Geomagnetically Induced Current Sensitivity to Assumed Substation Grounding Resistance

Uyen Bui, Thomas J. Overbye, Komal Shetye, Hao Zhu
University of Illinois at Urbana-Champaign
Urbana, IL, USA
uyenbui2@illinois.edu, overbye@illinois.edu, shetye1@illinois.edu, haozhu@illinois.edu,

James Weber
PowerWorld Corporation
Champaign, IL, USA
weber@powerworld.com

***Abstract*—This paper considers the sensitivity of the calculated geomagnetically induced currents (GICs) in an electrical power system model to the assumed substation grounding resistances. Substation grounding resistances are not contained in standard power system models, and approximate values are often used in GIC studies. The paper provides an algorithm to quantify the degree of dependence of the GICs at any given substation. Case study results using the 20-bus GIC test system and a model of the North American Eastern Interconnect indicate that the substation GICs can be quite dependent on the assumed substation grounding values.**

I. Introduction

Geomagnetic disturbances (GMDs), caused by solar activity, can impact the power grid by producing quasi-dc (with frequencies much below 1 Hz) geomagnetically induced currents (GICs) to flow in the high voltage transmission grid [1]. As noted in [2] GICs can cause half cycle saturation in the transformers producing harmonics and increased reactive power demand. If sufficiently severe this can lead to loss of reactive power support leading to voltage collage, and increased transformer heating resulting in possible equipment damage.

The reactive power impacts of GICs on the power grid can be modeled in either the power flow [3], [4], [5] or the transient stability applications [6]. This is done by first modeling the GMD-induced electric field variation in the power grid as dc voltage sources in series with the transmission lines [7]. The GICs in the system can then be calculating by solving

$$\mathbf{V} = \mathbf{G}^{-1}\mathbf{I} \quad (1)$$

where vector **I** models the impact of the GMD-induced dc line voltages as Norton equivalent dc current injections. The structure of **G** is similar to the power system bus admittance matrix except 1) it is a real matrix with just conductance values, 2) conductance values are determined by the parallel combination of the three individual phase resistances, 3) **G** is augmented to include the substation neutral buses and substation grounding resistance values, 4) transmission lines with series capacitive compensation are omitted since series capacitors block dc flow, and 5) transformers are modeled with their winding resistance to the substation neutral in the case of autotransformers.

When solved, the voltage vector **V** contains entries for the s substation neutral dc voltages and the m bus dc voltages. Throughout this paper it is assumed that the s substations are ordered as the first entries in **V**, and the m buses are ordered as the s+1 to s+m entries. The **V** vector can be used to calculate all the GICs in the system. The coupling between the GICs and the positive sequence model in the power flow or transient stability is accomplished by representing the GIC-related transformer reactive power losses as a function of the GICs through the transformer as in [1], [8], [9], with [10] noting that reactive losses vary linearly with the terminal voltage.

This paper focuses on the sensitivity of these GICs to the modeling assumptions used to determine **G**. Much of the data needed to setup this matrix is already contained in standard power flow models, or can be readily estimated. Such data includes the network topology, the bus voltage levels, the resistance of the transmission lines, and the presence of transmission line series compensation. For transformers, the power flow model contains the total series resistance of the transformer but does not contain the resistance of the individual windings. When available the actual winding resistance should be used. Otherwise the individual coil winding resistances can be easily estimated using the approach from [5]. Transformer winding configurations (e.g., wye or delta) and grounding are not usually included in the power flow model, but they can either be determined from short circuit data or by estimated.

However, the substation grounding resistance, a key piece of information needed to construct **G,** is usually not

The authors gratefully acknowledge support for this work provided by the US Department of Energy through "The Future Grid to Enable Sustainable Energy Systems: An Initiative of the Power Systems Engineering Research Center (PSERC)" and the Illinois Center for a Smarter Electric Grid (ICSEG).

accurately available. The substation grounding resistance field is used to represent the effective grounding resistance of the substation, consisting of its grounding mat and the ground paths emanating out from the substation such as due to shield wires grounding [2,9]. This grounding resistance is calculated using the Fall of Potential Method and procedures as described in IEEE Standard 81-1983 [11] and the revision 81.2-1991 [12]. There is an excellent presentation available on this topic in [13]. The grounding resistance is less a function of the construction of the substation grounding grid than it is a function of the local soil and earth conditions. Resistances can vary by more than an order of magnitude from 0.05 to 1.5 ohms.

Therefore the focus of this paper is on determining the sensitivity of the resultant GICs to the assumptions about these values. Previous work on this topic is contained in [14], and [15], with [14] providing test results on a model of the Finnish 400 kV grid and [15] providing a theoretical derivation and a test system model. The present paper builds on these results by using the concept of driving point impedance to quantify the dependence on the GICs on substation grounding resistance, using a sparse matrix/vector formulation.

The paper is organized as follows. Section II provides problem motivation and an algorithm for single substation parameter sensitivity. Section III presents a methodology for determining the sensitivity of the GICs to variation in the substation grounding resistance at a single substation, providing results for both the 20-bus GIC test system of [16] and a 62,500 bus model of the North America Eastern Interconnect (EI). Section IV then addresses the issue of sensitivity to the variation of the assumed grounding resistance at multiple substations. The last section provides a summary and directions for future research.

II. Motivation and Single Substation Algorithm

To motivate the problem, consider the two generator, four bus network shown in Fig. 1 with Bus 1 and its generator (Bus 3) in Substation A, and Bus 2 with its generator (Bus 4) in Substation B. Buses 1 and 2 are connected by a 765 kV line that has a per phase resistance of 3Ω, the per phase resistance of the high side (grounded side) coil of each of the two transformers is 0.3Ω, a grounding resistance of 0.2 Ω for Substation A and 0.3Ω for Substation B. Assume the substations are at the same latitude, separated by 150 km in the east-west direction, with a given electric field of 1 V/km in the east-west direction. This gives an induced voltage in the transmission line of 150V.

The GICs can then be determined by solving a simple dc circuit. From a GIC perspective, the three phases for the transmission line and transformers are in parallel, so the total three phase resistance for the 765 kV line is (3/3)Ω = 1Ω, and (0.3/3)Ω = 0.1Ω for each of the transformers. These resistance values are then in series with the Substation A and B grounding resistance, which leads to

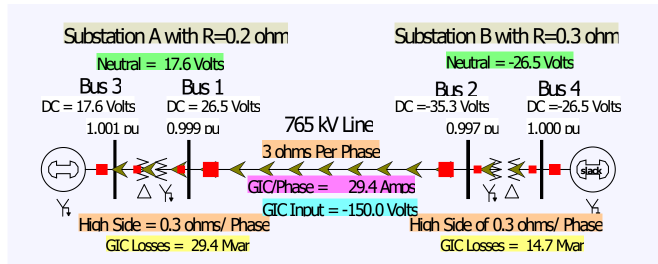

Fig. 1 Two substation, four bus GIC example

$$I_{GIC} = \frac{150\,\mathrm{V}}{(1+0.1+0.1+0.2+0.3)\Omega} = 88.26\,\mathrm{A} \qquad (2)$$

The 88.26A result gives the total current in all three phases (i.e., 29.4A per phase). This current flows from ground through the Substation B grounding resistance into the high side coil of the Substation B transformer, down the 765 kV line into the high voltage coil in Substation A and back into the ground through the Substation A grounding resistance. In the Fig. 1, the direction and size of the arrows are used to visualize the direction and magnitude of the GIC flow.

Using (2), it is possible to obtain the sensitivity of I_{GIC} to each of the model parameters. Focusing on the sensitivity of I_{GIC} with respect to the Substation A grounding resistance, R_A, (2) can be rewritten as

$$I_{GIC,A} = \frac{150\,\mathrm{V}}{(1+0.1+0.1+0.3+R_A)\Omega} = \frac{V_{TH,A}}{(R_A + R_{TH,A})} \qquad (3)$$

where $V_{TH,A}$ is the Thevenin equivalent voltage looking into the network from Substation A, and $R_{TH,A}$ is the corresponding Thevenin equivalent resistance. For this example, their values are 150V and 1.5Ω respectively; $I_{GIC,A}$ is the current flowing into the ground through the Substation A resistance. The sensitivity of $I_{GIC,A}$ to the variation in the assumed value for R_A is calculated by differentiating (3) with respect to R_A,

$$\frac{\partial I_{GIC,A}}{\partial R_A} = \frac{-V_{TH,A}}{(R_A + R_{TH,A})^2} \qquad (4)$$

which shows that $I_{GIC,A}$ can be changed equally by a variation in either R_A or in $R_{TH,A}$.

However, an important observation is that these quantities are often known with potentially vastly different degrees of accuracy. The substation grounding resistance is often an approximation with a large degree of uncertainty. In contrast, $R_{TH,A}$ is mostly based on values known with a relatively high degree of precision, including the transmission line and transformer resistances. While [9] makes the important observation that wire resistance is temperature dependence, this variation of about 0.4% per degree C is known and therefore can be included in a study by using approximate temperature profile. Also, it is apparent in this simple example that $R_{TH,A}$ depends upon the assumed resistance of the other substation.

Using (4) we can obtain the sensitivity of the substation GIC to variation in assumed substation resistance in terms of a normalized variation. That is, the percent variation in the current in terms of the percent variation in the grounding resistance. For the general case of an arbitrary substation i, this can be expressed as

$$\frac{\partial(\% I_{GIC,i})}{\partial(\% R_i)} = \frac{\partial I_{GIC,i} / I_{GIC,i}}{\partial R_i / R_i} = \frac{-R_i}{R_i + R_{TH,i}} \tag{5}$$

Note that this sensitivity is always negative since an increase in assumed resistance will always result in a decrease in the magnitude of the current. Hence, it is convenient to define the negative of this ratio as

$$\Re_i := \frac{R_i}{R_i + R_{TH,i}} \tag{6}$$

If the $\Re_i$ is small (i.e., the Thevenin resistance is substantially larger than the substation resistance), as in this example, then an accurate estimate of substation resistance is less important. Conversely, if $\Re_i$ approaches unity then the value of the substation resistance dominates in the determination of $I_{GIC,i}$.

Returning to the four bus example,

$$\Re_A = \frac{0.2}{0.2 + 1.5} = 0.1176 \tag{7}$$

which indicates that a 1.0 percentage error in R_i results in a 0.1176 percentage error in $I_{GIC,A}$. So if the assumed value of R_A is increased by say 10% (from 0.2 to 0.22Ω) the magnitude of $I_{GIC,A}$ decreases by about (0.1176)*(10%) = 1.176%. Of course, this is only a linearization about the currently estimate of R_i. For example, if the value of R_A were assumed to increase by 100% (changing to 0.4Ω) the value of $I_{GIC,A}$ would only decrease by about 10.6% (to 78.9A).

Since the Thevenin equivalence resistance can be obtained for any substation in a network by calculating the diagonal element of $\mathbf{G}^{-1}$ corresponding to the substation, this approach can be generalized to systems of any size. Define the resistance matrix as

$$\mathbf{R} = \mathbf{G}^{-1} \tag{8}$$

Then, with the assumption that the s substations are ordered as the first s entries in **G**, the driving point resistance for the i^{th} substation is $\mathbf{R}_{ii}$. Since for large systems **G** is quite sparse, the diagonal elements of **R** can be calculated with great computational efficiently using sparse vector methods [17]. It is important to emphasize that the entire matrix **G** is never explicitly inverted!

Since the substation resistances are directly connected to ground, the driving point resistance is the parallel combination of substation resistance and the Thevenin resistance, given by

$$\mathbf{R}_{ii} = \frac{1}{\frac{1}{R_i} + \frac{1}{R_{TH,i}}} \tag{9}$$

where R_i is the grounding resistance of the i^{th} substation, and $R_{TH,i}$ is the Thevenin resistance looking into the network from the same substation. Solving (9) for $R_{TH,i}$ gives

$$R_{TH,i} = \frac{1}{\frac{1}{\mathbf{R}_{ii}} - \frac{1}{R_i}} \tag{10}$$

Then the Thevenin voltage for the i^{th} substation, $V_{TH,i}$, is given by

$$V_{TH,i} = \left(R_i + R_{TH,i}\right) I_{GIC,i}. \tag{11}$$

With the Thevenin voltage and resistance, the impact of assumed changes in substation resistance on the substation current are easily determined by solving

$$\tilde{I}_{GIC,i} = \frac{V_{TH,i}}{\left(\tilde{R}_i + R_{TH,i}\right)} \tag{12}$$

where $\tilde{R}_i$ is the new grounding resistance and $\tilde{I}_{GIC,i}$ is the new current.

Before demonstrating the matrix approach on the four bus system and moving on to larger systems, several observations are warranted. First, the Thevenin voltages are dependent upon the **I** vector used in (1), which means they do depend upon the particular GMD scenario under consideration. Second and conversely, the Thevenin resistances values are independent of **I**, depending only upon **G**. Third, the substations for which accurate resistance values are most needed are those that have both high GIC values and high ratio values ($\Re$).

To finish the four bus example, its G values are given in Table 1. Since the low-side generator buses (Buses 3 and 4) are delta-connected, they do not have an impact on the GICs and have been omitted from the table entries. The resultant derived values are shown in Table 2, with the driving point resistances determined by (8), the Thevenin resistances by (10), the ratio $\Re$ by (6), and the Thevenin voltages by (11).

Using these values the impact of changes in the assumed substation grounding resistance on the substation GIC can be easily calculated by (12). While the values are straightforward for this simple example, its usefulness is demonstrated in the next section on the twenty bus test system of [16] and a 62,500 bus EI model.

Table 1: **G** matrix (in Siemens) for the four bus system

	Sub A	Sub B	Bus 1	Bus 2
Sub A	15.00	0.00	-10.00	0.00
Sub B	0.00	13.33	0.00	-10.00
Bus 1	-10.00	0.00	11.00	-1.00
Bus 2	0.00	-10.00	-1.00	11.00

Table 2: Equivalent values for four bus system

	Sub A	Sub B
Grounding resistance (Ω)	0.2	0.3
Driving point resistance (Ω)	0.1765	0.2471
Thevenin resistance (Ω)	1.5	1.4
$\Re$	0.1176	0.1765
GIC (A)	88.2	-88.2
Thevenin voltage (V)	150.0	-150.0

The last topic to consider before moving on to the larger system examples is the relationship between a variation in the substation GIC and the GICs in the transformers. Since (1) is a linear system, superposition holds and network sensitivity factors [18, App. 11.A] can be used to determine the incremental change in the GIC voltages due to a change in the flow between the substation neutral and ground. Hence for a change in the substation i GIC the system-wide changes can be determined by solving,

$$\Delta \mathbf{V} = [\mathbf{G}]^{-1} \Delta \mathbf{I} \tag{13}$$

with $\Delta\mathbf{I}$ set to all zeros except the location corresponding to substation i. It is important to recognize that the substation neutral currents are the total for all three phases in parallel whereas the transformer GICs are usually reported as per phase (e.g., as in [16]). This issue is easily resolved by dividing the substation currents by a factor of three to get equivalent per phase values.

III. Application of Single Substation Algorithm to Larger Systems

The algorithm is first demonstrated using the 20-bus test system from [16]. The one-line diagram of the system is shown in Fig. 2. The arrows in Fig. 2 represent the flow of the GICs for the 1 V/km eastward field, while the size of an arrow is proportional to the magnitude of the GIC on each of the lines. The locations of the eight substations in the case are labeled in Fig. 2. The algorithm is applied to the two GMD scenarios considered in [16], namely, a uniform 1V/km eastward field and a uniform 1 V/km northward field.

For convenience, the assumed substation grounding resistance values and the calculated GIC flows for the two scenarios (from Tables I and VII of [16]) are given in Table 3. Notice that Substation 1 is modeled with a GIC blocking device so its grounding resistance is assumed to be infinite. Substation 7 models a series capacitor location that has no connection to ground.

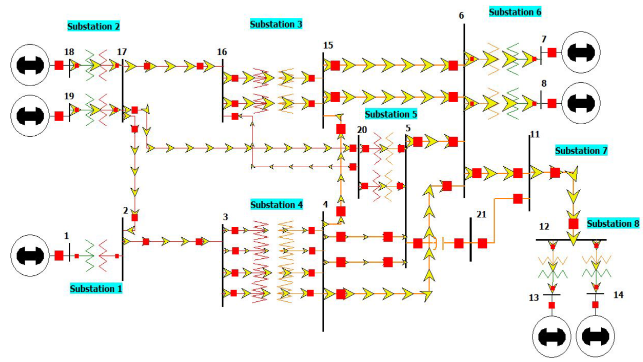

Fig. 2 20-bus GIC test system one-line showing 1 V/km eastward values

Table 3: 20-bus substation resistances and GIC flows

	Grounding resistance (Ω)	Eastward field GIC (A)	Northward field GIC (A)
Sub. 1	0.2 (but blocked)	0.00	0.00
Sub. 2	0.2	-189.29	115.63
Sub. 3	0.2	-109.49	139.85
Sub. 4	1.0	-124.58	19.98
Sub. 5	0.1	-65.46	-279.09
Sub. 6	0.1	354.52	-57.29
Sub. 7	No ground path	0.00	0.00
Sub. 8	0.1	134.30	60.9

Using the approach from the previous section, the values for the 20-bus system are given in Table 4 (since grounding resistance plays no role for Substations 1 and 7 they are omitted from the Table). The relatively low $\Re$ values for all the substations except 3 and 4 indicate that the substation GICs are not particularly dependent on the assumed substation resistance. In contrast, the GIC at Substation 4 is highly dependent on its grounding resistance value.

Table 4: 20-bus substation equivalent values

	Driving point resistance. (Ω)	R_{TH}, (Ω)	$\Re$	V_{TH} eastward (V)	V_{TH} northward (V)
Sub. 2	0.158	0.750	0.210	-179.88	109.90
Sub. 3	0.115	0.272	0.424	-51.61	65.95
Sub. 4	0.198	0.246	0.802	-155.28	24.90
Sub. 5	0.076	0.321	0.239	-27.53	-117.41
Sub. 6	0.075	0.302	0.249	142.50	-23.02
Sub. 8	0.093	1.365	0.068	196.69	89.20

For example, if the Substation 4 grounding resistance value is assumed to decrease by 50% (from 1.0 to 0.5Ω), then using (12) the new GIC for an eastward field would change from -124.6A to -208.1A (the magnitude increases by 67.01%). In contrast, at the less sensitive Substation 2 if its grounding resistance was also reduced by 50% (from 0.2 to 0.1Ω) the Substation 2 GIC for the eastward field would only change from -189.3 to -211.6A (the magnitude increases by 11.78%). The above sensitivity analysis confirms that $\Re$ is indeed a sensitivity indicator.

If desired, (13) could be used to determine the network shift factors for this change in the substation GIC. For example, Substation 2 has two identical generator step-up transformers (GSUs), both with shift factors of 0.5. Hence for the above 22.3A increase in the current out of the ground into the Substation 2 neutral, the current going into each GSU changes by 11.15A total for all three phases or 3.72A per phase.

Next the algorithm is applied to a 62,500 bus, 27,600 substation model of the EI used in [5]. Since this is an actual power system model, the line and transformer resistance values were determined from the power flow data with good accuracy. However, the substation grounding resistances were not known (except for at a handful of buses) and were estimated using a rather simplistic model.

In this model the assumed resistance depended upon the highest substation voltage level and its assumed size (based on the number of lines coming into the substation), with larger, higher voltage substations having lower values. Soil resistivity, which certainly can have an impact, was not known and hence not included in this simplistic model. Fig. 3 plots the sorted assumed substation grounding resistances used in this example. While the figure contains a handful of actual utility provided data values, the vast majority of it is estimated.

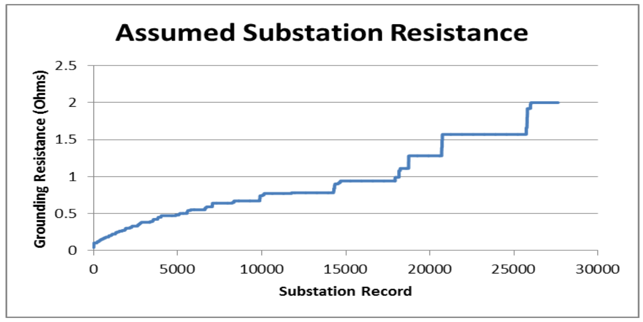

Fig. 3 EI sorted assumed substation grounding resistance

For this example, a uniform 1 V/km eastward electric field is applied to the entire EI system, then (1) is used to calculate the GICs. The purpose of applying a uniform electric field is not meant to imply such a field would represent a realistic GMD storm scenario; it certainly would not. Rather it was used solely to generate example GICs. As noted earlier the **G** matrix and hence the derived Thevenin resistance values are independent of the assumed **I** vector used in (1). Table 5 shows the data for the ten substations with the highest GICs (labeled Sub A to Sub I to maintain data confidentiality).

The result from this analysis is the substation GICs, and hence the associated transformer GICs, are quite dependent upon the estimated substation grounding resistance values. Note that more than half of the entries in Table 5 have $\Re$ values at or above 0.5, indicating that the assumed substation resistance dominates in determining the GIC for a particular substation. And this trend is not restricted just to the top ten substations. The average of $\Re$ is 0.47 for the top 100 substations and 0.51 for the top 500.

Table 5: EI substations with the largest GICs for a uniform eastward field

	Assumed grounding resistance (Ω)	GIC (A)	R_{TH}, (Ω)	$\Re$	V_{TH} eastward (V)
Sub. A	0.24	-294.85	0.38	0.39	-183.09
Sub. B	0.10	238.14	0.59	0.14	164.30
Sub. C	0.10	-228.69	0.08	0.54	-42.27
Sub. D	0.15	213.23	0.15	0.50	63.55
Sub. E	0.10	192.66	0.28	0.27	74.12
Sub. F	0.27	178.58	0.23	0.55	88.86
Sub. G	0.10	-151.93	0.11	0.49	-31.25
Sub. H	0.12	150.41	0.08	0.58	29.86
Sub. I	0.17	-144.39	0.10	0.63	-38.64
Sub. J	0.10	143.20	0.14	0.52	34.85

To illustrate, consider Substation D which has about 1500 MW of generation connected at 345 and 138 kV. With an assumed grounding resistance of 0.15Ω, its GIC of 213.2A for a 1V/km field indicates that it is highly susceptible to GIC on its GSUs. If the assumed grounding resistance is increased from 0.15 to 0.5Ω then this value drops to 97.8A.

IV. Multiple Substation Sensitivity Considerations

Before concluding, it is important to at least briefly comment on the issue of the dependence of the Thevenin resistance for a particular substation on the assumed grounding resistances of the other substations. This is clearly seen in the four bus example that

$$R_{TH,A} = 1.2 + R_{Ground,B} \tag{14}$$

For larger systems the dependence is more complex but is certainly still present. In [14] this network wide sensitivity is explored by simultaneously changing the substation resistances either by fixed percentages, fixed amounts, by random percentages (within a range) or by setting them all to zero (the "perfect-earthing" case).

To illustrate such an approach on the Thevenin resistance method, Table 6 compares the Thevenin resistance values of the original 20-bus system with values obtained either by doubling the assumed grounding resistances or by assuming the grounding resistance tended towards zero (representing them by just 0.01 Ω). Clearly as more resistance is added to the model, the Thevenin values will increase. Conversely, when the substation grounding resistance is neglected the Thevenin resistances will decrease, providing an upper bound on the magnitude of the substation GICs. Whether this upper bound is overly restrictive is somewhat system specific, and certainly an area for future research.

Table 6: Variation in 20-bus substation resistance values (Ω)

	Original Grounding Resistance	Original R_{TH},	Doubled Grounding Resistance	R_{TH}, Doubled Resistance	R_{TH}, Resistance of 0.01 Ω
Sub. 2	0.2	0.750	0.4	0.795	0.689
Sub. 3	0.2	0.272	0.4	0.308	0.224
Sub. 4	1.0	0.246	2.0	0.281	0.210
Sub. 5	0.1	0.321	0.2	0.372	0.223
Sub. 6	0.1	0.302	0.2	0.372	0.255
Sub. 8	0.1	1.365	0.2	1.418	1.304

V. Conclusion And Future Directions

The paper has addressed the issue of the sensitivity of the GICs to the assumed substation grounding resistance, providing an algorithm suitable for large system use to quantify this sensitivity. The conclusion of the paper is that the GICs can indeed be quite dependent on these values, with example results provided for the 20-bus GIC test system and the EI.

Another contribution of the paper is providing a methodology for identifying the substations that need accurate grounding resistance values. Those substations that have high GICs and high $\Re$ values. Of course ideally utility engineers would have easy access to data sets that provide accurate values for all substations in a network. However, this can be difficult in practice as was discussed in the introduction. The methodology introduced in the paper can help them focus on the locations in which accurate information is most needed.

The paper also suggests several directions for future research. First, the single substation algorithm presented in the paper needs to be further refined and tested on additional networks. Second, additional research in the dependence of the Thevenin resistances on the grounding values for other substations is needed. The upper bound approach of assuming low resistances needs to be further considered. Last, the paper certainly highlights the need for additional work in the area of GIC analysis validation using actual system data and measurements. Such measurements could, perhaps, provide a mechanism to directly estimate the actual substation grounding resistance values. With the advent of more direct measurement of power system GICs, there is a great potential for much better results in the near future.

References

[1] V.D. Albertson, J.M. Thorson Jr., R.E. Clayton, S.C. Tripathy, "Solar-Induced-Currents in Power Systems: Cause and Effects," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-92, no.2, pp. 471-477, March/April 1973.

[2] "2012 Special Reliability Assessment Interim Report: Effects of Geomagnetic Disturbances on the Bulk Power System," NERC, Feb. 2012.

[3] V.D. Albertson, J.G. Kappenman, N. Mohan, and G.A. Skarbakka, "Load-Flow Studies in the Presence of Geomagnetically-Induced Currents," *IEEE Trans. on Power Apparatus and Systems*, vol. PAS-100, pp. 594-606, Feb. 1981.

[4] J. Kappenman, "Geomagnetic Storms and Their Impacts on the U.S. Power Grid," Metatech Corporation Report Meta-R-319, Jan. 2010.

[5] T.J. Overbye, T.R. Hutchins, K. Shetye, J. Weber, S. Dahman, "Integration of Geomagnetic Disturbance Modeling into the Power Flow: A Methodology for Large-Scale System Studies," *Proc. 2012 North American Power Symposium*, September 2012, Champaign, IL.

[6] T.J. Overbye, K.S. Shetye, Y.Z. Hughes, J.D. Weber, "Preliminary Consideration of Voltage Stability Impacts of Geomagnetically Induced Currents," Accepted for presentation at IEEE PES 2013 General Meeting, Vancouver, BC, July 2013.

[7] D.H. Boteler, R.J. Pirjola, "Modeling Geomagnetically Induced Currents Produced by Realistic and Uniform Electric Fields," *Proc. IEEE Trans. on Power Delivery*, vol. 13, Oct. 1998, pp. 1303-1308.

[8] X. Dong, Y. Liu, J.G. Kappenman, "Comparative Analysis of Exciting Current Harmonics and Reactive Power Consumption from GIC Saturated Transformers," *Proc. IEEE 2001 Winter Meeting*, Columbus, OH, pp. 318-322, Jan. 2001.

[9] K. Zheng, D.H. Boteler, R. Pirjola, L.G. Liu, R. Becker, L. Marti, S. Boutilier, S., Guillon, "Influence of System Characteristics on the Amplitudes

of Geomagnetically Induced Currents," Submitted to *IEEE Transactions on Power Delivery*.
[10] R.A. Walling, A.H. Khan, "Characteristics of Transformer Exciting Current during Geomagnetic Disturbances", *IEEE Trans. on Power Delivery*, vol. 6, pp. 1707-1713, October 1991.
[11] ANSI/IEEE Standard 81-1983, IEEE Guide for Measuring Earth Resistivity, Ground Impedance and Earth Surface Potentials of a Ground System
[12] ANSI/IEEE Standard 81.2-1991, IEEE Guide for Measurement of Impedance and Safety Characteristics of Large, Extended or Interconnected Grounding Systems.
[13] "Testing and Evaluation of Grounding Systems: The Revision of the IEEE Std 81", Presentation given by Professor Sakis Meliopoulos of Georgia Tech at an IEEE Industrial Applications Society Atlanta Chapter meeting in January 19, 2010. http://ewh.ieee.org/r3/atlanta/ias/Testing%20and%20Evaluation%20of%20Grounding%20Systems.pdf
[14] R. Pirjola, "Study of effects of changes of earthing resistancs on geomagnetically induced currents in an electric power transmission system," *Radio Science*, vol. 43, 20008, RS1004.
[15] R. Pirjola, "Properties of matrices included in the calculation of geomagnetically induced currents (GICs) in power systems and introduction of a test model for GIC computation algorithms," *Earth Planets Space*, vol. 61, pp. 263-272, 2009.
[16] R. Horton, D.H. Boteler, T.J. Overbye, R.J. Pirjola, R. Dugan, "A Test Case for the Calculation of Geomagnetically Induced Currents," *IEEE Transactions on Power Delivery*, vol. 27, pp. 2368-2373, Oct. 2012.
[17] W.F. Tinney, V. Brandwajn, S.M. Chan, "Sparse Vector Methods," *IEEE Trans. on Power App. And Syst.*, vol. PAS-104, pp. 295-301, Aug. 1985.
[18] A.J. Wood, B.F. Wollenberg, *Power Generation and Control*, 2nd Edition, John Wiley & Sons, New York, 1996.