue Parameter Sensitivity

- This is simplified by noting that $\mathbf{w}_i^t(\mathbf{A} - \lambda_i \mathbf{I}) = 0$ by the definition of $\mathbf{w}_i$ being a left eigenvector
- Therefore

$$\mathbf{w}_i^t \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_i = \frac{\partial \lambda_i}{\partial A_{kj}}$$

- Since all elements of $\frac{\partial \mathbf{A}}{\partial A_{kj}}$ are zero, except the k^{th} row, j^{th} column is 1
- Thus $\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji}$

Sensitivity Example

- In the previous example we had

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \lambda_{1,2} = 5, -2, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

- Then the sensitivity of λ_1 and λ_2 to changes in **A** are

$$\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji} \rightarrow \frac{\partial \lambda_1}{\partial \mathbf{A}} = \frac{1}{7}\begin{bmatrix} 3 & 3 \\ 4 & 4 \end{bmatrix}, \quad \frac{\partial \lambda_2}{\partial \mathbf{A}} = \frac{1}{7}\begin{bmatrix} 4 & -3 \\ -4 & 3 \end{bmatrix}$$

- For example with $\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 2.1 \end{bmatrix}, \quad \hat{\lambda}_{1,2} = 5.057, -1.957$

$$\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}, \quad \hat{\lambda}_{1,2} = 5.61, -1.61,$$

Participation Factors

- The participation factors, P_{ki}, are used to determine how much the k^{th} state variable participates in the i^{th} mode

$$P_{ki} = V_{ki} W_{ki}$$

- The sum of the participation factors for any mode or any variable sum to 1
- The participation factors are quite useful in relating the eigenvalues to portions of a model

Participation Factors

- For the previous example with $P_{ki} = V_{ki}W_{ik}$ and

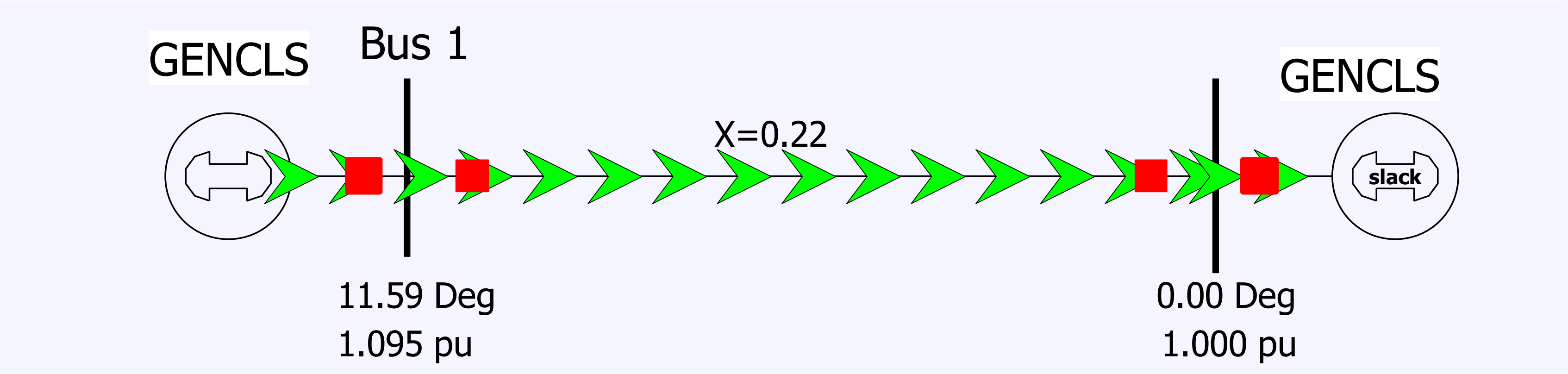
$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

- We get

$$\mathbf{P} = \frac{1}{7}\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

SSA Two Generator Example

- Consider the two bus, two classical generator system from lectures 18 and 20 with $X_{d1}'=0.3$, $H_1=3.0$, $X_{d2}'=0.2$, $H_2=6.0$



- Essentially everything needed to calculate the **A**, **B**, **C** and **D** matrices was covered in lecture 15

SSA Two Generator Example

- The **A** matrix is calculated differentiating **f**(**x**,**y**) with respect to **x** (where **x** is δ_1, $\Delta\omega_1$, δ_2, $\Delta\omega_2$)

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - P_{E1} - D_1\Delta\omega_{1.pu}\right)$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - P_{E2} - D_2\Delta\omega_{1.pu}\right)$$

$$\mathrm{P}_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di}\right)G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi}\right)G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di}\right)B_i$$

$$E_{Di} + jE_{Qi} = E_i'\left(\cos\delta_i + j\sin\delta_i\right)$$

SSA Two Generator Example

- Giving

$$\mathbf{A} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.761 & 0 & 0 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0 & 0 & -0.389 & 0 \end{bmatrix}$$

- **B**, **C** and **D** are as calculated previously for the implicit integration, except the elements in B are not multiplied by $\Delta t/2$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.2889 & 0.6505 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0833 & 0.3893 \end{bmatrix}$$

SSA Two Generator Example

- The **C** and **D** matrices are

$$\mathbf{C} = \begin{bmatrix} -3.903 & 0 & 0 & 0 \\ -1.733 & 0 & 0 & 0 \\ 0 & 0 & -4.671 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 7.88 & 0 & -4.54 \\ -7.88 & 0 & 4.54 & 0 \\ 0 & -4.54 & 0 & 9.54 \\ 4.54 & 0 & -9.54 & 0 \end{bmatrix}$$

- Giving

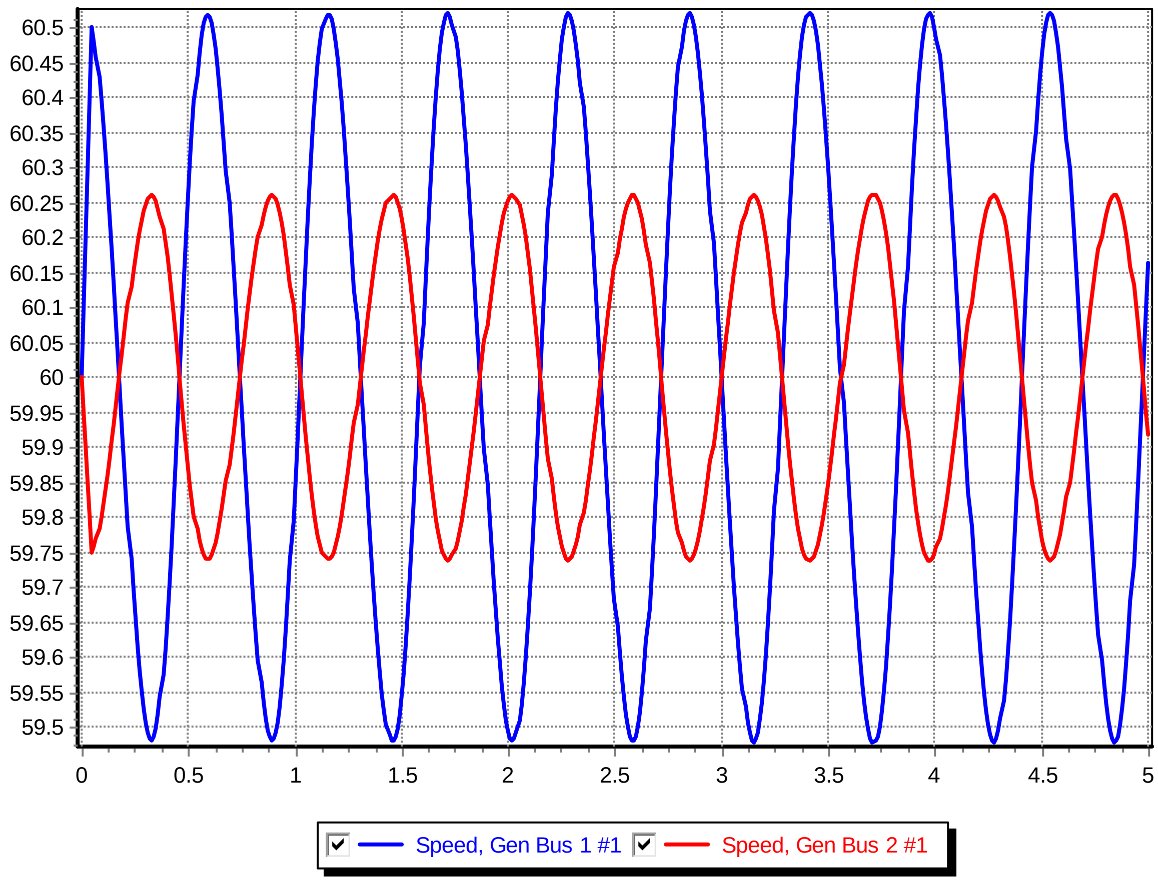
$$\mathbf{A}_{sys} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.229 & 0 & 0.229 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0.114 & 0 & -0.114 & 0 \end{bmatrix}$$

SSA Two Generator

- Calculating the eigenvalues gives a complex pair and two zero eigenvalues
- The complex pair, with values of +/- j11.39 corresponds to the generators oscillating against each other at 1.81 Hz
- One of the zero eigenvalues corresponds to the lack of an angle reference
  - Could be rectified by redefining angles to be with respect to a reference angle (see book 226) or we just live with the zero
- Other zero is associated with lack of speed dependence in the generator torques

SSA Two Generator Speeds

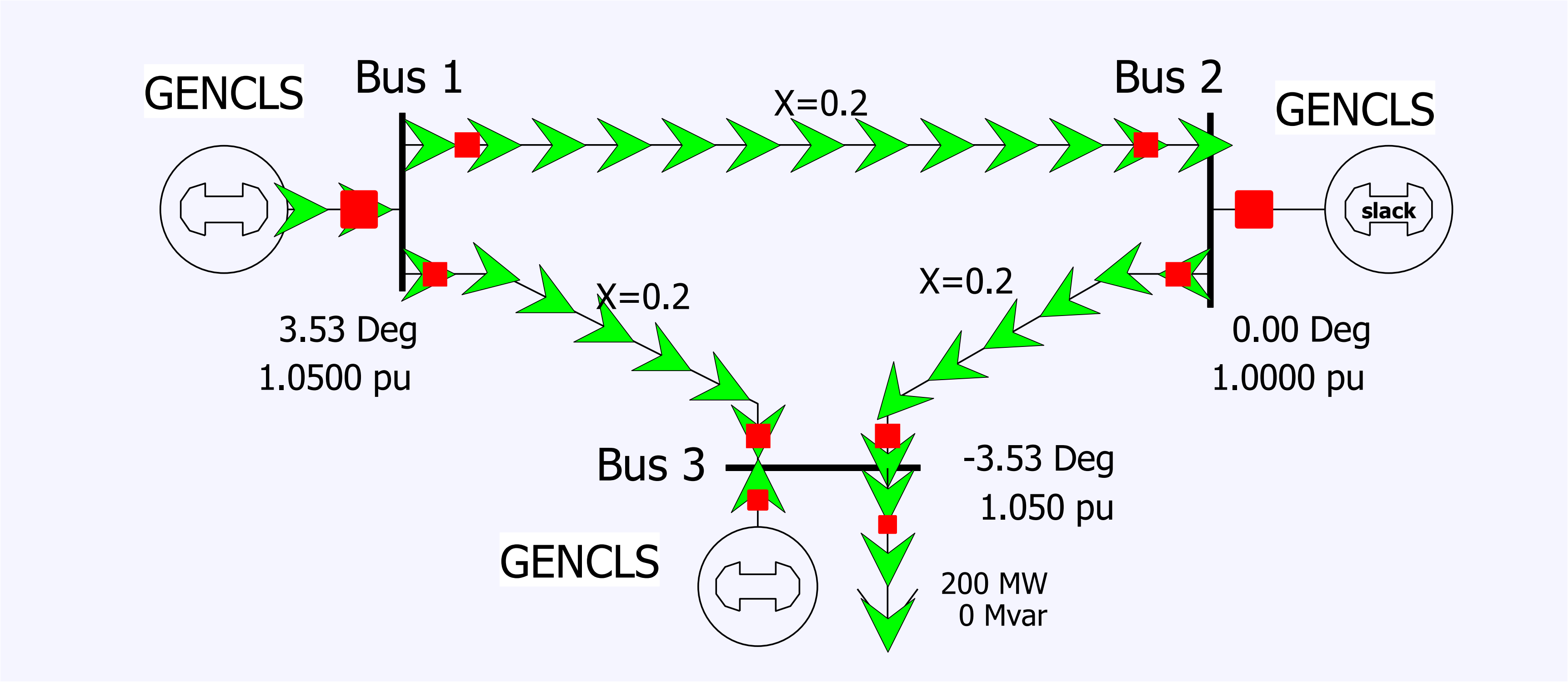
- The two generator system response is shown below for a small disturbance



Notice the actual response closely matches the calculated frequency

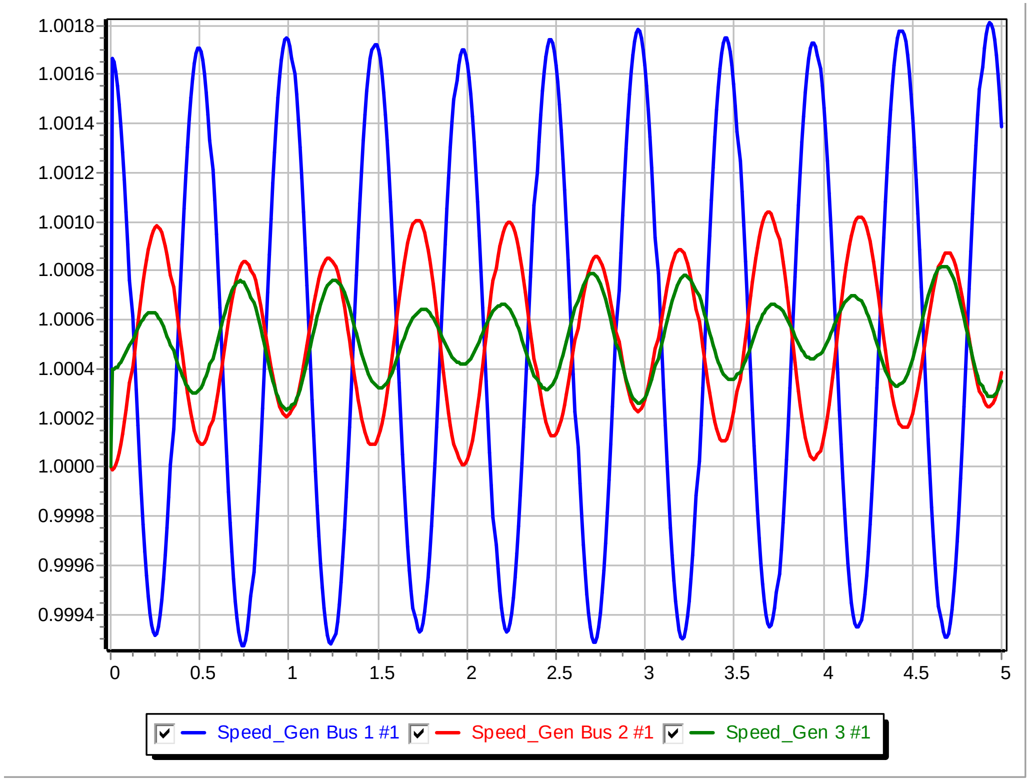
SSA Three Generator Example

- The two generator system is extended to three generators with the third generator having H_3 of 8 and $X_{d3}'=0.3$



SSA Three Generator Example

- Using SSA, two frequencies are identified: one at 2.02 Hz and one at 1.51 Hz



The oscillation is started with a short, self-clearing fault

Shortly we'll discuss modal analysis to determine the contribution of each mode to each signal

PowerWorld case **B2_CLS_3Gen_SSA**

Large System Studies

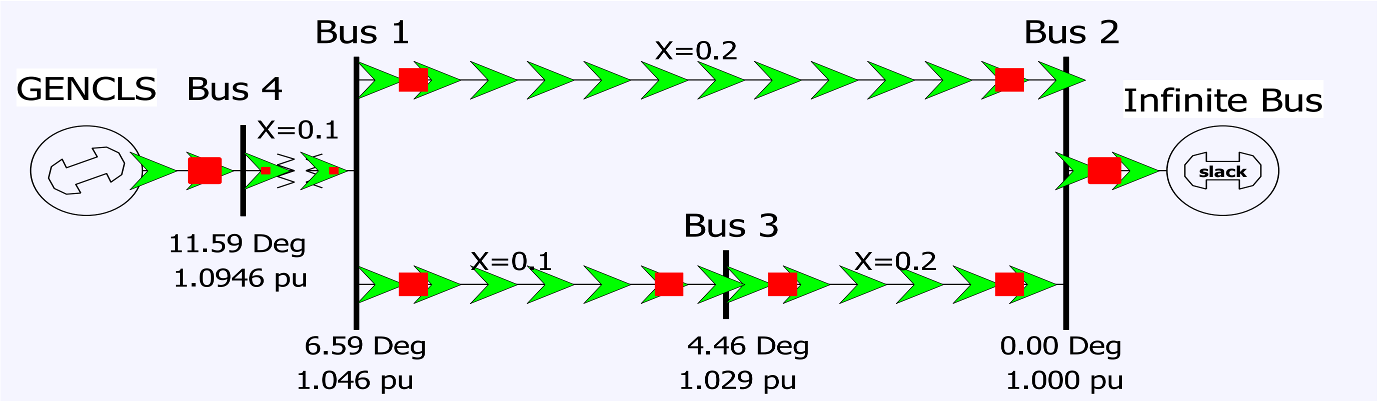
- The challenge with large systems, which could have more than 100,000 states, is the shear size
  - Most eigenvalues are associated with the local plants
  - Computing all the eigenvalues is computationally challenging, order n^3
- Specialized approaches can be used to calculate particular eigenvalues of large matrices
  - See Kundur, Section 12.8 and associated references

Single Machine Infinite Bus

- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
  - The Z_{ii} value can be calculated quite quickly using sparse vector methods
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

Small SMIB Example

- As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



- To get the SMIB for bus 4, first calculate Z_{44}

$$Y_{bus} = j\begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

Z_{44} is Z_{th} in parallel with $jX'_{d,4}$ (which is j0.3) so Z_{th} is j0.22

Small SMIB Example

- The infinite bus voltage is then calculated so as to match the bus i terminal voltage and current

$$\bar{V}_{\text{inf}} = \bar{V}_i - Z_i \bar{I}_i$$

$$\text{where } \left(\frac{P_i + jQ_i}{\bar{V}_i} \right)^* = \bar{I}_i$$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any size system

- In the example we have

$$\left(\frac{P_4 + jQ_4}{\bar{V}_4} \right)^* = \left(\frac{1 + j0.572}{1.072 + j0.220} \right)^* = 1 - j0.328$$

$$\bar{V}_{\text{inf}} = (1.072 + j0.220) - (j0.22)(1 - j0.328)$$

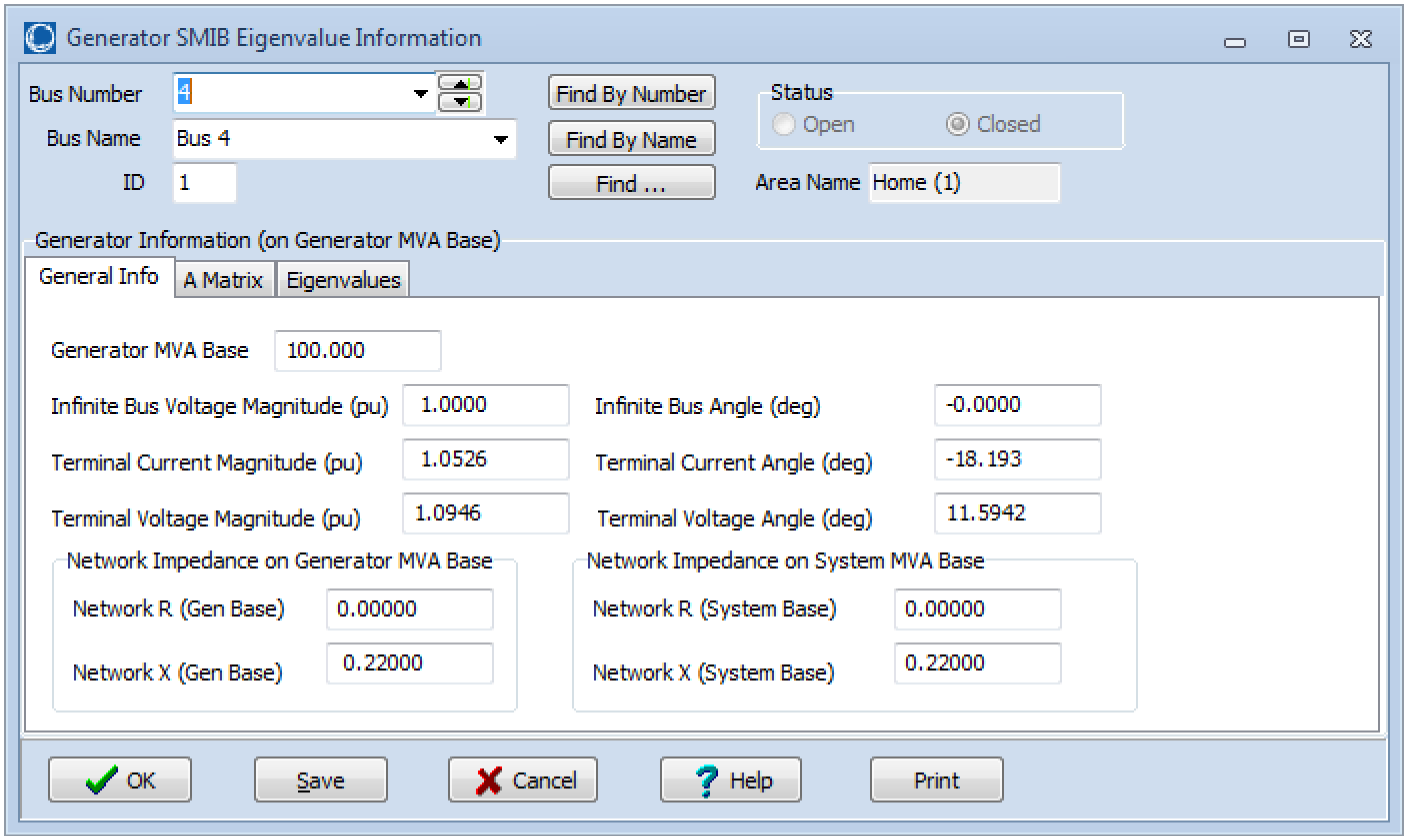
$$\bar{V}_{\text{inf}} = 1.0$$

Calculating the A Matrix

- The SMIB model **A** matrix can then be calculated either analytically or numerically
  - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting Transient Stability, SMIB Eigenvalues
  - Select Run SMIB to perform an SMIB analysis for all the generators in a case
  - Right click on a generator on the SMIB form and select Show SMIB to see the Generator SMIB Eigenvalue Dialog
  - These two bus equivalent networks can also be saved, which can be quite useful for understanding the behavior of individual generators

Example: Bus 4 SMIB Dialog

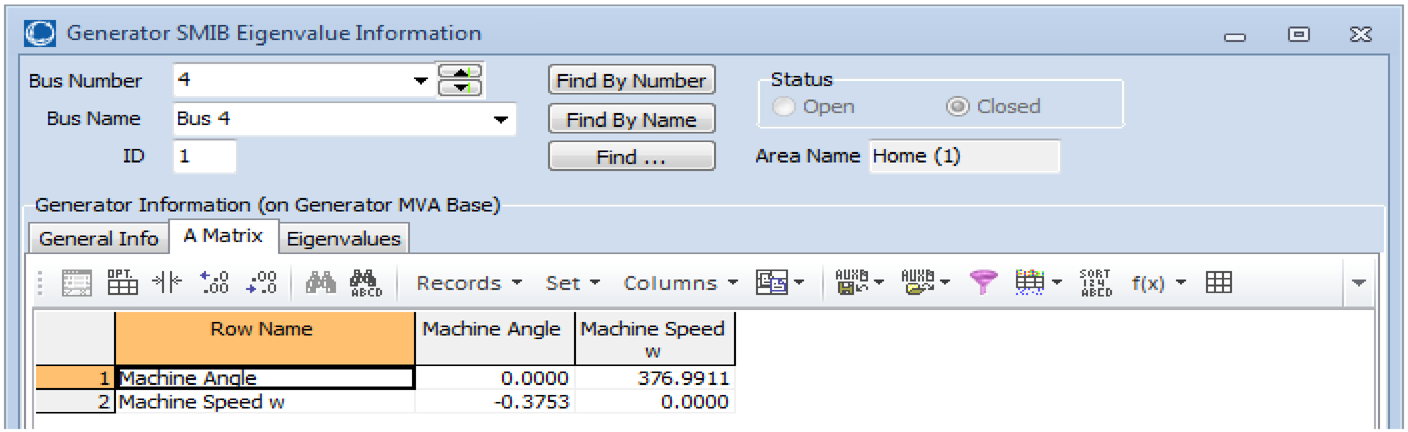
- On the SMIB dialog, the General Information tab shows information about the two bus equivalent



PowerWorld case **B4_SMIB**

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the **A** Matrix tab shows the $\mathbf{A}_{sys}$ matrix for the SMIB generator



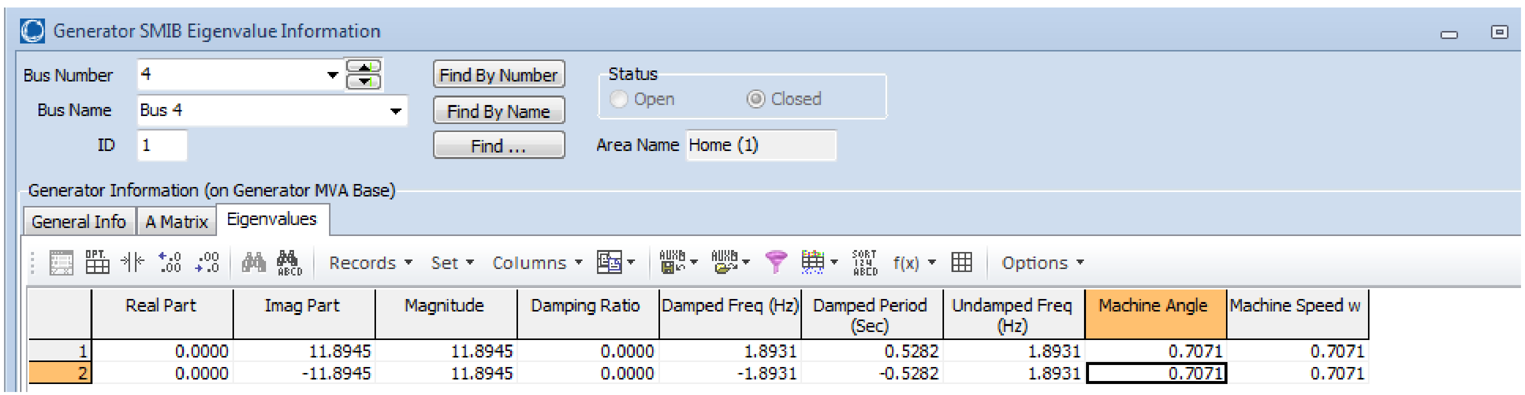
- In this example A_{21} is showing

$$\frac{\partial \Delta\omega_{4,pu}}{\partial \delta_4} = \frac{1}{2H_4}\left(\frac{-\partial P_{E,4}}{\partial \delta_4}\right) = -\left(\frac{1}{6}\right)\left(\left(\frac{-1}{0.3+0.22}\right)\left(-1.2812\cos\left(23.94^\circ\right)\right)\right)$$

$$= -0.3753$$

Example: Bus 4 SMIB Dialog

- On the SMIB dialog, the Eigenvalues tab shows the $\mathbf{A}_{sys}$ matrix eigenvalues and participation factors (which we'll cover shortly)



	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w
1	0.0000	11.8945	11.8945	0.0000	1.8931	0.5282	1.8931	0.7071	0.7071
2	0.0000	-11.8945	11.8945	0.0000	-1.8931	-0.5282	1.8931	0.7071	0.7071

- Saving the two bus SMIB equivalent, and putting a short, self-cleared fault at the terminal shows the 1.89 Hz, undamped response