ECEN 615
Methods of Electric Power Systems Analysis

Lecture 13: Power Flow Sensitivities

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Announcements

- Read Chapter 7
- Homework 4 should be done before the first exam, but need not be turned in
- Exam 1 is during class on Oct 13
  - Closed book, notes. One 8.5 by 11 inch notesheet and calculators allowed
  - Distance education students should make arrangements with Sanjana with HonorLock one approach
  - Exam 1 from 2020 is available in Canvas; solutions will be posted as we get closer in

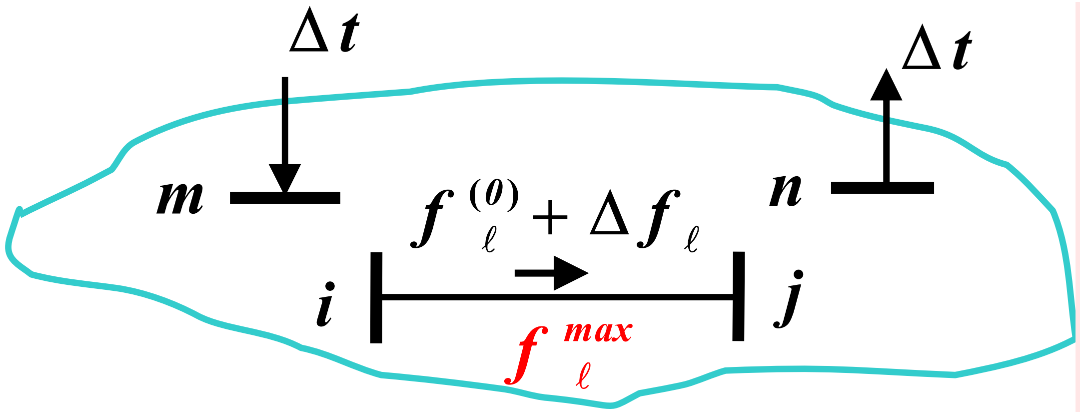
Power Flow Sensitivity Analysis

- The idea of power flow sensitivity analysis is to get an estimate of how some set of values would change with respect to a change in a set of control values
  - Need to keep in mind which control responses are implicitly modeled, such as P and Q changes at the slack, Q at PV buses
- The approach works by linearizing a system about an operating point; its usefulness depends on the validity of this approximation
- Sensitivities are widely used in power system analysis, with some algorithms doing sequential linearizations
  - They are most valid for real power, less useful for reactive power

Analysis Example: Available Transfer Capability

- The power system available transfer capability or ATC is defined as the maximum additional MW that can be transferred between two specific areas, while meeting all the specified pre- and post-contingency system conditions
- ATC impacts measurably the market outcomes and system reliability and, therefore, the ATC values impact the system and market behavior
- Total transfer capability (TTC)
  - Amount of real power that can be transmitted across an interconnected transmission network in a reliable manner, including considering contingencies
- ATC is the amount that is actually available; we'll just look at TTC
  - A useful reference on ATC is *Available Transfer Capability Definitions and Determination* from NERC, June 1996 (available online)

Total Transfer Capability (TTC) Evaluation



$$U_{m,n} = \max \ \Delta t$$

s.t.

$$f_\ell^{(j)} + \Delta f_\ell \leq f_\ell^{max} \quad \forall \ell \in L$$

for the base case j = 0 and each contingency case

$$j = 1,2 \ \ldots \ , J$$

Goal is to load the lines up to their limits, though only when also considering contingencies

Conceptual Solution Algorithm

1. Solve the initial power flow, corresponding to the initial system dispatch (i.e., existing commitments); set the change in transfer $\Delta t^{(0)} = 0$, $k=0$; set step size δ; j is used to indicate either the base case ($j=0$) or a contingency, $j= 1,2,3\ldots J$
2. Compute $\Delta t^{(k+1)} = \Delta t^{(k)} + \delta$
3. Solve the power flow for the new $\Delta t^{(k+1)}$
4. Check for limit violations: if violation is found set $U^{j}_{m,n} = \Delta t^{(k)}$ and stop; else set $k=k+1$, and goto 2

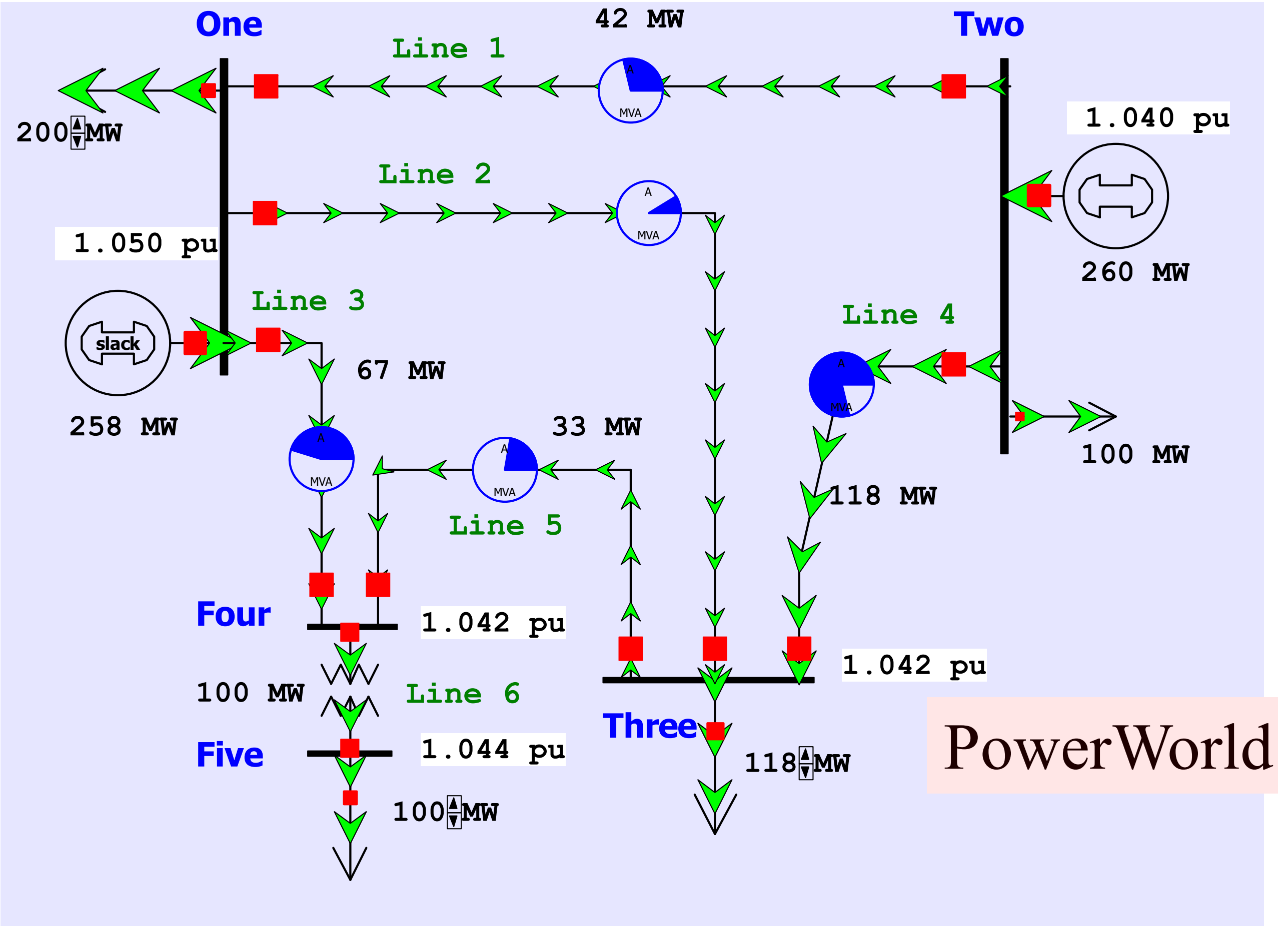
Conceptual Solution Algorithm, cont.

- This algorithm is applied for the base case (j=0) and each specified contingency case, j=1,2,..J
- The final TTC, $U_{m,n}$ is then determined by

$$U_{m,n} = \min_{0 \leq j \leq J} \left\{ U_{m,n}^{(j)} \right\}$$

- This algorithm can be easily performed on parallel processors since each contingency evaluation is independent of the others

Five Bus Example: Reference



PowerWorld Case: **B5_DistFact**

Five Bus Example: Reference

ℓ	i	j	g_ℓ	b_ℓ	$f_\ell^{max}\ (MW)$
ℓ_1	1	2	0	6.25	150
ℓ_2	1	3	0	12.5	400
ℓ_3	1	4	0	12.5	150
ℓ_4	2	3	0	12.5	150
ℓ_5	3	4	0	12.5	150
ℓ_6	4	5	0	10	1,000

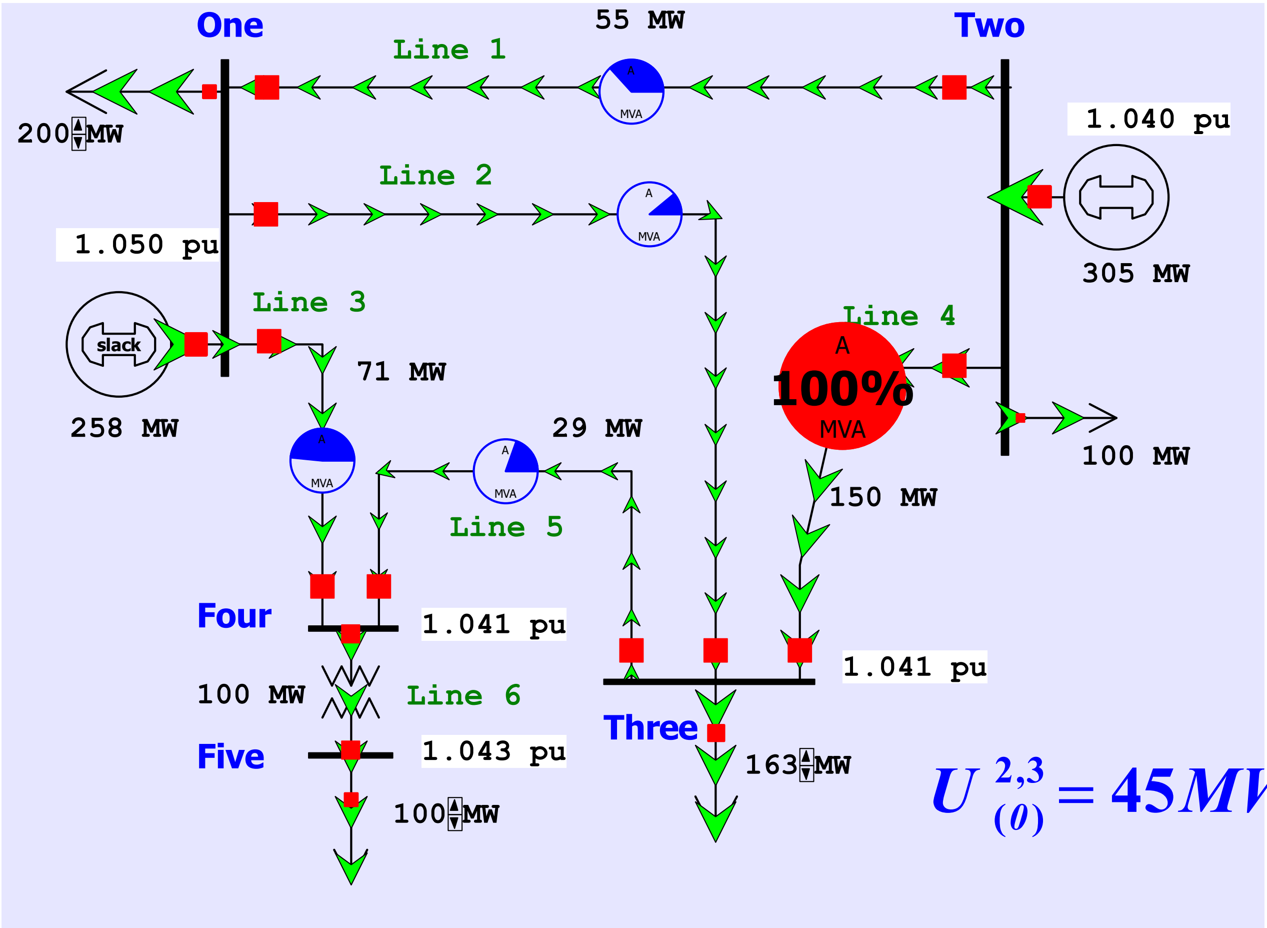
Five Bus Example

- We evaluate $U_{2,3}$ using the previous procedure
  - Gradually increase generation at Bus 2 and load at Bus 3
- We consider the base case and the single contingency with line 2 outaged (between 1 and 3): $J = 1$
- Simulation results show for the base case that

$$U_{2,3}^{(0)} = 45\ MW$$

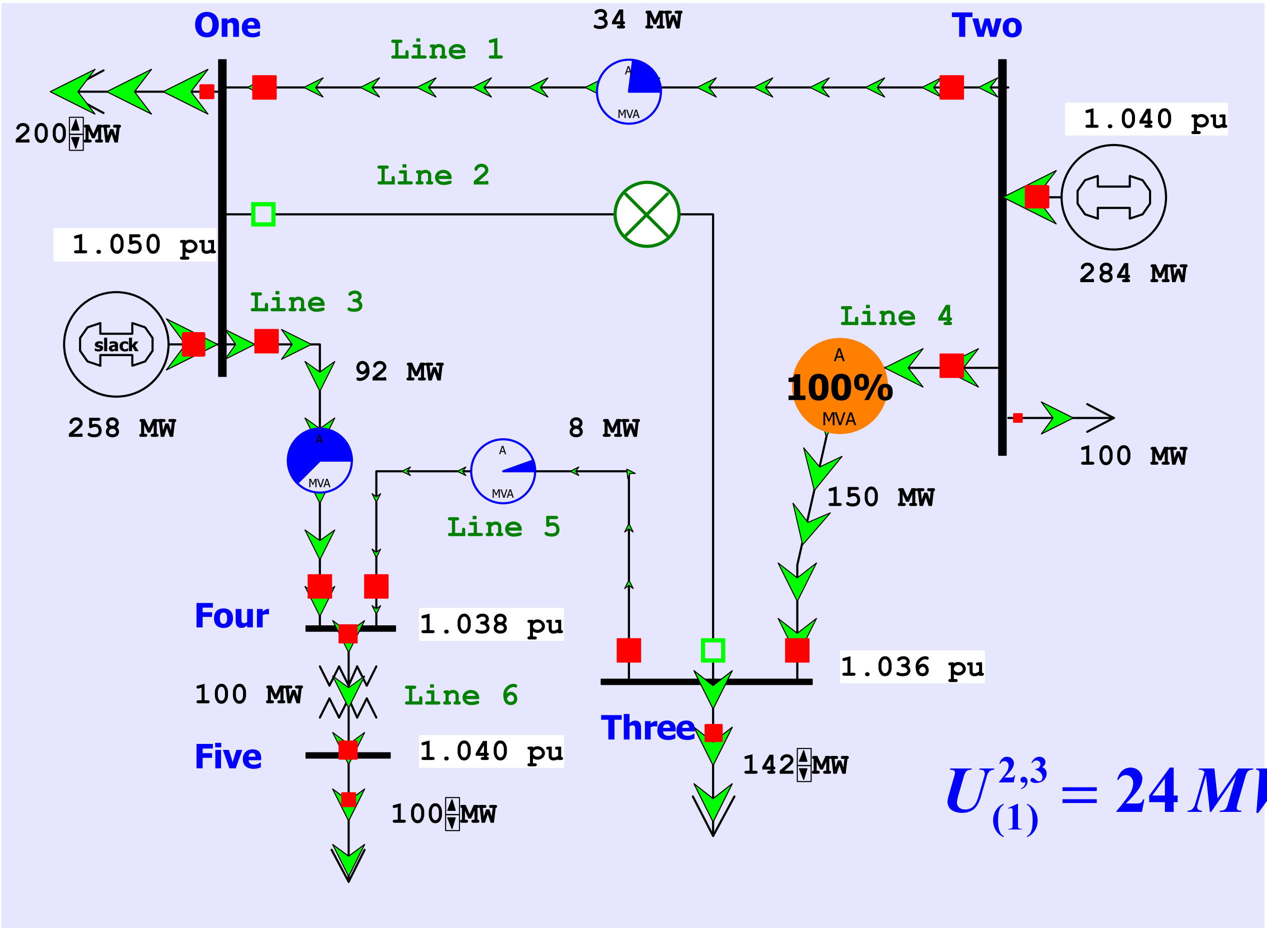
- And for the contingency that $U_{2,3}^{(1)} = 24\ MW$
- Hence $U_{2,3} = min\left\{U_{2,3}^{(0)}, U_{2,3}^{(1)}\right\} = 24\ MW$

Five Bus: Maximum Base Case Transfer



$$U^{2,3}_{(0)} = 45MW$$

Five Bus: Maximum Contingency Transfer



$$U_{(1)}^{2,3} = 24\,MW$$

Computational Considerations

- Obviously such a brute force approach can run into computational issues with large systems
- Consider the following situation:
  - 10 iterations for each case
  - 6,000 contingencies
  - 2 seconds to solve each power flow
- It will take over 33 hours to compute a single UTC for the specified transfer direction from m to n.
- Consequently, there is an acute need to develop fast tools that can provide satisfactory estimates

Sensitivity Problem Formulation

- Denote the system state by

$$\mathbf{x} \; @ \begin{bmatrix} \boldsymbol{\theta} \\ \mathbf{V} \end{bmatrix} \qquad \boldsymbol{\theta} \; @ \; [\theta_1, \theta_2, \cdots, \theta_N]^T \qquad \mathbf{V} \; @ \; [V_1, V_2, \cdots, V_N]^T$$

The **V** values are the voltage magnitudes

- Denote the conditions corresponding to the existing commitment/dispatch by $\mathbf{s}^{(0)}$, $\mathbf{p}^{(0)}$ and $\mathbf{f}^{(0)}$ so that

$$\begin{cases} \mathbf{g}(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) = \mathbf{0} \\ \mathbf{f}^{(0)} = \mathbf{h}(\mathbf{x}^{(0)}) \end{cases}$$

the power flow equations

line real power flow vector

- Define the angle difference as $\boldsymbol{\theta}_{jk} \, @ \, \theta_j - \theta_k$

Sensitivity Problem Formulation

$$\mathbf{g}(\mathbf{x},\mathbf{p}) = \begin{bmatrix} \mathbf{g}^{P}(\mathbf{x},\mathbf{p}) \\ \mathbf{g}^{Q}(\mathbf{x},\mathbf{p}) \end{bmatrix}$$

g includes the real and reactive power balance equations

$$g_k^P\ (\underline{s},\underline{p}) = V_k \sum_{m=1}^{N} \Big(V_m \big[G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km} \big] \Big) - p_k$$

$$g_k^Q (\underline{s},\underline{p}) = V_k \sum_{m=1}^{N} \Big(V_m \big[G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km} \big] \Big) - q_k$$

$$h_\ell(\underline{s}) = g_\ell \Big[\big(V_i\big)^2 - V_i V_j \cos\theta_{ij} \Big] - b_\ell V_i V_j \sin\theta_{ij} \ , \ell = \big(i, j\big)$$

Sensitivity Problem Formulation

- For a small change, $\Delta\mathbf{p}$, that moves the injection from $\mathbf{p}^{(0)}$ to $\mathbf{p}^{(0)} + \Delta\mathbf{p}$, we have a corresponding change in the state $\Delta\mathbf{x}$ with

$$\mathbf{g}\,(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \boldsymbol{p}^{(0)} + \Delta\mathbf{p}) = \mathbf{0}$$

- We then apply a first order Taylor's series expansion

$$\mathbf{g}\left(\mathbf{x}^{(0)} + \Delta\mathbf{x}, \mathbf{p}^{(0)} + \Delta\mathbf{p}\right) = \mathbf{g}\left(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}\right) + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\mathbf{x}$$

$$+ \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\boldsymbol{p} + \boldsymbol{h.o.t.}$$

Sensitivity Problem Formulation

- We consider this to be a "small signal" change, so we can neglect the higher order terms (h.o.t.) in the expansion
- Hence we should still be satisfying the power balance equations with this perturbation; so

$$\left.\frac{\partial \mathbf{g}}{\partial \mathbf{x}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta\mathbf{x} + \left.\frac{\partial \mathbf{g}}{\partial \mathbf{p}}\right|_{\left(\mathbf{x}^{(0)}\mathbf{p}^{(0)}\right)} \Delta \boldsymbol{p} \approx \mathbf{0}$$

Sensitivity Problem Formulation

- Also, from the power flow equations, we obtain

$$\frac{\partial \mathbf{g}}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{p}} \\ \hdashline \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{p}} \end{bmatrix} = \begin{bmatrix} -\mathbf{I} \\ \hdashline \mathbf{0} \end{bmatrix}$$

and then just the power flow Jacobian

$$\frac{\partial \mathbf{g}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{g}^{P}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{P}}{\partial \mathbf{V}} \\ \dfrac{\partial \mathbf{g}^{Q}}{\partial \boldsymbol{\theta}} & \dfrac{\partial \mathbf{g}^{Q}}{\partial \mathbf{V}} \end{bmatrix} = \mathbf{J}(\mathbf{x},\mathbf{p})$$

Sensitivity Problem Formulation

- With the standard assumption that the power flow Jacobian is nonsingular, then

$$\Delta \mathbf{x} \approx \left[\mathbf{J}(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) \right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

- We can then compute the change in the line real power flow vector

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right]^{T} \Delta \mathbf{s} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}} \right]^{T} \left[J(\mathbf{x}^{(0)}, \mathbf{p}^{(0)}) \right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

Sensitivity Comments

- Sensitivities can easily be calculated even for large systems
  - If $\Delta\mathbf{p}$ is sparse (just a few injections) then we can use a fast forward; if sensitivities on a subset of lines are desired, we could also use a fast backward
- Sensitivities are dependent upon the operating point
  - They also include the impact of marginal losses
- Sensitivities could easily be expanded to include additional variables in $\mathbf{x}$ (such as phase shifter angle), or additional equations, such as reactive power flow

Sensitivity Comments, cont.

- Sensitivities are used in the optimal power flow; in that context a common application is to determine the sensitivities of an overloaded line to injections at all the buses
- In the below equation, how could we quickly get these values?

$$\Delta \mathbf{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \mathbf{x}}\right]^T \Delta \boldsymbol{f} \approx \left[\frac{\partial \mathbf{h}}{\partial \boldsymbol{x}}\right]^T \left[\boldsymbol{J}(\mathbf{x}^{(\boldsymbol{0})}, \mathbf{p}^{(\boldsymbol{0})})\right]^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p}$$

  – A useful reference is O. Alsac, J. Bright, M. Prais, B. Stott, "Further Developments in LP-Based Optimal Power Flow," IEEE. Trans. on Power Systems, August 1990, pp. 697-711; especially see equation 3.

Sensitivity Example in PowerWorld

- Open case **B5_DistFact** and then Select **Tools, Sensitivities, Flow and Voltage Sensitivities**
  - Select **Single Meter, Multiple Transfers**, **Buses** page
  - Select the **Device Type (Line/XFMR)**, **Flow Type (MW)**, then select the line (from Bus 2 to Bus 3)
  - Click **Calculate Sensitivities**; this shows impact of a single injection going to the slack bus (Bus 1)
  - For our example of a transfer from 2 to 3 the value is the result we get for bus 2 (0.5440) minus the result for bus 3 (-0.1808) = 0.7248
  - With a flow of 118 MW, we would hit the 150 MW limit with (150-118)/0.7248 =44.1MW, close to the limit we found of 45MW

Sensitivity Example in PowerWorld

- If we change the conditions to the anticipated maximum loading (changing the load at 2 from 118 to 118+44=162 MW) and we re-evaluate the sensitivity we note it has changed little (from -0.7248 to -0.7241)
  - Hence a linear approximation (at least for this scenario) could be justified
- With what we know so far, to handle the contingency situation, we would have to simulate the contingency, and reevaluate the sensitivity values
  - We'll be developing a quicker (but more approximate) approach next

Linearized Sensitivity Analysis

- By using the approximations from the fast decoupled power flow we can get sensitivity values that are independent of the current state. That is, by using the **B**' and **B**'' matrices
- For the real power line flow we can approximate

$$h_\ell(\underline{s}) = g_\ell\left[\left(V_i\right)^2 - V_i V_j \cos\theta_{ij}\right] - b_\ell V_i V_j \sin\theta_{ij}, \ell = \left(i, j\right)$$

By using the FDPF appxomations

$$h_\ell(\underline{s}) \approx -b_\ell\theta_{ij} = \frac{\theta_{ij}}{X_\ell}, \; \ell = \left(i, j\right)$$

Linearized Sensitivity Analysis

- Also, for each line ⍰

$$\frac{\partial h_\ell}{\partial \boldsymbol{\theta}} \approx -b_\ell \mathbf{a}_\ell \qquad\qquad \frac{\partial h_\ell}{\partial \mathbf{V}} \approx \mathbf{0}$$

and so,

$$\frac{\partial \mathbf{h}}{\partial \mathbf{x}} = \begin{bmatrix} \dfrac{\partial \mathbf{h}}{\partial \boldsymbol{\theta}} \\[2ex] \dfrac{\partial \mathbf{h}}{\partial \mathbf{V}} \end{bmatrix} = -\left[\begin{array}{c:c:c} b_{\ell_1}\mathbf{a}_1 & \cdots & b_{\ell_L}\mathbf{a}_L \\ & \cdots & \\ \hdashline \mathbf{0} & & \mathbf{0} \end{array}\right] \approx \left[\begin{array}{c} \mathbf{A}^T\tilde{\mathbf{B}} \\ \hdashline \mathbf{0} \end{array}\right]$$

Sensitivity Analysis: Recall the Matrix Notation

- The series admittance of line ℓ is $g_{\ell} + jb_{\ell}$ and we define

$$\tilde{\mathbf{B}} \; @ \; -\,diag\left\{b_1, b_2, \cdots, b_L\right\}$$

- We define the L×N incidence matrix

$$\mathbf{A} \; @ \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \vdots \\ \mathbf{a}_L^T \end{bmatrix}$$

where the component j of $\mathbf{a}_i$ is nonzero whenever line ℓ_i is coincident with node j. Hence **A** is quite sparse, with at most two nonzeros per row

Linearized Active Power Flow Model

- Under these assumptions the change in the real power line flows are given as

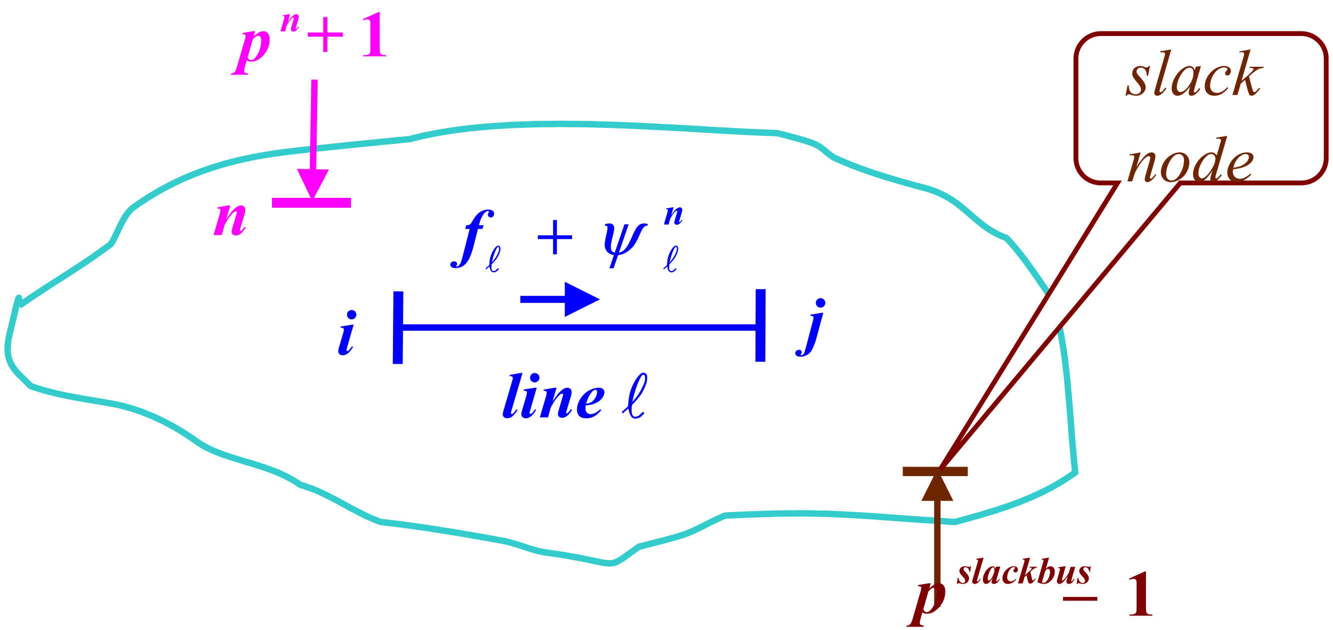
$$\Delta \mathbf{f} \approx \begin{bmatrix} \tilde{\mathbf{B}}\,\mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{B}' & \mathbf{0} \\ \mathbf{0} & \mathbf{B}'' \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix} \Delta \mathbf{p} = \underbrace{\tilde{\mathbf{B}}\,\mathbf{A}\left[\mathbf{B}'\right]^{-1} \Delta \mathbf{p}} = \boldsymbol{\Psi} \Delta \mathbf{p}$$

- The constant matrix $\boldsymbol{\Psi} \quad @ \quad \tilde{\mathbf{B}}\,\mathbf{A}\left[\mathbf{B}'\right]^{-1}$ is called the injection shift factor matrix (ISF)

Injection Shift Factors (ISFs)

- The element ψ_{ℓ}^{n} in row ⍰ and column n of ⍰ is called the injection shift factor (*ISF)* of line ⍰ with respect to the injection at node *n*
  - Absorbed at the slack bus, so it is slack bus dependent
- Terms generation shift factor (GSF) and load shift factor (LSF) are also used (such as by NERC)
  - Same concept, just a variation in the sign whether it is a generator or a load
  - Sometimes the associated element is not a single line, but rather a combination of lines (an interface)
- Terms used in North America are defined in the NERC glossary (http://www.nerc.com/files/glossary_of_terms.pdf)

ISF Interpretation



ψ_ℓ^n is the fraction of the additional 1 *MW* injection at node n that goes though line ℓ

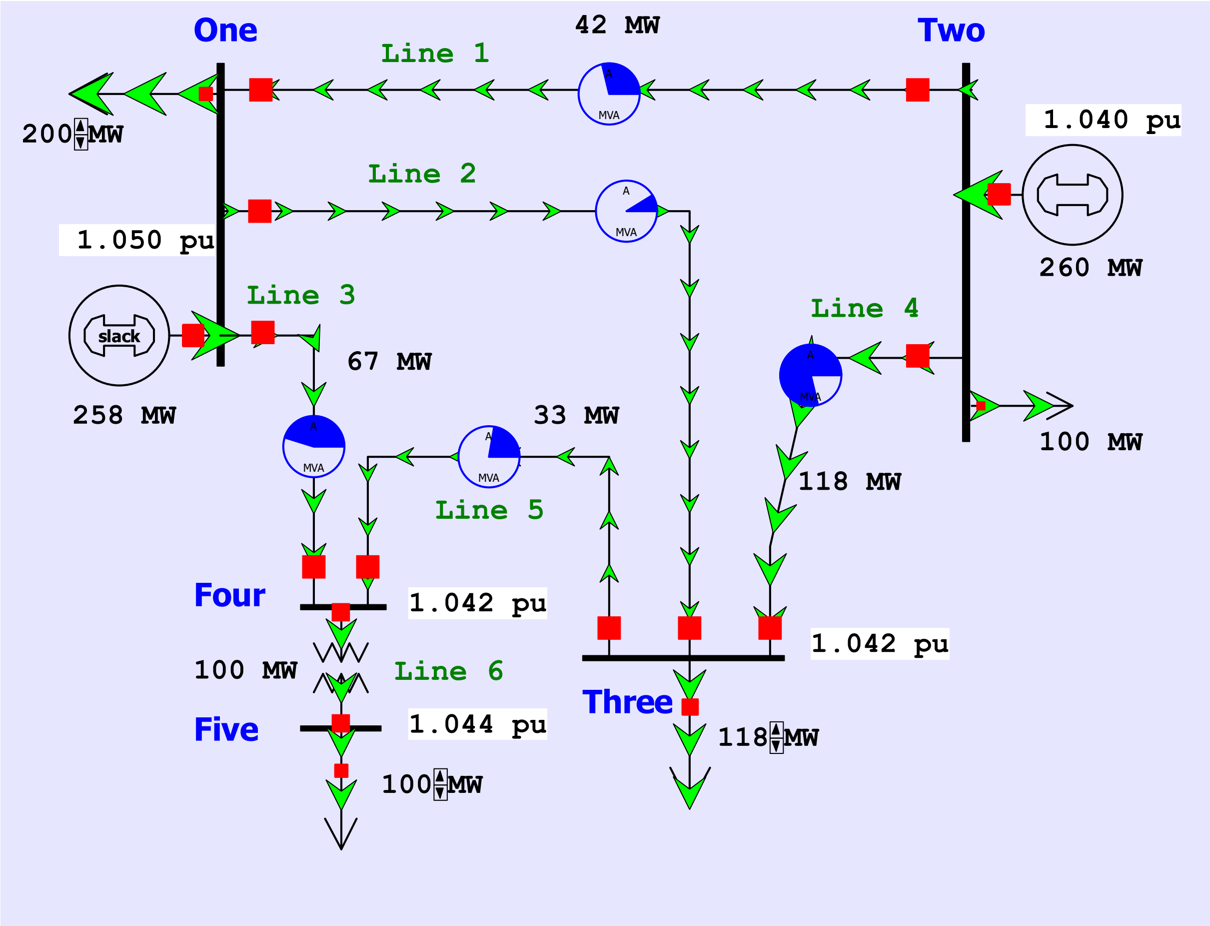
ISF Properties

- By definition, ψ_{ℓ}^{n} depends on the location of the slack bus
- By definition, $\psi_{\ell}^{slackbus} \equiv \boldsymbol{0}$ for $\forall \ell \in L$ since the injection and withdrawal buses are identical in this case and, consequently, no flow arises on any line ▯
- The magnitude of ψ_{ℓ}^{n} is at most 1 since

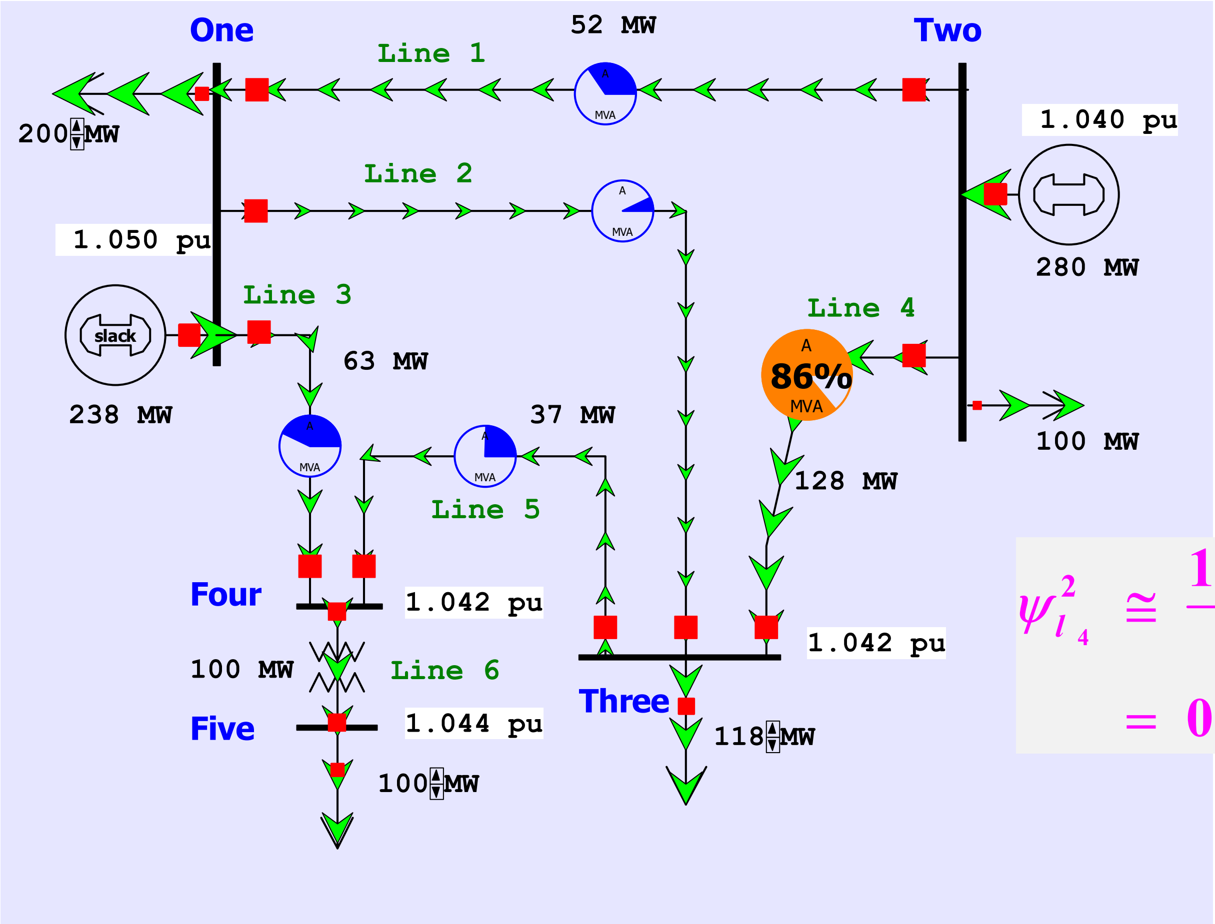
$$-\mathbf{1} \leq \psi_{\ell}^{n} \leq \mathbf{1}$$

Note, this is strictly true only for the linear (lossless) case. In the nonlinear case, it is possible that a transaction decreases losses. Hence a 1 MW injection could change a line flow by more than 1 MW.

Five Bus Example Reference



Five Bus ISF, Line 4, Bus 2 (to Slack)



$$\psi_{l_4}^{2} \cong \frac{128-118}{20} = 0.5$$

Five Bus Example

$$\tilde{\mathbf{B}} = -\,diag\{\mathbf{6.25}, \quad \mathbf{12.5}, \quad \mathbf{12.5}, \quad \mathbf{12.5}, \quad \mathbf{12.5}, \quad \mathbf{10}\}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{-1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{-1} & \mathbf{0} \\ \mathbf{1} & \mathbf{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-1} \end{bmatrix}$$

The row of **A** correspond to the lines and transformers, the columns correspond to the non-slack buses (buses 2 to 5); for each line there is a 1 at one end, a -1 at the other end (hence an assumed sign convention!). Here we put a 1 for the lower numbered bus, so positive flow is assumed from the lower numbered bus to the higher number

Five Bus Example

$$\mathbf{B}' = \mathbf{A}^T \tilde{\mathbf{B}}\,\mathbf{A} = \begin{bmatrix} -18.75 & 12.5 & 0 & 0 \\ 12.5 & -37.5 & 12.5 & 0 \\ 0 & 12.5 & -35 & 10 \\ 0 & 0 & 10 & -10 \end{bmatrix}$$

$$\underline{\Psi} = \underline{\tilde{B}}\,\underline{A}\left[\underline{B}'\right]^{-1} = \begin{bmatrix} -0.4545 & -0.1818 & -0.0909 & -0.0909 \\ -0.3636 & -0.5455 & -0.2727 & -0.2727 \\ -0.1818 & -0.2727 & -0.6364 & -0.6364 \\ 0.5455 & -0.1818 & -0.0909 & -0.0909 \\ 0.1818 & 0.2727 & -0.3636 & -0.3636 \\ 0 & 0 & 0 & -1.0000 \end{bmatrix}$$

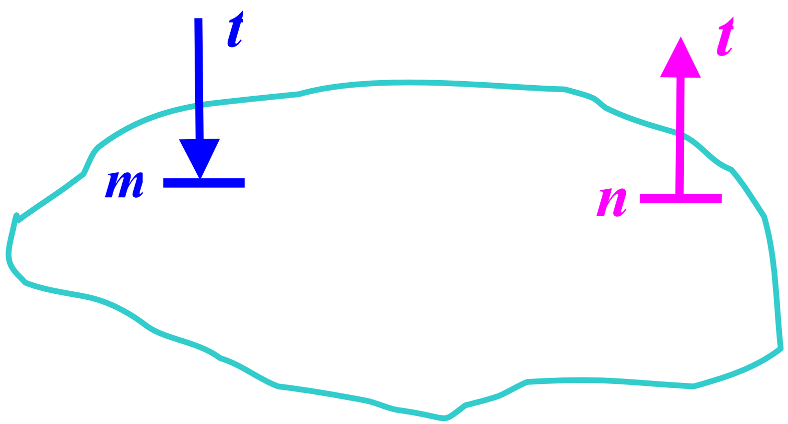
With bus 1 as the slack, the buses (columns) go for 2 to 5

Five Bus Example Comments

- At first glance the numerically determined value of (128-118)/20=0.5 does not match closely with the analytic value of 0.5455; however, in doing the subtraction we are losing numeric accuracy
  - Adding more digits helps (128.40 – 117.55)/20 = 0.5425
- The previous matrix derivation isn't intended for actual computation; ⍰ is a full matrix so we would seldom compute all of its values
- Sparse vector methods can be used if we are only interested in the ISFs for certain lines and certain buses

Distribution Factors and Basic Transactions

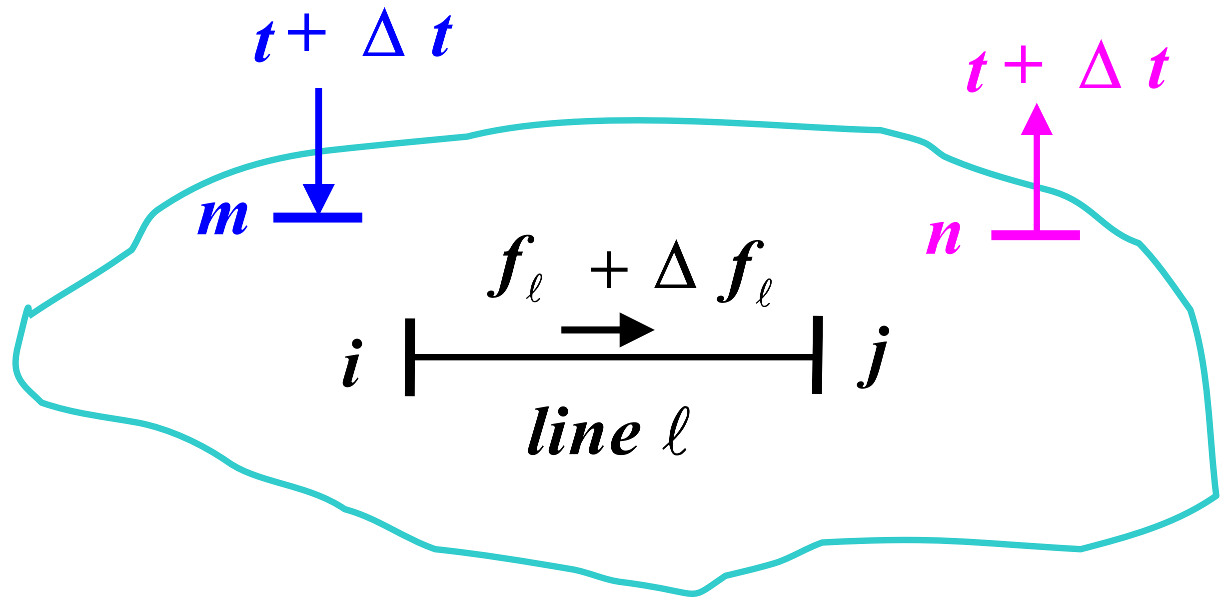
- Various additional distribution factors may be defined
  - power transfer distribution factor (PTDF)
  - line outage distribution factor (LODF)
  - line addition distribution factor (LADF)
  - outage transfer distribution factor (OTDF)
- These factors may be derived from the ISFs making judicious use of the superposition principle
- A basic transaction involves the transfer of a specified amount of power t from an injection node m to a withdrawal node n
- A basic transaction: $w = (m,n,t)$



Definition: PTDF

- NERC defines a PTDF as
  - "In the pre-contingency configuration of a system under study, a measure of the responsiveness or change in electrical loadings on transmission system Facilities due to a change in electric power transfer from one area to another, expressed in percent (up to 100%) of the change in power transfer"
  - Transaction dependent
- We'll use the notation $\varphi_{\ell}^{(w)}$ to indicate the PTDF on line ℓ with respect to basic transaction *w*
- In the lossless formulation presented here (and commonly used) it is slack bus independent

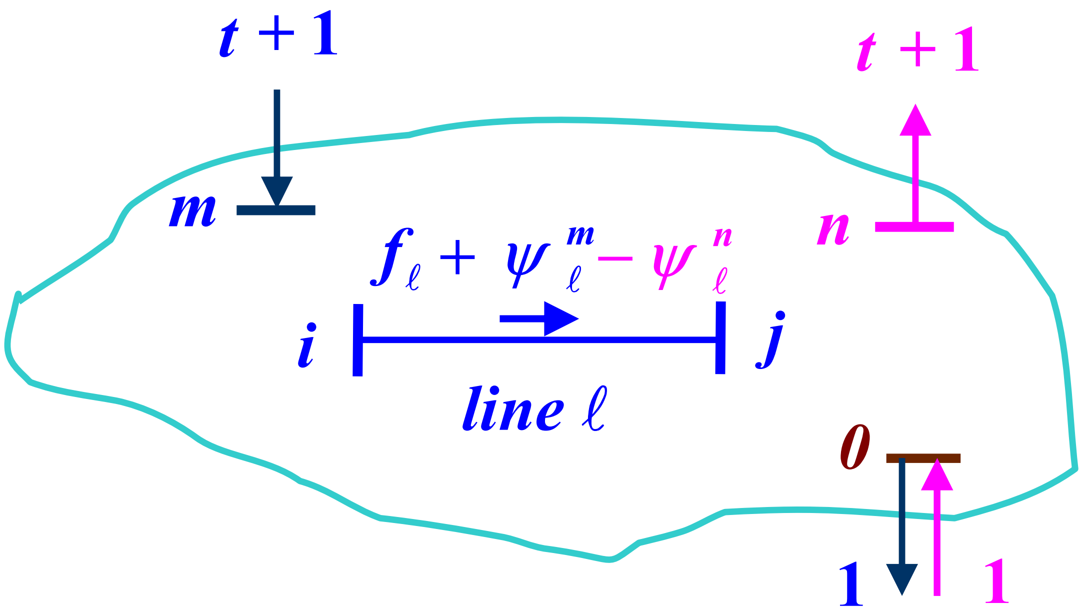
PTDFs



$$\varphi_{\ell}^{(w)} @ \frac{\Delta f_{\ell}}{\Delta t}$$

Note, the PTDF is independent of the amount Δt; which is often expressed as a percent

PTDF Evaluation



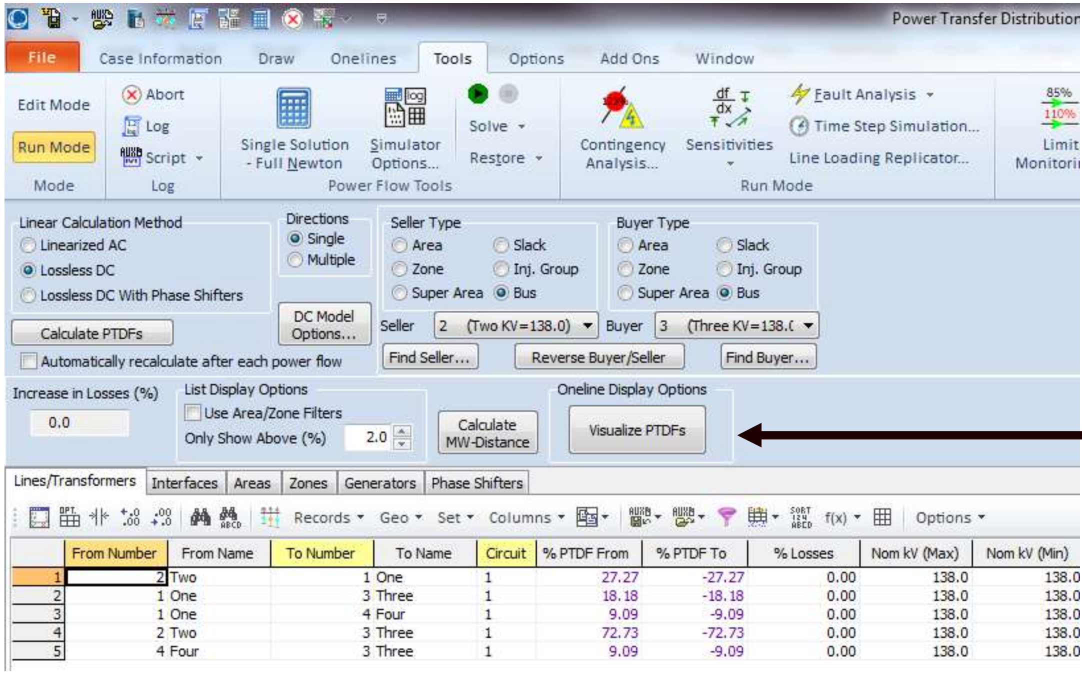
Defined in terms of the injection shift factors (ISFs); the slack bus dependence in each cancels out

$$\boxed{\varphi_\ell^{(w)} = \psi_\ell^m - \psi_\ell^n}$$

The PTDFs to the slack bus are the ISFs

Calculating PTDFs in PowerWorld

- PowerWorld provides a number of options for calculating and visualizing PTDFs
  - Select Tools, Sensitivities, Power Transfer Distribution Factors (PTDFs)

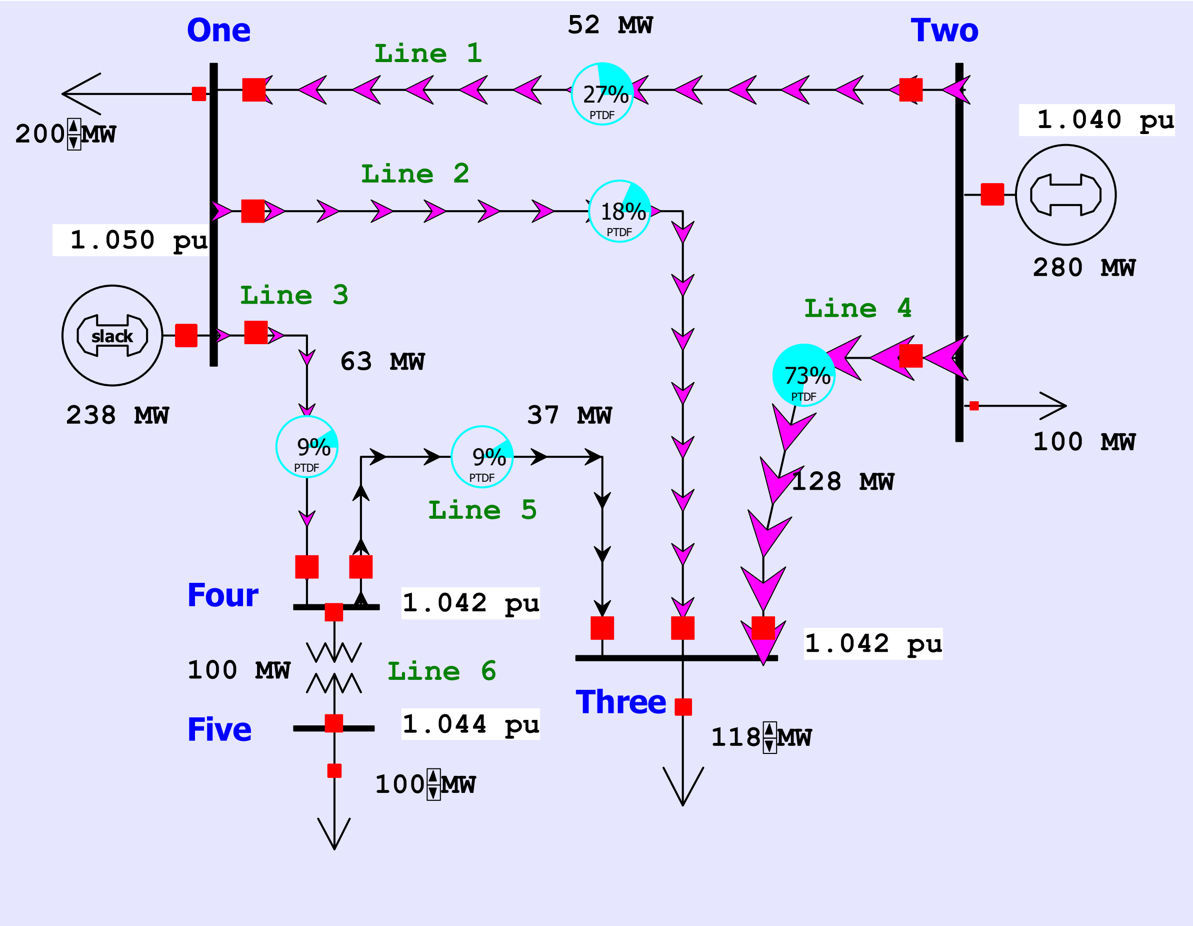


	From Number	From Name	To Number	To Name	Circuit	% PTDF From	% PTDF To	% Losses	Nom kV (Max)	Nom kV (Min)
1	2	Two	1	One	1	27.27	-27.27	0.00	138.0	138.0
2	1	One	3	Three	1	18.18	-18.18	0.00	138.0	138.0
3	1	One	4	Four	1	9.09	-9.09	0.00	138.0	138.0
4	2	Two	3	Three	1	72.73	-72.73	0.00	138.0	138.0
5	4	Four	3	Three	1	9.09	-9.09	0.00	138.0	138.0

Results are shown for the five bus case for the Bus 2 to Bus 3 transaction

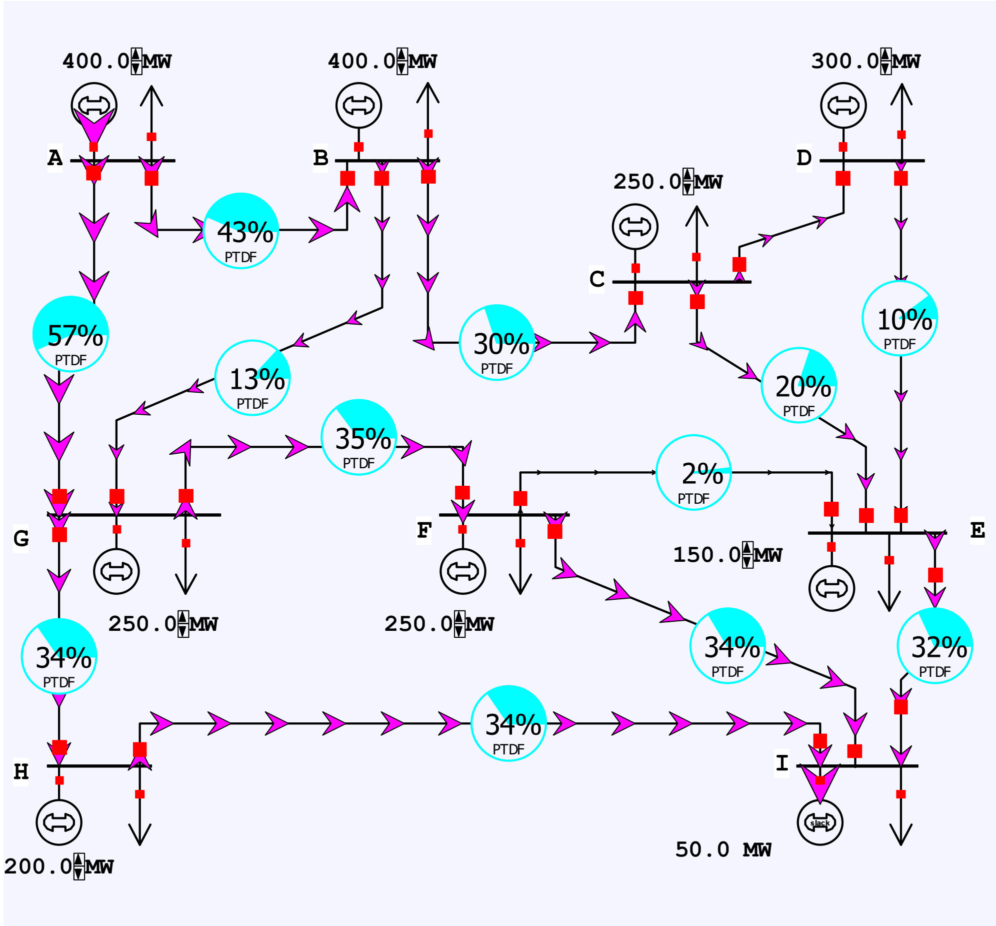
There is a button to visualize the PTDFs

Five Bus PTDF Visualization



PowerWorld Case: **B5_DistFact_PTDF**

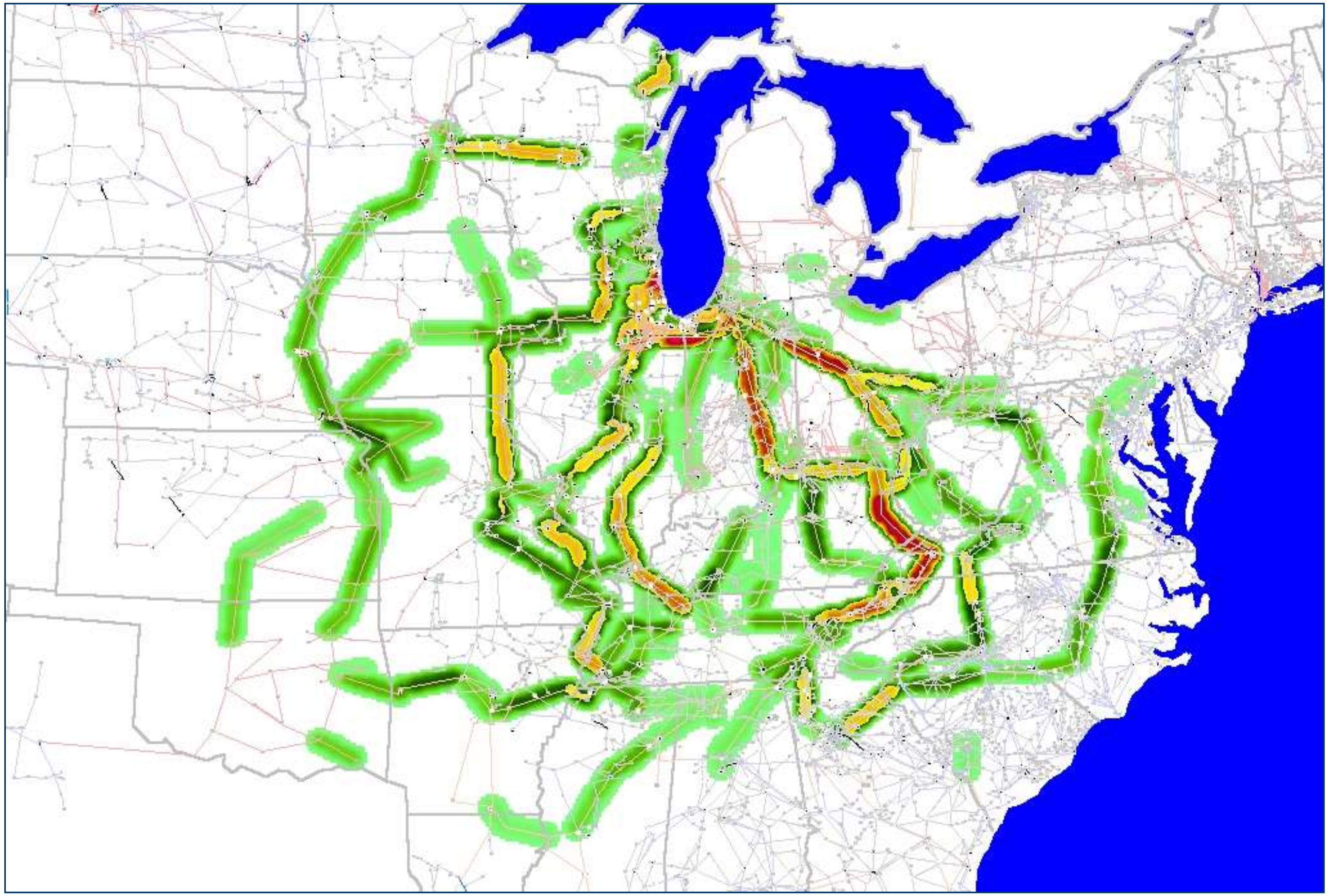
Nine Bus PTDF Example



Display shows the PTDFs for a basic transaction from Bus A to Bus I. Note that 100% of the transaction leaves Bus A and 100% arrives at Bus I

PowerWorld Case: **B9_PTDF**

Eastern Interconnect Example: Wisconsin Utility to TVA PTDFs



In this example multiple generators contribute for both the seller and the buyer

Contours show lines that would carry at least 2% of a power transfer from Wisconsin to TVA