ECEN 615
Methods of Electric Power Systems Analysis

Lecture 20: Voltage Stability, Economic Dispatch, Unit Commitment

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Announcements

- Starting reading Chapter 8
- Homework 6 is now due on Thursday Nov 17 but it counts as two regular homeworks.
- Economic dispatch is covered in Chapter 3, and Unit Commitment is in Chapter 4
- We'll be hosting the 2022 Resilient Electric Consortium of North America Symposium (RECONS 2022) on Thursday Nov 17, 2022 at CIR. All 615 students are invited (with a regular class at 8am that day)
- Register here: smartgridcenter.tamu.edu/index.php/recons-2022/
  - For free registration use the code TAMURECONS22

Power System Voltage Stability

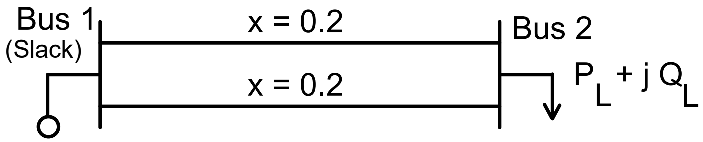
- **Voltage Stability**: The ability to maintain system voltage so that both power and voltage are controllable. System voltage responds as expected (i.e., an increase in load causes proportional decrease in voltage).
- **Voltage Instability**: Inability to maintain system voltage. System voltage and/or power become uncontrollable. System voltage does not respond as expected.
- **Voltage Collapse**: Process by which voltage instability leads to unacceptably low voltages in a significant portion of the system; typically results in loss of system load.

Voltage Stability

- Two good references are
  - P. Kundur, et. al., "Definitions and Classification of Power System Stability," *IEEE Trans. on Power Systems*, pp. 1387-1401, August 2004.
  - T. Van Cutsem, "Voltage Instability: Phenomena, Countermeasures, and Analysis Methods," *Proc. IEEE*, February 2000, pp. 208-227.
- Classified by either size of disturbance or duration
  - Small or large disturbance: small disturbance is just perturbations about an equilibrium point (power flow)
  - Short-term (several seconds) or long-term (many seconds to minutes) (covered in ECEN 667)

Small Disturbance Voltage Stability

- Small disturbance voltage stability can be assessed using a power flow (maximum loadability)
- Depending on the assumed load model, the power flow can have multiple (or no) solutions
- PV curve is created by plotting power versus voltage



Assume V_{slack}=1.0

$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

Where B is the line susceptance =-10,
V∠θ is the load voltage

Small Disturbance Voltage Stability

- Question: how do the power flow solutions vary as the load is changed?
- A Solution: Calculate a series of power flow solutions for various load levels and see how they change
- Power flow Jacobian

$$\mathbf{J}(\theta, V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$

$$\det \mathbf{J}(\theta, V) = VB^2\left(2V\cos\theta - \cos^2\theta - \sin^2\theta\right)$$

Singular when $\left(2V\cos\theta - 1\right) = 0$

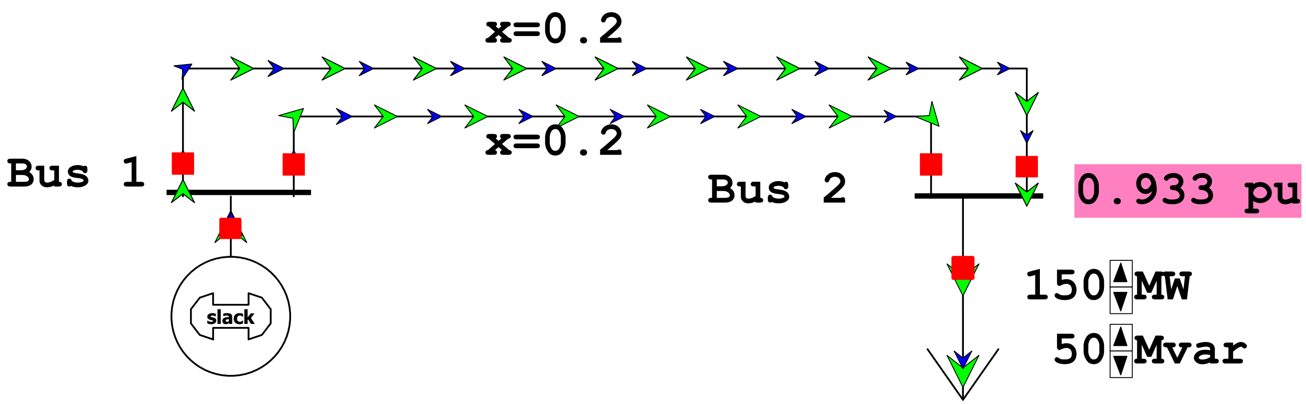
Maximum Loadability When Power Flow Jacobian is Singular

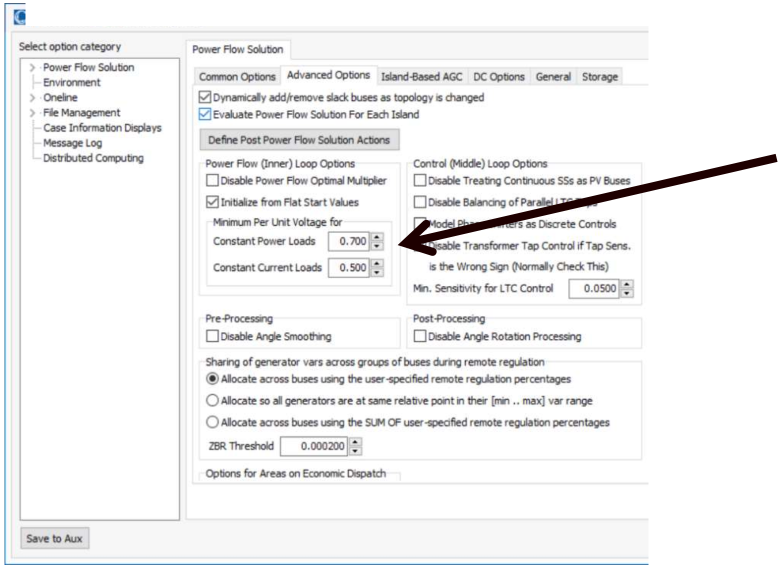
- An important paper considering this was by Sauer and Pai from IEEE Trans. Power Systems in Nov 1990, "Power system steady-state stability and the load-flow Jacobian"
- Other earlier papers were looking at the characteristics of multiple power flow solutions
- The power flow Jacobian depends on the assumed load model (we'll see the impact in a few slides)

Small Disturbance Voltage Collapse

- At constant frequency (e.g., 60 Hz) the complex power transferred down a transmission line is $S=VI^*$
  - V is phasor voltage, I is phasor current
  - This is the reason for using a high voltage grid
- Line real power losses are given by RI^2 and reactive power losses by XI^2
  - R is the line's resistance, and X its reactance; for a high voltage line X >> R
- Increased reactive power tends to drive down the voltage, which increases the current, which further increases the reactive power losses

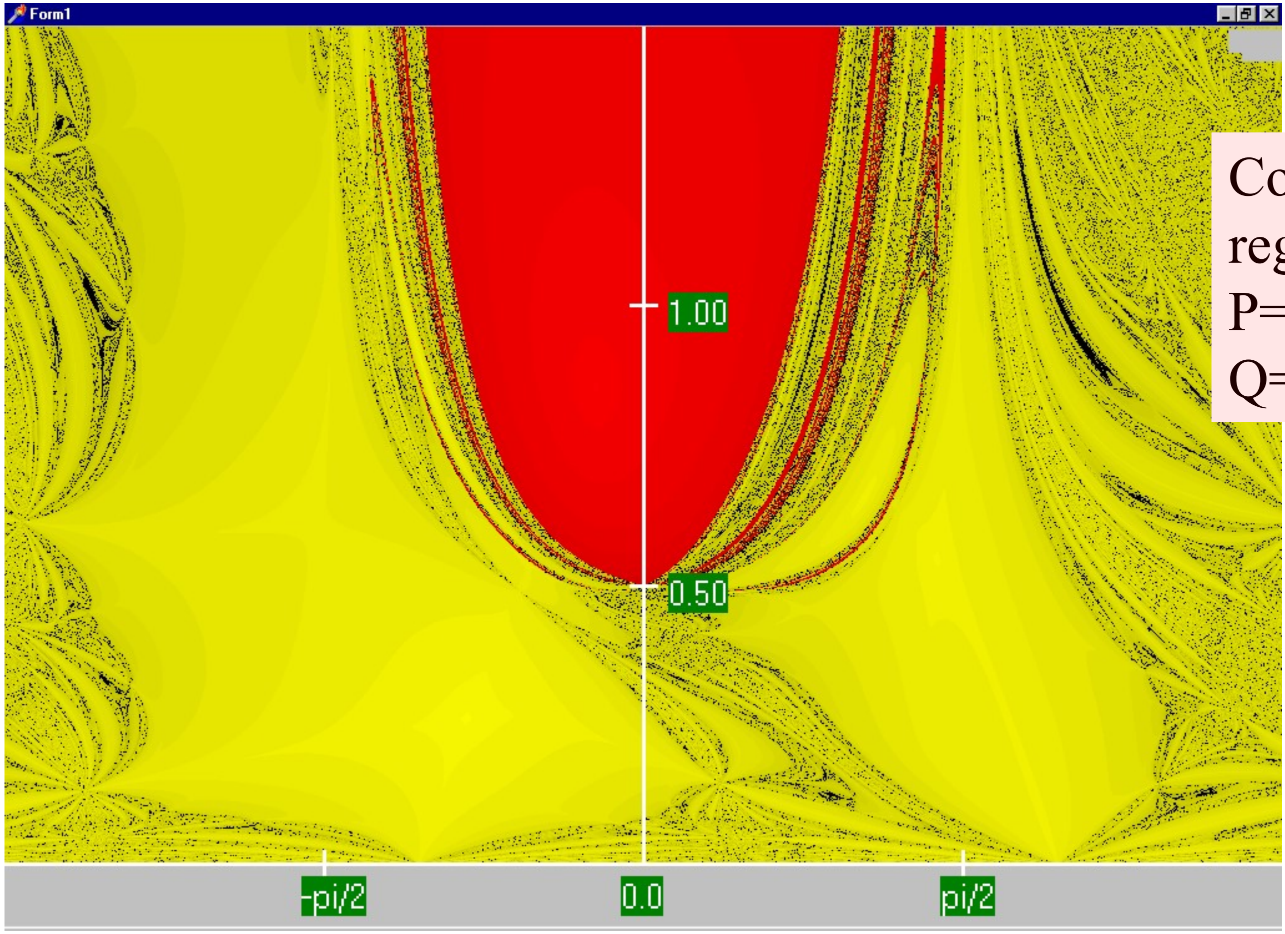
PowerWorld Two Bus Example





Commercial power flow software usually auto converts constant power loads at low voltages; set these fields to zero to disable this conversion

Power Flow Region of Convergence

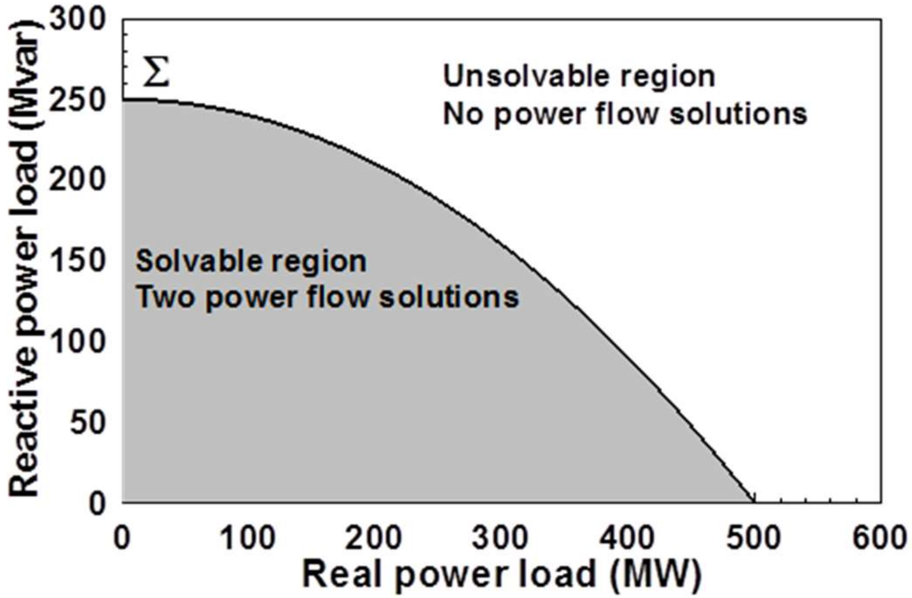


Convergence regions with P=100 MW, Q=0 Mvar

Load Parameter Space Representation

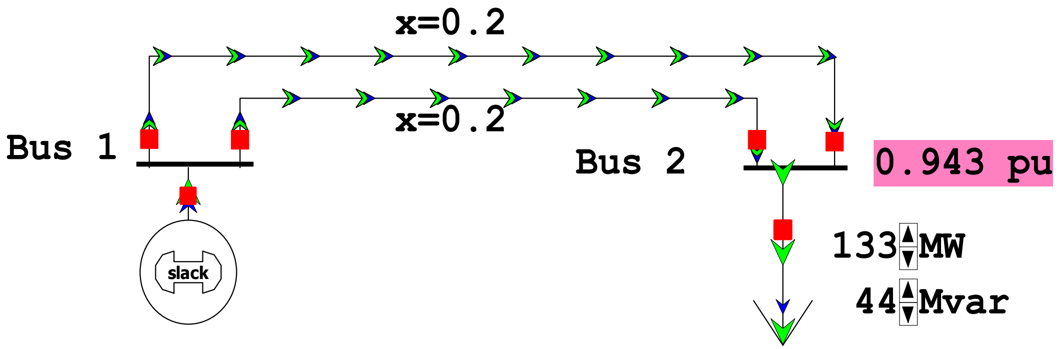
- With a constant power model there is a maximum loadability surface, Σ
  - Defined as point in which the power flow Jacobian is singular
  - For the lossless two bus system it can be determined as

$$-\frac{P_L^2}{B}+Q_L+\frac{1}{4}B=0$$



Load Model Impact

- With a static load model regardless of the voltage dependency the same PV curve is traced
  - But whether a point of maximum loadability exists depends on the assumed load model
    - If voltage exponent is > 1 then multiple solutions do not exist (see B.C. Lesieutre, P.W. Sauer and M.A. Pai "Sufficient conditions on static load models for network solvability,"NAPS 1992, pp. 262-271)



Change load to constant impedance; hence it becomes a linear model

ZIP Model Coefficients

- One popular static load model is the ZIP; lots of papers on the "correct" amount of each type

TABLE I
ZIP COEFFICIENTS FOR EACH CUSTOMER CLASS

Class	Z_p	I_p	P_p	Z_q	I_q	P_q
Large commercial	0.47	-0.53	1.06	5.30	-8.73	4.43
Small commercial	0.43	-0.06	0.63	4.06	-6.65	3.59
Residential	0.85	-1.12	1.27	10.96	-18.73	8.77
Industrial	0	0	1	0	0	1

TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13

Table 1 from M. Diaz-Aguilo, et. al., "Field-Validated Load Model for the Analysis of CVR in Distribution Secondary Networks: Energy Conservation," *IEEE Trans. Power Delivery*, Oct. 2013

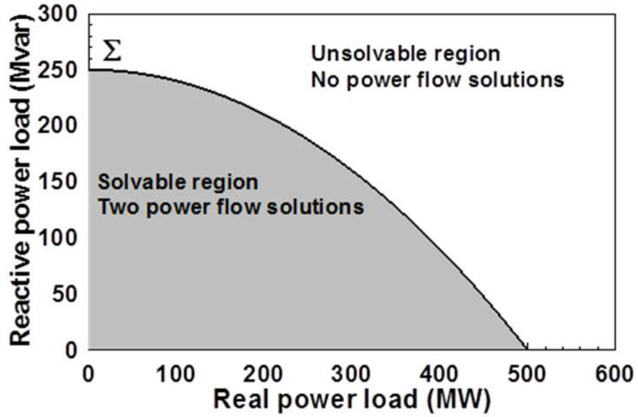
Table 7 from A, Bokhari, et. al., "Experimental Determination of the ZIP Coefficients for Modern Residential, Commercial, and Industrial Loads," *IEEE Trans. Power Delivery*, June. 2014

Application: Conservation Voltage Reduction (CVR)

- If the "steady-state" load has a true dependence on voltage, then a change (usually a reduction) in the voltage should result in a total decrease in energy consumption
- If an "optimal" voltage could be determined, then this could result in a net energy savings
- Some challenges are 1) the voltage profile across a feeder is not constant, 2) the load composition is constantly changing, 3) a decrease in power consumption might result in a decrease in useable output from the load, and 4) loads are dynamic and an initial decrease might be balanced by a later increase

Determining a Metric to Voltage Collapse

- The goal of much of the voltage stability work was to determine an easy to calculate metric (or metrics) of the current operating point to voltage collapse
  - PV and QV curves (or some combination) can determine such a metric along a particular path
  - Goal was to have a path independent metric. The closest boundary point was considered, but this could be quite misleading if the system was not going to move in that direction



  - Any linearization about the current operating point (i.e., the Jacobian) does not consider important nonlinearities like generators hitting their reactive power limits

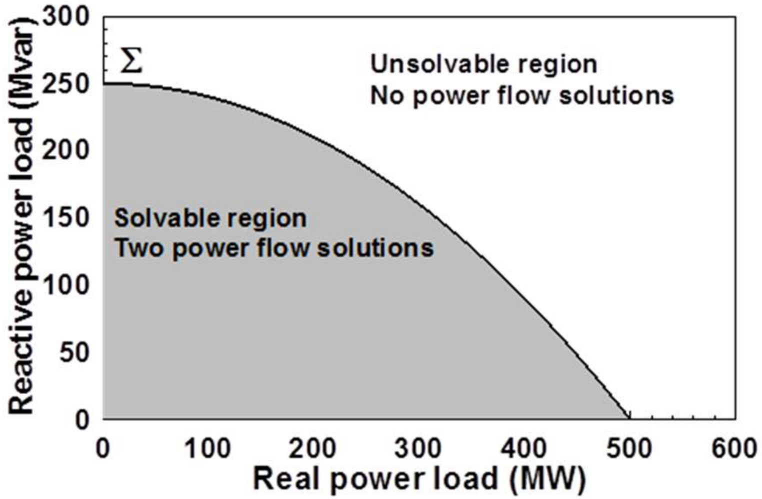
Determining a Metric to Voltage Collapse

- A paper by Dobson in 1992 (see below) noted that at a saddle node bifurcation, in which the power flow Jacobian is singular, that
  - The right eigenvector associated with the Jacobian zero eigenvalue tells the direction in state space of the voltage collapse
  - The left eigenvector associated with the Jacobian zero eigenvalue gives the normal in parameter space to the boundary Σ. This can then be used to estimate the minimum distance in parameter space to bifurcation.

I. Dobson, "Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems," IEEE Trans. Circuits and Systems, March 1992

Determining a Metric to Voltage Collapse Example

- For the previous two bus example we had



$$P_L - BV\sin\theta = 0$$

$$Q_L + BV\cos\theta - BV^2 = 0$$

$$\mathbf{J}(\theta,V) = \begin{bmatrix} -BV\cos\theta & -B\sin\theta \\ -BV\sin\theta & B\cos\theta - 2BV \end{bmatrix}$$

Singular when $\left(2V\cos\theta - 1\right) = 0$

So consider $B = -10,\ V = 0.6,\ \theta = -33.56°$, then $P_L = 3.317$, $Q_L = 1.400$

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

Determining a Metric to Voltage Collapse Example

- Calculating the right and left eigenvectors associated with the zero eigenvalue we get

$$\mathbf{J} = \begin{bmatrix} 5 & -5.528 \\ -3.317 & 3.667 \end{bmatrix}$$

$$\mathbf{v} = \begin{bmatrix} 0.742 \\ 0.671 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 0.553 \\ 0.833 \end{bmatrix}$$

The left eigenvector is telling the best way to vary the P and Q to restore solveability

Quantifying Power Flow Unsolvability

- Since lack of power flow convergence can be a major problem, it would be nice to have a measure to quantify the degree of unsolvability of a power flow
  - And then figure out the best way to restore solvabiblity
- T.J. Overbye, "A Power Flow Measure for Unsolvable Cases," IEEE Trans. Power Systems, August 1994

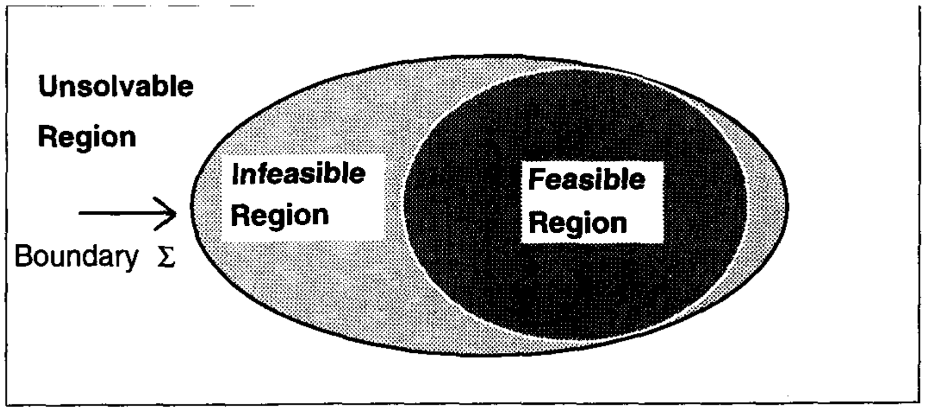


Figure 1 : Power Flow Security Regions

Aside: Power Flow using the Optimal Multiplier

- Classic reference on power flow optimal multiplier is S. Iwamoto, Y. Tamura, "A Load Flow Calculation Method for Ill-Conditioned Power Systems," *IEEE Trans. Power App. and Syst.*, April 1981
- Another paper is J.E. Tate, T.J. Overbye, "A Comparison of the Optimal Multiplier in Power and Rectangular Coordinates," *IEEE Trans. Power Systems*, Nov. 2005
- Key idea is once NR method has selected a direction, we can analytically determine the distance to move in that direction to minimize the norm of the mismatch
  - Goal is to help with stressed power systems

Aside: Power Flow using the Optimal Multiplier

- Consider an n bus power system with **f(x)** = **S** where **S** is the vector of the constant real and reactive power load minus generation at all buses except the slack, **x** is the vector of the bus voltages in rectangular coordinates: $V_i = e_i + jf_i$, and **f** is the function of the power balance constraints

$$f_{pi} = \sum_{j=1}^{n} \left(e_i \left(e_j G_{ij} - f_j B_{ij} \right) + f_i \left(f_j G_{ij} + e_j B_{ij} \right) \right)$$

$$f_{qi} = \sum_{j=1}^{n} \left(f_i \left(e_j G_{ij} - f_j B_{ij} \right) - e_i \left(f_j G_{ij} + e_j B_{ij} \right) \right)$$

G + j**B** is the bus admittance matrix

Aside: Power Flow using the Optimal Multiplier

- With a standard NR approach we would get

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \Delta\mathbf{x}^{k}$$

$$\Delta\mathbf{x}^{k} = -\mathbf{J}(\mathbf{x}^{k})^{-1}\left(\mathbf{f}(\mathbf{x}^{k}) - \mathbf{S}\right)$$

- If we are close enough to the solution the iteration converges quickly, but if the system is heavily loaded it can diverge
- Optimal multiplier approach modifies the iteration as

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \mu\,\Delta\mathbf{x}^{k}$$

$$\Delta\mathbf{x}^{k} = -\mathbf{J}(\mathbf{x}^{k})^{-1}\left(\mathbf{f}(\mathbf{x}^{k}) - \mathbf{S}\right)$$

Aside: Power Flow using the Optimal Multiplier

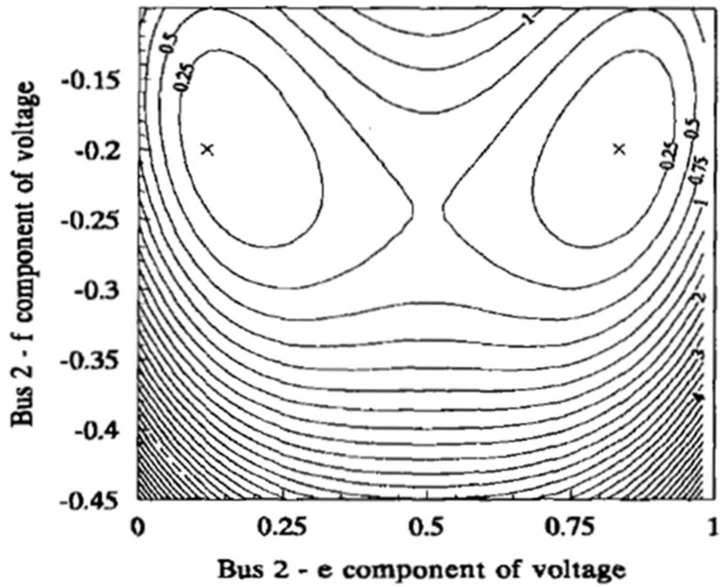
- The scalar μ is chosen to minimize the norm of the mismatch F in direction $\Delta\mathbf{x}$

$$F\left(\mathbf{x}^{k+1}\right)=\frac{1}{2}\left[\mathbf{f}\left(\mathbf{x}^{k}+\mu\Delta\mathbf{x}^{k}\right)-\mathbf{S}\right]^{T}\left[\mathbf{f}\left(\mathbf{x}^{k}+\mu\Delta\mathbf{x}^{k}\right)-\mathbf{S}\right]$$

- Paper by Iwamoto, Y. Tamura from 1981 shows μ can be computed analytically with little additional calculation when rectangular voltages are used
- Determination of μ involves solving a cubic equation, which gives either three real solutions, or one real and two imaginary solutions

Aside: Power Flow using the Optimal Multiplier

- A 1989 PICA paper by Iba ("A Method for Finding a Pair of Multiple Load Flow Solutions in Bulk Power Systems") showed that NR tends to converge along line joining the high and a low voltage solution
  - However there are some model restrictions, particularly associated with the load model
- We are currently doing research looking at whether this can be used to restore a power flow that has converged to an alternative solution



Quantifying Power Flow Unsolvability

- To setup the problem, first consider the power flow iteration without and with the optimal multiplier

$$\mathbf{x}^{k+1} = \mathbf{x}^{k} + \Delta\mathbf{x}^{k}$$

$$\Delta\mathbf{x}^{k} = -\mathbf{J}(\mathbf{x}^{k})^{-1}\left(\mathbf{f}(\mathbf{x}^{k}) - \mathbf{S}\right)$$

With the optimal multiplier we are minimizing

$$\mathrm{F}(\mathbf{x}^{k+1}) = \frac{1}{2}\left(\mathbf{f}(\mathbf{x}^{k}) + \mu\Delta\mathbf{x}^{k} - \mathbf{S}\right)^{T}\left(\mathbf{f}(\mathbf{x}^{k}) + \mu\Delta\mathbf{x}^{k} - \mathbf{S}\right)$$

When there is a solution $\mu \rightarrow 1$ and the cost function goes to zero

Quantifying Power Flow Unsolvability

$$\det(J) \quad = \quad B_{12}(B_{12} + 2eB_{22}) \quad = \quad 0 \qquad (12)$$

Here, where $B_{12} = -B_{22}$, the solution of (12) is e = 0.5. Substituting this solution for e into (10b) and using (10a) to solve for the f component of the bus 2 voltage, one gets Σ to be the set of all points where

$$\frac{P^2}{B_{12}} + Q - \frac{1}{4}B_{12} = 0 \qquad (13)$$

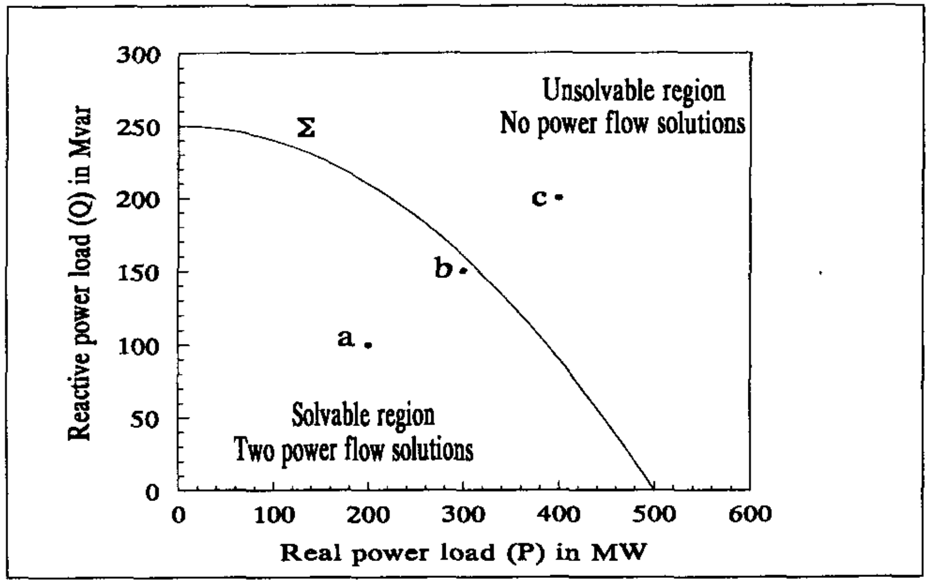


Figure 2 : Solvable and Unsolvable Regions in Parameter Space

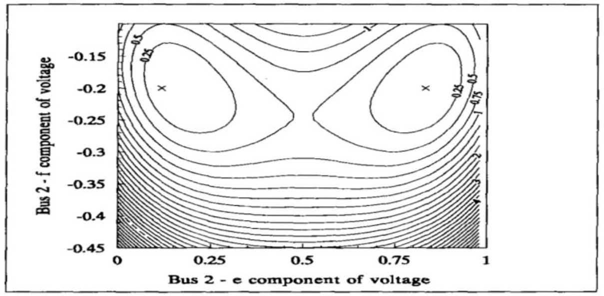


Figure 3a : Two Bus Cost Contours - Load of 200 MW and 100 Mvar

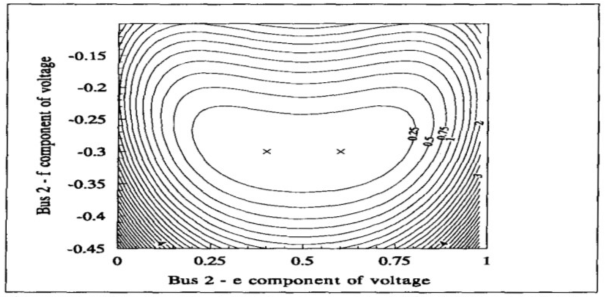


Figure 3b : Two Bus Cost Contours - Load of 300 MW and 150 Mvar

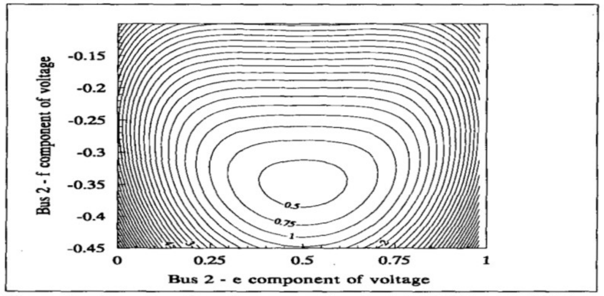


Figure 3c : Two Bus Cost Contours - Load of 400 MW and 200 Mvar

Quantifying Power Flow Unsolvability

- However, when there is no solution the standard power flow would diverge. But the approach with the optimal multiplier tends to point in the direction of minimizing $F(x^{k+1})$. That is,

$$\nabla F(\mathbf{x}^k) = \left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^k)$$

Also

$$\Delta\mathbf{x}^k = -\mathbf{J}(\mathbf{x}^k)^{-1}\left[\mathbf{f}(\mathbf{x}^k) - \mathbf{S}\right]$$

where how far to move in this direction is limited by μ.

Quantifying Power Flow Unsolvability

- The only way we cannot reduce the cost function some would be if the two directions were perpendicular, hence with a zero dot product. So

$$\frac{\nabla F\left(\mathbf{x}^k\right)\cdot\Delta\mathbf{x}^k}{\left\|\mathbf{x}^k\right\|} = \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \mathbf{J}(\mathbf{x}^{\mathrm{k}})\mathbf{J}(\mathbf{x}^{\mathrm{k}})^{-1}\left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

$$= \frac{= \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]^T \left[\mathbf{f}(\mathbf{x}^{\mathrm{k}}) - \mathbf{S}\right]}{\left\|\mathbf{x}^k\right\|}$$

(provided the Jacobian is not singular). As we approach singularity this goes to zero. Hence we converge to a point on the boundary Σ, but not necessarilty at the closest boundary point.

Quantifying Power Flow Unsolvability

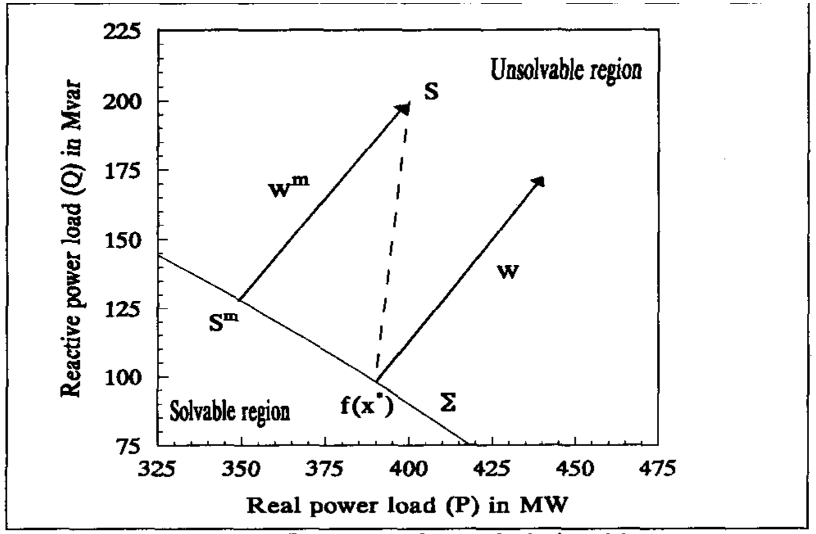


Figure 5 : Parameter Space Relationships

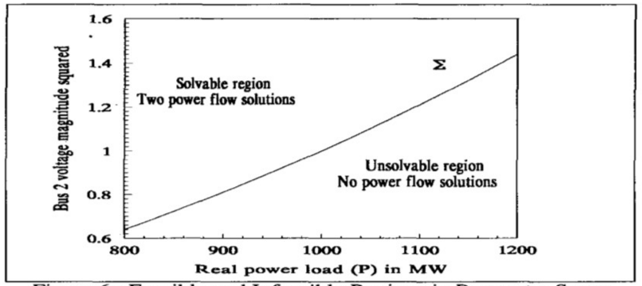


Figure 6 : Feasible and Infeasible Regions in Parameter Space

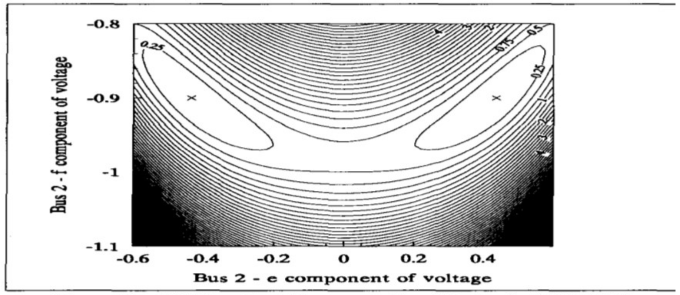


Figure 7a : PV Bus Cost Contours - Feasible load of 900 MW

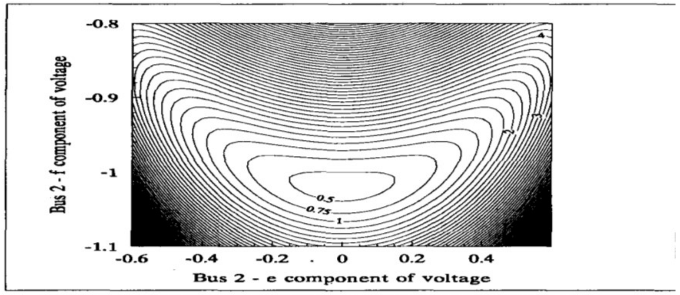


Figure 7b : PV Bus Cost Contours - Infeasible load of 1100 MW

If Σ were flat then **w** is parallel to **w**^{m}

Quantifying Power Flow Unsolvability

- The left eigenvector associated with the zero eigenvalue of the Jacobian (defined as $\mathbf{w}^{i*}$) is perpendicular to Σ (as noted in the early 1992 Dobson paper)
- We can get the closest point on the Σ just by iterating, updating the **S** Vector as

$$\mathbf{S}^{i+1} = \mathbf{S} + [(\mathbf{f}(\mathbf{x}^{i^*}) - \mathbf{S}) \cdot \mathbf{w}^{i^*}]\, \mathbf{w}^{i^*}$$

(here **S** is the initial power injection, $\mathbf{x}^{i*}$ a boundary solution)

- Converges when

$$\left\|(\mathbf{f}(\mathbf{x}^{i^*}) - \mathbf{S}^{i})\right\| < \varepsilon$$

Challenges

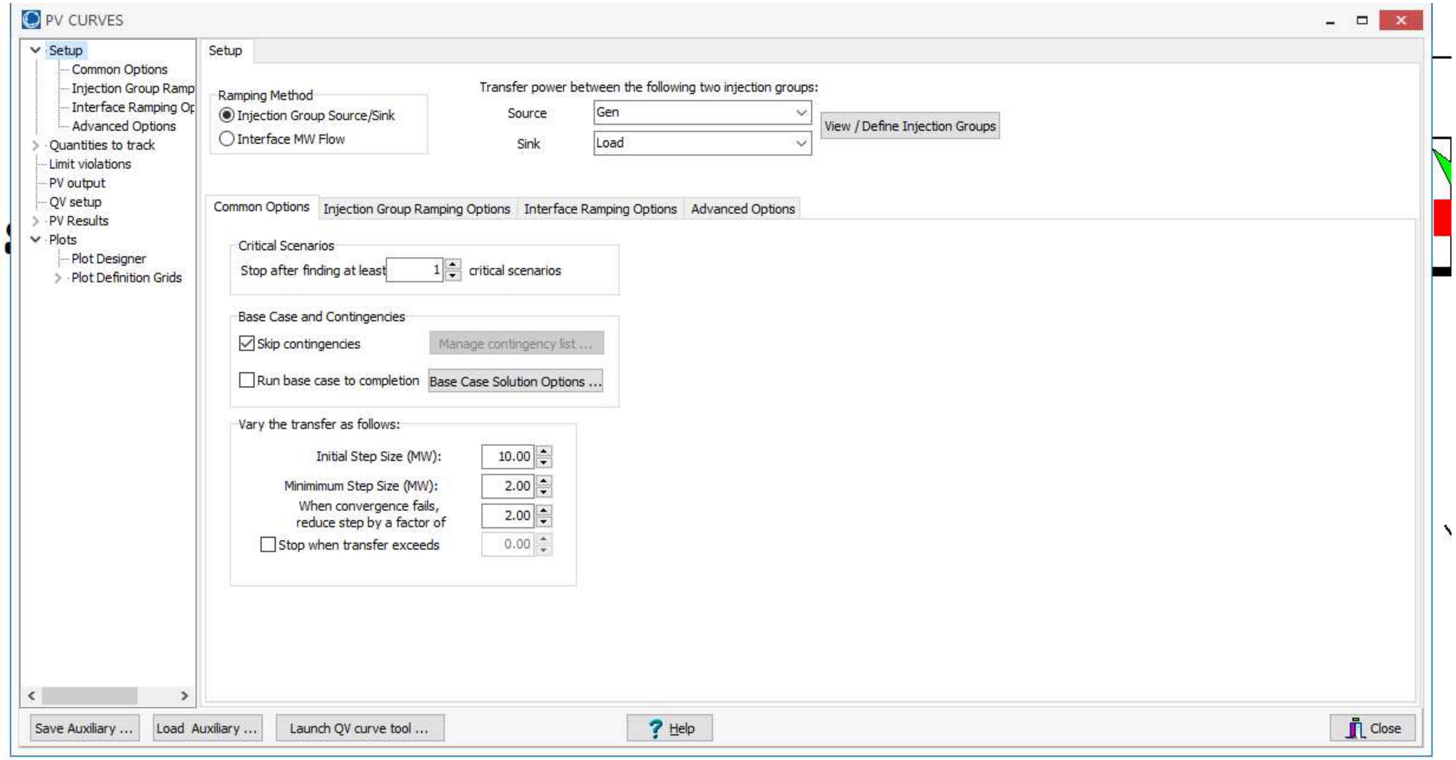
- The key issues is actual power systems are quite complex, with many nonlinearities. For example, generators hitting reactive power limits, switched shunts, LTCs, phase shifters, etc.
- Practically people would like to know how far some system parameters can be changed before running into some sort of limit violation, or maximum loadability.
  - The system is changing in a particular direction, such as a power transfer; this often includes contingency analysis
- Line limits and voltage magnitudes are considered
  - Lower voltage lines tend to be thermally constrained
- Solution is to just to trace out the PV or QV curves

PV and QV Analysis in PowerWorld

- Requires setting up what is known in PowerWorld as an injection group
  - An injection group specifies a set of objects, such as generators and loads, that can inject or absorb power
  - Injection groups can be defined by selecting **Case Information, Aggregation, Injection Groups**
- The PV and/or QV analysis then varies the injections in the injection group, tracing out the PV curve
- This allows optional consideration of contingencies
- The PV tool can be displayed by selecting **Add-Ons, PV**

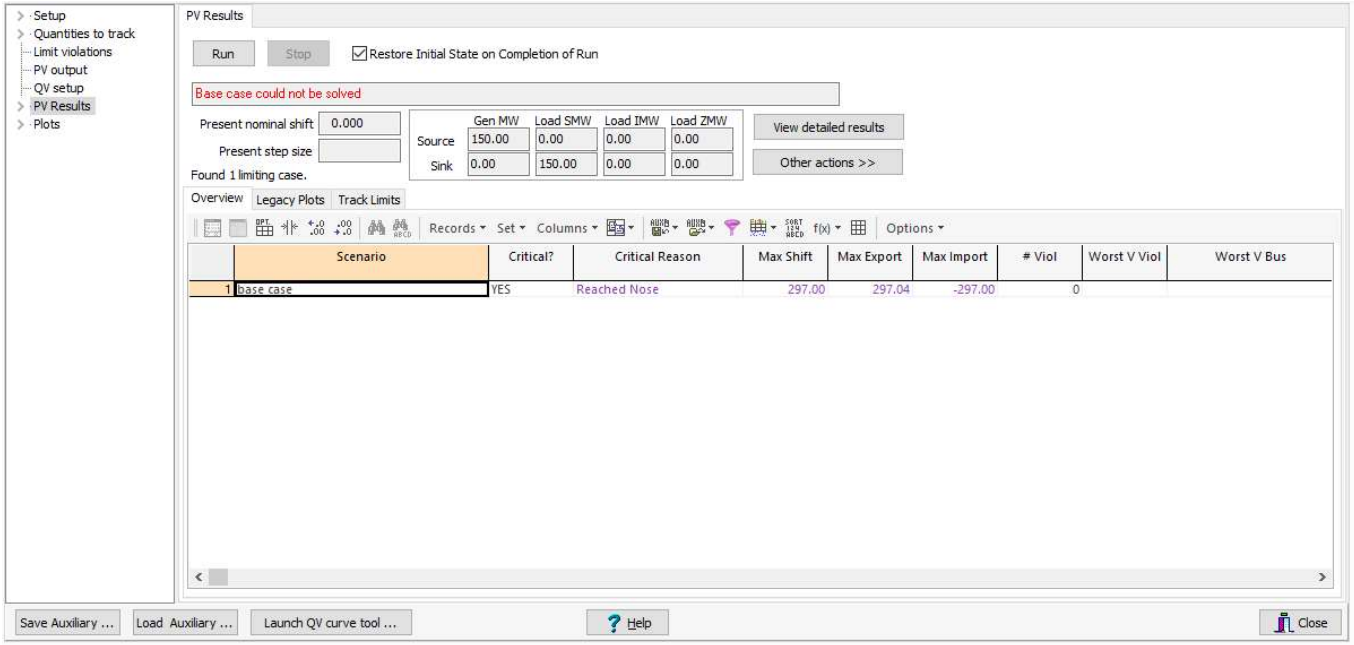
PV and QV Analysis in PowerWorld: Two Bus Example

- Setup page defines the source and sink and step size



PV and QV Analysis in PowerWorld: Two Bus Example

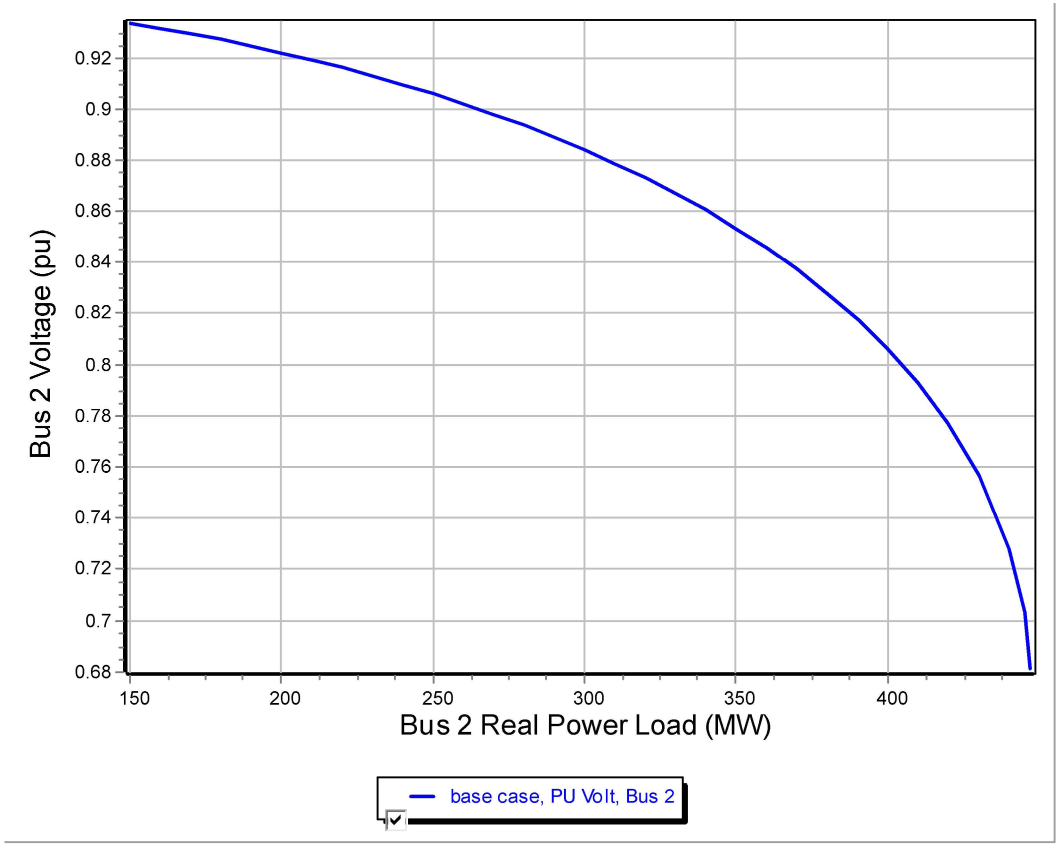
- The PV Results Page does the actual solution
  - Plots can be defined to show the results
  - **Other Actions**, **Restore initial state** restores the pre-study state



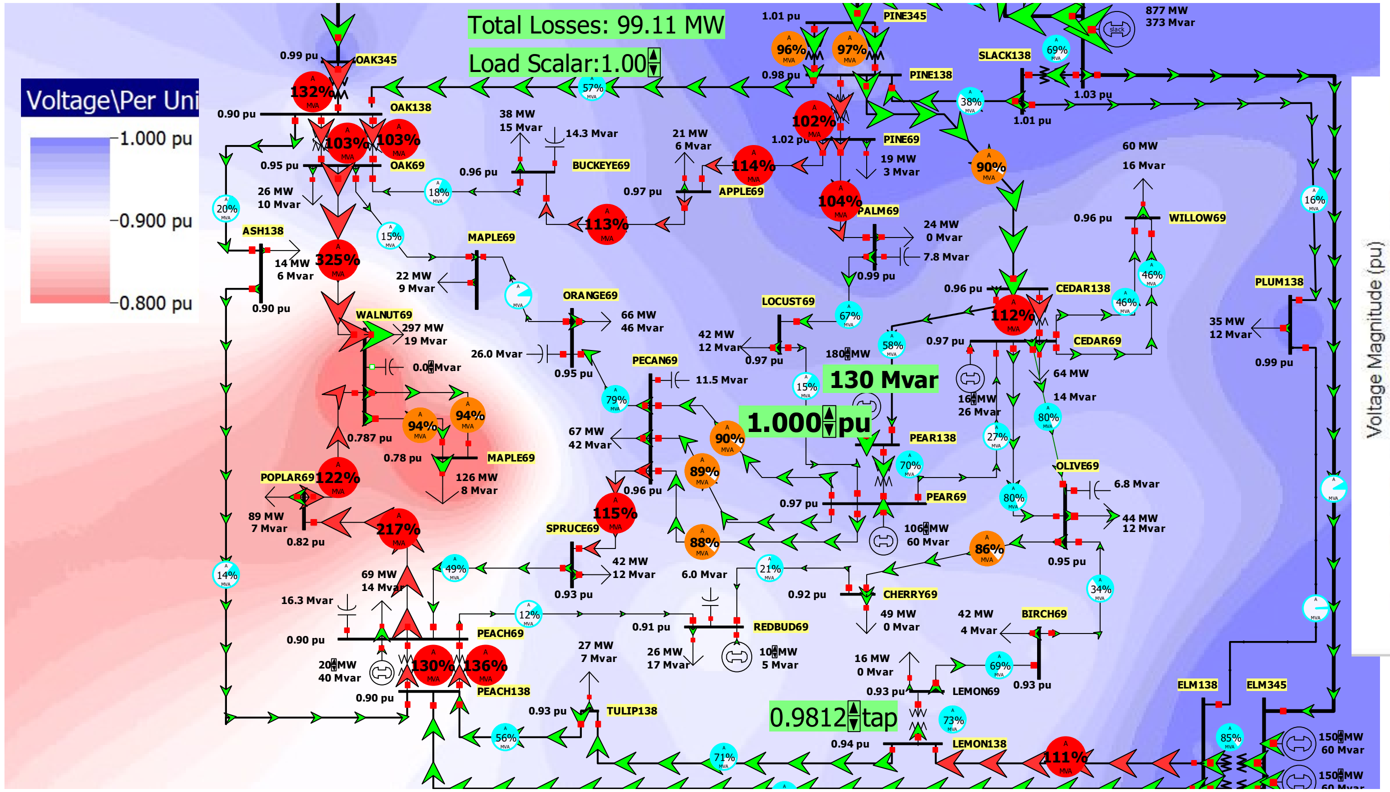
Click the Run button to run the PV analysis;

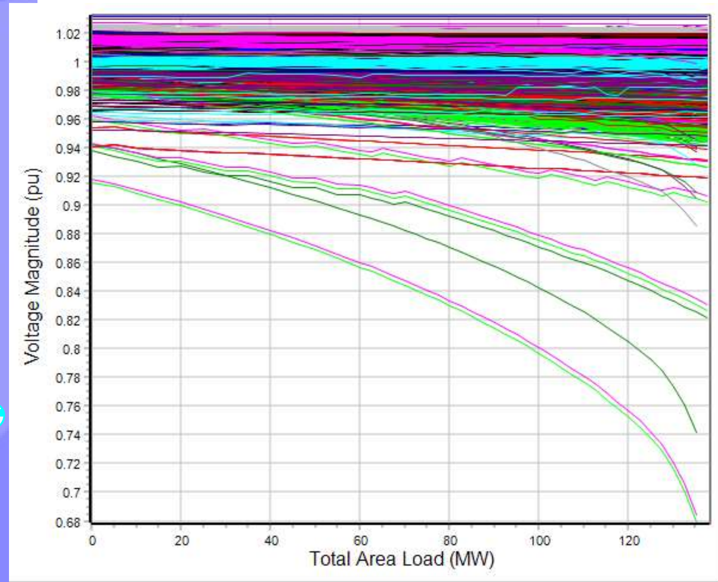
Check the **Restore Initial State on Completion of Run** to restore the pre-PV state (by default it is not restored)

PV and QV Analysis in PowerWorld: Two Bus Example



PV and QV Analysis in PowerWorld: 37 Bus Example





Usually other limits also need to be considered in doing a realistic PV analysis

Power System Economic Dispatch

- Generators can have vastly different incremental operational costs
  - Some are essentially free or low cost (wind, solar, hydro, nuclear)
  - Because of the large amount of natural gas generation, electricity prices are very dependent on natural gas prices
- Economic dispatch is concerned with determining the best dispatch for generators without changing their commitment
- Unit commitment focuses on optimization over several days. It is discussed in Chapter 4 of the book, but will just be briefly covered here

Unit Commitment: Quick Coverage (Chapter 4)

- Unit commitment is used to determine which generator units should be committed to meet the load
- The electric load varies substantially so there is almost always more generator capacity available than load
- Units have availability constraints
  - Minimum up time, time to start, cost to start
  - Minimum down time, time to shutdown, cost to shutdown
  - Ramp rates, minimum MW output
  - Scheduled and unscheduled outages
- System constraints including load, reserve, emissions, network

Solving Unit Commitment

- Unit commitment involves a potentially large number of integer and continuous variables
  - Not just the status of each unit, but also the amount of time it has been in a particular state (i.e., off or on)
- Solved for a set of discrete time periods, which at each time period there are lots of different potential states
- Solution approaches include
  - Dynamic programming
  - Lagrangian relaxation
  - Mixed Integer Programming (currently state-of-the-art)

Longer Term Optimization: Quicker Coverage (Chapter 5)

- Longer term optimization is a key consideration in hydro systems with significant reservoir storage
  - Use the water when it is the most valuable taking into account potentially many other constraints
- Generator maintenance scheduling
- Building generation often involves large upfront capital costs to create an asset that will last 20 to 40 years; long-term contracts provide a way to share the risk
- Take-or-pay contracts obligate a purchaser to purchase so much of a product over a given time period

Example: Prairie State Energy Campus

- The Prairie State Energy Campus (PSEC) is a 1600 MW coal plant in Southern Illinois with its own coal mine that opened in 2012
  - It is owned by municipals and coops (my former coop got >60% of the energy from PSEC)
  - While relatively efficient, it is one of the US's largest sources of CO_2 emissions
  - It cost an estimate $4 billion to build; if it sells its power at $30/MWh then maximum yearly income would be $30*1600*8760=$420 million

- Illinois's new clean energy law requires PSEC to reduce carbon emissions by 45% by 1/1/35 and be 100% carbon free by the end of 2045

Image: Pantagraph.com