ECEN 667
Power System Stability

Lecture 21: Measurement-Based Modal Analysis

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Announcements

- Read Chapter 8
- Homework 6 is due today
- Homework 7 will be assigned soon and due on Nov 30

Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately we are finding

$$y_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

- The $\boldsymbol{\alpha}$ is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Quickly Determining the b Vectors

- A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal k

$$\mathbf{y}_k = \mathbf{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k$$

And then the residual is minimized by selecting $\mathbf{b}_k = \mathbf{\Phi}(\boldsymbol{\alpha})^+ \mathbf{y}_k$

where $\mathbf{\Phi}(\boldsymbol{\alpha})$ is the m by n matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_i corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\mathbf{\Phi}(\boldsymbol{\alpha})^+$ is the pseudoinverse of $\mathbf{\Phi}(\boldsymbol{\alpha})$

Where m is the number of measurements and n is the number of modes

A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013

Aside: Pseudoinverse of a Matrix

- The pseudoinverse of a matrix generalizes concept of a matrix inverse to an m by n matrix, in which m >= n
  - Specifically this is a Moore-Penrose Matrix Inverse
- Notation for the pseudoinverse of $\mathbf{A}$ is $\mathbf{A}^+$
- Satisfies $\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$
- If $\mathbf{A}$ is a square matrix, then $\mathbf{A}^+ = \mathbf{A}^{-1}$
- Quite useful for solving the least squares problem since the least squares solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \mathbf{A}^+\mathbf{b}$
- Can be calculated using an SVD $\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$

$$\mathbf{A}^+ = \mathbf{V}\boldsymbol{\Sigma}^+\mathbf{U}^T$$

Least Squares Matrix Pseudoinverse Example

- Assume we wish to fix a line ($mx + b = y$) to three data points: (1,1), (2,4), (6,4)
- Two unknowns, m and b; hence $\mathbf{x} = [m \;\; b]^T$
- Setup in form of $\mathbf{Ax} = \mathbf{b}$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \quad \text{so} \;\; \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix}$$

Least Squares Matrix Pseudoinverse Example, cont.

- Doing an economy SVD

$$\mathbf{A} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T = \begin{bmatrix} -0.182 & -0.765 \\ -0.331 & -0.543 \\ -0.926 & 0.345 \end{bmatrix} \begin{bmatrix} 6.559 & 0 \\ 0 & 0.988 \end{bmatrix} \begin{bmatrix} -0.976 & -0.219 \\ 0.219 & -0.976 \end{bmatrix}$$

- Computing the pseudoinverse

$$\mathbf{A}^+ = \mathbf{V}\,\boldsymbol{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.976 & 0.219 \\ -0.219 & -0.976 \end{bmatrix} \begin{bmatrix} 0.152 & 0 \\ 0 & 1.012 \end{bmatrix} \begin{bmatrix} -0.182 & -0.331 & -0.926 \\ -0.765 & -0.543 & 0.345 \end{bmatrix}$$

$$\mathbf{A}^+ = \mathbf{V}\,\boldsymbol{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix}$$

In an economy SVD the $\boldsymbol{\Sigma}$ matrix has dimensions of m by m if m < n or n by n if n < m

Least Squares Matrix Pseudoinverse Example, cont.

- Computing $\mathbf{x} = [\mathrm{m}\ \mathrm{b}]^T$ gives

$$\mathbf{A}^{+}\mathbf{b} = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.429 \\ 1.71 \end{bmatrix}$$

- With the pseudoinverse approach we immediately see the sensitivity of the elements of **x** to the elements of **b**
  - New values of m and b can be readily calculated if **y** changes
- Computationally the SVD is order mn^2+n^3 (with $n < m$)
  - In this example it means it scales linearly with the number of points; matrices with m >> n are common

Computational Considerations

- When there is just one signal, the procedure scales with the cube of the number of measurements
  - This value is usually relatively small, say 20 seconds of data sampled at 10 Hz for 200 measurements
- If multiple signals are included, it scales linearly with the number of signals
- However, a key insight is once $\boldsymbol{\alpha}$ has been determined, each $\mathbf{b}_k$ can be determined with a matrix multiply of a matrix with dimensions of the number of modes and number of measurements

$$\mathbf{y}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k \rightarrow \mathbf{b}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})^+ \mathbf{y}_k$$

$\boldsymbol{\Phi}(\boldsymbol{\alpha})^+$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

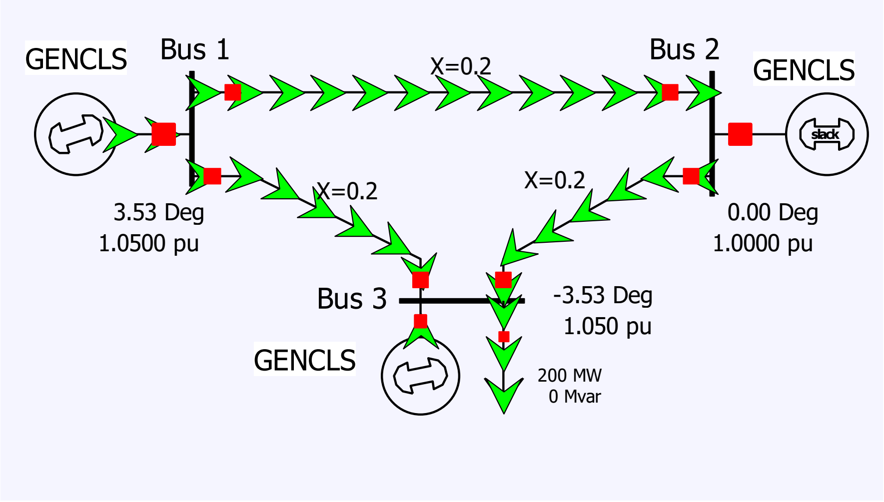
We can quickly get how well $\boldsymbol{\alpha}$ matches each signal

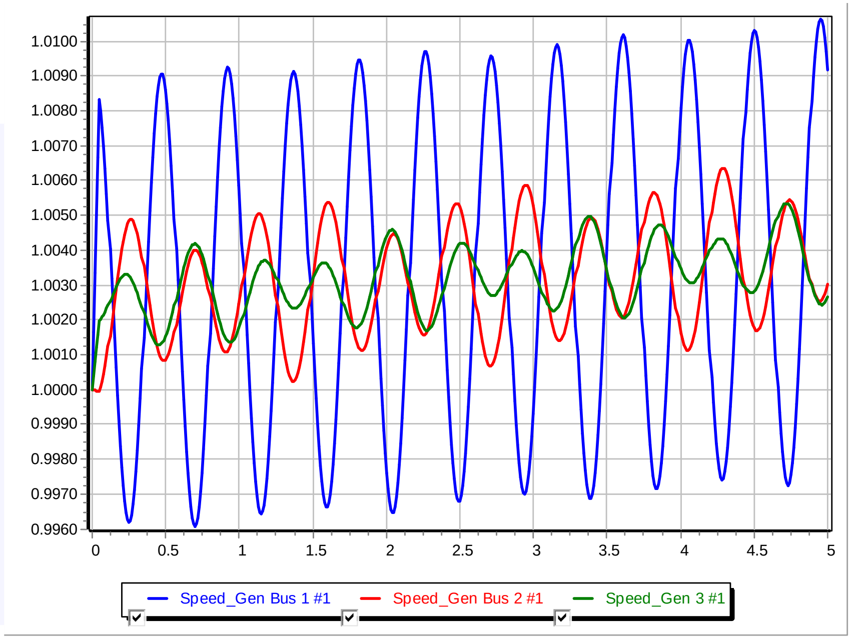
Modal Analysis in PowerWorld

- Goal is to make modal analysis easy to use, and easy to visualize the results
- Provided tool can be used with either transient stability results or actual system signals (e.g., from PMUs)
- Three ways to access in PowerWorld
  - From the Modal Analysis button (in **Add-Ons**)
  - On the Transient Stability Analysis form left menu, **Modal Analysis** (right below SMIB Eigenvalues)
  - By right-clicking on a transient stability or plot case information display, and selecting **Modal Analysis Selected Columns** or **Modal Analysis All Columns**

Modal Analysis: Three Generator Example

- A short fault at t=0 gets the below three generator case oscillating with multiple modes (mostly clearly visible for the red and the green curve)



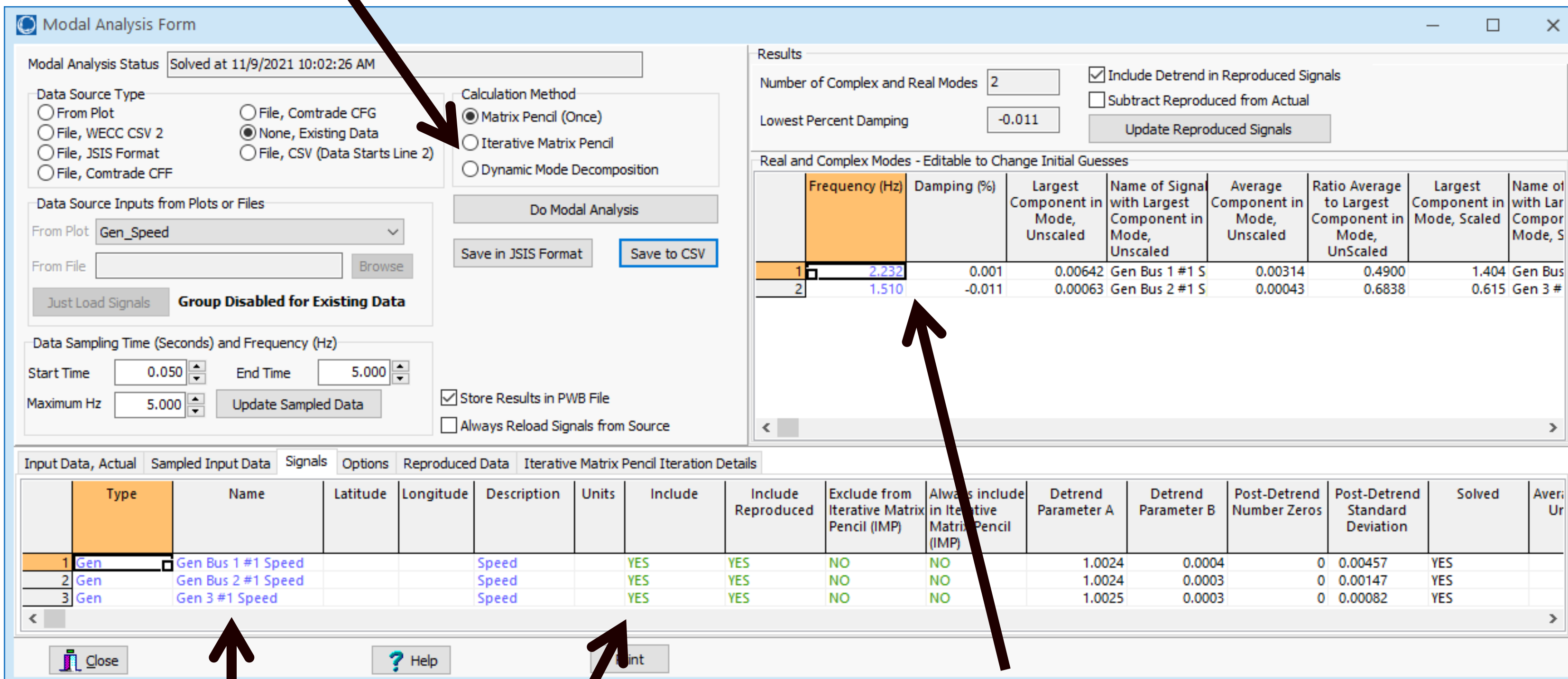


Modal Analysis: Three Generator Example

- Open the case **B3_CLS_UnDamped**
  - This system has three classical generators without damping; the default event is a self clearing fault at bus 1
- Run the transient stability for 5 seconds
- To do modal analysis, on the Transient Stability page select Results from RAM, view just the generator speed fields, right-click and select **Modal Analysis All Columns**
  - This display the Modal Analysis Form

Modal Analysis Form

First click on **Do Modal Analysis** to run the modal analysis



Right-click on signal to view its dialog

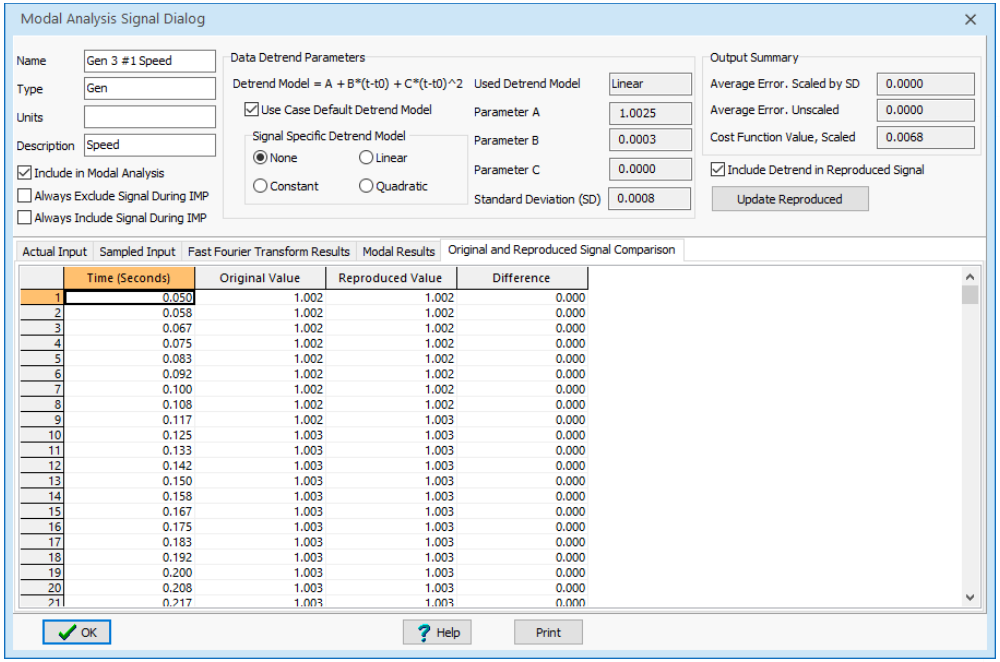
Signals to include

Key results are shown in the upper-right of the form. There are two main modes, one at 2.23Hz and one at 1.51; both have very little damping.

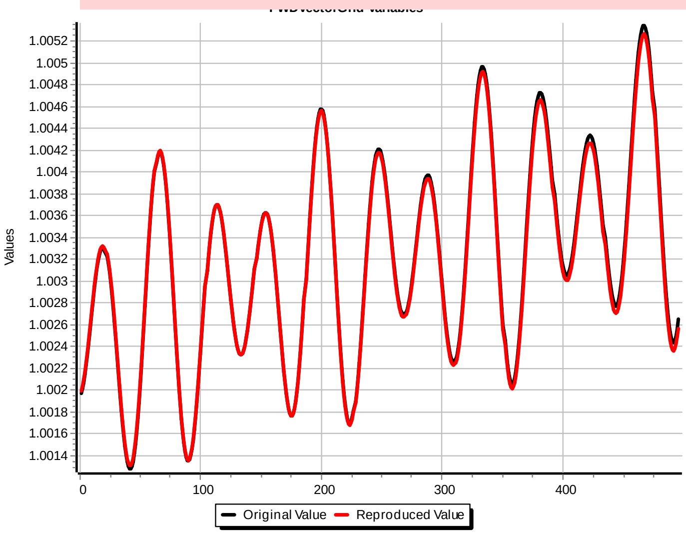
Three Generator Example: Signal Dialog

- The **Signal Dialog** provides details about each signal, including its modal components and a comparison between the original and reproduced signals (example for gen 3)

Plotting the original and reproduced signals shows a near exact match

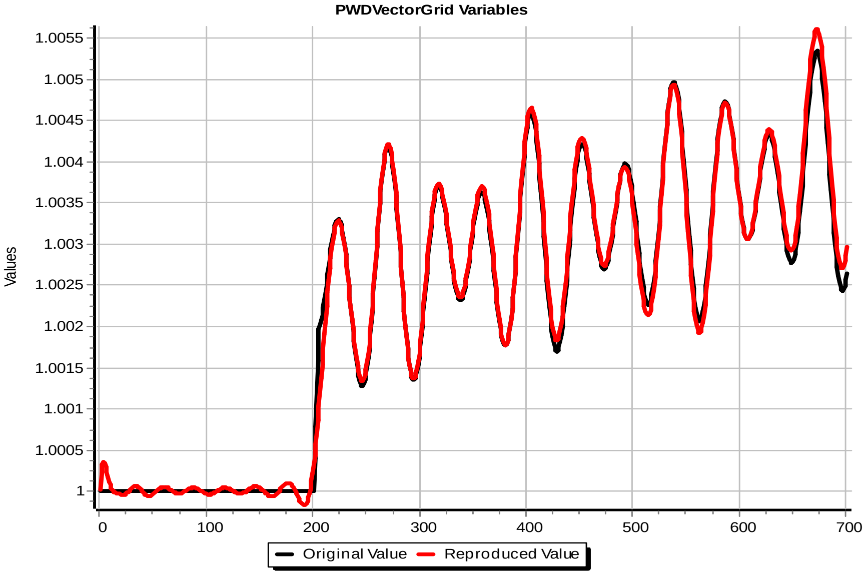


	Time (Seconds)	Original Value	Reproduced Value	Difference
1	0.050	1.002	1.002	0.000
2	0.058	1.002	1.002	0.000
3	0.067	1.002	1.002	0.000
4	0.075	1.002	1.002	0.000
5	0.083	1.002	1.002	0.000
6	0.092	1.002	1.002	0.000
7	0.100	1.002	1.002	0.000
8	0.108	1.002	1.002	0.000
9	0.117	1.002	1.003	0.000
10	0.125	1.003	1.003	0.000
11	0.133	1.003	1.003	0.000
12	0.142	1.003	1.003	0.000
13	0.150	1.003	1.003	0.000
14	0.158	1.003	1.003	0.000
15	0.167	1.003	1.003	0.000
16	0.175	1.003	1.003	0.000
17	0.183	1.003	1.003	0.000
18	0.192	1.003	1.003	0.000
19	0.200	1.003	1.003	0.000
20	0.208	1.003	1.003	0.000
21	0.217	1.003	1.003	0.000



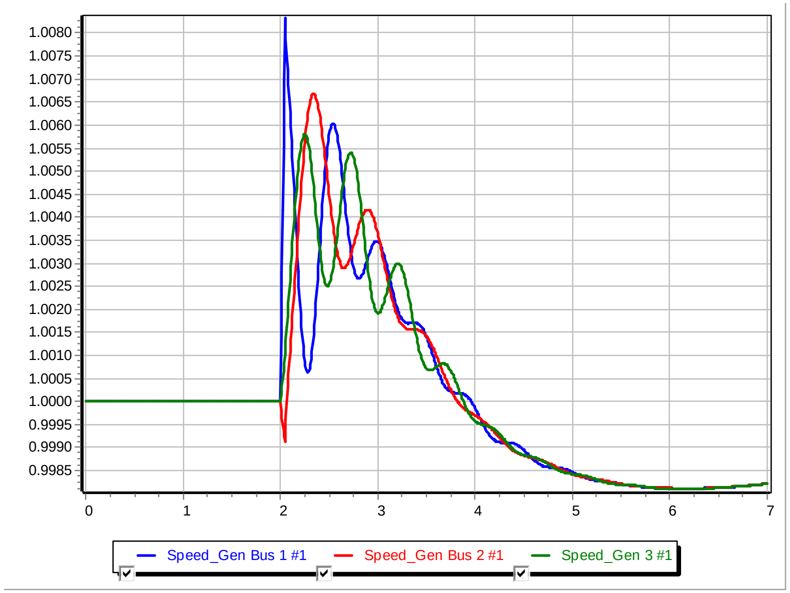
Caution: Setting Time Range Incorrectly Can Result in Unexpected Results!

- Assume the system is run with no disturbance for two seconds, and then the fault is applied and the system is run for a total of seven seconds (five seconds post-fault)
  - The incorrect approach would be to try to match the entire signal; rather just match from after the fault
  - Trying to match the full signal between 0 and 7 seconds required eleven modes!
  - By default the Modal Analysis Form sets the default start time to immediately after the last event



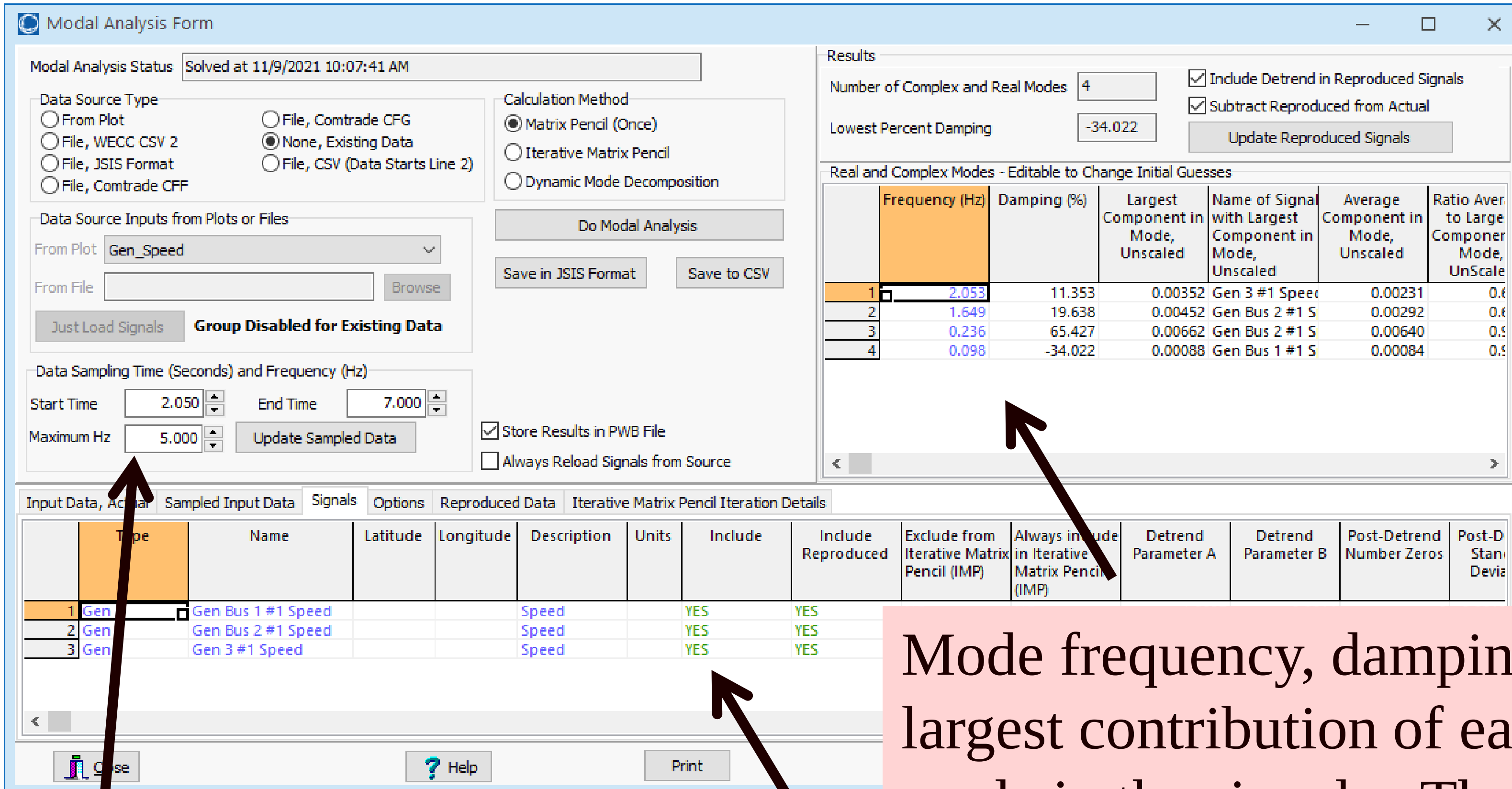
GENROU Example with Damping

- Open the case **B3_GENROU**, which changes the GENCLS to GENROU models, adding damping
  - Also each has an EXST1 exciter and a TGOV1 governor
  - The simulation runs for seven seconds, with the fault occurring at two seconds; modal analysis is done from the time the fault is cleared until the end of the simulation.



The image shows the generator speeds. The initial rise in the speed is caused by the load dropping during the fault, causing a power mismatch; this is corrected by the governors. Note the system now has damping; modal analysis tells us how much.

GENROU Example with Damping



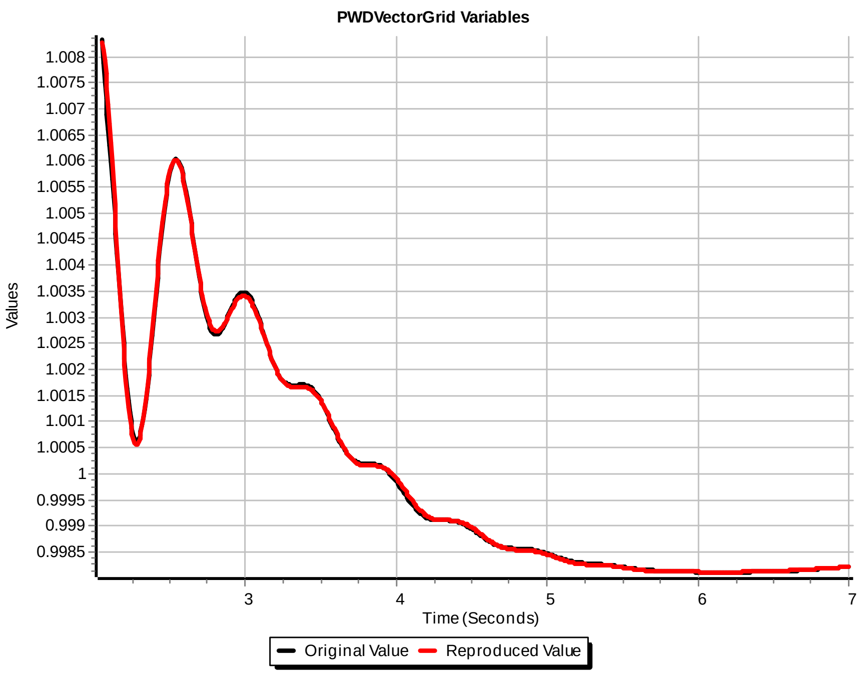
Start time default value

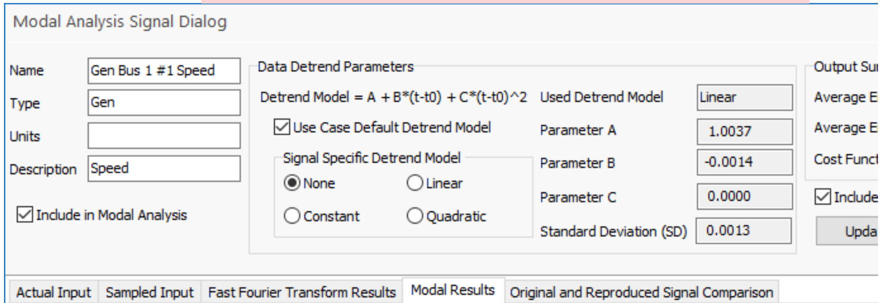
Mode frequency, damping, and largest contribution of each mode in the signals. The slower mode is associated with the governors.

GENROU Example with Damping

- Left image show how well the speed for generator 1 is approximated by the modes

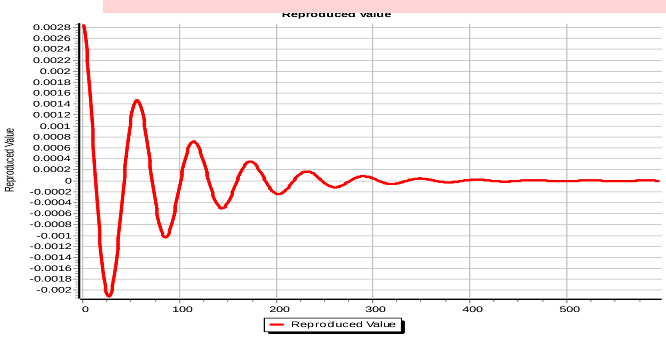
More signal details





	Damping (%)	Frequency (Hz)	Magnitude Scaled by SD	Magnitude, Unscaled	Angle (Deg)	Lambda	Include in Reproduced Signal
1	11.353	2.053	2.300	0.003	13.82	-1.474	YES
2	19.638	1.649	2.038	0.003	10.46	-2.075	YES
3	65.427	0.236	4.757	0.006	-91.36	-1.283	YES
4	-34.022	0.098	0.689	0.001	135.64	0.222	YES

Just the 2.05 Hz mode



Dealing with Multiple Signals

- When there are many signals, usually they are at least somewhat correlated, so we do not need to include all the signals in the calculation of α.
- Based on the previous quick calculation of $\mathbf{b}_k$, we can determine how well the signals match the α.
- A natural algorithm for improving is to include the signals that do not match α well. That is, have high residuals.
- This gave rise to what is called the Iterative Matrix Pencil algorithm.

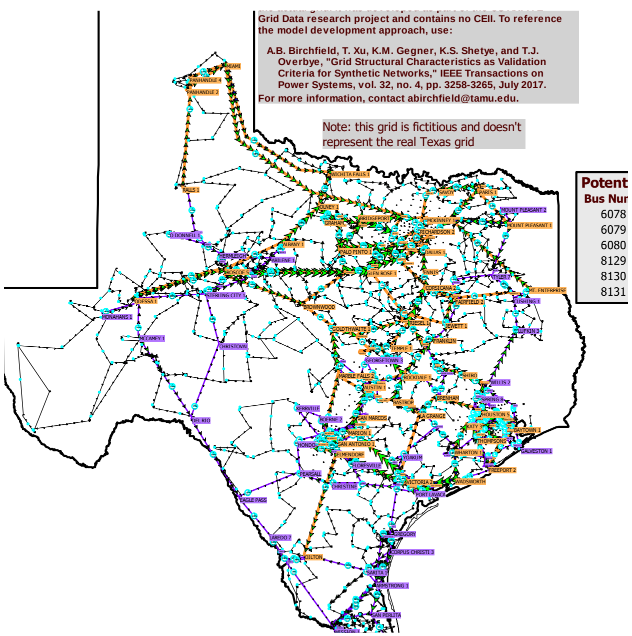
Iterative Matrix Pencil Method

- When there are a large number of signals the iterative matrix pencil method works by
  - Selecting an initial signal to calculate the $\boldsymbol{\alpha}$ vector
  - Quickly calculating the **b** vectors for all the signals, and getting a cost function for how closely the reconstructed signals match their sampled values
  - Selecting a signal that has a high cost function, and repeating the above adding this signal to the algorithm to get an updated $\boldsymbol{\alpha}$

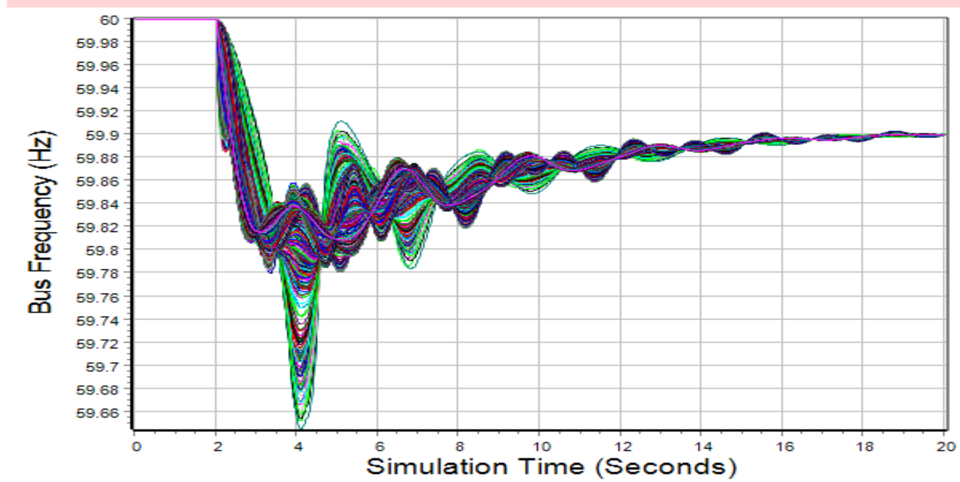
An open access paper describing this is W. Trinh, K.S. Shetye, I. Idehen, T.J. Overbye, "Iterative Matrix Pencil Method for Power System Modal Analysis," *Proc. 52nd Hawaii International Conference on System Sciences*, Wailea, HI, January 2019; available at scholarspace.manoa.hawaii.edu/handle/10125/59803

Texas 2000 Bus Synthetic Grid Example

- For this example we'll again use the Texas 2000 bus grid, saved as **TSGC_2000_GenDrop**
- We'll use the Iterative Matrix Pencil Method to examine its modes
  - The contingency is the loss of two large generators (at bus 7098 and 7099)

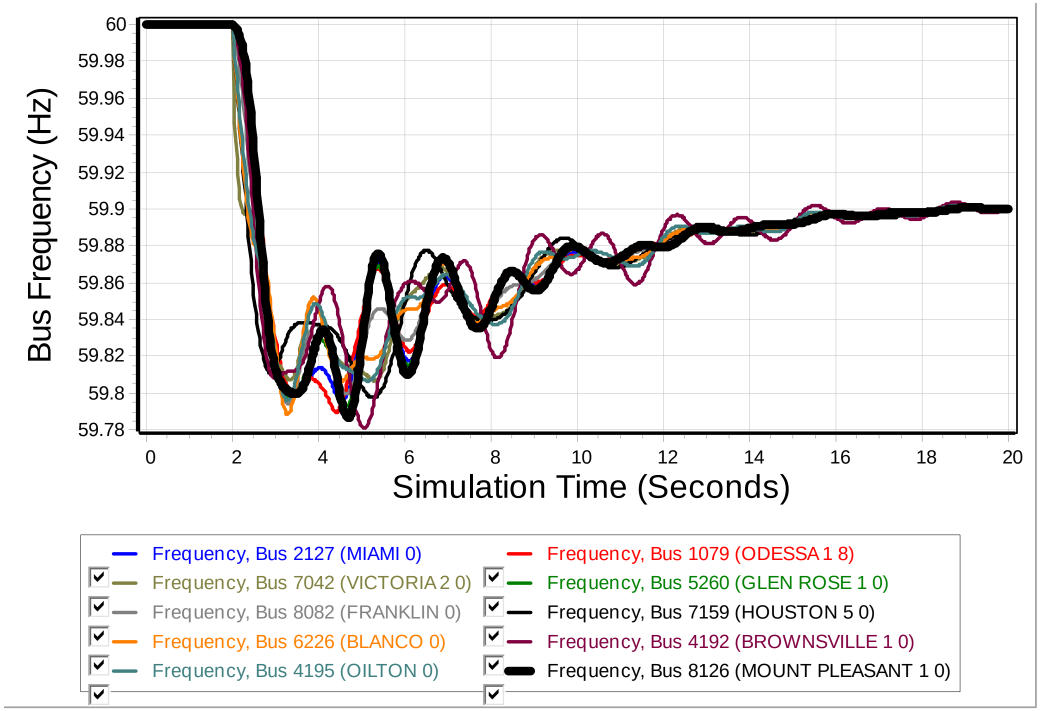


The measurements will be the frequencies at all 2000 buses



2000 Bus System Example, Initially Just One Signal

- Initially our goal is to understand the modal frequencies and their damping
- First we'll consider just one of the 2000 signals; arbitrarily I selected bus 8126 (Mount Pleasant)



Some Initial Considerations

- The input is a dynamics study running using a ½ cycle time step; data was saved every 3 steps, so at 40 Hz
  - The contingency was applied at time = 2 seconds
- We need to pick the portion of the signal to consider and the sampling frequency
  - Because of the underlying SVD, the algorithm scales with the cube of the number of time points (in a single signal)
- I selected between 2 and 17 seconds
- I sampled at ten times per second (so a total of 150 samples)

2000 Bus System Example, One Signal

- The results from the Matrix Pencil Method are

Number of Complex and Real Modes 6

Lowest Percent Damping 10.137

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

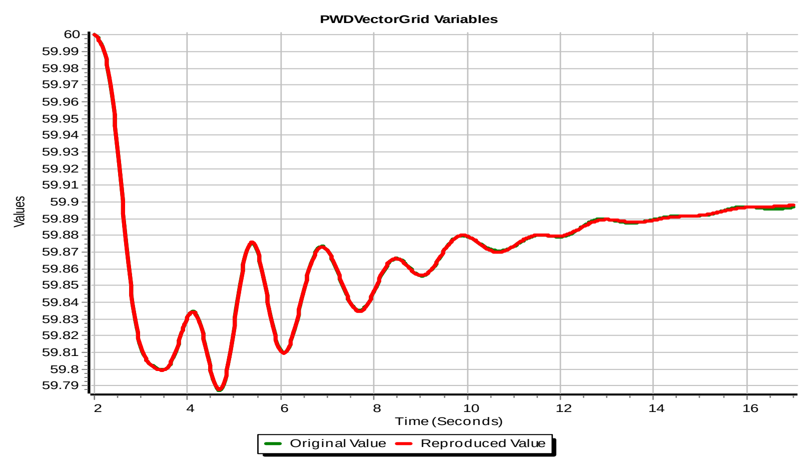
Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include in Reproduced Signal
1	0.383	32.011	0.44275	Bus 1073 (ODE	12.224	Bus 7310 (WHA	-0.8136	YES
2	0.670	24.191	0.38466	Bus 2120 (PARI	11.549	Bus 8078 (MT. E	-1.0490	YES
3	0.665	10.705	0.23093	Bus 2115 (PARI	6.801	Bus 2115 (PARI	-0.4501	YES
4	0.312	14.397	0.16911	Bus 1073 (ODE	4.954	Bus 7310 (WHA	-0.2855	YES
5	0.971	10.137	0.08179	Bus 1051 (MON	2.551	Bus 6147 (SAN	-0.6215	YES
6	0.052	41.828	0.04603	Bus 1074 (ODE	1.063	Bus 3035 (CHER	-0.1506	YES

Calculated mode information

Verification of results

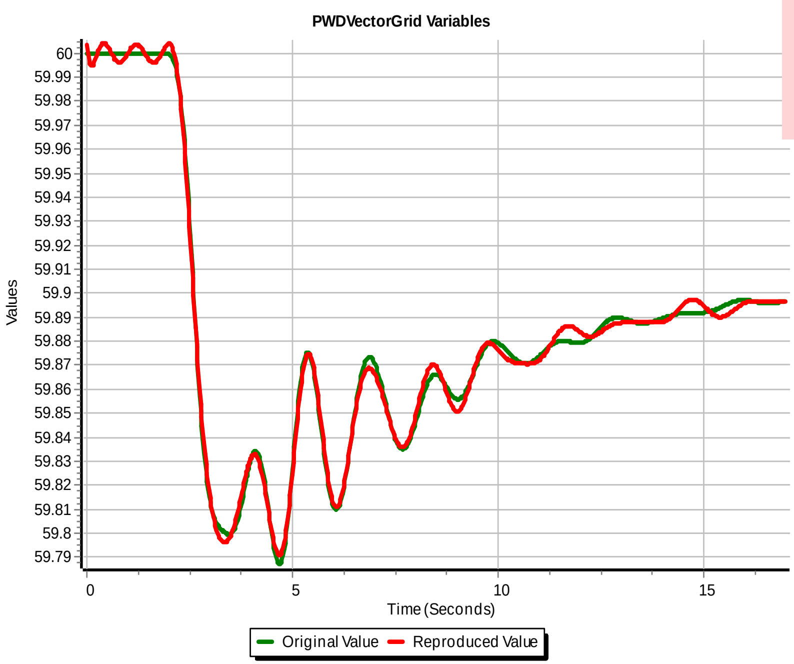


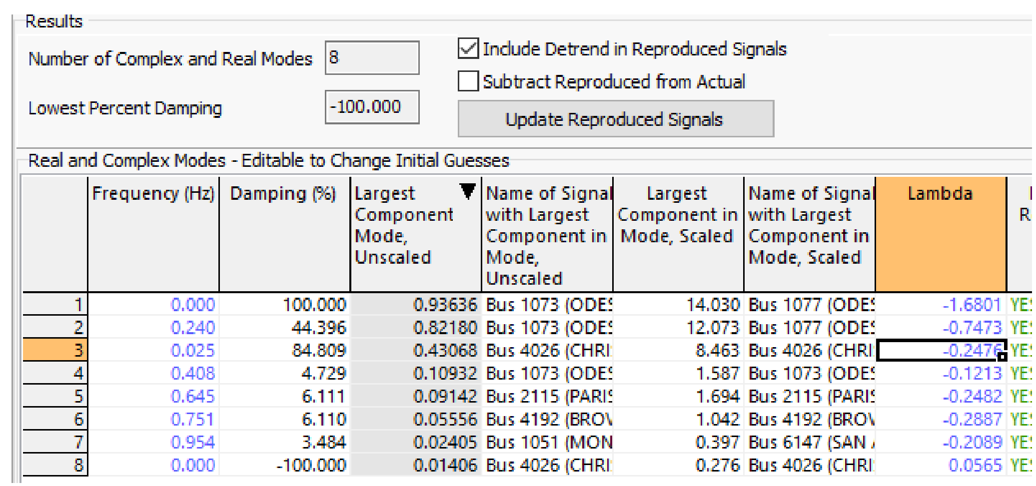
Some Observations

- These results are based on the consideration of just one signal
- The start time **should** be at or after the event!

If it isn't then…

The results show the algorithm trying to match the first two flat seconds; this should not be done!!





	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	R
1	0.000	100.000	0.93636	Bus 1073 (ODE	14.030	Bus 1077 (ODE	-1.6801	YE
2	0.240	44.396	0.82180	Bus 1073 (ODE	12.073	Bus 1077 (ODE	-0.7473	YE
3	0.025	84.809	0.43068	Bus 4026 (CHRI	8.463	Bus 4026 (CHRI	-0.2476	YE
4	0.408	4.729	0.10932	Bus 1073 (ODE	1.587	Bus 1073 (ODE	-0.1213	YE
5	0.645	6.111	0.09142	Bus 2115 (PARI	1.694	Bus 2115 (PARI	-0.2482	YE
6	0.751	6.110	0.05556	Bus 4192 (BRO	1.042	Bus 4192 (BRO	-0.2887	YE
7	0.954	3.484	0.02405	Bus 1051 (MON	0.397	Bus 6147 (SAN	-0.2089	YE
8	0.000	-100.000	0.01406	Bus 4026 (CHRI	0.276	Bus 4026 (CHRI	0.0565	YE

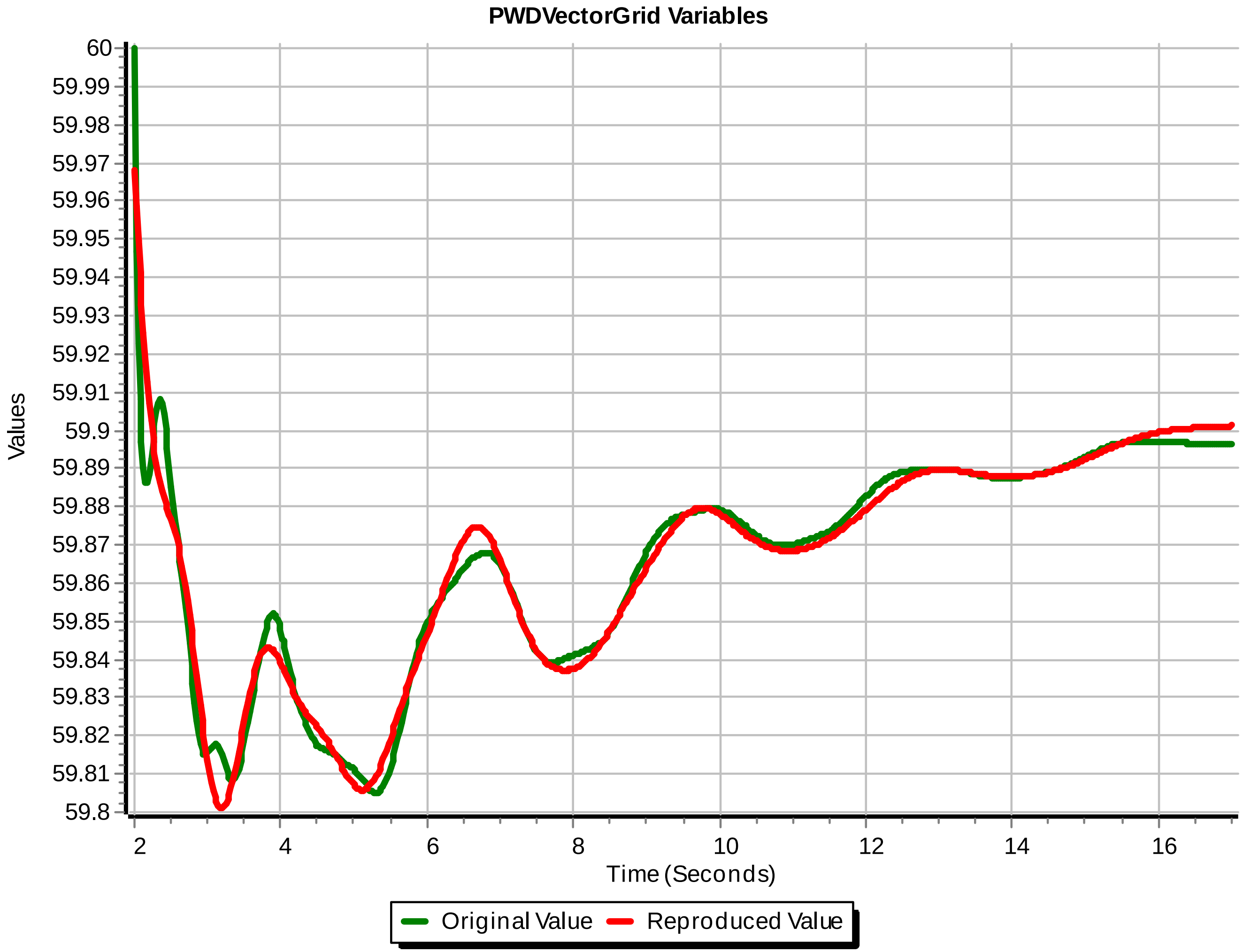
2000 Bus System Example, One Signal Included, Cost for All

- Using the previously discussed pseudoinverse approach, for a given set of modes ($\boldsymbol{\alpha}$) the $\mathbf{b}_k$ vectors for all the signals can be quickly calculated

$$\mathbf{b}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})^+ \mathbf{y}_k$$

  - The dimensions of the pseudoinverse are the number of modes by the number of sample points for one signal
- This allows each cost function to be calculated
- The Iterative Matrix Pencil approach sequentially adds the signals with the worst match (i.e., the highest cost function)

2000 Bus System Example, Worst Match (Bus 7061)



2000 Bus System Example, Two Signals

With two signals

Number of Complex and Real Modes: 9
Lowest Percent Damping: 7.359
☑ Include Detrend in Reproduced Signals
☐ Subtract Reproduced from Actual
Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component in Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambc
1	2.266	17.168	0.04028	Bus 7329 (NEW	1.730	Bus 7307 (WHA	-2
2	1.413	21.844	0.10763	Bus 4030 (FANN	4.475	Bus 4030 (FANN	
3	0.958	7.359	0.04666	Bus 6147 (SAN A	1.801	Bus 6147 (SAN A	
4	0.701	11.705	0.21220	Bus 1051 (MON	5.762	Bus 8077 (MT. E	
5	0.630	13.361	0.20903	Bus 2120 (PARIS	6.350	Bus 4192 (BROV	
6	0.352	36.405	0.44679	Bus 1051 (MON	13.024	Bus 7311 (WHA	
7	0.322	14.403	0.19570	Bus 1073 (ODES	5.372	Bus 7311 (WHA	
8	0.000	100.000	0.09305	Bus 1051 (MON	1.767	Bus 1051 (MON	
9	0.064	36.756	0.02993	Bus 1073 (ODES	1.182	Bus 7307 (WHA	

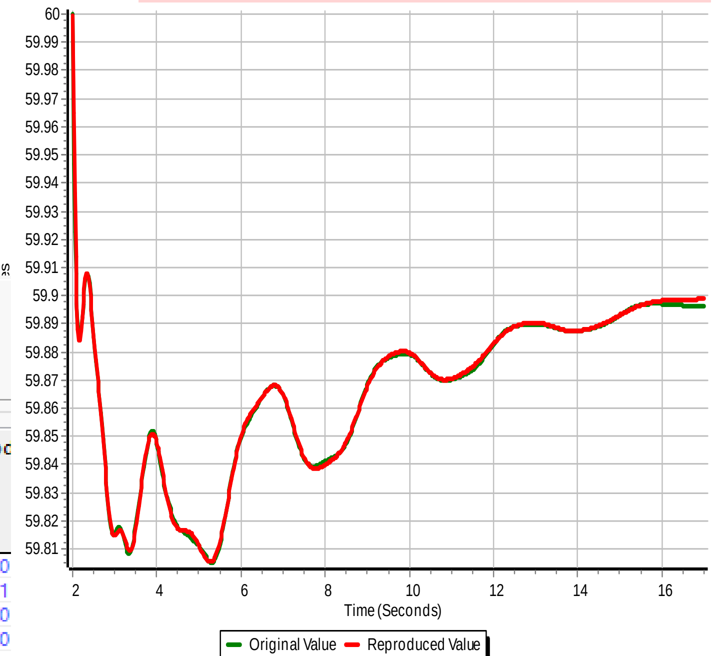
With one signal

Number of Complex and Real Modes: 6
Lowest Percent Damping: 10.137
☑ Include Detrend in Reproduced Signals
☐ Subtract Reproduced from Actual
Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (%)	Largest Component Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambc
1	0.383	32.011	0.44275	Bus 1073 (ODES	12.224	Bus 7310 (WHA	-0
2	0.670	24.191	0.38466	Bus 2120 (PARIS	11.549	Bus 8078 (MT. E	-1
3	0.665	10.705	0.23093	Bus 2115 (PARIS	6.801	Bus 2115 (PARIS	-0
4	0.312	14.397	0.16911	Bus 1073 (ODES	4.954	Bus 7310 (WHA	-0
5	0.971	10.137	0.08179	Bus 1051 (MON	2.551	Bus 6147 (SAN A	-0
6	0.052	41.828	0.04603	Bus 1074 (ODES	1.063	Bus 3035 (CHEF	-0

The new match on the bus that was previously worst (Bus 7061) is now quite good!



2000 Bus System Example, Iterative Matrix Pencil

- The Iterative Matrix Pencil intelligently adds signals until a specified number is met
  - Doing ten iterations takes about four seconds

Number of Complex and Real Modes: 11

Lowest Percent Damping: 6.082

☑ Include Detrend in Reproduced Signals

☐ Subtract Reproduced from Actual

Update Reproduced Signals

Real and Complex Modes - Editable to Change Initial Guesses

	Frequency (Hz)	Damping (% ▲	Largest Component in Mode, Unscaled	Name of Signal with Largest Component in Mode, Unscaled	Largest Component in Mode, Scaled	Name of Signal with Largest Component in Mode, Scaled	Lambda	Include in Reproduced Signal
1	0.631	6.082	0.10313	Bus BROWNSVI	3.292	Bus BROWNSVI	-0.2415	YES
2	0.959	7.068	0.04897	Bus SAN ANTOI	1.890	Bus SAN ANTOI	-0.4269	YES
3	1.364	7.246	0.03780	Bus ODESSA 1 :	1.420	Bus CHRISTINE	-0.6228	YES
4	0.593	7.897	0.07205	Bus BROWNSVI	2.300	Bus BROWNSVI	-0.2949	YES
5	1.602	8.562	0.04887	Bus FANNIN 2 F	2.032	Bus FANNIN 2 F	-0.8650	YES
6	0.732	11.936	0.21348	Bus MONAHAN	4.054	Bus MONAHAN	-0.5529	YES
7	0.324	14.207	0.19906	Bus ODESSA 1 :	5.268	Bus WHARTON	-0.2917	YES
8	0.324	39.346	0.55936	Bus MONAHAN	12.994	Bus WHARTON	-0.8722	YES
9	0.060	39.972	0.03815	Bus ODESSA 1 :	1.196	Bus POINT COM	-0.1645	YES
10	0.964	57.683	0.61264	Bus ODESSA 1 :	18.504	Bus POINT COM	-4.2760	YES
11	0.000	100.000	0.59650	Bus ODESSA 1 :	14.434	Bus WHARTON	-2.5257	YES

Takeaways So Far

- Modal analysis can be quickly done on a large number of signals
  - Computationally is an $O(N^3)$ process for one signal, where N is the number of sample points; it varies linearly with the number of included signals
  - The number of sample points can be automatically determined from the highest desired frequency (the Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency)
  - Determining how all the signals are manifested in the modes is quite fast!!