Abstract—Visualization of electric power transmission systems centers on the one-line diagram, which is a simplified logical depiction of the circuit’s connections and elements. Such a graph drawing is useful for supporting the study, analysis, and presentation of power system data. This paper proposes an automated way to generate one-line diagrams, given substation identifiers with geographic tags and the system bus-branch model. Two approaches to drawing the substations are given: a force-directed approach and a greedy approach, which in different ways modify the position and size of substations as drawn. For transmission line routing between substations, a Delaunay-based method is employed, which segments the transmission line drawings to avoid overlapping with substations and other lines. These methods show good visual properties in spacing, geographic context, relatively straight lines, and computational speed. The result is that large power systems, real and synthetic, can be easily and quickly visualized with a diagram that merges geographic context with logical clarity.

Index Terms—Delaunay triangulation, force-directed graph drawing, one-line diagrams, power system analysis, power system visualization, synthetic power grids.

I. INTRODUCTION

DiAGRAMS for electric power networks aid system analysis; they complement numerical data with a visual context. One-line diagrams are often carefully maintained through extensive labor, detailing system substations, generators, loads, and the transmission branches which connect them. Engineers studying a transmission system which has a high-quality system diagram have an additional platform to diagnose problems, skim large datasets for abnormalities, and communicate their results clearly. The target platform here is a computer-navigated diagram that can be zoomed and panned in planning and analysis software. The National Academy of Engineering has recognized the need for improved visualization techniques [1].

This paper addresses the problem of quickly creating automated single line diagrams for large bus-branch power system models, with substations defined and geographically tagged.

The present work employs graph drawing approaches in a geographic context to arrange substations and route transmission branches. The methodology is useful for a quick look at power flow cases which do not otherwise have a diagram, such as those which are dynamically created by an analysis tool or synthetic grid-making algorithm. It can also provide an excellent starting point for higher-grade diagrams of networks real or synthetic.

In drawing a single line diagram, physical correctness should balance logical function. The approach of this paper reflects physical layout mainly in the substation positioning, keeping the substations drawn in a similar spatial arrangement as they are on a map. Latitude and longitude data is now becoming routinely available, driven in part by geomagnetic disturbance studies [2]. The challenge addressed here is to implement this balance in placing and sizing the substations, with enough room to show the substation configuration and interconnections and minimal distortion to the geographic positioning.

The second part of the paper’s methodology, the line routing algorithm, puts the system bus and branches in and between the drawn substations. This step does not consider the actual right-of-way positioning of the transmission lines, since the system functionality can be displayed well without it, and often this data is not available. The objective in this step is to route the branches to show the network topology in a way which is visually understandable and logically accurate. The approach builds on previous work, with a geometric-based method for transmission line branches which relies on the Delaunay triangulation.

Section II provides background references to previous work in the fields of graph drawing, power system visualization, and synthetic power grids. Then Section III and Section IV detail the algorithms for drawing substations and transmission lines, respectively. In Section V four large diagrams built with these methods are evaluated for quantitative visual quality metrics, leading to the conclusion in Section VI.

II. BACKGROUND

Ways to draw a graph are as diverse as the types of graphs there are to be drawn. From computer networking and social networking to life sciences and microelectronics, researchers of all sorts have graphs complex and informative, and they need them to be visualized [3]. The method depends on the properties of the graph, and involves trade-offs between the desired characteristics of the drawing and computational complexity [4].
The force-directed approach to graph drawing [5], [6] shows how an analogy to the physics of particle interactions can be used to position nodes in a way that shows graphs aesthetically. These drawings show edges as straight lines and vertices as freely-mobile points that come to a visually pleasing equilibrium where edge distances are short but node spacing is comfortable. Extensions to this approach abound, for better computation [7], clustered graphs [8], graphs with many edges that should be bundled [9] or spaced [10]. Methods for non-straight lines include using general segmented paths [11], curves [12], and orthogonal segmented paths [13]. Computational geometry concepts such as the Delaunay triangulation are sometimes used for geometrically-constrained methods [14]. With large graphs, interactive viewing that scales between levels [15] can be an appropriate strategy. Keeping aesthetics in mind, competing objectives for a graph drawing include symmetry, minimal crossing, and minimal bends [16], [17].

Power system models are graphs of various kinds, depending on whether transmission or distribution is considered, whether the representation is bus-branch or node-breaker, and any equivalencing done in a particular case. Studies of the graph theory properties of power networks show that power systems are sparse connected graphs, with an exponential node degree distribution averaging 2–3, a high clustering coefficient and a small topological diameter [18]–[21]. Recent work has documented that transmission lines have a short and consistent distribution of topological distance along the geographic Delaunay triangulation graph, since power systems are geographically constrained [22], [23].

In power systems visualization, early work on one-line diagrams includes [24], which discusses the usefulness of diagrams of individual substations for evaluating the security of these substations. One approach is to automatically visualize the neighborhood of a bus of interest [25], to help a user get the context of the issue being addressed. One-line diagrams can be augmented with additional visualization techniques [26] to show line loading, power transfer distribution factors, and voltage magnitude contours superimposed on the network. For node-breaker models, [27] and [28] propose methods to diagram substations automatically. For distribution systems, which are more radial in nature, and numerous, [29] proposes a layout technique that scales up well. Reference [30] points out the benefits of hierarchical diagramming with relative coordinates. Some work applies methods similar to the force-directed graph drawing approach to draw networks [31], including using geographic information to initialize the positioning [32]. Reference [33] discusses power-flow related graphical objectives that improve the utility of diagrams, and [34] uses a multidimensional scaling of electric distance metrics to visualize a network’s power flow characteristics.

Several of the example systems shown in this paper are synthetic power grids—fictional test cases built algorithmically to match characteristics of actual grids, avoiding any confidentiality concerns [22], [23]. The evaluation test cases in this paper include the 2000 bus test case on the footprint of Texas [23] and large 10,000 and 25,000 bus test cases with the same method. None of these grids corresponds to any actual grid, and they are available publicly online [35].

III. SUBSTATION DRAWING AND SPACING

A. Challenges in Spacing and Size

Drawing substations as rectangles in their geographic location works for much of the system, but challenges come when substations are large enough and close enough together that they overlap. Substations are drawn much larger than they actually are, and to show rural substations without extensive zooming and whitespace requires a size that in urban areas results in an indiscernible mess.

The automated approaches discussed by this section determine the substation size and location to best display the functionality of the grid while keeping geographic context. That is, they maximize visual spacing between neighboring substations while minimizing distortion from their actual coordinates. Two approaches are discussed: one which is based on the force-directed graph drawing approach (Section III-C) and one which uses a greedy approach (Section III-E).

Note that substations are drawn on a coordinate system corresponding to a standard unit Mercator projection, where (0, 0) is the intersection of the prime meridian with the equator, the longitude scales linearly to ±1 at the international date line, and the latitude scales non-linearly to ±1 at about the eighty-fifth parallels.

B. Internal Layout of Substations

The spacing algorithm must know as an input the dimensions of the substation, which depends on how the substation is laid out internally. The substation will be scaled from there, but it is better to scale all or most of the substations uniformly so that the features appear the same size on a diagram. In addition, for routing the transmission lines in Section IV, this internal layout will determine the starting points for entering and exiting the substation.

The bus-branch data for substation configuration is often aggregated from the underlying node-breaker model, so no attempt is made to show the spatial layout of the substation (this is not always available anyway). Rather, the logical layout is used. Thin rectangles are the typical symbol for buses, and lines of varying thickness and color show the branches, aggregating three-phase conductors and any neutral wires. Branches which are transformers are shown symbolically, and loads, generators, and shunt devices are indicated with symbols attaching to the corresponding bus. These conventions are all retained in the present approach, with a box identifying the substation. This box contains all elements in the substation, with inter-substation branches crossing the boundary, as shown in Fig. 1.

As in [26], the internal drawing of each substation is decoupled; that is, each one can be done independently. Although a co-optimization with the line routing might improve the routing by determining, for example, which side of the substation a line ought to exit, such considerations can also make the internal...
The force-directed approach is a general method common to graph drawing problems [6], [7] that models each vertex as a particle subject to physical forces, which, when simulated to equilibrium, balance the various constraints desired for a graph drawing. The common basic formulation is to use a Coulomb-like inverse square repulsion force between vertex-particles to enforce spacing and a Hooke-like spring force attracting connected nodes. The problem addressed by this paper is different in that a reasonably good starting spot is known for each substation, and a key constraint is to minimize its displacement from that point. So the formulation proposed here is to keep the Coulomb repulsion and add a Hooke attraction between each point and its actual location. As is typical, static equilibrium is desired so the forces are modeled as inducing velocity rather than Newtonian acceleration. The algorithm steps are as follows:

Initialize each substation for $i = 1 : N$

$$X_i = X_{i0}$$

$$Y_i = Y_{i0}$$

Loop of $M$ iterations

For $i = 1 : N$

$$F_{X_i} = C_1 \cdot (X_{i0} - X_i)$$

$$F_{Y_i} = C_1 \cdot (Y_{i0} - Y_i)$$

For $i, j \in [1, N], i \neq j$

$$F_{mag} = \frac{C_2}{\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2}}$$

$$F_{angle} = \text{ATan2}(Y_j - Y_i, X_i - X_j)$$

$$F_{xi+} = F_{mag} \cdot \cos(F_{angle})$$

$$F_{yi+} = F_{mag} \cdot \sin(F_{angle})$$

For $i = 1 : N$

$$X_{i+} = C_3 \cdot F_{xi}$$

$$Y_{i+} = C_3 \cdot F_{yi}$$

In the equations above, there are $N$ substations with properties for substation $i$ of position $(X_i, Y_i)$, original position $(X_{i0}, Y_{i0})$, and force $(F_{xi}, F_{yi})$. $F_{mag}$ and $F_{angle}$ are the magnitude and angle of the force. The parameters of the algorithm are $M$, the number of iterations, $C_1$, the Hooke constant, $C_2$, the Coulomb constant, and $C_3$, the mass constant. An experimental process was used to tune the algorithm constants, using the 10,000 bus test case. Setting $C_1 = C_2 = R$, where $R$ is the desired spacing between substations (in the same units as $X$ and $Y$), allows the quadratic repulsion to balance the linear attraction at approximately the desired spacing. $C_3$ affects the convergence and speed, with 0.1R being the final value used in this paper. Larger values can cause divergence of the algorithm, and smaller values require more iterations to converge. This implementation converged for about $M = 100$. To reduce the $N^2$ computational order of the repulsion force checking, each particle need only check neighbors within a radius of $6R$.

Figs. 2 and 3 illustrate the original spacing and results after the force-directed approach comes to an equilibrium.

D. Multiple Scales

The size of substations shown in Fig. 2 is good for most of the system, where substations are sparse, but for crowded areas (such as Fig. 2 shows) the substation locations must be either significantly distorted in their location or shrink to allow viewing them and routing lines between them. This section discusses a shrinking process in preparation for the greedy approach discussed in Section III-E.

A substation’s size is defined in blocks, where, for example, the substations of Fig. 1 are sized 12-by-15 and 20-by-30 blocks. The scale of a substation defines how large a block is on the screen. The default scale is 1.0e-5, measured on the unit Mercator projection. This scale is such that at the latitude of San Francisco a 12-by-15 substation would cover about 1.2-by-1.5 km of map space. Two other scales are defined at 4.0e-6

Fig. 1. Example substation internal layouts from 10 K case [28], showing buses, lines, transformer, shunts, generators, and loads.
Fig. 2. Original substation placement for a portion of the synthetic 10,000 bus case, showing rectangles overlapping and crowded at many points. Black lines show outline of San Francisco Bay for context.

Fig. 3. Force-directed approach to spacing the substations of Fig. 2. and 1.5e-6, about two-thirds reduction in size each, for crowded parts of the grid.

The scale for each substation is determined by a simple check of the number of neighboring substations in a square radius of 2.5e-3 (on unit Mercator measure, approximately equal to 25-by-25 km), reducing to the second level if there are 30 substations in this area and to the third level if there are at least 200. Then the differences in size only appear at the intersection of the crowded regions and sparse regions. At an appropriate zoom level, similar elements will appear the same size within these regions. This method is general enough to be appropriate for various densities in transmission and sub-transmission modeling.

E. Greedy Approach to Substation Spacing

Once an appropriate scale for each substation has been found, the following algorithm is used to place each substation. It takes the greedy approach of assigning a location for each substation closest to its actual location without violating a buffer region around any of its neighbors that have already been placed.

For each substation for $i = 1 : N$

For $j, k \in [-10, 10]$

$X_{iT} = X_{i0} + j \cdot R$

$Y_{iT} = Y_{i0} + k \cdot R$

If $(X_{iT}, Y_{iT})$ intersects with spacing buffer of any already-placed substation $(X_i, Y_i) = (X_{iT}, Y_{iT})$

only if closer than existing and no feasible point found yet

Else continue

Else if $(X_{iT}, Y_{iT})$ is closer to $(X_{i0}, Y_{i0})$

than $(X_i, Y_i)$ is

$(X_i, Y_i) = (X_{iT}, Y_{iT})$

In this algorithm, 121 points $(X_{iT}, Y_{iT})$ are tested for overlap with already-placed substations and the closest acceptable one to the original location is chosen. An example of the results is shown in Fig. 4. As will be shown in Section V, this method has several advantages over the force-based approach, and is the one selected when continuing to the transmission line routing algorithm of the next section.

IV. TRANSMISSION LINE ROUTING

A. Goals and Preliminary Considerations

Given substations properly spaced with the greedy algorithm above, the next step is to draw the transmission lines which connect them. If straight lines are drawn to connect them, these lines
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Fig. 5. A straight-line approach to drawing transmission lines, showing the crowding and overlap that can occur.

Fig. 6. A Delaunay approach to drawing transmission lines, which avoids overlapping substations but causes too many bends in the high-voltage lines.

Fig. 7. A Two-layer Delaunay approach to drawing transmission lines, which avoids both overlapping substations and excessive bends in the high-voltage lines.

Fig. 8. A Delaunay triangulation of substations and a transmission line’s route through the channels. The red dots are waypoints which the channels will adjust to improve the spacing.

can cross over substations and produce a web that is challenging to understand. The goals for line routing are to add segmented waypoints to avoid overlapping with substations and each other, while minimizing the increased length of the line and avoiding sharp bends. Figs. 5–7 show examples of the straight-line approach and the Delaunay approaches explained below.

One heuristic that it used as a first step is applied when a line is headed the opposite direction as its exit from the origin substation. Since the line must route around its origin anyway, a first orthogonal corner is made to route the line north or south around the substation.

B. Delaunay-Based Approach

From computational geometry, the Delaunay triangulation is a planar graph which connects geometric points into nicely-shaped triangles, where nicely-shaped is defined as having the triangle’s circumcircle empty of other points. This algorithm uses segments of the substation Delaunay triangulation as routing channels through which the spacing of the lines routes is managed and the lines avoid intersecting with the substations. The steps are given below:

1) Set Up the Delaunay Triangulation: The Delaunay triangulation can be computed for a set of points (in this case the substation centers) in $O(n \log n)$ time. Each edge of this graph is established as a routing channel that will record lines which pass through it, so in the data structure it is connected with its neighboring edges on each end.

2) Route Lines Straight Through the Delaunay Triangulation: Lines have an initial point which is located near to a substation within a triangle in the Delaunay triangulation. The next step is to trace each line’s path through the triangulation, inserting a waypoint at each intersection with a Delaunay edge. This path can be traced quickly since from each waypoint there are only two possible edges which could form the next one. Fig. 8 illustrates this step.

3) Adjust the Waypoints to Ensure Good Spacing: Next, the routing channels, which correspond to Delaunay edges, are each analyzed in turn. There may be one or more waypoints registered on it, and the goal is to minimize the changing of these waypoints while keeping a buffer around the substations on either end and to keep multiple lines adequately spaced along the corridor. Ordered from one end of the channel, each waypoint is given the spot closest to its desired location (given by the point along the straight line between its two adjacent waypoints on the same transmission line) which is acceptably far from either substation and any already placed line. Thus this step takes a greedy approach to spacing the channels.

4) Iterate the Process: Step 3 is repeated at least 10 times, allowing neighboring channels to iteratively coordinate.

C. Two-Level Delaunay Approach

The results of the Delaunay approach can be seen in Fig. 6, where the lines are routed to avoid collision with each other and the substations. But the high-voltage lines (light green and orange) snake back and forth to avoid lower-voltage substations, which causes too much distraction for the more important visual
components. The approach proposed by this paper does the algorithm of Section IV-B twice: once using only extra-high-voltage (200 + kV) substations and lines, then again using all substations and remaining lines. The result (Fig. 7) has a much more understandable high-voltage network without much cost in overlap.

V. RESULTS AND EVALUATION

A. Cases Studied and Performance

This section evaluates the one-line diagrams built by the algorithms of Sections III and IV. Three large synthetic test systems are used: the ACTIVSg2000 case [23], with 2000 buses situated on the geographic footprint of Texas; the ACTIVSg10 K case with 10,000 buses on the western United States; and ACTIVSg25 K, 25,000 buses on the northeastern United States. In addition, a diagram was built for an actual planning case of the Eastern Interconnect in North America, which is modeled with 62,605 buses. The three synthetic systems are available with their diagrams online [35].

The algorithms above were implemented to write diagram text files for data exchange with various power flow and visualization packages. Table I gives computation times for the four cases with each of the algorithms discussed above. In the substation spacing algorithm, the force directed method is slower because each of the iterations is approximately $O(N \log N)$ and there are at least 100 iterations. The greedy algorithm is faster, and scales about linearly. For the line routing algorithm, the algorithm runs $O(L + N \log N)$ where $L$ is the number of transmission lines.

While computation time is not a crucial concern in this offline problem, it is clear that all four algorithms are reasonably fast for building diagrams and scalable to the largest size systems.

B. Evaluation Metrics for Substation Spacing

Substation spacing involves balancing two metrics: displacement from a substation’s original location and spacing between it and a nearest neighbor. The first is examined in units of km, since there is an intuitive grasp of how far on the map that corresponds to. Regardless of the drawing scale, the desired closeness to the original location remains constant. The spacing itself is measured relative to a substation’s drawn size; that is, the center-to-center distance to the nearest neighbor substation divided by the shorter rectangle side.

Figs. 9 and 10 show substation displacement and spacing for the four diagrams. Fig. 9 shows that the greedy algorithm is much less disruptive to the geographic context, with substations never moving more than 5 km from their original location and only 3% at most for each case moved more than 1 km. More than 90% of substations in each case are not moved at all with the greedy approach. The force-directed algorithm keeps all substations the same size, but that comes at the cost of more significant displacement. Almost no substations remain untouched by that algorithm, which moves some substations up to 30 km away.
Fig. 11. Distribution of transmission line overlap. The lines are plotted by fraction of overlap with substations, in sorted order. Dotted lines show the original overlap, dashed lines and solid lines show the Delaunay algorithm one-level and two-level.

<table>
<thead>
<tr>
<th>METRICS OF THE LINE ROUTING ALGORITHM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACTIVSg 2000</td>
</tr>
<tr>
<td>No. Lines</td>
</tr>
<tr>
<td>Overlaps, original</td>
</tr>
<tr>
<td>Overlaps, Delaunay</td>
</tr>
<tr>
<td>Bends, Delaunay</td>
</tr>
<tr>
<td>Bends, Two-level</td>
</tr>
<tr>
<td>Sharp bends, Delaunay</td>
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<tr>
<td>Sharp bends, Two-level</td>
</tr>
</tbody>
</table>

Fig. 10 shows in the dotted lines how congested the original layout is. Hundreds of substations are closer to a neighbor than 0.2 times their own width. The dashed lines show that the force-directed method has some success in reducing this clutter, but on the two largest cases still has a few dozen substations that end up quite crowded. The greedy algorithm, shown in solid lines, never goes below 0.9, keeping all substations adequately spaced.

C. Evaluation Metrics for Line Routing

In transmission line routing, one goal is to reduce the overlap with substations. So two metrics are studied for this: the number of substations crossed by a line and the percent of a line which crosses other substations. Balancing this objective are two considerations met perfectly by a straight-line approach: the relative line length and number of bends. The relative length is the ratio of the transmission line’s drawn path length to the straight-line distance between its two endpoints. Bends are evaluated as any waypoint with an angle of more than one degree. Sharp angles are defined as more than sixty degrees, which should be especially avoided.

Fig. 12. Distribution of transmission line drawn length, relative to a straight-line distance. The lines are plotted in sorted order. Dashed lines and solid lines show the Delaunay algorithm one- and two-level for the four cases.

Fig. 13. Distribution of transmission line bends. The lines are plotted by number of bends in sorted order. Dashed lines show the single-level Delaunay algorithm and solid lines show the two-level Delaunay algorithm for the four cases.

Figs. 11–13 and Table II show these metrics for the four diagrams. The overlaps between transmission lines and other substations are numerous in the original method, as shown in both the dotted lines of Fig. 11 and the third row of Table II. These are reduced significantly by the Delaunay approach, regardless of whether a one-layer or two-layer method is used. In fact, the number of overlaps went slightly down with the two-layer approach, since despite the fact that higher-voltage transmission lines were not trying to avoid lower substations, they also did not crowd the routing channels.

The Delaunay approach does have a visual cost, in that the lines are longer and bends are introduced relative to a straight-line approach. The plot of normalized length in Fig. 12 shows that most lines are not longer than about 120% of their original length, and even in the largest cases only a hundred or so are...
more than double that length. There are many bends, as Table II shows, but most are less than 60°. Fig. 12 shows the main quantitative difference between a one-level and two-level approach, in that the lines which originally had the most bends are in general straighter—these tend to be the long, extra-high-voltage lines.

VI. CONCLUSION

This paper presents several new techniques for automatically drawing large-scale power system diagrams for viewing in computer simulation and planning software. Though the force-directed approach is effective for spacing similarly-sized substations, a greedy approach that resizes groups of substations and then places each one in the best available slot seems to be preferable in both computation speed and visual effect. This greedy approach enforces desired spacing with minimal distortion to geographic context. The Delaunay approach for line routing given in this paper cuts down on the overlapping congestion without introducing too many bends or making the lines too long. This paper’s methodology shows many benefits for auto-generating single line diagrams in power systems. These visualizations can serve as a quick look at a system or serve a starting point for building a detailed diagram, and the properties of speed and visual appeal can aid engineers in understanding and interpreting the cases they work with.

REFERENCES


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