ECEN 615
Methods of Electric Power Systems Analysis

Lecture 22: Linear Programming, Optimal Power Flow,

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Announcements

- Read Chapter 8
- Read the Chapter 3 appendices (3A covers optimization with constraints, 3B covers linear programming, 3D covers dynamic programming, and 3E convex optimization
- An excellent book on optimization is Linear and Nonlinear Programming by Luenberger and Ye (the 5th edition came out in 2021)
- Homework 6 is due today
- Exam 2 is on Thursday Dec 1 during class (for the on campus students); it will be comprehensive, but with more emphasis on the material after the first exam

Quick Coverage of Linear Programming

- LP is probably the most widely used mathematical programming technique
- It is used to solve linear, constrained minimization (or maximization) problems in which the objective function and the constraints can be written as linear functions

Example Problem 1 (mentioned in Lecture 21)

- Assume that you operate a lumber mill which makes both construction-grade and finish-grade boards from the logs it receives. Suppose it takes 2 hours to rough-saw and 3 hours to plane each 1000 board feet of construction-grade boards. Finish-grade boards take 2 hours to rough-saw and 5 hours to plane for each 1000 board feet. Assume that the saw is available 8 hours per day, while the plane is available 15 hours per day. If the profit per 1000 board feet is $100 for construction-grade and $120 for finish-grade, how many board feet of each should you make per day to maximize your profit?

Problem 1 Setup

Let x_1=amount of cg, x_2= amount of fg

$$\begin{aligned} \text{Maximize} \quad & 100x_1 + 120x_2 \\ \text{s.t.} \quad & 2x_1 + 2x_2 \le 8 \\ & 3x_1 + 5x_2 \le 15 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Notice that all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are seeking to determine the values of x_1 and x_2

Example Problem 2 (Nutritionist Problem)

- A nutritionist is planning a meal with 2 foods: A and B. Each ounce of A costs \$ 0.20, and has 2 units of fat, 1 of carbohydrate, and 4 of protein. Each ounce of B costs \$0.25, and has 3 units of fat, 3 of carbohydrate, and 3 of protein. Provide the least cost meal which has no more than 20 units of fat, but with at least 12 units of carbohydrates and 24 units of protein.

Problem 2 Setup

Let x_1=ounces of A, x_2= ounces of B

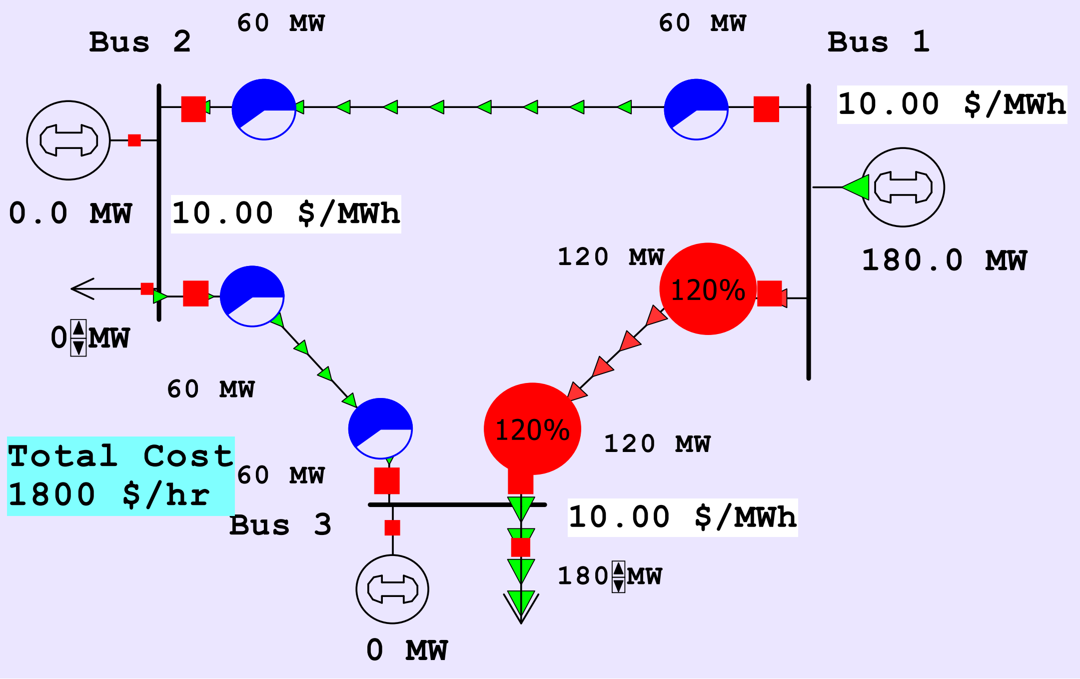
Minimize $\quad 0.20x_1 + 0.25x_2$

s.t.
$$2x_1 + 3x_2 \le 20$$
$$x_1 + 3x_2 \ge 12$$
$$4x_1 + 3x_2 \ge 24$$
$$x_1, x_2 \ge 0$$

Again all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are again seeking to determine the values of x_1 and x_2; notice there are also more constraints than solution variables

Three Bus Case Formulation

- For the earlier three bus system given the initial condition of an overloaded transmission line, minimize the cost of generation such that the change in generation is zero, and the flow on the line between buses 1 and 3 is not violating its limit
- Can be setup considering the change in generation, $(\Delta P_{G1}, \Delta P_{G2}, \Delta P_{G3})$



Three Bus Case Problem Setup

Let $x_1 = \Delta P_{G1}$, $x_2 = \Delta P_{G2}$, $x_3 = \Delta P_{G3}$

Minimize $\quad 10x_1 + 12x_2 + 20x_3$

s.t. $\quad \frac{2}{3}x_1 + \frac{1}{3}x_2 \leq -20$

Line flow constraint

$x_1 + x_2 + x_3 = 0$

Power balance constraint

enforcing limits on $x_1,\ x_2,\ x_3$

LP Standard Form

The standard form of the LP problem is

Minimize $\mathbf{c}\,\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}$

Maximum problems can be treated as minimizing the negative

where $\mathbf{x}$ = n-dimensional column vector

$\mathbf{c}$ = n-dimensional row vector

$\mathbf{b}$ = m-dimensional column vector

$\mathbf{A}$ = m×n matrix

For the LP problem usually n>> m

The previous examples were not in this form!

Replacing Inequality Constraints with Equality Constraints

- The LP standard form does not allow inequality constraints
- Inequality constraints can be replaced with equality constraints through the introduction of slack variables, each of which must be greater than or equal to zero

$$\ldots \le b_i \rightarrow \ldots + y_i = b_i \;\; \text{with } y_i \ge 0$$

$$\ldots \ge b_i \rightarrow \ldots - y_i = b_i \;\; \text{with } y_i \ge 0$$

- Slack variables have no cost associated with them; they merely tell how far a constraint is from being binding, which will occur when its slack variable is zero

Lumber Mill Example with Slack Variables

- Let the slack variables be x_3 and x_4, so

Minimize the negative

$$\begin{aligned} \text{Minimize} \quad & -(100x_1 + 120x_2) \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 = 8 \\ & 3x_1 + 5x_2 + x_4 = 15 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

LP Definitions

This is a key LP concept!

A vector $\mathbf{x}$ is said to be basic if

1. $\mathbf{Ax} = \mathbf{b}$
2. At most m components of $\mathbf{x}$ are non-zero; these are called the basic variables; the rest are non basic variables; if there are less than m non-zeros then $\mathbf{x}$ is called degenerate

$\mathbf{A}_\mathrm{B}$ is called the basis matrix

Define $\mathbf{x} = \begin{bmatrix} \mathbf{x}_\mathrm{B} \\ \mathbf{x}_\mathrm{N} \end{bmatrix}$ (with $\mathbf{x}_\mathrm{B}$ basic) and $\mathbf{A} = \begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix}$

With $\begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_\mathrm{B} \\ \mathbf{x}_\mathrm{N} \end{bmatrix} = \mathbf{b}$ so $\mathbf{x}_\mathrm{B} = \mathbf{A}_B^{-1}\left(\mathbf{b} - \mathbf{A}_N \mathbf{x}_\mathrm{N}\right)$

Fundamental LP Theorem

- Given an LP in standard form with **A** of rank m then
  - If there is a feasible solution, there is a basic feasible solution
  - If there is an optimal, feasible solution, then there is an optimal, basic feasible solution
- Note, there could be a LARGE number of basic, feasible solutions
  - Simplex algorithm determines the optimal, basic feasible solution usually very quickly

LP Graphical Interpretation

- The LP constraints define a polyhedron in the solution space
  - This is a polytope if the polyhedron is bounded and nonempty
  - The basic, feasible solutions are vertices of this polyhedron
  - With the linear cost function the solution will be at one of vertices

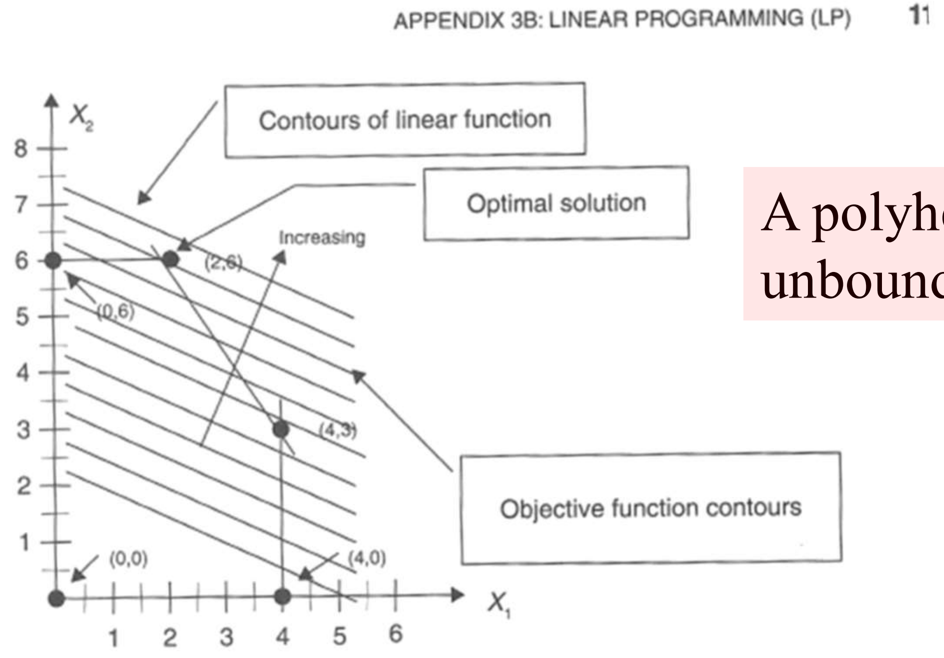


FIGURE 3.26 x_1, x_2 plane with cost contours and the optimal solution shown.

A polyhedron can be unbounded

Image: Figure 3.26 from course text

Simplex Algorithm

- The key is to move intelligently from one basic feasible solution (i.e., a vertex) to another, with the goal of continually decreasing the cost function
- The algorithm does this by determining the “best” variable to bring into the basis; this requires that another variable exit the basis, while always retaining a basic, feasible solution
- This is called pivoting

Determination of Variable to Enter the Basis

- To determine which non-basic variable should enter the basis (i.e., one which currently 0), look at how the cost function changes w.r.t. to a change in a non-basic variable (i.e., one that is currently zero)

Define $\mathbf{z} = \mathbf{c}\,\mathbf{x} = [\mathbf{c}_B \quad \mathbf{c}_N]\begin{bmatrix} \mathbf{x}_B \\ \mathbf{x}_N \end{bmatrix}$

With $\mathbf{x}_\mathrm{B} = \mathbf{A}_B^{-1}\left(\mathbf{b} - \mathbf{A}_N\mathbf{x}_\mathrm{N}\right)$

Then $\mathbf{z} = \mathbf{c}_B\mathbf{A}_B^{-1}\mathbf{b} + \left(\mathbf{c}_N - \mathbf{c}_B\mathbf{A}_B^{-1}\mathbf{A}_N\right)\mathbf{x}_\mathrm{N}$

Elements of $\mathbf{x}_\mathrm{n}$ are all zero, but we are looking to change one to decrease the cost

Determination of Variable to Enter the Basis, cont.

- Define the reduced (or relative) cost coefficients as

$$\mathbf{r} = \mathbf{c}_N - \mathbf{c}_B \mathbf{A}_B^{-1} \mathbf{A}_N$$

r is an n-m dimensional row vector

- Elements of this vector tell how the cost function will change for a change in a currently non-basic variable
- The variable to enter the basis is usually the one with the most negative relative cost
- If all the relative costs are nonnegative then we are at an optimal solution

Determination of Variable to Exit Basis

- The new variable entering the basis, say a position j, causes the values of all the other basic variables to change. In order to retain a basic, feasible solution, we need to insure no basic variables become negative. The change in the basic variables is given by

$$\tilde{\mathbf{x}}_B = \mathbf{x}_B - \mathbf{A}_B^{-1}\mathbf{a}_j\,\varepsilon$$

where ε is the value of the variable entering the basis, and $\mathbf{a}_j$ is its associated column in $\mathbf{A}$

Determination of Variable to Exit Basis, cont.

We find the largest value ε such

$$\tilde{\mathbf{x}}_B = \mathbf{x}_B - \mathbf{A}_B^{-1}\mathbf{a}_j\varepsilon \geq \mathbf{0}$$

If no such ε exists then the problem is unbounded; otherwise at least one component of $\tilde{\mathbf{x}}_B$ equals zero. The associated variable exits the basis.

Canonical Form

- The Simplex Method works by having the problem in what is known as canonical form
- Canonical form is defined as having the m basic variables with the property that each appears in only one equation, its coefficient in that equation is unity, and none of the other basic variables appear in the same equation
- Sometime canonical form is readily apparent

$$\begin{aligned} &\text{Minimize} \quad -(100x_1 + 120x_2) \\ &\text{s.t.} \quad 2x_1 + 2x_2 + x_3 = 8 \\ &\qquad 3x_1 + 5x_2 + x_4 = 15 \\ &\qquad x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Note that with x_3 and x_4 as basic variables $\mathbf{A}_B$ is the identity matrix

Canonical Form

- Other times canonical form is achieved by initially adding artificial variables to get an initial solution
- Example of the nutrition problem in canonical form with slack and artificial variables (denoted as **y**) used to get an initial basic feasible solution

Let x_1=ounces of A, x_2= ounces of B

Minimize $y_1+y_2+y_3$

s.t.

$$2x_1+3x_2+x_3+y_1=20$$

$$x_1+3x_2-x_4+y_2=12$$

$$4x_1+3x_2-x_5+y_3=24$$

$$x_1,x_2,x_3,x_4,x_5,y_1,y_2,y_3\geq 0$$

Note that with y_1, y_2, and y_3 as basic variables $\mathbf{A}_B$ is the identity matrix

LP Tableau

- With the system in canonical form, the Simplex solution process can be illustrated by forming what is known as the LP tableau
  - Initially this corresponds to the **A** matrix, with a column appended to include the **b** vector, and a row added to give the relative cost coefficients; the last element is the negative of the cost function value
  - Define the tableau as **Y**, with elements $\mathbf{Y}_{ij}$
  - In canonical form the last column of the tableau gives the values of the basic variables
- During the solution the tableau is updated by pivoting

LP Tableau for the Nutrition Problem with Artificial Variables

- When in canonical form the relative costs vector is

$$\mathbf{r} = \mathbf{c}_N - \mathbf{c}_B \mathbf{A}_B^{-1} \mathbf{A}_N = \mathbf{c}_B \mathbf{A}_N$$

$$\mathbf{r} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T - \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 & 0 & 0 \\ 1 & 3 & 0 & -1 & 0 \\ 4 & 3 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -7 \\ -9 \\ -1 \\ 1 \\ 1 \end{bmatrix}^T$$

- The initial tableau for the artificial problem is then

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	
2	3	1	0	0	1	0	0	20
1	3	0	–1	0	0	1	0	12
4	3	0	0	–1	0	0	1	24
–7	–9	–1	1	1	0	0	0	–56

Note the last column gives the values of the basic variables

LP Tableau Pivoting

- Pivoting is used to move from one basic feasible solution to another
  - Select the pivot column (i.e., the variable coming into the basis, say q) as the one with the most negative relative cost
  - Select the pivot row (i.e., the variable going out of the basis) as the one with the smallest ratio of x_i/Y_{iq} for $Y_{iq} > 0$; define this as row p (x_i is given in the last column)

That is, we find the largest value ε such

$$\tilde{\mathbf{x}}_B = \mathbf{x}_B - \mathbf{A}_B^{-1}\mathbf{a}_q\varepsilon \geq \mathbf{0}$$

If no such ε exists then the problem is unbounded;

otherwise at least one component of $\tilde{\mathbf{x}}_B$ equals zero.

The associated variable exits the basis.

LP Tableau Pivoting for Nutrition Problem

- Starting at

$$\begin{array}{ccccccccc}
x_1 & x_2 & x_3 & x_4 & x_5 & y_1 & y_2 & y_3 & \\
2 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 20 \\
1 & 3 & 0 & -1 & 0 & 0 & 1 & 0 & 12 \\
4 & 3 & 0 & 0 & -1 & 0 & 0 & 1 & 24 \\
-7 & -9 & -1 & 1 & 1 & 0 & 0 & 0 & -56
\end{array}$$

- Pivot on column q=2; for row get minimum of {20/3, 12/3, 24/3), which is row p=2

LP Tableau Pivoting

- Pivoting on element Y_{pq} is done by
  - First dividing row p by Y_{pq} to change the pivot element to unity.
  - Then subtracting from the kth row Y_{kq}/Y_{pq} times the pth row for all rows with $Y_{kq} <> 0$

$$
\begin{array}{ccccccccc}
x_1 & x_2 & x_3 & x_4 & x_5 & y_1 & y_2 & y_3 & \\
2 & 3 & 1 & 0 & 0 & 1 & 0 & 0 & 20 \\
1 & \mathbf{3} & 0 & -1 & 0 & 0 & 1 & 0 & 12 \\
4 & 3 & 0 & 0 & -1 & 0 & 0 & 1 & 24 \\
-7 & -9 & -1 & 1 & 1 & 0 & 0 & 0 & -56
\end{array}
$$

I'm only showing fractions with two ROD digits

Pivoting gives

$$
\begin{array}{ccccccccc}
x_1 & x_2 & x_3 & x_4 & x_5 & y_1 & y_2 & y_3 & \\
1 & 0 & 1 & 1 & 0 & 1 & -1 & 0 & 8 \\
0.33 & 1 & 0 & -0.33 & 0 & 0 & 0.33 & 0 & 4 \\
3 & 0 & 0 & 1 & -1 & 0 & -1 & 1 & 12 \\
-4 & 0 & -1 & -2 & 1 & 0 & 3 & 0 & -20
\end{array}
$$

LP Tableau Pivoting, Example, cont.

- Next pivot on column 1, row 3

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	
1	0	1	1	0	1	–1	0	8
0.33	1	0	–0.33	0	0	0.33	0	4
3	0	0	1	–1	0	–1	1	12
–4	0	–1	–2	1	0	3	0	–20

- Which gives

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	
0	0	1	0.67	0.33	1	–0.67	–0.33	4
0	1	0	–0.44	0.11	0	0.44	–0.11	2.67
1	0	0	0.33	–0.33	0	–0.33	0.33	4.0
0	0	–1	–0.67	–0.33	0	1.67	1.33	–4

LP Tableau Pivoting, Example, cont.

- Next pivot on column 3, row 1

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	
0	0	**1**	0.67	0.33	1	–0.67	–0.33	4
0	1	0	–0.44	0.11	0	0.44	–0.11	2.67
1	0	0	0.33	–0.33	0	–0.33	0.33	4
0	0	–1	–0.67	–0.33	0	1.67	1.33	–4

Since there are no negative relative costs we are done (with getting a starting solution)

- Which gives

x_1	x_2	x_3	x_4	x_5	y_1	y_2	y_3	
0	0	1	0.67	0.33	1	–0.67	–0.33	4
0	1	0	–0.44	0.11	0	0.44	–0.11	2.67
1	0	0	0.33	–0.33	0	–0.33	0.33	4
0	0	0	0	0	1	1	1	0

LP Tableau Full Problem

- The tableau from the end of the artificial problem is used as the starting point for the actual solution
  - Remove the artificial variables
  - Update the relative costs with the costs from the original problem and update the bottom right-hand corner value

$$\mathbf{c} = [0.2 \quad 0.25 \quad 0 \quad 0 \quad 0]$$

$$\mathbf{r} = \mathbf{c}_N - \mathbf{c}_B \mathbf{A}_B^{-1} \mathbf{A}_N = \mathbf{c}_B \mathbf{A}_N$$

$$\mathbf{r} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}^T - [0 \quad 0.25 \quad 0.2] \begin{bmatrix} 0.67 & 0.33 \\ -0.44 & 0.11 \\ 0.33 & -0.33 \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.04 \end{bmatrix}^T$$

- Since none of the relative costs are negative we are done with $x_1=4$, $x_2=2.7$ and $x_3=4$

Marginal Costs of Constraint Enforcement in LP

If we would like to determine how the cost function will change for changes in **b**, assuming the set of basic variables does not change then we need to calculate

$$\frac{\partial z}{\partial \mathbf{b}} = \frac{\partial(\mathbf{c}_B \mathbf{x}_B)}{\partial \mathbf{b}} = \frac{\partial(\mathbf{c}_B \mathbf{A}_B^{-1} \mathbf{b})}{\partial \mathbf{b}} = \mathbf{c}_B \mathbf{A}_B^{-1} = \boldsymbol{\lambda}$$

So the values of $\boldsymbol{\lambda}$ tell the marginal cost of enforcing each constraint.

The marginal costs will be used to determine the OPF locational marginal costs (LMPs)

Nutrition Problem Marginal Costs

- In this problem we had basic variables 1, 2, 3; nonbasic variables of 4 and 5

$$\mathbf{x}_{\mathrm{B}} = \mathbf{A}_{\mathrm{B}}^{-1}\left(\mathbf{b} - \mathbf{A}_{\mathrm{N}}\mathbf{x}_{\mathrm{N}}\right) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 0 \\ 4 & 3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 12 \\ 24 \end{bmatrix} = \begin{bmatrix} 4 \\ 2.67 \\ 4 \end{bmatrix}$$

$$\boldsymbol{\lambda} = \mathbf{c}_{\mathrm{B}}\mathbf{A}_{\mathrm{B}}^{-1} = \begin{bmatrix} 0.2 & 0.25 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 0 \\ 4 & 3 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 \\ 0.044 \\ 0.039 \end{bmatrix}$$

There is no marginal cost with the first constraint since it is not binding; values tell how cost changes if the **b** values were changed

Lumber Mill Example Solution

$$\begin{aligned} \text{Minimize} \quad & -(100x_1 + 120x_2) \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 = 8 \\ & 3x_1 + 5x_2 + x_4 = 15 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

An initial basic feasible solution is $x_1 = 0, x_2 = 0, x_3 = 8, x_4 = 15$

The solution is $x_1 = 2.5, x_2 = 1.5, x_3 = 0, x_4 = 0$

Then $\boldsymbol{\lambda} = \begin{bmatrix} 100 & 120 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$

Economic interpretation of λ is the profit is increased by 35 for every hour we up the first constraint (the saw) and by 10 for every hour we up the second constraint (plane)

Complications

- Often variables are not limited to being ≥ 0
  - Variables with just a single limit can be handled by substitution; for example if $x \geq 5$ then $x-5=z \geq 0$
  - Bounded variables, $\text{high} \geq x \geq 0$ can be handled with a slack variable so $x + y = \text{high}$, and $x,y \geq 0$
- Unbounded conditions need to be detected (i.e., unable to pivot); also the solution set could be null

$$\text{Minimize} \quad x_1 - x_2 \ \text{ s.t. } \ x_1 + x_2 \geq 8$$

$$\rightarrow x_1 + x_2 - y_1 = 8 \rightarrow x_2 = 8 \text{ is a basic feasible solution}$$

$$\begin{array}{cccc} x_1 & x_2 & y_1 & \\ 1 & 1 & -1 & 8 \\ 2 & 0 & -1 & 8 \end{array}$$

Complications

- Degenerate Solutions
  - Occur when there are less than m basic variables > 0
  - When this occurs the variable entering the basis could also have a value of zero; it is possible to cycle, anti-cycling techniques could be used
- Nonlinear cost functions
  - Nonlinear cost functions could be approximated by assuming a piecewise linear cost function
- Integer variables
  - Sometimes some variables must be integers; known as integer programming; we'll discuss after some power examples

LP Optimal Power Flow

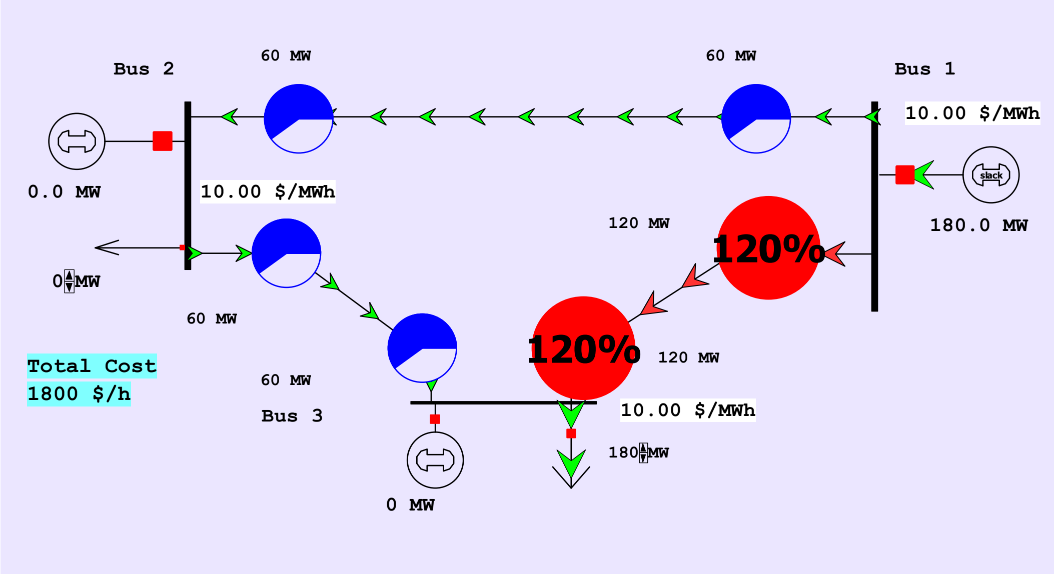
- LP OPF was introduced in
  - B. Stott, E. Hobson, “Power System Security Control Calculations using Linear Programming,” (Parts 1 and 2) *IEEE Trans. Power App and Syst.*, Sept/Oct 1978
  - O. Alsac, J. Bright, M. Prais, B. Stott, “Further Developments in LP-based Optimal Power Flow,” *IEEE Trans. Power Systems*, August 1990
- It is a widely used technique, particularly for real power optimization; it is the technique used in PowerWorld

LP Optimal Power Flow

- Idea is to iterate between solving the power flow, and solving an LP with just a selected number of constraints enforced
- The power flow (which could be ac or dc) enforces the standard power flow constraints
- The LP equality constraints include enforcing area interchange, while the inequality constraints include enforcing line limits; controls include changes in generator outputs
- LP results are transferred to the power flow, which is then resolved

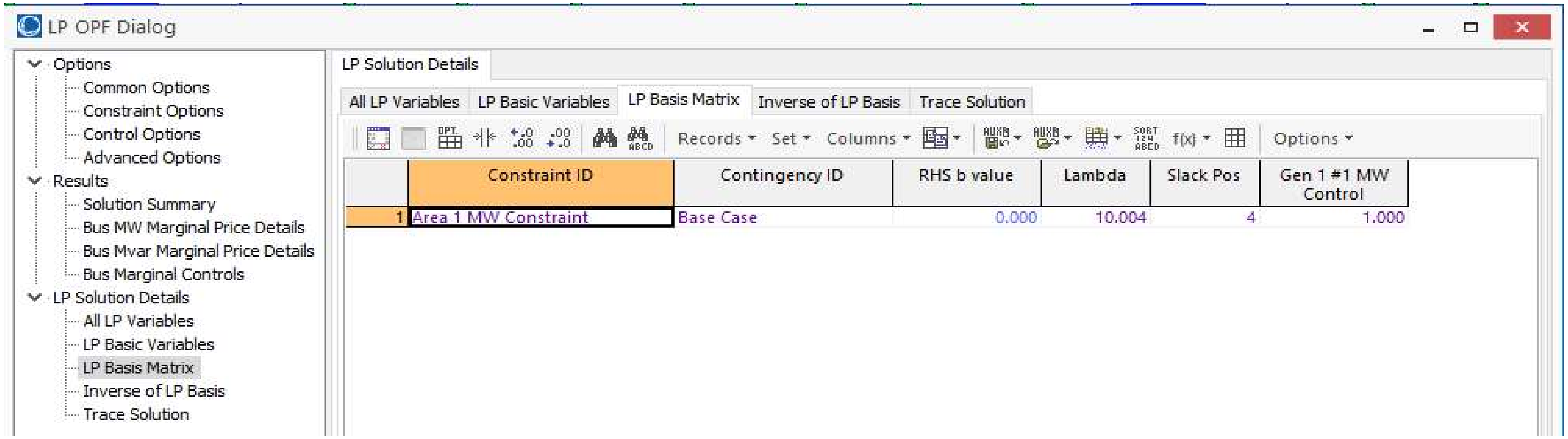
LP OPF Introductory Example

- In PowerWorld load the **B3LP** case and then display the LP OPF Dialog (select **Add-Ons, OPF Case Info, OPF Options and Results)**
- Use **Solve LP OPF** to solve the OPF, initially with no line limits enforced; this is similar to economic dispatch with a single power balance equality constraint

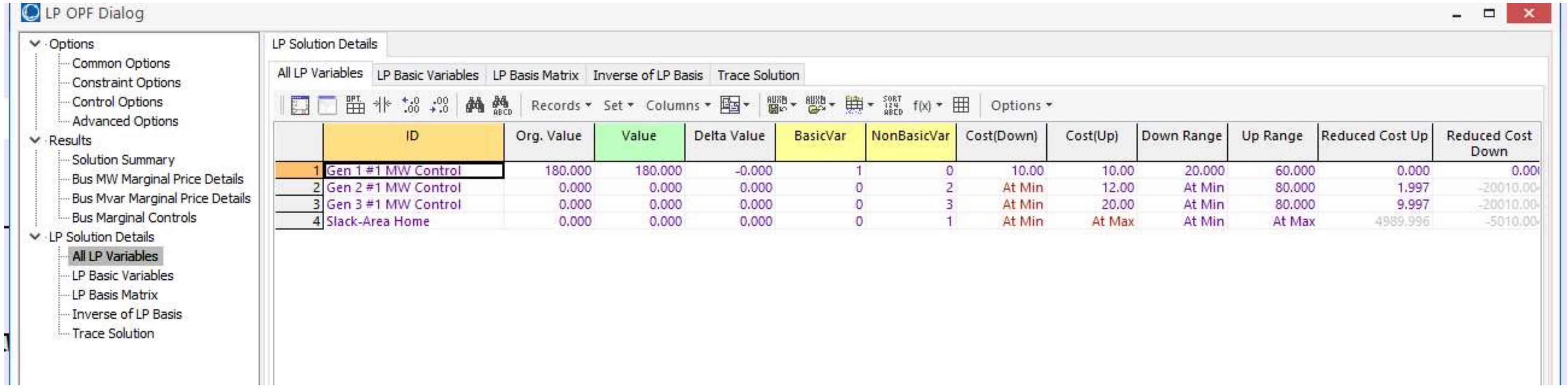


- The LP results are available from various pages on the dialog

LP OPF Introductory Example, cont



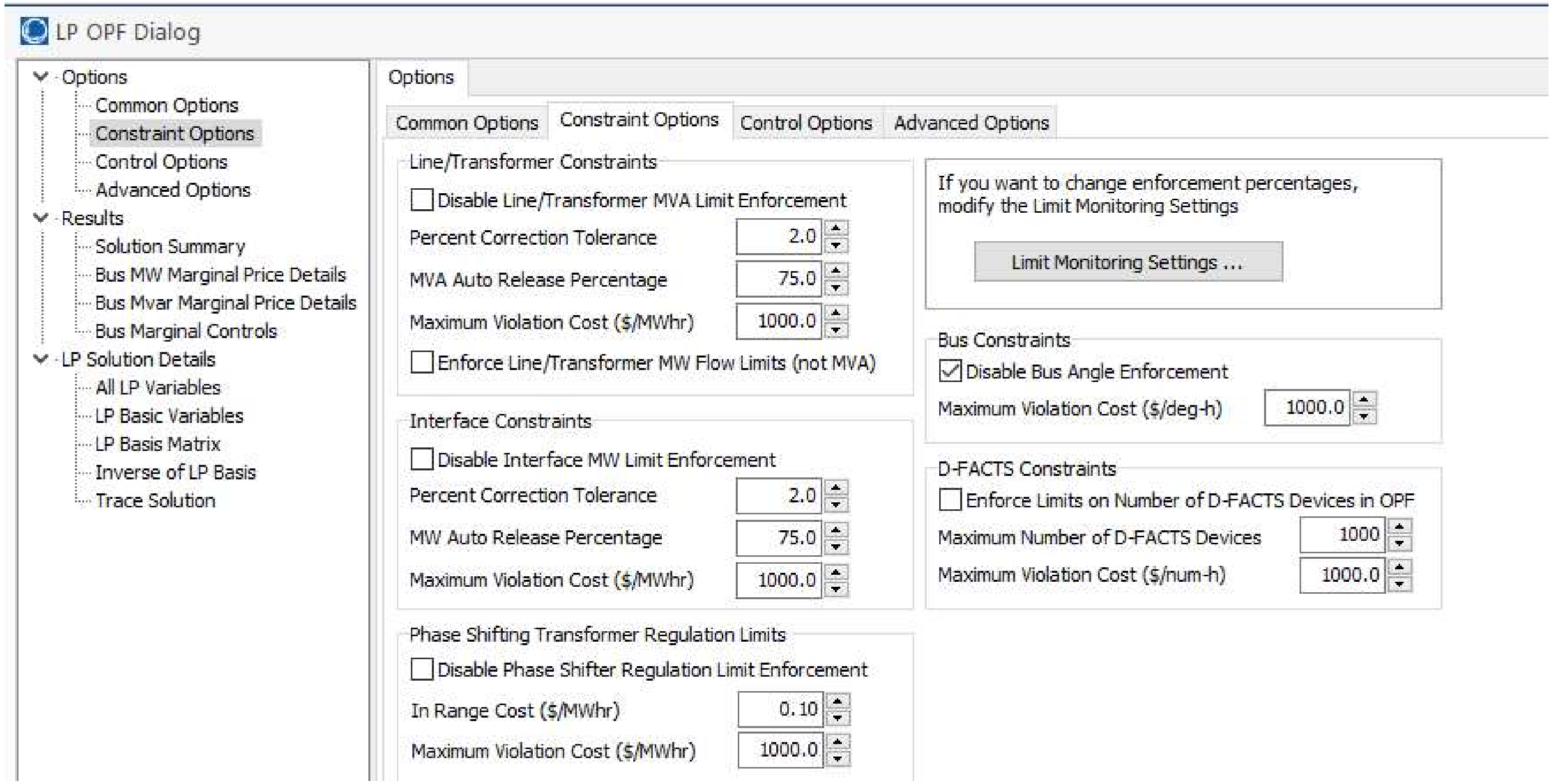
	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.000	10.004	4	1.000



	ID	Org. Value	Value	Delta Value	BasicVar	NonBasicVar	Cost(Down)	Cost(Up)	Down Range	Up Range	Reduced Cost Up	Reduced Cost Down
1	Gen 1 #1 MW Control	180.000	180.000	-0.000	1	0	10.00	10.00	20.000	60.000	0.000	0.00
2	Gen 2 #1 MW Control	0.000	0.000	0.000	0	2	At Min	12.00	At Min	80.000	1.997	-20010.00
3	Gen 3 #1 MW Control	0.000	0.000	0.000	0	3	At Min	20.00	At Min	80.000	9.997	-20010.00
4	Slack-Area Home	0.000	0.000	0.000	0	1	At Min	At Max	At Min	At Max	4989.996	-5010.00

LP OPF Introductory Example, cont

- On use **Options, Constraint Options** to enable the enforcement of the Line/Transformer MVA limits



Line/Transformer Constraints

☐ Disable Line/Transformer MVA Limit Enforcement

Percent Correction Tolerance: 2.0

MVA Auto Release Percentage: 75.0

Maximum Violation Cost ($/MWhr): 1000.0

☐ Enforce Line/Transformer MW Flow Limits (not MVA)

If you want to change enforcement percentages, modify the Limit Monitoring Settings

Interface Constraints

☐ Disable Interface MW Limit Enforcement

Percent Correction Tolerance: 2.0

MW Auto Release Percentage: 75.0

Maximum Violation Cost ($/MWhr): 1000.0

Bus Constraints

☑ Disable Bus Angle Enforcement

Maximum Violation Cost ($/deg-h): 1000.0

D-FACTS Constraints

☐ Enforce Limits on Number of D-FACTS Devices in OPF

Maximum Number of D-FACTS Devices: 1000

Maximum Violation Cost ($/num-h): 1000.0

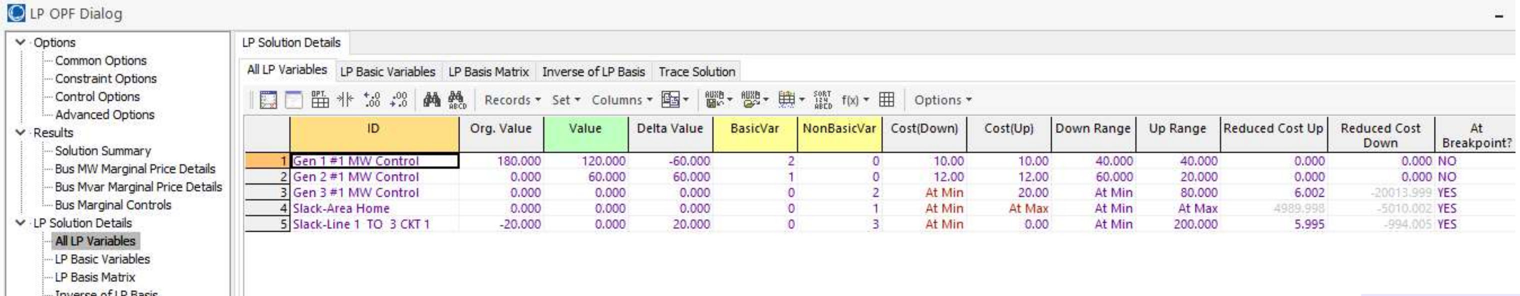
Phase Shifting Transformer Regulation Limits

☐ Disable Phase Shifter Regulation Limit Enforcement

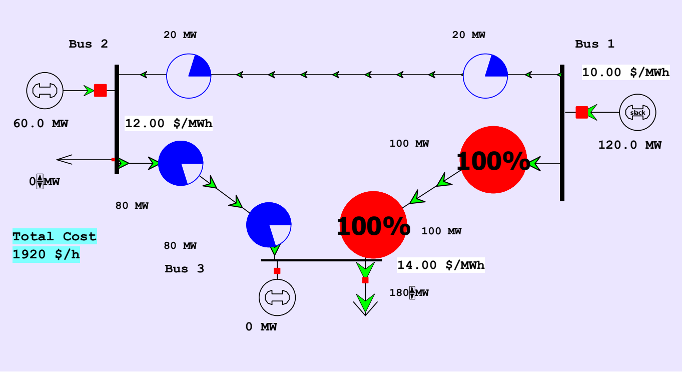
In Range Cost ($/MWhr): 0.10

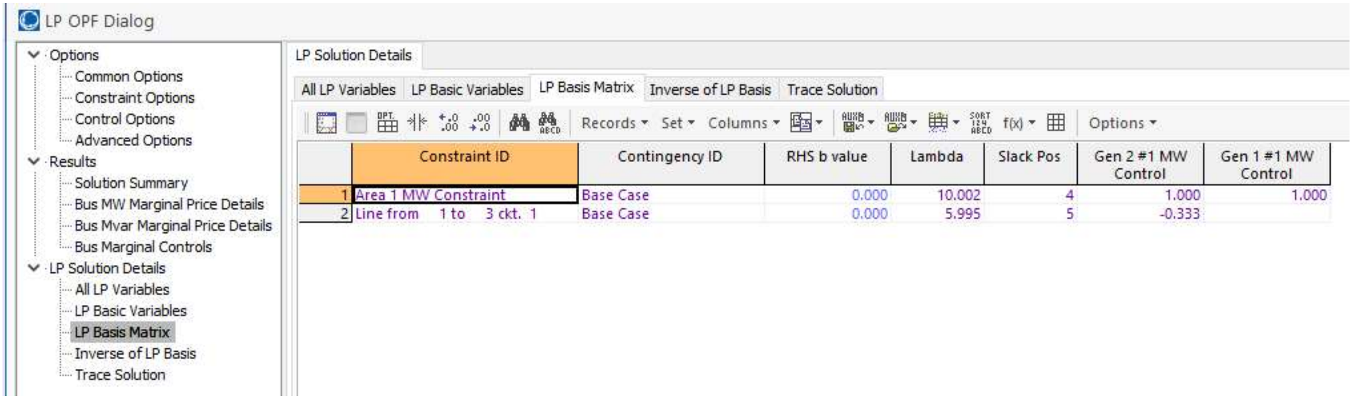
Maximum Violation Cost ($/MWhr): 1000.0

LP OPF Introductory Example, cont.



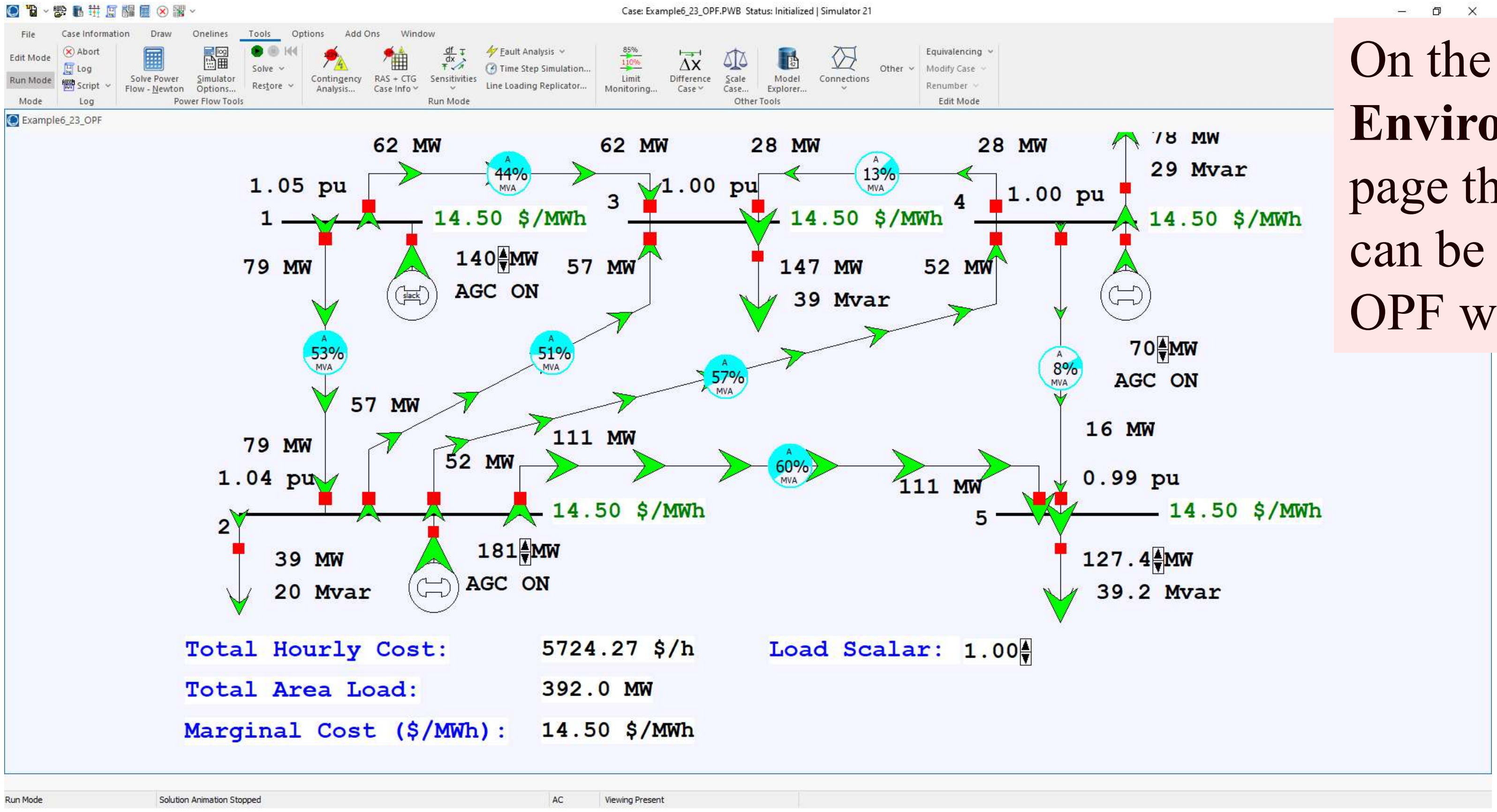
	ID	Org. Value	Value	Delta Value	BasicVar	NonBasicVar	Cost(Down)	Cost(Up)	Down Range	Up Range	Reduced Cost Up	Reduced Cost Down	At Breakpoint?
1	Gen 1 #1 MW Control	180.000	120.000	-60.000	2	0	10.00	10.00	40.000	40.000	0.000	0.000	NO
2	Gen 2 #1 MW Control	0.000	60.000	60.000	1	0	12.00	12.00	60.000	20.000	0.000	0.000	NO
3	Gen 3 #1 MW Control	0.000	0.000	0.000	0	2	At Min	20.00	At Min	80.000	6.002	-20013.999	YES
4	Slack-Area Home	0.000	0.000	0.000	0	1	At Min	At Max	At Min	At Max	4989.998	-5010.002	YES
5	Slack-Line 1 TO 3 CKT 1	-20.000	0.000	20.000	0	3	At Min	0.00	At Min	200.000	5.995	-994.005	YES





	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 2 #1 MW Control	Gen 1 #1 MW Control
1	Area 1 MW Constraint	Base Case	0.000	10.002	4	1.000	1.000
2	Line from 1 to 3 ckt. 1	Base Case	0.000	5.995	5	-0.333	

Example 6_23 Optimal Power Flow



Total Hourly Cost: 5724.27 $/h

Total Area Load: 392.0 MW

Marginal Cost ($/MWh): 14.50 $/MWh

Load Scalar: 1.00

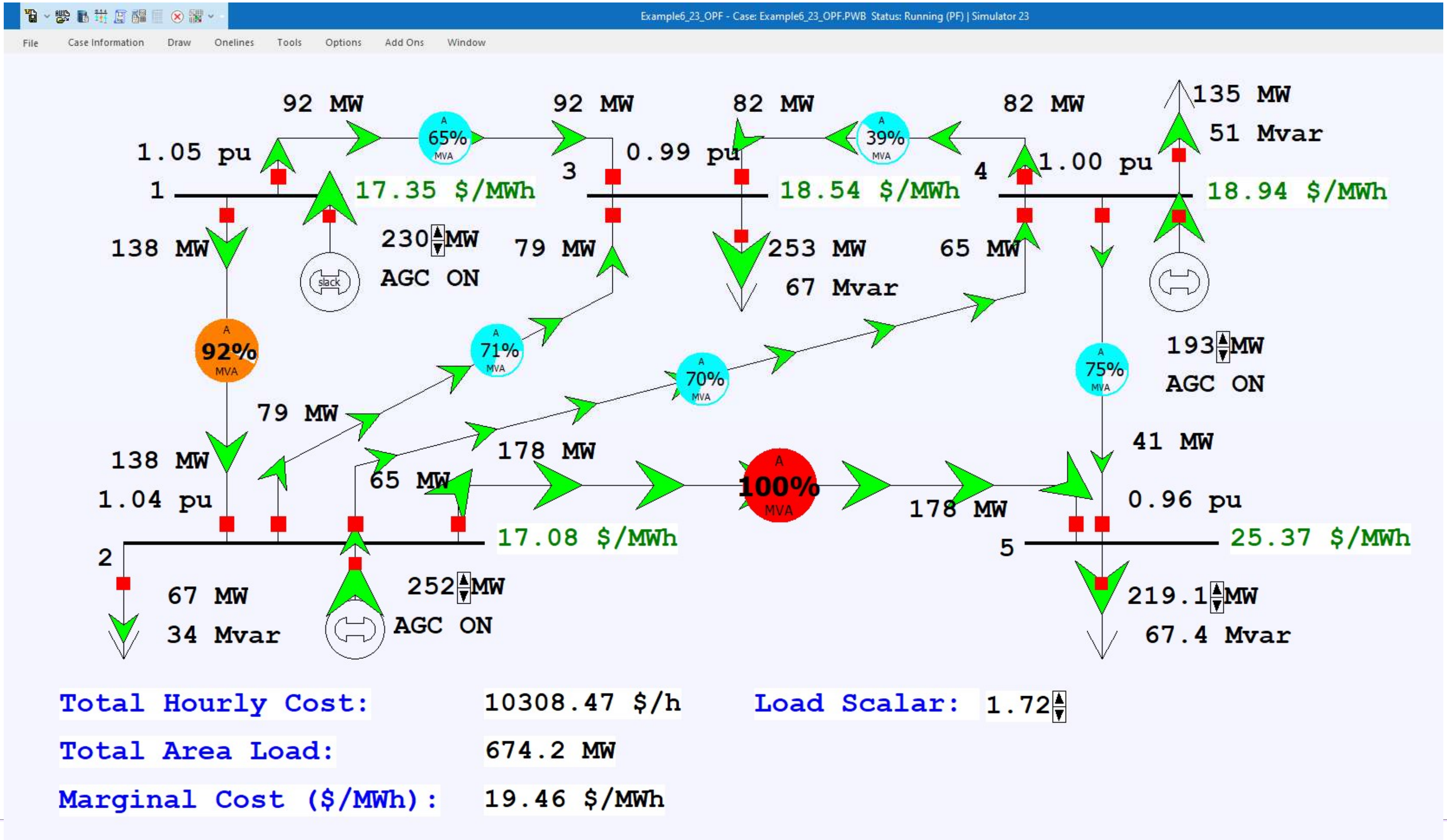
On the **Options, Environment** page the simulation can be set to solve an OPF when simulating

Open the case **Example6_23_OPF.** In this example the load is gradually increased

Locational Marginal Costs (LMPs)

- In an OPF solution, the bus LMPs tell the marginal cost of supplying electricity to that bus
- The term “congestion” is used to indicate when there are elements (such as transmission lines or transformers) that are at their limits; that is, the constraint is binding
- Without losses and without congestion, all the LMPs would be the same
- Congestion or losses causes unequal LMPs
- LMPs are often shown using color contours; a challenge is to select the right color range!

Example 6_23 Optimal Power Flow with Load Scale = 1.72



Total Hourly Cost: 10308.47 $/h

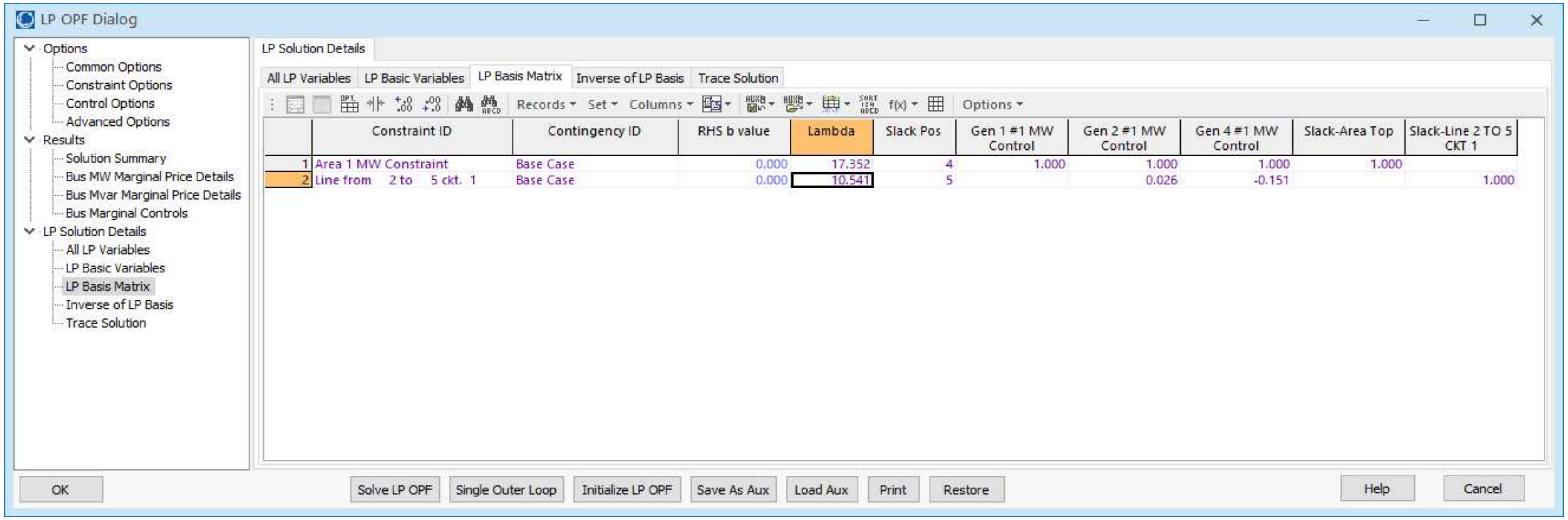
Load Scalar: 1.72

Total Area Load: 674.2 MW

Marginal Cost ($/MWh): 19.46 $/MWh

Example 6_23 Optimal Power Flow with Load Scale = 1.72

- LP Sensitivity Matrix (**A** Matrix)

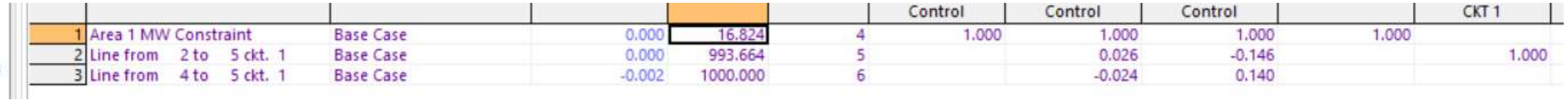


	Constraint ID	Contingency ID	RHS b value	Lambda	Slack Pos	Gen 1 #1 MW Control	Gen 2 #1 MW Control	Gen 4 #1 MW Control	Slack-Area Top	Slack-Line 2 TO 5 CKT 1
1	Area 1 MW Constraint	Base Case	0.000	17.352	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	10.541	5		0.026	-0.151		1.000

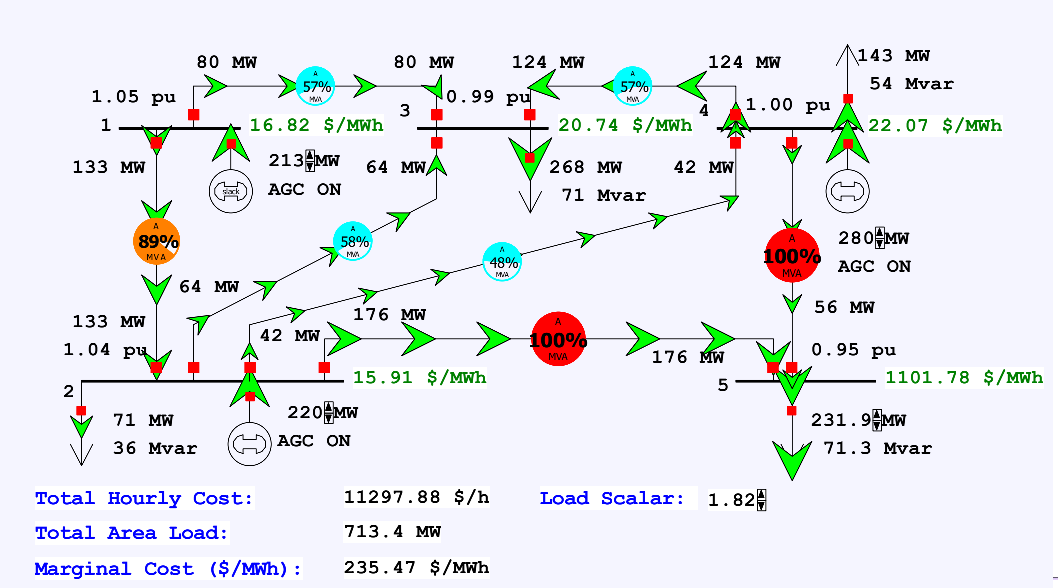
The first row is the power balance constraint, while the second row is the line flow constraint. The matrix only has the line flows that are being enforced.

Example 6_23 Optimal Power Flow with Load Scale = 1.82

- This situation is infeasible, at least with available controls. There is a solution because the OPF is allowing one of the constraints to violate (at high cost)

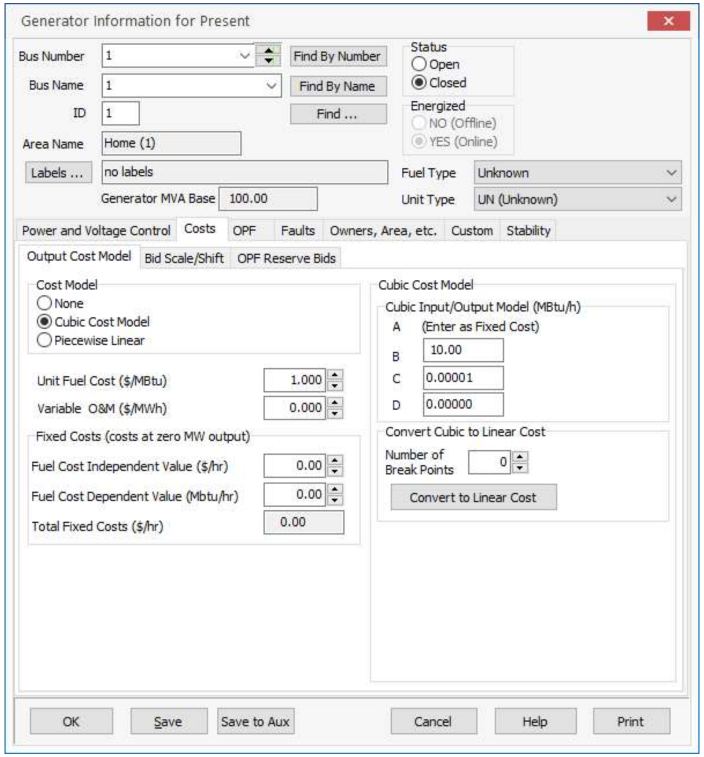


						Control	Control	Control		CKT 1
1	Area 1 MW Constraint	Base Case	0.000	16.824	4	1.000	1.000	1.000	1.000	
2	Line from 2 to 5 ckt. 1	Base Case	0.000	993.664	5		0.026	-0.146		1.000
3	Line from 4 to 5 ckt. 1	Base Case	-0.002	1000.000	6		-0.024	0.140		



Generator Cost Curve Modeling

- LP algorithms require linear cost curves, with piecewise linear curves used to approximate a nonlinear cost function
- Two common ways of entering cost information are
  - Quadratic function
  - Piecewise linear curve
- The PowerWorld OPF supports both types



Generator Information for Present

Bus Number: 1

Bus Name: 1

ID: 1

Area Name: Home (1)

Labels: no labels

Generator MVA Base: 100.00

Status: ○ Open ● Closed

Energized: ○ NO (Offline) ● YES (Online)

Fuel Type: Unknown

Unit Type: UN (Unknown)

Cost Model: ○ None ● Cubic Cost Model ○ Piecewise Linear

Unit Fuel Cost ($/MBtu): 1.000

Variable O&M ($/MWh): 0.000

Fixed Costs (costs at zero MW output)

Fuel Cost Independent Value ($/hr): 0.00

Fuel Cost Dependent Value (Mbtu/hr): 0.00

Total Fixed Costs ($/hr): 0.00

Cubic Cost Model

Cubic Input/Output Model (MBtu/h)

A (Enter as Fixed Cost)

B: 10.00

C: 0.00001

D: 0.00000

Convert Cubic to Linear Cost

Number of Break Points: 0

Security Constrained OPF

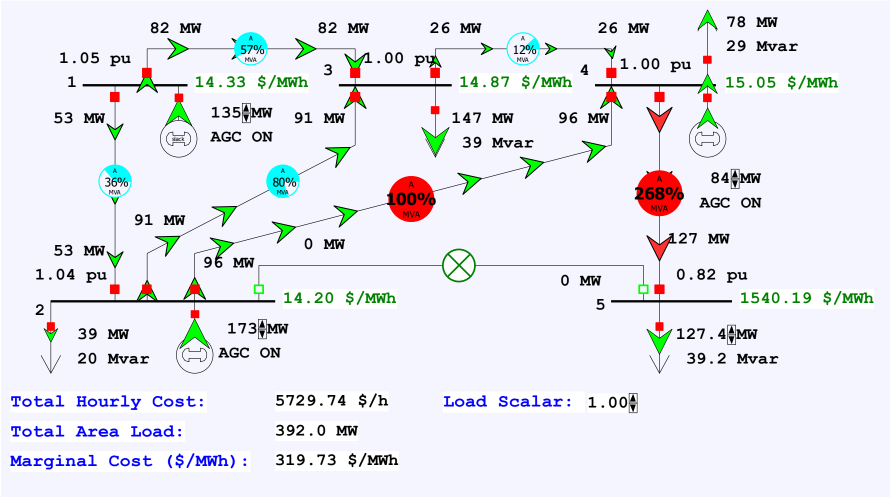
- Security constrained optimal power flow (SCOPF) is similar to OPF except it also includes contingency constraints
  - Again the goal is to minimize some objective function, usually the current system cost, subject to a variety of equality and inequality constraints
  - This adds significantly more computation, but is required to simulate how the system is actually operated (with N-1 reliability)
- A common solution is to alternate between solving a power flow and contingency analysis, and an LP

Security Constrained OPF, cont.

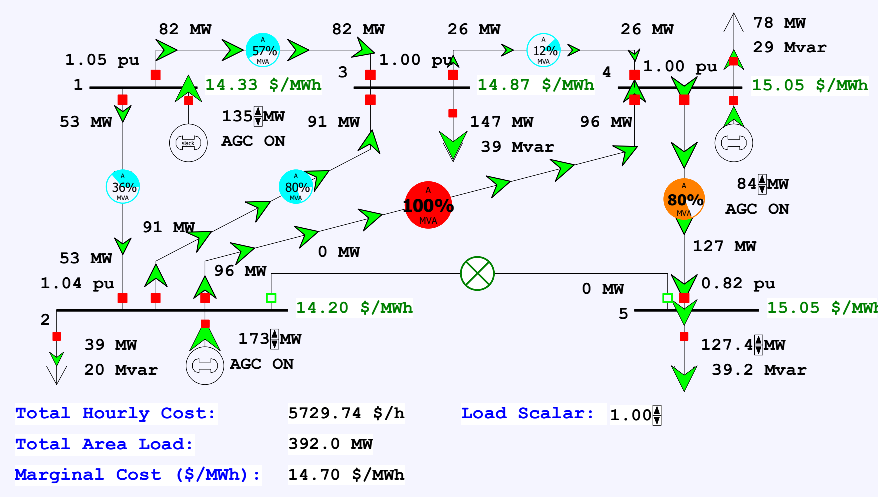
- With the inclusion of contingencies, there needs to be a distinction between what control actions must be done pre-contingent, and which ones can be done post-contingent
  - The advantage of post-contingent control actions is they would only need to be done in the unlikely event the contingency actually occurs
- Pre-contingent control actions are usually done for line overloads, while post-contingent control actions are done for most reactive power control and generator outage re-dispatch

SCOPF Example

- We'll again consider Example 6_23, except now it has been enhanced to include contingencies and we've also greatly increased the capacity on the line between buses 4 and 5; named Bus5_SCOPF_DC



Original with line 4-5 limit of 60 MW with 2-5 out

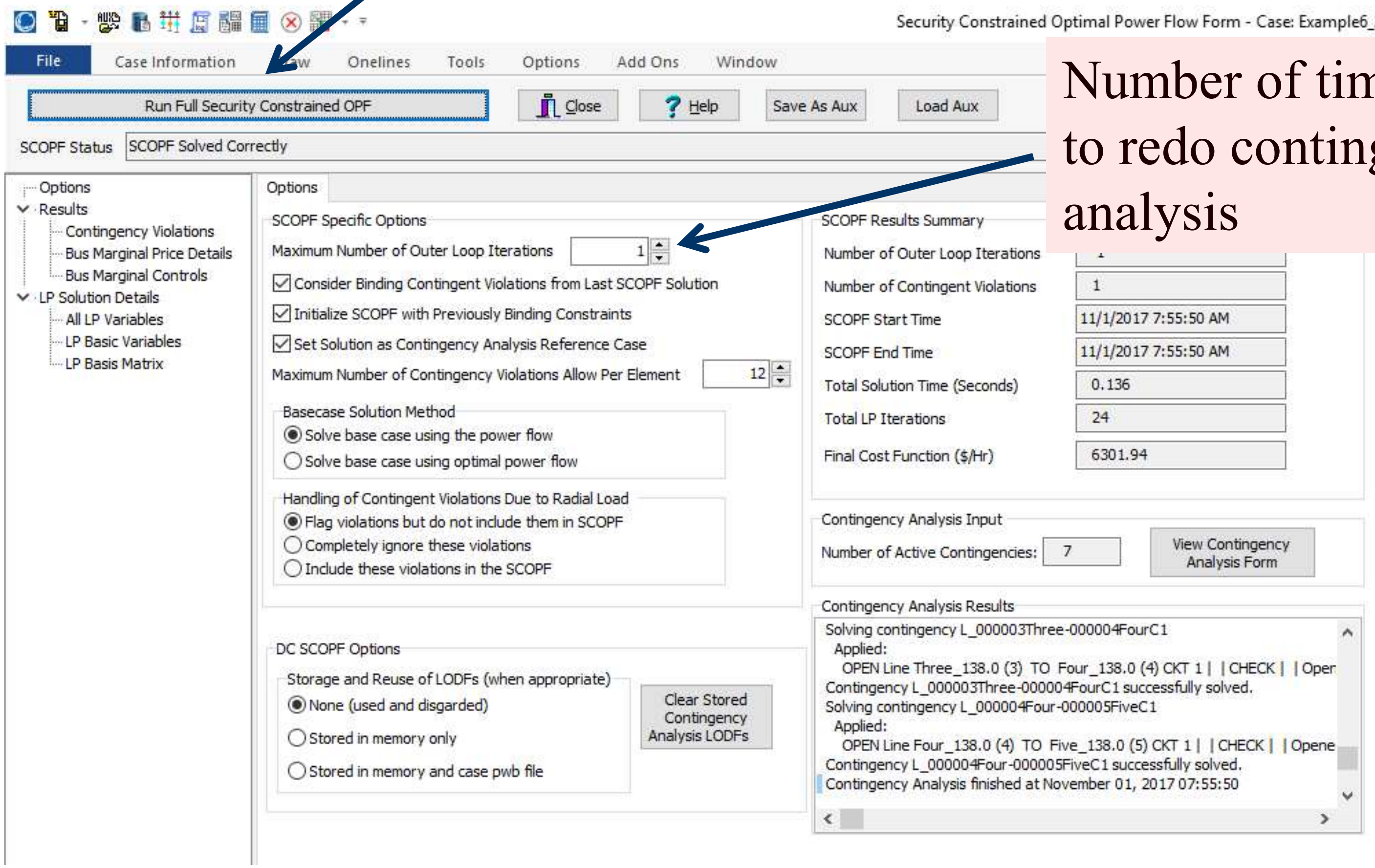


Modified with line 4-5 limit of 200 MVA with 2-5 out

PowerWorld SCOPF Application

Just click the button to solve

Number of times
to redo contingency
analysis



SCOPF Status: SCOPF Solved Correctly

SCOPF Specific Options

Maximum Number of Outer Loop Iterations: 1

☑ Consider Binding Contingent Violations from Last SCOPF Solution

☑ Initialize SCOPF with Previously Binding Constraints

☑ Set Solution as Contingency Analysis Reference Case

Maximum Number of Contingency Violations Allow Per Element: 12

Basecase Solution Method
● Solve base case using the power flow
○ Solve base case using optimal power flow

Handling of Contingent Violations Due to Radial Load
● Flag violations but do not include them in SCOPF
○ Completely ignore these violations
○ Include these violations in the SCOPF

DC SCOPF Options

Storage and Reuse of LODFs (when appropriate)
● None (used and disgarded)
○ Stored in memory only
○ Stored in memory and case pwb file

SCOPF Results Summary

Number of Outer Loop Iterations:

Number of Contingent Violations: 1

SCOPF Start Time: 11/1/2017 7:55:50 AM

SCOPF End Time: 11/1/2017 7:55:50 AM

Total Solution Time (Seconds): 0.136

Total LP Iterations: 24

Final Cost Function ($/Hr): 6301.94

Contingency Analysis Input

Number of Active Contingencies: 7

Contingency Analysis Results

```
Solving contingency L_000003Three-000004FourC1
  Applied:
    OPEN Line Three_138.0 (3)  TO  Four_138.0 (4) CKT 1 |  | CHECK |  | Oper
Contingency L_000003Three-000004FourC1 successfully solved.
Solving contingency L_000004Four-000005FiveC1
  Applied:
    OPEN Line Four_138.0 (4)  TO  Five_138.0 (5) CKT 1 |  | CHECK |  | Opene
Contingency L_000004Four-000005FiveC1 successfully solved.
Contingency Analysis finished at November 01, 2017 07:55:50
```

LP OPF and SCOPF Issues

- The LP approach is widely used for the OPF and SCOPF, particularly when implementing a dc power flow approach
- A key issue is determining the number of binding constraints to enforce in the LP tableau
  - Enforcing too many is time-consuming, enforcing too few results in excessive iterations
- The LP approach is limited by the degree of linearity in the power system
  - Real power constraints are fairly linear, reactive power constraints much less so