Homework 1. Solutions.

2. The following nonlinear equations contain terms that are often found in the power flow equations.

$f_1(\mathbf{x}) = 10\, x_1 \sin x_2 + 1.2 = 0$

$f_2(\mathbf{x}) = 10\,(x_1)^2 - 10\, x_1 \cos x_2 + 0.4 = 0$

Using the Newton-Raphson method, determine a solution. Start with an initial guess of $x_1(0) = 1$ and $x_2(0) = 0$ radians, and a stopping criteria of $\varepsilon = 10^{-4}$.

Question 2

$$f_1(x) = 10x_1 \sin x_2 + 1.2 = 0$$

$$f_2(x) = 10(x_1)^2 - 10x_1\cos(x_2) + 0.4 = 0$$

initial guess $\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (radians)

mismatch (tolerance) for power flow eqns $= 10^{-4}$

Newton Raphson method applied to power flow equations:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{new} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{old} - \left[J(x^0)\right]^{-1} \cdot \left[f(x^0)\right] \quad — (1)$$

where

$[J(x)]$ is the jacobian matrix.

$$[f(x)] = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$$

$$= \begin{bmatrix} 10x_1 \sin x_2 + 1.2 \\ 10x_1^2 - 10x_1\cos x_2 + 0.4 \end{bmatrix} \quad — (2)$$

$$[J(x)] = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 10\sin(x_2) & 10x_1\cos(x_2) \\ 20x_1 - 10\cos(x_2) & 10x_1\sin(x_2) \end{bmatrix} \quad — (3)$$

Applying (2), (3) and initial conditions in (1);

iteration 1:

$$x^{new} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{new} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}\begin{bmatrix} 1.2 \\ 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.96 \\ -0.12 \end{bmatrix}$$

$$\text{mismatch} = f(x^{new}) = \begin{bmatrix} 0.0507 \\ 0.0850 \end{bmatrix}$$

mismatch > tolerance

hence we do iteration 2.

Iteration 2:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{new} = \begin{bmatrix} 0.96 \\ -0.12 \end{bmatrix} - \begin{bmatrix} 0.0132 & 0.10956 \\ 0.1065 & 0.0137 \end{bmatrix}\begin{bmatrix} 0.0507 \\ 0.08503 \end{bmatrix}$$

$$= \begin{bmatrix} 0.950 \\ -0.126 \end{bmatrix}$$

$$\text{mismatch} = \begin{bmatrix} 0.00067 \\ 0.00112 \end{bmatrix}$$

Since mismatch > tolerance

Iteration 3:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^{new} = \begin{bmatrix} 0.95 \\ -0.126 \end{bmatrix} - \begin{bmatrix} 0.0142 & 0.112 \\ 0.107 & 0.015 \end{bmatrix} \begin{bmatrix} 0.00616 \\ 0.00031 \end{bmatrix}$$

$$= \begin{bmatrix} 0.9498 \\ -0.1266 \end{bmatrix}$$

$$\text{mismatch} = \begin{bmatrix} 1.0831 \times 10^{-6} \\ 2.1645 \times 10^{-6} \end{bmatrix}$$

mismatch < tolerance

$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0.949 \\ -0.126 \end{bmatrix}$ is the solution for given non-linear equations.

Homework 1 Solutions.

question 1

1. Use the Newton-Raphson method to find one solution to the polynomial equation $f(x) = x^3 - 9x^2 - 14x - 20 = 0$. Start with an intial guess of 0 and continue until the mismatch is below a tolerance of $\varepsilon = 0.001$.

$f(x) = x^3 - 9x^2 - 14x - 20 = 0$

initial guess $x^0 = 0$

mismatch tolerance $= 0.001$

General form of problem:

find $\hat{x}$ such that $f(\hat{x}) = 0$

let x^v be the guess of $\hat{x}$

then $\Delta x^v = \hat{x} - x^v$

using taylor series expansion;

$f(\hat{x}) = f(x^v) + \frac{df(x^v)}{dx} \cdot \Delta x^v + \frac{1}{2} \frac{d^2 f(x^v)}{dx^2} (\Delta x^v)^2 +$ higher order terms.

neglecting all terms having degree greater than 1.
(i.e. linearizing about x^v)

$$f(\hat{x}) = 0 \approx f(x^v) + \frac{df(x^v)}{dx} \cdot \Delta x^v$$

$$\therefore \Delta x^v = -\left[\frac{df(x^v)}{dx}\right]^{-1} \cdot f(x^v)$$

and $x^{v+1} = x^v + \Delta x^v$

$$f(x) = x^3 - 9x^2 - 14x - 20$$

$$\frac{df(x)}{dx} = 3x^2 - 18x - 14$$

iteration 0 ($v=0$)

$$x^0 = 0$$

$$f(x^0) = -20$$

$$\Delta x^1 = \text{mismatch} = \cancel{-}\frac{1}{\cancel{-}14}(-20) = -1.4286$$

iteration 1 ($v=1$)

$$x^1 = x^0 + \Delta x^1$$
$$= 0 - 1.4286$$
$$= -1.4286$$

$$f(x^1) = (-1.4286)^3 - 9(-1.4286) - 14(-1.4286) - 20$$

$$= -21.2827$$

mismatch $= -1.42857$ > tolerance

(∴ we go for iteration 3)

calculations of x^{ν}, $f(x^{\nu})$ and Δx^{ν} is done for multiple iterations till mismatch < tolerance value.

Below table provides respective values for each iteration upto iteration number 14.

ν	x^{ν}	$f(x^{\nu})$	Δx^{ν}
Iteration 0	x = 0	f(x) = -20	dx = -1.42857
Iteration 1	x = -1.42857	f(x) = -21.28277	dx = -1.42857
Iteration 2	x = -0.23537	f(x) = -17.21645	dx = 1.1932
Iteration 3	x = -2.02928	f(x) = -37.0084	dx = -1.79391
Iteration 4	x = -0.96829	f(x) = -15.79006	dx = 1.06099
Iteration 5	x = 1.56137	f(x) = -59.99364	dx = 2.52966
Iteration 6	x = -0.16303	f(x) = -17.96112	dx = -1.7244
Iteration 7	x = -1.79798	f(x) = -29.73526	dx = -1.63495
Iteration 8	x = -0.73835	f(x) = -14.97207	dx = 1.05963
Iteration 9	x = 15.43403	f(x) = 1296.56925	dx = 16.17238
Iteration 10	x = 12.36752	f(x) = 321.9356	dx = -3.06651
Iteration 11	x = 10.919	f(x) = 55.92593	dx = -1.44852
Iteration 12	x = 10.53889	f(x) = 3.37729	dx = -0.38011
Iteration 13	x = 10.51281	f(x) = 0.01517	dx = -0.02608
Iteration 14	x = 10.51269	f(x) = -0.00023	dx = -0.00012

after 14 iterations,

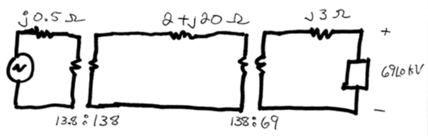
$x^{14} = 10.51$

and $|f(x)| = 0.00023$

Homework 1 Solutions.

question 3.

3. Assume the below diagram models a balanced three-phase system in which a 100 + j50 MVA load (total for all three phases) is supplied at 69 kV (line-to-line). First, redraw the network using a per unit representation with a 100 MVA base, and a 69 kV voltage base for the load. Then solve the circuit to determine how much real and reactive power is being supplied by the generator (source) on the left?



Load for all 3 phases = 100 + j50 MVA

Supply voltage = 69 kV

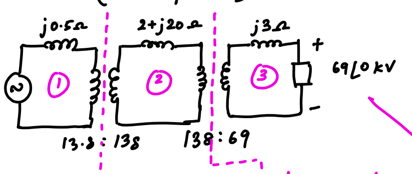
base MVA = 100 MVA

base voltage = 69 kV

→ convert network to pu

→ find real (P) and reactive (Q) power:

network (from question)



$S_L = 100 + j50$ MVA

$S_B = 100$ MVA

$Z_B = \frac{V_B^2}{S_B}$ (⇒ V_B in kV)

$V_{B1} = V_{B2} \times \frac{13.8\text{kV}}{138\text{ kV}} = \cancel{138} \times \frac{13.8\text{ kV}}{\cancel{138}\text{ kV}} = 13.8\text{ kV}$

$Z_{B1} = \frac{V_{B1}^2}{S_B} = 1.9\,\Omega$

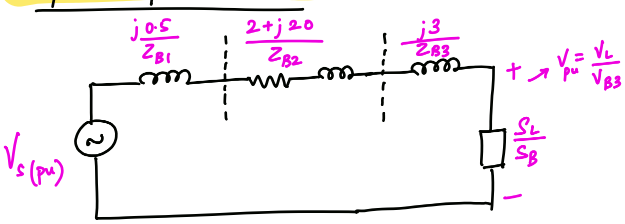
$V_{B2} = V_{B3} \times \frac{138\text{ kV}}{69\text{ kV}} = \cancel{69} \times \frac{138\text{ kV}}{\cancel{69}\text{ kV}} = 138\text{ kV}$

$Z_{B2} = \frac{V_{B2}^2}{S_B} = 190.4\,\Omega$

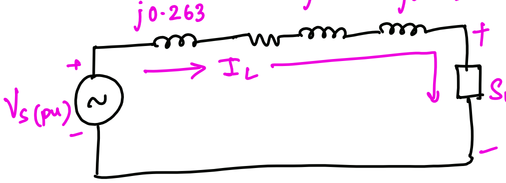
$V_{B3} = 69\text{ kV}$

$Z_{B3} = \frac{V_{B3}^2}{S_B} = 47.6\,\Omega$

p.u. representation:



that is,



$V_{L(pu)} = 1$ pu (since system $V_B = 69$ kV)

$S_{L(pu)} = 1 + j0.5$ (since system $S_B = 100$ MVA)

applying KVL;

$$V_{s(pu)} = V_{L(pu)} + I_{L(pu)}(Z_{total})$$

where;

$$I_{L(pu)} = \left(\frac{S_{L(pu)}}{V_{L(pu)}}\right)^* \quad (\text{since } S = VI^*)$$

$$= \left(\frac{1 + j0.5}{1\angle 0}\right)^* = 1 - j0.5 \text{ pu}$$

using KVL equation;

$$V_{s(pu)} = 1\angle 0 + (1-j0.5)(j0.263 + 0.01 + j0.105 + j0.063)$$

$$= 1\angle 0 + (1-j0.5)(0.01 + j0.431)$$

$$= 1.225 + j0.426 \text{ pu}.$$

power generated by the source;

Apparent power

$$S_{s(pu)} = V_{s(pu)} \cdot I^{*}_{L(pu)}$$

$$= (1.225 + j0.426)(1-j0.5)^{*}$$

$$= 1.0125 + j1.0387 \text{ pu}$$

$$S_{s(pu)} = P_{s(pu)} + Q_{s(pu)}$$

$$\therefore P_{s(pu)} = 1.0125 \text{ pu} \approx 101.25 \text{ MW} \quad (\text{since } S_B = 100 \text{ MVA})$$

$$Q_{s(pu)} = 1.0387 \text{ pu} \approx 103.87 \text{ MVar}$$