ECEN 667 Homework #7

Problem #1 Book Problem 8.8

Find the participation factors of the eigenvalues for the following system, where $\dot{x} = Ax$.

Part A

$$A = \begin{bmatrix} 3 & 8 \\ 2 & 3 \end{bmatrix}$$

First, we need to get the eigenvalues.

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0 \implies \begin{vmatrix} 3-\lambda & 8 \\ 2 & 3-\lambda \end{vmatrix} = 0$$
$$\implies \lambda_1 = 7 \qquad \lambda_2 = -1$$

Then, we can calculate the right eigenvectors:

$$\boldsymbol{A}\boldsymbol{v}_1 = \lambda_1 \boldsymbol{v}_1 \implies \boldsymbol{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \boldsymbol{A}\boldsymbol{v}_2 = \lambda_2 \boldsymbol{v}_2 \implies \boldsymbol{v_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Then the left eigenvectors:

$$\boldsymbol{w}_1^t \boldsymbol{A} = \boldsymbol{w}_1^t \lambda_1 \implies \boldsymbol{w}_1^t = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \boldsymbol{w}_2^t \boldsymbol{A} = \boldsymbol{w}_2^t \lambda_1 \implies \boldsymbol{w}_2^t = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

However, we need $w_i^t v_i = 1$, so we scale every element of w_i by $\frac{1}{4}$.

$$\implies \boldsymbol{p} = \boldsymbol{v}\left(\frac{1}{4}\boldsymbol{w}\right) = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

Part B

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{bmatrix}$$

We apply the same procedure as before, and get the following results:

$$\lambda_1 = -2,\ \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix},\ \boldsymbol{w}_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \qquad \lambda_2 = 1,\ \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \boldsymbol{w}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \lambda_3 = 4,\ \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \boldsymbol{w}_3 = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$$

Scaling $\boldsymbol{w}$ to get $w_i^t v_i = 1$, we get:

$$\implies \boldsymbol{p} = \boldsymbol{v}\boldsymbol{w} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 \end{bmatrix}$$

Problem #2   Book Problem 9.1

A single machine connected to an infinite bus has the following faulted and post-fault equations:

$$\textbf{Faulted: } 0.0133\frac{d^2\delta}{dt^2} = 0.91 \quad 0 < t \le t_{c\ell}$$
$$\textbf{PostFault: } 0.0133\frac{d^2\delta}{dt^2} = 0.91 - 3.24\sin\delta \quad t > t_{c\ell}$$

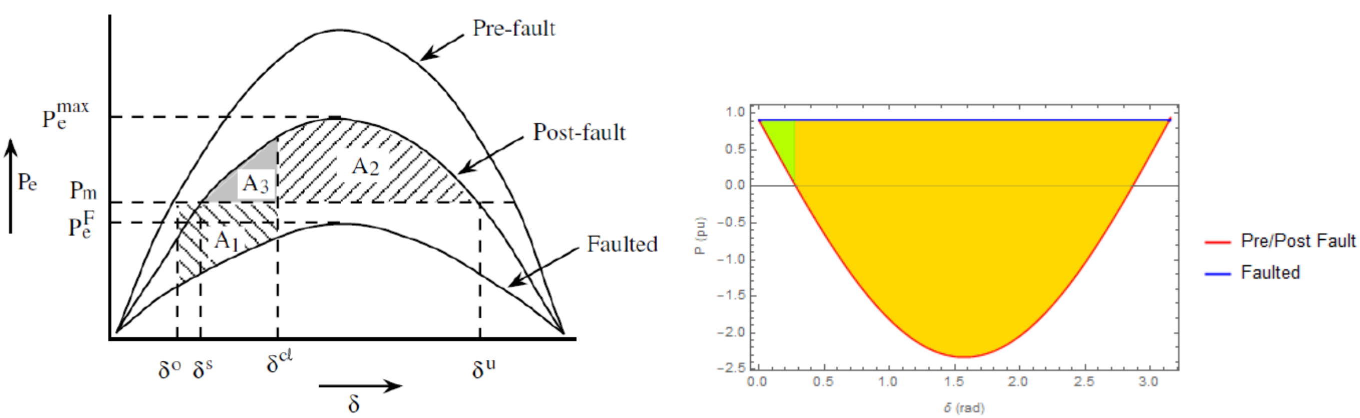
(a) Find $V(\delta,\omega)$ and V_{cr} using the u.e.p formulation. We know that $V(\delta,\omega) = \frac{1}{2}M\omega^2 + V_{pe}(\delta)$, where:

$$M = 0.0133 \qquad V_{pe}(\delta) = -P_m\delta - P_e^{max}\cos\delta \qquad P_m = 0.91 \qquad P_e^{max} = 3.24$$
$$\implies V(\delta,\omega) = 0.00665\omega^2 - 0.91\delta - 3.24\cos\delta$$
$$\implies V_{cr} = -P_m(\pi - 2\delta^s) + 2P_e^{max}\cos(\delta^s)$$
$$= 3.878$$

(b) Explain stability test of (a) using the equal-area criterion. Sketch the areas A_1, A_2, and A_3.

Using Figure 1a as our sample, we note that stability occurs when $A_1 < A_2$. However, in our case, since the faulted system is the same as the mechanical power, and our pre and post fault systems are identical, A_1 has no value, as it's the area between P_m and the faulted system. In Figure 1b, A_2 is the area in yellow, which spans from δ^s onwards, while A_3 is the area in green, which spans from $\delta = 0$ to $\delta = \delta^s$.

$$V(\delta,\omega) < 3.878$$



(a) Example Equal-Area Criterion

(b) Equal-Area Criterion for Our System

Figure 1: Equal Area Criterion Plots

(c) Find t_{cr} using V_{cr}.

To solve for t_{cr} using V_{cr} in the u.e.p method, we must find when $V(\delta,\omega) = 3.878$. This means taking closed forms for δ and ω, and evaluating $V(\delta,\omega)$ at each point.

$$\implies \omega(t) = \frac{d\delta}{dt} = \int \frac{d^2\delta}{dt^2}dt = \frac{P_m}{M}t$$
$$\implies \delta(t) = \int \frac{d\delta}{dt}dt = \frac{P_m}{2M}t^2 + \delta^s$$
$$V(\delta,\omega) = \frac{1}{2}M\omega^2 - P_m(\delta - \delta^s) - P_e^{max}(\cos\delta - \cos\delta^s)$$

Using this method, we get $t_{cr} \approx 0.215s$.

(d) Find t_{cr} using the PEBS method.

Using the PEBS method, we get the plot seen in Figure 2. Here, we look for the maximum of $V_{PE}(\theta)$, and find the time t at which this occurs for $V(\delta, \omega)$. Doing so, we get $t_{cr} \approx 0.210s$.

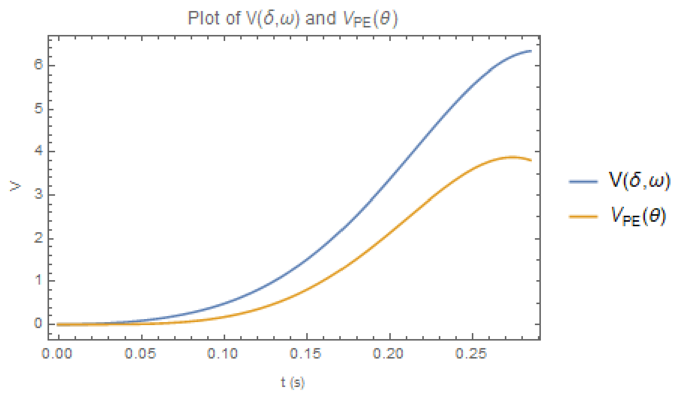


Figure 2: PEBS Plot

Problem #3

Using the **HW7_Prob3** case, we modify the Ka value, which results in the plot seen in Figure 3. Thus, the system becomes stable around a Ka value of approximately 125. When comparing the dominant mode from SMIB analysis, we see that there is a mode at 1.2845 Hz with damping of 5%. Comparing this to the modal analysis of Generator 2's rotor angle, we see somewhat close results, where we have a mode with 1.304 Hz frequency, and a damping of 4.645%.

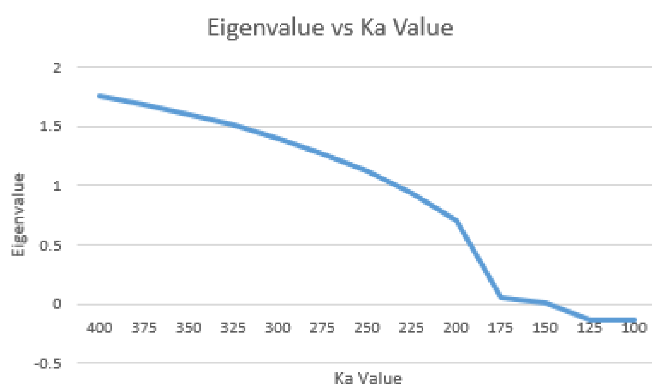


Figure 3: Plot of Eigenvalues vs Ka value

Problem #4

Using the **HW7_Prob4** case, we change the H values of the generators to the values seen in Table 1a. Performing the Iterative Matrix Pencil method with 4 iterations results in the modes seen in Table 1b.

Table 1: Tables for Problem #4

(a) Inertia Values for Generators

Gen #	UIN Digit	H Value
1	8	8
2	0	10
3	5	5
4	0	10

(b) Frequency Modes

Frequency (Hz)	Damping (%)
5.01	59.116
0.023	17.532

Problem #5

We begin by manipulating a case in which there are no stabilizers in the system, and changing the generator inertia constants based on my UIN, 625008050. If we look at the modes of the rotor angles using the Matrix Pencil method, we get the modes seen in Table 2a. From this, we select the 0.915 Hz mode, with 4.918% damping. Then, we applying a SIGNALSTAB stabilizer to Generator 2, and set the input signal to have a magnitude of 0.05 pu, at a frequency of 0.915 Hz. Applying this to the case, we perform modal analysis on the statistics for Generator 2, and get the angles seen in Table 2b. From here, we need to get the angle necessary to damp out that single, and use it to calculate α.

Table 2: Tables for Problem #5

(a) Initial System Modes

Frequency (Hz)	Damping (%)
1.226	11.341
0.915	4.918
0.063	65.711
0	-100

(b) Angles for SIGNALSTAB

Angles (°)	
Gen 2 Vpu	19.493
Gen 2 Vstab	118.085
Gen 2 MW	130
Gen 2 Speed	-138.596

(c) Final Frequency Modes

Frequency (Hz)	Damping (%)
1.055	13.054
0.919	31.788
0.195	72.805
0.036	25.629

From Table 2b, we get that we need to have approximately 76.681° of compensation. Split between two lead-lag blocks, we get that $\phi = 38.3405°$.

$$\alpha = \frac{1 - \sin\phi}{1 + \sin\phi} = 0.234$$
$$\implies T_1 = \frac{1}{2\pi f\sqrt{\alpha}} = 0.2682$$
$$\implies T_2 = \alpha T_1 = 0.0628$$

Finally, we also have to tune K_s, which we get to be about 40. This is from the fact that a K_s value of 120 results in almost instability, and we divide that by 3. Taking all these values into account, we get the results in Table 2c, which shows our frequency now has damping of about 31.788%.

Problem #6

The approximate minimum of stabilizers that need to be enabled for a stable response is roughly 5. This is geographically dependent, however. In order to test this, all of the stabilizers in the Far West region were enabled, which showed a stable response. Each stabilizer was then deactivated one by one until we reached an unstable response. At 5 stabilizers enabled, the resulting frequency behavior is seen in Figure 4. However, upon enabling all of the stabilizers in the northern region while disabling those in the Far West, we do not get a stable response, indicating some sort of geographical dependence.

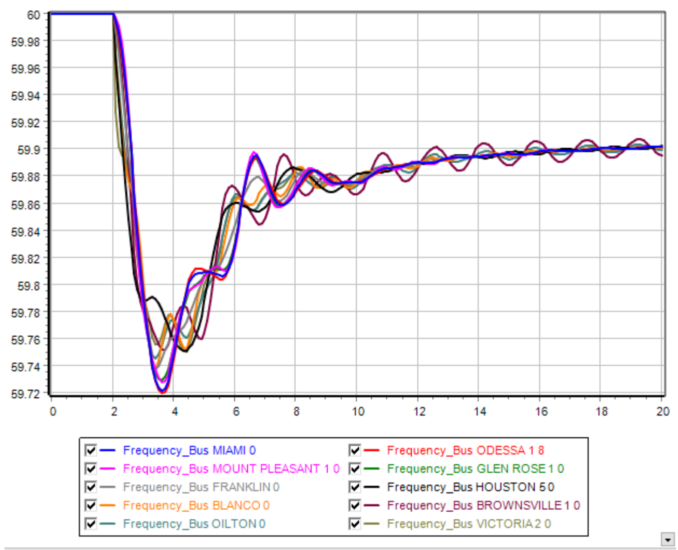


Figure 4: Frequency Response with 5 Stabilizers