ECEN 667
Power System Stability

Lecture 3: Electromagnetic Transients

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Announcements

- Start reading Chapters 1 and 2 from the book (Chapter 1 is Introduction, Chapter 2 is Electromagnetic Transients)
- EPG Dinner is on September 9 at 5pm; please RSVP using the link that was emailed to all.
- Homework 1 is due on Thursday September 7
- Classic reference paper on EMTP is H.W. Dommel, "Digital Computer Solution of Electromagnetic Transients in Single- and Multiphase Networks," *IEEE Trans. Power App. and Syst.*, vol. PAS-88, pp. 388-399, April 1969

Multistep Methods

- Euler's and Runge-Kutta methods are single step approaches, in that they only use information at **x**(t) to determine its value at the next time step
- Multistep methods take advantage of the fact that using we have information about previous time steps **x**(t-Δt), **x**(t-2Δt), etc
- These methods can be explicit or implicit (dependent on **x**(t+Δt) values; we'll just consider the explicit Adams-Bashforth approach

Multistep Motivation

- In determining **x**(t+Δt) we could use a Taylor series expansion about **x**(t)

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\Delta t\,\dot{\mathbf{x}}(t)+\frac{\Delta t^2}{2}\ddot{\mathbf{x}}(t)+O(\Delta t^3)$$

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\Delta t\,\mathbf{f}(t)+\frac{\Delta t^2}{2}\left(\frac{\mathbf{f}(\mathbf{x}(t))-\mathbf{f}(\mathbf{x}(t-\Delta t))}{\Delta t}+O(\Delta t)\right)$$

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\Delta t\left(\frac{3}{2}\mathbf{f}\left(\mathbf{x}(t)\right)-\frac{1}{2}\mathbf{f}\left(\mathbf{x}(t-\Delta t)\right)\right)+O(\Delta t^3)$$

(note Euler's is just the first two terms on the right-hand side)

Adams-Bashforth

- What we derived is the second order Adams-Bashforth approach. Higher order methods are also possible, by approximating subsequent derivatives. Here we also present the third order Adams-Bashforth

Second Order

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\frac{\Delta t}{2}\left(3\mathbf{f}(\mathrm{x}(t))-\mathbf{f}(x(t-\Delta t))\right)+O(\Delta t^{3})$$

Third Order

$$\mathbf{x}(t+\Delta t)=\mathbf{x}(t)+\frac{\Delta t}{12}\left(23\mathbf{f}(\mathrm{x}(t))-16\mathbf{f}(x(t-\Delta t))+5\mathbf{f}(x(t-2\Delta t))\right)+O(\Delta t^{4})$$

Adams-Bashforth Versus Runge-Kutta

- The key Adams-Bashforth advantage is the approach only requires one function evaluation per time step while the RK methods require multiple evaluations
- A key disadvantage is when discontinuities are encountered, such as with limit violations
  - In some simulations limits can be hit often
  - Another method needs to be used until there are sufficient past solutions
- They also have difficulties if variable time steps are used

Numerical Instability

- All explicit methods can suffer from numerical instability if the time step is not correctly chosen for the problem eigenvalues

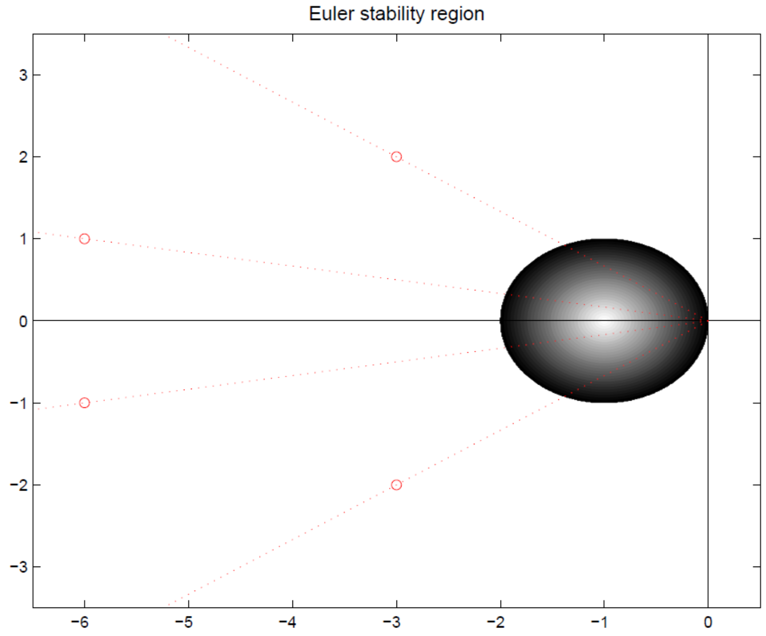


Figure 10.2: The spectrum of A is scaled by h. Stability of the origin is recovered if $h\lambda$ is in tl region of absolute stability $|1 + z| < 1$ in the complex plane.

Values are scaled by the time step; the shape for RK2 has similar dimensions but is closer to a square. Key point is to make sure the time step is small enough relative to the eigenvalues.

Image source: http://www.staff.science.uu.nl/~frank011/Classes/numwisk/ch10.pdf

Stiff Differential Equations

- Stiff differential equations are ones in which the desired solution has components the vary quite rapidly relative to the solution
- Stiffness is associated with solution efficiency: in order to account for these fast dynamics we need to take quite small time steps

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -1000x_1 - 1001x_2$$

$$\dot{\mathbf{x}} \rightarrow = \begin{bmatrix} 0 & 1 \\ -1000 & -1000 \end{bmatrix} \mathbf{x}$$

$$x_1(t) = Ae^{-t} + Be^{-1000t}$$

Stiff differential equations are common in power systems, but there are efficient techniques for handling them

Implicit Methods

- Implicit solution methods have the advantage of being numerically stable over the entire left half plane
- Only methods considered here are the is the Backward Euler and Trapezoidal

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \mathbf{A}\mathbf{x}(t))$$

Initially we'll assume linear equations

Then using backward Euler

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \Delta t\,\mathbf{A}(\mathbf{x}(t+\Delta t))$$

$$\left[I - \Delta t\,\mathbf{A}\right]\mathbf{x}(t+\Delta t) = \mathbf{x}(t)$$

$$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1}\mathbf{x}(t)$$

Backward Euler Cart Example

- Returning to the cart example

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \mathbf{x}(t))$$

Then using backward Euler with $\Delta t = 0.25$

$$\mathbf{x}(t+\Delta t) = [I - \Delta t\,\mathbf{A}]^{-1}\mathbf{x}(t) = \begin{bmatrix} 1 & -0.25 \\ 0.25 & 1 \end{bmatrix}^{-1} \mathbf{x}(t)$$

Backward Euler Cart Example

- Results with $\Delta t = 0.25$ and 0.05

time	actual $x_1(t)$	$x_1(t)$ with $\Delta t=0.25$	$x_1(t)$ with $\Delta t=0.05$
0	1	1	1
0.25	0.9689	0.9411	0.9629
0.50	0.8776	0.8304	0.8700
0.75	0.7317	0.6774	0.7185
1.00	0.5403	0.4935	0.5277
2.00	-0.416	-0.298	-0.3944

Note: Just because the method is numerically stable doesn't mean it is error free! RK2 is more accurate than backward Euler.

Trapezoidal Linear Case

- For the trapezoidal with a linear system we have

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \mathbf{A}\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[\mathbf{A}(\mathbf{x}(t)) + \mathbf{A}(\mathbf{x}(t+\Delta t))\right]$$

$$\left[I - \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t+\Delta t) = \left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

$$\mathbf{x}(t+\Delta t) = \left[I - \Delta t\,\mathbf{A}\right]^{-1}\left[I + \frac{\Delta t}{2}\mathbf{A}\right]\mathbf{x}(t)$$

Trapezoidal Cart Example

- Results with $\Delta t = 0.25$, comparing between backward Euler and trapezoidal

time	actual $x_1(t)$	Backward Euler	Trapezoidal
0	1	1	1
0.25	0.9689	0.9411	0.9692
0.50	0.8776	0.8304	0.8788
0.75	0.7317	0.6774	0.7343
1.00	0.5403	0.4935	0.5446
2.00	-0.416	-0.298	-0.4067

The Best Numerical Integration Approach Depends on the Application

- There is no single best numerical integration method, with all approaches having advantages and disadvantages
- Issues to consider include
  - Speed
  - Accuracy
  - Numerical stability
  - Code complexity; with power system stability this includes the ability to support a wide, and growing list of models
- Explicit methods are commonly used with great success, with numerical instability methods managed through effective engineering
  - An analogy is airplane, which through engineering can be made to effectively fly even though there are conditions in which they can crash

Electromagnetic Transients

- The modeling of very fast power system dynamics (much less than one cycle) is known as electromagnetics transients program (EMTP) analysis
  - Covers issues such as lightning propagation and switching surges; they can also be used with inverter-based controls
- Concept originally developed by Prof. Hermann Dommel for his PhD in the 1960's (now emeritus at Univ. British Columbia)
  - After his PhD work Dr. Dommel worked at BPA where he was joined by Scott Meyer in the early 1970's
  - Alternative Transients Program (ATP) developed in response to commercialization of the BPA code

Power System Time Frames

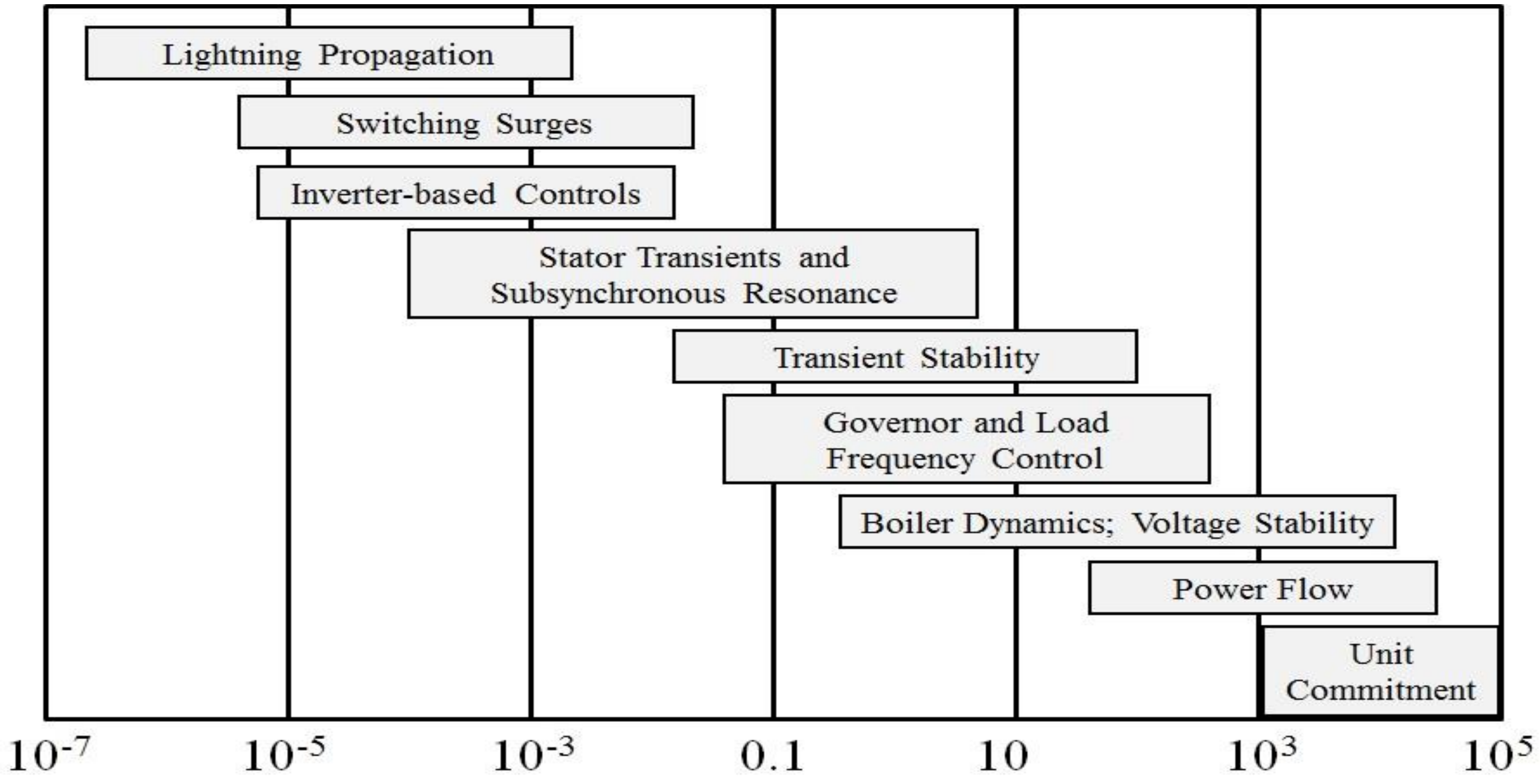


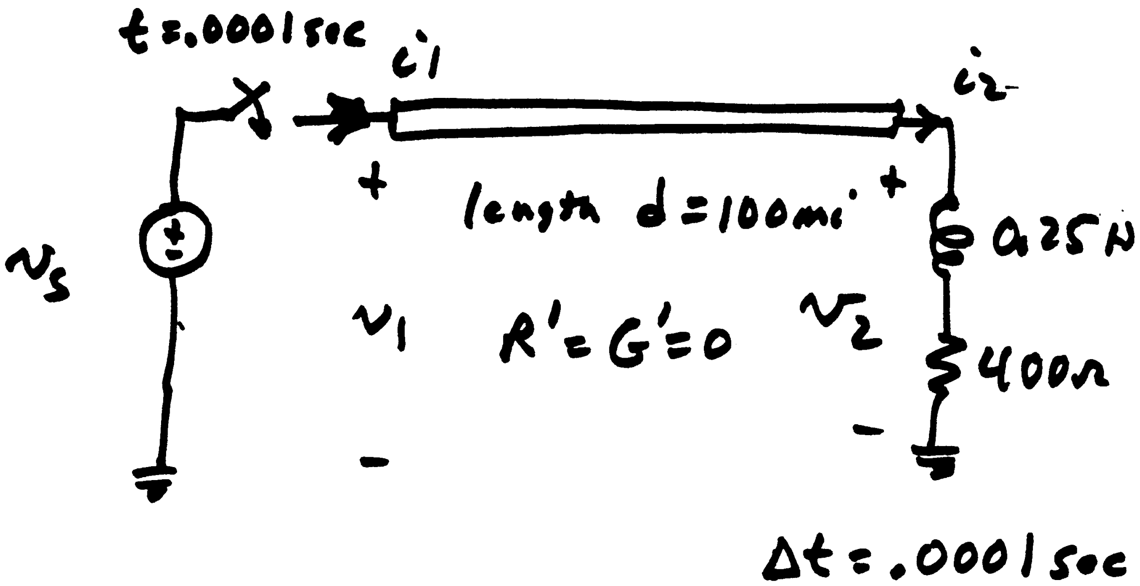
Image source: P.W. Sauer, M.A. Pai, Power System Dynamics and Stability, 1997, Fig 1.2, modified

Transmission Line Modeling

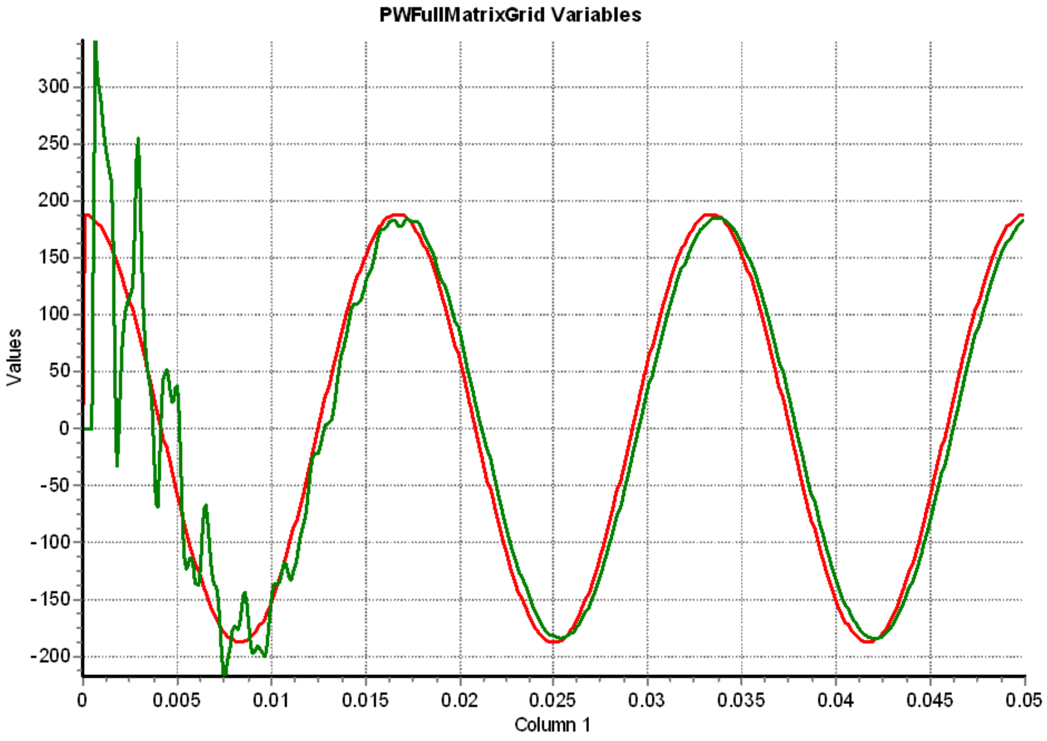
- In power flow and transient stability transmission lines are modeled using a lumped parameter approach
  - Changes in voltages and current in the line are assumed to occur instantaneously
  - Transient stability time steps are usually a few ms (1/4 cycle is common, equal to 4.167ms for 60Hz)
- In EMTP time-frame this is no longer the case; speed of light is 300,000km/sec or 300km/ms or 300m/μs
  - Change in voltage and/or current at one end of a transmission cannot instantaneously affect the other end

Need for EMTP

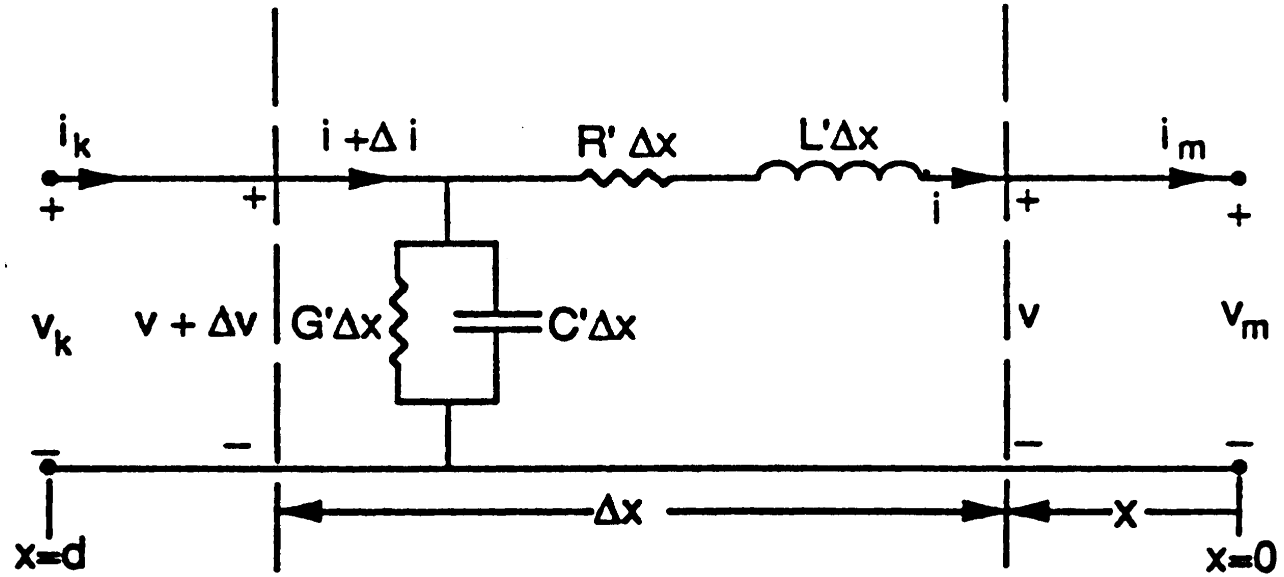
- The change isn't instantaneous because of propagation delays, which are near the speed of light; there also wave reflection issues



Red is the v_s end, green the v_2 end



Incremental Transmission Line Modeling



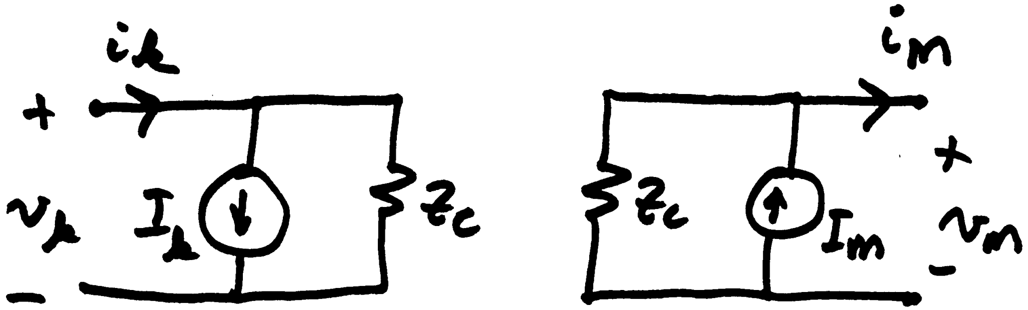
$$\Delta v = R'\Delta x i + L'\Delta x \frac{\partial i}{\partial t}$$

$$\Delta i = G'\Delta x\left(v+\Delta v\right) + C'\Delta x \frac{\partial}{\partial t}\left(v+\Delta v\right)$$

Define the receiving end as bus m (x=0) and the sending end as bus k (x=d)

Where We Will End Up

- Goal is to come up with model of transmission line suitable for numeric studies on this time frame



Both ends of the line are represented by Norton equivalents

$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

Assumption is we don't care about what occurs along the line

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Incremental Transmission Line Modeling

We are looking to determine v(x,t) and i(x,t)

Recall $\Delta i = G'\Delta x\left(v + \Delta v\right) + C'\Delta x \frac{\partial}{\partial t}\left(v + \Delta v\right)$

Substitute $\Delta v = \Delta x\left(R'i + L'\frac{\partial i}{\partial t} \right)$

Into the equation for Δi and divide both by Δx

$$\frac{\Delta i}{\Delta x} = G'v + G'\left(R'\Delta x i + L'\Delta x \frac{\partial i}{\partial t} \right) + C'\frac{\partial v}{\partial t}$$

$$+C'\left[R'\Delta x \frac{\partial i}{\partial t} + L'\Delta x \frac{\partial^2 i}{\partial t^2} \right]$$

Incremental Transmission Line Modeling

Taking the limit we get

$$\lim_{\Delta x \to 0} \frac{\Delta v}{\Delta x} = \boxed{\frac{\partial v}{\partial x} = R'i + L'\frac{\partial i}{\partial t}}$$

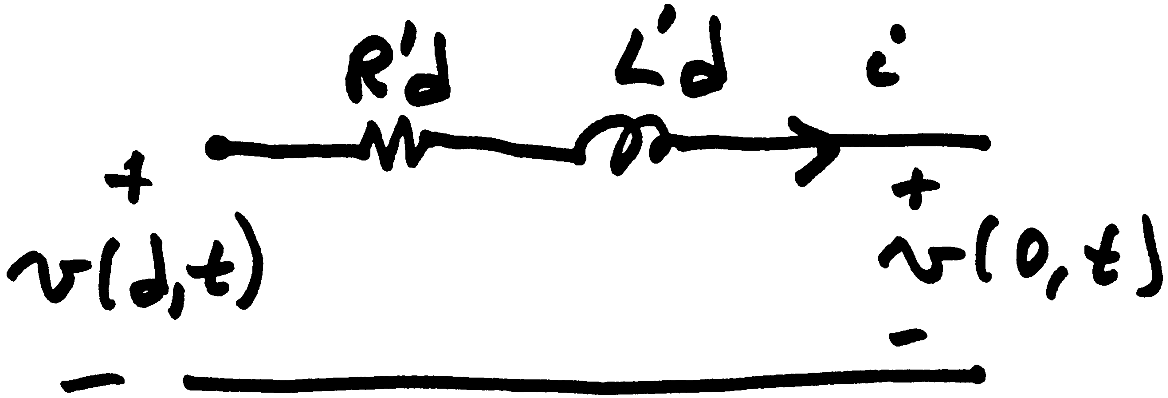
$$\lim_{\Delta x \to 0} \frac{\Delta i}{\Delta x} = \boxed{\frac{\partial i}{\partial x} = G'v + C'\frac{\partial v}{\partial t}}$$

Some authors have a negative sign with these equations; it just depends on the direction of increasing x; note that the values are function of both x and t

Special Case 1

C' = G' = 0 (neglect shunts)

$$v(x,t) = v(0,t) + R'x_i + L'x\frac{di}{dt}$$



This just gives a lumped parameter model, with all electric field effects neglected

Special Case 2: Wave Equation

The lossless line (R'=0, G'=0), which gives

$$\frac{\partial v}{\partial x} = L' \frac{\partial i}{\partial t}, \quad \frac{\partial i}{\partial x} = C' \frac{\partial v}{\partial t}$$

This is the wave equation with a general solution of

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$z_c = \sqrt{L'/C'}, \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Z_c is the characteristic impedance and v_p is the velocity of propagation

Special Case 2: Wave Equation

- This can be thought of as two waves, one traveling in the positive x direction with velocity v_p, and one in the opposite direction
- The values of f_1 and f_2 depend upon the boundary (terminal) conditions

$$i(x,t) = -f_1\left(x - v_p t\right) - f_2\left(x + v_p t\right)$$

$$v(x,t) = z_c f_1\left(x - v_p t\right) - z_c f_2\left(x + v_p t\right)$$

$$z_c = \sqrt{L' / C'} \ , \quad v_p = \frac{1}{\sqrt{L'C'}}$$

Boundaries are receiving end with x=0 and the sending end with x=d

Calculating v_p

- To calculate v_p for a line in air we go back to the definition of L' and C'

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right), \quad C' = \frac{2\pi\varepsilon_0}{\ln D/r}$$

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu_0\varepsilon_0 \frac{\ln D/r'}{\ln D/r}}} = c\frac{1}{\sqrt{\frac{\ln D/r'}{\ln D/r}}}$$

With r'=0.78r this is very close to the speed of light

Important Insight

- The amount of time for the wave to go between the terminals is $d/v_p = \tau$ seconds
  - To an observer traveling along the line with the wave, $x+v_p t$, will appear constant
- What appears at one end of the line impacts the other end τ seconds later

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$v(x,t) + z_c i(x,t) = -2 z_c f_2(x + v_p t)$$

Both sides of the bottom equation are constant when $x+v_p t$ is constant

Determining the Constants

- If just the terminal characteristics are desired, then an approach known as Bergeron's method can be used.
- Knowing the values at the receiving end m (x=0) we get

$$i(x,t) = -f_1(x - v_p t) - f_2(x + v_p t)$$

$$v(x,t) = z_c f_1(x - v_p t) - z_c f_2(x + v_p t)$$

$$i_m(t) = i(0,t) = -f_1(-v_p t) - f_2(v_p t)$$

$$v_m(t) = z_c f_1(-v_p t) - z_c f_2(v_p t)$$

This can be used to eliminate f_1

Determining the Constants

- Eliminating f_1 we get

$$v_m(t) = z_c f_1\left(-v_p\, t\right) - z_c f_2\left(v_p\, t\right)$$

$$f_1\left(-v_p\, t\right) = \frac{v_{m(t)}}{z_c} + f_2\left(v_p\, t\right)$$

$$i_m(t) = -\frac{v_m}{z_c} - 2 f_2\left(v_p\, t\right)$$

Solve for f_1 and replace it in the equation from the previous slide

Determining the Constants

- To solve for f_2 we need to look at what is going on at the sending end (i.e., k at which x=d) $\tau = d/v_p$ seconds in the past

$$i_k\left(t-\frac{d}{v_p}\right) = -f_1\left(d - v_p\left(t-\frac{d}{v_p}\right)\right) - f_2\left(d + v_p\left(t-\frac{d}{v_p}\right)\right)$$

$$i_k\left(t-\frac{d}{v_p}\right) = -f_1\left(2d - v_p t\right) - f_2\left(v_p t\right)$$

$$v_k\left(t-\frac{d}{v_p}\right) = z_c f_1\left(2d - v_p t\right) - z_c f_2\left(v_p t\right)$$

Determining the Constants

- Dividing v_k by z_c, and then adding it with i_k gives

$$i_k\left(t-\frac{d}{v_p}\right)+\frac{v_k}{z_c}\left(t-\frac{d}{v_p}\right)=-2f_2\left(v_p t\right)$$

- Then substituting for f_2 in i_m gives

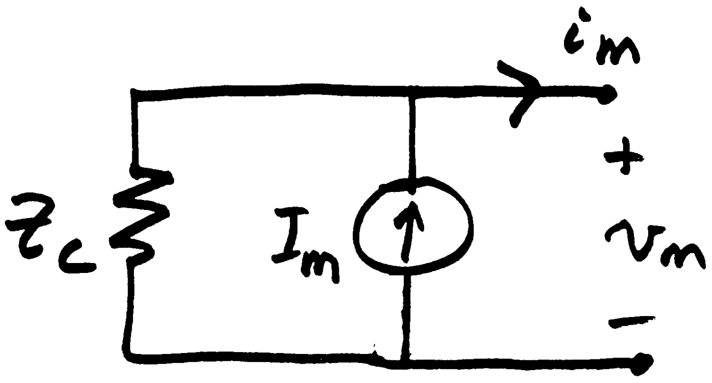
$$i_m\left(t\right)=-\frac{v_m\left(t\right)}{z_c}+i_k\left(t-\frac{d}{v_p}\right)+\frac{1}{z_c}v_k\left(t-\frac{d}{v_p}\right)$$

Hence $i_m(t)$ depends on current conditions at m and past conditions at k

Equivalent Circuit Representation

- The receiving end can be represented in circuit form as

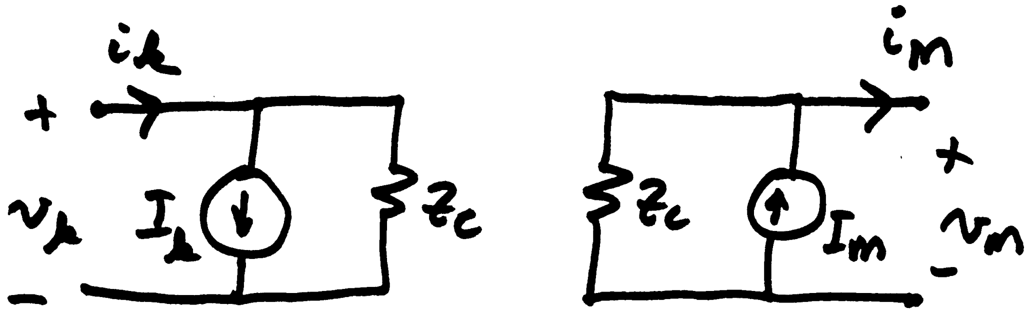
$$i_m(t) = -\frac{v_m(t)}{z_c} + i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right) \longrightarrow I_m$$



Since $\tau = d/v_p$, I_m just depends on the voltage and current at the other end of the line from τ seconds in the past. Since these are known values, it looks like a time-varying current source.

Repeating for the Sending End

- The sending end has a similar representation



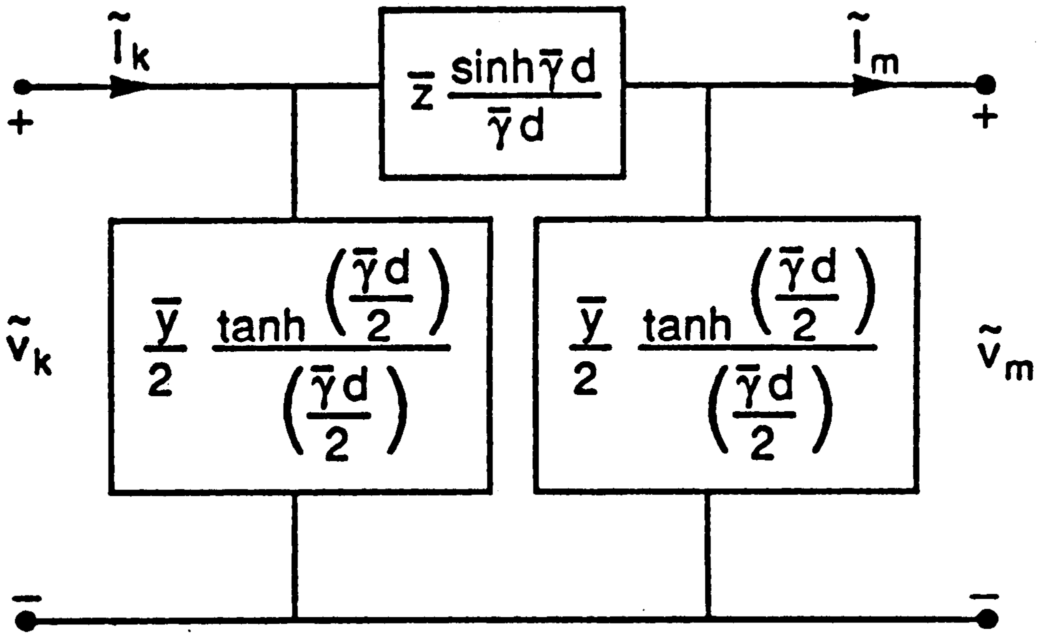
Both ends of the line are represented by Norton equivalents

$$I_k = i_m\left(t - \frac{d}{v_p}\right) - \frac{1}{z_c} v_m\left(t - \frac{d}{v_p}\right)$$

$$I_m = i_k\left(t - \frac{d}{v_p}\right) + \frac{1}{z_c} v_k\left(t - \frac{d}{v_p}\right)$$

Lumped Parameter Model

- In the special case of constant frequency, book shows the derivation of the common lumped parameter model



This is used in power flow and transient stability; in EMTP the frequency is not constant

Including Line Resistance

- An approach for adding line resistance, while keeping the simplicity of the lossless line model, is to just to place ½ of the resistance at each end of the line
  - Another, more accurate approach, is to place ¼ at each end, and ½ in the middle
- Standalone resistance, such as modeling the resistance of a switch, is just represented as an algebraic equation

$$i_{k,m} = \frac{1}{R}\left(v_k - v_m\right)$$

Numerical Integration with Trapezoidal Method

- Numerical integration is often done using the trapezoidal method discussed last time
  - Here we show how it can be applied to inductors and capacitors
- For a general function the trapezoidal approach is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[f(\mathbf{x}(t)) + f(\mathbf{x}(t+\Delta t))\right]$$

- Trapezoidal integration introduces error on the order of Δt^3, but it is numerically stable

Trapezoidal Applied to Inductor with Resistance

- For a lossless inductor,

$$v = L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{v}{L} \qquad i(0) = i^0$$

This is a linear equation

$$i(t+\Delta t) = i(t) + \frac{\Delta t}{2L}\left(v(t) + v(t+\Delta t)\right)$$

- This can be represented as a Norton equivalent with current into the equivalent defined as positive (the last two terms are the current source)

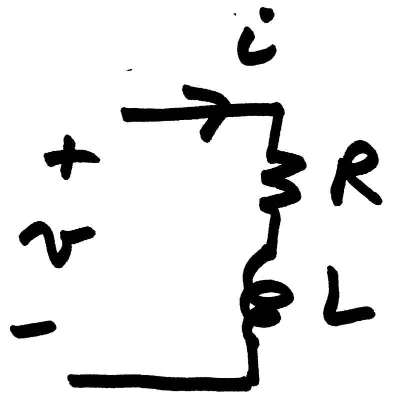
$$i(t+\Delta t) = \frac{v(t+\Delta t)}{2L/\Delta t} + i(t) + \frac{v(t)}{2L/\Delta t}$$

Trapezoidal Applied to Inductor with Resistance

- For an inductor in series with a resistance we have

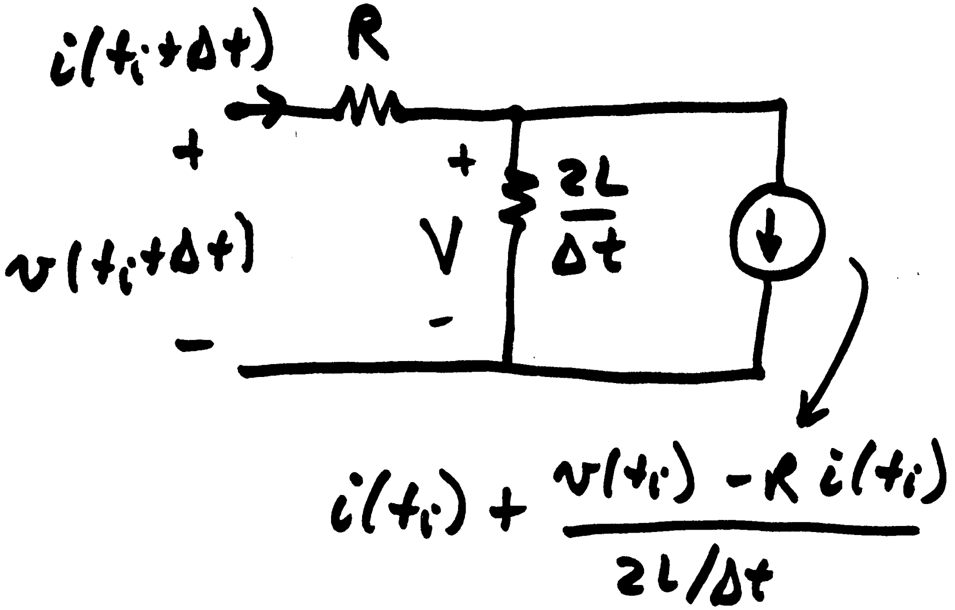
$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$



Trapezoidal Applied to Inductor with Resistance

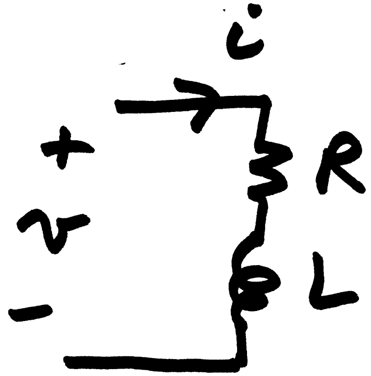
$$i(t_i+\Delta t) \approx i(t_i) + \frac{\Delta t}{2}\left[-\frac{R}{L}i(t_i) + \frac{1}{L}v(t_i) - \frac{R}{L}i(t_i+\Delta t) + \frac{1}{L}v(t_i+\Delta t)\right]$$



This also becomes a Norton equivalent. A similar expression will be developed for capacitors.

RL Example

- Assume a series RL circuit with an open switch with R= 200Ω and L = 0.3H, connected to a voltage source with $v = 133{,}000\sqrt{2}\cos\left(2\pi 60t\right)$
- Assume the switch is closed at t=0
- The exact solution is



$$i = -712.4e^{-667t} + 578.8\sqrt{2}\cos\left(2\pi 60t - 29.5^0\right)$$

$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$

R/L=667, so the dc offset decays relatively quickly

RL Example Trapezoidal Solution

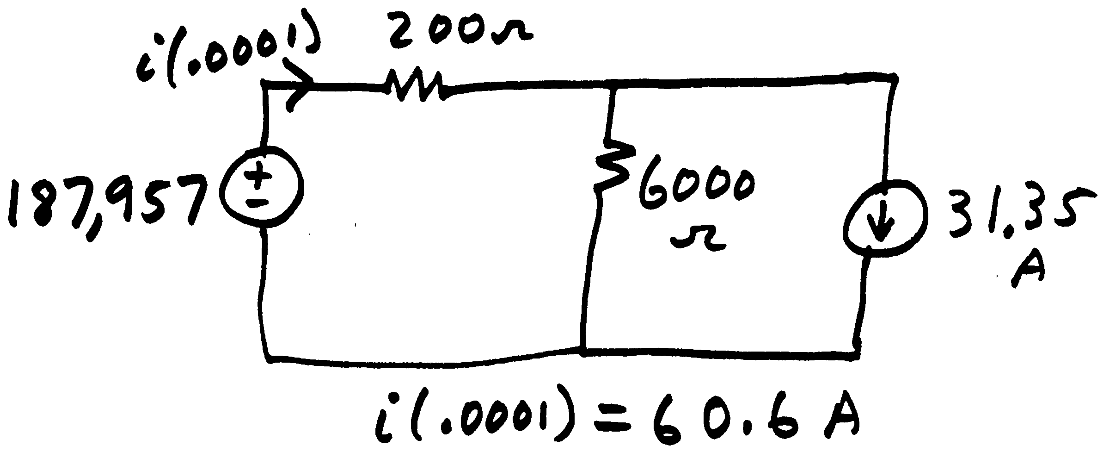
$$\frac{2L}{\Delta t} = \frac{2*0.3}{0.0001} = 6000$$

$$\Delta t = 0.0001 \text{sec}$$

$$t = 0 \quad i(0) = 0$$

$$t = 0.0001$$

$$i(0) + \frac{v(0) - Ri(0)}{6000} = 31.35\text{A}$$



Numeric solution: $i(0.0001) = \frac{187{,}957}{6200} + \frac{31.35 \times 6000}{6200} = 60.65\text{A}$

Exact solution:

$$i(0.0001) = -712.4e^{-.0677} + 578.8\sqrt{2}\cos\left(2\pi 60 \times .0001 - 29.5\frac{\pi}{180}\right)$$

$$= -666.4 + 727.0 = 60.6A$$

RL Example Trapezoidal Solution

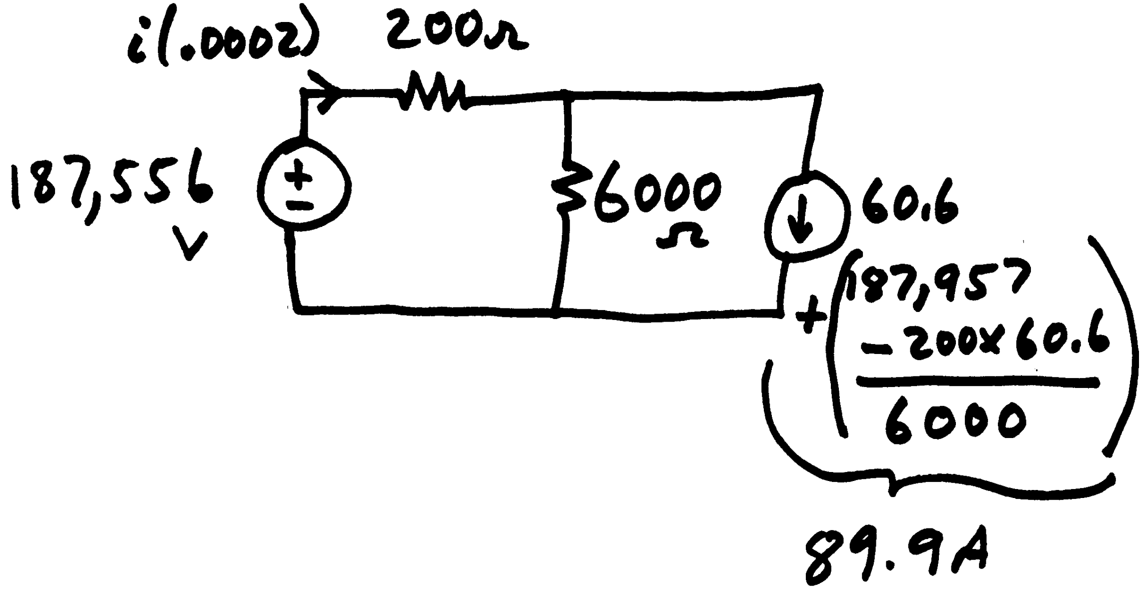
$t = 0.0002$

Solving for i(0.0002)

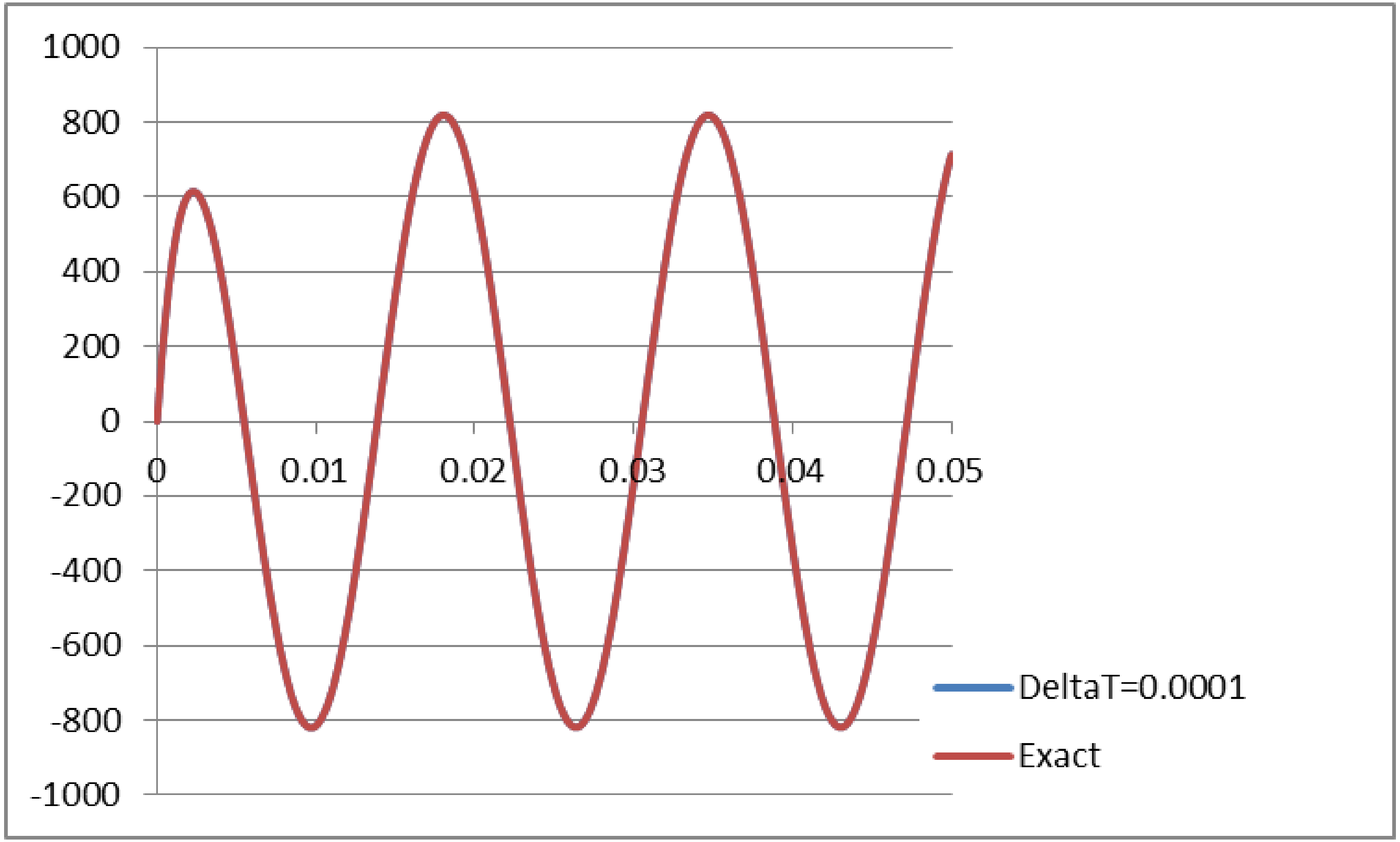
$i(0.0002) = 117.3\text{A}$

Compare to the exact solution

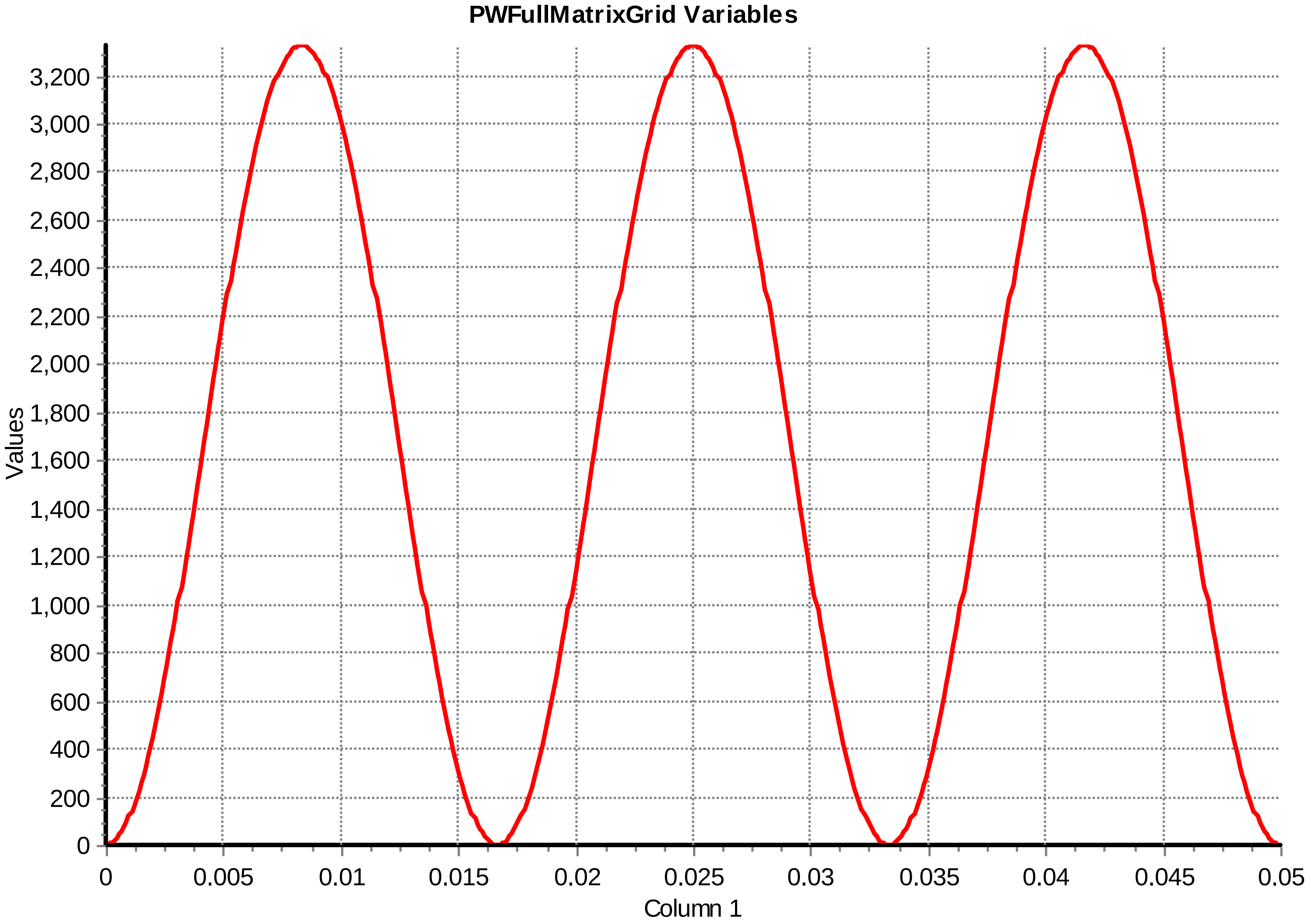
$i(0.0002) = 117.3\text{A}$



Full Solution Over Three Cycles



A Favorite Problem: R=0 Case, with v(t) = Sin(2*pi*60)



Note that the current is never negative!

Lumped Capacitance Model

- The trapezoidal approach can also be applied to model lumped capacitors

$$i(t) = C\frac{dv(t)}{dt}$$

- Integrating over a time step gives

$$v(t+\Delta t) = v(t) + \frac{1}{C}\int_{t}^{t+\Delta t} i(t)$$

- Which can be approximated by the trapezoidal as

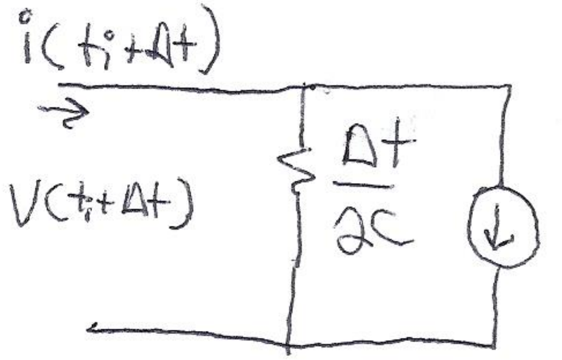
$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

Lumped Capacitance Model

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

$$i(t+\Delta t) = \frac{v(t+\Delta t)}{\Delta t/2C} - \frac{v(t)}{\Delta t/2C} - i(t)$$

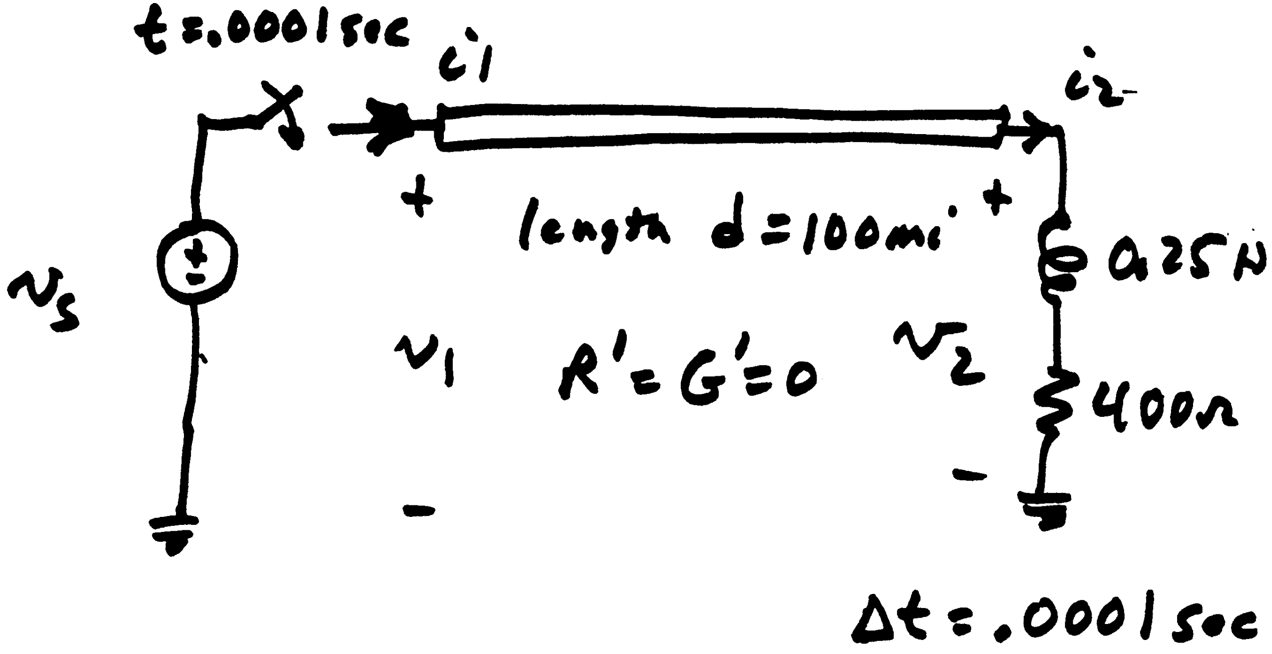
- Hence we can derive a circuit model similar to what was done for the inductor



$$-\frac{v(t)}{\Delta t/2C} - i(t)$$

This is a current source that depends on the past values

Example 2.1: Line Closing



Switch is closed at time t = 0.0001 sec

$$L' = 1.5 \times 10^{-3} H/mi$$

$$C' = 0.02 \times 10^{-6} F/mi$$

Example 2.1: Line Closing

Initial conditions: $i_1 = i_2 = v_1 = v_2 = 0$

for $t < 0.0001$ sec

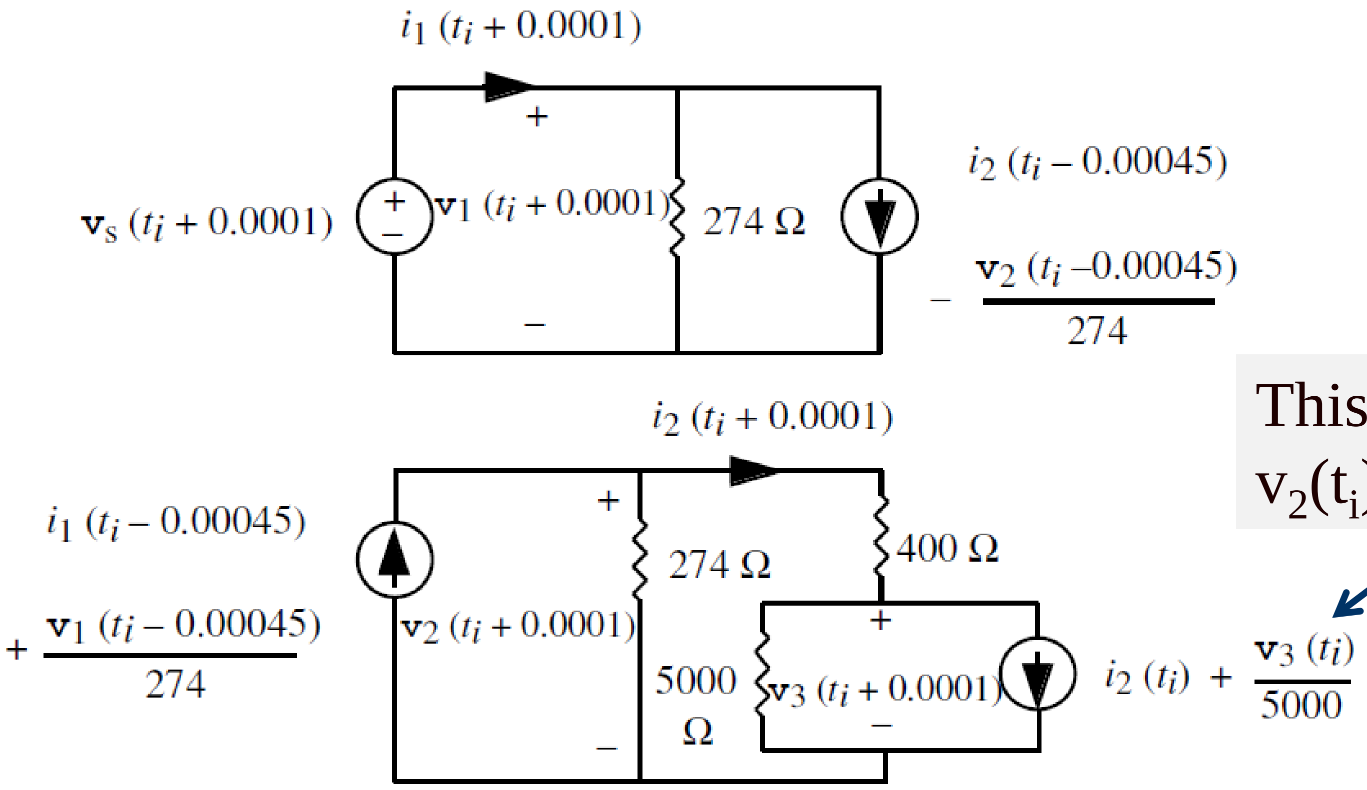
$$z_c = \sqrt{\frac{L'}{C'}} = 274\Omega \qquad v_p = \frac{1}{\sqrt{L'C'}} = 182{,}574\,mi/\sec$$

$$\frac{d}{v_p} = 0.00055\sec$$

$$\frac{2L}{\Delta t} = 5000\Omega$$

Because of finite propagation speed, the receiving end of the line will not respond to energizing the sending end for at least 0.00055 seconds.

Example 2.1: Line Closing



This is $v_3(t_i) = v_2(t_i) - 400*i_2(t_i)$

Figure 2.8: *Single line and R-L load circuit at* $t = t_i + 0.0001$

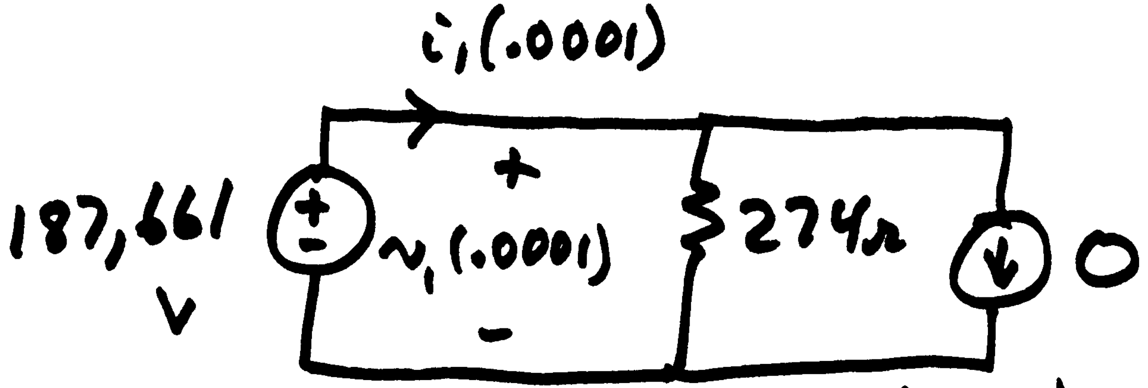
Note we have two separate circuits, coupled together only by past values.

Example 2.1: t=0.0001

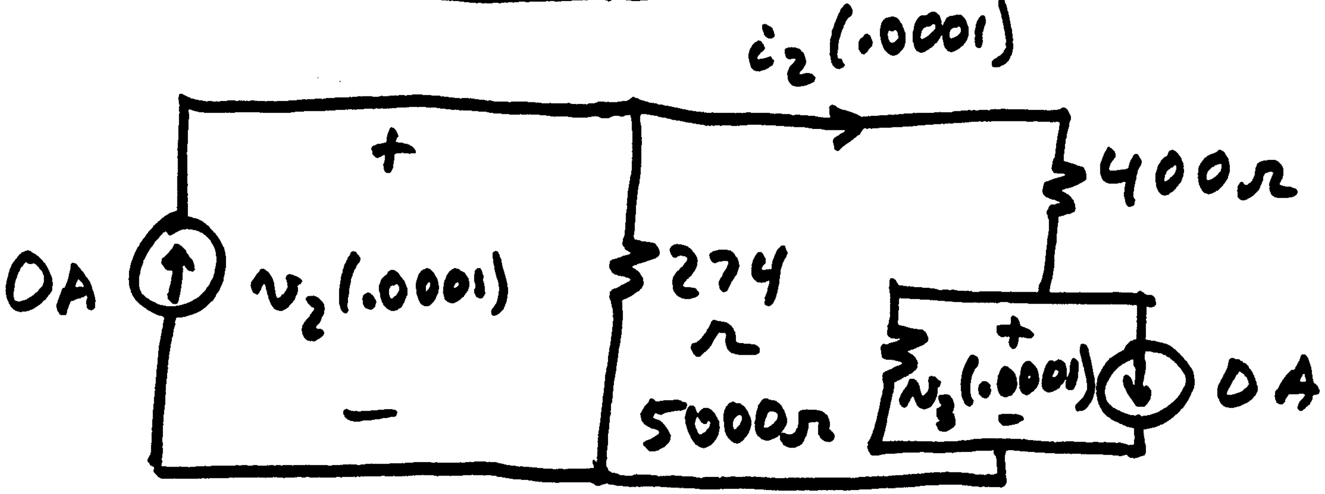
Need: $i_1(-0.00045),\ v_1(-0.00045),\ i_2(-0.00045),$
$v_2(-0.00045),\ i_2(0),\ v_3(0),\ v_s(0.0001)$

$$i_1(-0.00045) = 0 \quad i_2(0) = 0$$
$$v_1(-0.00045) = 0 \quad v_3(0) = 0$$
$$i_2(-0.00045) = 0 \quad v_2(-0.00045) = 0$$
$$v_s(0.0001) = 230{,}000\sqrt{\frac{2}{3}}\cos(2\pi 60 \times 0.0001) = 187{,}661\,V$$

Example 2.1: t=0.0001



Sending End



Receiving End

Example 2.1: t=0.0001

$$i_1(0.0001) = 685A$$

$$v_1(0.0001) = 187,661V$$

$$i_2(0.0001) = 0$$

$$v_2(0.0001) = 0$$

$$v_3(0.0001) = 0$$

Instantaneously changed from zero at t = 0.0001 sec.

Example 2.1: t=0.0002

Need:

$$i_1(-0.00035)=0$$
$$v_1(-0.00035)=0$$
$$i_2(-0.00035)=0$$
$$v_2(-0.00035)=0$$
$$i_2(0.0001)=0$$
$$v_3(0.0001)=0$$
$$v_s(0.0002)=187,261V$$

$$i_1(0.0002)=683A$$
$$v_1(0.0002)=187,261V$$
$$i_2(0.0002)=0.$$
$$v_2(0.0002)=0.$$
$$v_3(0.0002)=0.$$

Circuit is essentially the same

Wave is traveling down the line

Example 2.1: t=0.0002 to 0.006

$$\frac{d}{v_p} = 0.00055 \qquad \Delta t = 0.0001$$

$$t_i = 0$$
$$t_i = 0.0001$$
$$= 0.0002$$
$$= 0.0003$$
$$= 0.0004$$
$$= 0.0005$$
$$= 0.0006$$

$$t = 0.0001 \leftarrow$$ switch closed

$$t = 0.0002$$
$$= 0.0003$$
$$= 0.0004$$
$$= 0.0005$$
$$= 0.0006 \leftarrow$$
$$= 0.0007 \leftarrow$$

With interpolation receiving end will see wave

Example 2.1: t=0.0007

Need: $i_1(.00015)$

$v_1(.00015),\ v_2(.00015)$

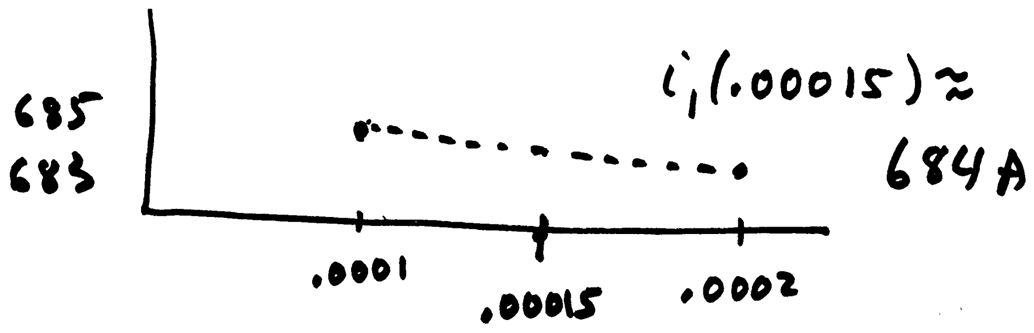
$i_2(.0006),\ v_3(.0006),\ v_s(.0007)$

$$i_1(.0001) = 685A$$

$$i_1(.0002) = 683A$$

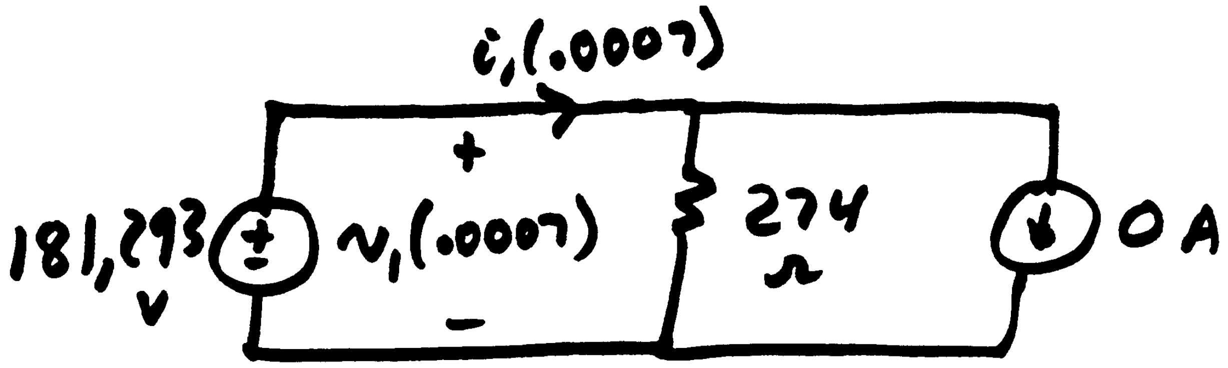
(linear interpolation)

$$i_1(.00015) \approx i_1(.0001) + \frac{.00015 - .0001}{.0002 - .0001} \times \left(i_1(.0002) - i_1(.0001)\right)$$



Example 2.1: t=0.0007

For t_i = .0006 (t = .0007 sec) at the sending end



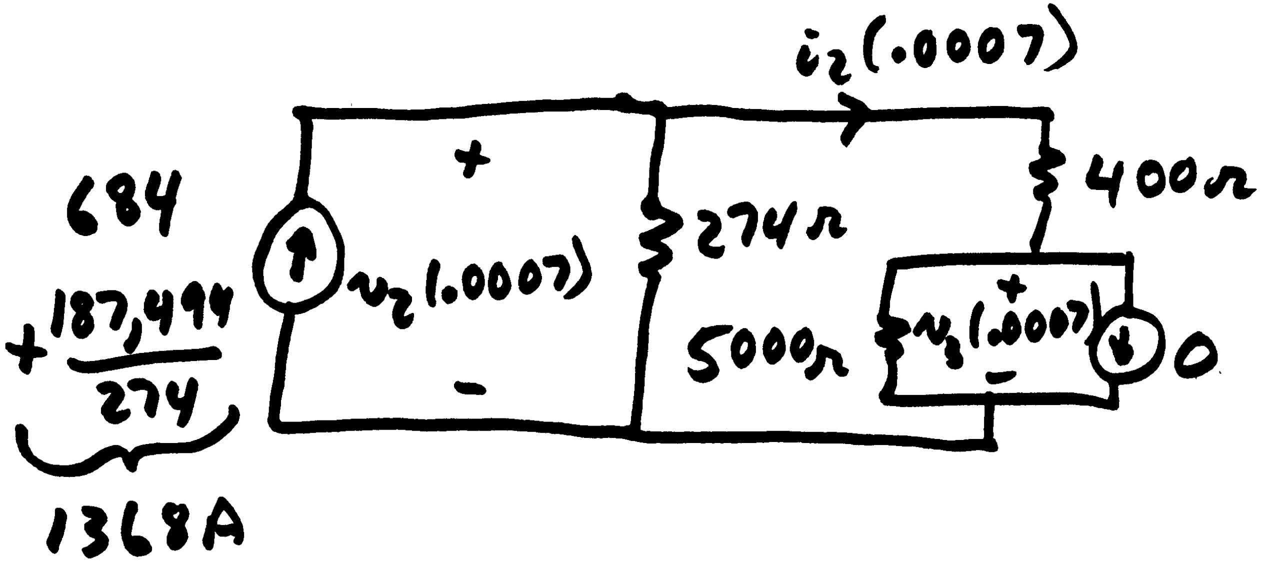
This current source will stay zero until we get a response from the receiving end, at about 2τ seconds

$$i_1(.0007) = 662A$$

$$v_1(.0007) = 181{,}293V$$

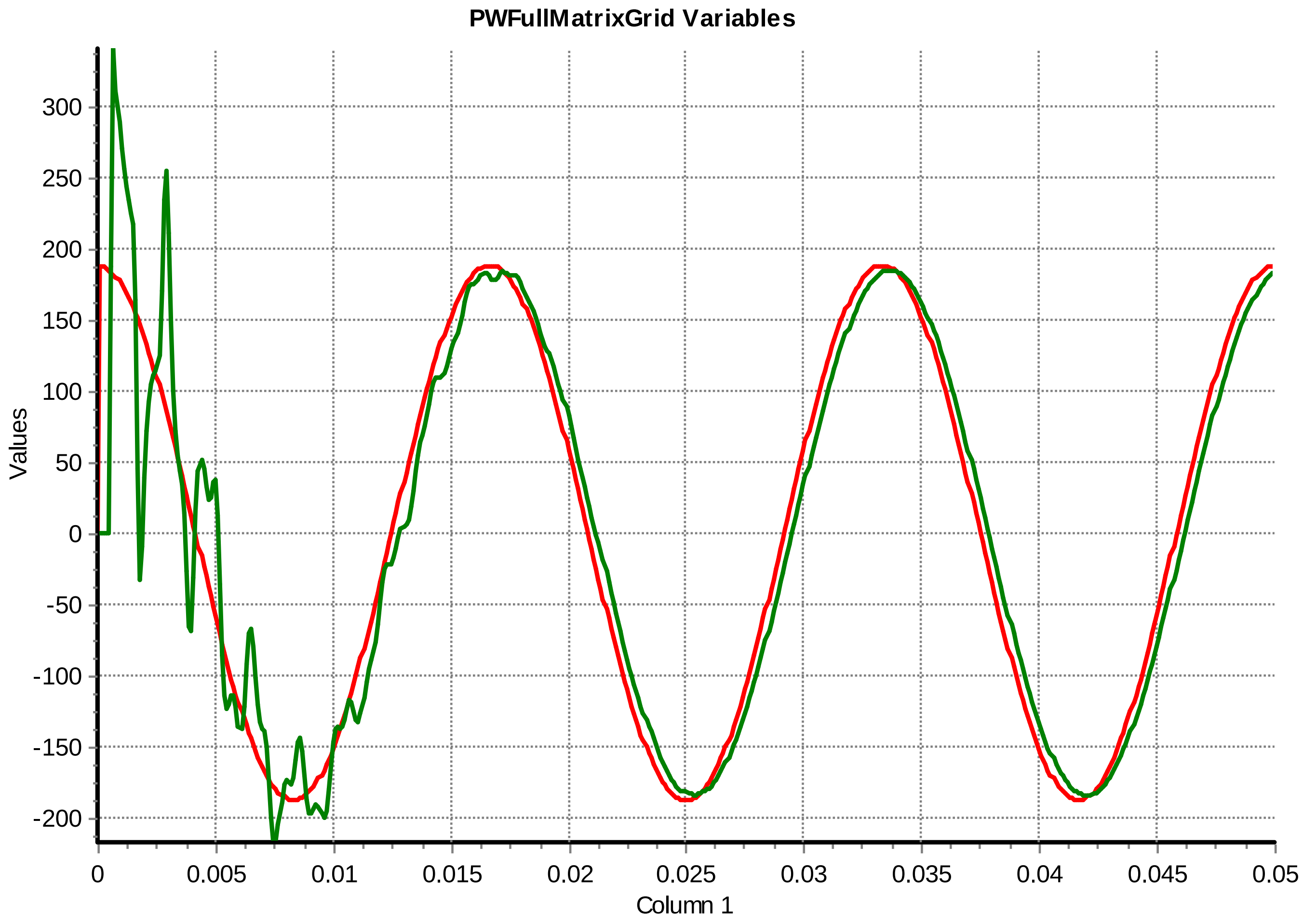
Example 2.1: t=0.0007

For t_i = .0006 (t = .0007 sec) at the receiving end



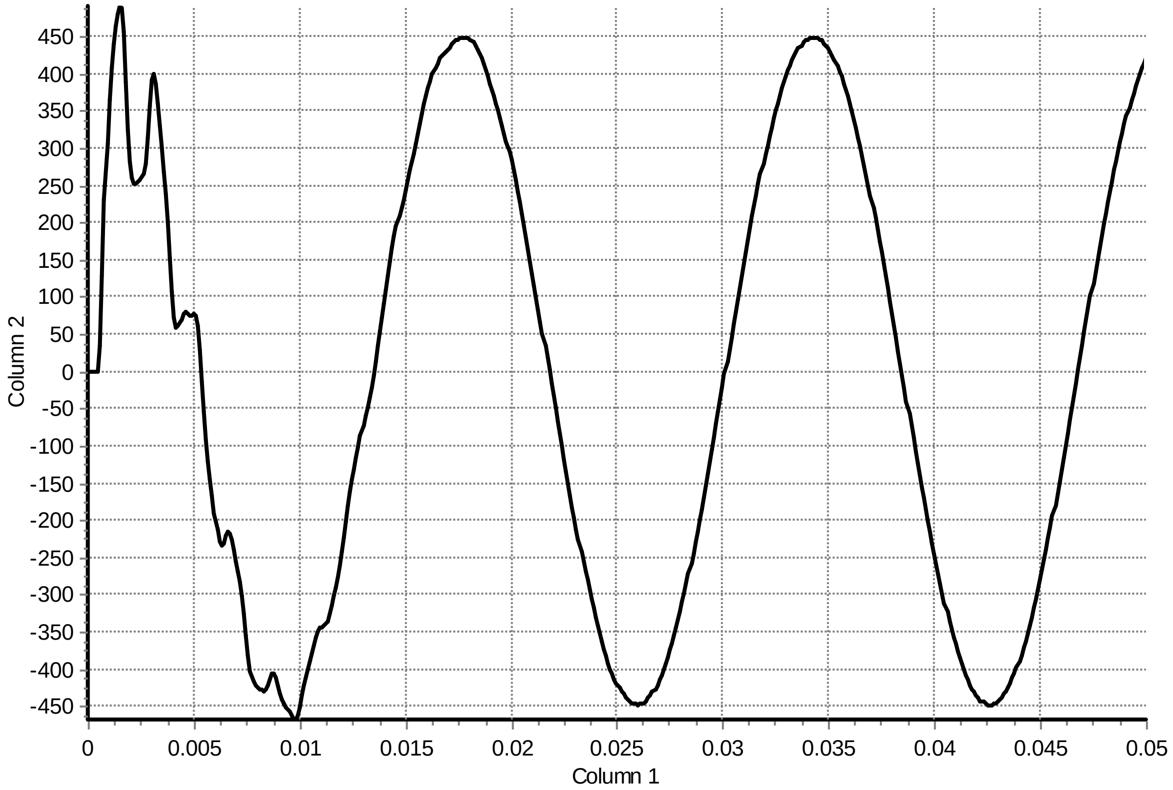
$$v_2(.0007) = 356,731V$$
$$i_2(.0007) = 66A$$

Example 2.1: First Three Cycles



Red is the sending end voltage (in kv), while green is the receiving end voltage. Note the approximate voltage doubling at the receiving end.

Example 2.1: First Three Cycles



Graph shows the current (in amps) into the RL load over the first three cycles.

To get a **ballpark** value on the expected current, solve the simple circuit assuming the transmission line is just an inductor

$$I_{load,rms} = \frac{230{,}000/\sqrt{3}}{400 + j94.2 + j56.5} = 311\angle -20.6°,$$ hence a peak value of 439 amps