ECEN 615 Methods of Electric Power Systems Analysis

Lecture 17: State Estimation

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Announcements

- Read Chapter 9
- Skim Chapters 3, 4 and 5
- Starting reading Chapter 8
- The classic book in this area is Power System State Estimation: Theory and Implementation by Ali Abur and Antonio Gomez-Exposito (2004)
- Homework 5 is due today
- Homework 6 is due on Thursday Nov 10

State Estimation for Linear Functions

- First we'll consider the linear problem. That is where

$$\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x}) = \mathbf{z}^{meas} - \mathbf{H}\mathbf{x}$$

- Let **R** be defined as the diagonal matrix of the variances (square of the standard deviations) for each of the measurements

$$\mathbf{R} = \begin{bmatrix} \sigma_1^2 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_m^2 \end{bmatrix}$$

State Estimation for Linear Functions

- We then differentiate J(**x**) w.r.t. **x** to determine the value of **x** that minimizes this function

$$J(\mathbf{x}) = \left[\mathbf{z}^{meas} - \mathbf{H}\mathbf{x}\right]^T \mathbf{R}^{-1} \left[\mathbf{z}^{meas} - \mathbf{H}\mathbf{x}\right]$$

$$\nabla J(\mathbf{x}) = -2\mathbf{H}^T\mathbf{R}^{-1}\mathbf{z}^{meas} + 2\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\mathbf{x}$$

At the minimum we have $\nabla J(\mathbf{x}) = \mathbf{0}$. So solving for $\mathbf{x}$ gives

$$\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1} \mathbf{H}^T\mathbf{R}^{-1}\mathbf{z}^{meas}$$

Simple DC System Example

- Say we have a two bus power system that we are solving using the dc approximation. Say the line's per unit reactance is j0.1. Say we have power measurements at both ends of the line. For simplicity assume **R**=**I**. We would then like to estimate the bus angles. Then

$$z_1 = 2.2,\ f_1(\mathbf{x}) = \frac{\theta_1 - \theta_2}{0.1}, \quad z_2 = -2.0,\ f_2(\mathbf{x}) = \frac{\theta_2 - \theta_1}{0.1}$$

$$\mathbf{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix}, \mathbf{H}^T\mathbf{H} = \begin{bmatrix} 200 & -200 \\ -200 & 200 \end{bmatrix}$$

We have a problem since $\mathbf{H}^T\mathbf{H}$ is singular.
This is because of lack of an angle reference.

Simple DC System Example, cont.

- Say we directly measure θ_1 (with a PMU) to be zero; set this as the third measurement. Then

$$z_1 = 2.2,\ f(\mathbf{x}) = \frac{\theta_1 - \theta_2}{0.1}, \quad z_2 = -2.0,\ f_2(\mathbf{x}) = \frac{\theta_2 - \theta_1}{0.1}, \quad z_3 = 0,\ f_3(\mathbf{x}) = 0$$

$$\mathbf{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 2.2 \\ -2 \\ 0 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 10 & -10 \\ -10 & 10 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{H}^T\mathbf{H} = \begin{bmatrix} 201 & -200 \\ -200 & 200 \end{bmatrix}$$

$$\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\mathbf{z}^{meas}$$

$$\mathbf{x} = \begin{bmatrix} 201 & -200 \\ -200 & 200 \end{bmatrix}^{-1} \begin{bmatrix} 10 & -10 & 1 \\ -10 & 10 & 0 \end{bmatrix} \begin{bmatrix} 2.2 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.21 \end{bmatrix}$$

Note that the angles are in radians

Nonlinear Formulation

- A regular ac power system is nonlinear, so we need to use an iterative solution approach. This is similar to the Newton power flow. Here assume m measurements and n state variables (usually bus voltage magnitudes and angles) Then the Jacobian is the **H** matrix

$$\mathbf{H}(\mathbf{x}) = \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{x_1} & \cdots & \frac{\partial f_1}{x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_m}{x_1} & \cdots & \frac{\partial f_m}{x_n} \end{bmatrix}$$

Measurement Example

- Assume we measure the real and reactive power flowing into one end of a transmission line; then the z_i-$f_i(\mathbf{x})$ functions for these two are

$$P_{ij}^{meas} - \left[-V_i^2 G_{ij} + V_i V_j \left(G_{ij} \cos\left(\theta_i - \theta_j\right) + B_{ij} \sin\left(\theta_i - \theta_j\right) \right) \right]$$

$$Q_{ij}^{meas} - \left[V_i^2 \left(B_{ij} + \frac{B_{cap}}{2} \right) + V_i V_j \left(G_{ij} \sin\left(\theta_i - \theta_j\right) - B_{ij} \cos\left(\theta_i - \theta_j\right) \right) \right]$$

  – Two measurements for four unknowns

- Other measurements, such as the flow at the other end, and voltage magnitudes, add redundancy

SE Iterative Solution Algorithm

- We then make an initial guess of **x**, $\mathbf{x}^{(0)}$ and iterate, calculating $\Delta\mathbf{x}$ each iteration

$$\Delta\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\begin{bmatrix} z_1 - f_1(\mathbf{x}) \\ \vdots \\ z_m - f_m(\mathbf{x}) \end{bmatrix}$$

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \Delta\mathbf{x}$$

This is exactly the least squares form developed earlier with $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}$ an n by n matrix. This could be solved with Gaussian elimination, but this isn't preferred because the problem is often ill-conditioned

Keep in mind that **H** is no longer constant, but varies as x changes, and is often ill-conditioned

Nonlinear SE Solution Algorithm, Book Figure 9.11

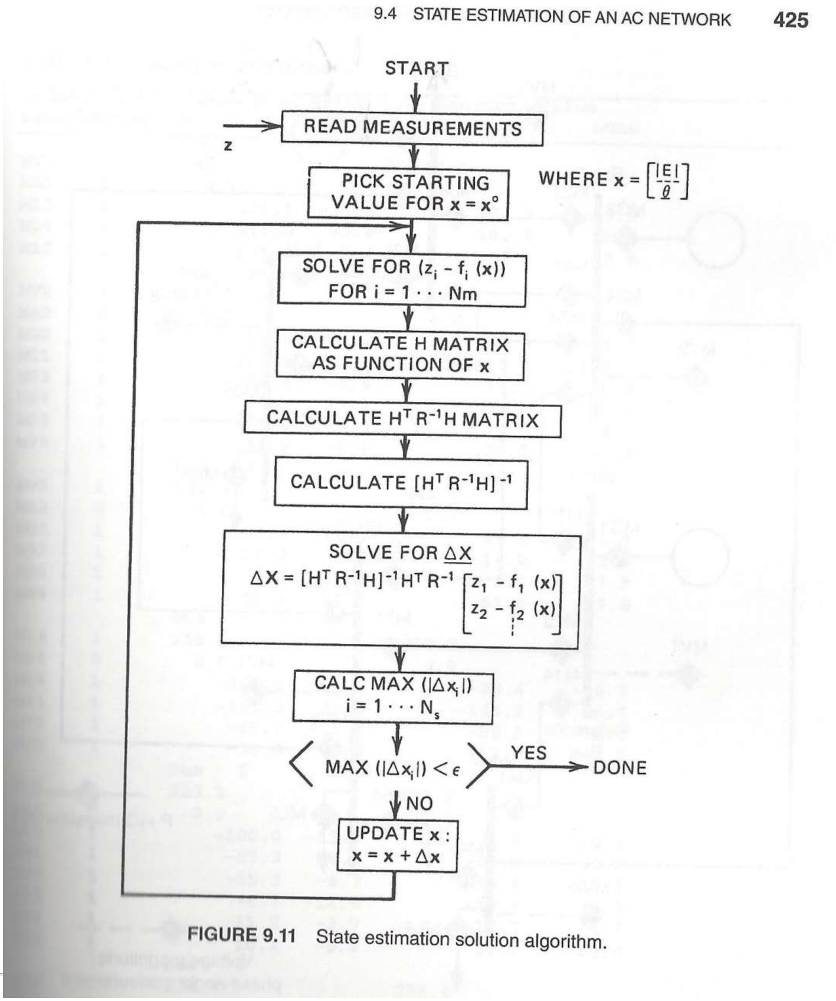


FIGURE 9.11 State estimation solution algorithm.

Example: Two Bus Case

- Assume a two bus case with a generator supplying a load through a single line with x=0.1 pu. Assume measurements of the p/q flow on both ends of the line (into line positive), and the voltage magnitude at both the generator and the load end. So $B_{12} = B_{21}=10.0$

$$P_{ij}^{meas} - \left[V_i V_j \left(B_{ij} \sin\left(\theta_i - \theta_j \right) \right) \right]$$

$$Q_{ij}^{meas} - \left[V_i^2 B_{ij} + V_i V_j \left(-B_{ij} \cos\left(\theta_i - \theta_j \right) \right) \right]$$

$$V_i^{meas} - V_i = 0$$

We need to assume a reference angle unless we directly measuring phase

Example: Two Bus Case

- Let

$$\mathbf{Z}^{meas} = \begin{bmatrix} P_{12} \\ Q_{12} \\ P_{21} \\ Q_{21} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 1.01 \\ 0.87 \end{bmatrix} \quad x^0 = \begin{bmatrix} V_1 \\ \theta_2 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \sigma_i = 0.01$$

We assume an angle reference of $\theta_1=0$

$$H(\mathbf{x}) = \begin{bmatrix} V_2 10\sin(-\theta_2) & -V_1V_2 10\cos(-\theta_2) & V_1 10\sin(-\theta_2) \\ 20V_1 - V_2 10\cos(-\theta_2) & -V_1V_2 10\sin(-\theta_2) & -V_1 10\cos(-\theta_2) \\ V_2 10\sin(\theta_2) & V_1V_2 10\cos(\theta_2) & V_1 10\sin(\theta_2) \\ -V_2 10\cos(\theta_2) & V_1V_2 10\sin(\theta_2) & 20V_2 - V_1 10\cos(\theta_2) \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example: Two Bus Case

- With a flat start guess we get

$$H(\mathbf{x}^0) = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & -10 \\ 0 & 10 & 0 \\ -10 & 0 & 10 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{z} - \mathbf{f}(\mathbf{x}^0) = \begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0001 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0001 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

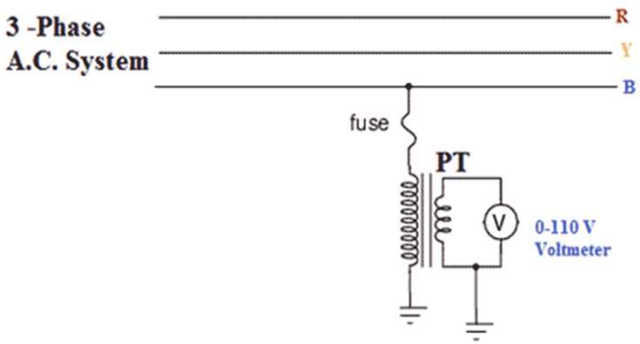
Example: Two Bus Case

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = 1e^6 \times \begin{bmatrix} 2.01 & 0 & -2 \\ 0 & 2 & 0 \\ -2 & 0 & 2.01 \end{bmatrix}$$

$$\mathbf{x}^1 = \mathbf{x}^0 + \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\begin{bmatrix} 2.02 \\ 1.5 \\ -1.98 \\ -1 \\ 0.01 \\ -0.13 \end{bmatrix} = \begin{bmatrix} 1.003 \\ -0.2 \\ 0.8775 \end{bmatrix}$$

Electric Grid Measurements

- The two major types of measurements are voltages and currents
  - The challenge for both types are doing these measurements at the high electric grid voltage levels
- Potential transformers (PTs) are used to measure voltage, using a transformer sometimes with a set of series capacitors to drop the voltage

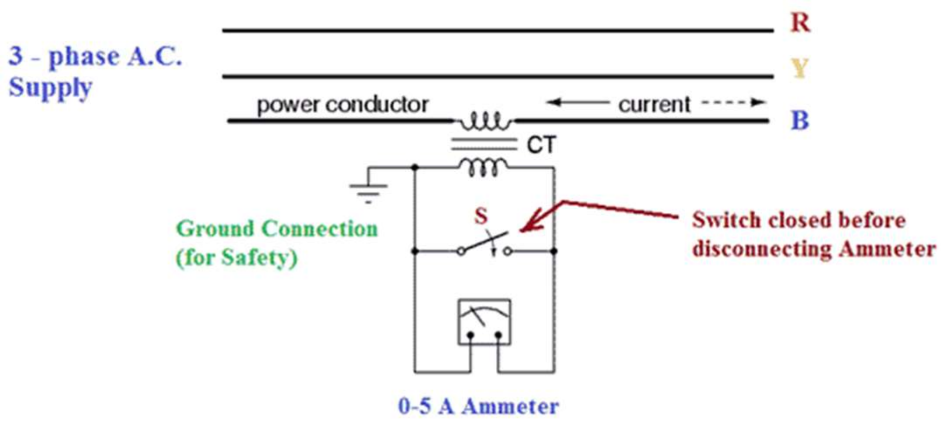


Potential Transformer (P.T.)

Image source: www.electrical4u.com/instrument-transformers/

Electric Grid Measurements

- Current transformers (CTs) are used to measure current, with the primary often consisting of the transmission wire itself; the secondary then has its number of turns set to give a specified current (say 5A) at a specified line current
  - Many CTs are used in the protection system so these need to be calibrated to correctly measure fault current; others are used to give more accurate load current values
- All meters have errors



Current Transformer (C.T.)

Image source: www.electrical4u.com/instrument-transformers/

Phasor Measurement Units (PMUs)

- All AC signals have a magnitude and phase. It is very easy to measure the phase angle differences between local signals (e.g., at an electrical substation)
  - These differences are used to calculate power values
- However, it had been challenging to measure phase angle differences between signals at different locations
  - This requires access to a precise time source
  - At 60 Hz one cycle takes 16.67 ms, which means one degree takes 46 μs.

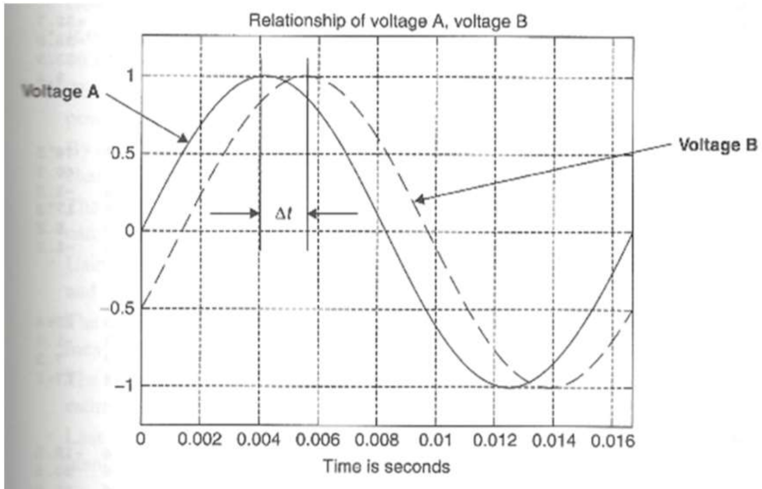


FIGURE 9.19 Phase difference as a function of time difference.

Phasor Measurement Units (PMUs)

- Widespread access to precise time became available in the 1980's when civilian use of the GPS was allowed
- PMUs use the GPS signals to determine the phase angles of voltages and currents (relative to some global reference)
  - The inputs to PMUs come from the CTs and PTs
- PMUs sample the system at rates on the order of 30 times per second
- PMU values are being used in SE algorithms

Assumed SE Measurement Accuracy

- The assumed measurement standard deviations can have a significant impact on the resultant solution, or even whether the SE converges
- The assumption is a Gaussian (normal) distribution of the error with no bias

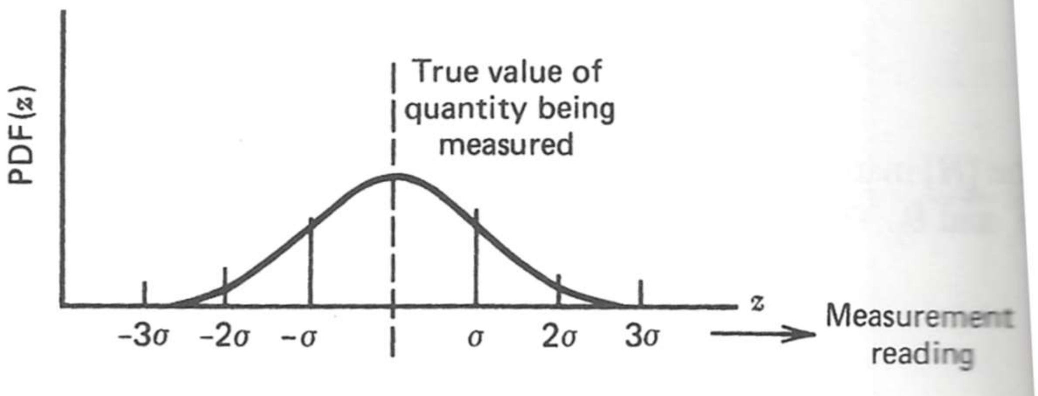


FIGURE 9.8 Normal distribution of meter errors.

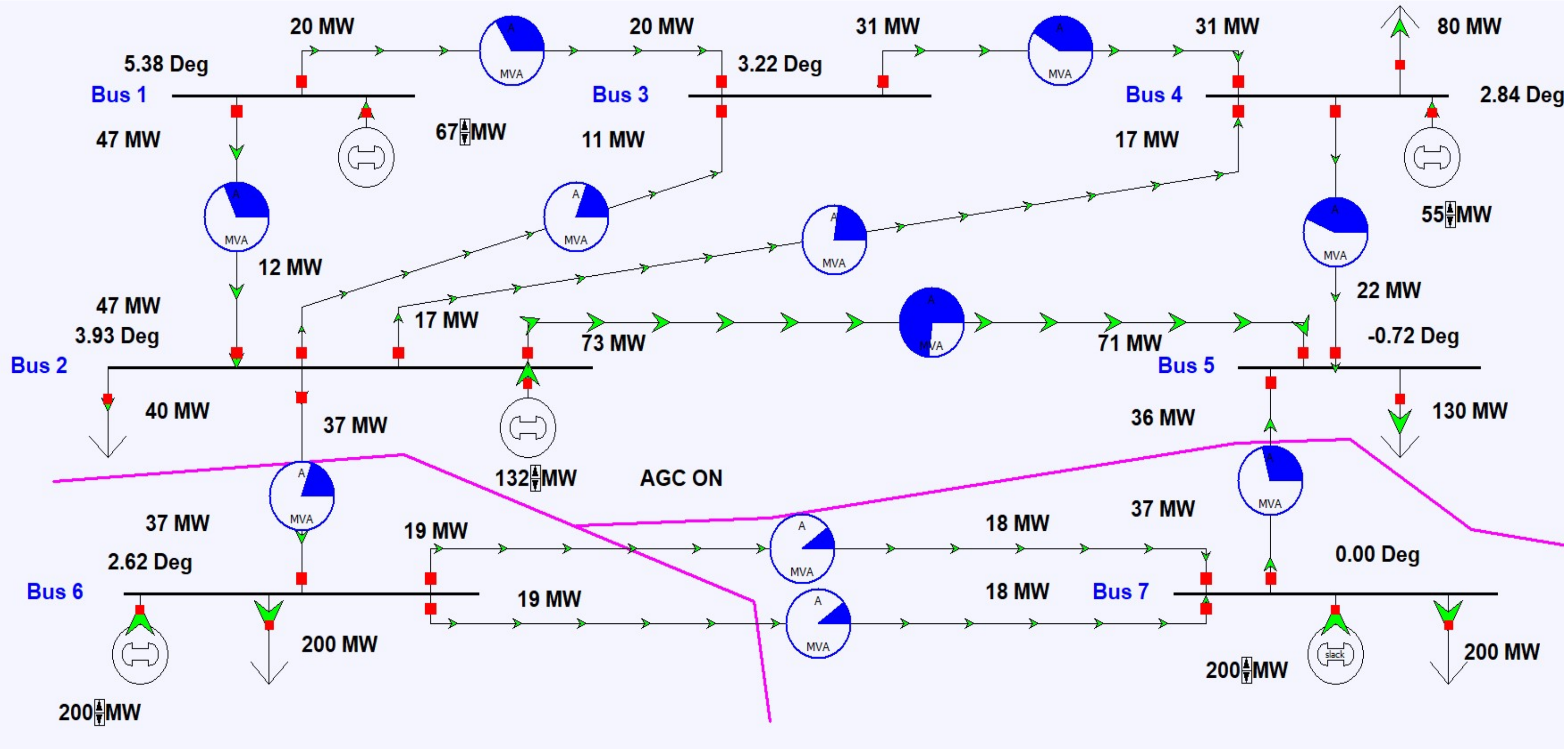
SE Observability

- In order to estimate all n states we need at least n measurements. However, where the measurements are located is also important, a topic known as observability
  - In order for a power system to be fully observable usually we need to have a measurement available no more than one bus away
  - At buses we need to have at least measurements on all the injections into the bus except one (including loads and gens)
  - Loads are usually flows on feeders, or the flow into a transmission to distribution transformer
  - Generators are sometimes just injections from the generator step-up transformer (GSU)

Pseudo Measurements

- Pseudo measurements are used at buses in which there is no load or generation; that is, the net injection into the bus is know with high accuracy to be zero
  - In order to enforce the net power balance at a bus we need to include an explicit net injection measurement
- To increase observability sometimes estimated values are used for loads, shunts and generator outputs
  - These "measurements" are represented as having a higher much standard deviation

SE Observability Example



SE Bad Data Detection

- The quality of the measurements available to an SE can vary widely, and sometimes the SE model itself is wrong. Causes include
  - Modeling Errors: perhaps the assumed system topology is incorrect, or the assumed parameters for a transmission line or transformer could be wrong
  - Data Errors: measurements may be incorrect because of in correct data specifications, like the CT ratios or even flipped positive and negative directions
  - Transducer Errors: the transductors may be failing or may have bias errors
  - Sampling Errors: SCADA does not read all values simultaneously and power systems are dynamic

SE Bad Data Detection

- The challenge for SE is to determine when there is likely a bad measurement (or multiple ones), and then to determine the particular bad measurements
- J(**x**) is random number, with a probability density function (PDF) known as a chi-squared distribution, $\chi^2(K)$, where K is the degrees of freedom, K=m-n
- It can be shown the expected mean for J(**x**) is K, with a standard deviation of $\sqrt{2K}$
  - Values of J(**x**) outside of several standard deviations indicate possible bad measurements, with the measurement residuals used to track down the likely bad measurements
- SE can be re-run without the bad measurements

QR Factorization

- Used in SE since it handles ill-conditioned m by n matrices (with m >= n)
- Can be used with sparse matrices
- We will first split the $\mathbf{R}^{-1}$ matrix

$$\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = \mathbf{H}^T\mathbf{R}^{-1/2}\mathbf{R}^{-1/2}\mathbf{H} = \mathbf{H}'^T\mathbf{H}'$$

- QR factorization represents the m by n **H'** matrix as

$$\mathbf{H}' = \mathbf{Q}\,\mathbf{U}$$

with **Q** an m by m orthonormal matrix and **U** an upper triangular matrix (most books use **Q R** but we use **U** to avoid confusion with the previous **R**)

Orthonormal Matrices

- The term orthogonal is used with vectors to indicate their dot product is zero (i.e., they are perpendicular to each other)
- Orthonormal is used to indicate they are orthogonal and each has unit length (magnitude of 1)
- The definition of an orthogonal matrix is $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
  - This implies its inverse always exists
- Its determinant is 1
- They can be used for transformations such as an angular rotation

QR Factorization

- If $\mathbf{H}' = \mathbf{Q}\,\mathbf{U}$ then $\mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\mathbf{Q}^T\mathbf{Q}\,\mathbf{U}$
- But since **Q** is an orthonormal matrix, $\mathbf{Q}^T\mathbf{Q} = \mathbf{I}$
- Hence we have $\mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\,\mathbf{U}$

Originally $\Delta\mathbf{x} = \left[\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

Q is an m by m matrix

With $\mathbf{H}^T\mathbf{R}^{-1}\mathbf{H} = \mathbf{H}'^T\mathbf{H}' = \mathbf{H}'^T\mathbf{H}' = \mathbf{U}^T\mathbf{U}$

Let $\hat{\mathbf{z}} = \mathbf{Q}^T\mathbf{R}^{-1/2}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right]$

$$\Delta\mathbf{x} = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{H}^T\mathbf{R}^{-1/2}\mathbf{R}^{-1/2}\left[\mathbf{z}^{meas} - \mathbf{f}(\mathbf{x})\right] = \left[\mathbf{U}^T\mathbf{U}\right]^{-1}\mathbf{U}^T\hat{\mathbf{z}}$$

$$\mathbf{U}^T\mathbf{U}\Delta\mathbf{x} = \mathbf{U}^T\hat{\mathbf{z}} \rightarrow \Delta\mathbf{x} = \mathbf{U}^{-1}\hat{\mathbf{z}}$$

QR Factorization

- When factored the **U** matrix (i.e., what most call the **R** matrix) will be an m by n upper triangular matrix
- Several methods are available including the Householder method and the Givens method
- Givens is preferred when dealing with sparse matrices
- A good reference is Gene H. Golub and Charles F. Van Loan, “Matrix Computations,” third edition, Johns Hopkins University Press, 1996

Givens Algorithm for Factoring a Matrix A

- The Givens algorithm works by pre-multiplying the initial matrix, **A**, by a series of matrices and their transposes, starting with $\mathbf{G}_1\mathbf{G}_1^T$
  - If **A** is m by n, then each **G** is an m by m matrix
- The algorithm proceeds column by column, sequentially zeroing out elements in the lower triangle of **A**, starting at the bottom of each column

$$\mathbf{G}_1 \ldots \mathbf{G}_p \mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{Q}\mathbf{U}$$

$$\mathbf{G}_1 \ldots \mathbf{G}_p = \mathbf{Q}$$

$$\mathbf{G}_p^T \ldots \mathbf{G}_1^T \mathbf{A} = \mathbf{U}$$

If **A** is sparse, then we can take advantage of sparsity going up the column