EFFECTIVE CALCULATION OF POWER SYSTEM LOW-VOLTAGE SOLUTIONS

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Abstract

This paper develops a method for reliably determining the set of low-voltage solutions which are closest to the operable power flow solution. These solutions are often used in conjunction with techniques such as energy methods and the voltage instability proximity index (VIPI) for assessing system voltage stability. This paper presents an algorithm which provides good initial guesses for these solutions. The results are demonstrated on a small system and on larger systems with up to 2000 buses.

I. Introduction

One class of methods for determination of the voltage stability limits utilizes the distance between the operable power flow solution and an alternative (or low-voltage) solution. This includes the use of state space measures [1], [2], [3], a PV curve based method [4], and energy methods [5] for quasi-static voltage stability assessment. Additionally, low-voltage solutions are used in the determination of the exit point for dynamic voltage stability assessment [6], [7]. The common element of these methods is the need to determine at least one of these low-voltage solutions. It has long been recognized that the nonlinear power flow equations can have multiple solutions [8]. While the actual number of solutions depends upon system loading, the number of potential solutions can be quite large; the maximum number for an n+1 bus system is estimated to be 2^n in [9] and $\binom{2n}{n}$ in [10]. For these methods to be practical, there is a need for an efficient method to determine the appropriate low-voltage solutions reliably.

A number of methods have been proposed for determining some (or all) of these solutions. In [9] an algorithm is presented that attempts to determine all of the low-voltage solutions by systematically starting the power flow solution from different initial voltage guesses. The number of solutions actually found depends upon the convergence characteristics of the Newton-Raphson (N-R) power flow. In [10] a homotopy-based method is developed that offers a more rigorous method for determining all of the system solutions but requires computation proportional to $\binom{2n}{n}$. An improved homotopy method for finding all solutions is presented in [11] that requires computation proportional to the product of the system size and the actual number of solutions. Genetic algorithms for solving this problem are discussed in [12]. While these methods promise to find all of the system solutions, they require computation well beyond that considered reasonable for on-line usage.

Luckily, results from [13] indicate that the search for low-voltage solutions can be restricted to those solutions corresponding to equilibrium points with a single positive eigenvalue associated with the Jacobian of the linearized system dynamic equations (i.e., the type-one solutions). For certain realistic classes of load models, the stability of an equilibrium point can be determined from the eigenvalues of the polar form of the power flow Jacobian [14], [15], [16]. Furthermore, only those solutions associated with relatively low energy margins need to be determined. Nevertheless, determining these solutions quickly enough for on-line usage has proven difficult.

As the system approaches the loadability limit, the number of solutions decreases. At some point (usually very close to the limit) only the operable solution and a closely paired low-voltage solution remain. If one is concerned only with checking for this condition, [17] presents an efficient method that exploits the convergence characteristics of the rectangular N-R power flow. An alternative method is proposed in [18] that uses the Householder transform to check for the solution.

To assess security while a system is still a reasonable distance from the loadability limit, a number of type-one low-voltage solutions are usually required, with the different solutions providing measures of the security in different portions of the system [13]. In [9] a method that requires approximately n power flow solutions was suggested. The computation was further reduced in [13] and [19] by employing screening techniques using equivalent systems. The goal of this paper is to develop a more robust and efficient method for determining this set of low-voltage solutions "closest" to the operable solution. The method presented utilizes the structure of the power flow equations to determine good initial guesses for these solutions, and is general enough to determine more than just a single low-voltage solution. The results are demonstrated on both small systems and larger systems with up to 2000 buses.

II. Power Flow Low-Voltage Solutions

The problem is introduced by considering the low-voltage solution characteristics of the symmetric three-bus system shown in Figure 1 (line impedances are per unit using a 100 MVA base). Buses 1 and 2 are modeled as constant PQ load buses, while bus 3 is the system slack with a constant per unit voltage of 1.0 + j0.0. For the base case loading shown in the figure, the system has four real power flow solutions given in Table 1. Solution A corresponds to the operable solution, solutions B and C are type-one, while solution D is type-two.

As the system loading is varied, the locations of these solutions also vary. In particular, as loading increases towards a point of maximum loadability, the number of solutions decreases until at some point only a single type-one solution and the operable solution remain. Maximum loadability occurs when these two solutions coalesce at a point on the maximum loadability boundary (denoted by Σ) in a saddle-node bifurcation. System loadability can be monitored by tracking the distance between the operable solution and this type-one solution. However, a key point is that unless one knows a priori how system loading will change, it is impossible to know with absolute (and sometimes even reasonable) certainty the identity of this type-one solution. Therefore a number of solutions close to the operable solution must be determined.

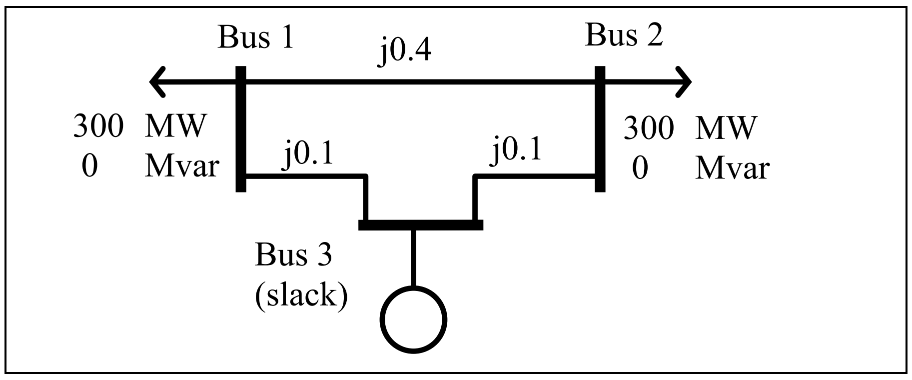

Figure 1: Three Bus System

Voltage	Solution A	Solution B	Solution C	Solution D
V_1	0.9487	0.2677	0.7477	0.3162
θ_1	-18.43°	-77.52°	-26.95°	-71.56°
V_2	0.9487	0.7471	0.2677	0.3162
θ_2	-18.43°	-26.95°	-77.52°	-71.56°

Table 1: Three Bus Base Case Solutions

The advantage of this approach is that it provides a method of loadability assessment that is independent of any assumed path to the boundary Σ. This quality can be extremely important in trying to assess system security accurately under the unusual operating conditions often associated with proximity to voltage instability. While many utilities have a good idea of load participation under normal operating conditions, this information is much more difficult to ascertain under such stressed conditions for which little historical data is available. Furthermore, as transmission systems become more open, there will be an increase in potentially harmful third party transactions of which the operator may have little knowledge and control.

For example, the following three figures show the solution variation for the three bus system in the V_1-V_2 plane as load is increased with different assumed participation factors. In Figure 2 the load increase at bus 1 is twice that at bus 2, while in Figure 3 the opposite occurs, with the load increase at bus 2 twice the value at bus 1. In the first case the heavy load increase at bus 1 ultimately causes solutions A and B to coalesce as maximum loadability is reached (at a total load of 965 MW). Note that solutions C and D also coalesce, but at a lower load of only 787 MW. The opposite occurs in the second case, with solutions A and C now coalescing at maximum loadability (now B and D coalesce at a lower value).

In the Figure 4 case the load participation is initially higher at bus 1, causing solutions A and B to initially move toward each other. However, when the bus 1 load is at 1.3 times the base value, the participation is switched so that most of the load increase occurs at bus 2. The maximum loadability boundary is then ultimately reached with solutions A and C coalescing. This example illustrates that even though a low-voltage solution may at some point be closest to the operable solution (as B is early on), changing system conditions can result in a final bifurcation with another solution. Hence the potential need to calculate more than one low-voltage solution.

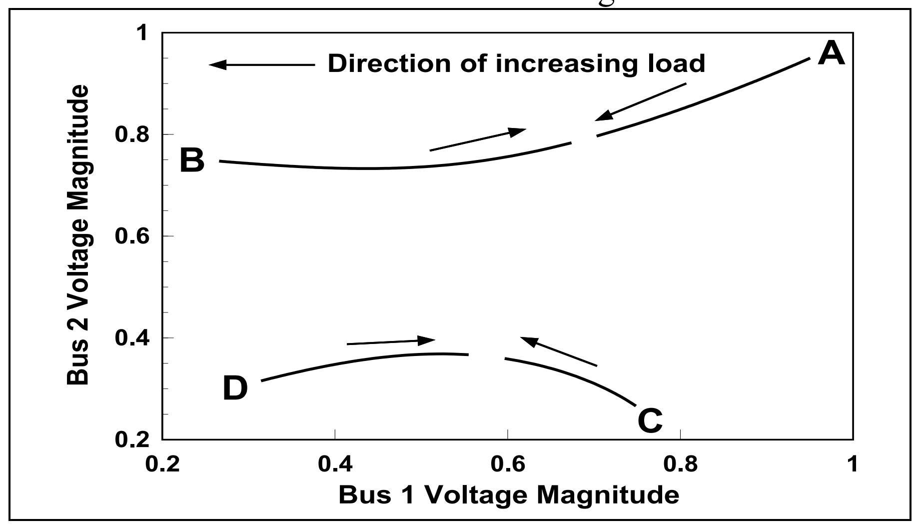

Figure 2: Maximum Load Participation at Bus 1

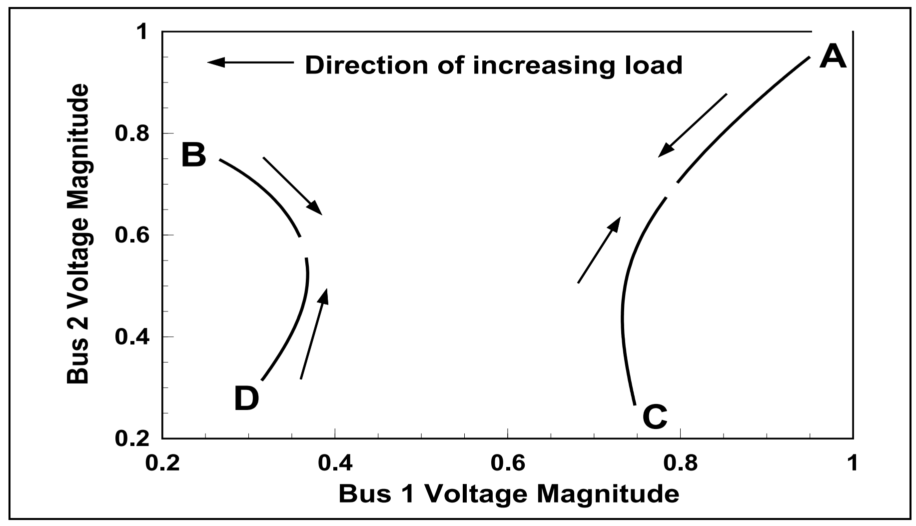

Figure 3: Maximum Load Participation at Bus 2

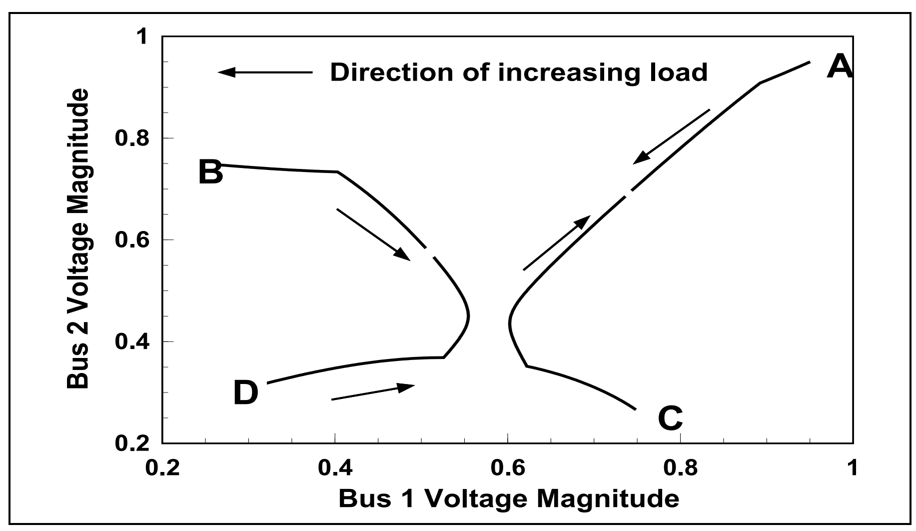

Figure 4: Varying Load Participation

The type-one low-voltage solutions are often identified by the bus number with the lowest voltage magnitude. Alternatively, and almost always equivalently, the solutions can be classified based upon information derived from an eigenvector of the polar form of the solution Jacobian. This is because at the point of bifurcation the Jacobian is singular, with the initial direction of voltage collapse along the eigenvector corresponding to the zero eigenvalue of the polar form of the power flow Jacobian [20], [21]. This eigenvector usually has relatively large components at only a few buses. Also, for loading significantly less than this critical value, the eigenvector associated with the positive eigenvalue of $J(\mathbf{x}^u)$ still approximates this direction [13]. The solution can be identified from the bus number corresponding to the largest magnitude in this eigenvector. This solution can then be used to obtain a measure of the voltage security in that portion of the system. For the three bus example, solution B is the bus 1 low-voltage solution, while solution C is the bus 2 solution (solution A is the operable solution, while D is type-two).

III. Relationships between Solutions

To develop the necessary relationships between the operable solution and the low-voltage solutions, consider the power flow equations for an n+1 bus system (with bus n+1 as the slack):

$$\mathbf{S} = \mathbf{f}(\mathbf{x}^*) \tag{1}$$

where **S** is a vector of the constant real and reactive power load minus generation at all the non-generator (PQ) buses, and the real power plus voltage magnitude equality constraints at all generator (PV) buses (except the slack). For convenience assume that the first m buses are PV, and the remainder are PQ:

$$\mathbf{S} = [P_1, ..., P_n, V_1^2, ..., V_m^2, Q_{m+1}, ..., Q_n]^T \tag{2}$$

with $\mathbf{x}^*$ the power flow solution in rectangular coordinates,

$$\mathbf{x} = [e_1, e_2, ..., e_n, f_1, f_2, ..., f_n]^T, \tag{3}$$

and **f** the function of the bus power/voltage constraints

$$\mathbf{f} = [f_{p1}(\mathbf{x}), ..., f_{pn}(\mathbf{x}), f_{q1}(\mathbf{x}), ..., f_{qn}(\mathbf{x})]^T \tag{4}$$

$$\mathbf{f}_{pi} = -\sum_{j=1}^{n} [\, e_i(e_jG_{ij} - f_jB_{ij}) + f_i(f_jG_{ij} + e_jB_{ij}) \,] \tag{5a}$$

and at the PV and PQ buses respectively

$$f_{qi} = e_i^2 + f_i^2 \tag{5b}$$

$$f_{qi} = -\sum_{j=1}^{n} [\, f_i(e_jG_{ij} - f_jB_{ij}) - e_i(f_jG_{ij} + e_jB_{ij}) \,] \tag{5c}$$

Because the power balance equations are a set of quadratic equations with no first order terms, the Taylor series expansion of (1) can be written exactly as [22]:

$$\mathbf{S} = \mathbf{f}(\mathbf{x}^*) = \mathbf{f}(\mathbf{x} + \Delta\mathbf{x}) = \mathbf{f}(\mathbf{x}) + \mathbf{J}(\mathbf{x})\,\Delta\mathbf{x} + \mathbf{f}(\Delta\mathbf{x}) \tag{6}$$

with $\Delta\mathbf{x}$ the difference between an arbitrary **x** and a solution $\mathbf{x}^*$, while $\mathbf{J}(\mathbf{x})$ is the power flow Jacobian. Additionally, since (1) is quadratic, for an arbitrary scalar μ we have

$$\mathbf{f}(\mu\Delta\mathbf{x}) = \mu^2\, \mathbf{f}(\Delta\mathbf{x}) \tag{7}$$

In order to develop an improved low voltage solution algorithm, we make four observations. First, consider two unique power flow solutions: $\mathbf{x}^s$ the operable power flow solution, and $\mathbf{x}^u$ a type-one low-voltage solution. Let $\mathbf{x}^m$ be the midpoint between the two solutions. Then define

$$\Delta\mathbf{x}^m = \mathbf{x}^s - \mathbf{x}^m = \mathbf{x}^m - \mathbf{x}^u \tag{8}$$

From (6) and (7) it is clear that

$$\mathbf{f}(\mathbf{x}^s) = \mathbf{f}(\mathbf{x}^m) + \mathbf{J}(\mathbf{x}^m)\,\Delta\mathbf{x}^m + \mathbf{f}(\Delta\mathbf{x}^m) = \mathbf{S} \tag{9}$$

$$\mathbf{f}(\mathbf{x}^u) = \mathbf{f}(\mathbf{x}^m) - \mathbf{J}(\mathbf{x}^m)\,\Delta\mathbf{x}^m + \mathbf{f}(\Delta\mathbf{x}^m) = \mathbf{S} \tag{10}$$

Subtracting (10) from (9) gives

$$\mathbf{0} = 2\,\mathbf{J}(\mathbf{x}^m)\,\Delta\mathbf{x}^m \tag{11}$$

Since by definition $\Delta\mathbf{x}^m \neq \mathbf{0}$, $\mathbf{J}(\mathbf{x}^m)$ is singular, indicating that $\mathbf{x}^m$ is a point on the maximum loadability boundary Σ, and that $\Delta\mathbf{x}^m$ is the eigenvector associated with the zero eigenvalue of $\mathbf{J}(\mathbf{x}^m)$. Since $\mathbf{x}^m$ is on the line segment joining $\mathbf{x}^s$ and $\mathbf{x}^u$, it is the boundary point closest (in state space) to both solutions; the load level for $\mathbf{x}^m$ is $\mathbf{f}(\mathbf{x}^m)$. Also, starting from the operable solution $\mathbf{x}^s$ and knowing $\mathbf{x}^m$, we could directly determine $\mathbf{x}^u$.

The second observation provides a method for approximating $\Delta\mathbf{x}^m$ (and hence $\mathbf{x}^m$). Again using (6) we can write

$$\mathbf{f}(\mathbf{x}^m) = \mathbf{f}(\mathbf{x}^s) - \mathbf{J}(\mathbf{x}^s)\,\Delta\mathbf{x}^m + \mathbf{f}(\Delta\mathbf{x}^m) \tag{12}$$

By subtracting (9) from (12), and recalling that $\mathbf{J}(\mathbf{x}^m)\,\Delta\mathbf{x}^m = \mathbf{0}$, we can directly solve for $\Delta\mathbf{x}^m$

$$\mathbf{f}(\mathbf{x}^m) - \mathbf{f}(\mathbf{x}^s) = -\mathbf{f}(\mathbf{x}^m) + \mathbf{f}(\mathbf{x}^s) - \mathbf{J}(\mathbf{x}^s)\,\Delta\mathbf{x}^m \tag{13}$$

$$2\,[\mathbf{J}(\mathbf{x}^s)]^{-1}\,[\mathbf{f}(\mathbf{x}^m) - \mathbf{f}(\mathbf{x}^s)] = \Delta\mathbf{x}^m \tag{14}$$

Thus, if we know the "midpoint mismatch", $[\mathbf{f}(\mathbf{x}^m) - \mathbf{f}(\mathbf{x}^s)]$ (or have a reasonable approximation), we can determine $\Delta\mathbf{x}^m$.

To develop this approximation, we require the use of the optimal multiplier result of [22] which states that once an arbitrary direction $\Delta\mathbf{x}$ has been found, one can directly and efficiently solve for a scalar optimal multiplier μ which minimizes the following cost function in that direction:

$$F(\mathbf{x}) = \frac{1}{2}\,[\mathbf{S} - \mathbf{f}(\mathbf{x}^s + \mu\Delta\mathbf{x})]^T\,[\mathbf{S} - \mathbf{f}(\mathbf{x}^s + \mu\Delta\mathbf{x})] \tag{15}$$

This result is derived by first using (6) to expand $\mathbf{f}(\mathbf{x})$ about $\mathbf{x}^s$

$$\mathbf{f}(\mathbf{x}^s + \mu\Delta\mathbf{x}) = \mathbf{f}(\mathbf{x}^s) + \mu\,\mathbf{J}(\mathbf{x}^s)\,\Delta\mathbf{x} + \mu^2\,\mathbf{f}(\Delta\mathbf{x})$$

Then define vectors **a**, **b**, **c**, and rewrite (15)

$$\mathbf{a} = \mathbf{S} - \mathbf{f}(\mathbf{x}^s) \tag{16}$$

$$\mathbf{b} = -\mathbf{J}(\mathbf{x}^s)\,\Delta\mathbf{x} \tag{17}$$

$$\mathbf{c} = -\mathbf{f}(\Delta\mathbf{x}) \tag{18}$$

$$F(\mathbf{x}) = \frac{1}{2}\,[\mathbf{a} + \mu\,\mathbf{b} + \mu^2\,\mathbf{c}]^T\,[\mathbf{a} + \mu\,\mathbf{b} + \mu^2\,\mathbf{c}] \tag{19}$$

The values of μ which minimize (15) are determined by differentiating (19) with respect to μ, which gives

$$g_0 + g_1\,\mu + g_2\,\mu^2 + g_3\,\mu^3 = 0 \tag{20}$$

where for the general problem

$$g_0 = \mathbf{a}^T\,\mathbf{b} \tag{21}$$

$$g_1 = \mathbf{b}^T\,\mathbf{b} + 2\,\mathbf{a}^T\,\mathbf{c} \tag{22}$$

$$g_2 = 3\,\mathbf{b}^T\,\mathbf{c} \tag{23}$$

$$g_3 = 2\,\mathbf{c}^T\,\mathbf{c} \tag{24}$$

However, for the specific problem under consideration here (of estimating $\mathbf{x}^u$ from $\mathbf{x}^s$), we note that since $\mathbf{a} = \mathbf{0}$ by definition (and hence $g_0 = 0$), (20) can be simplified to

$$\mu\,(g_1 + g_2\,\mu + g_3\,\mu^2) = 0 \tag{25}$$

Since any solution of the power flow problem is an absolute minima of (15), (25) always has a root $\mu = 0$, corresponding to $\mathbf{x}^s$. Whether (25) has two additional real roots depends upon the g coefficients (specifically we require $g_2^2 > 4\, g_1\, g_3$). If this condition is met, the root closest to zero is always a local maximum of (15) while the more distant root is always a local minimum. The third observation then is that since

$$\mathbf{x}^s + 2\,\Delta\mathbf{x}^m = \mathbf{x}^u \tag{26}$$

to determine $\mathbf{x}^u$ we need only determine a vector $\Delta\mathbf{x}$ which is collinear to $\Delta\mathbf{x}^m$. The optimal multiplier equation can be used to determine the actual distance to move in direction $\Delta\mathbf{x}$. Because of the linearity of (14), this implies that we do not actually need to determine the midpoint mismatch, $[\mathbf{f}(\mathbf{x}^m) - \mathbf{f}(\mathbf{x}^s)]$, but rather just a vector $\Delta\mathbf{S}$ which is collinear to it.

The last observation is used to develop an estimate of this collinear vector; this is the only approximation which is employed. Again consider the two solutions $\mathbf{x}^s$ and $\mathbf{x}^u$. The midpoint mismatch then provides one amount of load increase which would cause these two solutions to coalesce. Since $\mathbf{x}^m$ is the midpoint in state space on the line segment joining the two solutions, it can be thought of as the "closest" bifurcation point in state space for the two solutions. The midpoint mismatch is the amount of parameter variation necessary to reach this point. However, this point is not equivalent to the closest point in parameter space computed in [23] using an l_2-norm (Euclidean norm). It appears to be more closely related to the point which locally minimizes the l_1-norm.

Examination of the midpoint mismatch reveals that for type-one solutions it usually has a relatively large component only at a single bus, and then only in the reactive component. Furthermore, this bus corresponds to the bus number used earlier to identify the solution. This implies that the "quickest" way to voltage instability via a bifurcation between a particular type-one solution and the operable solution would be to increase just the reactive power at the identifying bus. For example, Table 2 shows the midpoint mismatches for the two type-one solutions from Table 1, with the bus 1 solution having a large Q_1 mismatch while the bus 2 solution has a large Q_2 mismatch. Thus, the vector $\Delta\mathbf{S}$ collinear to $[\mathbf{f}(\mathbf{x}^m) - \mathbf{f}(\mathbf{x}^s)]$ can be approximated by including a single nonzero at the location corresponding to the reactive power at a particular bus.

Midpoint Mismatch	Bus 1 Solution	Bus 2 Solution
P_1 (MW)	-4.3	4.3
Q_1 (Mvar)	220.2	9.5
P_2 (MW)	4.3	-4.3
Q_2 (Mvar)	9.5	220.2

Table 2: Midpoint Mismatches for Three Bus Solutions

IV. Low-voltage Solution Algorithm

The observations from the previous section are now used to develop an algorithm to reliably calculate one or more low-voltage solutions. The algorithm is repeatedly applied to each bus k in a set K consisting of all buses for which potential low-voltage solutions are desired. The operable solution $\mathbf{x}^s$ is assumed to be known. For each bus k in K Do

1. Set $\Delta\mathbf{S}$ with a single nonzero at the position of the reactive power mismatch for bus k.
2. Use (14) to calculate the search direction $\Delta\mathbf{x}$.
3. Attempt to determine the optimal multiplier using (25). If the result is imaginary, stop the bus k iteration; a second local minimum of the cost function does not exist in the search direction. Otherwise, the largest root μ determines how far to move in the direction $\Delta\mathbf{x}$. Set $\mathbf{x}^0 = \mathbf{x}^s + \mu\,\Delta\mathbf{x}$ as the initial estimate of $\mathbf{x}^u$.
4. The accuracy of the initial estimate can be quantified by calculating the norm of the initial mismatch $\mathbf{f}(\mathbf{x}^0) - \mathbf{S}$. If the norm is sufficiently low, solve the Newton-Raphson power flow using $\mathbf{x}^0$ as an initial guess of $\mathbf{x}^u$.

The computational requirements for solving for the initial guess $\mathbf{x}^0$ are quite reasonable. If the operable solution has been determined using the Newton-Raphson method, then $\mathbf{J}(\mathbf{x}^s)$ is already available in factored form. Otherwise it only needs to be calculated and factored once for the entire set K. Since $\Delta\mathbf{S}$ is a sparse vector, $\Delta\mathbf{x}$ can be calculated very efficiently in step 2 using sparse vector methods with a fast forward, full backward solution [24]. To determine the optimal multiplier in step 3, the vectors $\mathbf{b}$ and $\mathbf{c}$ are needed. However, by substituting (14) into (17), it is clear that $\mathbf{b} = -\Delta\mathbf{S}$. The computation of $\mathbf{c}$ is equivalent to a power flow mismatch calculation. All other operations to calculate μ are order n. If the optimal multiplier is real, then determining the norm of the initial mismatch is again equivalent to a power flow mismatch calculation.

As discussed in the next section, only those solutions with sufficiently small mismatches need to be solved in step 5. How quickly (or whether) the power flow converges to the bus k low solution depends upon the accuracy of the initial guess $\mathbf{x}^0$. The numerical testing discussed in the next section indicates that for low-voltage solutions close to the operable solution the results are quite good, resulting in rapid convergence.

The final issue that must be addressed is the determination of the set of buses likely to have low-voltage solutions of interest (i.e., set K). Since the voltage security in a particular area of the system is determined using a low voltage solution from a bus in that area, usually only a subset of the system buses needs to be searched. For example a utility might just select its own internal system and several nearby external buses; other portions of the system, where the utility has no operational control and only limited real-time knowledge, are excluded.

This set is further reduced by using a fast screening method to identify the locally weakest buses. One approach is to use screening methods based upon solving a smaller equivalenced system, such as the methods discussed in [19]. Another is to first calculate the $\partial V_i/\partial Q_i$ sensitivity for each bus in the internal system. While we do not advocate the use of these linear sensitivities as explicit measures of voltage security, testing indicates that they are useful for identifying the locally weakest buses, which are then included in set K. Using sparse vector methods these values can be computed very efficiently. Table 3 compares the times (in seconds) of a single power flow solution to the time to determine all the $\partial V_i/\partial Q_i$ sensitivities for a system (note that normally only sensitivities for the internal buses would need to be determined) (studies were done with a PC).

Test System	Power flow (secs)	Sensitivities (secs)
IEEE 118 bus	0.3	0.1
IEEE 300 bus	0.5	0.3

415 bus utility sys.	1.1	0.5
2000 bus utility sys.	14.3	23.6

Table 3 : Computation Time to Calculate Diagonal Sensitivities

V. Test System Results

The ability of the algorithm to determine low-voltage solutions over a wide range of system loading is first evaluated using the earlier three-bus system. Under light loading this system exhibits four solutions, two of which are type-one. However the identity of the type-one solution that ultimately coalesces with the stable equilibrium strongly depends on the load participation at buses 1 and 2. For the Figure 2 case (with load participation at bus 1 twice that at bus 2), Figure 5 shows the variation in the largest component of the initial mismatch, $\mathbf{f}(\mathbf{x}^0) - \mathbf{S}$, as a function of load. The solution found with k equal to bus 1 corresponds to solution B in Figure 2, while k equal to bus 2 corresponds to solution C. The method was able to trace the bus 1 solution all the way to the bifurcation boundary (Σ) and the bus 2 solution to the point where it coalesces with the type-two solution at a load of 787 MW. An increasing mismatch accompanies the loss of solution at bus 2, whereas at all load levels the bus 1 solution mismatch is rather small, with the value tending towards zero as Σ is approached. Figure 6 shows the variation in the $\partial V_i/\partial Q_i$ sensitivities as a function of load. While the sensitivities are useful for determining the more heavily loaded portion(s) of the system (i.e., bus 1 for this case), their relative insensitivity to changes in loading (almost until the point of bifurcation) make them unsuitable for direct voltage security assessment. Conversely, Figure 7 shows how the low-voltage solution(s) can be used with energy methods to provide smoothly varying measure(s) of system security [5].

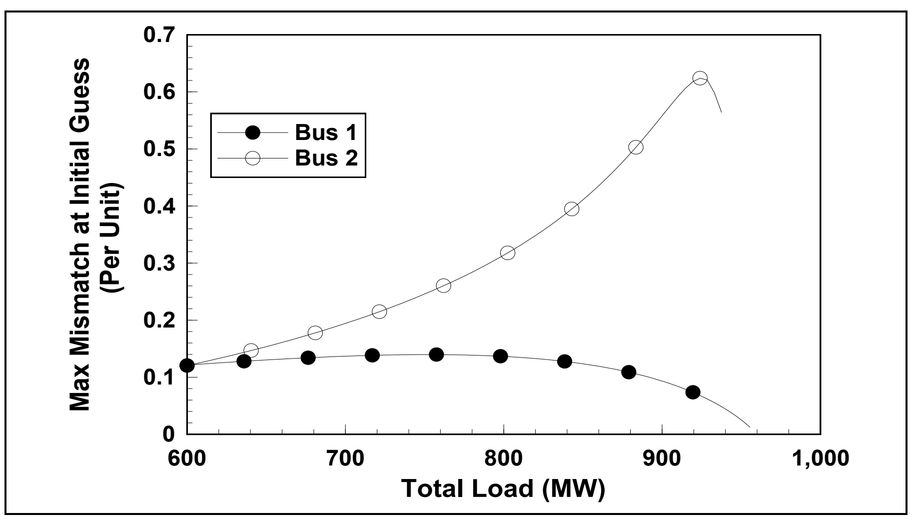

Figure 5: Largest Initial Low-Voltage Solution Mismatch

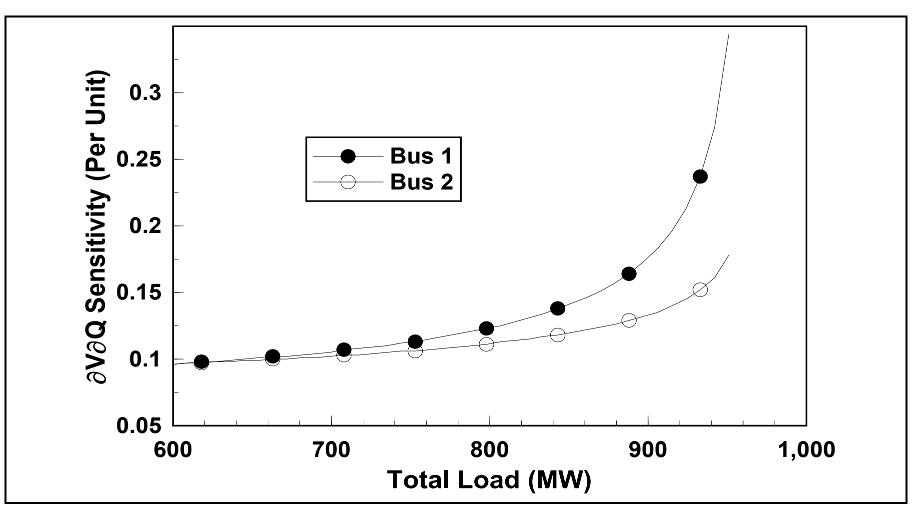

Figure 6: Three Bus $\partial V_i/\partial Q_i$ Sensitivities

The variation of the energy measures corresponding to the type-one solutions of the IEEE 118-Bus system has been investigated as a function of load in [19] using the simplified method of [9]. A similar analysis, this time using the proposed method, produced a nearly identical set of curves. Figure 8 illustrates the method's ability to track a number of low-energy solutions as load is increased uniformly at all buses toward maximum loadability. Experimentation has shown that the method performs equally well for a number of other load parametrizations. Moreover, Figure 9 shows that the mismatches at the initial low-solution guesses again are quite low, resulting in efficient convergence to the low solutions.

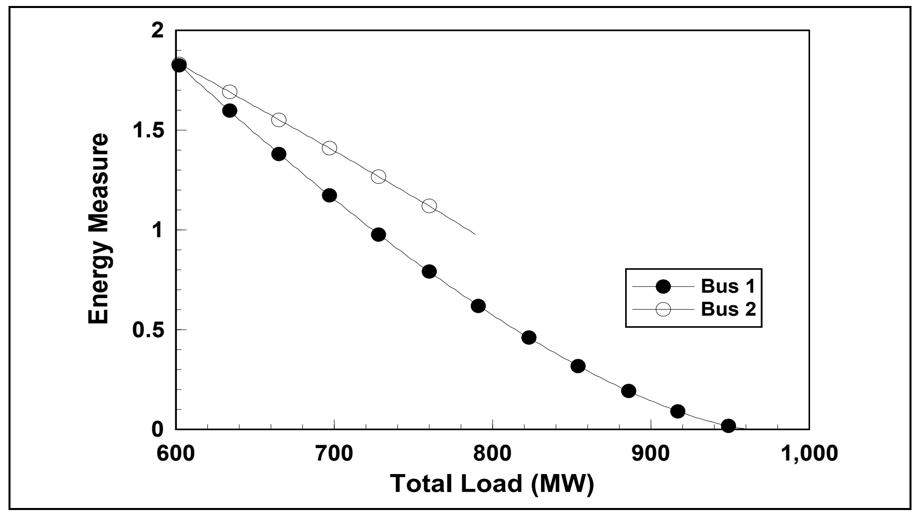

Figure 7: Energy Measures for 3-Bus System

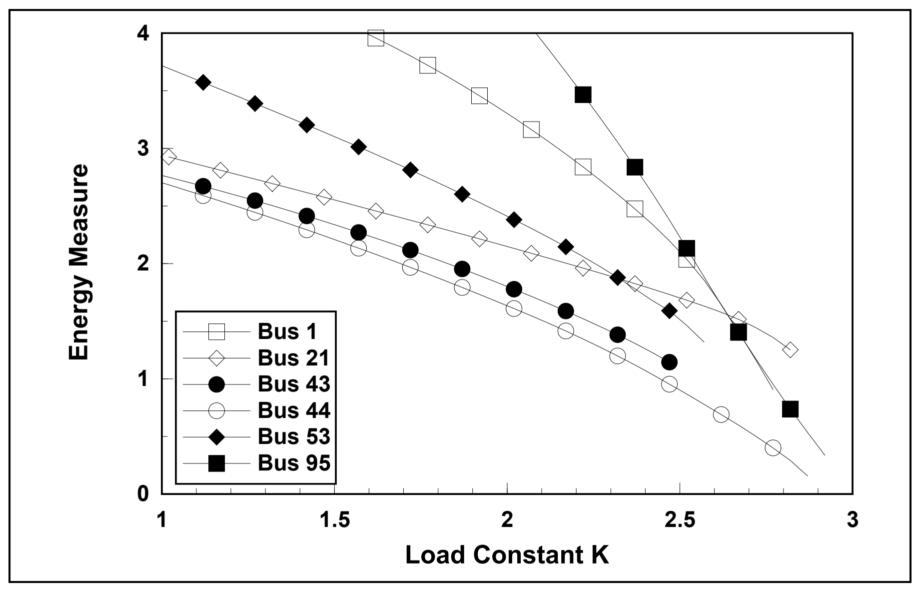

Figure 8: Energy Measures for 118-Bus System

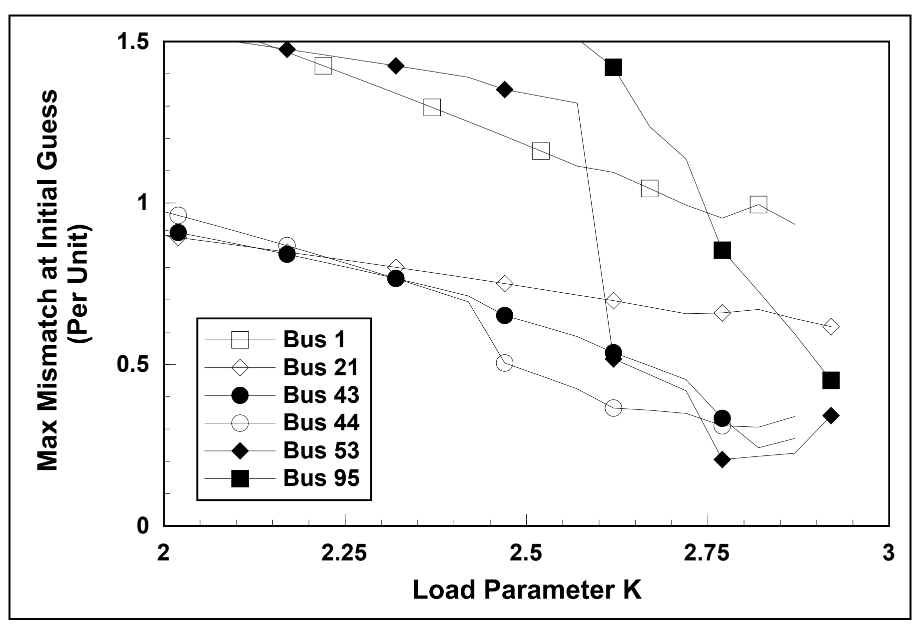

Figure 9: Largest Initial Low-Voltage Solution Mismatch

The use of $\partial V_i/\partial Q_i$ sensitivity as a screening tool was seen to be an expedititious, though not entirely foolproof, means of determining set K. By checking the six most sensitive buses at any given load level, the least-energy solution was always found. The identity of those buses leading to the five least-energy solutions could usually be found with reasonable certainty if the ten most sensitive buses were searched. Lastly, the method was tested on a 2000 bus utility system. As was the case for the earlier systems, the initial mismatches in $\mathbf{f}(\mathbf{x}^0) - \mathbf{S}$ were usually quite low, resulting in good convergence to the low-voltage solutions. Screening and determining the close by solutions for the entire system required computation equal to about 15 standard power flow solutions. However if only a portion of the system was screened (such as a 300 bus "internal system" in this case), this dropped to less than 10 times a standard power flow.

VI. Conclusion

A computationally efficient and reliable method for calculating the lowest-energy type-one solutions has been introduced and demonstrated on a number of systems. The algorithm poses little additional computation and converges to the low-voltage solution within a few iterations. Moreover, it is general enough to track several low-voltage solutions, and it provides a convenient screening facility for discerning which low-voltage solutions to investigate. Further research is needed on developing improved techniques to limit the size of set K.

VII. Acknowledgments

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VIII. References

[1] Y. Tamura, H. Mori, and S. Iwamoto, "Relationship between voltage instability and multiple load flow solutions in electric power systems," *IEEE Trans. on Power App. and Sys.*, vol. PAS-102, pp. 1115-1123, May 1983.

[2] Y. Tamura, H. Mori and S. Iwamoto, "Voltage instability proximity index (VIPI) based on multiple load flow solutions in ill-conditioned power systems," *Proc. 27th Conf. Decision and Control*, Austin, TX, Dec. 1988.

[3] N. Yorino, S. Harada, H. Sasaki and M. Kitagawa, "Use of multiple load flow solutions to approximate closest loadability limit," Bulk Power System Voltage Phenomena III Conference, Davos, Switzerland, Aug. 1994.

[4] S. Suzuki, S. Wada, M. Sato, T. Asano and Y. Kudo, "Newly developed voltage security monitoring system," *IEEE Trans. on Power Sys.*, vol. PWRS-7, pp. 965-973, Aug. 1992.

[5] C.L. DeMarco and T.J. Overbye, "An energy based security measure for assessing vulnerability to voltage collapse," *IEEE Trans. on Power Sys.*, vol. PWRS-5, pp. 582-591, May 1990.

[6] W. Xu and Y. Mansour, "Voltage stability analysis using generic dynamic load models," *IEEE Trans. on Power Sys.*, vol. PWRS-9, pp. 479-493, Feb. 1994.

[7] T.J. Overbye, "Computation of a practical method to restore power flow solvability," to appear, *IEEE Trans. on Power Sys.*, also IEEE 1994 PES Winter Meeting, 94WM245-1, New York, NY, Jan. 1994.

[8] A. Klos and A. Kerner, "The nonuniqueness of load flow solution," *Proc. PSCC-5*, 3.1/8, Cambridge, 1975.

[9] Y. Tamura, K. Iba, and S. Iwamoto, "A method for finding multiple load-flow solutions for general power systems," *Proc. IEEE 1980 PES Winter Meeting*, A80 043-0, New York, NY, Feb. 1980.

[10] F.M.A. Salam et. al., "Parallel processing for the load flow of power systems: the approach and applications," *Proc. 28th Conf. Decision and Control*, Tampa, FL, Dec. 1989.

[11] W. Ma and J.S. Thorp, "Solving for all the solutions of power systems equations," *Proc. of 1992 ISCAS*, San Diego, CA, May 1992, pp. 2537-2540.

[12] X. Yin and N. Germay, "Investigations on solving the load flow problem by genetic algorithms," *Electric Power Systems Research*, 1991, pp. 151-163.

[13] T.J. Overbye and C.L. DeMarco, "Improved techniques for power system voltage stability assessment using energy methods," *IEEE Trans. on Power Sys.*, vol. PWRS-6, pp. 1446-1452, Nov. 1991.

[14] A. Tiranuchit and R.J. Thomas, "A posturing strategy against voltage instabilities in electrical power systems," *IEEE Trans. on Power Sys.*, vol. 3, pp. 87-93, Feb. 1988.

[15] P.W. Sauer and M.A. Pai, "Power system steady-state stability and the load-flow Jacobian," *IEEE Trans. on Power Sys.*, vol. 5, pp. 1374-1383, Nov. 1990.

[16] T.J. Overbye, "Effects of load modeling on analysis of power system voltage stability," to appear, *International Journal of Electric Power and Energy Systems*.

[17] K. Iba, H. Suzuki, M. Egawa, and T. Watanabe, "A method for finding a pair of multiple load flow solutions in bulk power systems," *IEEE Trans. on Power Sys.*, vol. PWRS-5, pp. 582-591, May 1990.

[18] H. Mori, "Estimating multiple load flow solutions with the Householder transform," Bulk Power System Voltage Phenomena III Conference, Davos, Switzerland, Aug. 1994.

[19] T.J. Overbye, "Use of energy methods for on-line assessment of power system voltage security," *IEEE Trans. on Power Sys.*, vol. 8, pp. 452-458, May 1993.

[20] I. Dobson and H.-D. Chiang, "Towards a theory of voltage collapse in electric power systems," *Systems and Control Letters*, vol. 13, pp. 253-262, 1989.

[21] I. Dobson, "Observations on the geometry of saddle node bifurcation and voltage collapse in electric power systems," *IEEE Trans. on Circuits and Sys.,* Part 1, vol. 39, pp. 240-243, March 1992.

[22] S. Iwamoto and Y. Tamura, "A load flow calculation method for ill-conditioned power systems," *IEEE Trans. Power App. and Sys.*, vol. PAS-100, pp. 1736-1743, April 1981.

[23] I. Dobson and L. Lu, "New methods for computing a closest saddle node bifurcation and worst case load power margin for voltage collapse," *IEEE Trans. on Power Sys.*, vol. 8, pp. 905-913, Aug. 1993.

[24] W.F. Tinney, V. Brandwajn and S.M. Chan, "Sparse vector methods," *IEEE Trans. on Power App. and Sys.*, vol. PAS-104, pp. 295-301, Feb. 1985.

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