# ECEN 667
# Power System Stability

## Lecture 4: Electromagnetic Transients, Three-Phase Line Modeling, Stability Overview

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# Announcements

- Start reading Chapters 3
- Homework 1 is due on Thursday September 7
- Reference for modeling three-phase lines is W. Kersting, *Distribution System Modeling and Analysis*, 4th Edition, CRC Press, 2018
  - There is now a fifth edition that adds Robert Kerestes as an second author

# Numerical Integration with Trapezoidal Method

- Numerical integration is often done using the trapezoidal method discussed last time
  - Here we show how it can be applied to inductors and capacitors
- For a general function the trapezoidal approach is

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2}\left[ f(\mathbf{x}(t)) + f(\mathbf{x}(t+\Delta t))\right]$$

- Trapezoidal integration introduces error on the order of $\Delta t^3$, but it is numerically stable

# Trapezoidal Applied to Inductor with Resistance

- For a lossless inductor,

$$v = L\frac{di}{dt} \quad \rightarrow \quad \frac{di}{dt} = \frac{v}{L} \qquad i(0) = i^0$$

This is a linear equation

$$i(t+\Delta t) = i(t) + \frac{\Delta t}{2L}\left(v(t) + v(t+\Delta t)\right)$$

- This can be represented as a Norton equivalent with current into the equivalent defined as positive (the last two terms are the current source)

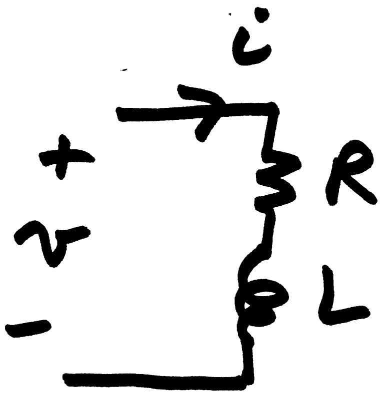
$$i(t+\Delta t) = \frac{v(t+\Delta t)}{2L/\Delta t} + i(t) + \frac{v(t)}{2L/\Delta t}$$

# Trapezoidal Applied to Inductor with Resistance

- For an inductor in series with a resistance we have

$$v = iR + L\frac{di}{dt}$$

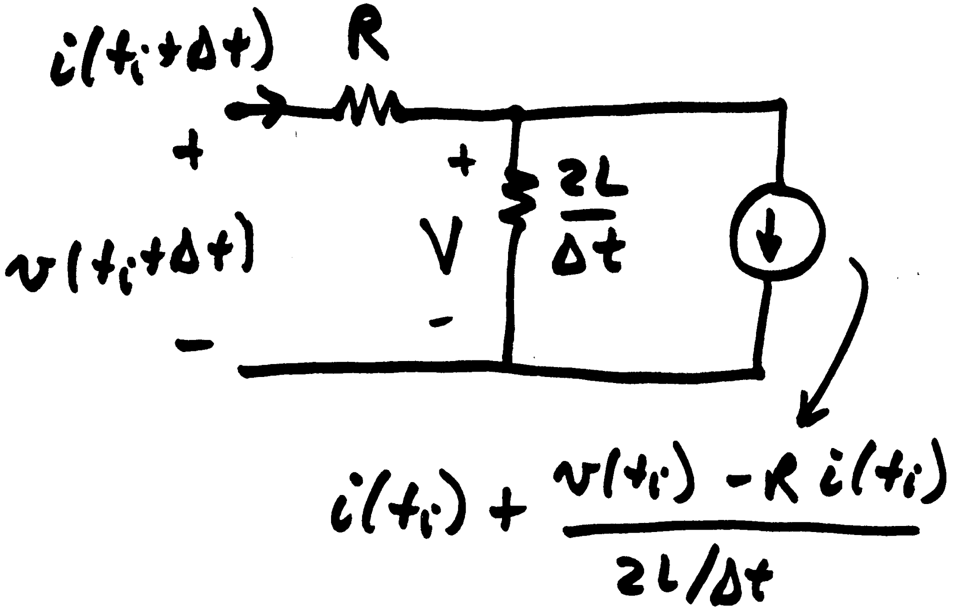
$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i(0) = i^0$$



# Trapezoidal Applied to Inductor with Resistance

$$i(t_i+\Delta t)\approx i(t_i)+\frac{\Delta t}{2}\left[-\frac{R}{L}i(t_i)+\frac{1}{L}v(t_i)\right.$$

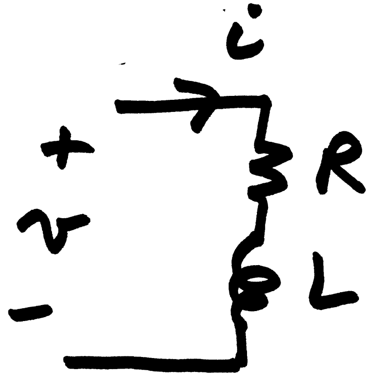
$$\left.-\frac{R}{L}i(t_i+\Delta t)+\frac{1}{L}v(t_i+\Delta t)\right]$$




This also becomes a Norton equivalent. A similar expression will be developed for capacitors.

# RL Example

- Assume a series RL circuit with an open switch with R= 200Ω and L = 0.3H, connected to a voltage source with $v = 133{,}000\sqrt{2}\cos\left(2\pi 60 t\right)$
- Assume the switch is closed at t=0
- The exact solution is




$$i = -712.4e^{-667t} + 578.8\sqrt{2}\cos\left(2\pi 60t - 29.5^{0}\right)$$

$$v = iR + L\frac{di}{dt}$$

$$\frac{di}{dt} = -\frac{R}{L}i + \frac{1}{L}v \qquad i\left(0\right) = i^{0}$$

R/L=667, so the dc offset decays relatively quickly

# RL Example Trapezoidal Solution

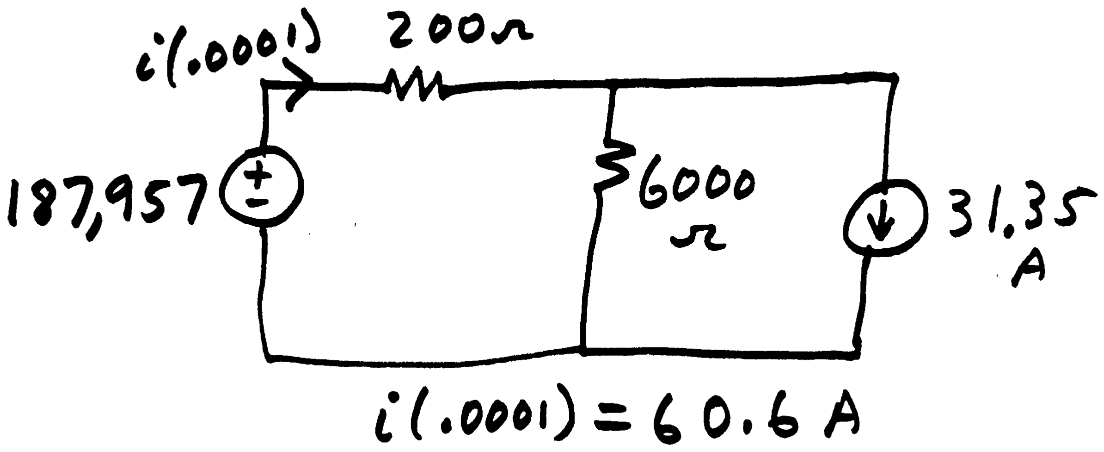
$$\frac{2L}{\Delta t} = \frac{2*0.3}{0.0001} = 6000$$

$$\Delta t = 0.0001\text{sec}$$

$$t = 0 \quad i(0) = 0$$

$$t = 0.0001$$

$$i(0) + \frac{v(0) - Ri(0)}{6000} = 31.35\text{A}$$




Numeric solution: $i(0.0001) = \frac{187{,}957}{6200} + \frac{31.35 \times 6000}{6200} = 60.65\text{A}$

Exact solution:

$$i(0.0001) = -712.4e^{-.0677} + 578.8\sqrt{2}\cos\left(2\pi 60 \times .0001 - 29.5\frac{\pi}{180}\right)$$

$$= -666.4 + 727.0 = 60.6A$$

# RL Example Trapezoidal Solution

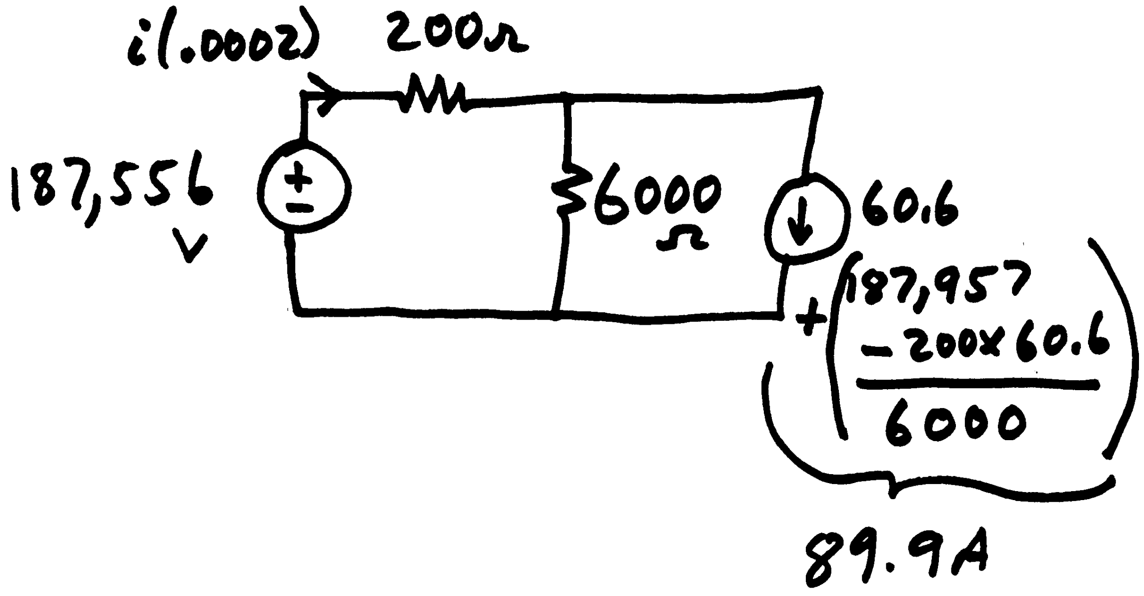
$t = 0.0002$

Solving for i(0.0002)

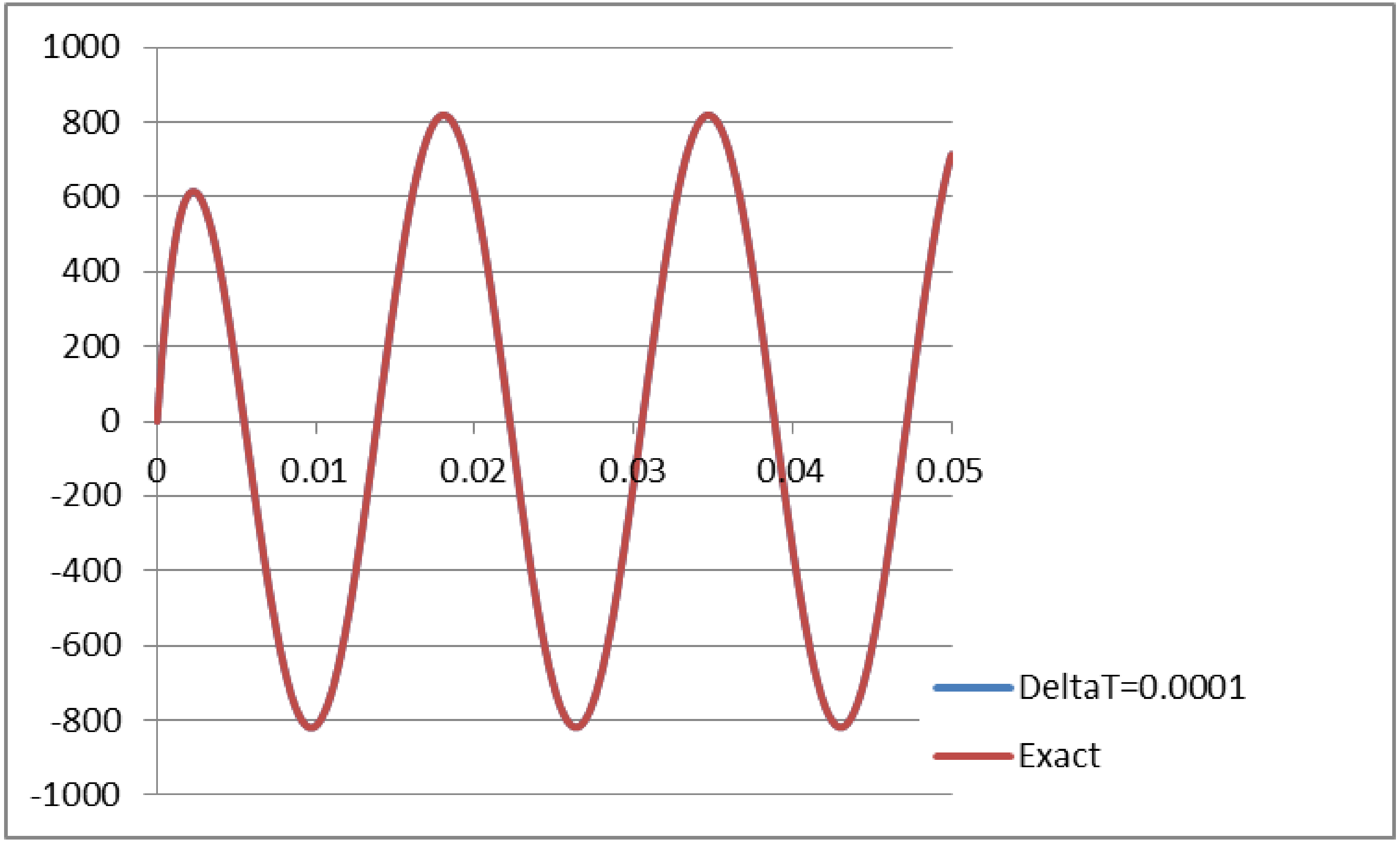
$i(0.0002) = 117.3\text{A}$

Compare to the exact solution

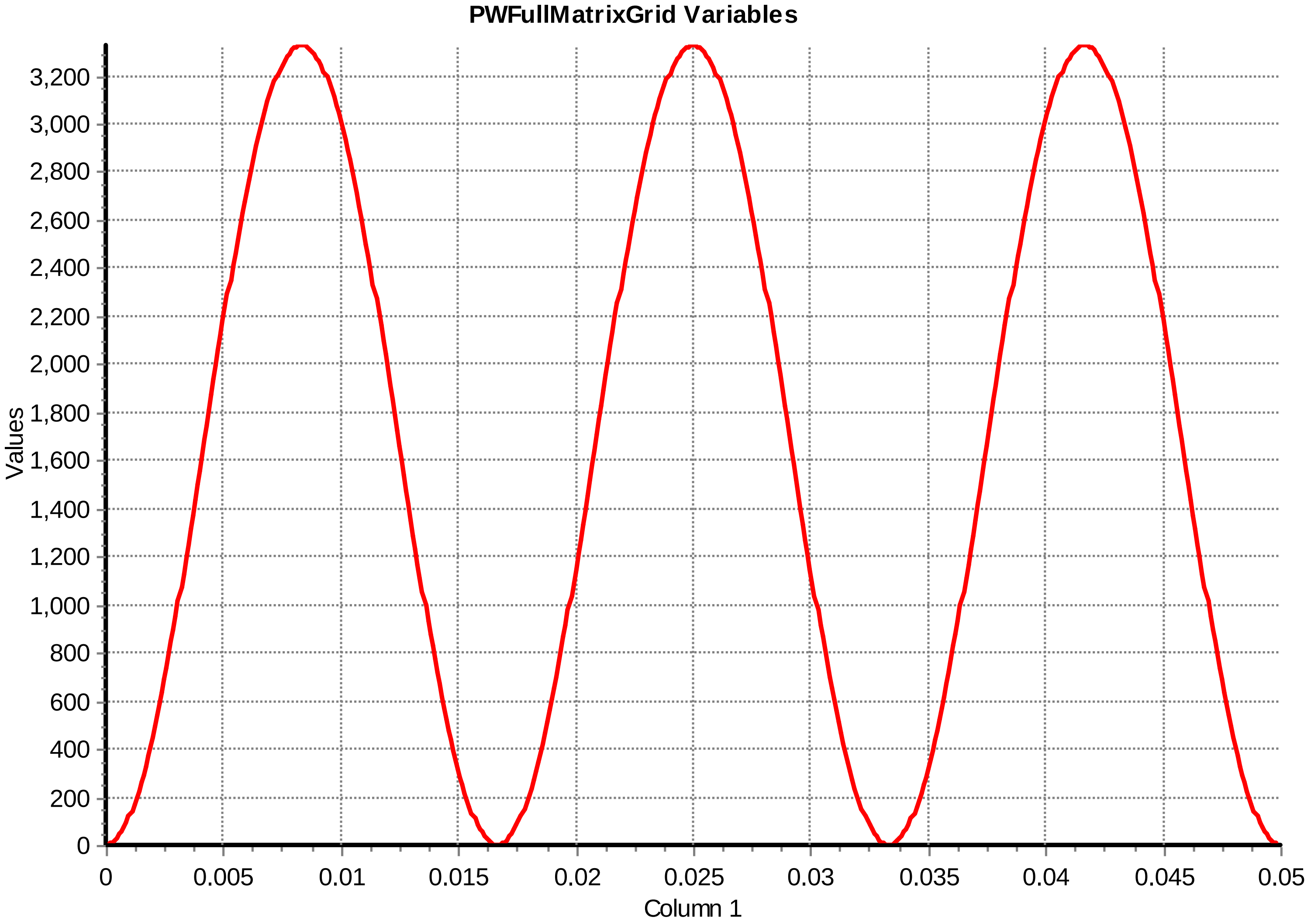
$i(0.0002) = 117.3\text{A}$



# Full Solution Over Three Cycles



# A Favorite Problem: R=0 Case, with v(t) = Sin(2*pi*60)




Note that the current is never negative!

# Lumped Capacitance Model

- The trapezoidal approach can also be applied to model lumped capacitors

$$i(t) = C\frac{dv(t)}{dt}$$

- Integrating over a time step gives

$$v(t+\Delta t) = v(t) + \frac{1}{C}\int_{t}^{t+\Delta t} i(t)$$

- Which can be approximated by the trapezoidal as

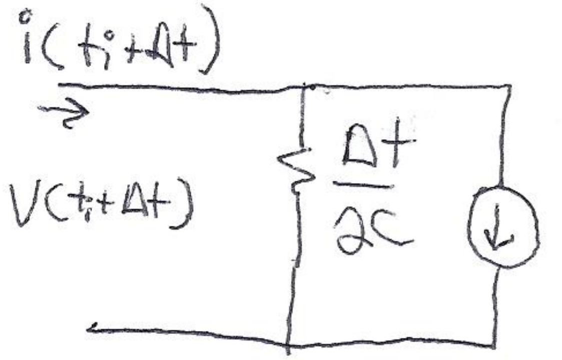
$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

# Lumped Capacitance Model

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2C}\left(i(t+\Delta t) + i(t)\right)$$

$$i(t+\Delta t) = \frac{v(t+\Delta t)}{\Delta t/2C} - \frac{v(t)}{\Delta t/2C} - i(t)$$

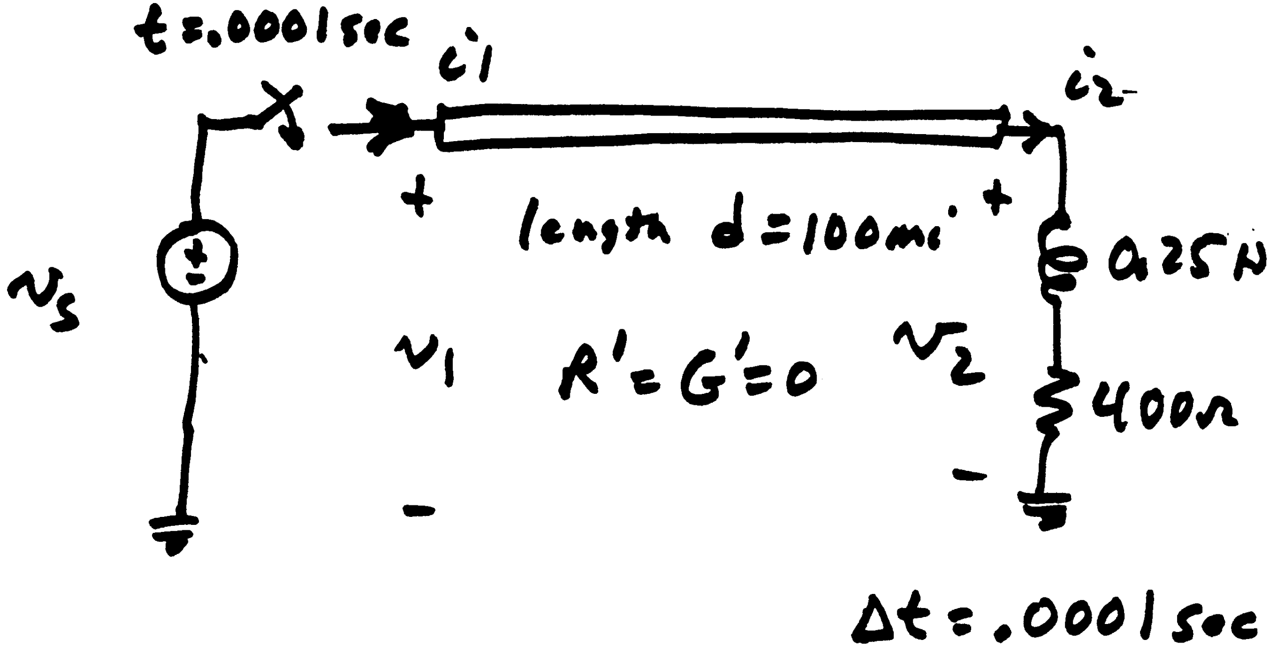
- Hence we can derive a circuit model similar to what was done for the inductor




$$-\frac{v(t)}{\Delta t/2C} - i(t)$$

This is a current source that depends on the past current and voltage values

# Example 2.1: Line Closing




Switch is closed at time t = 0.0001 sec

$$L' = 1.5 \times 10^{-3} H / mi$$

$$C' = 0.02 \times 10^{-6} F / mi$$

# Example 2.1: Line Closing

Initial conditions: $i_1 = i_2 = v_1 = v_2 = 0$

for $t < 0.0001$ sec

$$z_c = \sqrt{\frac{L'}{C'}} = 274\Omega \quad v_p = \frac{1}{\sqrt{L'C'}} = 182{,}574 mi/\sec$$

$$\frac{d}{v_p} = 0.00055 \sec$$

$$\frac{2L}{\Delta t} = 5000\Omega$$

Because of finite propagation speed, the receiving end of the line will not respond to energizing the sending end for at least 0.00055 seconds.

# Example 2.1: Line Closing

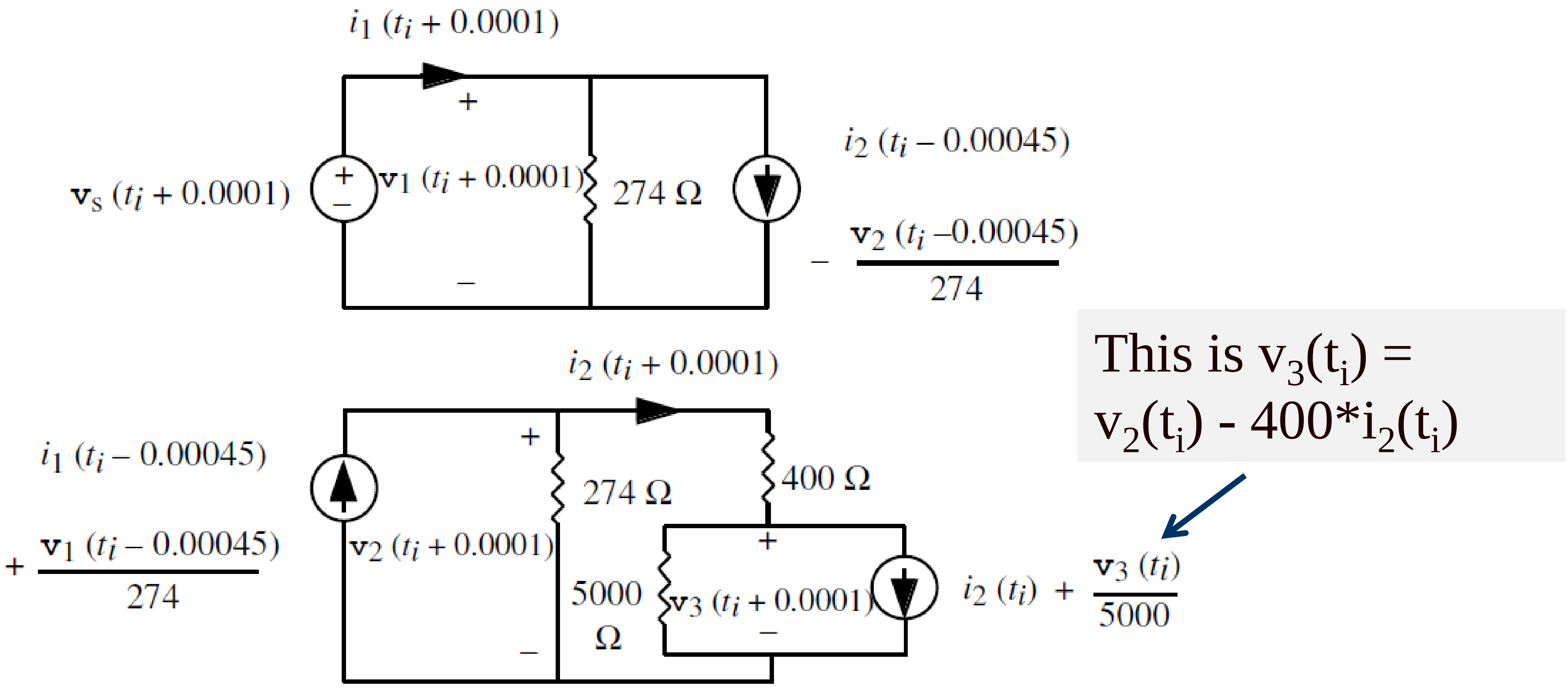



Figure 2.8: *Single line and R-L load circuit at* $t = t_i + 0.0001$

Note we have two separate circuits, coupled together only by past values.

# Example 2.1: t=0.0001

Need: $i_1(-0.00045),\ v_1(-0.00045),\ i_2(-0.00045),$

$v_2(-0.00045),\ i_2(0),\ v_3(0),\ v_s(0.0001)$

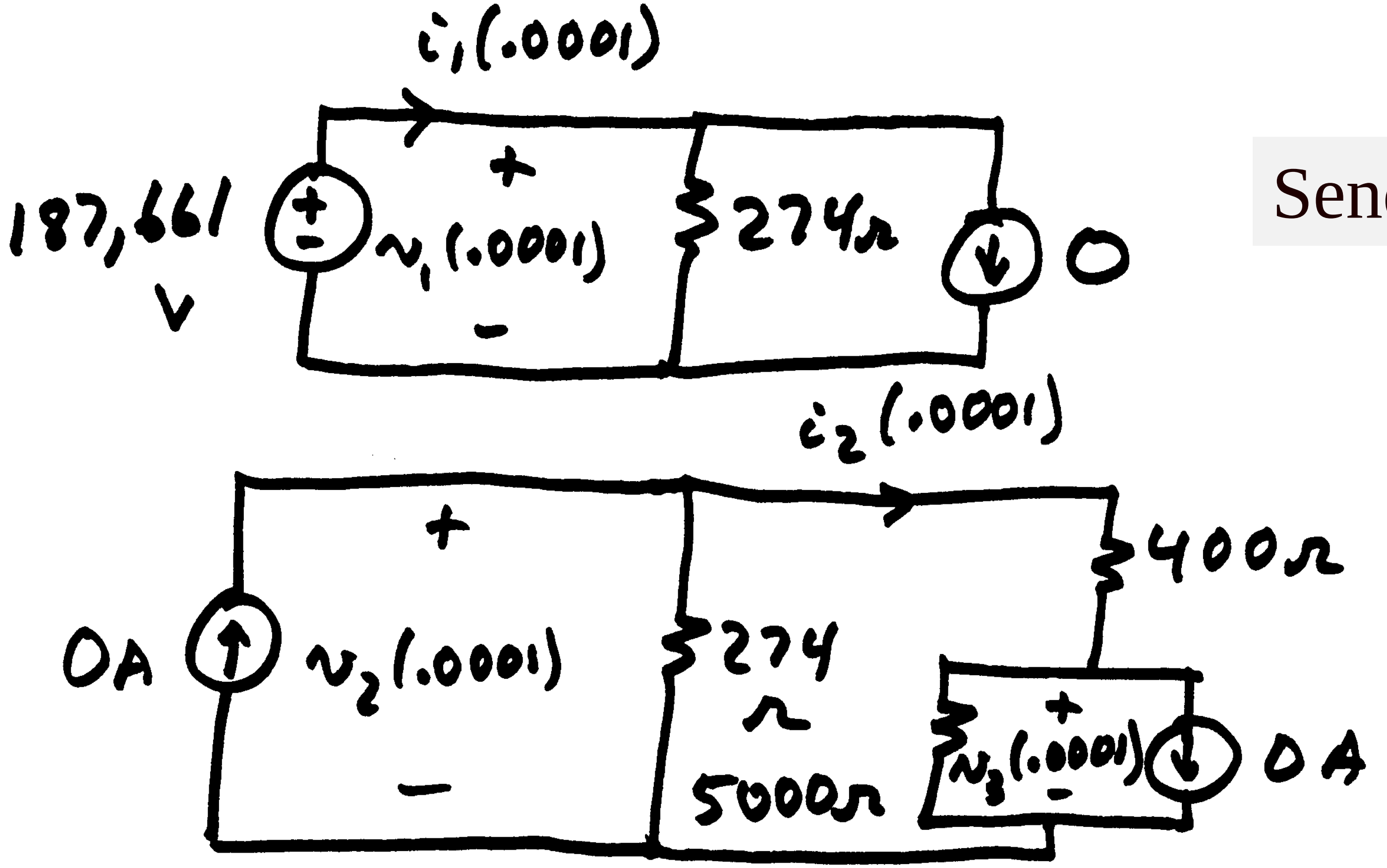
$$i_1(-0.00045)=0 \quad i_2(0)=0$$

$$v_1(-0.00045)=0 \quad v_3(0)=0$$

$$i_2(-0.00045)=0 \quad v_2(-0.00045)=0$$

$$v_s(0.0001)=230,000\sqrt{\frac{2}{3}}\cos(2\pi 60\times 0.0001)=187,661\,V$$

# Example 2.1: t=0.0001




Sending End

Receiving End

# Example 2.1: t=0.0001

$$i_1(0.0001) = 685A$$

$$v_1(0.0001) = 187,661V$$

$$i_2(0.0001) = 0$$

$$v_2(0.0001) = 0$$

$$v_3(0.0001) = 0$$

→ Instantaneously changed from zero at $t$ = 0.0001 sec.

# Example 2.1: t=0.0002

Need:

$$i_1(-0.00035) = 0$$

$$v_1(-0.00035) = 0$$

$$i_2(-0.00035) = 0$$

$$v_2(-0.00035) = 0$$

$$i_2(0.0001) = 0$$

$$v_3(0.0001) = 0$$

$$v_s(0.0002) = 187,261V$$

$$i_1(0.0002) = 683A$$

$$v_1(0.0002) = 187,261V$$

$$i_2(0.0002) = 0.$$

$$v_2(0.0002) = 0.$$

$$v_3(0.0002) = 0.$$

Circuit is essentially the same

Wave is traveling down the line

# Example 2.1: t=0.0002 to 0.0006

$$\frac{d}{v_p} = 0.00055 \qquad \Delta t = 0.0001$$

$$t_i = 0$$

$$t_i = 0.0001$$

$$= 0.0002$$

$$= 0.0003$$

$$= 0.0004$$

$$= 0.0005$$

$$= 0.0006$$

$$t = 0.0001 \leftarrow$$ switch closed

$$t = 0.0002$$

$$= 0.0003$$

$$= 0.0004$$

$$= 0.0005$$

$$= 0.0006 \leftarrow$$

$$= 0.0007 \leftarrow$$

With interpolation receiving end will see wave

# Example 2.1: t=0.0007

Need: $i_1(.00015)$

$v_1(.00015),\ v_2(.00015)$

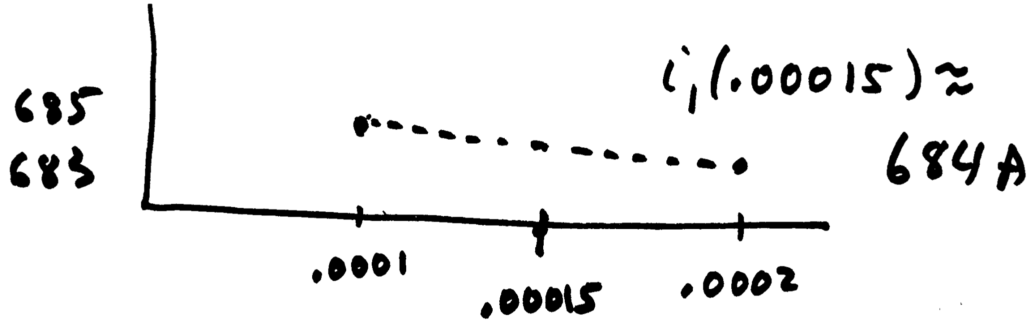
$i_2(.0006),\ v_3(.0006),\ v_s(.0007)$

$$i_1(.0001) = 685A$$

$$i_1(.0002) = 683A$$

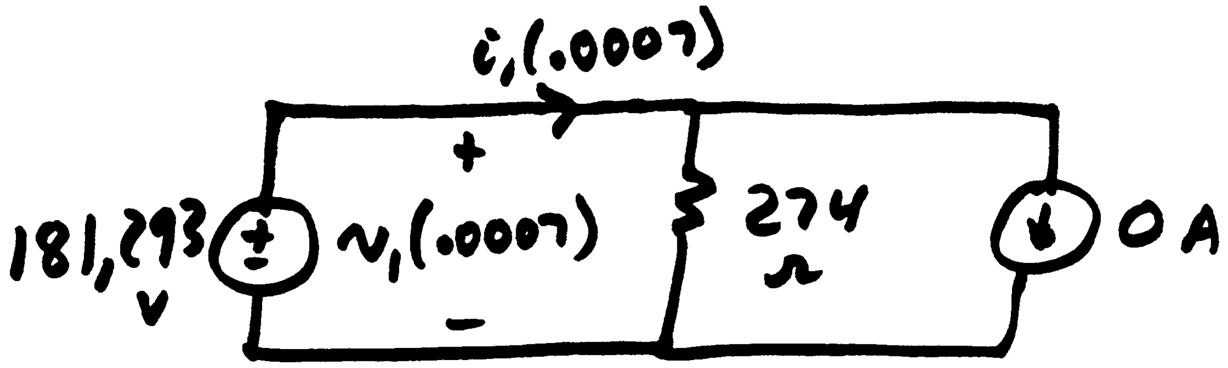
(linear interpolation)

$$i_1(.00015) \approx i_1(.0001) + \frac{.00015 - .0001}{.0002 - .0001} \times \left(i_1(.0002) - i_1(.0001)\right)$$



# Example 2.1: t=0.0007

For $t_i$ = .0006 ($t$ = .0007 sec) at the sending end



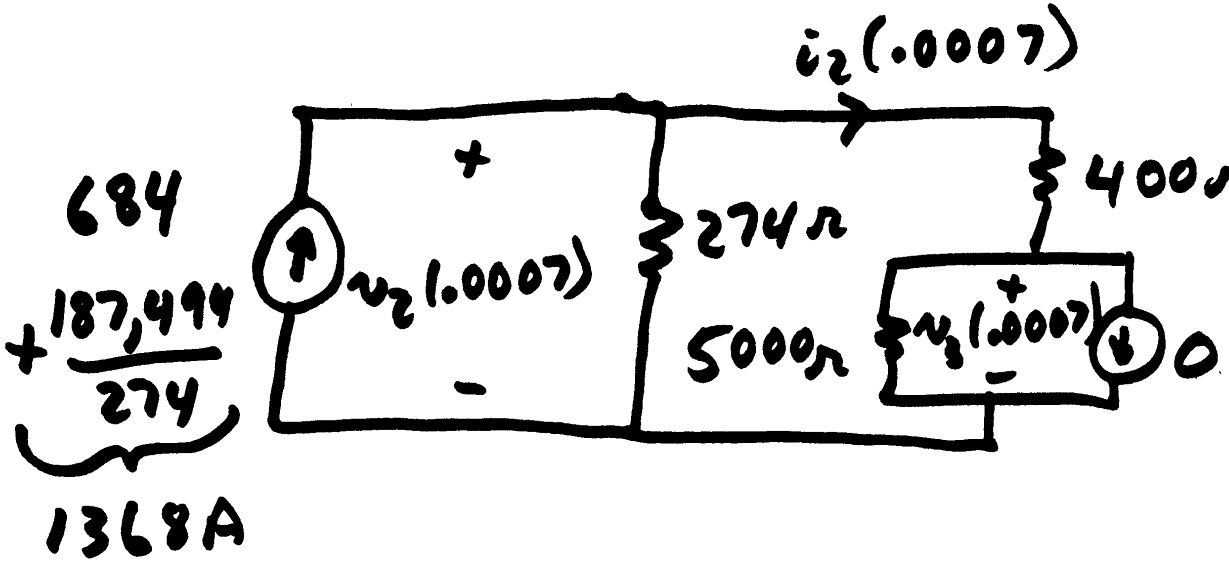

This current source will stay zero until we get a response from the receiving end, at about 2τ seconds

$$i_1(.0007) = 662A$$

$$v_1(.0007) = 181{,}293V$$

# Example 2.1: t=0.0007

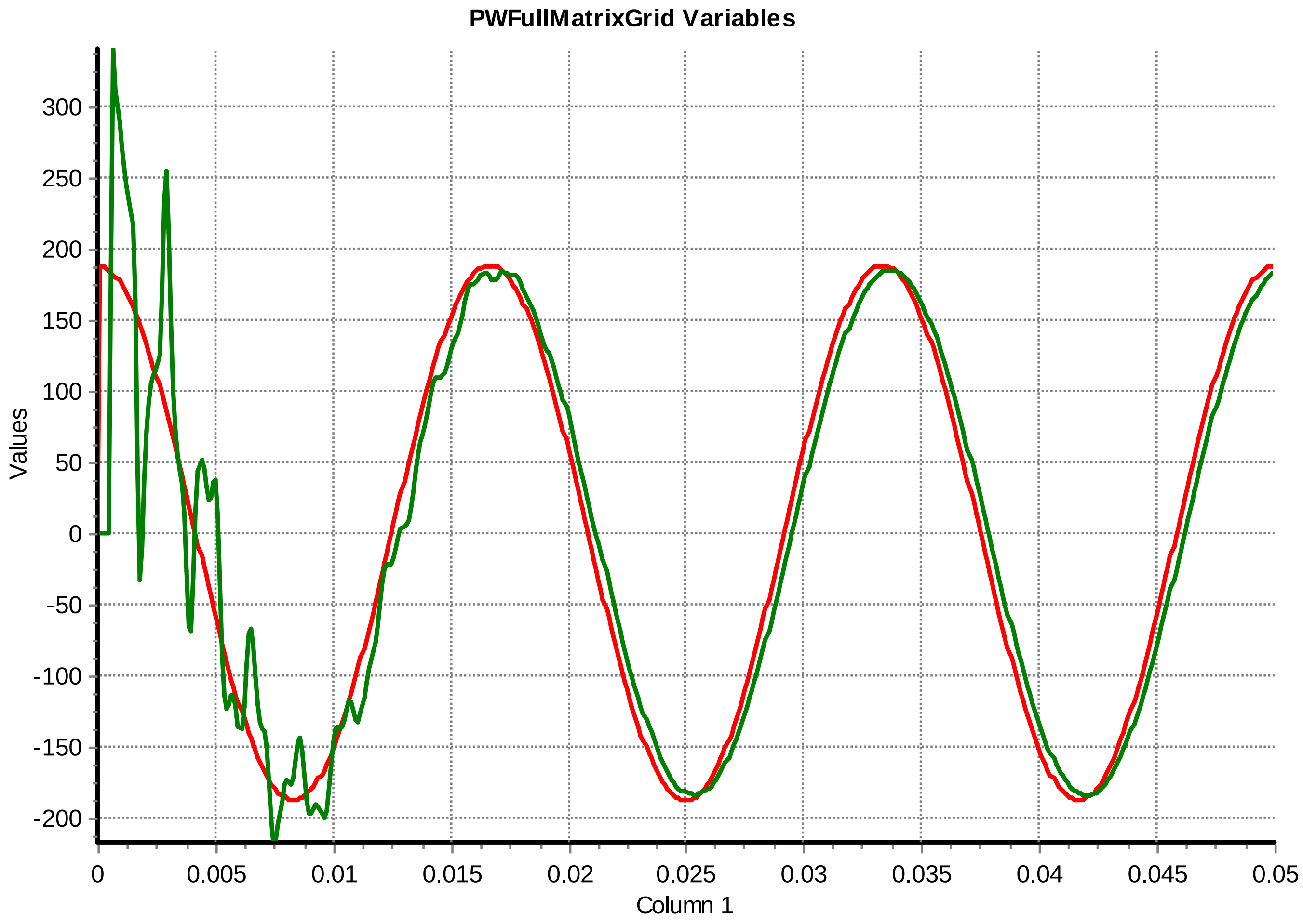
For $t_i$ = .0006 ($t$ = .0007 sec) at the receiving end




$$v_2(.0007) = 356{,}731V$$

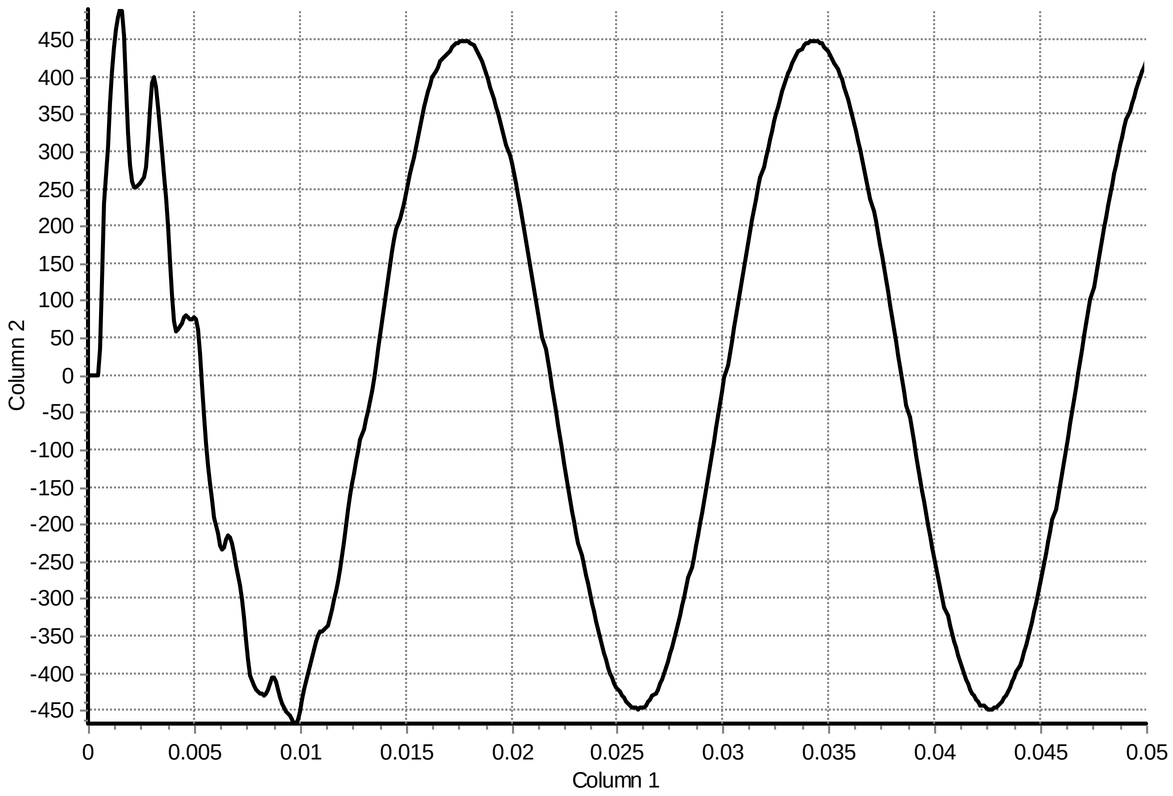
$$i_2(.0007) = 66A$$

# Example 2.1: First Three Cycles




Red is the sending end voltage (in kv), while green is the receiving end voltage. Note the approximate voltage doubling at the receiving end.

# Example 2.1: First Three Cycles

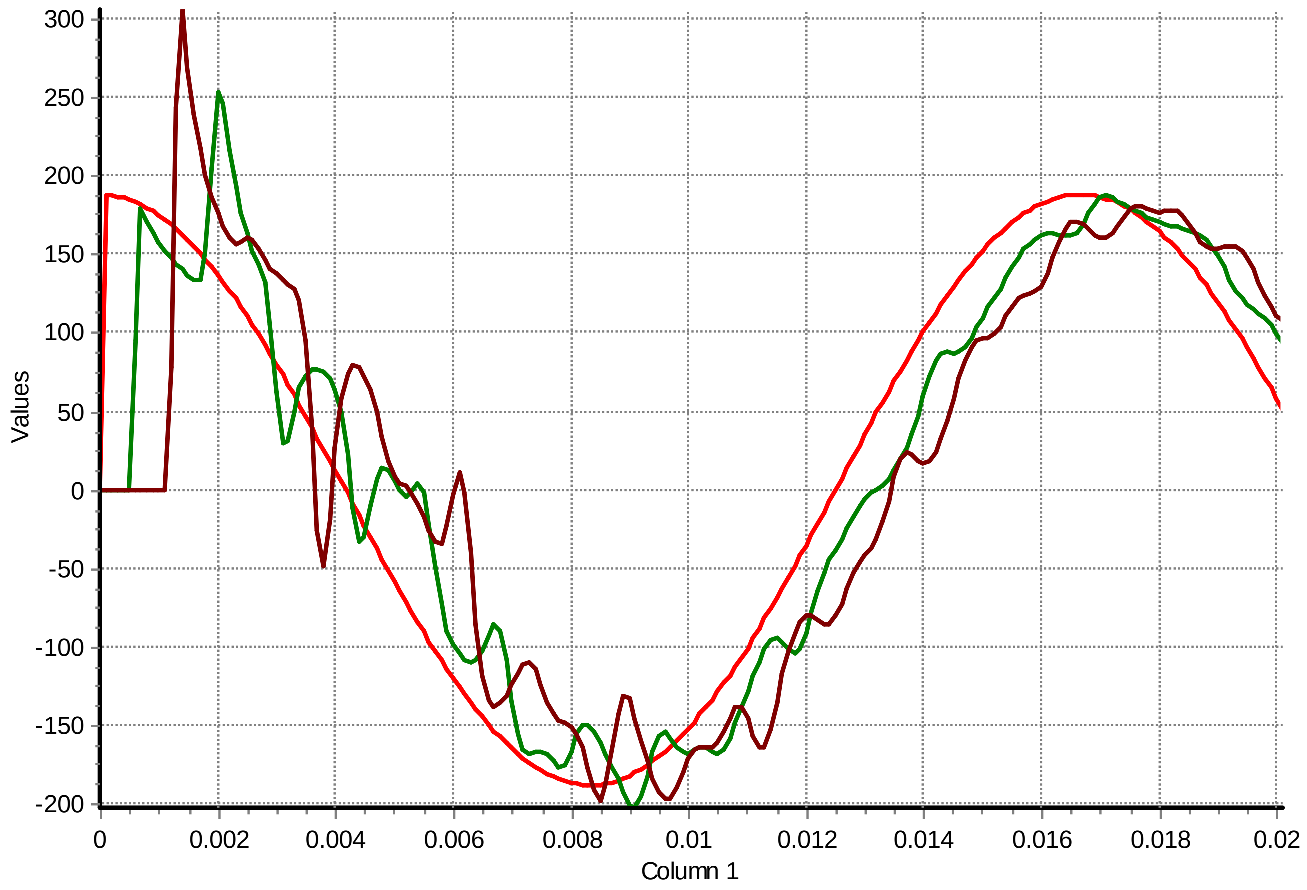



Graph shows the current (in amps) into the RL load over the first three cycles.

To get a **ballpark** value on the expected current, solve the simple circuit assuming the transmission line is just an inductor

$$I_{load,rms} = \frac{230{,}000/\sqrt{3}}{400 + j94.2 + j56.5} = 311\angle -20.6°,$$ hence a peak value of 439 amps

# Three Node, Two Line Example

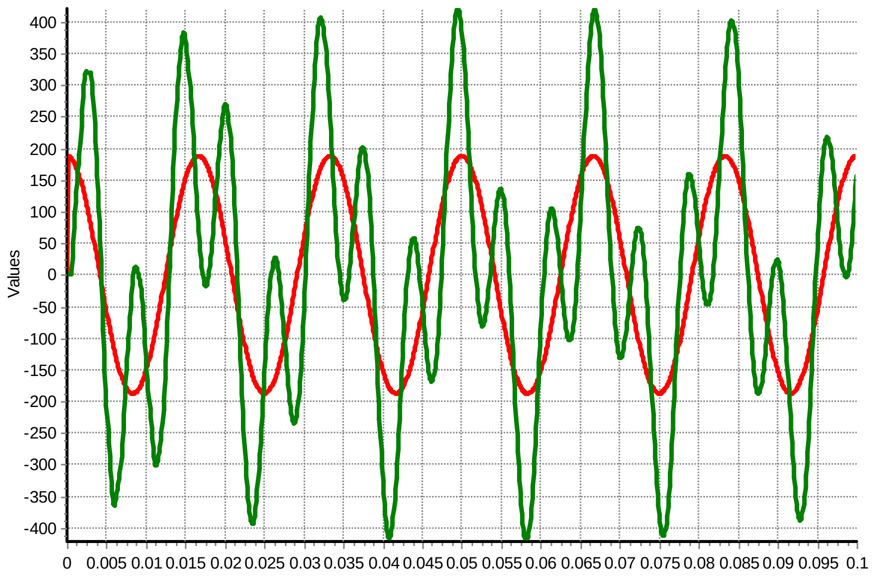


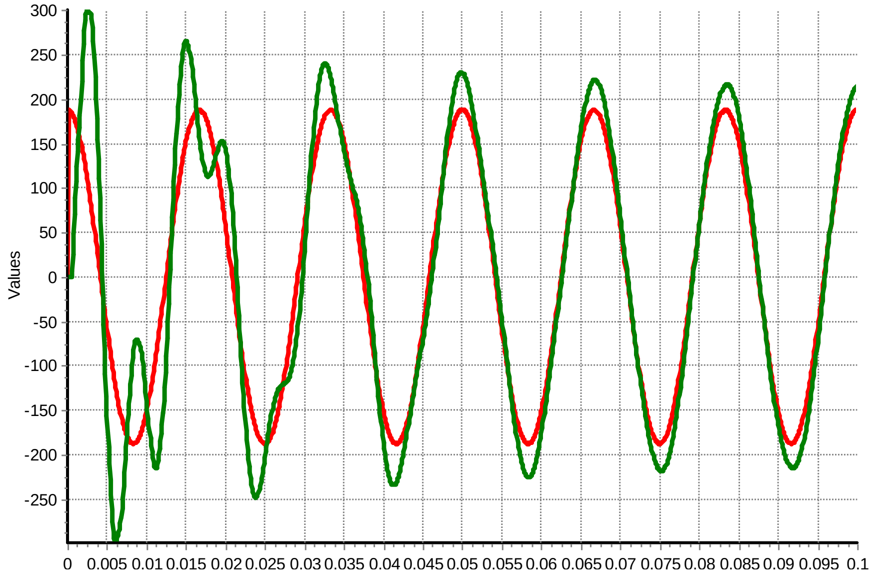

Graph shows the voltages for a total of 0.02 seconds for the Example 2.1 case extended to connect another 120 mile line to the receiving end with an identical load.

Note that there is no longer an initial overshoot for the receiving (green) end since wave continues into the second line

# Example 2.1 with Capacitance

- Below graph shows example 2.1 except the RL load is replaced by a 5 μF capacitor (about 100 Mvar)
- Graph on left is unrealistic case of no resistance in line
  - Since there is no resistance, there is no damp (dissipation)
- Graph on right has R=0.1 Ω/mile



# EMTP Network Solution

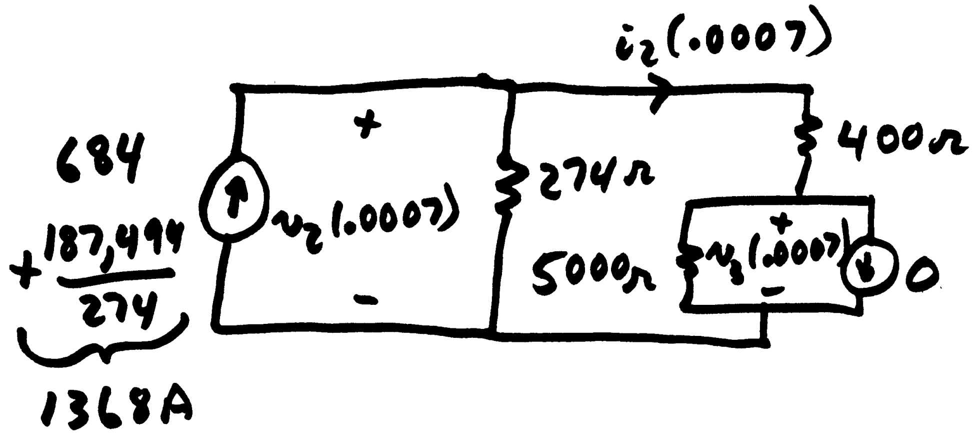
- The EMTP network is represented in a manner quite similar to what is done in the dc power flow or the transient stability network power balance equations or geomagnetic disturbance modeling (GMD)
- Solving set of dc equations for the nodal voltage vector **V** with

  $$\mathbf{V} = \mathbf{G}^{-1}\mathbf{I}$$

  where **G** is the bus conductance matrix and **I** is a vector of the Norton current injections

# EMTP Network Solution

- Fixed voltage nodes can be handled in a manner analogous to what is done for the slack bus: just change the equation for node i to $V_i = V_{i,fixed}$
- Because of the time delays associated with the transmission line models **G** is often quite sparse, and can often be decoupled
- Once all the nodal voltages are determined, the internal device currents can be set
  – E.g., in example 2.1 one we know $v_2$ we can determine $v_3$



# Three-Phase EMTP

- What we just solved was either just for a single-phase system, or for a balanced three-phase system
  - That is, per phase analysis (positive sequence)
- EMTP type studies are often done on either balanced systems operating under unbalanced conditions (i.e., during a fault) or on unbalanced systems operating under unbalanced conditions
  - Lightning strike studies
- In this introduction to EMTP will just covered the balanced system case (but with unbalanced conditions)
  - Solved with symmetrical components

# Modeling Transmission Lines

- Undergraduate power classes usually derive a per phase model for a uniformly transposed transmission line

$$L = \frac{\mu_0}{2\pi}\ln\frac{D_m}{R_b} = 2\times10^{-7}\ln\frac{D_m}{R_b} \quad \text{H/m}$$

$$C = \frac{2\pi\varepsilon}{\ln {D_m}/{R_b^c}}$$

$$\text{D}_\text{m} = \left[d_{ab}d_{ac}d_{bc}\right]^{1/3} \qquad R_b = (\text{r}'d_{12}\cdots d_{1n})^{1/n}$$

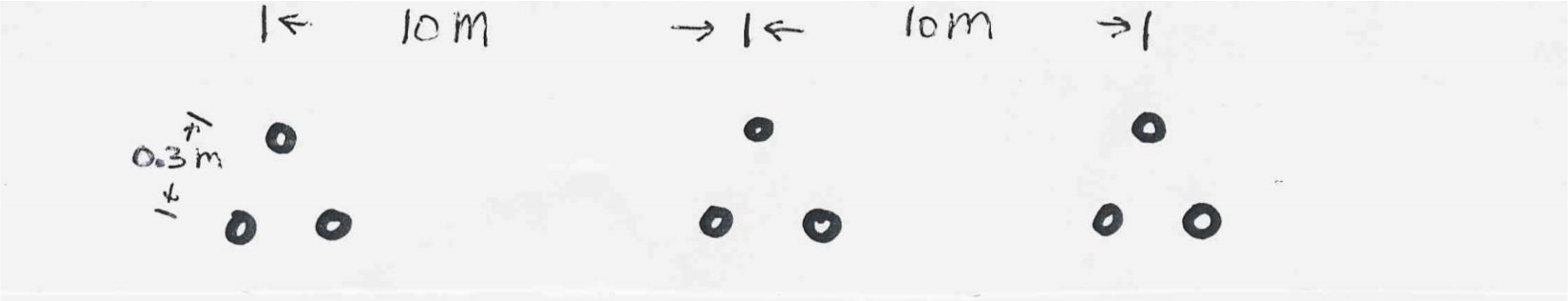
$$\text{R}_\text{b}^\text{c} = (rd_{12}\cdots d_{1n})^{1/n} \quad \text{(note r NOT r')}$$

$$\varepsilon \text{ in air} = \varepsilon_\text{o} = 8.854\times10^{-12} \text{ F/m}$$

A good reference is the *Distribution System Modeling and Analysis*, by Kersting

# Modeling Transmission Lines

- Resistance is just the Ω per unit length times the length
- Calculate the per phase inductance and capacitance per km of a balanced 3ϕ, 60 Hz, line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3 m.  Assume the line is uniformly transposed and the conductors have a 1.5 cm radius and resistance = 0.06 Ω/km



# Modeling Transmission Lines

$$D_m = (10\times10\times20)^{1/3} = 12.6\text{m}$$

$$R_b = (0.78\times0.015\times0.3\times0.3)^{1/3} = 0.102\text{m}$$

$$L = 2\times10^{-7}\ln\frac{12.6}{0.102} = 9.63\times10^{-7}\text{H/m} = 9.63\times10^{-4}\text{H/km}$$

$$R_b^c = (0.015\times0.3\times0.3)^{1/3} = 0.1105\text{m}$$

$$C = \frac{2\pi\times8.854\times10^{-12}}{\ln 12.6/0{,}1105} = 1.17\times10^{-11}\text{F/m} = 1.17\times10^{-8}\text{F/km}$$

- Resistance is 0.06/3=0.02Ω/km
  - Divide by three because three conductors per bundle

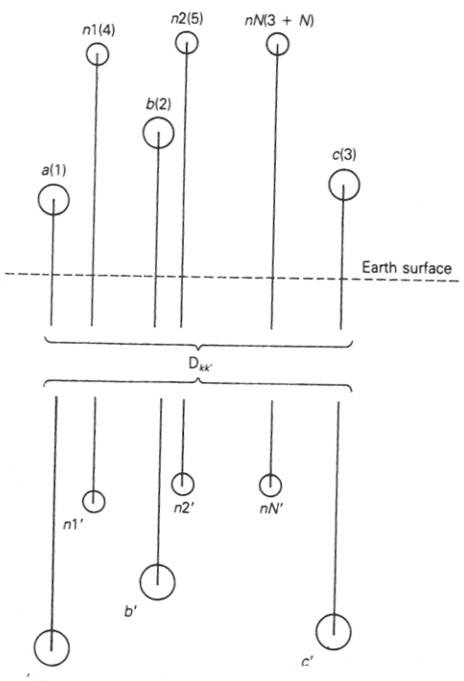
# Untransposed Lines with Ground Conductors

- To model untransposed lines, perhaps with grounded neutral wires, we use the approach of Carson (from 1926) of modeling the earth return with equivalent conductors located in the ground under the real wires
  - Earth return conductors have the same GMR of their above ground conductor (or bundle) and carry the opposite current
- Distance between conductors is

$$D_{kk'} = 658.5\sqrt{\rho / f} \text{ m}$$

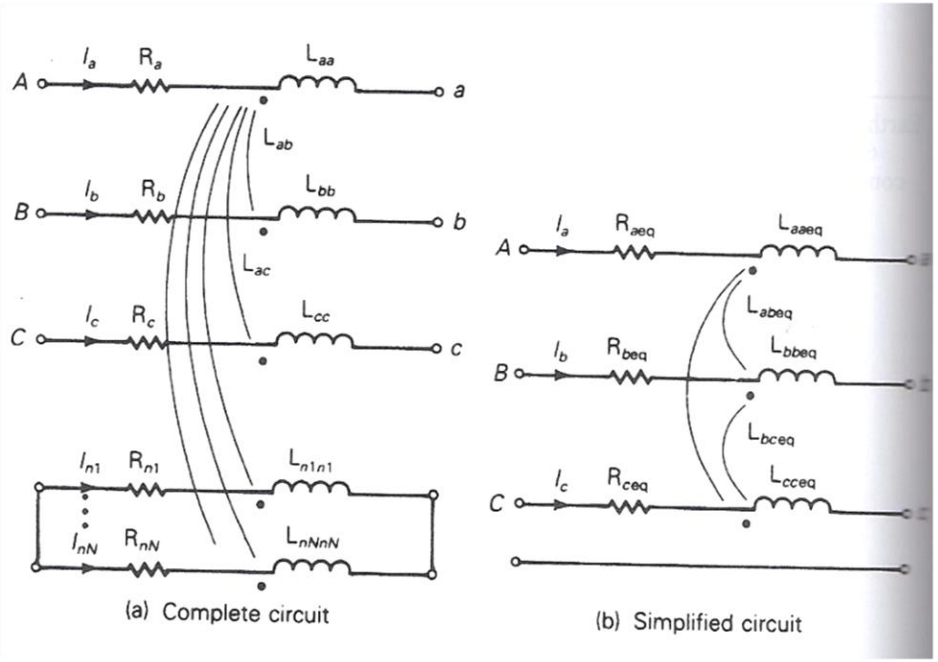
where $\rho$ is the earth resistivity in Ω-m with 100 Ω-m a typical value

Note this depends on frequency!



# Untransposed Lines with Ground Conductors

- The resistance of the equivalent conductors is $R_{k'}=9.869\times10^{-7}\times f$ Ω/m with f the frequency, which is also added in series to the R of the actual conductors
- Conductors are mutually coupled; we'll be assuming three phase conductors and N grounded neutral wires
- Total current in all conductors sums to zero



# Untransposed Lines with Ground Conductors

- The relationships between voltages and currents per unit length is

$$\begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \left(\mathbf{R} + j\omega\mathbf{L}\right) \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_{n1} \\ \vdots \\ InN \end{bmatrix}$$

- Where the diagonal resistance are the conductor resistance plus $R_{k'}$ and the off-diagonals are all $R_{k'}$
- The inductances are with $D_{kk}$ just the GMR for the conductor (or bundle)

$$L_{km} = 2 \times 10^{-7} \ln\left(\frac{D_{km'}}{D_{km}}\right)$$

$D_{kk'}$ is large so $D_{km'} \approx D_{kk'}$

# Untransposed Lines with Ground Conductors

- This then gives an equation of the form

$$
\begin{bmatrix} E_{Aa} \\ E_{Bb} \\ E_{Cc} \\ \hdashline 0 \\ \cdots \\ 0 \end{bmatrix}
=
\left[\begin{array}{ccc:ccc}
\multicolumn{3}{c:}{\overbrace{\hphantom{Z_{(3+N)1} \quad Z_{(3+N)2} \quad Z_{(3+N)3}}}^{Z_A}} & \multicolumn{3}{c}{\overbrace{\hphantom{Z_{(3+N)4} \quad \cdots \quad Z_{(3+N)(3+N)}}}^{Z_B}} \\
Z_{11} & Z_{12} & Z_{13} & Z_{14} & \cdots & Z_{1(3+N)} \\
Z_{21} & Z_{22} & Z_{23} & Z_{24} & \cdots & Z_{2(3+N)} \\
Z_{31} & Z_{32} & Z_{33} & Z_{34} & \cdots & Z_{3(3+N)} \\
\hdashline
Z_{41} & Z_{42} & Z_{43} & Z_{44} & \cdots & Z_{4(3+N)} \\
 & & & & & \\
Z_{(3+N)1} & Z_{(3+N)2} & Z_{(3+N)3} & Z_{(3+N)4} & \cdots & Z_{(3+N)(3+N)} \\
\multicolumn{3}{c:}{\underbrace{\hphantom{Z_{(3+N)1} \quad Z_{(3+N)2} \quad Z_{(3+N)3}}}_{Z_C}} & \multicolumn{3}{c}{\underbrace{\hphantom{Z_{(3+N)4} \quad \cdots \quad Z_{(3+N)(3+N)}}}_{Z_D}}
\end{array}\right]
\begin{bmatrix} I_a \\ I_b \\ I_c \\ \hdashline I_{n1} \\ I_{nN} \\ \vdots \end{bmatrix}
$$

- Which can be reduced to just the phase values

$$\mathbf{E}_p = \left[ \mathbf{Z}_A - \mathbf{Z}_B \mathbf{Z}_D^{-1} \mathbf{Z}_C \right] \mathbf{I}_p = \mathbf{Z}_p \mathbf{I}_p$$

- We'll use $\mathbf{Z}_p$ with symmetrical components

# Example (from 4.1 in Kersting Book)

- Given a 60 Hz overhead distribution line with the tower configuration (N=1 neutral wire) with the phases using Linnet conductors and the neutral 4/0 6/1 ACSR, determine $\mathbf{Z}_p$ in ohms per mile

  - Linnet has a GMR = 0.0244 ft, and R = 0.306 Ω/mile
  - 4/0 6/1 ACSR has GMR=0.00814 ft and R=0.592 Ω/mile
  - $R_{k'}=9.869\times10^{-7}\times f$ Ω/m is 0.0953 Ω/mile at 60 Hz
  - Phase R diagonal values are 0.306 + 0.0953 = 0.401 Ω/mile
  - The neutral R values are 0.592 + 0.0953 = 0.6873 Ω/mile

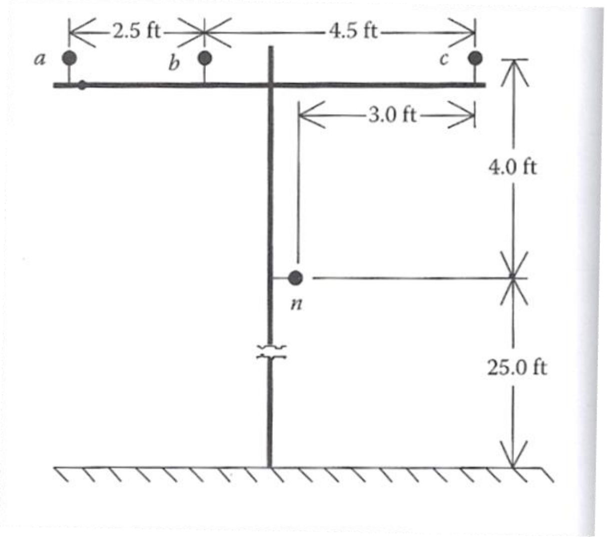



Figure 4.7 from Kersting

# Example (from 4.1 in Kersting Book)

- Example inductances are worked with $\rho$ = 100 $\Omega$-m

$$D_{kk'} = 658.5\sqrt{100/60}\ m = 850.1m = 2789\ \text{ft}$$

$$L_{km} = 2\times 10^{-7} \ln\left(\frac{D_{km'}}{D_{km}}\right) \approx 2\times 10^{-7} \ln\left(\frac{D_{kk'}}{D_{km}}\right)$$

- Note at 2789 ft, $D_{kk'}$ is much, much larger than the distances between the conductors, justifying the above assumption

# Example (from 4.1 in Kersting Book)

- Working some of the inductance values

$$L_{aa} = 2\times 10^{-7}\ln\left(\frac{2789}{0.0244}\right) = 2.329\times 10^{-6}\text{ H/m}$$

- Phases a and b are separated by 2.5 feet, while it is 5.66 feet between phase a and the ground conductor

$$L_{ab} = 2\times 10^{-7}\ln\left(\frac{2789}{2.5}\right) = 1.403\times 10^{-6}\text{ H/m}$$

$$L_{an} = 2\times 10^{-7}\ln\left(\frac{2789}{5.66}\right) = 1.240\times 10^{-6}\text{ H/m}$$

Even though the distances are worked here in feet, the result is in H/m because of the units on $\mu_0$

# Example (from 4.1 in Kersting Book)

- Continue to create the 4 by 4 symmetric **L** matrix
- Then **Z** = **R** + jω**L**

$$\mathbf{Z} = \begin{bmatrix} 0.4013 + j1.4133 & 0.0953 + j0.8515 & 0.0953 + j0.7266 & 0.0953 + j0.7524 \\ 0.0953 + j0.8515 & 0.4013 + j1.4133 & 0.0953 + j0.7802 & 0.0953 + j0.7865 \\ 0.0953 + j0.7266 & 0.0953 + j0.7802 & 0.4013 + j1.4133 & 0.0953 + j0.7674 \\ 0.0953 + j0.7524 & 0.0953 + j0.7865 & 0.0953 + j0.7674 & 0.6873 + j1.5465 \end{bmatrix}$$

- Partition the matrix and solve $\mathbf{Z}_p = \left[\mathbf{Z}_A - \mathbf{Z}_B \mathbf{Z}_D^{-1} \mathbf{Z}_C\right]$
- The result in Ω/mile is

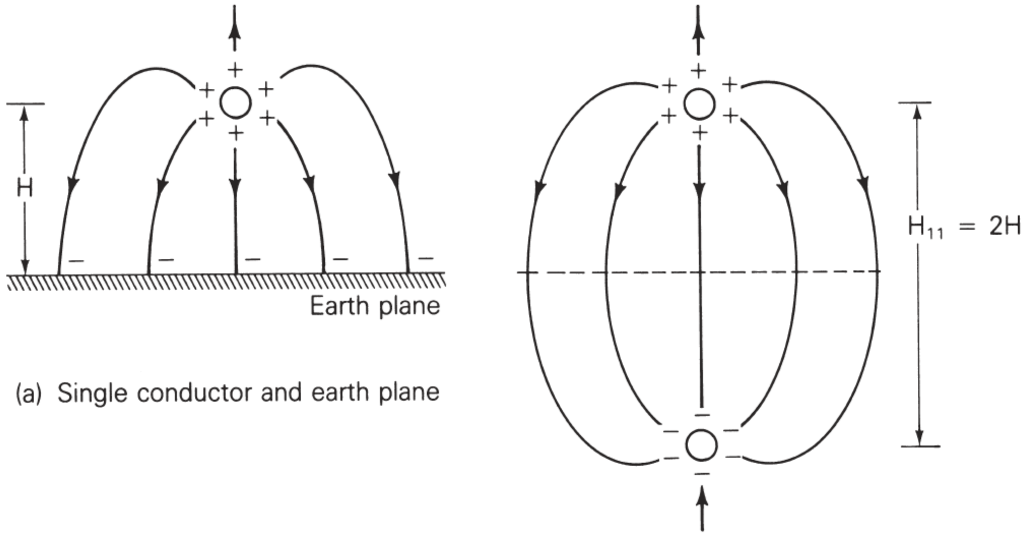
$$\mathbf{Z}_p = \begin{bmatrix} 0.4576 + 1.0780 & 0.1560 + j0.5017 & 0.1535 + j0.3849 \\ 0.1560 + j0.5017 & 0.4666 + j1.0482 & 0.1580 + j0.4236 \\ 0.1535 + j0.3849 & 0.1580 + j0.4236 & 0.4615 + j1.0651 \end{bmatrix}$$

# Modeling Line Capacitance

- For capacitance the earth is typically modeled as a perfectly conducting horizontal plane; then the earth plane is replaced by mirror image conductors
  - If conductor is distance H above ground, mirror image conductor is distance H below ground, hence their distance apart is 2H

**FIGURE 4.23**

Method of images



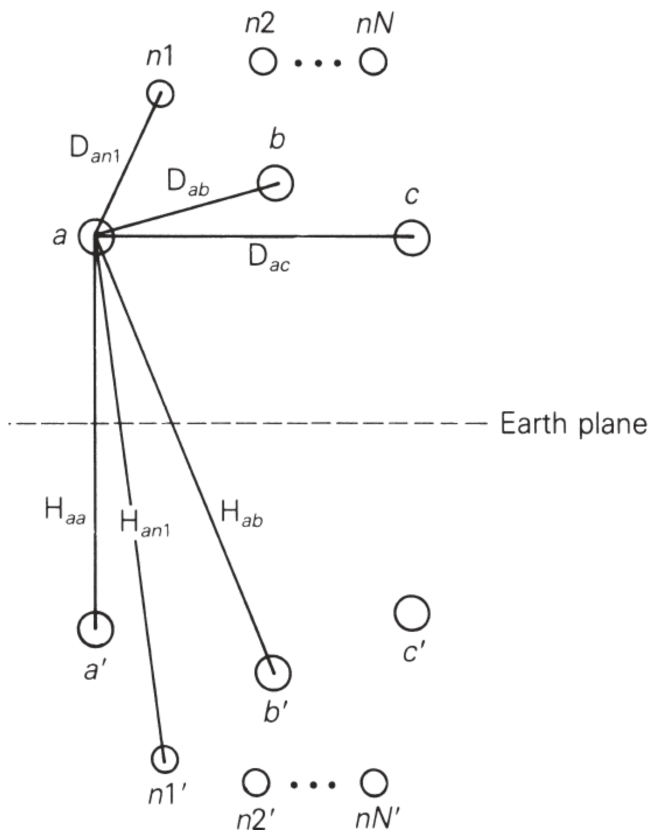

In 667 you don't need to know how to do line capacitance

# Modeling Line Capacitance

- The relationship between the voltage to neutral and charges are then given as

$$V_{kn} = \frac{1}{2\pi\varepsilon}\sum_{m=a}^{nN} q_m \ln\frac{H_{km}}{D_{km}} = \sum_{m=a}^{nN} q_m P_{km}$$

$$P_{km} = \frac{1}{2\pi\varepsilon}\ln\frac{H_{km}}{D_{km}}$$




- P's are called potential coefficients
- Where $D_{km}$ is the distance between the conductors, $H_{km}$ is the distance to a mirror image conductor and $D_{kk} = R_b^c$

# Modeling Line Capacitance

- Then we setup the matrix relationship

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} = \left[\begin{array}{ccc|ccc} \overbrace{\begin{matrix} P_{aa} & P_{ab} & P_{ac} \end{matrix}}^{\mathbf{P}_A} & & & \overbrace{\begin{matrix} P_{an1} & \cdots & P_{anN} \end{matrix}}^{\mathbf{P}_B} & & \\ P_{ba} & P_{bb} & P_{bc} & P_{bn1} & \cdots & P_{bnN} \\ P_{ca} & P_{cb} & P_{cc} & P_{cn1} & \cdots & P_{cnN} \\ \hline P_{n1a} & P_{n1b} & P_{n1c} & P_{n1n1} & \cdots & P_{n1nN} \\ \vdots & & & & & \\ \underbrace{\begin{matrix} P_{nNa} & P_{nNb} & P_{nNc} \end{matrix}}_{\mathbf{P}_C} & & & \underbrace{\begin{matrix} P_{nNn1} & \cdots & P_{nNnN} \end{matrix}}_{\mathbf{P}_D} & & \end{array}\right] \begin{bmatrix} q_a \\ q_b \\ q_c \\ \hline q_{n1} \\ \vdots \\ q_{nN} \end{bmatrix}$$

- And solve

$$\mathbf{V}_p = \left[\mathbf{P}_A - \mathbf{P}_B \mathbf{P}_D^{-1} \mathbf{P}_C\right] \mathbf{Q}_p$$

$$\mathbf{C}_p = \left[\mathbf{P}_A - \mathbf{P}_B \mathbf{P}_D^{-1} \mathbf{P}_C\right]^{-1}$$

# Continuing the Previous Example

- In example 4.1, assume the below conductor radii

  For the phase conductor $R_b^c = 0.0300$ ft

  For the neutral conductor $R_n^c = 0.0235$ ft

- Calculating some values

$$\varepsilon_0 = 8.85\times 10^{-12}\text{ F/m} = 1.424\times 10^{-2}\ \mu\text{F/mile}$$

$$P_{aa} = \frac{1}{2\pi\varepsilon_0}\ln\left(\frac{2\times 29.0}{0.0300}\right) = 11.177\ln\left(\frac{2\times 29.0}{0.0300}\right) = 84.57\text{mile/}\mu\text{F}$$

$$P_{ab} = 11.177\ln\left(\frac{58.05}{2.5}\right) = 35.15\text{mile/}\mu\text{F}$$

$$P_{an} = 11.177\ln\left(\frac{54.148}{5.6569}\right) = 25.25\text{mile/}\mu\text{F}$$

# Continuing the Previous Example

- Solving we get

$$\mathbf{P}_p = \left[\mathbf{P}_A - \mathbf{P}_B \mathbf{P}_D^{-1} \mathbf{P}_C\right] = \begin{bmatrix} 77.12 & 26.79 & 15.84 \\ 26.79 & 75.17 & 19.80 \\ 15.87 & 19.80 & 76.29 \end{bmatrix} \text{ mile/}\mu\text{F}$$

$$\mathbf{C}_p = \left[\mathbf{P}_p\right]^{-1} = \begin{bmatrix} 0.0150 & -0.0049 & -0.0018 \\ -0.0049 & 0.0158 & -0.0030 \\ -0.0018 & -0.0030 & 0.0137 \end{bmatrix} \mu\text{F/mile}$$

# Frequency Dependence

- We might note that the previous derivation for **L** assumed a frequency. For steady-state and transient stability analysis this is just the power grid frequency
- As we have seen in EMTP there are a number of difference frequencies present, particularly during transients
  - Coverage is beyond the scope of this class
  - An early paper is J.K. Snelson, "Propagation of Travelling on Transmission Lines: Frequency Dependent Parameters," *IEEE Trans. Power App. and Syst*., vol. PAS-91, pp. 85-91, 1972

# Power System Overvoltages

- Line switching can cause transient overvoltages
  - Resistors (200 to 800 Ω) are preinserted in EHV circuit breakers to reduce over voltages, and subsequently shorted
- Common overvoltage cause is lightning strikes
  - Lightning strikes themselves are quite fast, with rise times of 1 to 20 μs, with a falloff to ½ current within less than 100 μs
  - Peak current is usually less than 100kA
  - Shield wires above the transmission line greatly reduce the current that gets into the phase conductors
  - EMTP studies can show how these overvoltage propagate down the line

# Insulation Coordination

- Insulation coordination is the process of correlating electric equipment insulation strength with expected overvoltages
- The expected overvoltages are time-varying, with a peak value and a decay characteristic
- Transformers are particularly vulnerable
- Surge arrestors are placed in parallel (phase to ground) to cap the overvoltages
    - They have high impedance during normal voltages, and low impedance during overvoltages; airgap devices have been common, though gapless designs are also used