Homework 7

Problem 1

X_1 = amount of construction grade boards.

X_2 = amount of finish grade boards

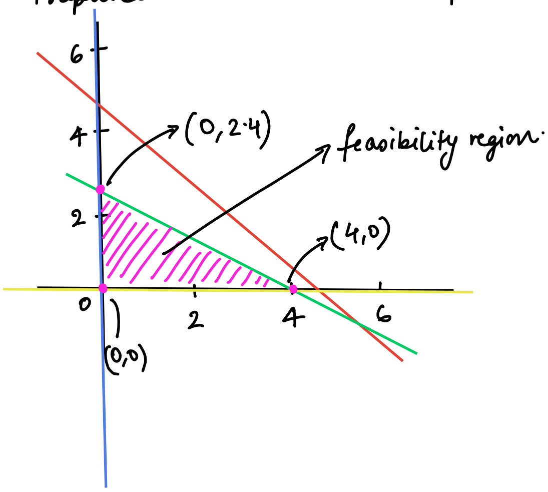
$$\max_{x_1,x_2} \; 100x_1 + 120x_2$$

$$2x_1 + 2x_2 \leq 9$$

$$3x_1 + 5x_2 \leq 12$$

$$x_1 > 0 \quad x_2 > 0$$

Graphical Solution to the problem



The point (4,0) maximizes the objective function.

Producing 4 ft of cg boards everyday maximizes profit.

Problem 3

$$\min_{x_1,x_2} \; 2x_1^2 + x_2^2$$

$$\text{s.t. } 3x_1 + x_2 \geq 2$$

initial guess z^0

$$z^1 = z^0 - [H(z)]^{-1} \nabla L(z)$$

(quadratic function with linear constraints — so no iteration required)

Assume constraint is binding;

$$L(x,\lambda) = 2x_1^2 + x_2^2 + \lambda(3x_1 + x_2 - 2)$$

$$\frac{\partial L}{\partial x_1} = 4x_1 + 3\lambda$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda$$

$$\nabla L(x,\lambda) = \begin{bmatrix} 4x_1 + 3\lambda \\ 2x_2 + \lambda \\ 3x_1 + x_2 - 2 \end{bmatrix}$$

$$\frac{dL}{d\lambda} = (3x_1 + x_2 - 2)$$

$$H(x,\lambda) = \begin{bmatrix} 4 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 3 \\ 0 & 2 & 1 \\ 3 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix} \rightarrow \left[z^{k+1} = z^k - [H]^{-1} \cdot \nabla L(z^k) \right]$$

initial guess.

$$\begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 0.545 \\ 0.363 \\ -0.727 \end{bmatrix}$$

Problem 2

let $P_{G1} = x_1$ $P_{G2} = x_2$, $P_{G3} = x_3$

the optimization problem can be formulated as:

$$\min_{x_1, x_2, x_3} \; (450 + 20x_1 + 25x_2 + 30x_3)$$

s.t.

$0 \leq x_1 \leq 200$
$0 \leq x_2 \leq 100$
$0 \leq x_3$

$\rightarrow$ $x_1 \leq 200$, $x_2 \leq 100$ and $x_1, x_2, x_3 \geq 0$

$x_1 + x_2 + x_3 = 300$ [Generation = load (assuming No losses]

$x_2 - x_1 \leq 150$

in "standard form" this becomes;

$$\min \; z = 20x_1 + 25x_2 + 30x_3 + 450$$

s.t.

$x_1 + s_1 = 200$ — (1)

$x_2 + s_2 = 100$ — (2)

$x_1 + x_2 + x_3 = 300$ — (3)

$x_2 - x_1 + s_3 = 150$ — (4)

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

For this LP, s_1, s_2 and s_3 can constitute initial basic variables associated with constraint (1), (2) and (4)

Constraint (3) does not have any slack variable.

Thus, we introduce artificial variable y

$$\min \; z = y$$

s.t.

$x_1 + s_1 = 200$
$x_2 + s_2 = 100$
$x_1 + x_2 + x_3 + y = 300$
$-x_1 + x_2 + s_3 = 150$

$\rightarrow$ $x_1 + 3x_2 + x_3 + s_1 + s_2 + s_3 + y = 750$

y is an artificial variable.

$-750 = -x_1 - 3x_2 - x_3 - s_1 - s_2 - s_3$

$S_1, S_2, S_3, y \rightarrow$ basic variables

$x_1, x_2, x_3 \rightarrow$ non basic variables.

$$r = C_N - C_B A_B^{-1} A_N = C_B A_N.$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}^T_{3\times1} - \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}}_{1\times4} \begin{array}{ccc} x_1 & x_2 & x_3 \\ \left[\begin{array}{c} 1 \\ 0 \\ -1 \\ 1 \end{array}\right. & \begin{array}{c} 0 \\ 1 \\ 1 \\ 1 \end{array} & \left.\begin{array}{c} 0 \\ 0 \\ 0 \\ 1 \end{array}\right] \end{array}_{4\times3}$$

$$= \begin{bmatrix} -1 \\ -3 \\ -1 \end{bmatrix}^T$$

initial tableau will be;

x_1	x_2	x_3	S_1	S_2	S_3	y	
1	0	0	1	0	0	0	200
0	1	0	0	1	0	0	100
-1	1	0	0	0	1	0	150
1	1	1	0	0	0	1	300
-1	-3	-1	0	0	0	0	-750

(b) Using simplex method

select pivot column (variable with most negative relative cost) - here column of variable x_2. $(q=2)$

select pivot row (smallest ratio y_i / y_{iq} for $y_{iq} > 0$ ($x_i \rightarrow$ last column of tableau). $(p=2)$

For column $x_2 \rightarrow$ 0, 1, 1, 1

x, $\frac{100}{1}$, $\frac{150}{1}$, $\frac{300}{1}$

smallest ratio.

(row 2)

the pivot element will be; $\boxed{y_{22} = 1}$

x_1	x_2	x_3	S_1	S_2	S_3	y		
1	0	0	1	0	0	0	200	← k=1
0	1	0	0	1	0	0	100	← p^{th} row
-1	1	0	0	0	1	0	150	← k=3
1	1	1	0	0	0	1	300	← k=4
-1	-3	-1	0	0	0	0	-750	← k=5

q^{th} column.

subtract $\frac{y_{k2}}{y_{22}} \times p^{th}$ row from the remaining K rows.

$$\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & s_1 & s_2 & s_3 & y & \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 200 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 100 \\
-1 & 0 & 0 & 0 & -1 & 1 & 0 & 50 \\
1 & 0 & 1 & 0 & -1 & 0 & 1 & 200 \\
-1 & 0 & -1 & 0 & 3 & 0 & 0 & -450
\end{array}$$

Value of cost function after first pivot.

For row 1 $\frac{y_{12}}{y_{22}} = 0$

For row 3 $\frac{y_{32}}{y_{22}} = 1$

For row 4 $\frac{y_{42}}{y_{22}} = 1$

For row 5 $\frac{y_{52}}{y_{22}} = -3$

(c) $\overset{100}{x_2}, \overset{200}{s_1}, \overset{50}{s_3}, \overset{200}{y} \rightarrow$ basic variables after first pivot.
(value of basic variables is just the last column)

$\underset{0}{x_1}, \underset{0}{x_3}, \underset{0}{s_2} \rightarrow$ non basic variables after the first pivot.
(value of all non basic variables is zero)

Since we still have negative relative costs, this is <u>not</u> the optimal solution!
(last row has negative values!)