# ECEN 667
# Power System Stability

## Lecture 8: Synchronous Machine Modeling

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

TEXAS A&M UNIVERSITY

# Announcements

- Read Chapter 5; look at Appendix A
- Homework 2 is due today; Homework 3 is due on September 21
  - Good writing is a key engineering skill! There are many books to help. I like, *The Handbook of Technical Writing* by Alred, Brusaw and Oliu (now in the 12th Edition)
- First exam is on Tuesday October 3 during class (except for the distance education students)
- Energy and Power Group seminar speaker on Friday is Maryam Kazerooni giving a talk titled, "Application of Machine Learning in Energy Trading." It is at 11:30 to 12:20 p.m. in Zach 244.
- Maryam will also be giving a special tutorial on Friday from 2 to 4 p.m. titled, "An Introduction to Power Trading with a Focus on US Markets." The location will be announced soon.

# In the News: ERCOT Frequency on 9/6/23

- ERCOT entered emergency operations as a result of this frequency decline; there was also a sudden drop of 1300 MW in wind power

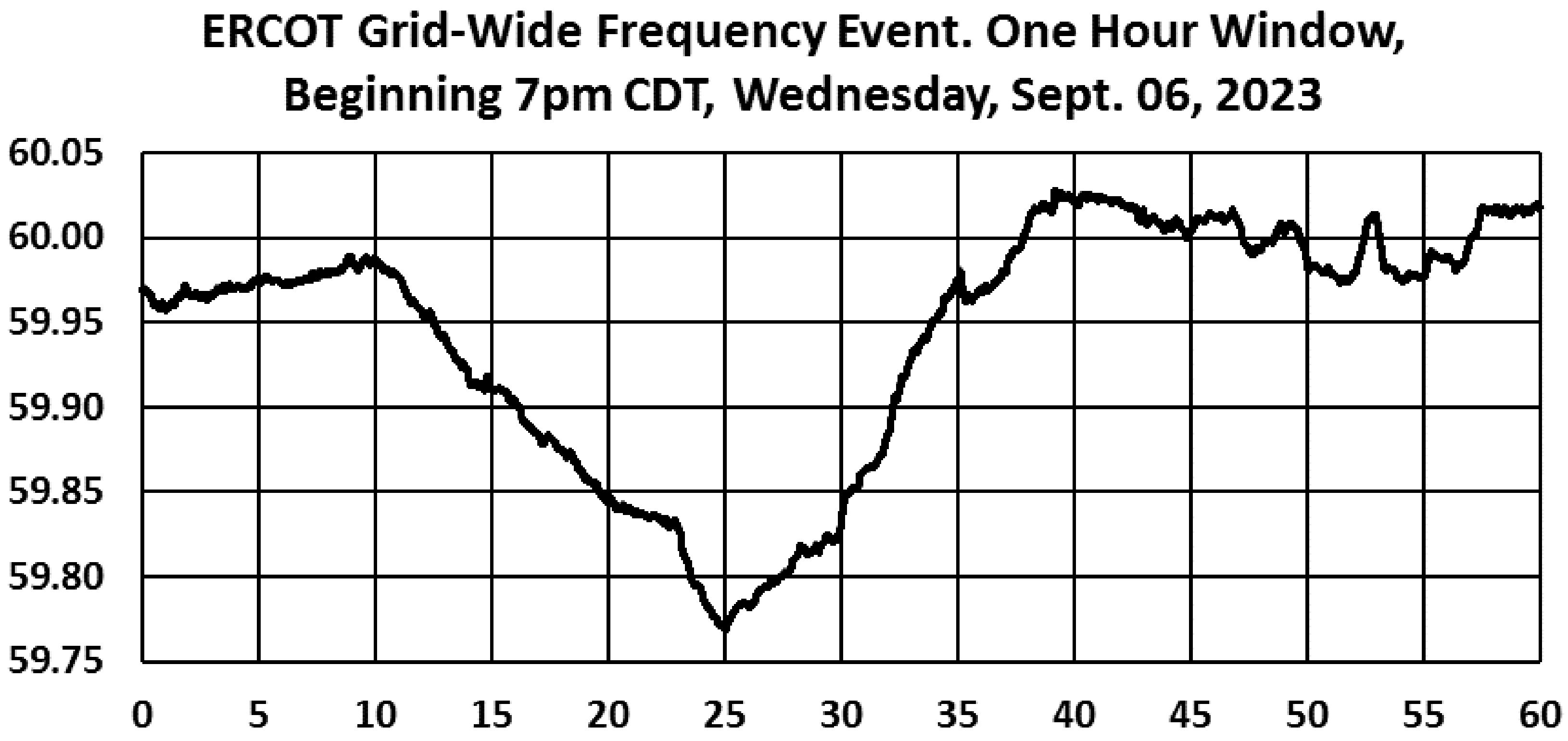
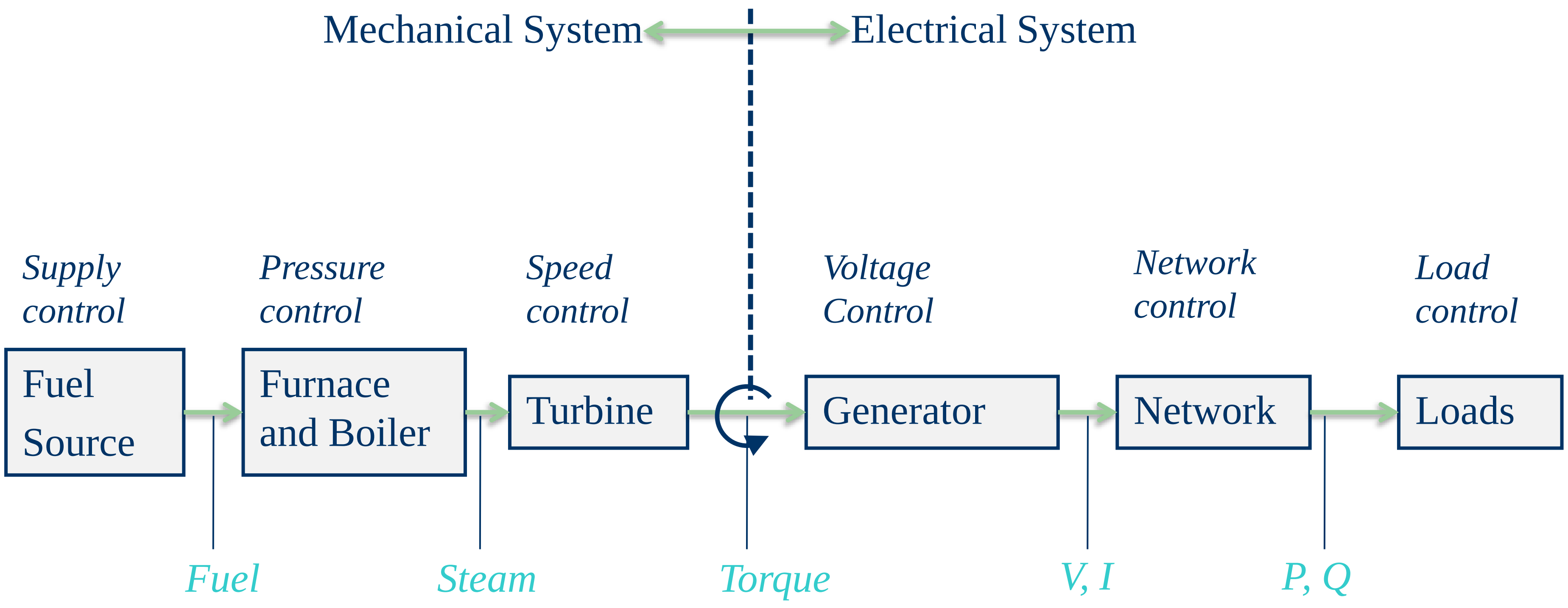


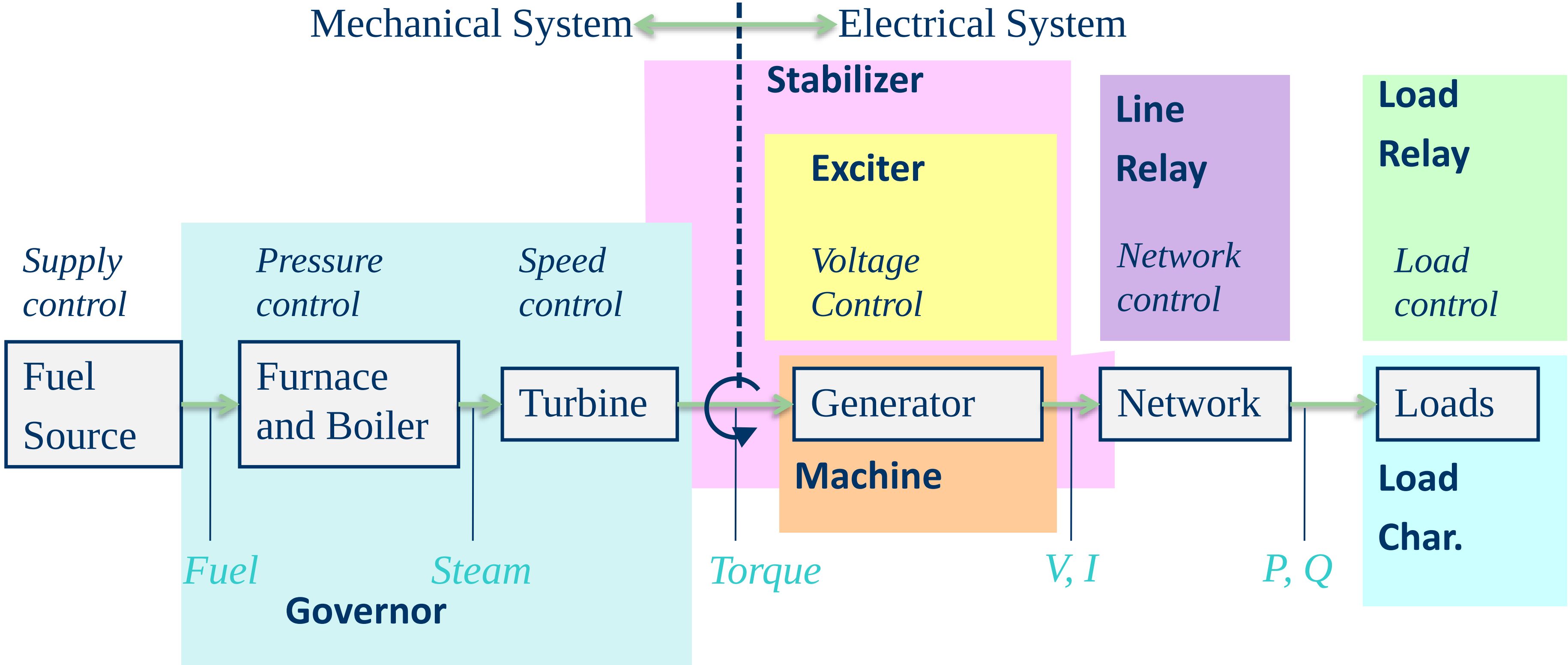

Image Source: Dr. Mack Grady, Baylor University

# Physical Structure Power System Components




P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

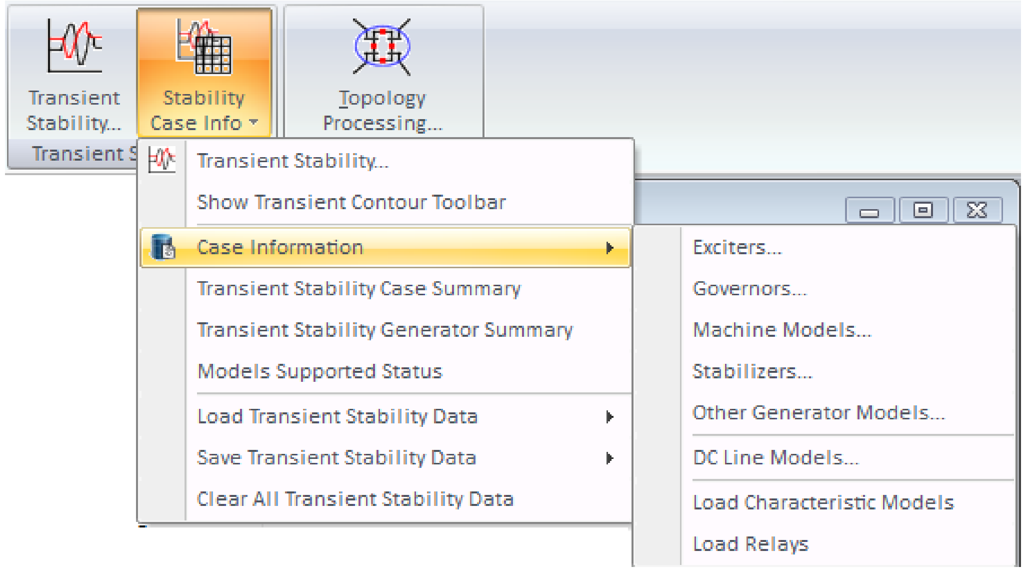
# Dynamic Models in the Physical Structure



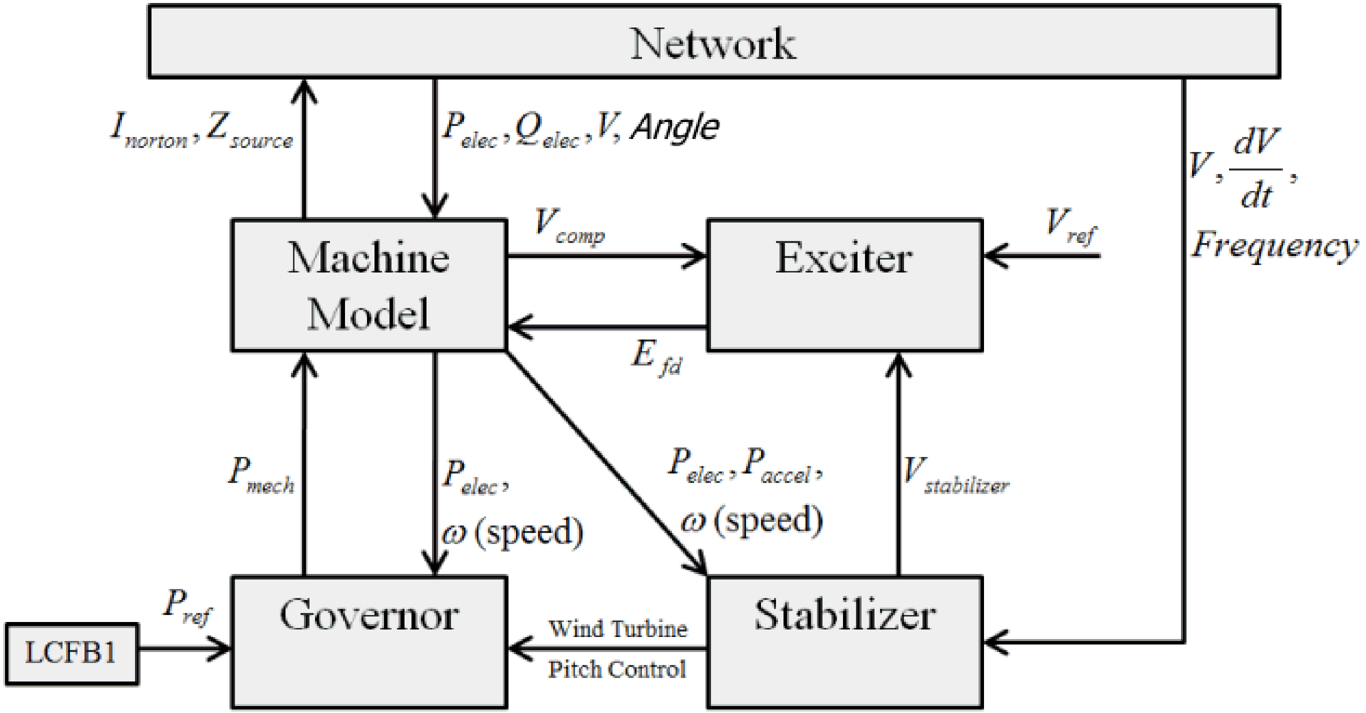

P. Sauer and M. Pai, *Power System Dynamics and Stability*, Stipes Publishing, 2006.

# Generator Models

- Generators can have several classes of models assigned to them
  - Machine Models
  - Exciter
  - Governors
  - Stabilizers
- Others also available
  - Excitation limiters, voltage compensation, turbine load controllers, and generator relay model



# Generator Models




$P_{elec}$ = Electrical Power

$Q_{elec}$ = Electrical Reactive Power

$V$ = Voltage at Terminal Bus

$\frac{dV}{dt}$ = Derivate of Voltage

$V_{comp}$ = Compensated Voltage

$P_{mech}$ = Mechanical Power

$\omega$(speed) = Rotor Speed (often it's deviation from nominal speed)

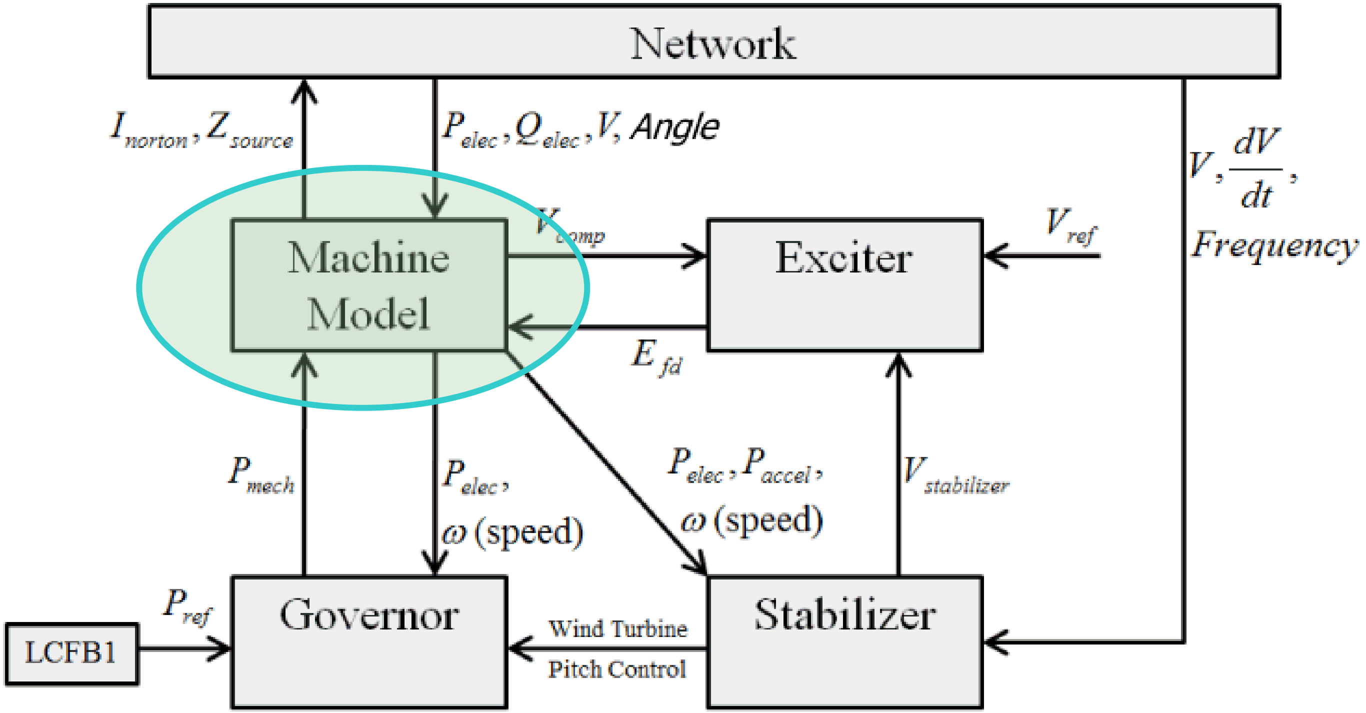
$P_{accel}$ = Accelerating Power

$V_{stabilizer}$ = Output of Stabilizer

$V_{ref}$ = Exciter Control Setpoint (determined during initialization)

$P_{ref}$ = Governor Control Setpoint (determined during initialization)

# Machine Models

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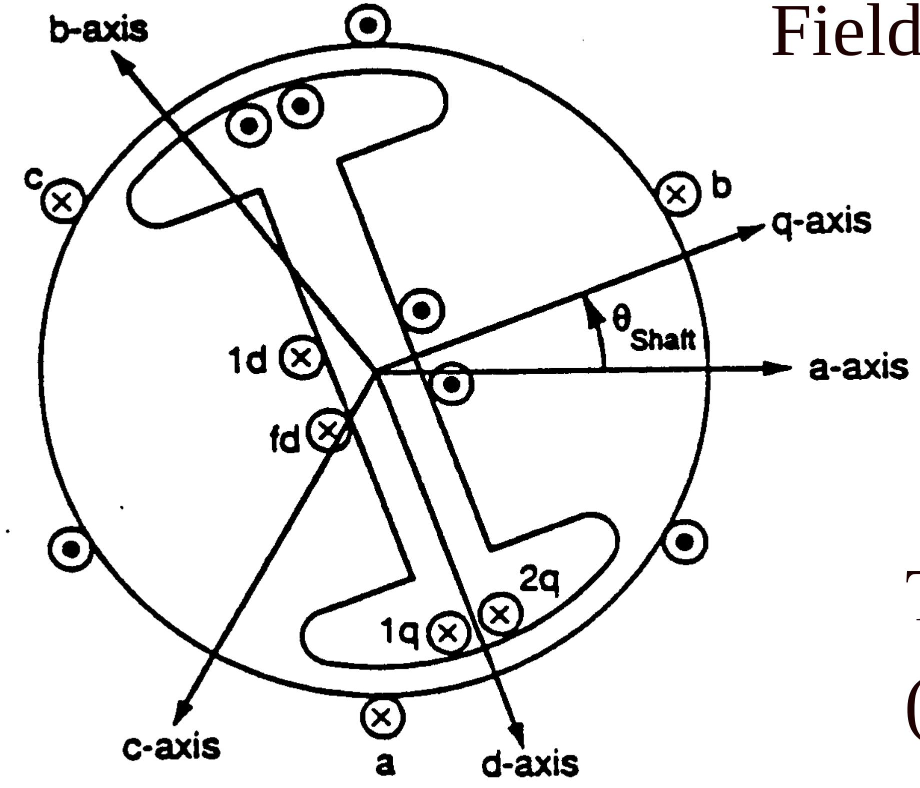
# Synchronous Machine Modeling

- Electric machines are used to convert mechanical energy into electrical energy (generators) and from electrical energy into mechanical energy (motors)
  - Many devices can operate in either mode, but are usually customized for one or the other
- Vast majority of electricity is generated using synchronous generators and some is consumed using synchronous motors, so we'll start there
- There is much literature on subject, and sometimes it is overly confusing with the use of different conventions and nomenclature

# Synchronous Machine Modeling

3ϕ bal. windings (a,b,c) – stator




Field winding (fd) on rotor

Damper in "d" axis
(1d) on rotor

Two dampers in "q" axis
(1q, 2q) on rotor

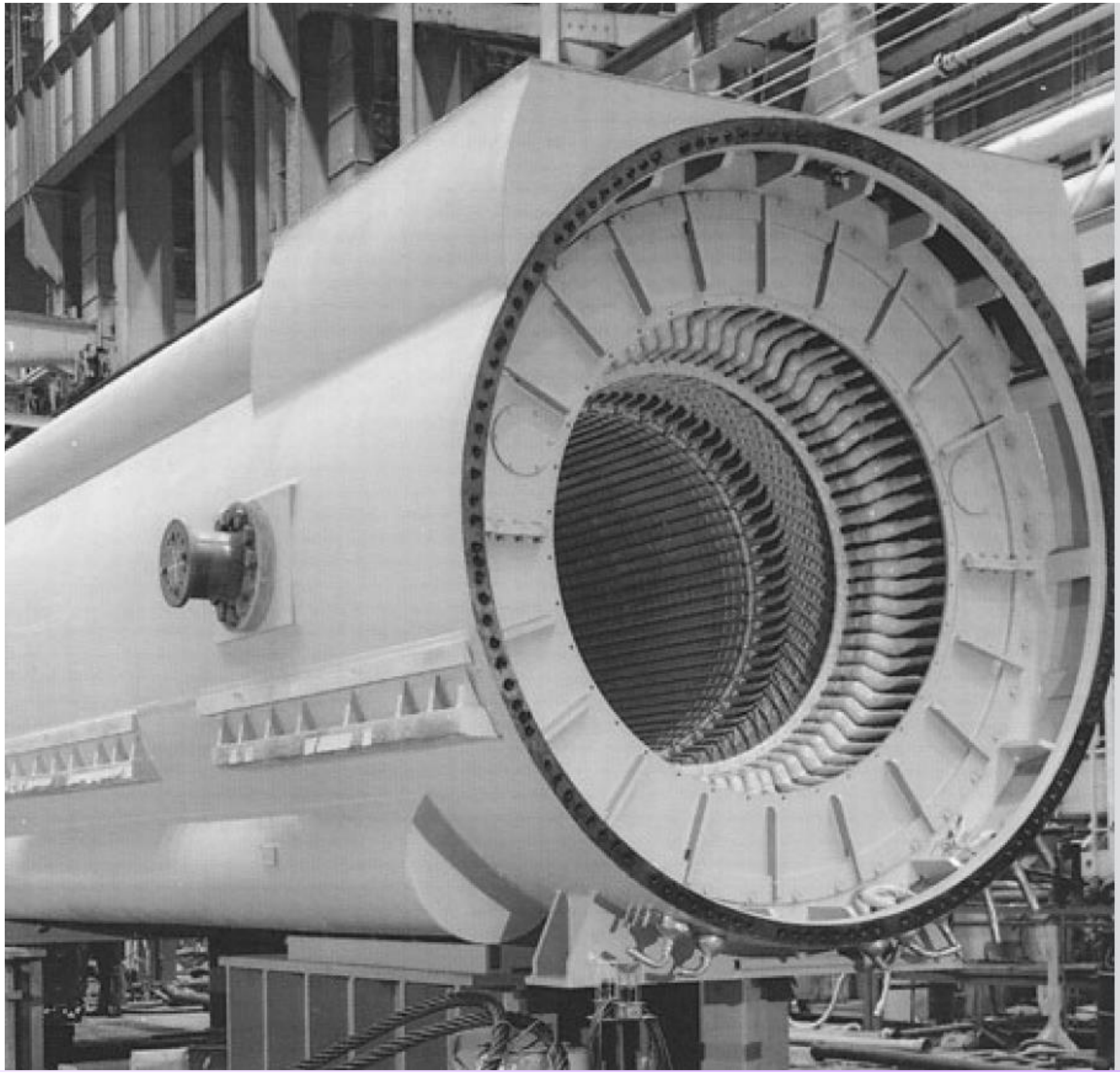
# Two Main Types of Synchronous Machines

- Round Rotor
  - Air-gap is constant, used with higher speed machines
- Salient Rotor (often called Salient Pole)
  - Air-gap varies circumferentially
  - Used with many pole, slower machines such as hydro
  - Narrowest part of gap in the d-axis and the widest along the q-axis

# Dq0 Reference Frame

- Stator is stationary, rotor is rotating at synchronous speed
- Rotor values need to be transformed to fixed reference frame for analysis
- Done using Park's transformation into what is known as the dq0 reference frame (direct, quadrature, zero)
  - Parks' 1929 paper voted 2nd most important power paper of 20th century at the 2000 NAPS Meeting
    (1st was Fortescue's symmetrical components)
- Convention used here is the q-axis leads the d-axis (which is the IEEE standard)

# Synchronous Machine Stator

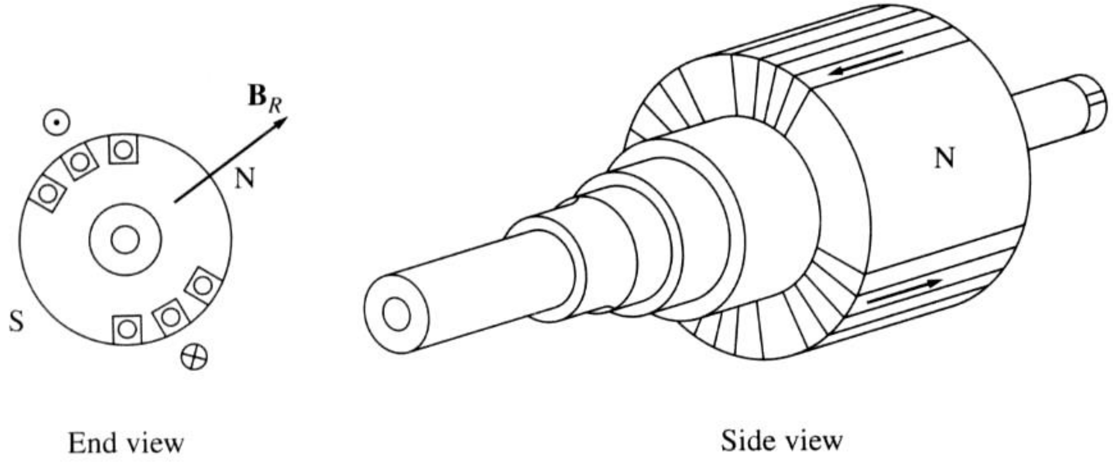


Generator stator showing completed windings for a 757-MVA, 3600-RPM, 60-Hz synchronous generator (Courtesy of General Electric.)

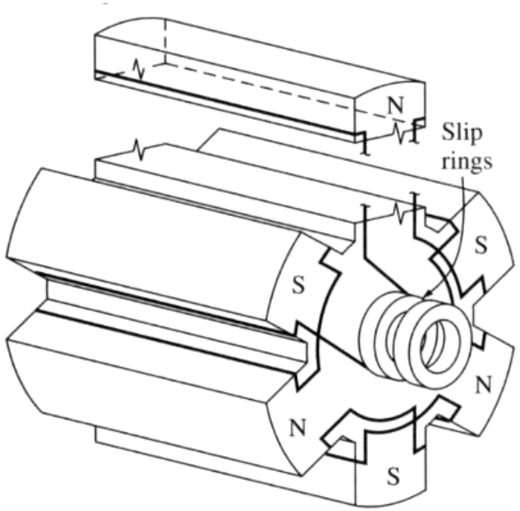
Image Source: Glover/Overbye/Sarma Book, Sixth Edition, Beginning of Chapter 8 Photo

# Synchronous Machine Rotors

- Rotors are essentially electromagnets




Two pole (P) round rotor




Six pole salient rotor

Images Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

# Synchronous Machine Rotor

High pole salient rotor

Shaft

Part of exciter, which is used to control the field current

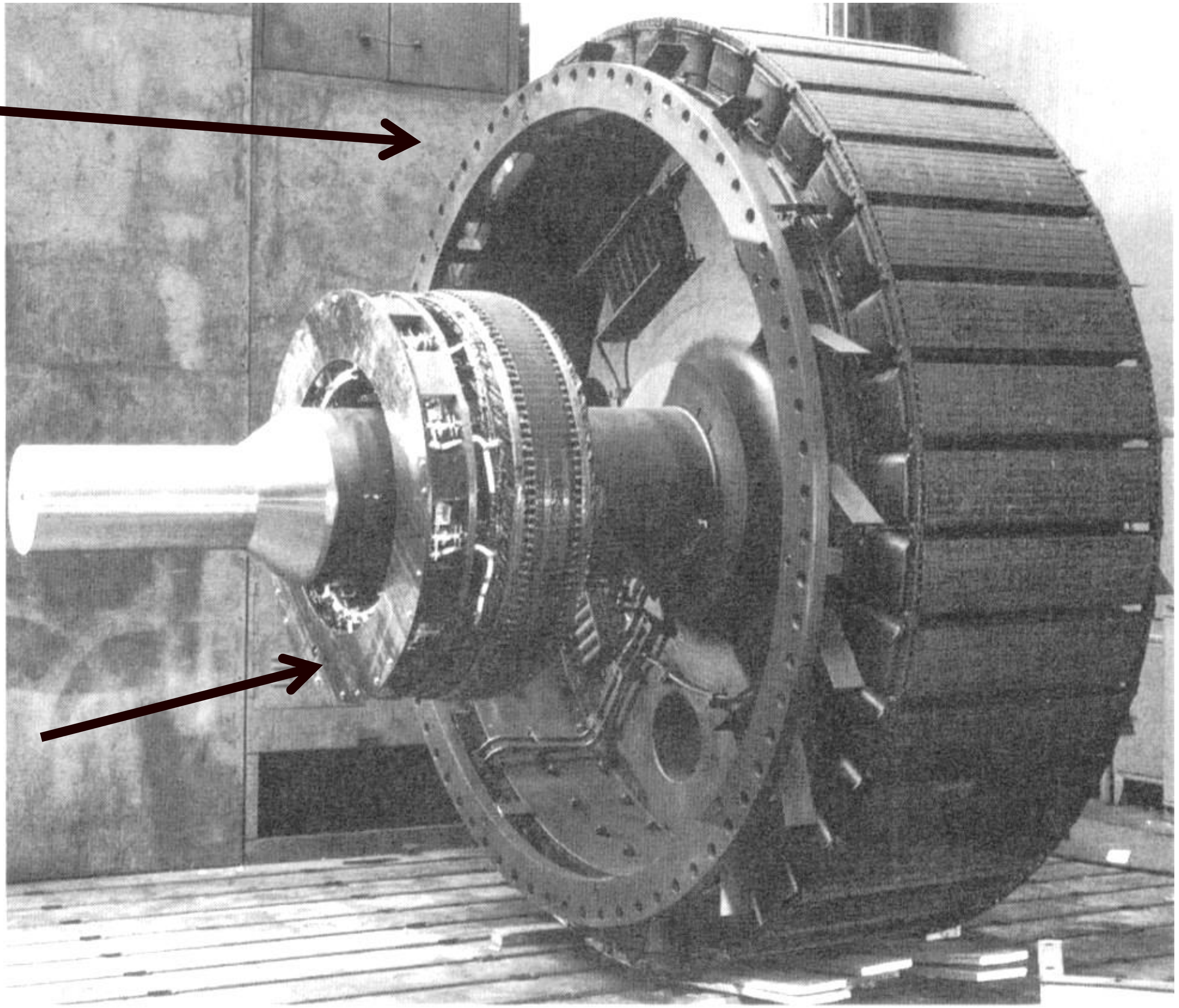


Image Source: Dr. Gleb Tcheslavski, ee.lamar.edu/gleb/teaching.htm

# Fundamental Laws

- Kirchhoff's Voltage Law, Ohm's Law, Faraday's Law, Newton's Second Law

Stator

$$v_a = i_a r_s + \frac{d\lambda_a}{dt}$$

$$v_b = i_b r_s + \frac{d\lambda_b}{dt}$$

$$v_c = i_c r_s + \frac{d\lambda_c}{dt}$$

Rotor

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{\text{shaft}}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

The rotor has the field winding and up to three damper windings (added to provide damping)

# Dq0 Transformations

$$\begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} \triangleq T_{dqo} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{or} \quad i, \lambda$$

In the next few slides we'll quickly go through how these basic equations are transformed into the standard machine models; the point is to show the physical basis for the models.

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = T_{dqo}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix}$$

# Dq0 Transformations

$$T_{dqo} \triangleq \frac{2}{3}\begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \cos\frac{P}{2}\theta_{shaft} & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Note that the transformation depends on the shaft angle.

with the inverse,

$$T_{dqo}^{-1} = \begin{bmatrix} \sin\frac{P}{2}\theta_{shaft} & \cos\frac{P}{2}\theta_{shaft} & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}-\frac{2\pi}{3}\right) & 1 \\ \sin\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & \cos\left(\frac{P}{2}\theta_{shaft}+\frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

# Transformed System

Stator

$$v_d = r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt}$$

$$v_q = r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt}$$

$$v_o = r_s i_o + \frac{d\lambda_o}{dt}$$

Rotor

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt}$$

Shaft

$$\frac{d\theta_{shaft}}{dt} = \frac{2}{P}\omega$$

$$J\frac{2}{P}\frac{d\omega}{dt} = T_m - T_e - T_{f\omega}$$

We are now in the dq0 space

# Electrical & Mechanical Relationships

Electrical system: $v = iR + \frac{d\lambda}{dt}$ (voltage)

$$vi = i^2 R + i\frac{d\lambda}{dt} \quad \text{(power)}$$

P is the number of poles (e.g., 2,4,6); $T_{fw}$ is the friction and windage torque

Mechanical system:

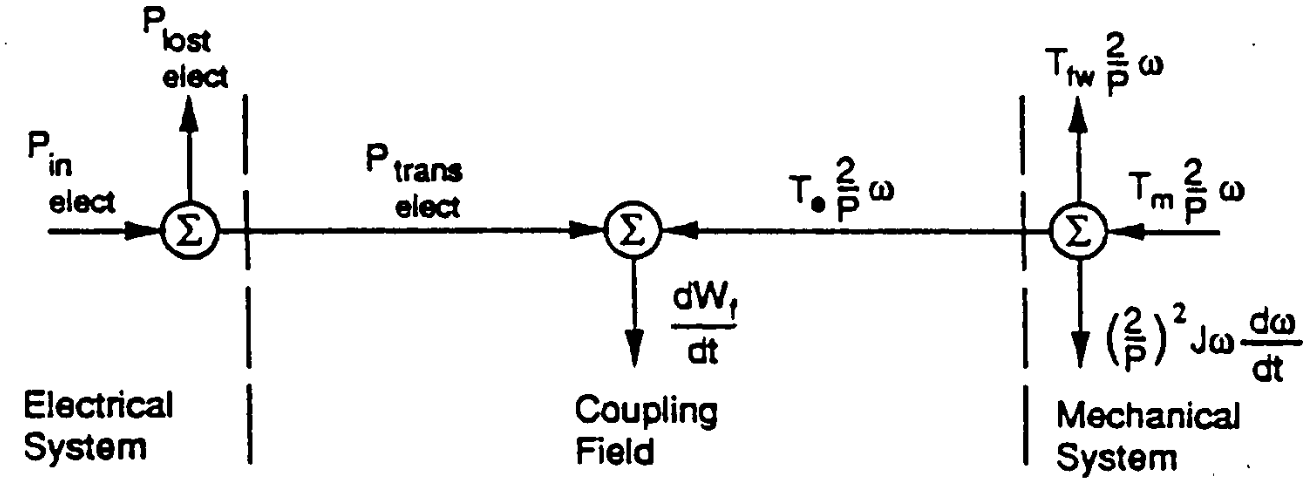
$$J\left(\frac{2}{P}\right)\frac{d\omega}{dt} = T_m - T_e - T_{fw} \quad \text{(torque)}$$

$$J\left(\frac{2}{P}\right)^2 \omega\frac{d\omega}{dt} = \frac{2}{P}\omega T_m - \frac{2}{P}\omega T_e - \frac{2}{P}\omega T_{fw} \quad \text{(power)}$$

# Torque Derivation

- Torque is derived by looking at the overall energy balance in the system
- Three systems: electrical, mechanical and the coupling magnetic field
  - Electrical system losses are in the form of resistance
  - Mechanical system losses are in the form of friction
- Coupling field is assumed to be lossless, hence we can track how energy moves between the electrical and mechanical systems

# Energy Conversion




The coupling field stores and discharges energy but has no losses

Look at the instantaneous power:

$$v_a i_a + v_b i_b + v_c i_c = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o$$

# Change to Conservation of Power

$$P_{\substack{in \\ elect}} = v_a i_a + v_b i_b + v_c i_c + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{\substack{lost \\ elect}} = r_s \left( i_a^2 + i_b^2 + i_c^2 \right) + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{\substack{trans \\ elect}} = i_a \frac{d\lambda_a}{dt} + i_b \frac{d\lambda_b}{dt} + i_c \frac{d\lambda_c}{dt} + i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt} + i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

We are using $v = d\lambda/dt$

# With the Transformed Variables

$$P_{\substack{in\\elect}} = \frac{3}{2} v_d i_d + \frac{3}{2} v_q i_q + 3 v_o i_o + v_{fd} i_{fd} + v_{1d} i_{1d} + v_{1q} i_{1q} + v_{2q} i_{2q}$$

$$P_{\substack{lost\\elect}} = \frac{3}{2} r_s i_d^2 + \frac{3}{2} r_s i_q^2 + 3 r_s i_o^2 + r_{fd} i_{fd}^2 + r_{1d} i_{1d}^2 + r_{1q} i_{1q}^2 + r_{2q} i_{2q}^2$$

$$P_{\substack{trans\\elect}} = -\frac{3}{2} \frac{P}{2} \frac{d\theta_{shaft}}{dt} \lambda_q i_d + \frac{3}{2} i_d \frac{d\lambda_d}{dt} + \frac{3}{2} \frac{P}{2} \frac{d\theta_{shaft}}{dt} \lambda_d i_q + \frac{3}{2} i_q \frac{d\lambda_q}{dt} + 3 i_o \frac{d\lambda_o}{dt}$$

$$+ i_{fd} \frac{d\lambda_{fd}}{dt} + i_{1d} \frac{d\lambda_{1d}}{dt} + i_{1q} \frac{d\lambda_{1q}}{dt} + i_{2q} \frac{d\lambda_{2q}}{dt}$$

# Change in Coupling Field Energy

$$\frac{dW_f}{dt} = \boxed{T_e \frac{2}{P}} \frac{d\theta}{dt} + \boxed{i_a} \frac{d\lambda_a}{dt} + \boxed{i_b} \frac{d\lambda_b}{dt}$$

$$+ \boxed{i_c} \frac{d\lambda_c}{dt} + \boxed{i_{fd}} \frac{d\lambda_{fd}}{dt} + \boxed{i_{1d}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{i_{1q}} \frac{d\lambda_{1q}}{dt} + \boxed{i_{2q}} \frac{d\lambda_{2q}}{dt}$$

First term on right is what is going on mechanically, other terms are what is going on electrically

This requires the lossless coupling field assumption

# Change in Coupling Field Energy

For independent states $\theta, \lambda_a, \lambda_b, \lambda_c, \lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q}$

$$\frac{dW_f}{dt} = \boxed{\frac{\partial W_f}{\partial \theta}} \frac{d\theta}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_a}} \frac{d\lambda_a}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_b}} \frac{d\lambda_b}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_c}} \frac{d\lambda_c}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{fd}}} \frac{d\lambda_{fd}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{1d}}} \frac{d\lambda_{1d}}{dt}$$

$$+ \boxed{\frac{\partial W_f}{\partial \lambda_{1q}}} \frac{d\lambda_{1q}}{dt} + \boxed{\frac{\partial W_f}{\partial \lambda_{2q}}} \frac{d\lambda_{2q}}{dt}$$

# Equate the Coefficients

$$T_e \frac{2}{P} = \frac{\partial W_f}{\partial \theta} \qquad i_a = \frac{\partial W_f}{\partial \lambda_a} \qquad \text{etc.}$$

There are eight such “reciprocity conditions for this model.

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

# Equate the Coefficients

$$\frac{\partial W_f}{\partial \theta_{shaft}} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) + T_e$$

$$\frac{\partial W_f}{\partial \lambda_d} = \frac{3}{2} i_d\,, \qquad \frac{\partial W_f}{\partial \lambda_q} = \frac{3}{2} i_q, \qquad \frac{\partial W_f}{\partial \lambda_o} = 3 i_o$$

$$\frac{\partial W_f}{\partial \lambda_{fd}} = i_{fd}\,, \qquad \frac{\partial W_f}{\partial \lambda_{1d}} = i_{1d}\,, \qquad \frac{\partial W_f}{\partial \lambda_{1q}} = i_{1q}, \qquad \frac{\partial W_f}{\partial \lambda_{2q}} = i_{2q}$$

These are key conditions – i.e. the first one gives an expression for the torque in terms of the coupling field energy.

# Coupling Field Energy

- The coupling field energy is calculated using a path independent integration
  - For integral to be path independent, the partial derivatives of all integrands with respect to the other states must be equal

For example, $$\frac{3}{2}\frac{\partial i_d}{\partial \lambda_{fd}} = \frac{\partial i_{fd}}{\partial \lambda_d}$$

- Since integration is path independent, choose a convenient path
  - Start with a de-energized system so variables are zero
  - Integrate shaft position while other variables are zero
  - Integrate sources in sequence with shaft at final value

# Define Unscaled Variables

$$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$

$\omega_s$ is the rated synchronous speed
$\delta$ plays an important role!

$$\frac{d\lambda_d}{dt} = -r_s i_d + \omega\lambda_q + v_d$$

$$\frac{d\lambda_q}{dt} = -r_s i_q - \omega\lambda_d + v_q$$

$$\frac{d\lambda_o}{dt} = -r_s i_o + v_o$$

$$\frac{d\lambda_{fd}}{dt} = -r_{fd} i_{fd} + v_{fd}$$

$$\frac{d\lambda_{1d}}{dt} = -r_{1d} i_{1d} + v_{1d}$$

$$\frac{d\lambda_{1q}}{dt} = -r_{1q} i_{1q} + v_{1q}$$

$$\frac{d\lambda_{2q}}{dt} = -r_{2q} i_{2q} + v_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$J\frac{2}{p}\frac{d\omega}{dt} = T_m + \left(\frac{3}{2}\right)\left(\frac{P}{2}\right)\left(\lambda_d i_q - \lambda_q i_d\right) - T_{f\omega}$$

# Synchronous Machine Equations in Per Unit

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + V_o$$

$$\frac{1}{\omega_s}\frac{d\psi_{fd}}{dt} = -R_{fd} I_{fd} + V_{fd}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1d}}{dt} = -R_{1d} I_{1d} + V_{1d}$$

$$\frac{1}{\omega_s}\frac{d\psi_{1q}}{dt} = -R_{1q} I_{1q} + V_{1q}$$

$$\frac{1}{\omega_s}\frac{d\psi_{2q}}{dt} = -R_{2q} I2 + V_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

The $\psi$ variables are in the $\lambda$ variables in per unit (see book 3.50 to 3.52)

Units of $H$ are seconds

# Sinusoidal Steady-State

$$V_a = \sqrt{2}V_s \cos(\omega_s t + \theta_{vs})$$

$$V_b = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} - \frac{2\pi}{3}\right)$$

$$V_c = \sqrt{2}V_s \cos\left(\omega_s t + \theta_{vs} + \frac{2\pi}{3}\right)$$

$$I_a = \sqrt{2}I_s \cos(\omega_s t + \theta_{is})$$

$$I_b = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} - \frac{2\pi}{3}\right)$$

$$I_c = \sqrt{2}I_s \cos\left(\omega_s t + \theta_{is} + \frac{2\pi}{3}\right)$$

Here we consider the application to balanced, sinusoidal conditions

# Simplifying Using δ

- Define $$\delta \triangleq \frac{P}{2}\theta_{shaft} - \omega_s t$$
- Hence $$V_d = V_s \sin(\delta - \theta_{vs})$$ $$V_q = V_s \cos(\delta - \theta_{vs})$$ $$I_d = I_s \sin(\delta - \theta_{is})$$ $$I_q = I_s \cos(\delta - \theta_{is})$$

The conclusion is if we know δ, then we can easily relate the phase to the dq values!

- These algebraic equations can be written as complex equations

$$\left(V_d + jV_q\right)e^{j(\delta - \pi/2)} = V_s e^{j\theta_{vs}}$$

$$\left(I_d + jI_q\right)e^{j(\delta - \pi/2)} = I_s e^{j\theta_{is}}$$

# Summary So Far

- The model as developed so far has been derived using the following assumptions
  - The stator has three coils in a balanced configuration, spaced 120 electrical degrees apart
  - Rotor has four coils in a balanced configuration located 90 electrical degrees apart
  - Relationship between the flux linkages and currents must reflect a conservative coupling field
  - The relationships between the flux linkages and currents must be independent of $\theta_{shaft}$ when expressed in the dq0 coordinate system

# Assuming a Linear Magnetic Circuit

- If the flux linkages are assumed to be a linear function of the currents then we can write

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \\ \hline \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} L_{ss}\left(\theta_{shaft}\right) & L_{sr}\left(\theta_{shaft}\right) \\ \hline L_{rs}\left(\theta_{shaft}\right) & L_{rr}\left(\theta_{shaft}\right) \end{array}\right] \begin{bmatrix} i_a \\ i_b \\ i_c \\ \hline i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

The rotor self-inductance matrix $L_{rr}$ is independent of $\theta_{shaft}$

# Conversion to dq0 for Angle Independence

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \\ \lambda_{fd} \\ \lambda_{1d} \\ \lambda_{1q} \\ \lambda_{2q} \end{bmatrix} = \left[\begin{array}{c|c} T_{dqo} L_{ss} T_{dqo}^{-1} & T_{dqo} L_{sr} \\ \hline L_{rs} T_{dqo}^{-1} & L_{rr} \end{array}\right] \begin{bmatrix} i_d \\ i_q \\ i_o \\ i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix}$$

# Conversion to dq0 for Angle Independence

$$\lambda_d = \left(L_{\ell s} + L_{md}\right) i_d + L_{sfd} i_{fd} + L_{s1d} i_{1d}$$

$$\lambda_{fd} = \frac{3}{2} L_{sfd} i_d + L_{fdfd} i_{fd} + L_{fd1d} i_{1d}$$

$$\lambda_{1d} = \frac{3}{2} L_{s1d} i_d + L_{fd1d} i_{fd} + L_{1d1d} i_{1d}$$

$$\lambda_q = \left(L_{\ell s} + L_{mq}\right) i_q + L_{s1q} i_{1q} + L_{s2q} i_{2q}$$

$$\lambda_{1q} = \frac{3}{2} L_{s1q} i_q + L_{1q1q} i_{1q} + L_{1q2q} i_{2q}$$

$$\lambda_{2q} = \frac{3}{2} L_{s2q} i_q + L_{1q2q} i_{1q} + L_{2q2q} i_{2q}$$

$$\lambda_o = L_{\ell s} i_o$$

$$L_{md} = \frac{3}{2}\left(L_A + L_B\right),$$

$$L_{mq} = \frac{3}{2}\left(L_A - L_B\right)$$

For a round rotor machine $L_B$ is small and hence $L_{md}$ is close to $L_{mq}$. For a salient pole machine $L_{md}$ is substantially larger. Note $L_A$ and $L_B$ are defined in book 3.95.

# Convert to Normalized at f = $\omega_s$

- Convert to per unit, and assume frequency of $\omega_s$
- Then define new per unit reactance variables

$$X_{\ell s} = \frac{\omega_s L_{\ell s}}{Z_{BDQ}}, \quad X_{md} = \frac{\omega_s L_{md}}{Z_{BDQ}}, \quad X_{mq} = \frac{\omega_s L_{mq}}{Z_{BDQ}}$$

$$X_{fd} = \frac{\omega_s L_{fdfd}}{Z_{BFD}}, \quad X_{1d} = \frac{\omega_s L_{1d1d}}{Z_{B1D}}, \quad X_{fd1d} = \frac{\omega_s L_{fd1d} L_{sfd}}{Z_{BFD} L_{s1d}}$$

$$X_{1q} = \frac{\omega_s L_{1q1q}}{Z_{B1Q}}, \quad X_{2q} = \frac{\omega_s L_{2q2q}}{Z_{B2Q}}, \quad X_{1q2q} = \frac{\omega_s L_{1q2q} L_{s1q}}{Z_{B1Q} L_{s2q}}$$

$$X_{\ell fd} = X_{fd} - X_{md}, \quad X_{\ell 1d} = X_{1d} - X_{md}$$

$$X_{\ell 1q} = X_{1q} - X_{mq}, \quad X_{\ell 2q} = X_{2q} - X_{mq}$$

$$X_d = X_{\ell s} + X_{md}, \quad X_q = X_{\ell s} + X_{mq}$$

# Key Simulation Parameters

- The key parameters that occur in most models can then be defined as

$$X'_d = X_{\ell s} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{\ell fd}}} = X_d - \frac{X_{md}^2}{X_{fd}}$$

$$X'_q = X_{\ell s} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{\ell 1q}}} = X_q - \frac{X_{mq}^2}{X_{1q}}$$

$$T'_{do} = \frac{Xfd}{\omega_s R_{fd}}, \quad T'_{qo} = \frac{X_{1q}}{\omega_s R_{1q}}$$

These values will be used in all the synchronous machine models

In a salient rotor machine $X_{mq}$ is small so $X_q = X'_{q;}$ also $X_{1q}$ is small so $T'_{q0}$ is small

# Key Simulation Parameters

- And the subtransient parameters

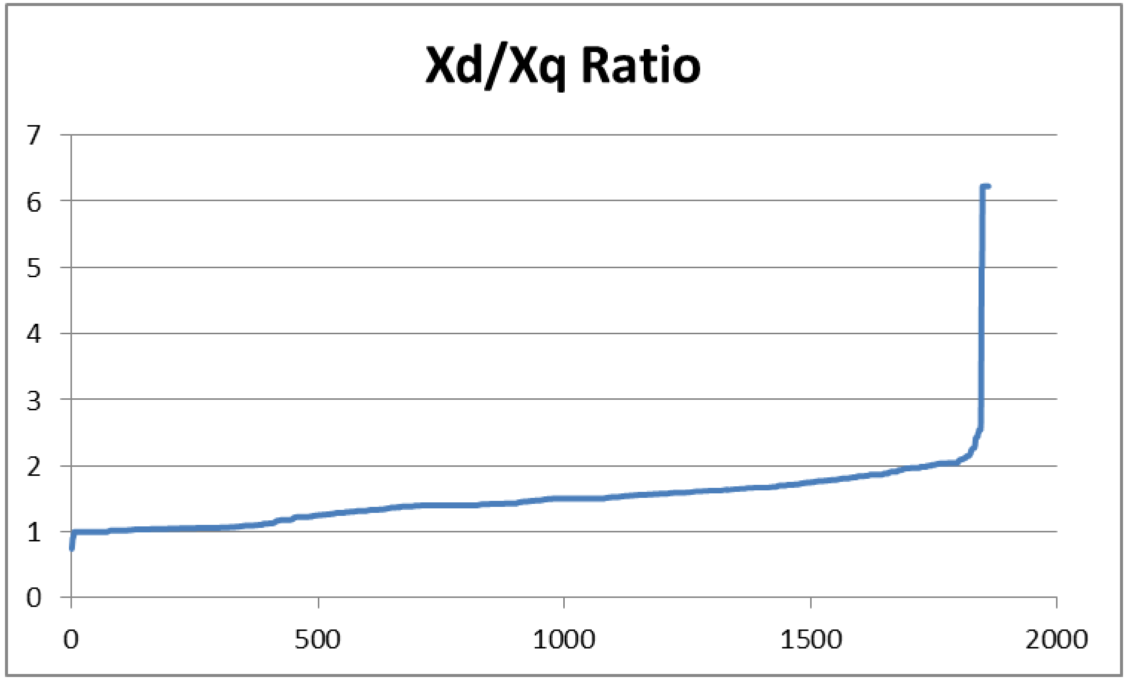
$$X''_d = X_{\ell s} + \frac{1}{\dfrac{1}{X_{md}} + \dfrac{1}{X_{\ell fd}} + \dfrac{1}{X_{\ell 1d}}}$$

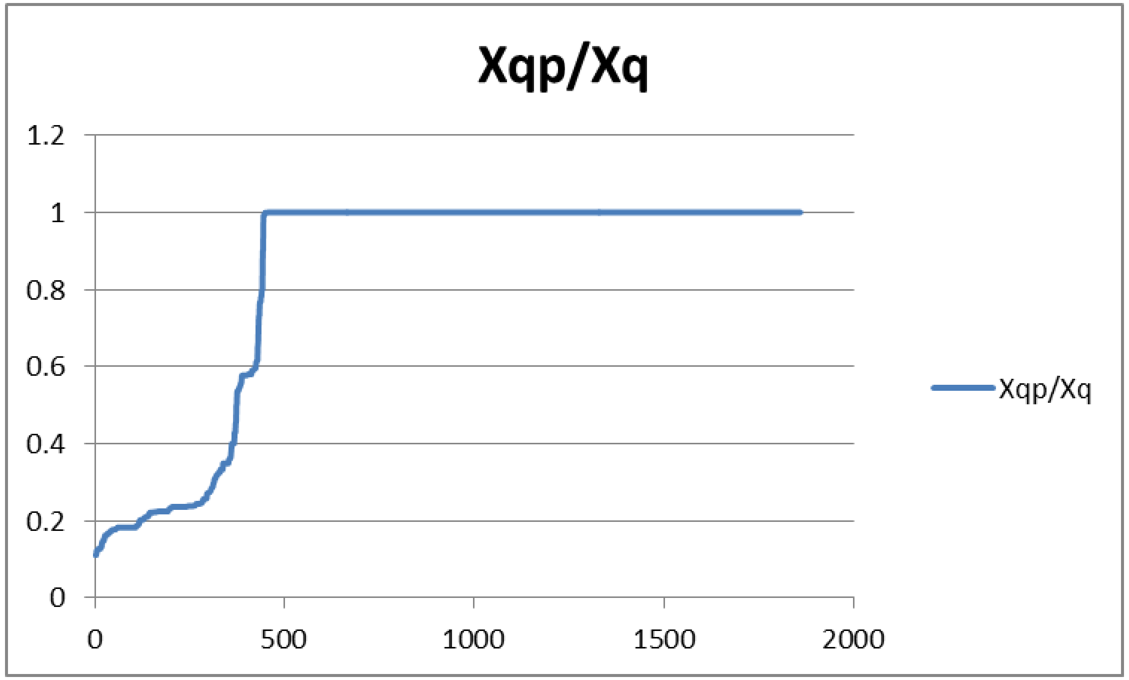
$$X''_q = X_{\ell s} + \frac{1}{\dfrac{1}{X_{mq}} + \dfrac{1}{X_{\ell 1q}} + \dfrac{1}{X_{\ell 2q}}}$$

These values will be used in the subtransient machine models. It is common to assume $X''_d = X''_q$

$$T''_{do} = \frac{1}{\omega_s R_{1d}} \left( X \right.$$ [redacted] $\omega_s R_{2q}$ $\frac{1}{X_{md}} +$ [redacted]

# Example Xd/Xq Ratios and X'q/Xq Ratios for a WECC Case




About 75% are Clearly Salient Pole Machines!

# Internal Variables

- Define the following variables, which are quite important in subsequent models

$$E'_q \triangleq \frac{X_{md}}{X_{fd}} \psi_{fd}$$

$$E'_d \triangleq \frac{X_{mq}}{X_{1q}} \psi_{1q}$$

$$E_{fd} \triangleq \frac{X_{md}}{R_{fd}} V_{fd}$$

Hence $E'_q$ and $E'_d$ are scaled flux linkages (with $E'_q$ associated with the field flux linkage and $E'_d$ the damper winding). $E_{fd}$ is the scaled field voltage.

# Dynamic Model Development

- In developing the dynamic model not all of the currents and fluxes are independent
  - In this formulation only seven out of fourteen are independent
- Approach is to eliminate the rotor currents, retaining the terminal currents ($I_d$, $I_q$, $I_0$) for matching the network boundary conditions

# Rotor Currents

- Use new variables to solve for the rotor currents

$$\psi_d = -X''_d I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$I_{fd} = \frac{1}{X_{md}}\left[E'_q + \left(X_d - X'_d\right)\left(I_d - I_{1d}\right)\right]$$

$$I_{1d} = \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left[\psi_{1d} + \left(X'_d - X_{\ell s}\right) I_d - E'_q\right]$$

$$\psi_q = -X''_q I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$I_{1q} = \frac{1}{X_{mq}}\left[-E'_d + \left(X_q - X'_q\right)\left(I_q - I_{2q}\right)\right]$$

$$I_{2q} = \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left[\psi_{2q} + \left(X'_q - X_{\ell s}\right) I_q + E'_d\right]$$

$$\psi_o = X_{\ell s}\left(-I_o\right)$$

# Final Complete Model

$$\frac{1}{\omega_s}\frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s}\psi_q + V_d$$

$$\frac{1}{\omega_s}\frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s}\psi_d + V_q$$

$$\frac{1}{\omega_s}\frac{d\psi_o}{dt} = R_s I_o + Vo$$

These first three equations define what are known as the stator transients; we will shortly approximate them as algebraic constraints

$$T'_{do}\frac{dE'_q}{dt} = -E'_q - \left(X_d - X'_d\right)\left[I_d - \frac{X'_d - X''_d}{\left(X'_d - X_{\ell s}\right)^2}\left(\psi_{1d} + \left(X'_d - X_{\ell s}\right)I_d - E'_q\right)\right] + E_{fd}$$

$$T'_{qo}\frac{dE'_d}{dt} = -E'_d + \left(X_q - X'_q\right)\left[I_q - \frac{X'_q - X''_q}{\left(X'_q - X_{\ell s}\right)^2}\left(\psi_{2q} + \left(X'_q - X_{\ell s}\right)I_q + E'_d\right)\right]$$

# Final Complete Model, cont.

$$T''_{do}\frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$T''_{qo}\frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_M - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = -X''_d I_d + \frac{\left(X''_d - X_{\ell s}\right)}{\left(X'_d - X_{\ell s}\right)} E'_q + \frac{\left(X'_d - X''_d\right)}{\left(X'_d - X_{\ell s}\right)} \psi_{1d}$$

$$\psi_q = -X''_q I_q - \frac{\left(X''_q - X_{\ell s}\right)}{\left(X'_q - X_{\ell s}\right)} E'_d + \frac{\left(X'_q - X''_q\right)}{\left(X'_q - X_{\ell s}\right)} \psi_{2q}$$

$$\psi_o = -X_{\ell s} I_o$$

$T_{FW}$ is the friction and windage component

# Single-Machine Steady-State

$$0 = R_s I_d + \psi_q + V_d \qquad (\omega = \omega_s)$$

$$0 = R_s I_q - \psi_d + V_q$$

$$0 = R_s I_o + V_o$$

$$0 = -E'_q - \left(X_d - X'_d\right) I_d + E_{fd}$$

$$0 = -\psi_{1d} + E'_q - \left(X'_d - X_{\ell s}\right) I_d$$

$$0 = -E'_d + \left(X_q - X'_q\right) I_q$$

$$0 = -\psi_{2q} - E'_d - \left(X'_q - X_{\ell s}\right) I_q$$

$$0 = \omega - \omega_s$$

$$0 = T_m - \left(\psi_d I_q - \psi_q I_d\right) - T_{FW}$$

$$\psi_d = E'_q - X''_d I_d$$

$$\psi_q = -X''_q I_q - E'_d$$

$$\psi_o = -X_{\ell s} I_o$$

The key variable we need to determine the initial conditions is actually δ, which doesn't appear explicitly in these equations!

# Field Current

- The field current, $I_{fd}$, is defined in steady-state as

$$I_{fd} = E_{fd} / X_{md}$$

- However, what is usually used in transient stability simulations for the field current is the product

$$I_{fd} X_{md}$$

- So the value of $X_{md}$ is not needed

# Single-Machine Steady-State

- Previous derivation was done assuming a linear magnetic circuit
- We'll consider the nonlinear magnetic circuit later but will first do the steady-state condition (3.6)
- In steady-state the speed is constant (equal to $\omega_s$), $\delta$ is constant, and all the derivatives are zero
- Initial values are determined from the terminal conditions: voltage magnitude, voltage angle, real and reactive power injection

# Determining δ without Saturation

- In order to get the initial values for the variables we need to determine δ
- We'll eventually consider two approaches: the simple one when there is no saturation, and then later a general approach for models with saturation
- To derive the simple approach we have

$$V_d = R_s I_d + E'_d + X'_q I_q$$

$$V_q = -R_s I_q + E'_q - X'_d I_d$$

# Determining δ without Saturation

Since $j = e^{j(\pi/2)}$

$$\tilde{E} = \left[ \left( X_q - X'_d \right) I_d + E'_q \right] e^{j\delta}$$

- In terms of the terminal values

$$\tilde{E} = \tilde{V}_{as} + (R_s + jX_q)\tilde{I}_{as}$$

$$\textit{The angle on } \tilde{E} = \delta$$