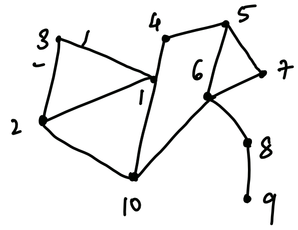
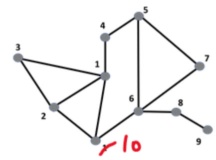
Homework 3

Solution 3



3. Use the Tinney 2 approach to order the following network. Give the permutation vector.



4. Using your reordered results from question 3, draw the full factorization path graph for the system.

1 — 4 4 4 3 2
2 — 3 3 3 2
3 — 2 2 2
4 — 2 2 2 2 2 2
5 — 3 3 3 3 3 3 3 2
6 — 4 4 3 3 3 3 3 2 1
7 — 2 2 2 2 2 2 2
8 — 2 1
9 — 1
10 — 3 3 3 3 2 2 2 2 1 1

due to fills.

due to fills

$$\text{rowPerm} = \begin{array}{cccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ \end{array}$$
$$\begin{bmatrix} 9 & 8 & 3 & 2 & 1 & 4 & 7 & 5 & 6 & 10 \end{bmatrix}$$

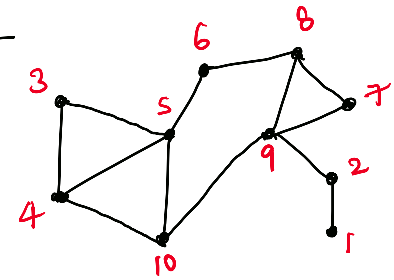
Solution 4

Matrix representation of relabeled graph.

	1	2	3	4	5	6	7	8	9	10
1	X	X								
2	X	X							X	
3			X	X	X					X
4			X	X	X					X
5			X	X	X	X				X
6					X	X		X		O
7							X	X	X	
8	–					X	X	X	X	O
9		X					X	X	X	X
10				X	X	O		O	X	X

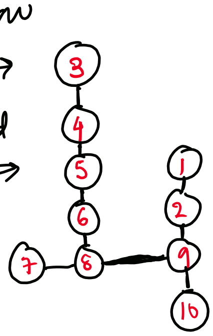
only consider the lower triangular matrix.

reordered graph.



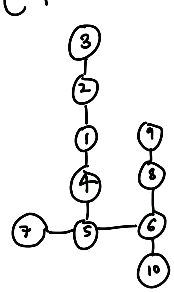
Factorization path. →

From reordered graph & connection matrix. ⇒



⇒

Factorization path for the original system (replace node numbers with original graph node numbers)



Solution 1

1. Manually do an LU factorization on the following matrix A. Then manually do a forward and backward substitution to solve for x in Ax=b

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ -1 & -1 & 5 & 2 \\ 0 & -1 & 2 & 4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ -1 & -1 & 5 & 2 \\ 0 & -1 & 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 2 \end{bmatrix}$$

initially $L = I_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_3 = R_3 + \frac{1}{2} \times R_1$

$$\begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ 0 & -1 & 9/2 & 2 \\ 0 & -1 & 2 & 4 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 = R_3 + R_2/4$

$$\begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ 0 & 0 & 17/4 & 7/4 \\ 0 & -1 & 2 & 4 \end{bmatrix} \Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1/2 & -1/4 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\rightarrow R_4 = R_4 + R_2/4$

$\rightarrow R_4 = R_4 - \frac{7R_3}{17}$

$$\begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ 0 & 0 & 17/4 & 7/4 \\ 0 & 0 & 0 & \frac{103}{34} \end{bmatrix} \Rightarrow U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{1}{2} & -\frac{1}{4} & 1 & 0 \\ 0 & -\frac{1}{4} & \frac{7}{17} & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -0.5 & -0.25 & 1 & 0 \\ 0 & -0.25 & 0.411 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 0 & -1 & 0 \\ 0 & 4 & -1 & -1 \\ 0 & 0 & 4.25 & 1.75 \\ 0 & 0 & 0 & 3.03 \end{bmatrix}$$

Use forward substitution to solve $Ly = b$.

$y = [1, 2, 4, 0.8529]$

Use backward substitution to calculate x; $Ux = y$.

$x = [0.9126, 0.7767, 0.8252, 0.2816]$

2 Code the LU factorization discussed in class for full matrices, along with the forward/backward substitution. To test your algorithm use it to factor and solve the above matrix from question 1. You do not need to code pivoting.

```
clc;

A = [2, 0, -1, 0; 0, 4, -1, -1; -1, -1, 5, 2; 0, -1, 2, 4;];
%A = input('Input matrix A:');

%performing LU decomposition
rows = 4;
col = 4;
v=[1 1 1 1];
L=diag(v); %initializing the lower triangular matrix as identity matrix
U=A; %initializing the upper triangular matrix to A
k=1; %inner loop for factorizing U matrix

%calculating LU decomposition
for i=2:1:rows
    for j=1:1:col
        if j<i
            L(i,j)=U(i,j)/U(j,j); %calculating scale factor
            for k=1:1:col
                if k>=j
                    U(i,k)=U(i,k)-(L(i,j)*U(j,k)); %performing row operation
                end
            end
        end
    end
end

result = L*U; %this is same as A

b = [1; 2; 3; 2];
%b = input('Input matrix b:');

y=inv(L)*b; %forward subsitution to calculate y
x=inv(U)*y; %backward substitution to calcculate x

display(y);
display(x);
```

```
y =

    1.0000
    2.0000
    4.0000
    0.8529


x =

    0.9126
    0.7767
    0.8252
    0.2816
```