ECEN 667
Power System Stability

Lecture 7: Stability Overview, Synchronous Machine Modeling

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Announcements

- Read Chapter 3
- Homework 2 is due on Thursday Sept 13.

An Interesting Video of an Uncleared Fault

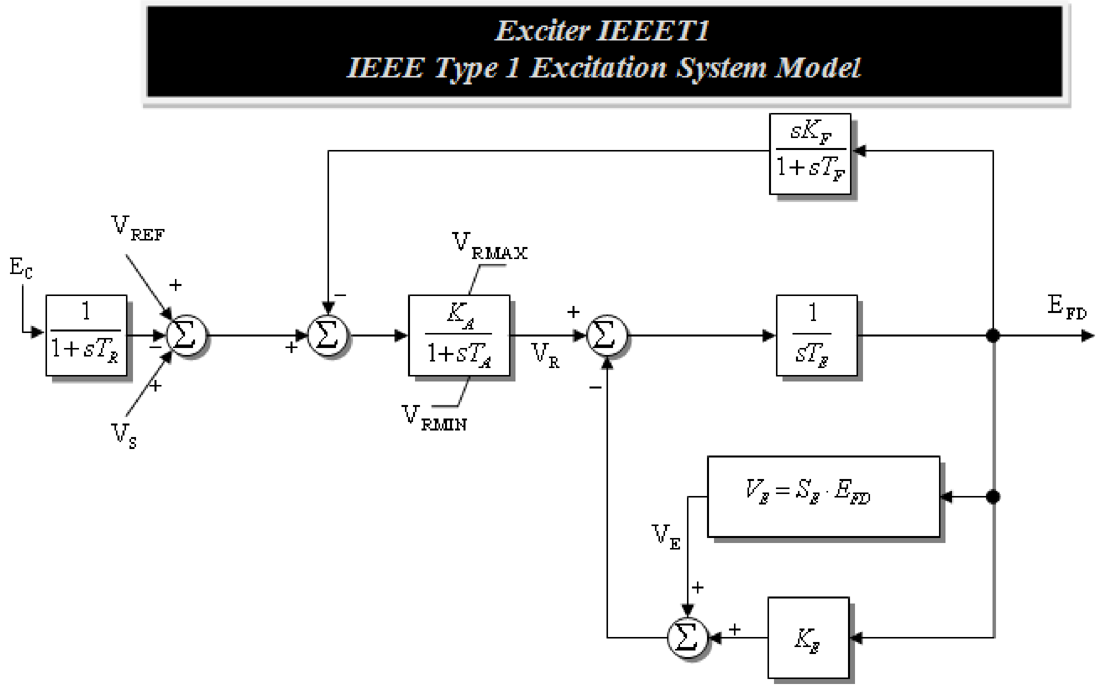
- The below video shows an event from September 2, 2022 in the Netherlands (Flevoland province) in which a transmission line fault was not cleared. Note the sag and smoke (steam?) coming from the line
- https://nos.nl/video/2443017-explosies-en-rokende-kabels-bij-verdeelstation-in-dronten

Adding a Generator Exciter

- The purpose of the generator excitation system (exciter) is to adjust the generator field current to maintain a constant terminal voltage.
- PowerWorld Simulator includes many different types of exciter models. One simple exciter is the IEEET1. To add this exciter to the generator at bus 4 go to the generator dialog, “Stability” tab, “Exciters” page. Click Insert and then select IEEET1 from the list. Use the default values.
- Exciters will be covered in the first part of Chapter 4

IEEET1 Exciter

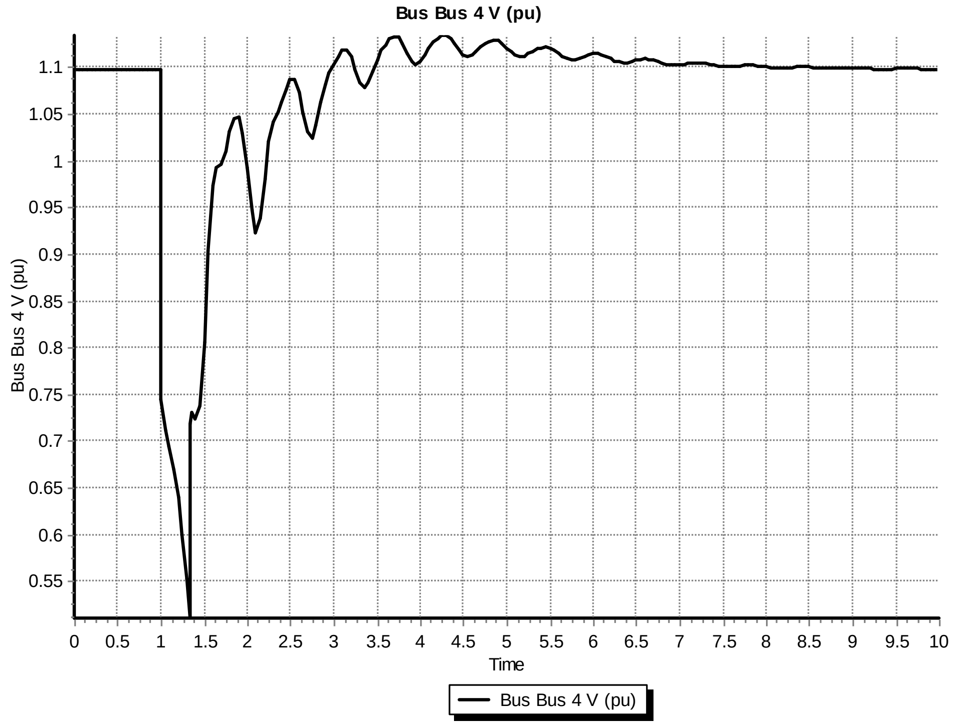
- Once you have inserted the IEEET1 exciter you can view its block diagram by clicking on the "Show Diagram" button. This opens a PDF file in Adobe Reader to the page with that block diagram. The block diagram for this exciter is also shown below.



The input to the exciter, E_c,is usually the terminal voltage. The output, E_{FD}, is the machine field voltage.

Voltage Response with Exciter

- Re-do the run. The terminal time response of the terminal voltage is shown below. Notice that now with the exciter it returns to its pre-fault voltage.



Case Name: **Example_13_4_GenROU_IEEET1**

Defining Plots

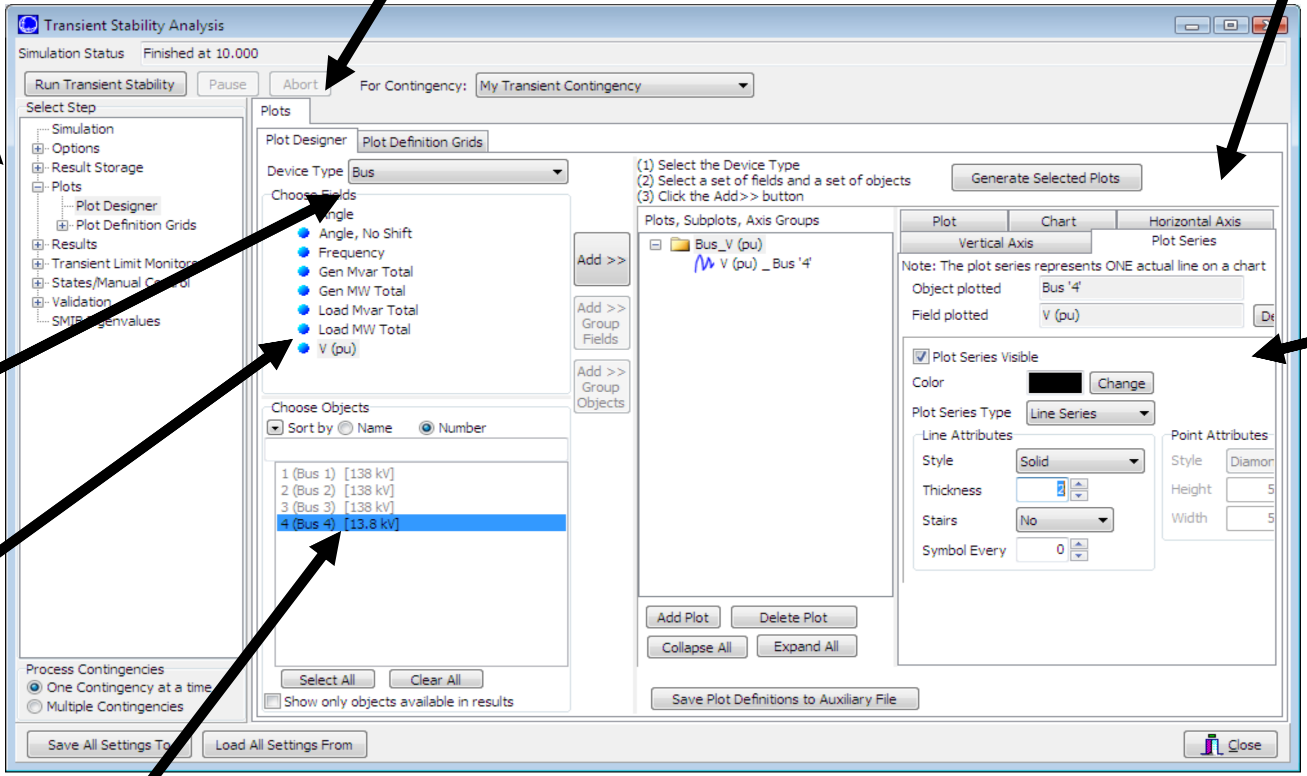
- Because time plots are commonly used to show transient stability results, PowerWorld Simulator makes it easy to define commonly used plots.
  - Plot definitions are saved with the case, and can be set to automatically display at the end of a transient stability run.
- To define some plots on the **Transient Stability Analysis** form select the **Plots** page. Initially we'll setup a plot to show the bus voltage.
  - Use the Plot Designer to choose a Device Type (Bus), Field, (Vpu), and an Object (Bus 4). Then click the **Add** button. Next click on the Plot Series tab (far right) to customize the plot's appearance; set Color to black and Thickness to 2.

Defining Plots

Plots Page

Plot Designer tab

Plot Series tab



Customize the plot line

Device Type

Field

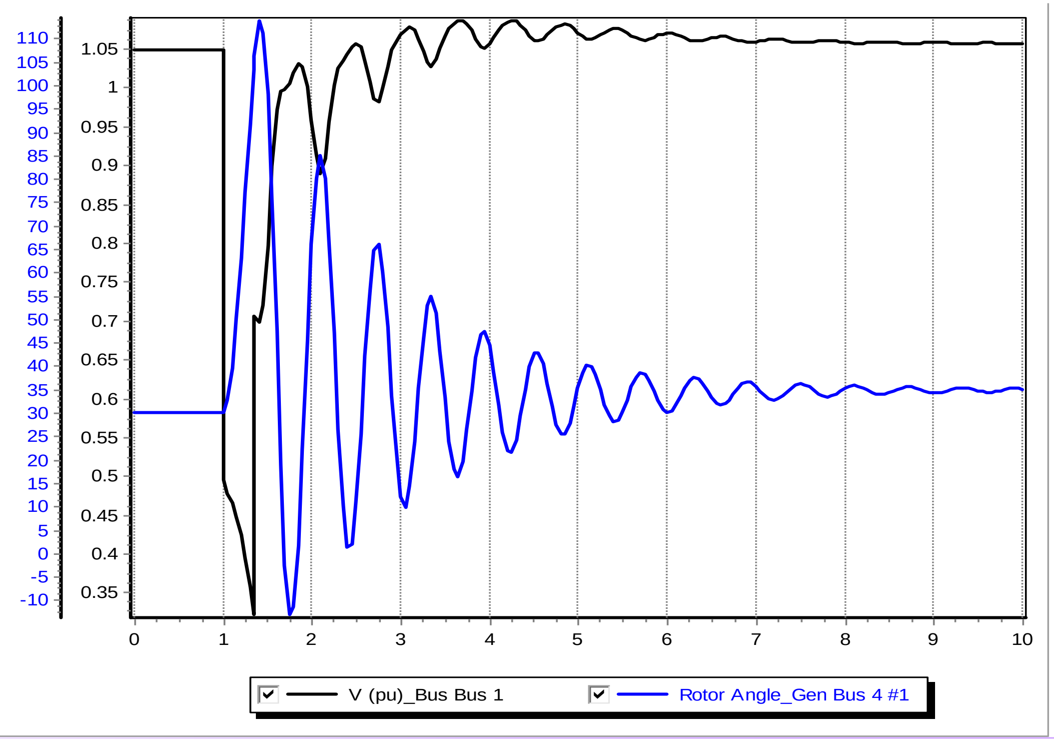
Object; note multiple objects and/or fields can be simultaneously selected.

Adding Multiple Axes

- Once the plot is designed, save the case and rerun the simulation. The plot should now automatically appear.
- In order to compare the time behavior of various fields an important feature is the ability to show different values using different y-axes on the same plot.
- To add a new Vertical Axis to the plot, close the plot, go back to the **Plots** page, select the Vertical Axis tab (immediately to the left of the Plot Series tab). Then click **Add Axis Group**. Next, change the Device Type to Generator, the Field to Rotor Angle, and choose the Bus 4 generator as the Object. Click the **Add** button. Customize as desired. There are now two axis groups.

A Two Axes Plot

- The resultant plot is shown below. To copy the plot to the windows clipboard, or to save the plot, right click towards the bottom of the plot. You can re-do the plot without re-running the simulation by clicking on "Generate Selected Plots" button.



Many plot options are available

This case is saved as **Example_13_4_WithPlot**

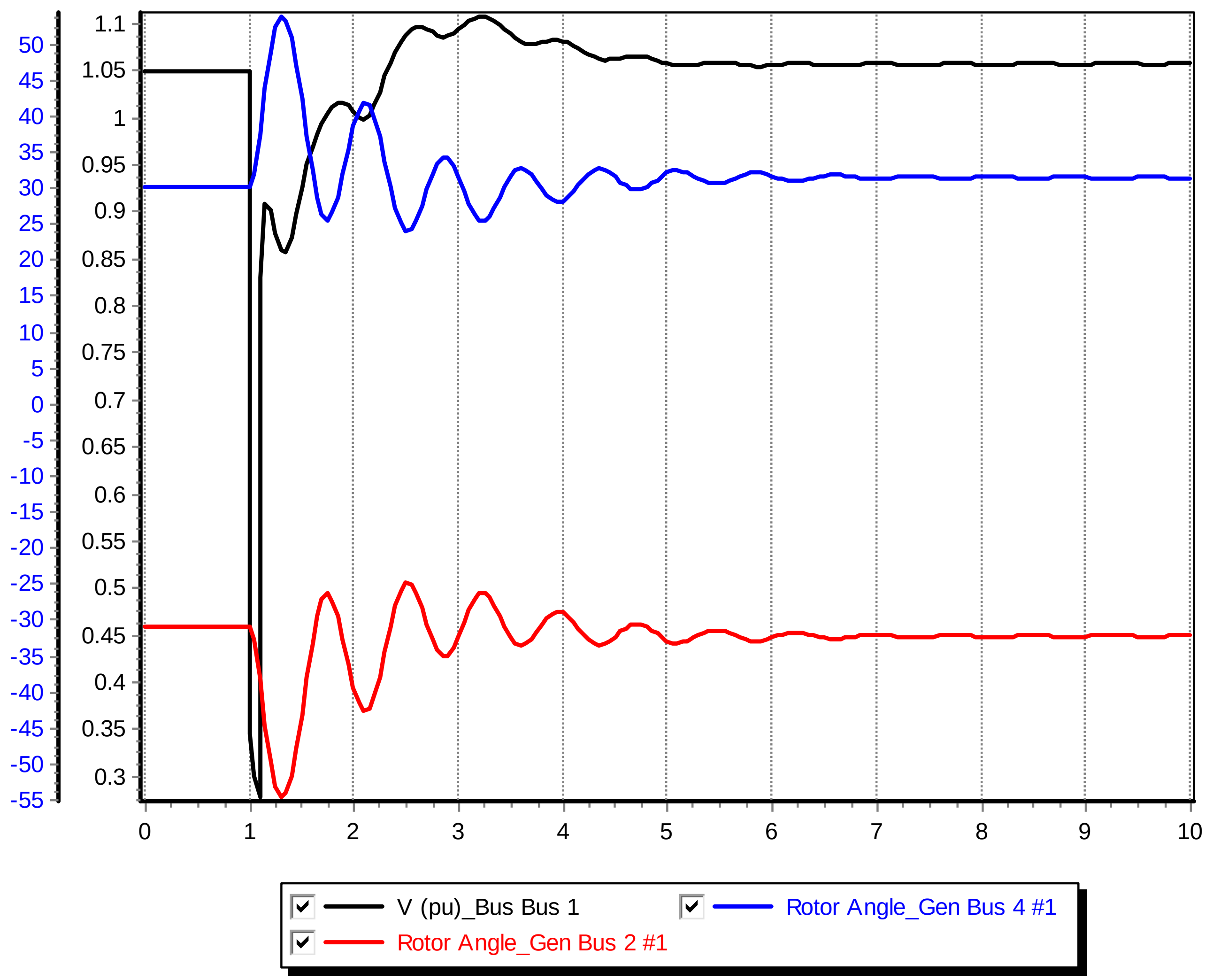
Setting the Angle Reference

- Infinite buses do not exist, and should not usually be used except for small, academic cases.
  - An infinite bus has a fixed frequency (e.g. 60 Hz), providing a convenient reference frame for the display of bus angles.
- Without an infinite bus the overall system frequency is allowed to deviate from the base frequency
  - With a varying frequency we need to define a reference frame
  - PowerWorld Simulator provides several reference frames with the default being average of bus frequency.
  - Go to the **Options**, **Power System Model** page. Change Infinite Bus Model to **No Infinite Buses**; Under **Options, Result Options**, set the **Angle Reference** to **Average of Generator Angles**.

Setting Models for the Bus 2 Gen

- Without an infinite bus we need to set up models for the generator at bus 2. Use the same procedure of adding a GENROU machine and an IEEET1 exciter.
  - Accept all the defaults, except set the H field for the GENROU model to 30 to simulate a large machine.
  - Go to the Plot Designer, click on PlotVertAxisGroup2 and use the "Add" button to show the rotor angle for Generator 2. Note that the object may be grayed out but you can still add it to the plot.
  - Without an infinite bus the case is no longer stable with a 0.34 second fault; on the main Simulation page change the event time for the opening on the lines to be 1.10 seconds (you can directly overwrite the seconds field on the display).
  - Case is saved as **Example_13_4_NoInfiniteBus**

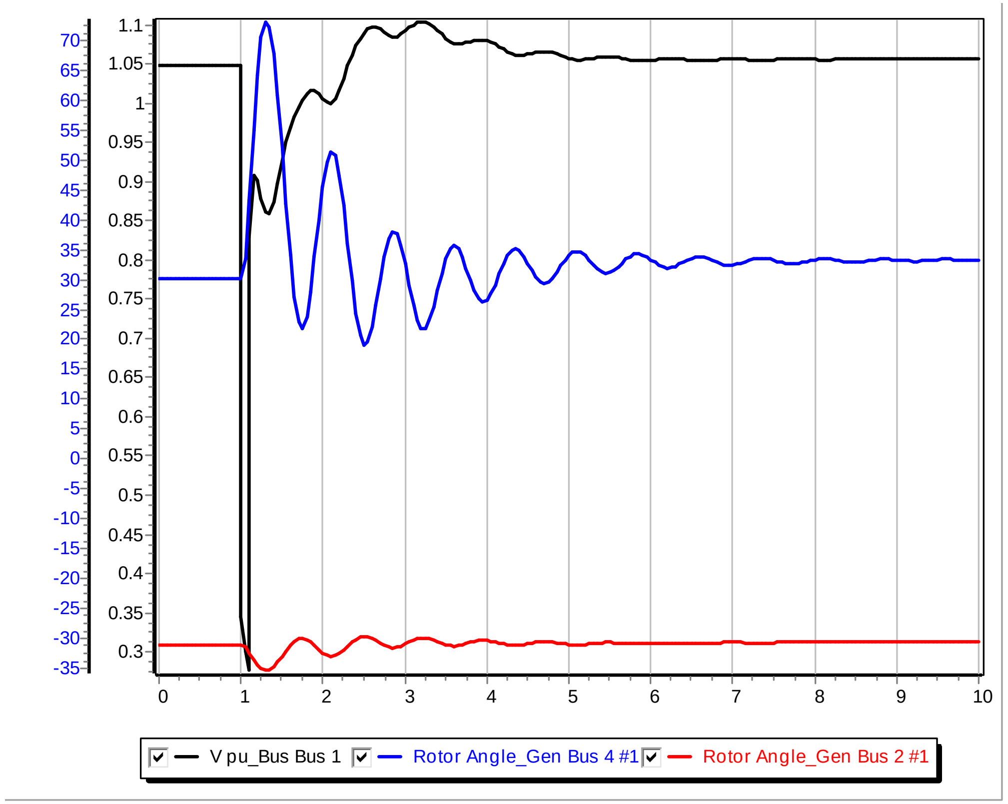
No Infinite Bus Case Results



Plot shows the rotor angles for the generators at buses 2 and 4, along with the voltage at bus 1. Notice the two generators are swinging against each other.

Impact of Angle Reference on Results

- To see the impact of the reference frame on the angles results, go to the “Options”, “Power System Model” page. Under “Options, Result Options”, set the Angle Reference to “Synchronous Reference Frame.”



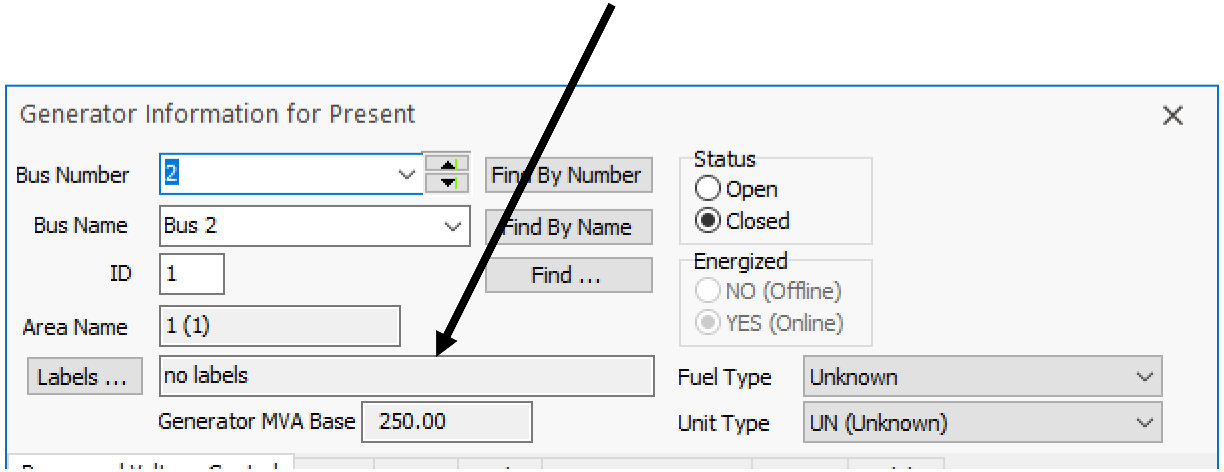
This shows the more expected results, but it is not “more correct.” Both are equally correct

WSCC Nine Bus, Three Machine Case

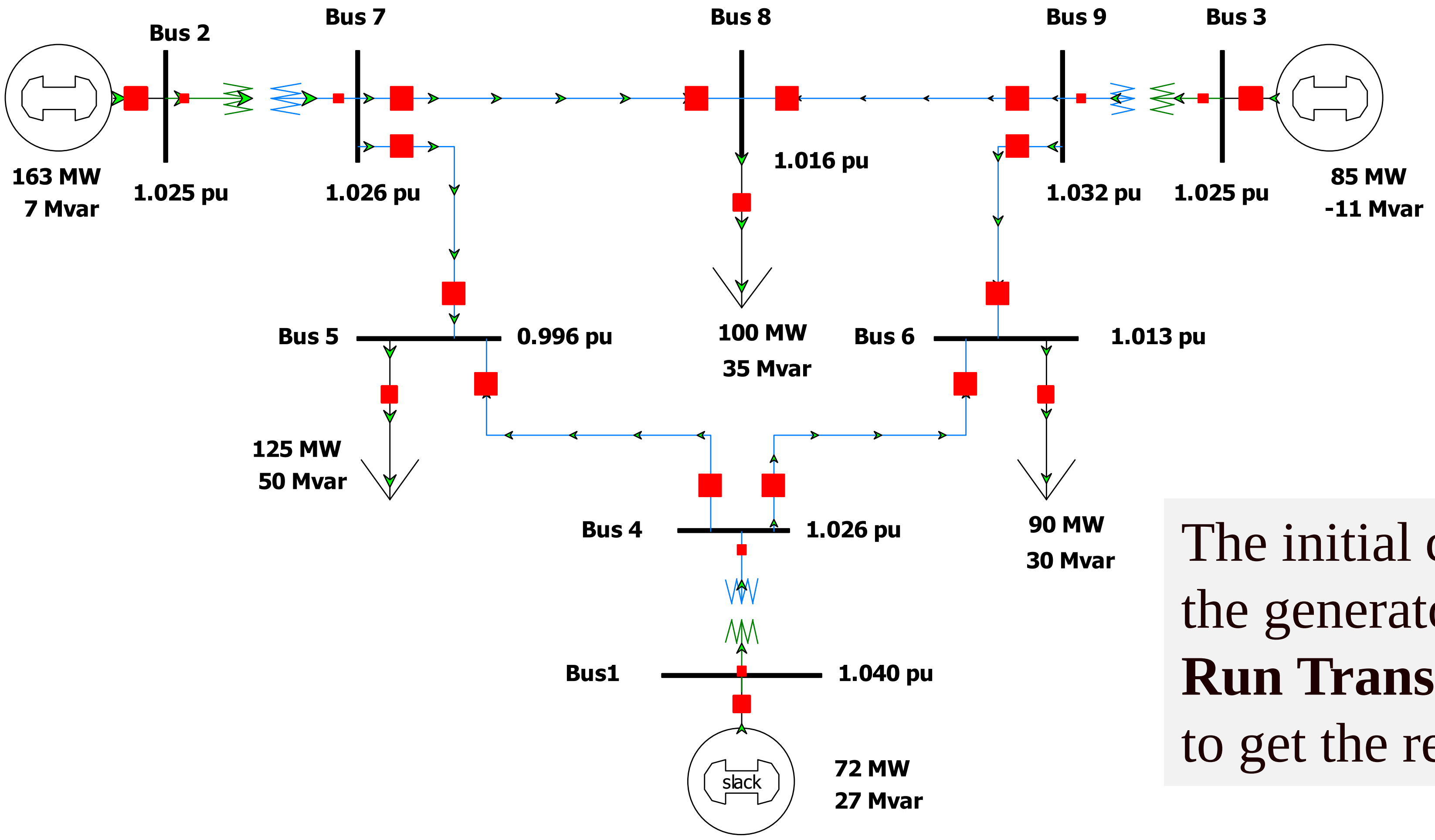
- As a next step in complexity we consider the WSCC (now WECC) nine bus case, three machine case.
  - This case is described in several locations including EPRI Report EL-484 (1977), the Anderson/Fouad book (1977). Here we use the case as presented as Example 7.1 in the Sauer/Pai text except the generators are modeled using the subtransient GENROU model, and data is in per unit on generator MVA base (see next slide).
  - The Sauer/Pai book contains a derivation of the system models, and a fully worked initial solution for this case.
- Case Name: **WSCC_9Bus**

Generator MVA Base

- Like most transient stability programs, generator transient stability data in PowerWorld Simulator is entered in per unit using the generator MVA base.
- The generator MVA base can be modified in the **Edit Mode** (upper left portion of the ribbon), using the **Generator Information Dialog**. You will see the MVA Base in the **Run Mode** but not be able to modify it.



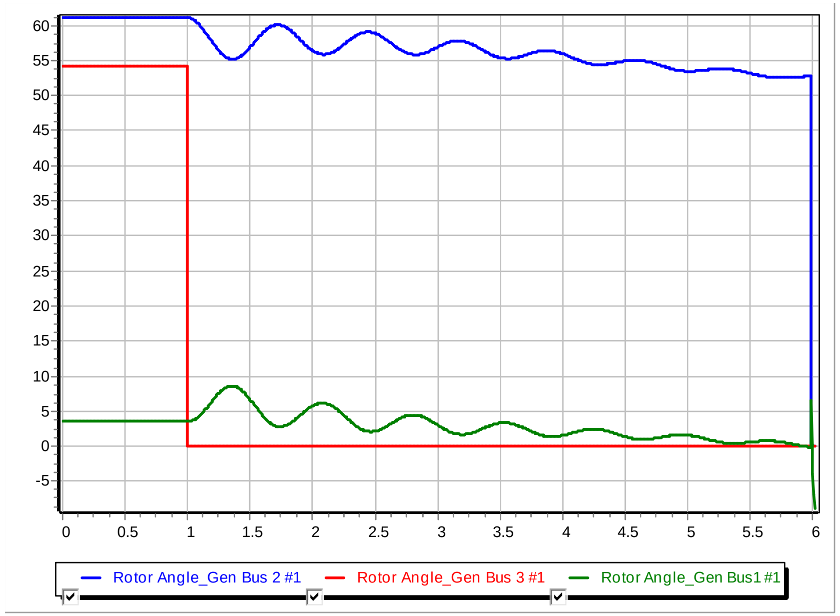
WSCC Case One-line

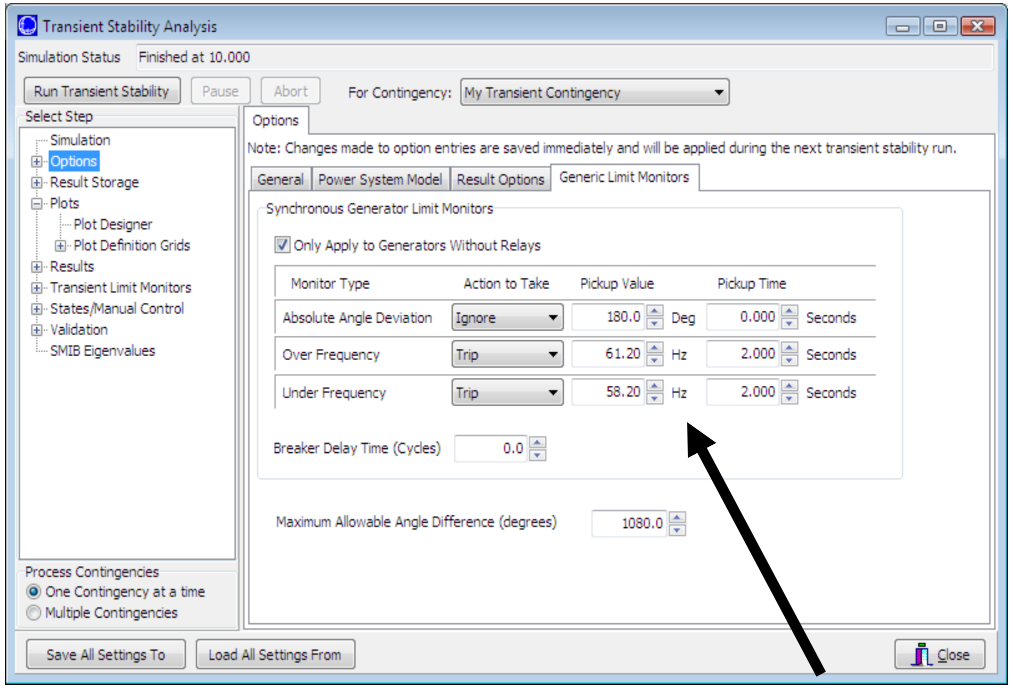


The initial contingency is to trip the generator at bus 3. Select **Run Transient Stability** to get the results.

Automatic Generator Tripping

Sometimes unseen errors may lurk in a simulation!

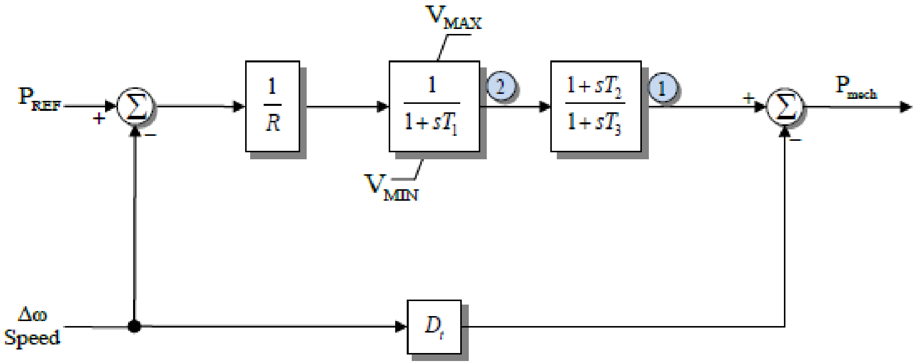




Because this case has no governors and no infinite bus, the bus frequency keeps rising throughout the simulation, even though the rotor angles are stable. Users may set the generators to automatically trip in “Options”, “Generic Limit Monitors”.

Generator Governors

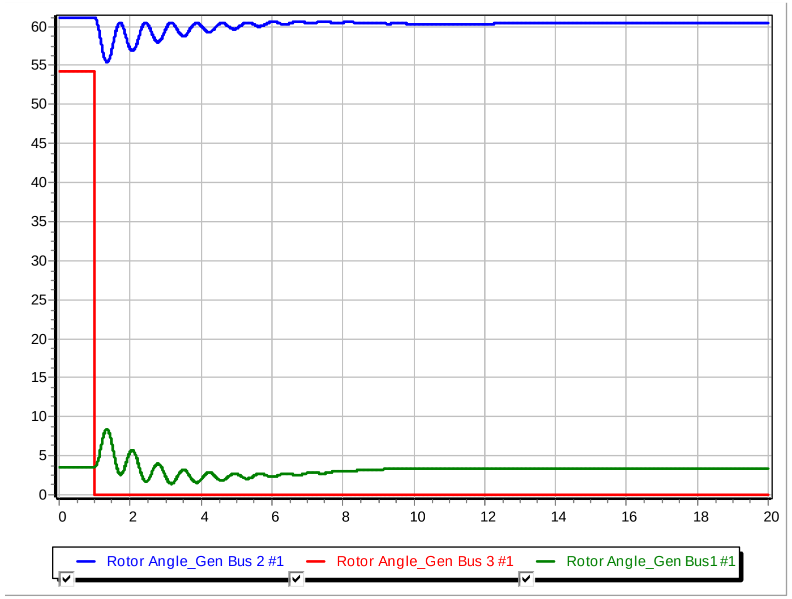
- Governors are used to control the generator power outputs, helping the maintain a desired frequency
- Covered in sections 4.4 and 4.5
- As was the case with machine models and exciters, governors can be entered using the Generator Dialog.
- Add TGOV1 models for all three generators using the default values.



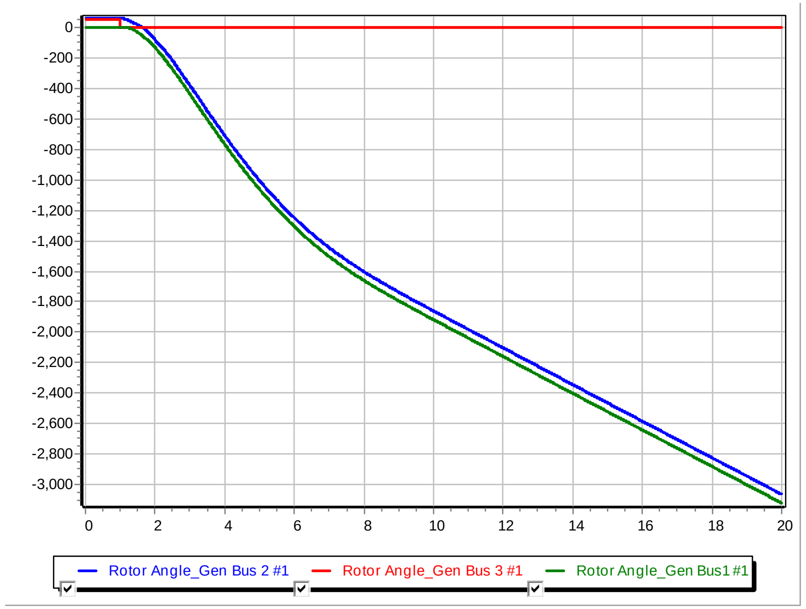
Additional WSCC Case Changes

- Use the “Add Plot” button on the plot designer to insert new plots to show 1) the generator speeds, and 2) the generator mechanical input power.
- Change contingency to be the opening of the bus 3 generator at time t=1 second. There is no “fault” to be cleared in this example, the only event is opening the generator. Run case for 20 seconds.
- Case Name: **WSCC_9Bus_WithGovernors**

Generator Angles on Different Reference Frames



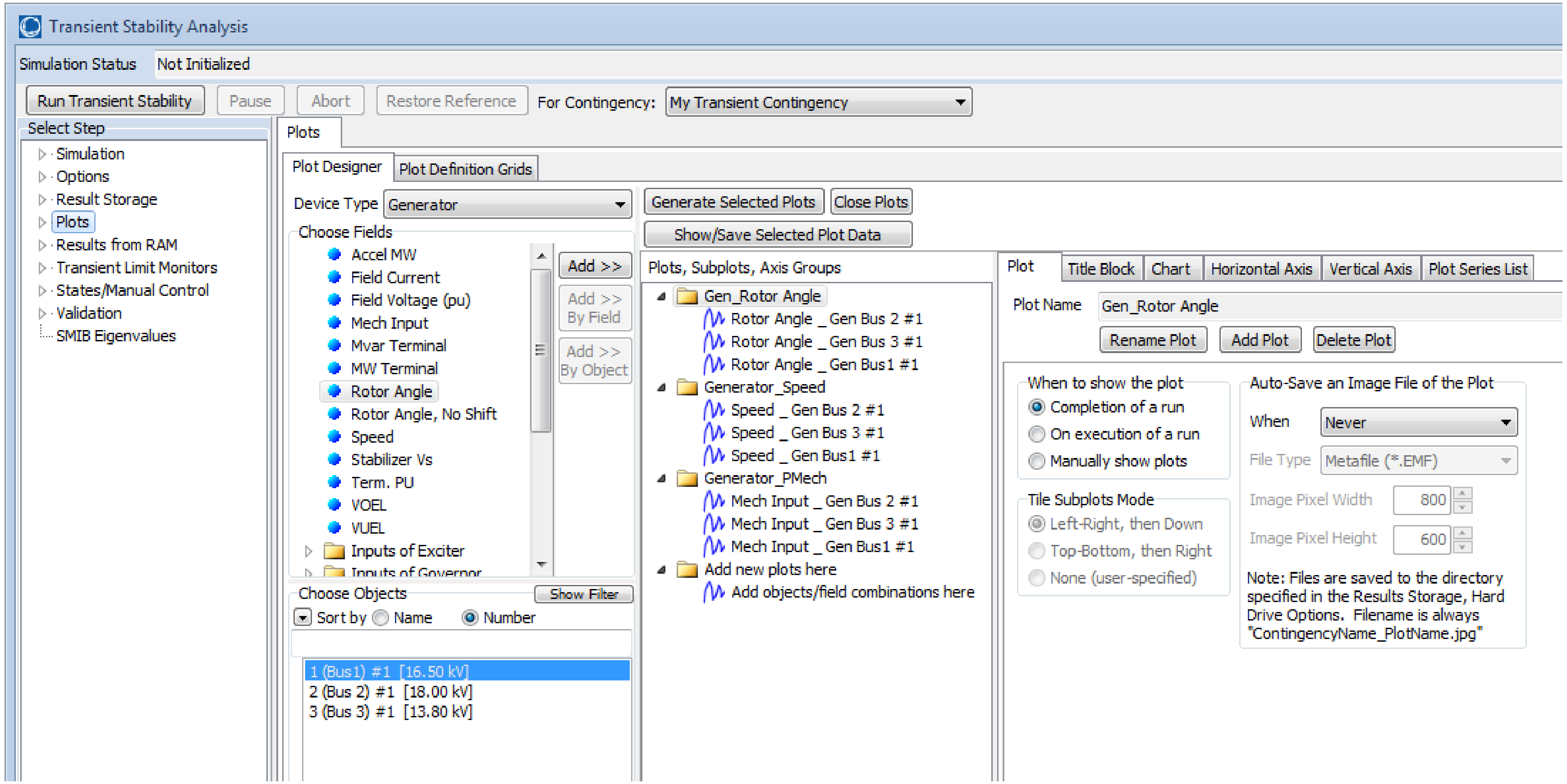
Average of Generator Angles Reference Frame



Synchronous Reference Frame

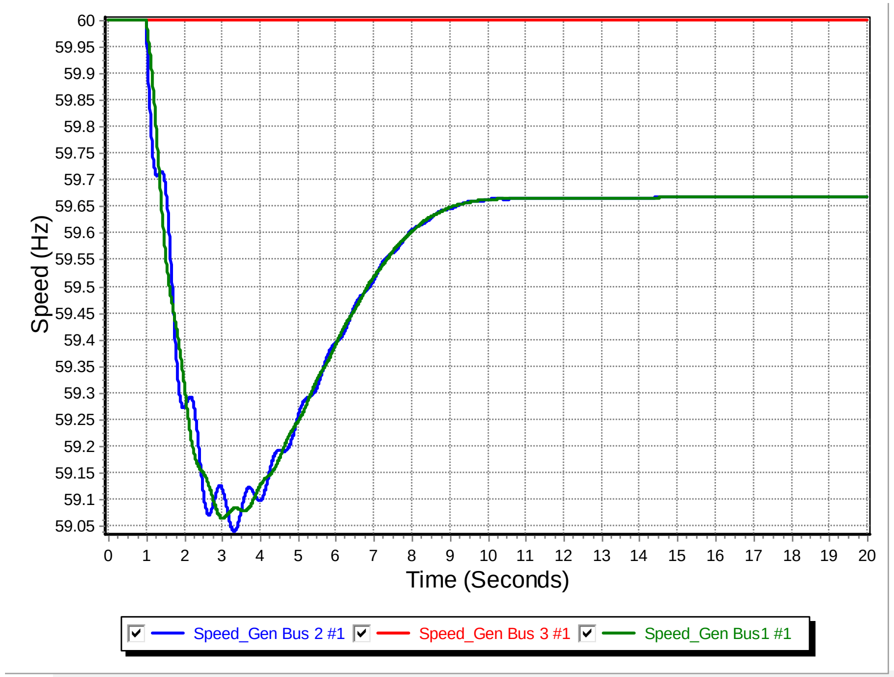
Both are equally “correct”, but it is much easier to see the rotor angle variation when using the average of generator angles reference frame

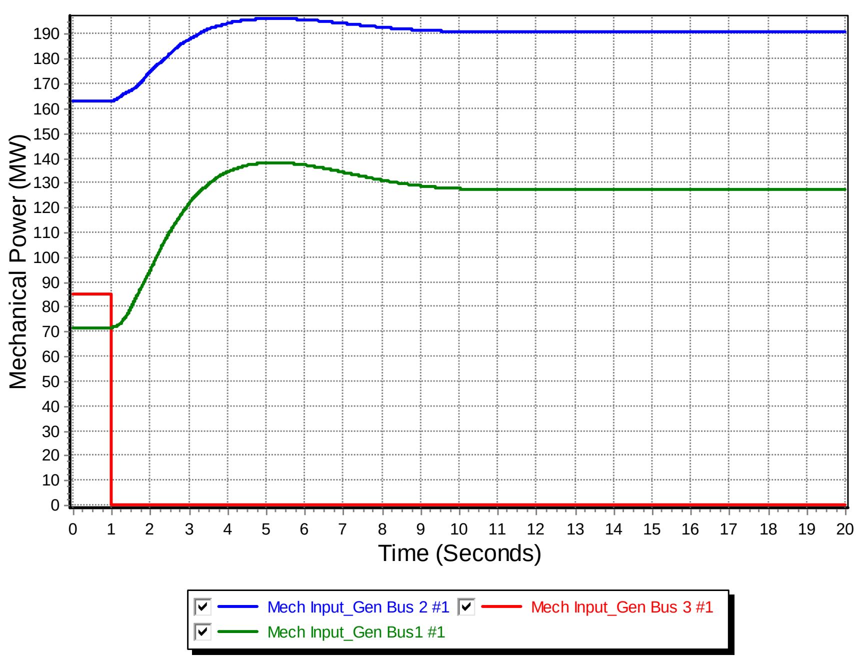
Plot Designer with New Plots



Note that when new plots are added using “Add Plot”, new Folders appear in the plot list. This will result in separate plots for each group

Gen 3 Open Contingency Results





The left figure shows the generator speed, while the right figure shows the generator mechanical power inputs for the loss of generator 3. This is a severe contingency since more than 25% of the system generation is lost, resulting in a frequency dip of almost one Hz. Notice frequency does not return to 60 Hz.

2007 CWLP Dallman Accident

- In 2007 there was an explosion at the Springfield, IL City Water Light Power (CWLP) 86 MW Dallman 1 generator. The explosion was eventually determined to be caused by a sticky valve that prevented the cutoff of steam into the turbine when the generator went off line. So the generator turbine continued to accelerate up to over 6000 rpm (3600 normal).
  - High speed caused parts of the generator to shoot out
  - Hydrogen escaped from the cooling system, and eventually escaped causing the explosion
  - Repairs took about 18 months, costing more than $52 million

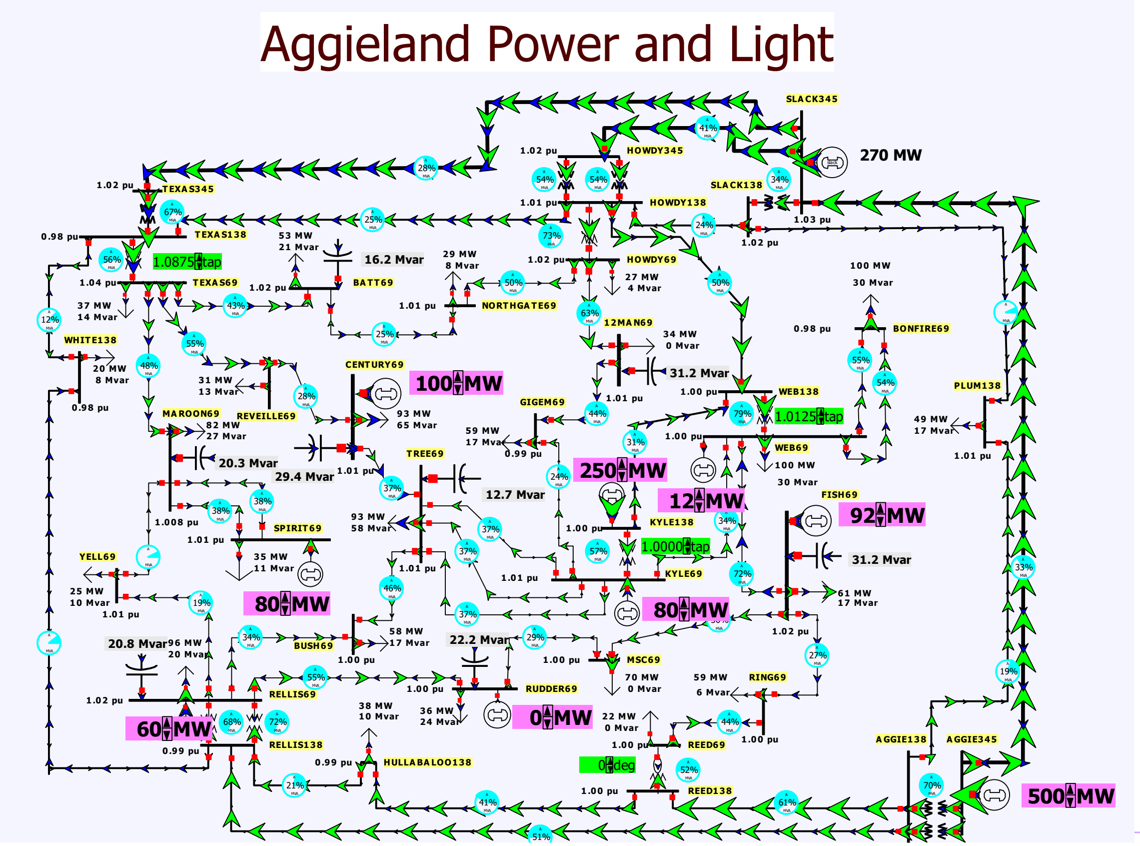
Dallman After the Accident

Outside of Dallman

CWLP retired Dallman 1 and 2 at the end of 2020. Dallman 4 is their newer coal unit, coming online in 2009. It is a 200 MW unit, with recent DOE funding looking doing CO2 capture at the location.

37 Bus System

- Next we consider a slightly larger, ten generator, 37 bus system. To view this system open case **AGL37_TS**. The system one-line is shown below.



To see summary listings of the transient stability models in this case select **Stability Case Info** from the ribbon, and then either **TS Generator Summary** or **TS Case Summary**

Transient Stability Case and Model Summary Displays

Right click on a line and select **Show Dialog** for more information.

Models in Use | Generators | Load Characteristics | Load Summary

Filter: Advanced | TSModelSummaryObject

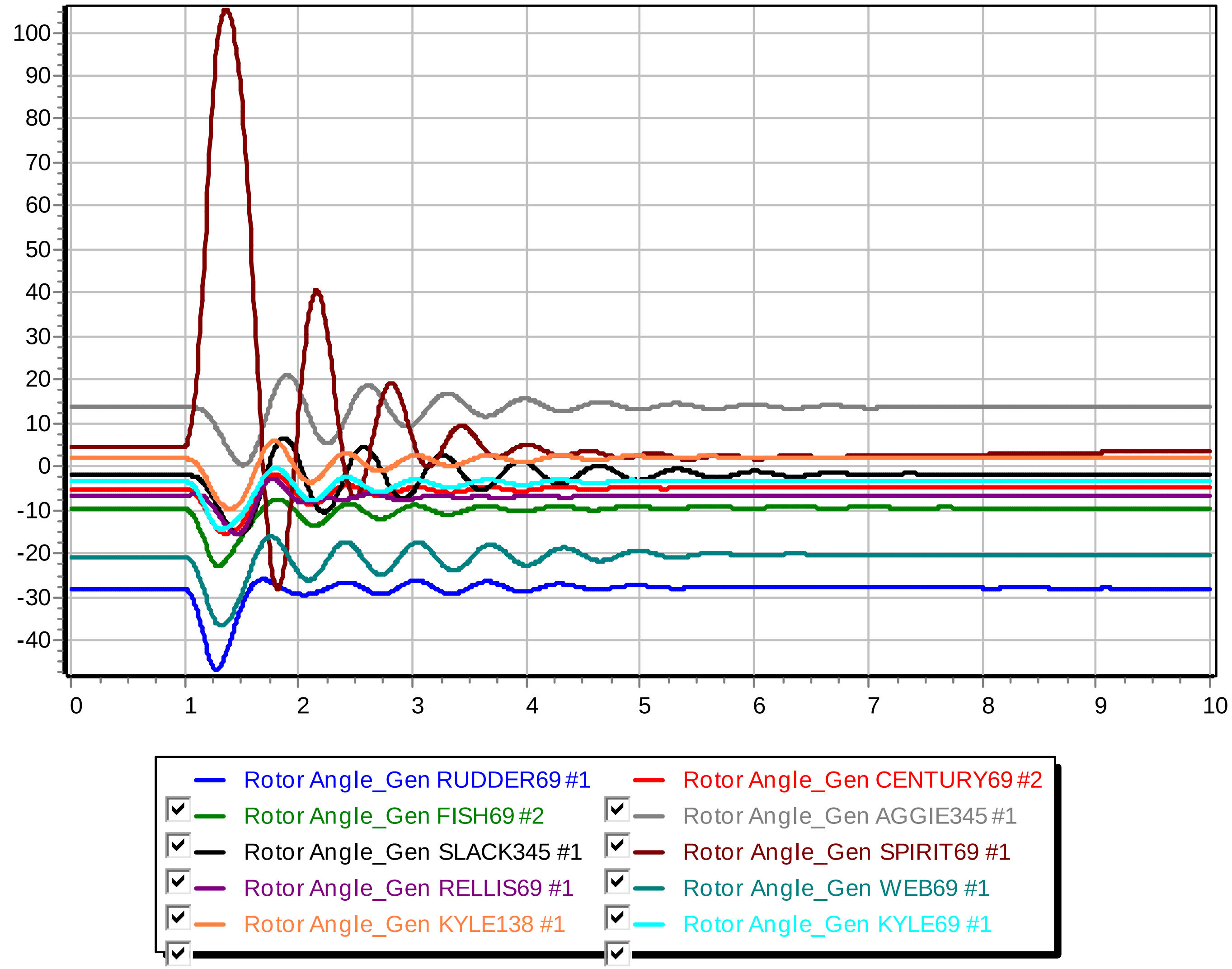
	Model Class	Object Type	Active and Online Count	Active Count	Inactive Count	Fully Supported
1	Machine Model	GENSAL	1	1	0	YES
2	Machine Model	GENROU	9	9	0	YES
3	Exciter	IEEET1	10	10	0	YES
4	Governor	TGOV1	10	10	0	YES

Generator Model Use | Model Summary | Generators | Load Characteristics | Load Summary

Filter: Advanced | Generator

	Number of Bus	Name of Bus	ID	Status	Gen MW	MVA Base	Machine	Exciter	Governor	Stabilizer	Other Model	Governor Response Limits	H (system base)	TS Rcom (system b
1	14	RUDDER69	1	Closed	0.00	50.00	GENROU	IEEET1	TGOV1			Normal	1.50000	0.0
2	16	CENTURY69	2	Closed	100.00	120.00	GENROU	IEEET1	TGOV1			Normal	3.60000	0.0
3	20	FISH69	2	Closed	91.75	130.00	GENROU	IEEET1	TGOV1			Normal	3.90000	0.0
4	28	AGGIE345	1	Closed	500.00	600.00	GENROU	IEEET1	TGOV1			Normal	36.00000	0.0
5	31	SLACK345	1	Closed	270.20	600.00	GENROU	IEEET1	TGOV1			Normal	36.00000	0.0
6	37	SPIRIT69	1	Closed	80.00	90.00	GENSAL	IEEET1	TGOV1			Normal	2.70000	0.0
7	44	RELLIS69	1	Closed	60.00	80.00	GENROU	IEEET1	TGOV1			Normal	2.40000	0.0
8	48	WEB69	1	Closed	12.30	80.00	GENROU	IEEET1	TGOV1			Normal	2.40000	0.0
9	53	KYLE138	1	Closed	250.00	300.00	GENROU	IEEET1	TGOV1			Normal	9.00000	0.0
10	54	KYLE69	1	Closed	80.00	100.00	GENROU	IEEET1	TGOV1			Normal	3.00000	0.0

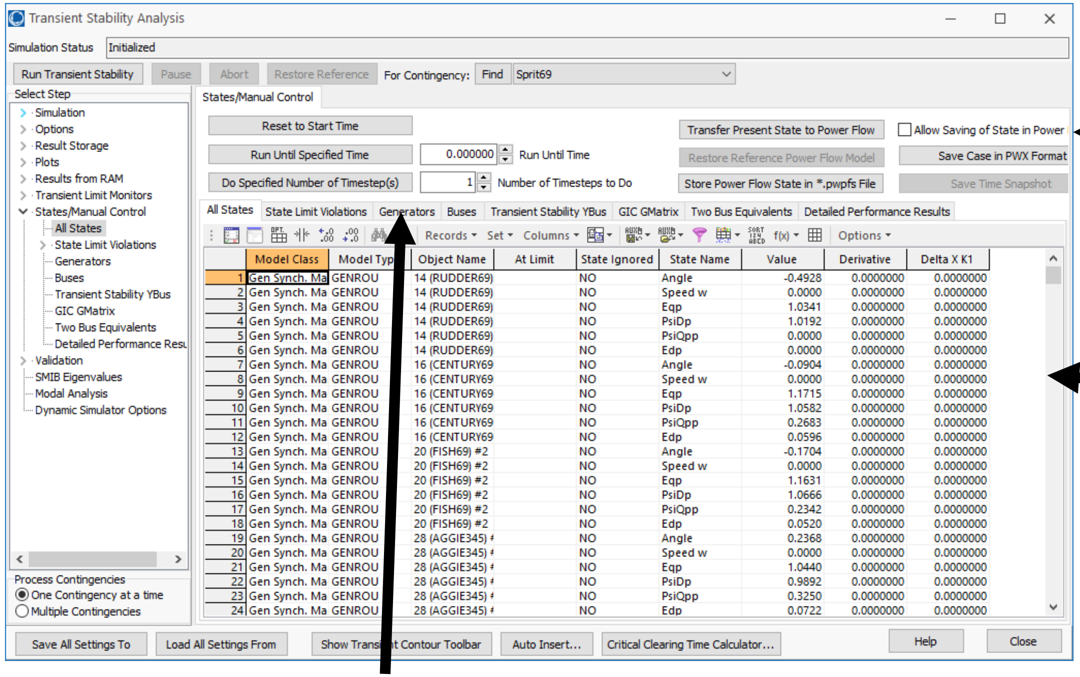
37 Bus Case Solution



Graph shows all the generator rotor angles following a fault on a transmission line

Stepping Through a Solution

- Simulator provides functionality to make it easy to see what is occurring during a solution. This functionality is accessed on the **States/Manual Control** page

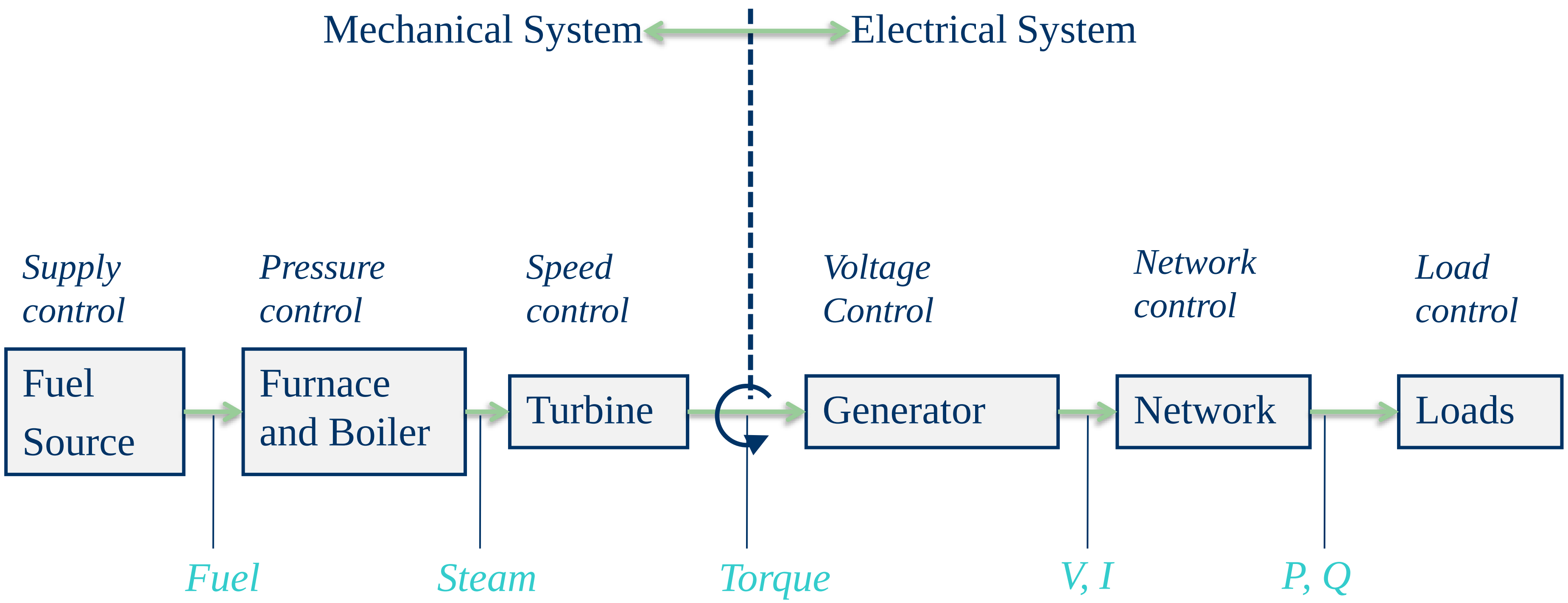


Transfer results to Power Flow to view using standard PowerWorld displays and one-lines

See detailed results at the Paused Time

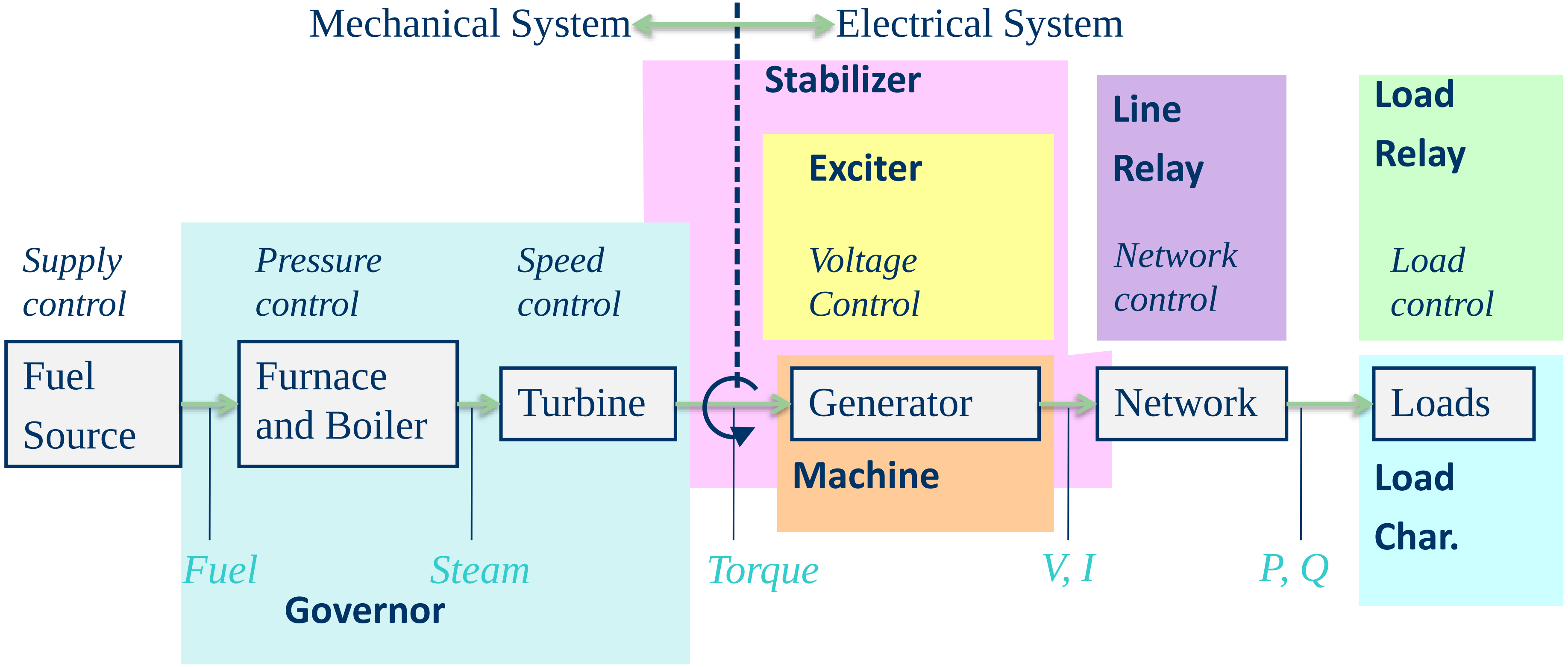
Run a Specified Number of Timesteps or Run Until a Specified Time, then Pause.

Physical Structure Power System Components



P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

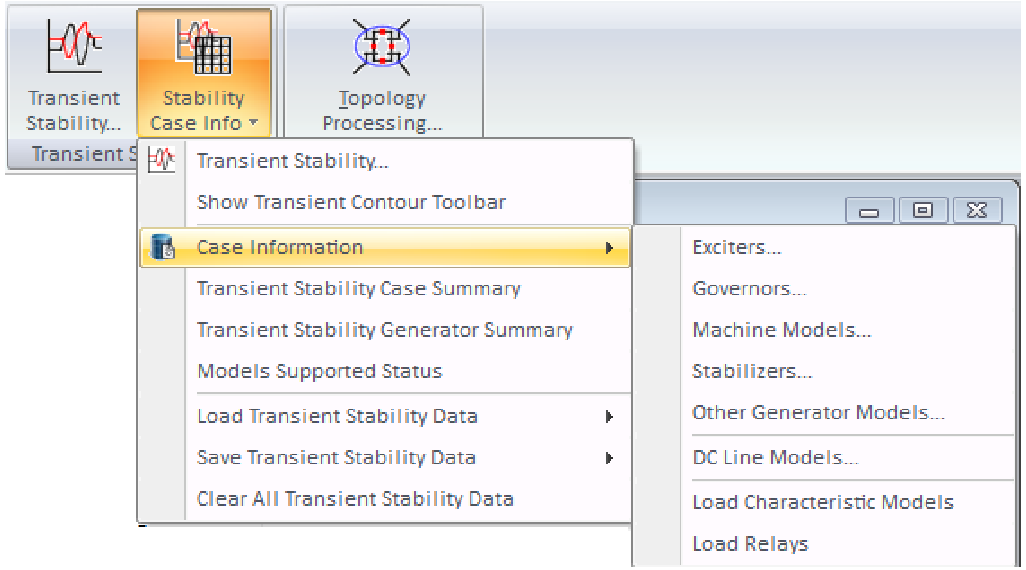
Dynamic Models in the Physical Structure



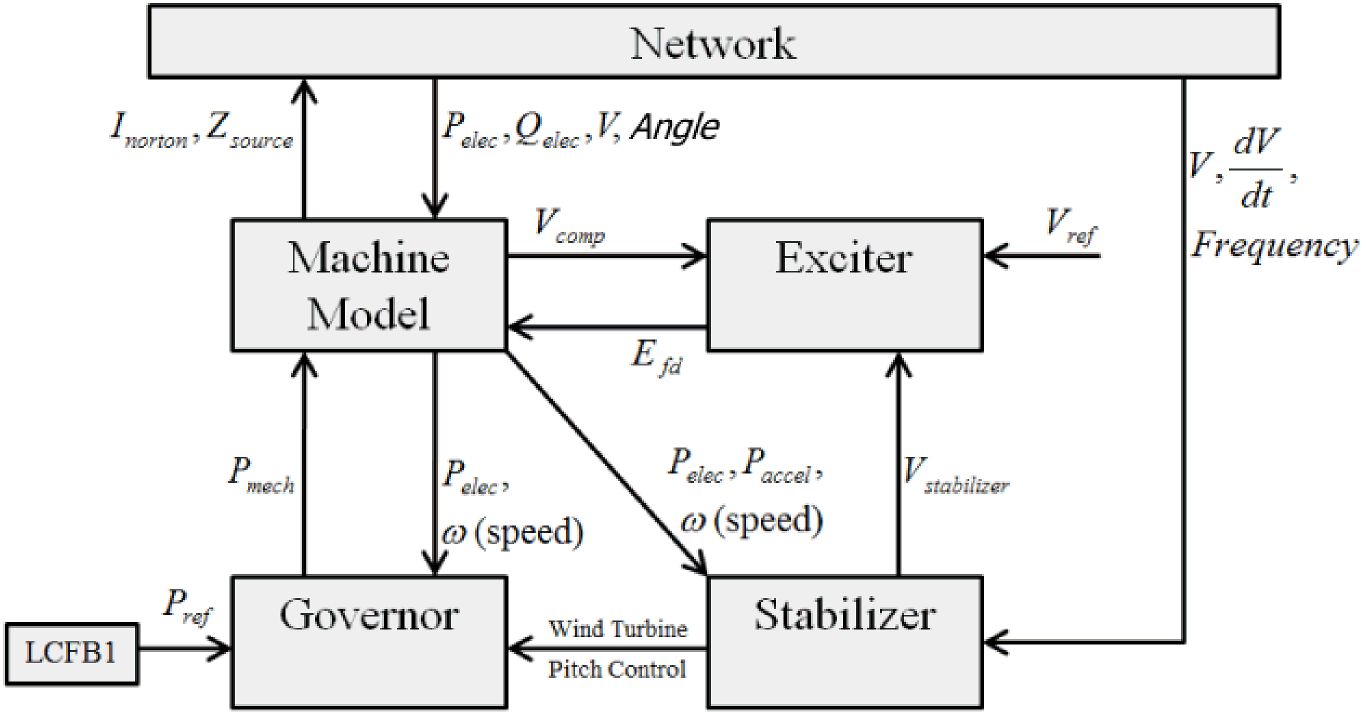
P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

Generator Models

- Generators can have several classes of models assigned to them
  - Machine Models
  - Exciter
  - Governors
  - Stabilizers
- Others also available
  - Excitation limiters, voltage compensation, turbine load controllers, and generator relay model



Generator Models



P_{elec} = Electrical Power

Q_{elec} = Electrical Reactive Power

V = Voltage at Terminal Bus

$\frac{dV}{dt}$ = Derivate of Voltage

V_{comp} = Compensated Voltage

P_{mech} = Mechanical Power

ω(speed) = Rotor Speed (often it's deviation from nominal speed)

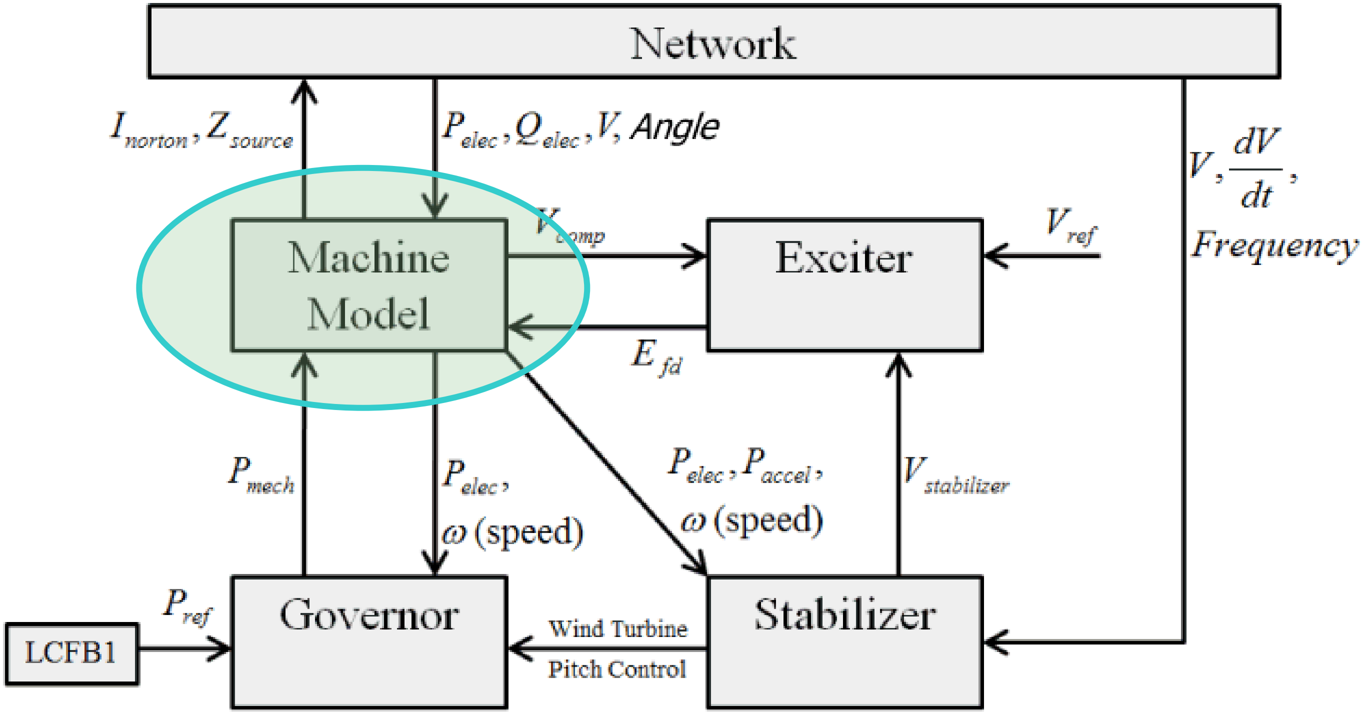
P_{accel} = Accelerating Power

$V_{stabilizer}$ = Output of Stabilizer

V_{ref} = Exciter Control Setpoint (determined during initialization)

P_{ref} = Governor Control Setpoint (determined during initialization)

Machine Models



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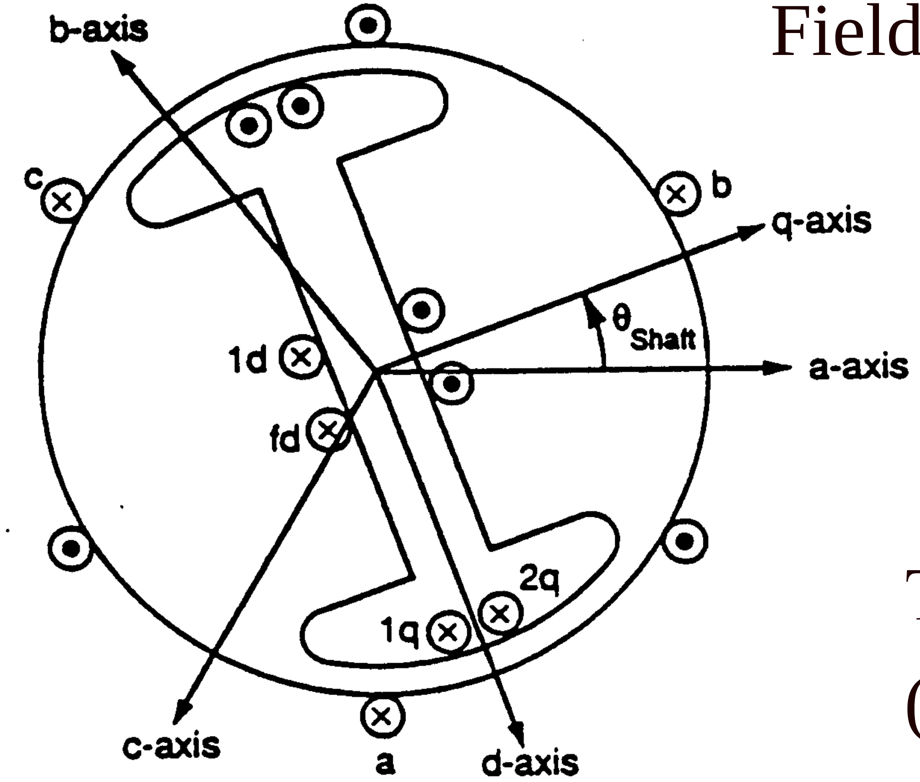
Synchronous Machine Modeling

- Electric machines are used to convert mechanical energy into electrical energy (generators) and from electrical energy into mechanical energy (motors)
  - Many devices can operate in either mode, but are usually customized for one or the other
- Vast majority of electricity is generated using synchronous generators and some is consumed using synchronous motors, so we'll start there
- There is much literature on subject, and sometimes it is overly confusing with the use of different conventions and nomenclature

Synchronous Machine Modeling

3ϕ bal. windings (a,b,c) – stator



Field winding (fd) on rotor

Damper in “d” axis (1d) on rotor

Two dampers in “q” axis (1q, 2q) on rotor

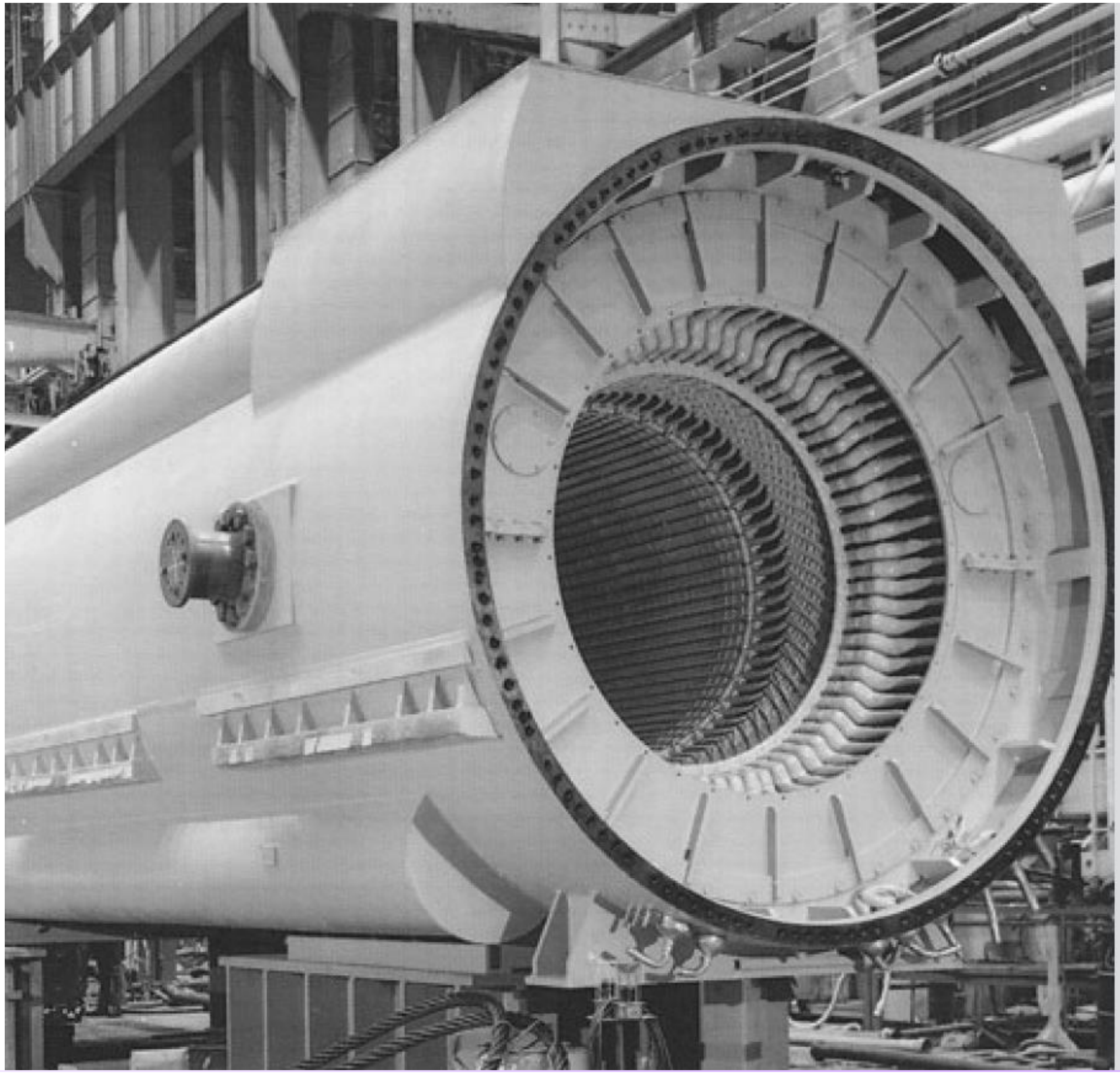
Two Main Types of Synchronous Machines

- Round Rotor
  - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
  - Air-gap varies circumferentially
  - Used with many pole, slower machines such as hydro
  - Narrowest part of gap in the d-axis and the widest along the q-axis

Dq0 Reference Frame

- Stator is stationary, rotor is rotating at synchronous speed
- Rotor values need to be transformed to fixed reference frame for analysis
- Done using Park's transformation into what is known as the dq0 reference frame (direct, quadrature, zero)
  - Parks' 1929 paper voted 2nd most important power paper of 20th century at the 2000 NAPS Meeting
    (1st was Fortescue's sym. components)
- Convention used here is the q-axis leads the d-axis (which is the IEEE standard)

Synchronous Machine Stator

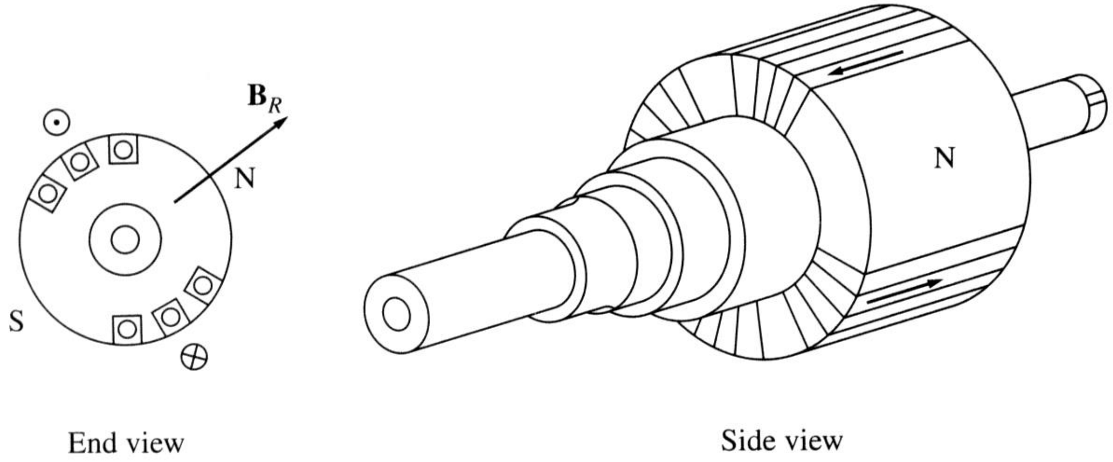


Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

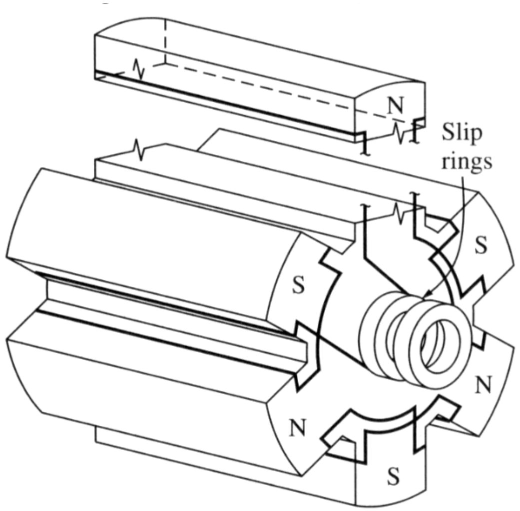
Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

Synchronous Machine Rotors

- Rotors are essentially electromagnets



Two pole (P) round rotor



Six pole salient rotor

Images Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

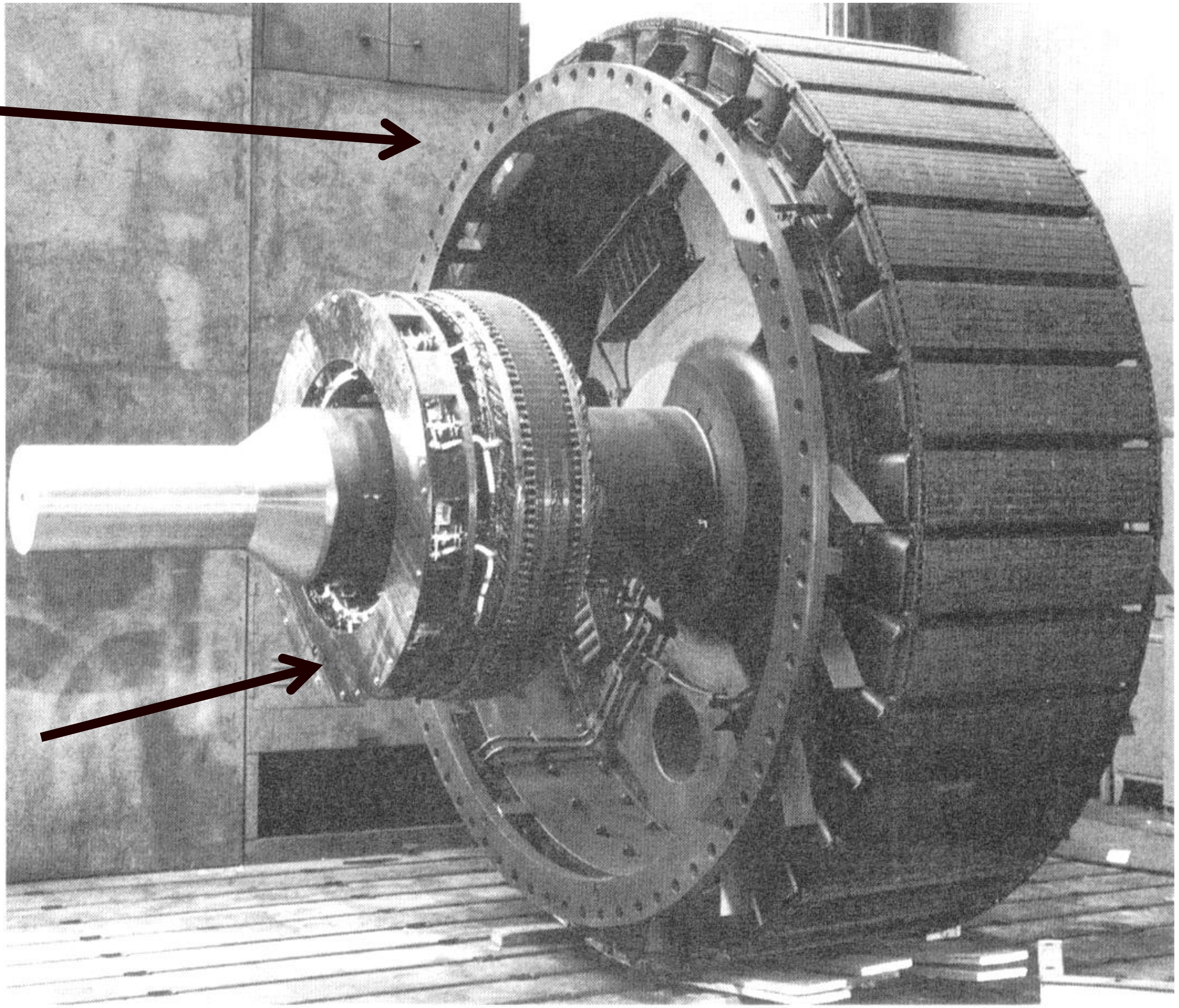


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

Fundamental Laws

- Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

The rotor has the field winding and up to three damper windings (added to provide damping)

Dq0 Transformations

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or} \quad i, \lambda$$

In the next few slides we'll quickly go through how these basic equations are transformedinto the standard machine models; the point is to show the physical basis for the models.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

Dq0 Transformations

$$T_{dqo} \triangleq \frac{2}{3}\begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \cos\frac{P}{2}\theta_{shaft} & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

with the inverse,

$$T_{dqo}^{-1} = \begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \cos\frac{P}{2}\theta_{shaft} & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

Transformed System

Stator

$$v_d = r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

We are now in the dq0 space

Electrical & Mechanical Relationships

Electrical system: $v = iR + \frac{d\lambda}{dt}$ (voltage)

$$vi = i^2R + i\frac{d\lambda}{dt} \quad \text{(power)}$$

P is the number of poles (e.g., 2,4,6); T_{fw} is the friction and windage torque

Mechanical system:

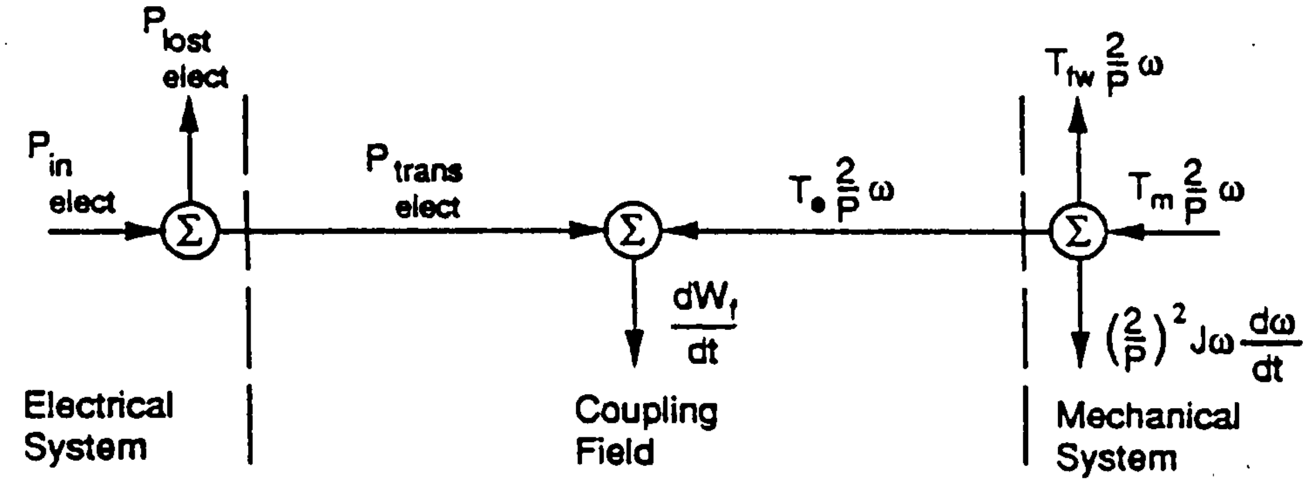
$$J\left(\frac{2}{P}\right)\frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad \text{(torque)}$$

$$J\left(\frac{2}{P}\right)^2 \omega\frac{d\omega}{dt} = \frac{2}{P}\omega T_m - \frac{2}{P}\omega T_e - \frac{2}{P}\omega T_{fw} \quad \text{(power)}$$

Torque Derivation

- Torque is derived by looking at the overall energy balance in the system
- Three systems: electrical, mechanical and the coupling magnetic field
  - Electrical system losses are in the form of resistance
  - Mechanical system losses are in the form of friction
- Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

Energy Conversion



The coupling field stores and discharges energy but has no losses

Look at the instantaneous power:

$$v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o$$

Change to Conservation of Power

$$P_{in\ elect} = v_a i_a + v_b i_b + v_c i_c + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{lost\ elect} = r_s\left(i_a^2 + i_b^2 + i_c^2\right) + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{trans\ elect} = i_a \frac{d\lambda_a}{dt} + i_b \frac{d\lambda_b}{dt} + i_c \frac{d\lambda_c}{dt} + i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt} + i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

We are using $v = d\lambda/dt$

With the Transformed Variables

$$P_{\substack{in \\ elect}} = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{\substack{lost \\ elect}} = \frac{3}{2} r_s i_d^2 + \frac{3}{2} r_s i_q^2 + 3 r_s i_o^2 + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{\substack{trans \\ elect}} = -\frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_q i_d + \frac{3}{2} i_d \frac{d\lambda_d}{dt} + \frac{3}{2}\frac{P}{2}\frac{d\theta_{shaft}}{dt}\lambda_d i_q + \frac{3}{2} i_q \frac{d\lambda_q}{dt} + 3 i_o \frac{d\lambda_o}{dt}$$

$$+ i_{fd}\frac{d\lambda_{fd}}{dt} + i_{1d}\frac{d\lambda_{1d}}{dt} + i_{1q}\frac{d\lambda_{1q}}{dt} + i_{2q}\frac{d\lambda_{2q}}{dt}$$

Change in Coupling Field Energy

$$\frac{dW_f}{dt} = \boxed{T_e \frac{2}{P}} \frac{d\theta}{dt} + \boxed{i_a} \frac{d\lambda_a}{dt} + \boxed{i_b} \frac{d\lambda_b}{dt}$$

$$+ \boxed{i_c} \frac{d\lambda_c}{dt} + \boxed{i_{fd}} \frac{d\lambda_{fd}}{dt} + \boxed{i_{1d}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{i_{1q}} \frac{d\lambda_{1q}}{dt} + \boxed{i_{2q}} \frac{d\lambda_{2q}}{dt}$$

First term on right is what is going on mechanically, other terms are what is going on electrically

This requires the lossless coupling field assumption

Change in Coupling Field Energy

For independent states $\theta, \lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q}$

$$\frac{dW_f}{dt} = \boxed{\frac{\partial W_f}{\partial \theta}} \frac{d\theta}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_a}} \frac{d\lambda_a}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_b}} \frac{d\lambda_b}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_c}} \frac{d\lambda_c}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{fd}}} \frac{d\lambda_{fd}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{1d}}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_{1q}}} \frac{d\lambda_{1q}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{2q}}} \frac{d\lambda_{2q}}{dt}$$

Equate the Coefficients

$$T_e \frac{2}{P} = \frac{\partial W_f}{\partial \theta} \qquad i_a = \frac{\partial W_f}{\partial \lambda_a} \qquad \text{etc.}$$

There are eight such “reciprocity conditions for this model.

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Equate the Coefficients

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e$$

$$\frac{\partial W_f}{\partial \lambda_d} = \frac{3}{2} i_d \,, \quad \frac{\partial W_f}{\partial \lambda_q} = \frac{3}{2} i_q, \quad \frac{\partial W_f}{\partial \lambda_o} = 3 i_o$$

$$\frac{\partial W_f}{\partial \lambda_{fd}} = i_{fd} \,, \quad \frac{\partial W_f}{\partial \lambda_{1d}} = i_{1d}, \quad \frac{\partial W_f}{\partial \lambda_{1q}} = i_{1q}, \quad \frac{\partial W_f}{\partial \lambda_{2q}} = i_{2q}$$

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

Coupling Field Energy

- The coupling field energy is calculated using a path independent integration
  - For integral to be path independent, the partial derivatives of all integrands with respect to the other states must be equal

For example, $$\frac{3}{2}\frac{\partial i_d}{\partial \lambda_{fd}} = \frac{\partial i_{fd}}{\partial \lambda_d}$$

- Since integration is path independent, choose a convenient path
  - Start with a de-energized system so variables are zero
  - Integrate shaft position while other variables are zero
  - Integrate sources in sequence with shaft at final value

Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

ω_s is the rated synchronous speed
δ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

Synchronous Machine Equations in Per Unit

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I2 + V_{2q}$$

The ψ variables are in the λ variables in per unit (see book 3.50 to 3.52)

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

Units of H are seconds

Sinusoidal Steady-State

$$V_a = \sqrt{2}V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2}I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application to balanced, sinusoidal conditions

Simplifying Using δ

- Define $$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$
- Hence
$$V_d = V_s \sin(\delta - \theta_{vs})$$
$$V_q = V_s \cos(\delta - \theta_{vs})$$
$$I_d = I_s \sin(\delta - \theta_{is})$$
$$I_q = I_s \cos(\delta - \theta_{is})$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

- These algebraic equations can be written as complex equations
$$(V_d + jV_q)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$
$$(I_d + jI_q)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of θ_{shaft} when expressed in the dq0 coordinate system