Example 9.1

(a) $M_{12} = \dfrac{\theta_1 - \theta_2}{0.2} \quad [\because X_{12} = 0.2]$

$M_{13} = \dfrac{\theta_1 - \theta_3}{0.4} \quad [\because X_{13} = 0.4]$

$M_{32} = \dfrac{\theta_3 - \theta_2}{0.25} \quad [\because X_{32} = 0.25]$

$$M_{12} = 5\theta_1 - 5\theta_2$$
$$M_{13} = 2.5\theta_1 - 2.5\theta_2 \Rightarrow H = \begin{array}{cc} \theta_1 & \theta_2 \\ \left[\begin{matrix} 5.0 \\ 2.5 \\ 0 \end{matrix}\right. & \left.\begin{matrix} -5.0 \\ 0 \\ -4.0 \end{matrix}\right] \end{array}$$
$$M_{32} = 4\theta_3 - 4\theta_2$$

$$Z_{meas} = \begin{bmatrix} 0.6 \\ 0.04 \\ 0.405 \end{bmatrix} pu.$$

$$R = \begin{bmatrix} 4\times10^{-4} & 0 & 0 \\ 0 & 1\times10^{-4} & 0 \\ 0 & 0 & 4\times10^{-6} \end{bmatrix}$$

$$x^{est} = \left[H' R^{-1} H\right]^{-1} H' R^{-1} Z^{meas}$$

$$x^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 0.01782 \\ -0.10126 \end{bmatrix}$$

(b) Residual $J(x) = J(\theta_1, \theta_2) = \dfrac{\left[Z_{12} - M_{12}(\theta_1,\theta_2)\right]^2}{\sigma_{12}^2} + \dfrac{\left[Z_{13} - M_{13}(\theta_1,\theta_3)\right]^2}{\sigma_{13}^2} + \dfrac{\left[Z_{32} - M_{32}(\theta_2,\theta_3)\right]^2}{\sigma_{32}^2}$

$$M_{12}(\theta_1, \theta_2) = \frac{\theta_1 - \theta_2}{0.2} = 0.5954$$

$$M_{13}(\theta_1, \theta_3) = \frac{\theta_1 - \theta_3}{0.4} = 0.0445$$

$$M_{32}(\theta_3, \theta_2) = \frac{\theta_3 - \theta_2}{0.25} = 0.405$$

$$= \frac{(0.6 - 0.5954)^2}{(0.02)^2} + \frac{[0.04 - 0.0445]^2}{(0.01)^2} + \frac{[0.405 - 0.405)^2}{(0.002)^2}$$

$$= \frac{0.00002116}{0.0004} + \frac{0.00002025}{0.0001} + 0$$

$$= 0.0529 + 0.2025 = 0.2554.$$

Degree of freedom $= N_m - N_s$

$= 3 - 2$

$= 1$

Using a chi distribution table, for a significant level ($\alpha = 0.01$) and $k = 1$, the threshold residual $= 6.635$.

Since $J(x) <<$ threshold $\rightarrow$ we can assume the likely absence of bad data in measurements.

Example 9.3.

(a)

$$M_{13} = \frac{\theta_1 - \theta_3}{0.5}, \quad M_{31} = \frac{\theta_3 - \theta_1}{0.5}, \quad M_{12} = \frac{\theta_1 - \theta_2}{0.25} \rightarrow H = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \end{bmatrix}$$

$$Z^{meas} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \end{bmatrix} \text{ p.u.}; \quad \theta_4 = 0$$

$$R = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}$$

$$H'R^{-1}H = \begin{bmatrix} 12000 & -40000 & -80000 \\ -4000 & 40000 & 0 \\ -8000 & 0 & 80000 \end{bmatrix}$$

$\rightarrow$ singular. (hence noninvertible) $\rightarrow$ the system is unobservable

(b)

$$H = \begin{bmatrix} 2 & 0 & -2 \\ -2 & 0 & 2 \\ 4 & -4 & 0 \\ -2 & 0 & 12 \end{bmatrix}$$

$$R = \begin{bmatrix} 10^{-4} & 0 & 0 & 0 \\ 0 & 10^{-4} & 0 & 0 \\ 0 & 0 & 4 \times 10^{-4} & 0 \\ 0 & 0 & 0 & 2.25 \times 10^{-4} \end{bmatrix}$$

$$Z^{meas} = \begin{bmatrix} -0.705 \\ 0.721 \\ 0.212 \\ 0.920 \end{bmatrix} \text{ p.u.}$$

$$H'R^{-1}H = \begin{bmatrix} 1.3778 & -0.4 & -1.8667 \\ -0.4 & 0.4 & 0 \\ -1.8667 & 0 & 7.2 \end{bmatrix} \times 10^5$$

$\rightarrow$ is full rank & thus observable.

$$X^{est} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} -0.3358 \\ -0.3888 \\ 0.0207 \end{bmatrix} \text{ radians.}$$

Problem 4

$A = \begin{bmatrix} 2 & 7 \\ 4 & 3 \\ 5 & 6 \end{bmatrix}$

Using Givens Rotation do QR factorization.

$z = -4/5 = -0.8$

First we zero out $A[3,1]$ $(a=4, b=5)$ $s = \frac{1}{\sqrt{1+z^2}} = 0.7808$ $c = sz = -0.6246$

$$G_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6247 & 0.7808 \\ 0 & -0.7808 & -0.6247 \end{bmatrix}$$

$$G_1^T A = \begin{bmatrix} 2 & 7 \\ -6.4028 & -6.5589 \\ 0 & -1.4058 \end{bmatrix}$$

Next we zero out $A[2,1]$ $(a=2, b=6.4028)$ $s = 0.9545$ $c = 0.2982$

$$G_2 = \begin{bmatrix} 0.2982 & 0.9545 & 0 \\ -0.9545 & 0.2982 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G_2^T G_1^T A = \begin{bmatrix} 6.7078 & 8.3478 \\ 0 & 4.725 \\ 0 & -1.4058 \end{bmatrix}$$

$z = -b/a = 0.2975$

Then zero out $A[3,2]$ $(a=-4.725, b=1.4058)$ $c = 0.9584$ $s = 0.2857$

$$G_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.9584 & 0.2851 \\ 0 & -0.2857 & 0.9584 \end{bmatrix}$$

$$G_3^T G_2^T G_1^T A = \begin{bmatrix} 6.7078 & 8.3478 \\ 0 & -4.929 \\ 0 & 0 \end{bmatrix}$$

$$G_1 G_2 G_3 G_3^T G_2^T G_1^T A = QU = \begin{bmatrix} 0.2982 & 0.914 & 0.272 \\ 0.5959 & -0.4011 & 0.695 \\ 0.7448 & -0.045 & -0.665 \end{bmatrix} \begin{bmatrix} 6.7078 & 8.3478 \\ 0 & -4.725 \\ 0 & 1.4058 \end{bmatrix}$$

Problem 5.

$y = a + bx + cx^2$

$Ax = y.$

$$\Rightarrow \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \\ 1 & x_4 & x_4^2 \\ 1 & x_5 & x_5^2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

For points $(1,1)$; $(2,5)$; $(3,10)$; $(4,6)$; $(5,5)$

$X = [1, 2, 3, 4, 5]$ $\quad y = [1, 5, 10, 6, 5]$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 5 & 25 \end{bmatrix}$$

$$X^* = \underbrace{(A^T A)^{-1} A^T}_{\text{pseudoinverse}} b = \begin{bmatrix} -6.8 \\ 9.043 \\ -1.357 \end{bmatrix}$$

Problem 8

$$Q_L = \frac{P_L^2}{B} - \frac{B}{4}$$

Let

$P_L = 4 + (56/100) = 4.56$

$B = -10$

$$\therefore Q_L = \frac{4.56^2}{-10} + \frac{10}{4} = 0.42$$

Problem 6.

Y_{bus} of original system is available from Powerworld.

Eliminate buses 1, 2 and 6.

$$Y_{ee} = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 6 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 6 \end{array} & \begin{bmatrix} -20.83 & 16.67 & 0 \\ 16.67 & -52.78 & 16.67 \\ 0 & 16.67 & -25 \end{bmatrix} \end{array}$$

$$Y_{es} = \begin{array}{cc} & \begin{array}{cccc} 3 & 4 & 5 & 7 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 6 \end{array} & \begin{bmatrix} 4.17 & 0 & 0 & 0 \\ 5.56 & 5.56 & 8.33 & 0 \\ 0 & 0 & 0 & 8.33 \end{bmatrix} \end{array}$$

$$Y_{se} = Y_{es}^T$$

$$Y_{ss} = \begin{array}{cc} & \begin{array}{cccc} 3 & 4 & 5 & 7 \end{array} \\ \begin{array}{c} 3 \\ 4 \\ 5 \\ 7 \end{array} & \begin{bmatrix} -43.06 & 33.33 & 0 & 0 \\ 33.33 & -43.06 & 4.17 & 0 \\ 0 & 4.17 & -29.17 & 16.67 \\ 0 & 0 & 16.67 & -25 \end{bmatrix} \end{array}$$

$$Y_{eq} = Y_{ss} - Y_{se} \cdot Y_{ee}^{-1} Y_{es}$$

$$= \begin{array}{cc} & \begin{array}{cccc} 3 & 4 & 5 & 7 \end{array} \\ \begin{array}{c} 3 \\ 4 \\ 5 \\ 7 \end{array} & \begin{bmatrix} -39.43 & 35.07 & 2.61 & 1.74 \\ 35.07 & -41.96 & 5.81 & 1.09 \\ 2.61 & 5.81 & -26.7 & 18.3 \\ 1.74 & 1.09 & 18.3 & -21.1 \end{bmatrix} \end{array}$$

yellow highlighted positions in the equivalent Y_{bus} are the new lines added in the system after removal of bus 1, 2 and 6.

$X_{35} = -0.383j$ pu

$X_{37} = -0.574j$ pu

$X_{74} = -0.917j$ pu

Problem 10

$$\min_{x_1 x_2} (x_1-3)^2+(x_2-2)^2 \quad \text{s.t. } x_1+x_2=8$$

$$L(x_1,x_2,\lambda)= (x_1-3)^2+(x_2-2)^2-\lambda(x_1+x_2-8)$$

$$\frac{dL}{dx_1} = 2(x_1-3)-\lambda = 0 \qquad 2(x_1-3)=3$$

$$x_1 = \frac{3}{2}+3 = 4.5$$

$$\frac{dL}{dx_2} = 2(x_2-2)-\lambda=0 \qquad 2(x_2-2)=3$$

$$\frac{dL}{d\lambda} = -(x_1+x_2-8)=0 \qquad x_2=\frac{3}{2}+2 = 3.5$$

$$2x_1-6-\lambda=0 \rightarrow 2x_1=6+\lambda$$

$$2x_2-4-\lambda=0 \rightarrow 2x_2=4+\lambda$$

$$x_1+x_2=8$$

$$6+\lambda+4+\lambda=16$$

$$10+2\lambda=16$$

$$5+\lambda=8$$

$$\lambda=3$$

$$x_1 = 4.5$$

$$x_2 = 8.5$$

Example 9.4.

$$M_{21} = \frac{\theta_2-\theta_1}{X_{21}} = 10\theta_2-10\theta_1$$

$$M_{32} = \frac{\theta_3-\theta_2}{X_{32}} = -4\theta_2$$

$$M_{23} = \frac{\theta_2-\theta_3}{X_{32}} = 4\theta_2$$

$$\Rightarrow H= \begin{array}{cc} \theta_1 & \theta_2 \end{array}$$
$$\begin{bmatrix} 0 & -4 \\ 0 & -4 \\ 0 & 4 \\ -10 & 14 \\ -10 & 10 \end{bmatrix}$$

$$M_3=M_{32} = -4\theta_2$$

$$M_2 = M_{23}+M_{21} = 14\theta_2-10\theta_1$$

$$R= (0.01)^2 \cdot I(5)$$

on 100 MVA base

$$Z = \begin{bmatrix} 1.05 \\ 0.98 \\ -1.35 \\ 0.49 \\ 1.48 \end{bmatrix} \begin{array}{l} M_3 \\ M_{32} \\ M_{23} \\ M_2 \\ M_{21} \end{array}$$

(a)

$$\theta = (H^TR^{-1}H)^{-1}H^TR^{-1}z$$

$$= \begin{bmatrix} -0.431 \\ -0.277 \end{bmatrix}$$

(b) Residual $J(x) = J(\theta_1,\theta_2) = 852.72$

$$= \sum \left[\frac{[Z_{meas} - M(\theta_1,\theta_2)]}{\sigma^2}\right]^2$$

(c) Estimated Generator output

$$M_2 = 14\theta_2-10\theta_1 = 0.432\text{pu} \rightarrow 43.2\text{MW}$$

$$M_3 = -4\theta_2 = 1.108 \text{ pu} \rightarrow 110.8\text{MW}$$

$$M_{12} = -154 \text{ MW}$$

$$M_{23} = -110.8\text{MW}$$

(d.) degree of freedom $= N_m - N_s$

$= 5 - 2$

$= 3$

using a chi-distribution table; for $\alpha = 0.01$ threshold $= 11.345$

$J = 835 \gg 11.345$.

$\therefore$ there exist bad measurements.

$$\Rightarrow Z^{meas} - Z^{cal} = \begin{bmatrix} 1.108 - 1.05 \\ 1.108 - 0.98 \\ -(1.108) + (1.35) \\ 0.432 - 0.49 \\ -1.48 + 1.54 \end{bmatrix} = \begin{bmatrix} 0.058 \\ 0.128 \\ 0.242 \\ -0.058 \\ +0.06 \end{bmatrix}$$

→ removing suspected bad measurements i.e. M_{23} & M_3

$$H = \begin{bmatrix} 0 & -4 \\ -10 & 14 \\ -10 & 10 \end{bmatrix} ; \quad Z = \begin{bmatrix} 0.98 \\ 0.49 \\ 1.48 \end{bmatrix}$$

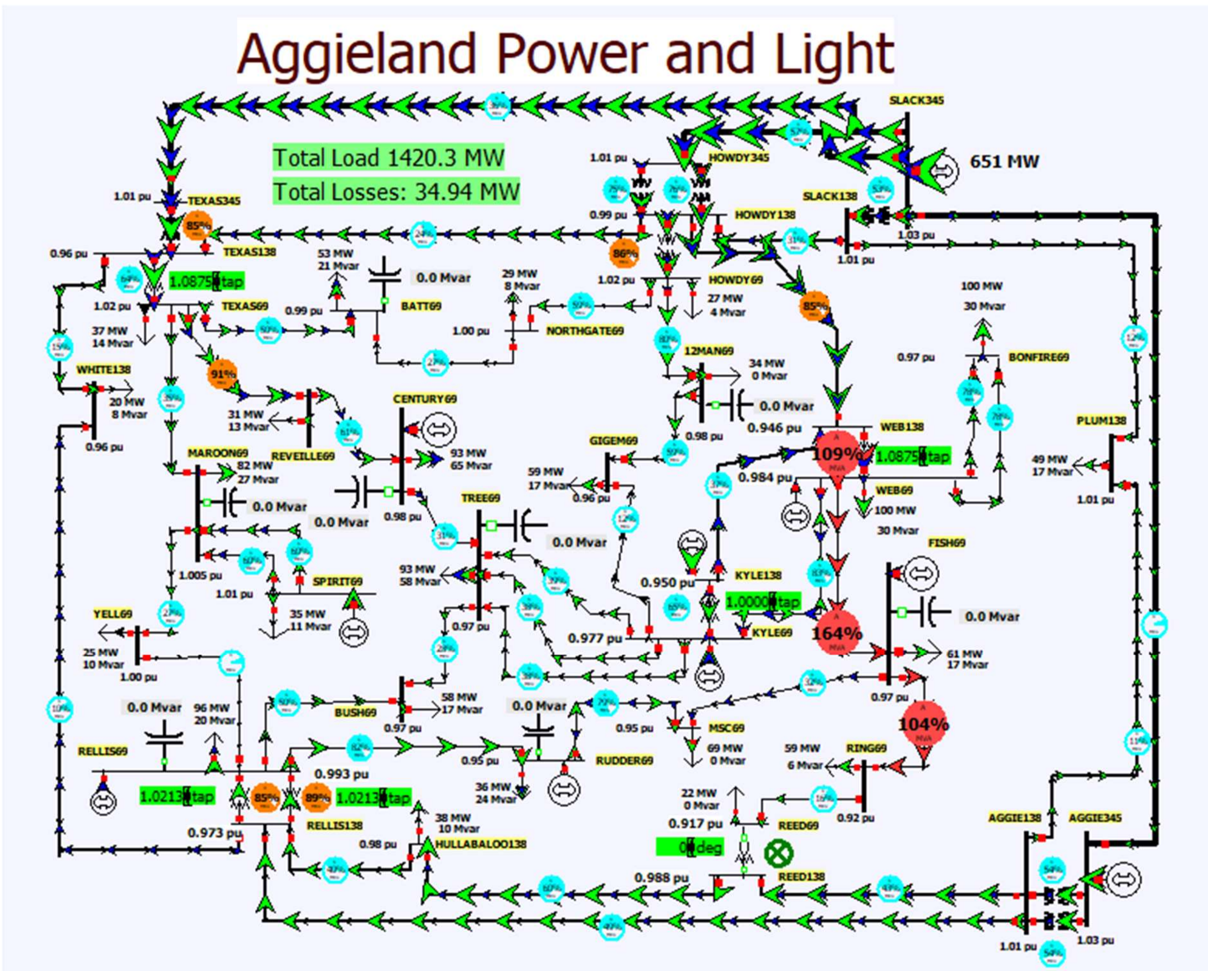
$$\theta = \begin{bmatrix} -0.394 \\ -0.245 \end{bmatrix}$$

$J = 0.333 <<< 6.635$ (from the chi-distribution table for new degree of freedom $= 1$)

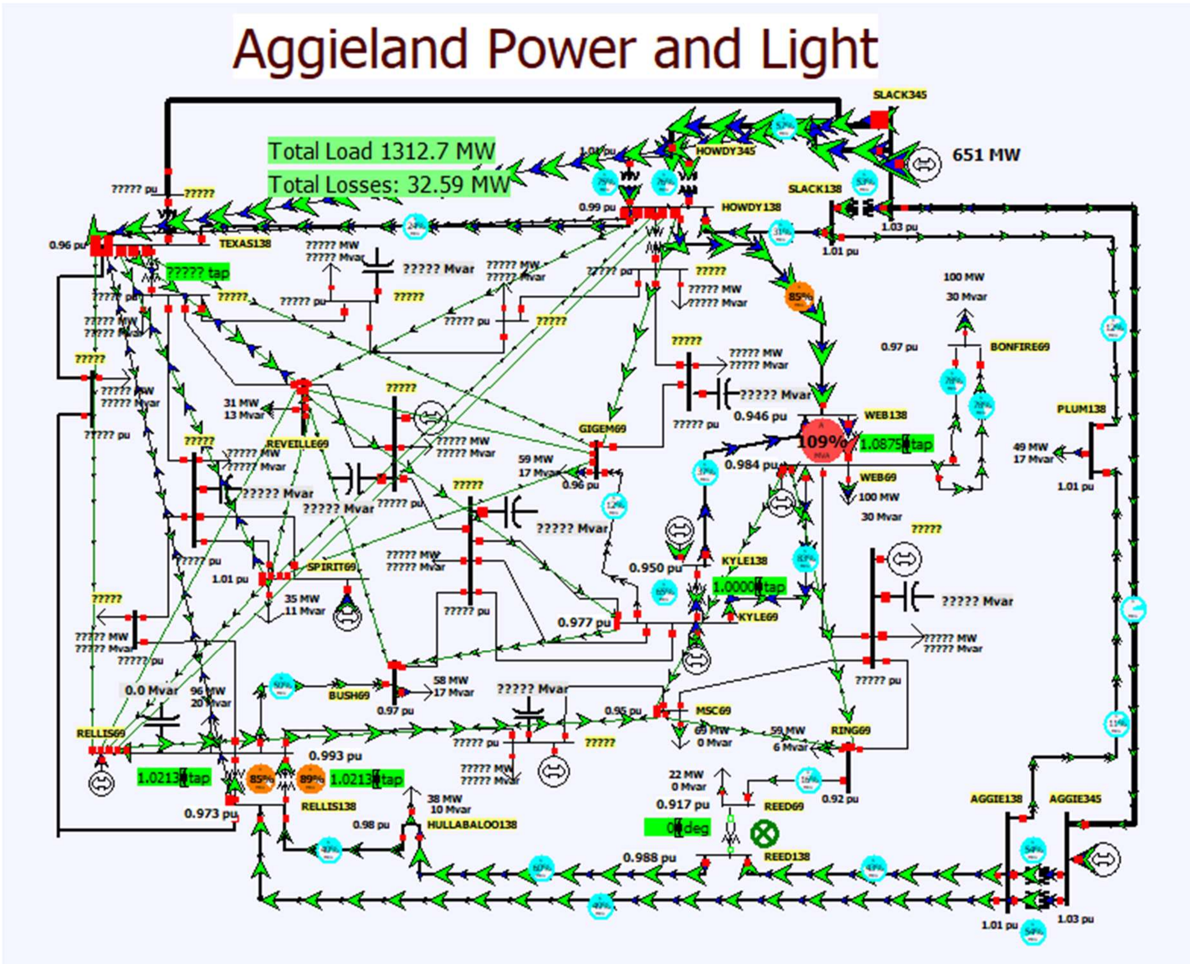
Problem 7:

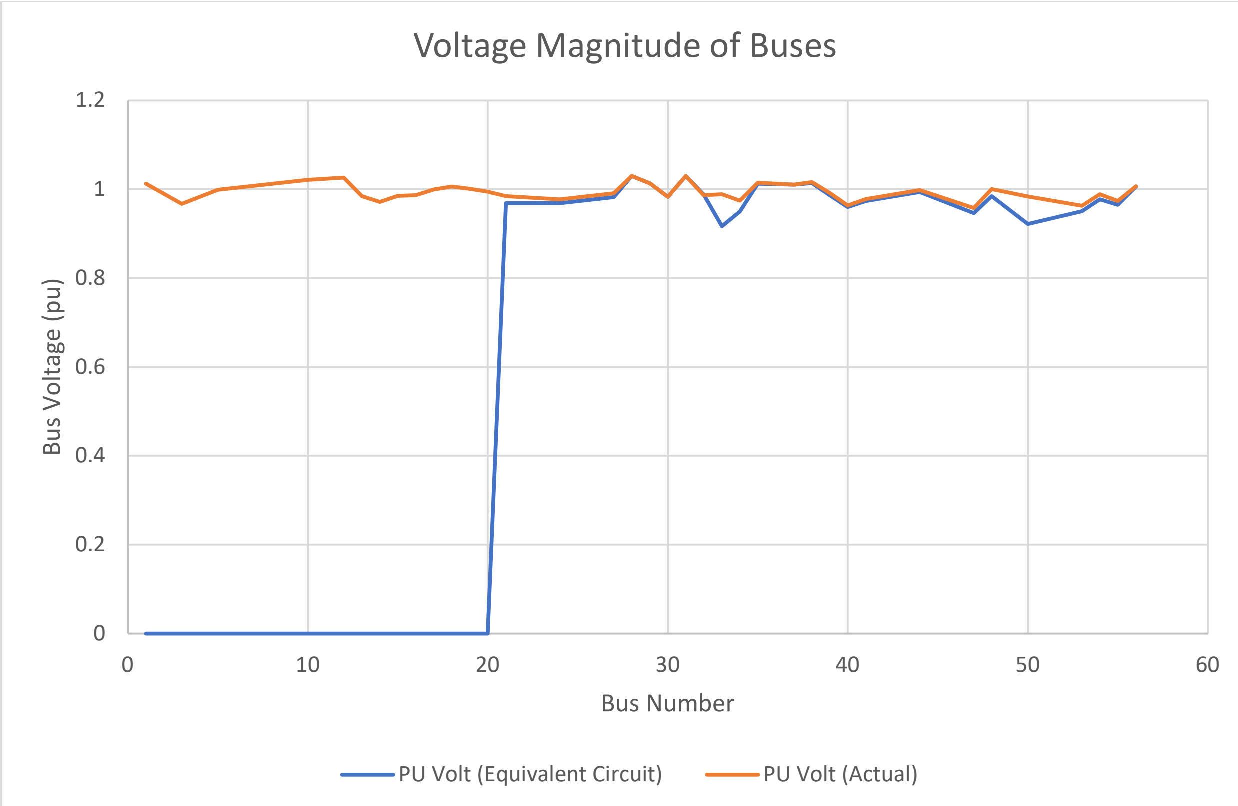
In PowerWorld Simulator using the Aggieland37 case, first calculate the line flows and bus voltage magnitudes for the contingent opening of the transformers between buses 32 and 33. You may wish to store these results in a spreadsheet. Then, reopen the case (i.e., without the contingency) and in PowerWorld create an equivalent eliminating all the buses with bus numbers less than 20. Then, repeat the previous contingency, and compare the results with the full system (obviously only comparing for the retained buses and lines).

Before Equivalencing (with Trafo between Bus 32 and Bus 33 open)



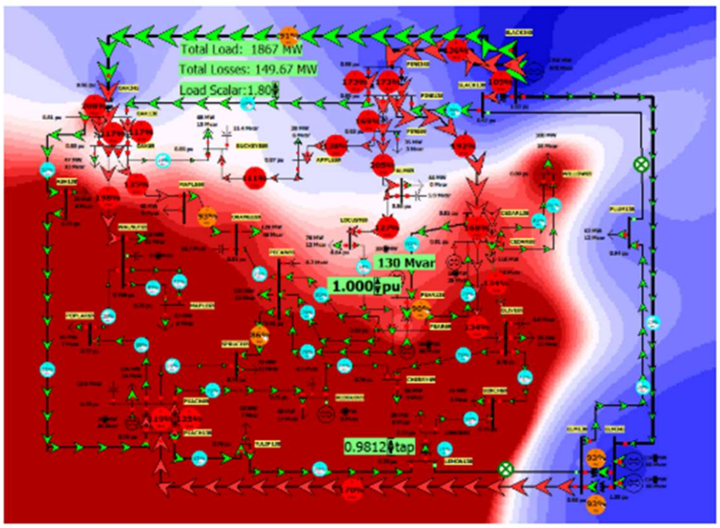
After Equivalencing (with Trafo between Bus 32 and Bus 33 open)




Voltage Magnitude of Buses
Bus Voltage (pu)
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40
50
60
Bus Number
PU Volt (Equivalent Circuit)
PU Volt (Actual)

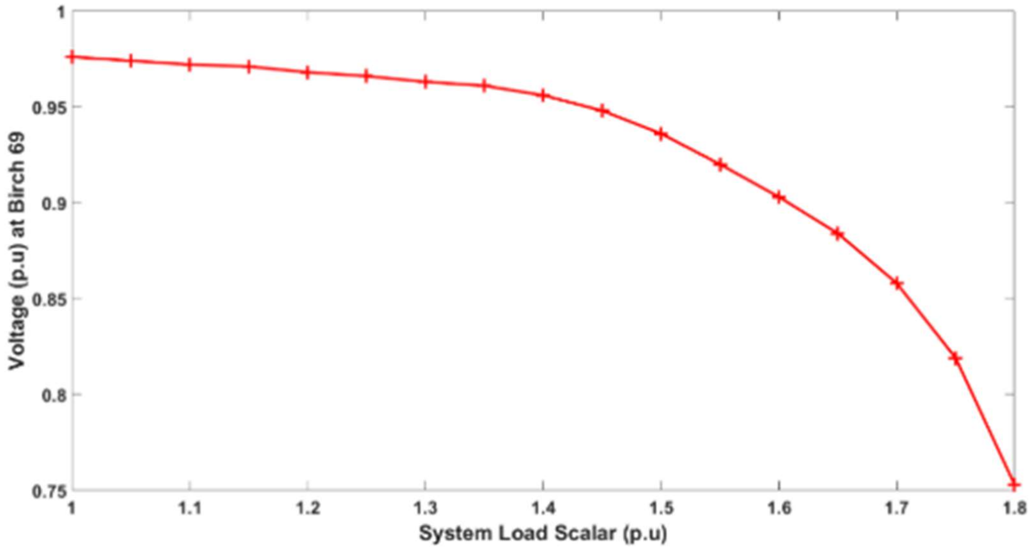
Problem 9:

In PowerWorld using the Bus37_PV_HW6 system first open two transmission lines and one generator. You may choose any two lines, except with the requirement that you not isolate any load or island the system. For the generator you may open any one, excepting the slack bus generator. Then, use the **Load Scalar** field to increase the system load until the system reaches voltage collapse. Plot the PV curve, with P being the total system load, and V being the voltage magnitude at the bus that has the lowest voltage magnitude at the point of voltage collapse. Your PV curve should have at least ten fairly uniformly spaced points.



Two transmission lines opened: Slack138-Plum 138, Lemon 138-Elm138
One generator disconnected: Gen at Redbud69

System Load Scalar	1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50	1.55	1.60	1.65	1.70	1.75	1.80
V(p.u) at Birch 69	0.976	0.974	0.972	0.971	0.968	0.966	0.963	0.961	0.956	0.948	0.936	0.920	0.903	0.884	0.858	0.819	0.753



Problem 11:

Using PowerWorld case Bus5_Losses with the Load Scalar equal to 1.0, determine the eneration dispatch that minimizes system losses. (*Hint:* manually vary the generation at buses 2 and 4 until their loss sensitivity values are zero.)