

# On the Existence of Distinct Inter-Area Electro-Mechanical Modes in North American Electric Grids

Thomas J. Overbye, Sanjana Kunkolienkar

Dept. of Electrical and Computer Engineering, Texas A&M University, College Station, TX, USA

overbye@tamu.edu, sanjanakunkolienkar@tamu.edu

**Abstract**—This paper examines the extent to which distinct inter-area electromechanical modes exist in large-scale electric grids. Electric grids oscillate, and these oscillations have often been described using the linear systems concept of modes. However, electric grids are nonlinear systems, and are becoming increasingly nonlinear with the growth of inverter-based controls, deadbands, and other devices that often operate at their limits. Hence adequate linearizations may no longer exist, calling into question the extent to which distinct modes exist. For a mode to exist at an operating point its frequency, damping and shape must be independent of the disturbance used to excite it. The paper shows for one synthetic and two actual models of North American electric grids that these criteria are not always met, particularly for the largest electric grids.

## I. INTRODUCTION

“Everything should be made as simple as possible, but no simpler” is a quote widely attributed to Einstein. While this is likely a paraphrase of something Einstein actually said [1], the quote still contains broadly useful wisdom that is germane to this paper’s focus on large-scale electric grid dynamic modeling and oscillations. Electric grids have dynamics over many different time-scales, with the focus here on dynamics in the transient stability time range – electrical cycles out to minutes [2]. Within this time scale electric grids oscillate, with the study of these oscillations an area of interest for many years [3], [4], [5]. Historically these oscillations have been associated with individual generators, but as systems interconnected, the idea of inter-area (or wide-area) oscillations developed. Inter-area oscillations typically have frequencies between 0.15 and 1.0 Hz and impact an interconnect; oscillations with frequencies between 1.0 and 5.0 Hz are known as intra-area or plant oscillations. The focus here is on inter-area oscillations, with a goal to make their analysis as simple as possible, but not simpler.

Over the years several techniques for analyzing these oscillations have developed with the initial digital computer approaches focused on eigenvalue analysis [6], [7], [8]. This requires linearizing the grid about an operating point. If such a model is valid, then eigenvalues then give the frequency and damping of the different modes, and their associated right eigenvectors then tells how the different electric grid devices participate in the mode (known as the mode shape [8]). The premise is the response of this linearized model can approximate the behavior of the actual grid. A key focus of eigenvalue analysis was identifying inter-area modes of the grid. An early paper looking at the modes in the North American Western Electricity Coordinating Council (WECC)

is given in [9] while [10] provides some initial work looking at the modes in the North American Eastern Interconnect (EI). More recent references include [11], [12], [13], and [14].

Over the last several decades there has been increased use of signal-based modal analysis techniques, with a number of different methods available. These methods are divided into two main classes: ring-down and ambient data. Here the focus is investigating the presence of modes using the ring-down approach. With this approach a disturbance (e.g., a generator outage) is applied to the grid, and then the technique reproduces one or more of the resultant signals (such as a bus frequency) using a set of basis functions, with the basis functions usually a set of exponentially scaled sinusoids. A number of different methods are available, with this paper utilizing the Iterative Matrix Pencil (IMP) [15], which provides a computationally tractable extension of the Matrix Pencil Method (MPM) [16] to large sets of signals. The resultant exponential functions then give the observed modes.

The purpose of the paper is to assess the degree to which the inter-area oscillatory behavior of large-scale electric grids can be well approximated by a relatively small set of distinct electromechanical modes. If a dynamic system is linear, then its modes can be calculated by eigenvalue analysis. However, electric grids are not linear systems, and they are likely becoming more non-linear, particularly with the growth of inverter-based renewable generation that is often operated at maximum power limits. The models used to represent the grid’s behavior are also becoming more non-linear. For example, with the modeling of deadbands, saturation, non-linear gain functions, and many more limits, some of which are binding during normal operation.

The paper presents a methodology and test results using North American electric grids for determining the degree to which the linear modal analysis technique can be used with current electric grid models. The remainder of the paper is organized as follows. The next section presents the methodology and the test grids. The third section then provides results on several different large-scale grids. The last section summarizes the paper and presents directions for future work. All calculations and visualizations are done using PowerWorld Simulator Version 23.

## II. APPROACH AND TEST GRIDS

The question addressed here is whether large-scale electric grids actually have distinct inter-area electromechanical modes. Or more specifically, whether the simulation results used to represent these grids have distinct modes. Modal

analysis can be done using either measurements from phasor measurement units (PMUs) or from electric grid simulations. While using actual data can certainly be beneficial, in the context of this paper’s question simulation results provide the following advantages. First, with simulations a number of different disturbances can be applied to the exact same operating point, allowing for easy comparison of results. Second, simulation signals are available for all locations in the grid, and these signals have no associated noise. Third, results can be provided for both models of actual grids and for synthetic electric grids [17], with a key synthetic grid advantage being the grid and results can be made fully public. This is not the case with the actual North American electric grid models since their detailed model parameters are considered to be critical energy/electricity infrastructure information (CEII) that cannot be publicly disclosed [18]. Of course, showing results from actual grid models is important as well, even if results cannot be fully released. Hence the paper provides results for a 2000 bus (2K) synthetic grid, and larger recent models of the WECC and EI grids.

The paper’s procedure for testing whether distinct modes exist is to apply a set of disturbances to the exact same operating point to see whether identical modes are observed. While the modes would be expected to change with variation in the operating point, they should not be dependent upon the type of the applied perturbation, provided the disturbance excites the mode. So if the observed mode’s frequency, damping or shape changes substantially based on the disturbance, then it is reasonable to question whether the mode exists, or whether the nonlinear electric grid is exhibiting a more complex type of behavior.

With this procedure each disturbance creates a set of signals, with the focus here on the bus frequencies. These signals then provide the inputs for the ring-down modal analysis, which is done here using the IMP [15]. With the IMP results, and the computation insight from [19], how each signal participates in each calculated mode can be readily determined to show the mode shape. The mode shapes can then be visualized using the techniques presented in [20], [21], [22] and improved here. If the grid actually has distinct modes, then the IMP calculated modes should be consistent in frequency, damping and shape between the disturbances.

Results are shown using three example grids. The first the 2K synthetic grid that covers a geographic footprint of most of the U.S. state of Texas using 500/230/161/115 kV transmission system; details on the creation of this grid are given in [17], [23], and [24]. For stability simulations the grid has 17 separate model types, 2685 model instances, and about 10,300 state variables when simulated. For the paper scenarios this grid has a total load of 67 GW. The online for this case is shown in Figure 1, with the green ovals showing the location of the generators using the geographic data view (GDV) approach [25] (with the area of the GDV proportional to the substation generation). The transmission lines are shown using the orange (500 kV), purple (230 kV) and black (< 200 kV) lines. The key bus numbers used in the results are also shown.

The second and third examples are using actual grid models for the WECC and the EI. In particular, the 2019 series grids from [26] are used, with the WECC grid having about 23,100 buses and the EI 87,000 buses, with most buses geo-mapped to aid with visualization. For the stability simulations the WECC grid has 124 separate model types, 16,000 model instances with about 90,400 state variables. For the EI there are 150 model types, 18,000 model instances, and about 104,000 state variables. While a disadvantage of using actual grids is they are not available publically, a key advantage is there are numerous publications detailing the modal behavior of these grids, facilitating comparisons.

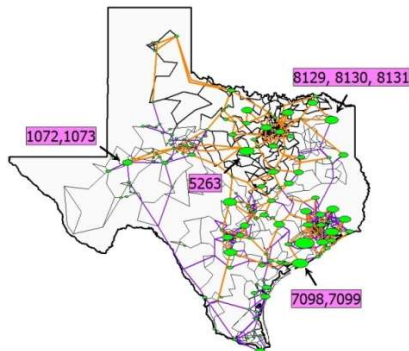


Figure 1: Texas 2000 Bus System Online

### III. RESULTS

This section provides specific examples considering the degree to which distinct inter-area modes actually exist in these grids. Starting with the 2K grid, Figure 2 shows the frequencies at all 2000 buses for the rather severe disturbance of opening the generators at buses 7098 and 7099 (a loss of 2590 MW) at simulation time of one second. With 2000 signals the figure is not meant to show the result of any particular bus frequency, but rather the overall envelope of the response. Next, the modal content of all the 2000 signals is determined using the IMP, with the analysis period from the beginning of the event (1.0 s) to the end (15 s) with a sample rate of 20 Hz; the calculated modes are shown in Table 1.

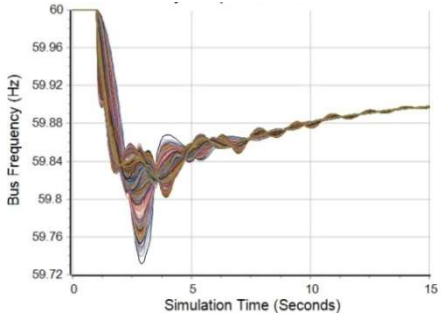


Figure 2: Texas 2000 Bus Frequency Contingency Results

A nice characteristic of the IMP is because of the computation insight of [19], each of the 2000 signals can be reconstructed from the IMP results (between 1.0 and 15 seconds), and then compared to the actual signal using a cost function [15]. The result is a quite close match, with a plot of all the 2000 reconstructed signals shown in Figure 3. To

further illustrate the high degree of match, Figure 4 compares the bus that had the worst match (i.e., the highest cost function) for the original signal and the reconstructed signal, with the result that even here it is a quite good match.

Table 1: Modal Frequency and Damping for the Figure 2 Signals

Freq (Hz)	Damping (%)	Average Magnitude (Hz)
0		0.0116
0.06	31.1	0.021
0.21	46.6	0.129
0.40	20.0	0.052
0.64	5.7	0.009
0.72	13.6	0.014
0.96	8.1	0.012
1.31	17.3	0.008

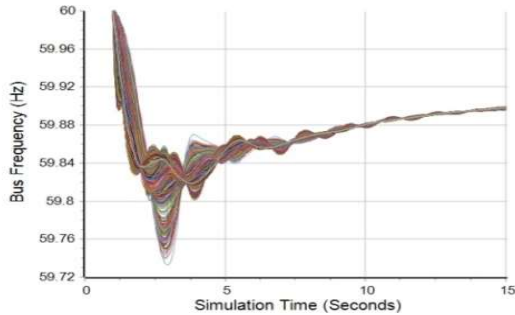


Figure 3: Reconstructed Figure 2 Signals

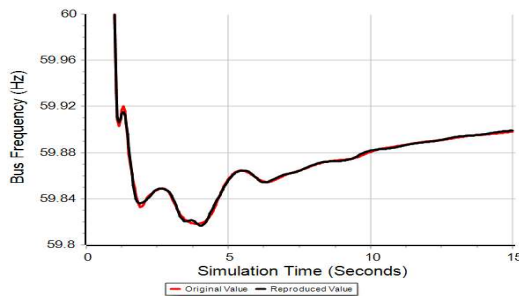


Figure 4: Worst Frequency Signal Match at Bus 7307

Finally, the shape of any calculated mode can be visualized using the GDV/contouring approach presented in [20], with the enhancement in this paper of using a color mapping from [27] that has an advantage of being perceptually uniform [28]. For example, Figure 5 visualizes the shape of the 0.40 Hz mode from Table 1 with a damping of 20.0% (this is a mode with fairly large magnitudes and not too high damping). In the figure the arrow lengths are proportional to the mode's amplitude at the locations, and the arrow directions, along with the contour, showing the phase angles (with 0 degrees to the left, 90 degrees up). Overall the image shows data for each of the 2000 buses, though the number of arrows is pruned to avoid display clutter. To aid in the comparison between contours for similar modes, the angle of a reference bus is chosen arbitrarily, with all other angles shifted according. Here the reference is the one with the largest magnitude in this mode, Bus 1073, and its angle is set to -45 degrees.

To move forward it is necessary to address a reasonable critique that applying generator outage contingencies alters the grid particularly for smaller grids, so the post-disturbance

operating points are different. Hence the observed change in the calculated mode could be due this operating point variation. To address this, the disturbance can be changed to one in which the pre and post-disturbance grids are essentially identical. While difficult to do with an actual grid (with the pulsing of a braking resistor from [12] a close equivalent), in simulations the state values can be arbitrarily perturbed. The approach used is to provide a step change to the speeds of one or more generators, then allowing the grid to settle down.

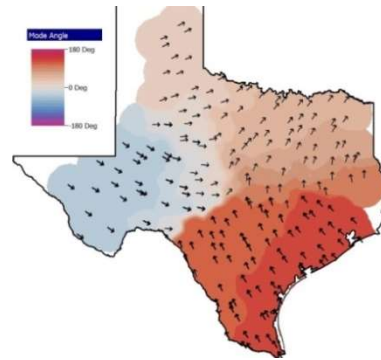


Figure 5: Gen 7098, 7099 Contingency 0.40 Hz Mode Shape

To determine if observed 0.4 Hz mode shape is independent of the disturbance, the disturbance is changed to opening just the Bus 7099 generator (a loss of 1350 MW). The IMP determines a mode at 0.36 Hz with 26.3% damping and the Figure 6 mode shape. Figure comparison indicates fairly similar results, though with a nontrivial change in damping.

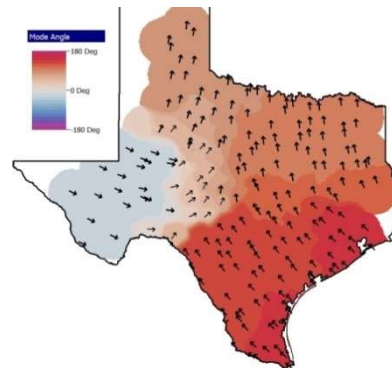


Figure 6: Gen 7099 Contingency 0.36 Hz Mode Shape

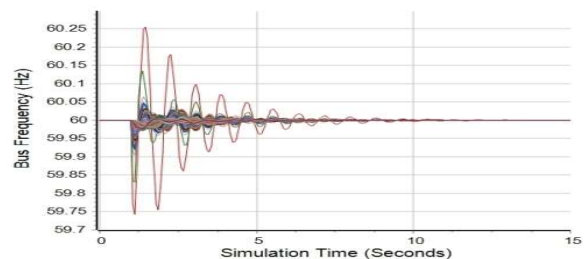


Figure 7: Generator 7098, 7099 Speed Perturbation Bus Frequencies

Figure 7 shows an example of the bus frequency response for a -0.5 Hz disturbance to the bus 7098 and 7099 generators. The same modal analysis approach can be applied to the results, with the left image in Figure 8 showing the 0.35 Hz,

23% damping mode for this disturbance, and the right image showing the 0.35 Hz, 26% damping mode calculated from a similar frequency disturbance for the generators at buses 1072 and 1073 (on the other side of the grid). While the frequency, damping and shape of the observed mode for these four disturbances does vary, the variation is relatively small, giving credence to the grid having a distinct mode with a frequency of about 0.36 Hz, and a damping of about 23%.

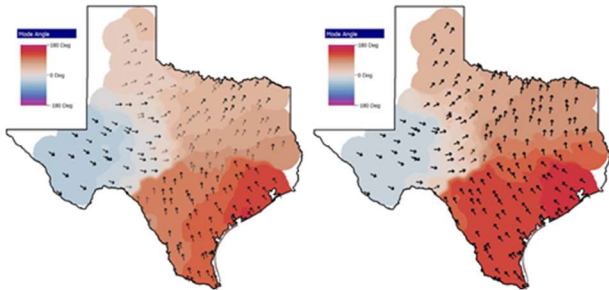


Figure 8: Left: Gens 7098, 7099 Disturbance 0.35 Hz Mode Shape; Right: Gens 1072, 1073 Disturbance

Similar analysis can be done for the other modes, with fairly reasonable results showing that the modes are consistent in frequency, damping and shape. Keeping with the approach of using the generator speed perturbation contingencies, the left image in Figure 9 shows the shape of the 0.96 mode from Table 1 when the disturbance is decreasing the speed by 0.5 Hz at the 7098 and 7099 generators; the calculated frequency is again 0.96 Hz with damping of 7.9% the figure arrows are set so the bus 7098 one points right). If the disturbance is moved to the far upper-right portion of the grid (buses 8129, 8130 and 8131) the frequency is 0.98 Hz with a damping of 9.0%, and a mode shape shown in the right half of Figure 9.

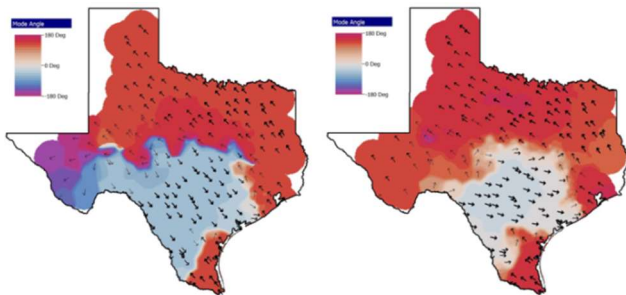


Figure 9: Left: Gens 7098, 7099 Disturbance 0.95 Hz Mode Shape; Right: Gens 8129, 8130, 8131 0.98 Hz Mode Shape

The previous example has demonstrated that the paper’s methodology can be used to obtain fairly consistent modes on the 2K grid. This result is not unexpected since the 2K has fairly simple dynamic models with few nonlinearities. The next example is the more complex and larger real WECC model. As has been noted in a number of references, the WECC has well known modes, with this paper focusing on the main ones as described in [11], [12] and [13]. For ease in comparison, the mode shape arrows are shifted to match the largest components of the visualizations in [13].

The first mode considered is the North-South Mode B (NSB), which is noted in [12] as having a frequency of between 0.35 and 0.45 HZ, being the most geographically widespread, having damping between 5 and 10%, and having Alberta (AB) swinging against British Columbia (BC) and the northern US. The issue considered isn’t whether such a mode is observed, nor whether its frequency and damping vary with operating point. Both certainly occur. Rather, the issue is whether at a particular operating point, regardless of disturbance, a distinct NSB mode observed. Using the previous generator speed disturbance approach, with all changes of -0.5 Hz, Figure 10 and Figure 11 show the mode shape for disturbances in Arizona (AZ), AB, BC and Montana (MT) respectively. As can be seen, the mode shape did vary, particularly across the US. The observed frequency varied between 0.32 and 0.37 Hz, while the damping was between 13 and 20%. The key issue here these variations were observed at the same operating point.

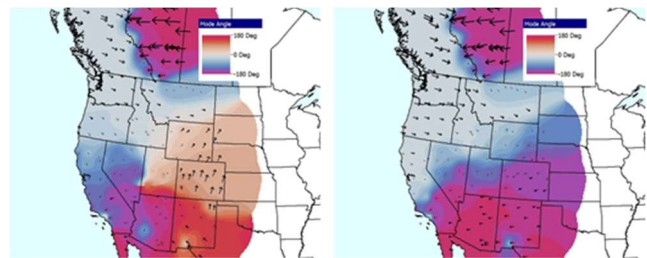


Figure 10: Left: WECC NSB for an Arizona (AZ) Disturbance; Right: WECC NSB for an Alberta (AB) Disturbance

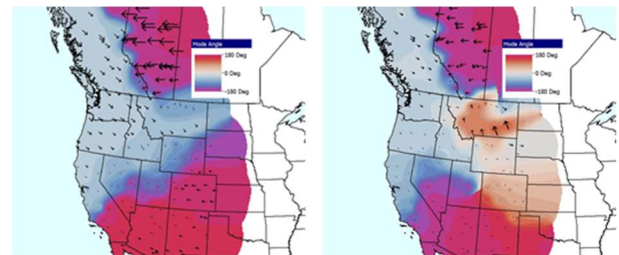


Figure 11: Left: WECC NSB for a British Columbia (BC) Disturbance; Right: WECC NSB for a Montana (MT) Disturbance

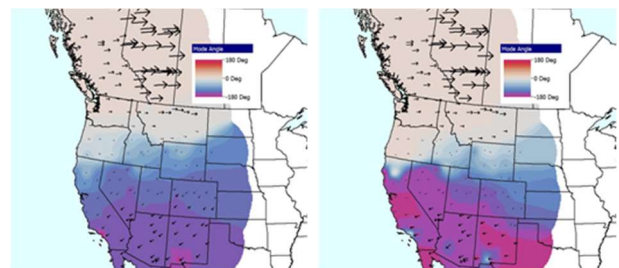


Figure 12: Left: WECC NSA for an Alberta Disturbance; Right: WECC NSA for an Arizona Perturbation

The second mode considered is the North-South Mode A (NSA), which is noted in [12] as having a frequency of between 0.2 and 0.3 HZ with Alberta swinging against the rest of the system. Figure 12 shows two visualizations of the calculated mode for a disturbance, first for Alberta (left) and then Arizona (right). Regardless of the disturbance this mode

tended to have a fairly consistent frequency (around 0.21 Hz), damping (about 30%) and shape. The last mode considered is the British Columbia (BC) one, with [12] mentioning two BC modes (BCA and BCB). For the simulations done here this mode's shape tended to vary the most, with it not observed under some scenarios; when observed its frequency was around 0.52 Hz with a damping of usually 13 to 14%.

The last examples are with the 87,000 bus EI grid, with the case representing heavy loading. As before the testing procedure is to apply a series of disturbances to the exact same operating point, and use the IMP to determine the best fit modes. Here all the disturbances are opening generation at a single location at a simulation time of one second, and running until 15 seconds. The modal analysis is done over using 12 seconds of data, between simulation time of two and 14 seconds, sampled again at 20 Hz.

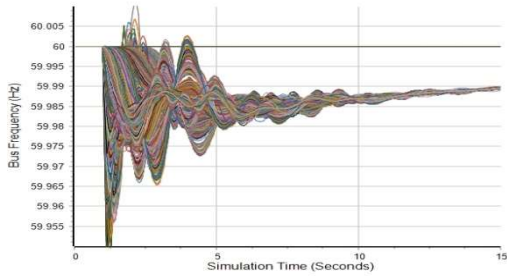


Figure 13: EI Frequency Response for a Missouri Generator Outage

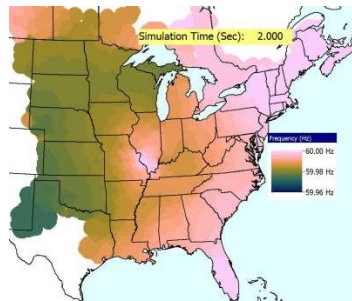


Figure 14: Spatial Frequency Variation for the Figure 13 Scenario

The first example is for opening a generator in the US state of Missouri. Figure 13 shows the frequency variation at all 87,000 buses, with the intent of the figure not to show any particular signal, but rather the envelope of the responses. At any point in the simulation the spatial variation in the frequencies could also be visualized, with Figure 14 showing an example at 2.0 seconds using the contouring approach of [29]. As noted in [20], movies can be made uses a series of such images. Table 2 shows the key calculated modes. Of these, the only one with a both a large magnitude and large geographic footprint is at 0.24 Hz. Using the previous approach, its shape is shown in Figure 15; to aid in comparison, in all the EI mode visualizations the vector's infinity norm is set to 1.0. The issue is again the degree to which the observed modes vary with the disturbances.

Table 2: Modal Frequency and Damping for the Figure 13 Signals

Freq (Hz)	Damping (%)	Average Magnitude (Hz)
0.24	18.2	0.0036
0.42	15.5	0.0021
0.59	6.6	0.0018

0.74	6.2	0.0013
0.90	4.64	0.0008

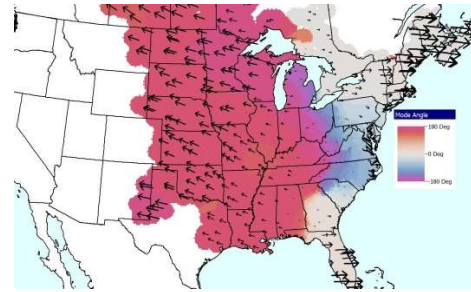


Figure 15: Missouri Scenario 0.24 Hz Mode Shape, 18.2% Damping

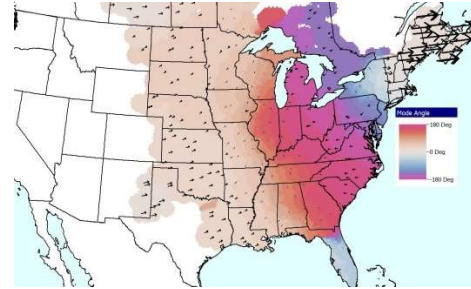


Figure 16: New England Scenario 0.33 Hz Mode Shape, 12.2% Damping

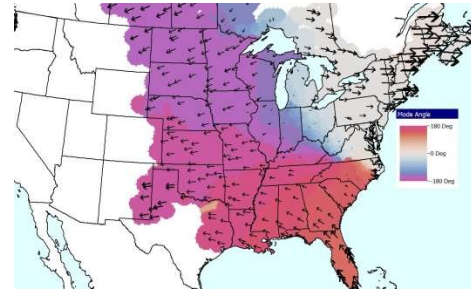


Figure 17: New England Scenario 0.21 Hz Mode Shape, 16.5 % Damping

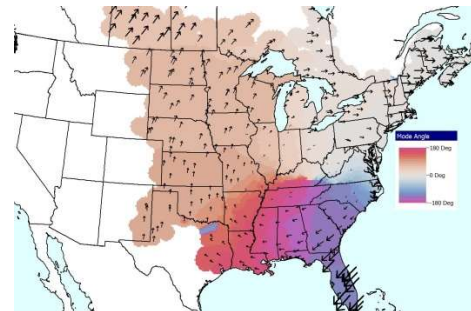


Figure 18: Florida Scenario 0.24 Hz Mode Shape, 17.2% Damping

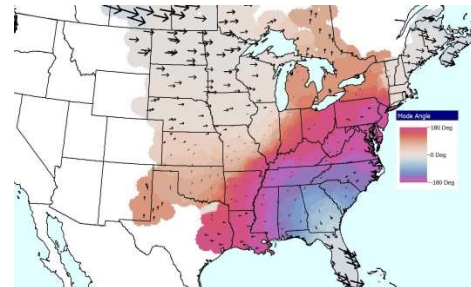


Figure 19: Florida Scenario 0.37 Hz Mode Shape, 13.8% Damping

In doing the research for this paper many different disturbances were considered, with results for a generator outage in the US New England region shown in Figures 16 and 17, one in Florida shown in Figures 18 and 19, and one in Georgia in Figure 20. There are three observations from this. First, the major frequencies and dampings are in the ranges given for the EI in [11] and [13], and often similar mode shapes are observed. Second, the damping, mode shape and to some extent frequency vary substantially based upon the disturbance. For example, with contrasting the 0.2-0.27 Hz shapes shown in Figures 15, 17, 18 and 20. Third, though not fully explored here, the errors in the IMP of fitting this large set of frequencies is higher than with the other grids. Based on these results, there is not strong evidence that a small set of distinct modes actually exist for the EI.

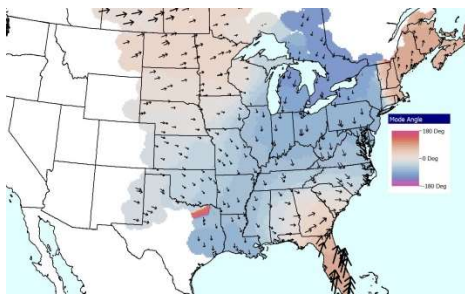


Figure 20: Georgia Scenario 0.27 Hz Mode Shape, 15.0% Damping

#### IV. SUMMARY AND FUTURE DIRECTIONS

This paper has examined the existence of distinct modes in three different electric grids, showing that sometimes distinct modes appear to exist regardless of the disturbance. However, sometimes that is not the case particularly for the EI. Moving forward as grids continue to change, the degree to which modes are a useful approximation needs to be more thoroughly examined, and for situations in which they are not new approaches need to be developed to continue to make things “as simple as possible, but no simpler” [1].

#### V. ACKNOWLEDGEMENTS

This work was supported in part by PSERC and the Texas A&M Smart Grid Center.

#### REFERENCES

[1] A. Robinson, “Einstein said that – didn’t he?”, *Nature*, vol. 557, pp. 30, May 2018.  
 [2] P.W. Sauer, M.A. Pai, J.H. Chow, *Power System Dynamics and Stability*, John Wiley & Sons Ltd., Hoboken, NJ, 2018.  
 [3] C. Concordia, S. B. Crary, J.M. Lyons, Stability Characteristics of Turbine Generators, *AIEE Transactions*, vol. 57, pp. 732-744, 1938.  
 [4] C. Concordia, G. Kron, *AIEE Transactions*, “Damping and Synchronizing Torques in Power Systems,” vol. 64, pp. 366-371, 1945  
 [5] F.R. Schleich, J.H. White, “Damping for the Northwest – Southwest Tieline Oscillations – An Analog Study,” *IEEE Trans. Power App. & Syst.*, vol. PAS-85, pp. 1239-1247, December 1966.

[6] R.T. Byerly, R.J Bennon, D.E. Sherman, “Eigenvalue Analysis of Synchronizing Power Flow Oscillations in Large Electric Power Systems, *IEEE Trans. Power App.* vol. 101, pp. 235-243, Jan. 1982.  
 [7] P. Kundur, G.J. Rogers, D.Y. Wong, L. Wang, M.G. Lauby, “A Comprehensive Computer Program Package for Small Signal Stability Analysis of Power Systems,” *IEEE Trans. Power Sys.*, vol. 5, pp. 1076-1083, Nov. 1990.  
 [8] P. Kundur, *Power System Stability and Control*, McGraw-Hill, Inc., New York, NY, 1994.  
 [9] R.J. Piwko, H.A. Othman, O.A. Alvarez, C.Y. Wu, “Eigenvalue and Frequency-Domain Analysis of the Intermountain Power Project and the WSCC Network, *IEEE Trans. on Power Systems*, Vol. 6, pp 238-244, Feb. 1991.  
 [10] P. Kundur, G.J. Rogers, D.Y. Wong, L. Wang, M.G. Lauby, “A Comprehensive Computer Program Package for Small Signal Stability Analysis in Power Systems,” *IEEE Trans. on Power Systems*, vol. 5, pp. 1076-1082, Nov. 1990.  
 [11] *Interconnection Oscillation Analysis*, NERC, July 2019  
 [12] *Modes of Inter-Area Oscillations in the Western Interconnection*, Western Interconnection Modes Review Group, WECC, 2021.  
 [13] R.T. Elliott, D.A. Schoenwald, “Visualizing the Inter-Area Modes of the Western Interconnection,” *IEEE PES 2022 GM*, Denver, CO, July 2022.  
 [14] J. Follum, N. Nayak, J. Eto, “Online Tracking of Two Dominant Inter-Area Modes of Oscillation in the Eastern Interconnect,” *56<sup>th</sup> Hawaii International Conference on System Sciences*, Lahaina, HI, Jan. 2023.  
 [15] W.C. Trinh, K.S. Shetye, I. Idehen, T.J. Overbye, “Iterative Matrix Pencil Method for Power System Modal Analysis,” *52nd Hawaii International Conference on System Sciences*, Waikoloa, HI, Jan 2019.  
 [16] M.L. Crow, A. Singh, “The Matrix Pencil for Power System Modal Extraction,” *IEEE Trans. Power Systems*, vol. 20, pp. 501-502, Feb. 2005.  
 [17] A.B. Birchfield, T. Xu, K. Gegner, K.S. Shetye, T.J. Overbye, “Grid Structural Characteristics as Validation Criteria for Synthetic Networks,” *IEEE Trans. Power Syst.*, vol. 32, pp. 3258-3265, July 2017.  
 [18] Critical Energy/Electric Infrastructure Information, US Federal Energy Regulatory Commission (FERC); available online at [www.ferc.gov/ceii](http://www.ferc.gov/ceii) .  
 [19] A.R. Borden, B.C. Lesieutre, J. Gronquist, “Power System Model Analysis Tool Developed for Industry Use,” *Proc. 2013 North American Power Symposium*, Sept. 2013.  
 [20] I. Idehen, B. Wang, K.S. Shetye, T.J. Overbye, J.D. Weber, “Visualization of Large-Scale Electric Grid Oscillation Modes,” *Proc. 50<sup>th</sup> North American Power Symposium*, Fargo, ND, September 2018.  
 [21] T.J. Overbye, K.S. Shetye, J. Wert, W. Trinh, A. Birchfield, T. Rolstad, J.D. Weber, “Techniques for Maintaining Situational Awareness During Large-Scale Electric Grid Simulations,” *Proc. 2021 Power and Energy Conference at Illinois*, Champaign, IL, April 2021.  
 [22] W. Trinh, S. Kunkolienkar, Y. Liu, and T. J. Overbye, “On the Sensitivity of Ring-Down Observed Inter-Area Modes in Large-Scale Electric Grids Using a Simulation Approach,” *IEEE. Power and Energy Conference at Illinois (PECI)*, Champaign, IL, March 2022  
 [23] T. Xu, A.B. Birchfield, K.S. Shetye, T.J. Overbye, “Creation of Synthetic Electric Grid Models for Transient Stability Studies,” *10<sup>th</sup> Bulk Power Sys. Dynamics & Control Symposium*, Espinho, Portugal, Sept. 2017.  
 [24] T. Xu, A. B. Birchfield, and T. J. Overbye, “Modeling, Tuning, and Validating System Dynamics in Synthetic Electric Grids,” *IEEE Trans. on Power Systems*, vol. 33, no. 6, pp. 6501–6509, Nov. 2018.  
 [25] T. J. Overbye, E. M. Rantanen and S. Judd, “Electric power control center visualization using Geographic Data Views,” *2007 iREP Symposium – Bulk Power System Dynamics and Control – VII. Revitalizing Operational Reliability*, Charleston, SC., Aug. 2007.  
 [26] T.J. Overbye, K. Shetye, J. Wert, H. Li, C. Cathey, and H. Scribner, “Stability Considerations for a Synchronous Interconnection of the North American Eastern and Western Electric Grids,” *55<sup>th</sup> Hawaii International Conference on System Sciences (HICSS)*, virtual, Jan. 2022.  
 [27] F. Cramer, “Scientific Color Maps”, 2018 , Zenodo, <http://doi.org/10.5281/zenodo.1243862>.  
 [28] F. Cramer, G.E. Shephard, P.J. Heron, “The Misuse of Color in Science Communication,” *Nature Communications*, Vol. 11, Oct. 2020.  
 [29] J.D. Weber, T.J. Overbye, “Voltage contours for power system visualization,” *IEEE Trans. on Power Syst.*, pp. 404-409, Feb., 2000.