

ECEN 667

Power System Stability

Lecture 9: Synchronous Machine Modeling

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Announcements



- Read Chapter 5; look at Appendix A
- Homework 3 should be done before the first exam, but does not need to be turned in.
- First exam is on Tuesday October 3 during class (except for the distance education students)
 - It is closed-book and closed-notes, but one 8.5 by 11 inch hand written note sheet and calculators allowed
 - My first exam from ECEN 667 in Fall 2021 has been posted to Canvas

ERCOT Opportunity



The graphic features a group of ten professionals in business attire standing in a modern office setting. The background is a stylized landscape with a wind turbine, overlaid with a network of blue lines and data points, suggesting a focus on technology and energy. The ERCOT logo is positioned in the lower right corner of the graphic area.

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EVENT DATE
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9:30 A.M. CDT

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- ◆ Internship & Engineer Development Program

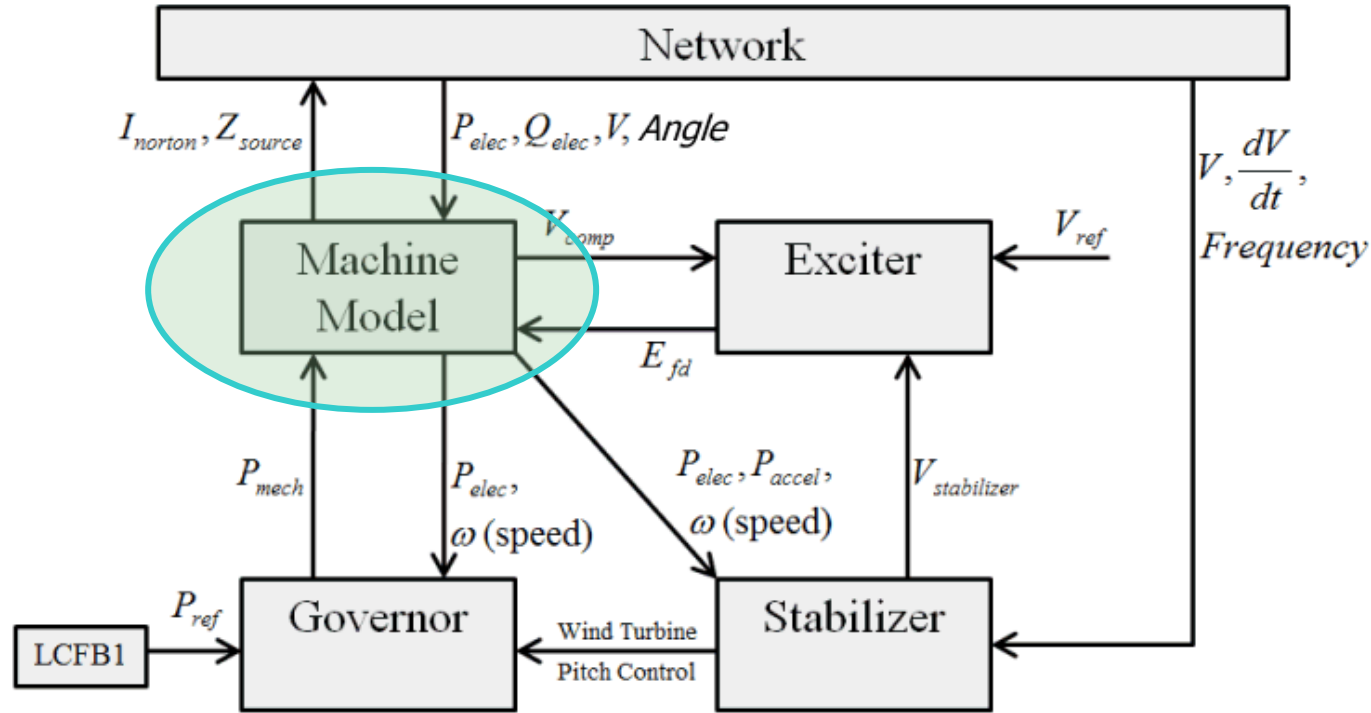
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If you have any questions, please email EDPReach@ercot.com

Machine Models



P_{elec} = Electrical Power
 Q_{elec} = Electrical Reactive Power
 V = Voltage at Terminal Bus
 $\frac{dV}{dt}$ = Derivate of Voltage
 V_{comp} = Compensated Voltage

P_{mech} = Mechanical Power
 $\omega(\text{speed})$ = Rotor Speed (often it's deviation from nominal speed)
 P_{accel} = Accelerating Power
 $V_{stabilizer}$ = Output of Stabilizer
 V_{ref} = Exciter Control Setpoint (determined during initialization)
 P_{ref} = Governor Control Setpoint (determined during initialization)

Final Complete Model



$$\frac{1}{\omega_s} \frac{d\psi_d}{dt} = R_s I_d + \frac{\omega}{\omega_s} \psi_q + V_d$$

$$\frac{1}{\omega_s} \frac{d\psi_q}{dt} = R_s I_q - \frac{\omega}{\omega_s} \psi_d + V_q$$

$$\frac{1}{\omega_s} \frac{d\psi_o}{dt} = R_s I_o + V_o$$

These first three equations define what are known as the stator transients; we will shortly approximate them as algebraic constraints

$$T'_{do} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d) \left[I_d - \frac{X'_d - X''_d}{(X'_d - X_{ls})^2} (\psi_{1d} + (X'_d - X_{ls}) I_d - E'_q) \right] + E_{fd}$$

$$T'_{qo} \frac{dE'_d}{dt} = -E'_d + (X_q - X'_q) \left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{ls})^2} (\psi_{2q} + (X'_q - X_{ls}) I_q + E'_d) \right]$$

Final Complete Model, cont.



$$T_{do}'' \frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{ls})I_d \quad \psi_d = -X_d''I_d + \frac{(X_d'' - X_{ls})}{(X'_d - X_{ls})}E'_q + \frac{(X'_d - X_{ls})}{(X'_d - X_{ls})}\psi_{1d}$$

$$T_{qo}'' \frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{ls})I_q \quad \psi_q = -X_q''I_q - \frac{(X_q'' - X_{ls})}{(X'_q - X_{ls})}E'_d + \frac{(X'_q - X_q'')}{(X'_q - X_{ls})}\psi_{2q}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\psi_o = -X_{ls}I_o$$

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = T_M - (\psi_d I_q - \psi_q I_d) - T_{FW}$$

T_{FW} is the friction and windage component

Single-Machine Steady-State



$$\begin{aligned} 0 &= R_s I_d + \psi_q + V_d \\ 0 &= R_s I_q - \psi_d + V_q \\ 0 &= R_s I_o + V_o \end{aligned} \quad \left(\omega = \omega_s \right)$$



$$\begin{aligned} \psi_d &= E'_q - X'_d I_d \\ \psi_q &= -X'_q I_q - E'_d \\ \psi_o &= -X_{\ell s} I_o \end{aligned}$$

The top two are equations 3.222 and 3.223 from the book

$$\begin{aligned} 0 &= -E'_q - (X_d - X'_d) I_d + E_{fd} \\ 0 &= -\psi_{1d} + E'_q - (X'_d - X_{\ell s}) I_d \\ 0 &= -E'_d + (X_q - X'_q) I_q \\ 0 &= -\psi_{2q} - E'_d - (X'_q - X_{\ell s}) I_q \\ 0 &= \omega - \omega_s \\ 0 &= T_m - (\psi_d I_q - \psi_q I_d) - T_{FW} \end{aligned}$$

The key variable we need to determine the initial conditions is actually δ , which doesn't appear explicitly in these equations!

Single-Machine Steady-State



- Previous derivation was done assuming a linear magnetic circuit
- We'll consider the nonlinear magnetic circuit later but will first do the steady-state condition (3.6)
- In steady-state the speed is constant (equal to ω_s), δ is constant, and all the derivatives are zero
- Initial values are determined from the terminal conditions: voltage magnitude, voltage angle, real and reactive power injection

Determining δ without Saturation



- In order to get the initial values for the variables we need to determine δ
- We'll eventually consider two approaches: the simple one when there is no saturation, and then later a general approach for models with saturation
- To derive the simple approach we have

$$V_d = R_s I_d + E'_d + X'_q I_q$$

$$V_q = -R_s I_q + E'_q - X'_d I_d$$

These are derived by combining

$$0 = R_s I_d + \psi_q + V_d$$

$$0 = R_s I_q - \psi_d + V_q$$

with

$$\psi_d = E'_q - X'_d I_d$$

$$\psi_q = -X'_q I_q - E'_d$$

Determining δ without Saturation

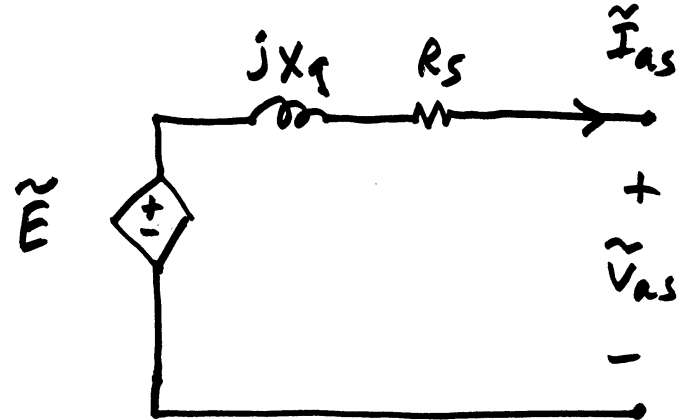
Since $j = e^{j(\pi/2)}$

$$\tilde{E} = \left[(X_q - X'_d) I_d + E'_q \right] e^{j\delta}$$

- In terms of the terminal values

$$\tilde{E} = \tilde{V}_{as} + (R_s + jX_q) \tilde{I}_{as}$$

The angle on $\tilde{E} = \delta$

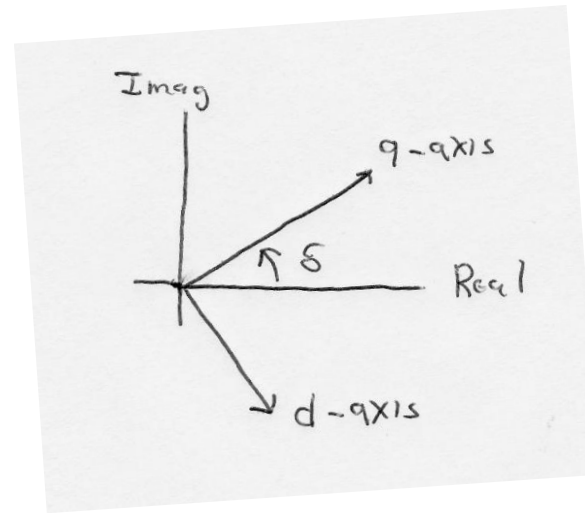


D-q Reference Frame

- Machine voltage and current are “transformed” into the d-q reference frame using the rotor angle, δ
- Terminal voltage in network (power flow) reference frame are
$$V_S = V_t = V_r + jV_i$$

$$\begin{bmatrix} V_r \\ V_i \end{bmatrix} = \begin{bmatrix} \sin \delta & \cos \delta \\ -\cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} V_d \\ V_q \end{bmatrix}$$

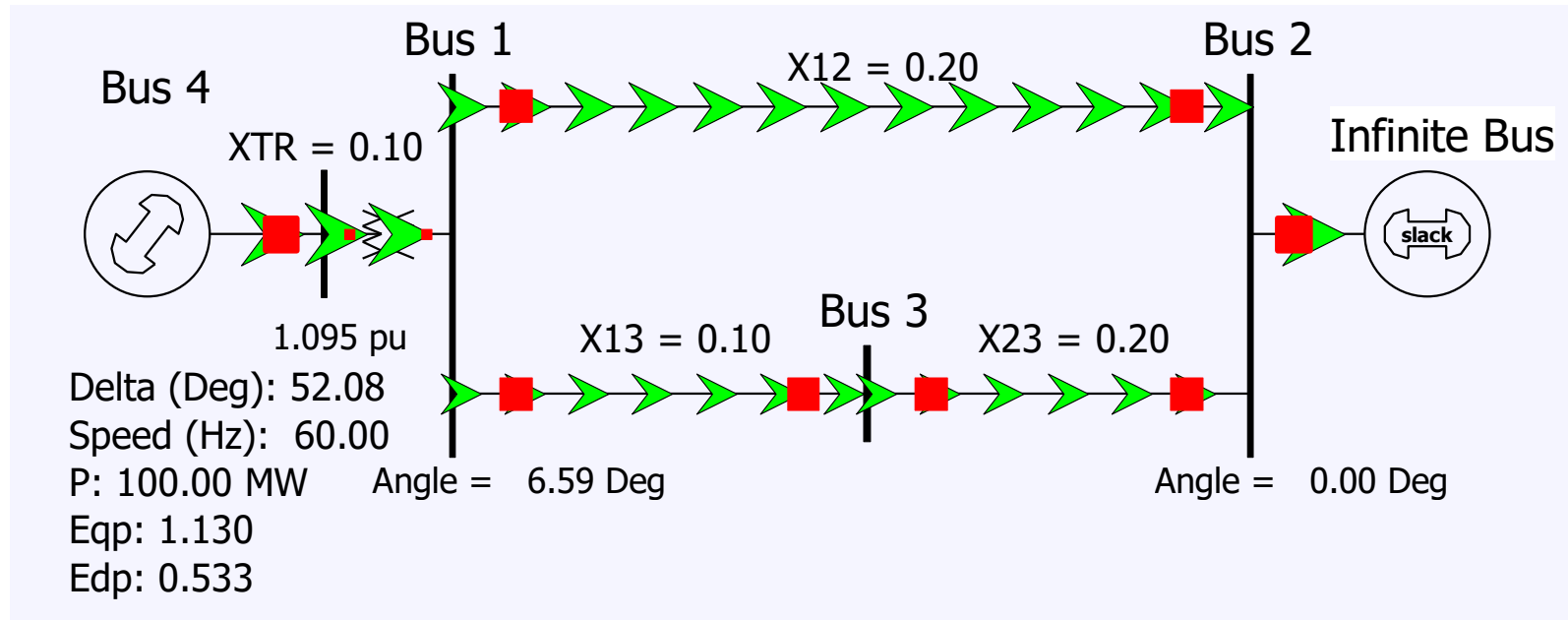
$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$



A Steady-State Example



- Assume a generator is supplying 1.0 pu real power at 0.95 pf lagging into an infinite bus at 1.0 pu voltage through the below network. Generator pu values are $R_s=0$, $X_d=2.1$, $X_q=2.0$, $X'_d=0.3$, $X'_q=0.5$



A Steady-State Example, cont.



- First determine the current out of the generator from the initial conditions, then the terminal voltage

$$\tilde{I} = 1.0526 \angle -18.20^\circ = 1 - j0.3288$$

$$\begin{aligned}\tilde{V}_s &= 1.0 \angle 0^\circ + (j0.22)(1.0526 \angle -18.20^\circ) \\ &= 1.0946 \angle 11.59^\circ = 1.0723 + j0.220\end{aligned}$$

A Steady-State Example, cont.



- We can then get the initial angle and initial dq values

$$\tilde{E} = 1.0946 \angle 11.59^\circ + (j2.0)(1.052 \angle -18.2^\circ) = 2.814 \angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

$$V_d + jV_q = V_s e^{j\theta} e^{j(\pi/2 - \delta)} = 1.0945 \angle (11.6 + 90 - 52.1)$$

$$= 1.0945 \angle 49.5^\circ = 0.710 + j0.832$$

A Steady-State Example, cont



- The initial state variables are determined by solving with the differential equations equal to zero.

$$E'_q = V_q + R_s I_q + X'_d I_d = 0.8326 + (0.3)(0.9909) = 1.1299$$

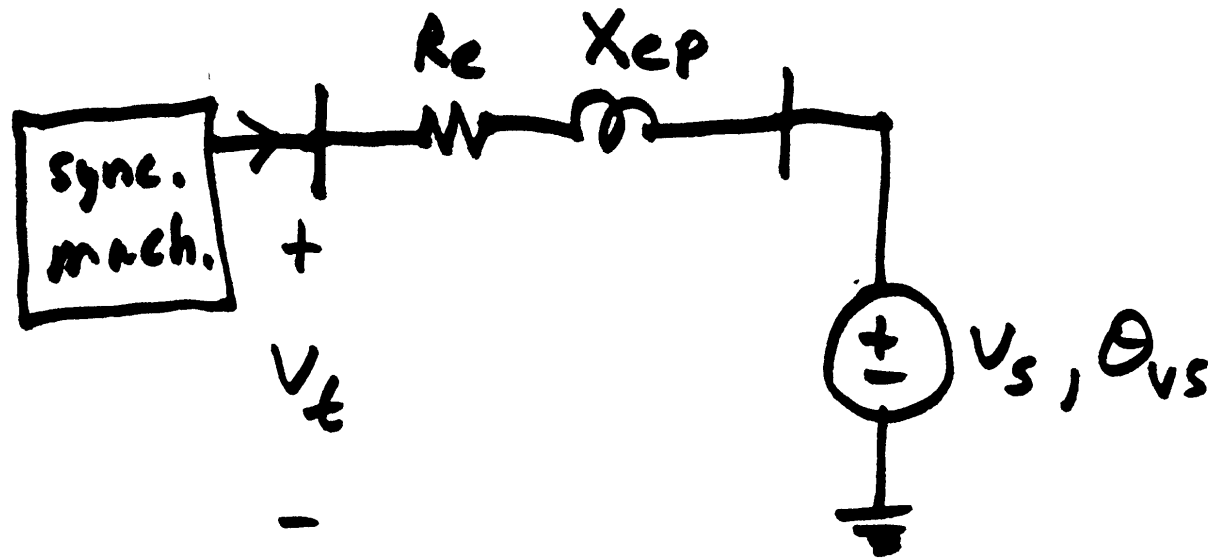
$$E'_d = V_d - R_s I_d - X'_q I_q = 0.7107 - (0.5)(0.3553) = 0.5330$$

$$E_{fd} = E'_q + (X_d - X'_d) I_d = 1.1299 + (2.1 - 0.3)(0.9909) = 2.9135$$

The value of E_{fd} is determined from the equilibrium condition

$$0 = -E'_q - (X_d - X'_d) I_d + E_{fd}$$

Single Machine, Infinite Bus System (SMIB)



Usually the infinite bus angle, θ_{vs} , is zero

This example can be simplified by combining machine values with line values

$$\Psi_{de} = \Psi_d + \Psi_{ed}$$

$$X_{de} = X_d + X_{ep} \quad \text{etc}$$

$$R_{se} = R_s + R_e$$

Introduce New Constants



$$\omega_t = T_s (\omega - \omega_s) \quad \text{“Transient Speed”}$$

$$T_s = \sqrt{\frac{2H}{\omega_s}} \quad \text{Mechanical time constant}$$

$$\varepsilon = \frac{1}{\omega_s} \quad \text{A small parameter}$$

We are ignoring the exciter and governor for now; they will be covered in detail later

Stator Flux Differential Equations



$$\varepsilon \frac{d\psi_{de}}{dt} = R_{se} I_d + \left(1 + \frac{\varepsilon}{T_s} \omega_t \right) \psi_{qe} + V_s \sin(\delta - \theta_{vs})$$

$$\varepsilon \frac{d\psi_{qe}}{dt} = R_{se} I_q - \left(1 + \frac{\varepsilon}{T_s} \omega_t \right) \psi_{de} + V_s \cos(\delta - \theta_{vs})$$

$$\varepsilon \frac{d\psi_{oe}}{dt} = R_{se} I_o$$

Elimination of Stator Transients



- If we assume the stator flux equations are much faster than the remaining equations, then letting ε go to zero allows us to replace the differential equations with algebraic equations

$$0 = R_{se} I_d + \psi_{qe} + V_s \sin(\delta - \theta_{vs})$$

$$0 = R_{se} I_q - \psi_{de} + V_s \cos(\delta - \theta_{vs})$$

$$0 = R_{se} I_o$$

This assumption might not be valid if we are considering faster dynamics on other devices (such as converter dynamics)

Impact on Studies

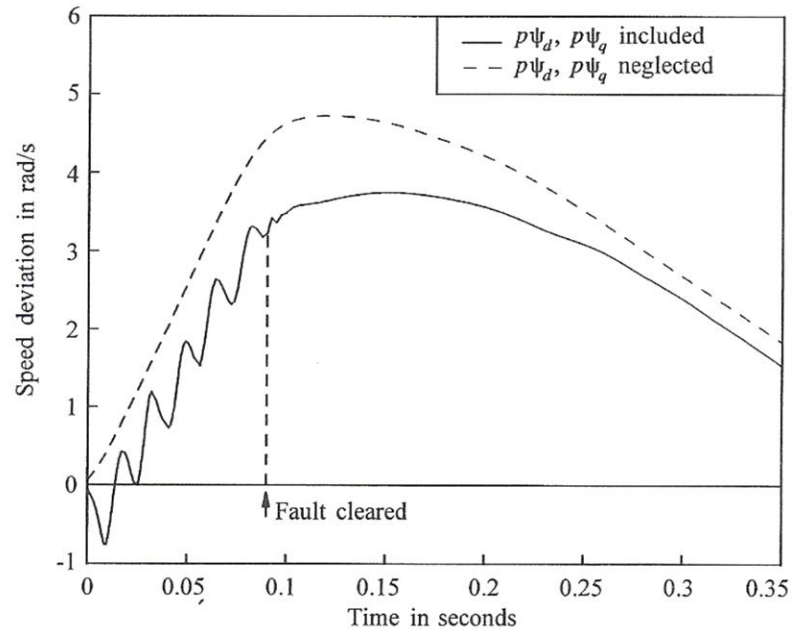


Figure 5.3 Effect of neglecting stator transients on speed deviation

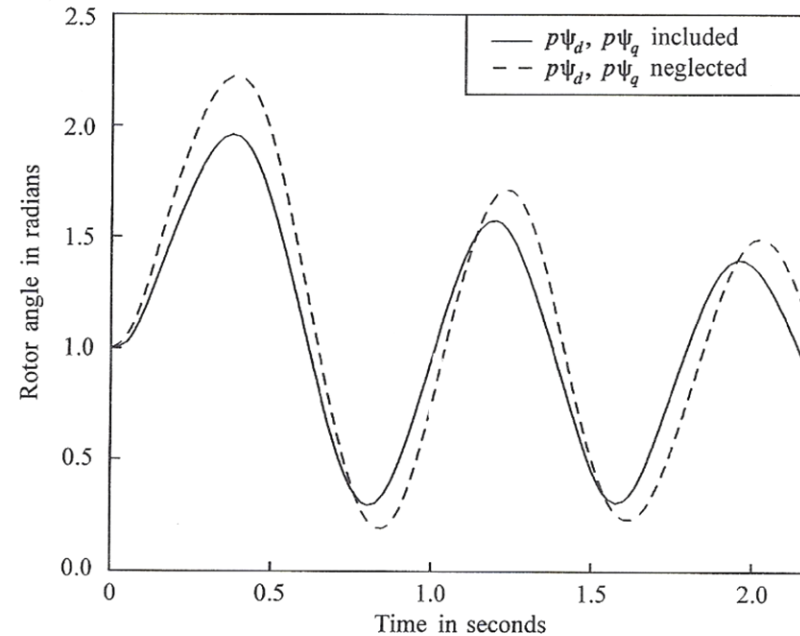


Figure 5.4 Effect of neglecting stator transients on rotor angle swings

Stator transients are not usually considered in stability studies

Machine Variable Summary



- Three fast dynamic states, now eliminated

$$\Psi_{de}, \Psi_{qe}, \Psi_{oe}$$

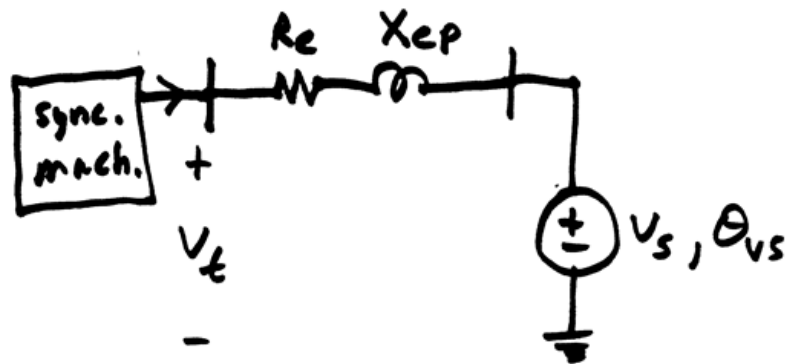
- Seven not so fast dynamic states

$$E'_q, \Psi_{1d}, E'_d, \Psi_{2q}, \delta, \omega_t, E_{fd}$$

- Eight algebraic states

$$I_d, I_q, I_o, V_d, V_q, V_t, \Psi_{ed}, \Psi_{eq}$$

We'll get to the exciter and governor shortly



$$V_t = \sqrt{V_d^2 + V_q^2}$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs})$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs})$$

Network Expressions



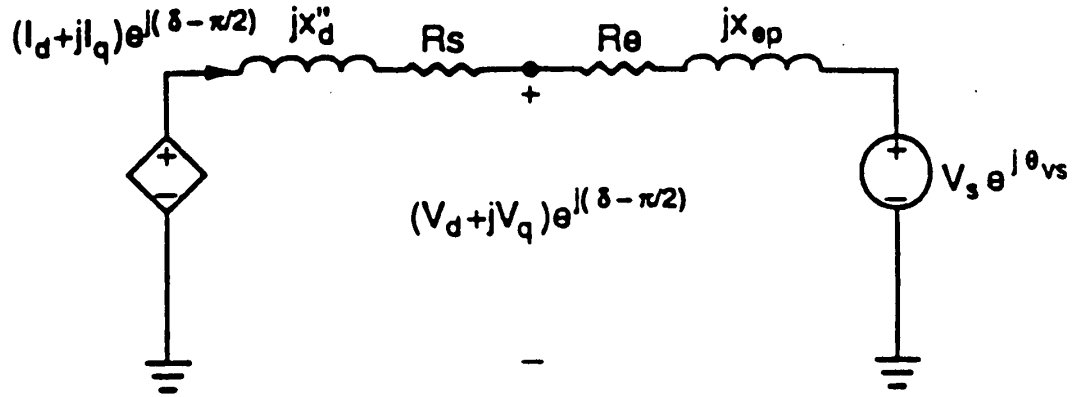
$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs})$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs})$$

These two equations can be written as one complex equation.

$$\begin{aligned} (V_d + jV_q)e^{j(\delta - \pi/2)} &= (R_e + jX_{ep})(I_d + jI_q)e^{j(\delta - \pi/2)} \\ &\quad + V_s e^{j\theta_{vs}} \end{aligned}$$

Subtransient Algebraic Circuit



$$\left[\left(\frac{(X_q'' - X_{ls})}{(X_q' - X_{ls})} E_d' - \frac{(X_q' - X_q'')}{(X_q' - X_{ls})} \psi_{2q} + (X_q'' - X_d'') I_q \right) \right]$$

$$+ j \left(\frac{(X_d'' - X_{ls})}{(X_d' - X_{ls})} E_q' + \frac{(X_d' - X_d'')}{(X_d' - X_{ls})} \psi_{1d} \right) \right] e^{j(\delta - \pi/2)}$$

Network Reference Frame



- In transient stability the initial generator values are set from a power flow solution, which has the terminal voltage and power injection
 - Current injection is just conjugate of Power/Voltage
- These values are on the network reference frame, with the angle given by the slack bus angle

$$\bar{V}_j = V_{r,j} + jV_{i,j} \quad \text{or} \quad \bar{V}_j = V_{Dj} + jV_{Qj}$$

- Voltages at bus j converted to d-q reference by

$$\begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix} = \begin{bmatrix} \sin \delta & -\cos \delta \\ \cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} \quad \begin{bmatrix} V_{r,j} \\ V_{i,j} \end{bmatrix} = \begin{bmatrix} \sin \delta & \cos \delta \\ -\cos \delta & \sin \delta \end{bmatrix} \begin{bmatrix} V_{d,j} \\ V_{q,j} \end{bmatrix}$$

Network Reference Frame



- Issue of calculating δ , which is key, will be considered for each model
- Starting point is the per unit stator voltages

$$V_d = -\psi_q \omega - R_s I_d$$

$$V_q = \psi_d \omega - R_s I_q$$

$$\text{Equivalently, } (V_d + jV_q) + R_s (I_d + jI_q) = \omega(-\psi_q + j\psi_d)$$

- Sometimes the scaling of the flux by the speed is neglected, but this can have a major solution impact
- In per unit the initial speed is unity

Simplified Machine Models



- Often more simplified models were used to represent synchronous machines
- These simplifications are becoming much less common but they are still used in some situations and can be helpful for understanding generator behavior
- Next several slides go through how these models can be simplified, then we'll cover the standard industrial models

Two-Axis Model



- If we assume the damper winding dynamics are sufficiently fast, then T''_{d0} and T''_{q0} go to zero, so there is an integral manifold (covered in Appendix A of the book) for their dynamic states

$$\psi_{1d} = E'_q - (X'_d - X_{\ell s})I_d$$

$$\psi_{2q} = -E'_d - (X'_q - X_{\ell s})I_q$$

Two-Axis Model



$$T''_{do} \frac{d\psi_{1d}}{dt} = -\psi_{1d} + E'_q - (X'_d - X_{\ell s}) I_d = 0$$

$$T'_{do} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d) \times$$

Note this entire term becomes zero using the equation from the previous slide

$$\left[I_d - \frac{X'_d - X''_d}{(X'_d - X_{\ell s})^2} \left(\psi_{1d} + (X'_d - X_{\ell s}) I_d - E'_q \right) \right] + E_{fd}$$

Which can be simplified to

$$T'_{do} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d) I_d + E_{fd}$$

Two-Axis Model



$$T''_{qo} \frac{d\psi_{2q}}{dt} = -\psi_{2q} - E'_d - (X'_q - X_{ls}) I_q = 0$$

$$T'_{qo} \frac{dE'_d}{dt} = -E'_d + (X_q - X'_q) \times$$

Likewise this entire term becomes zero

$$\left[I_q - \frac{X'_q - X''_q}{(X'_q - X_{ls})^2} \left(\psi_{2q} + (X'_q - X_{ls}) I_q + E'_d \right) \right]$$

Which can be simplified to

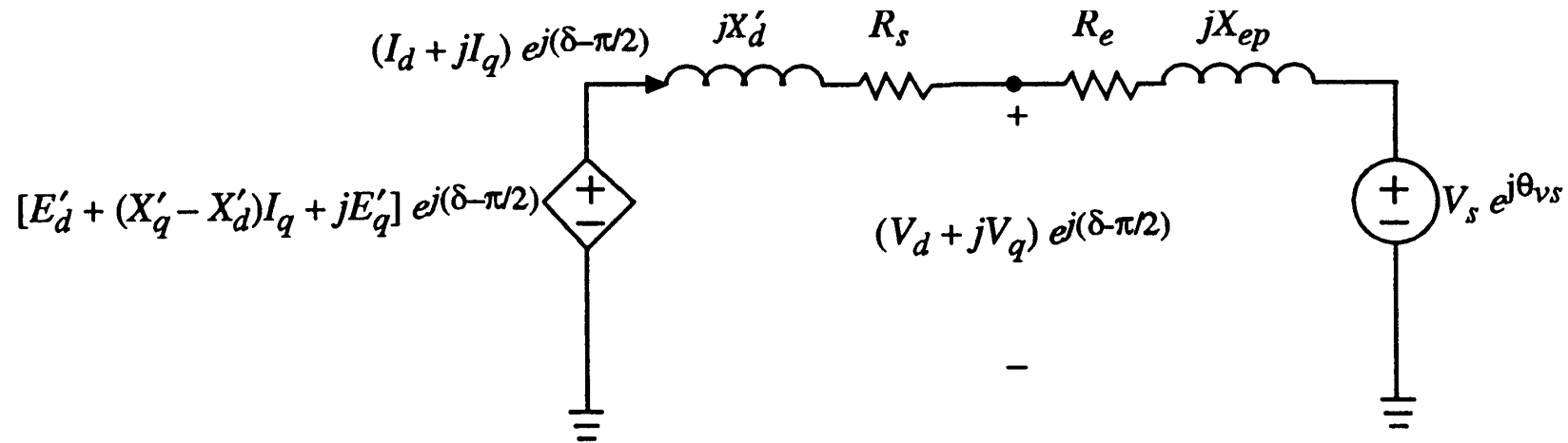
$$T'_{qo} \frac{dE'_d}{dt} = -E'_d + I_q (X_q - X'_q)$$

Two-Axis Model



$$0 = (R_s + R_e)I_d - (X'_q + X_{ep})I_q - E'_d + V_s \sin(\delta - \theta_{vs})$$

$$0 = (R_s + R_e)I_q + (X'_d + X_{ep})I_d - E'_q + V_s \cos(\delta - \theta_{vs})$$



Two-Axis Model



$$T'_{do} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd}$$

$$T'_{qo} \frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)I_q$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - (X'_q - X'_d) I_d I_q - T_{FW}$$

$$0 = (R_s + R_e)I_d - (X'_q + X_{ep})I_q - E'_d + V_s \sin(\delta - \theta_{vs})$$

$$0 = (R_s + R_e)I_q + (X'_d + X_{ep})I_d - E'_q + V_s \cos(\delta - \theta_{vs}) \setminus$$

$$V_d = R_e I_d - X_{ep} I_q + V_s \sin(\delta - \theta_{vs})$$

$$V_q = R_e I_q + X_{ep} I_d + V_s \cos(\delta - \theta_{vs})$$

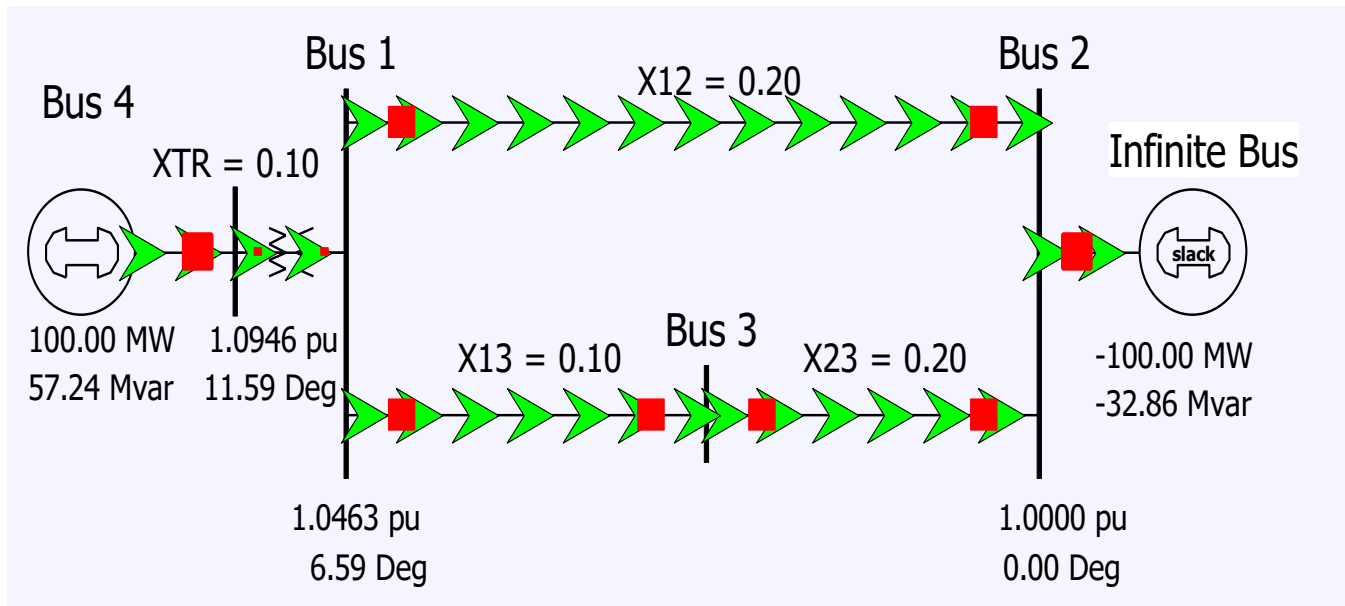
$$V_t = \sqrt{V_d^2 + V_q^2}$$

No saturation effects are included with this model

Example (Used for All Models)



- Below example will be used with all models. Assume a 100 MVA base, with gen supplying $1.0 + j0.3286$ power into infinite bus with unity voltage through network impedance of $j0.22$
 - Gives current of $1.0 - j0.3286 = 1.0526 \angle -18.19^\circ$
 - Generator terminal voltage of $1.072 + j0.22 = 1.0946 \angle 11.59^\circ$



Current sign convention is out of generator positive

Two-Axis Example



- For the two-axis model assume $H = 3.0$ per unit-seconds, $R_s=0$, $X_d = 2.1$, $X_q = 2.0$, $X'_d = 0.3$, $X'_q = 0.5$, $T'_{do} = 7.0$, $T'_{qo} = 0.75$ per unit using the 100 MVA base.
- Solving we get

$$\bar{E} = 1.0946 \angle 11.59^\circ + (j2.0)(1.0526 \angle -18.19^\circ) = 2.81 \angle 52.1^\circ$$

$$\rightarrow \delta = 52.1^\circ$$

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

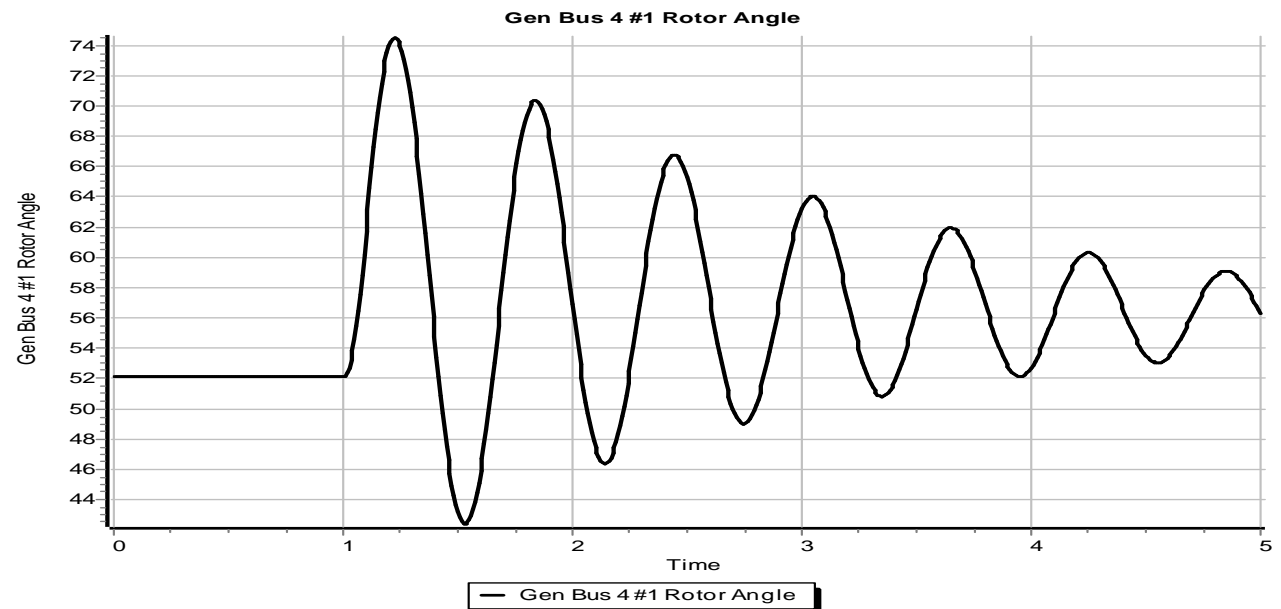
$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

Two-Axis Example

- And $E'_q = 0.8326 + (0.3)(0.9909) = 1.130$
 $E'_d = 0.7107 - (0.5)(0.3553) = 0.533$
 $E_{fd} = 1.1299 + (2.1 - 0.3)(0.9909) = 2.913$

Saved as case
B4_TwoAxis

- Assume a fault at bus 3 at time $t=1.0$, cleared by opening both lines into bus 3 at time $t=1.1$ seconds



Two-Axis Example



- PowerWorld allows the gen states to be easily stored

Result Storage

Where to Save/Store Results Save Results Every n Timesteps: 1

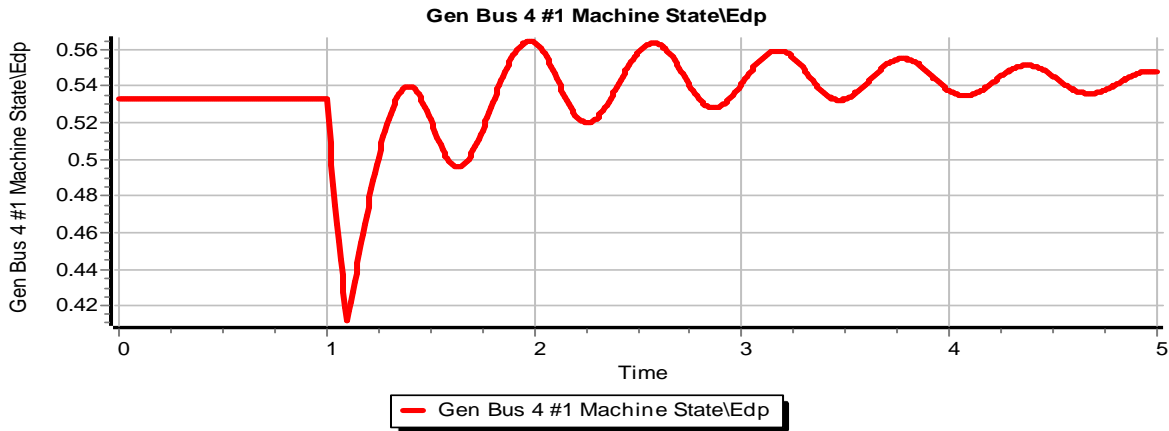
Store Results to RAM
 Save Results to Hard Drive
 Do Not Combine RAM Results with Hard Drive Results
 Save the Results stored to RAM in the PWB file
 Save the Min/Max Results stored to RAM in the PWB file

Store to RAM Options Save to Hard Drive Options

Note: All fields that are specified in a plot series of defined plot will also be stored to RAM.

Store Results for Open Devices Set All to NO for All Types Set Save All by Type ...

Generator	Bus	Load	Switched Shunt	Branch	Transformer	DC Transmission Line	VSC DC Line	Multi-Terminal DC Record	Multi-Terminal DC Converter	Area	Zone	Interf							
From Selection:		Save All	Save Rotor Angle	Save Rotor Angle No Shift	Save Speed	Save MW Mech	Save MW	Save MW Accel	Save Mvar	Save V pu	Save Efd	Save Ifd	Save Vstab	Save VOEL	Save VUEL	Save I pu	Save Status	Save Machine State	Save Exciter State
Make Plot	1	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO	NO
Make Plot	2	NO	YES	NO	YES	NO	YES	NO	YES	NO	NO	NO	NO	NO	NO	NO	NO	YES	NO



Graph shows variation in E_d'

Flux Decay Model



- If we assume T'_{q0} is sufficiently fast that its equation becomes an algebraic constraint

$$T'_{q0} \frac{dE'_d}{dt} = -E'_d + (X_q - X'_q) I_q = 0$$

In previous example
 $T'_{q0} = 0.75$

$$T'_{d0} \frac{dE'_q}{dt} = -E'_q - (X_d - X'_d) I_d + E_{fd}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = T_M - E'_d I_d - E'_q I_q - (X'_q - X'_d) I_d I_q - T_{FW}$$

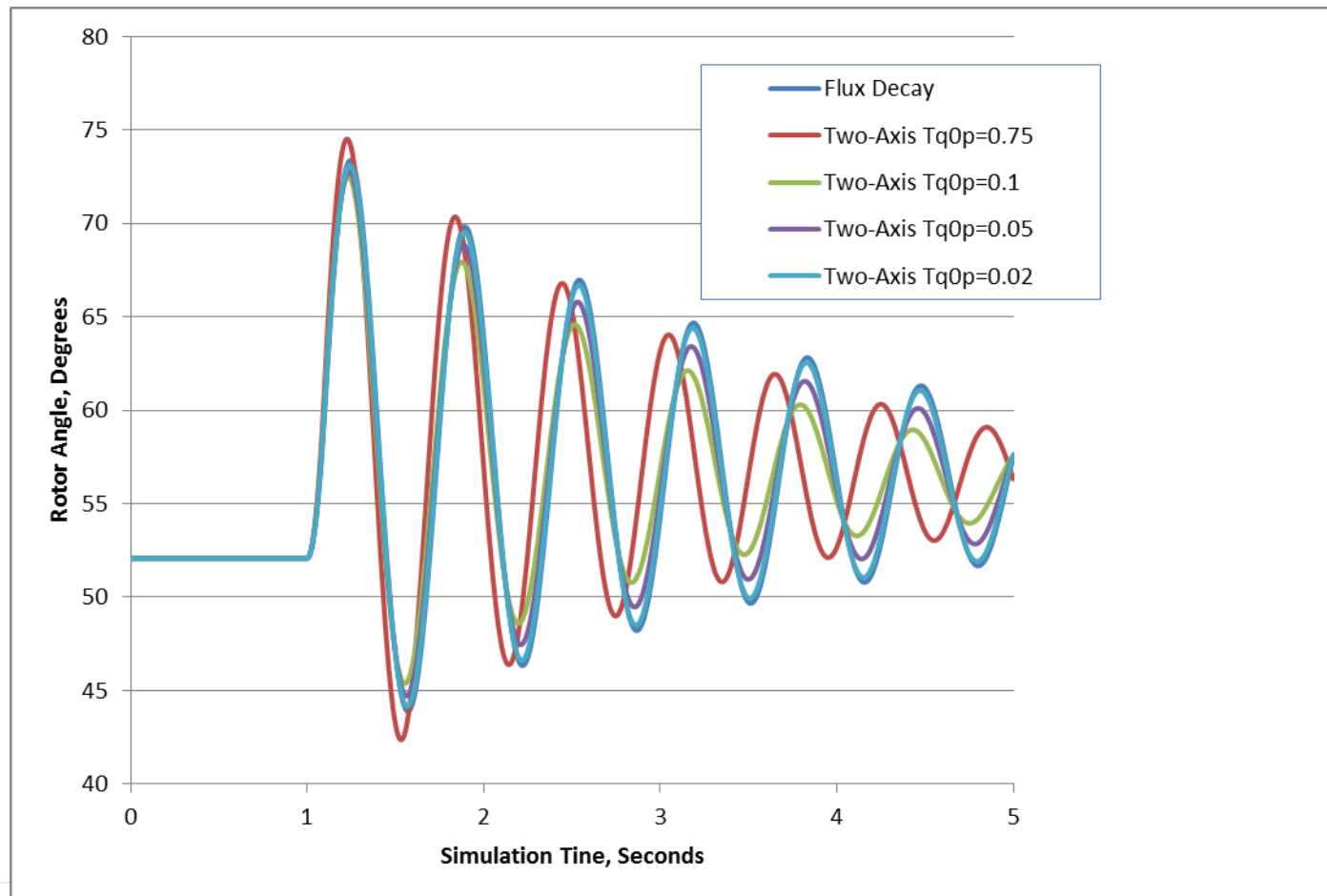
$$= T_M - (X_q - X'_q) I_q I_d - E'_q I_q - (X'_q - X'_d) I_d I_q - T_{FW}$$

$$= T_M - E'_q I_q - (X_q - X'_d) I_d I_q - T_{FW}$$

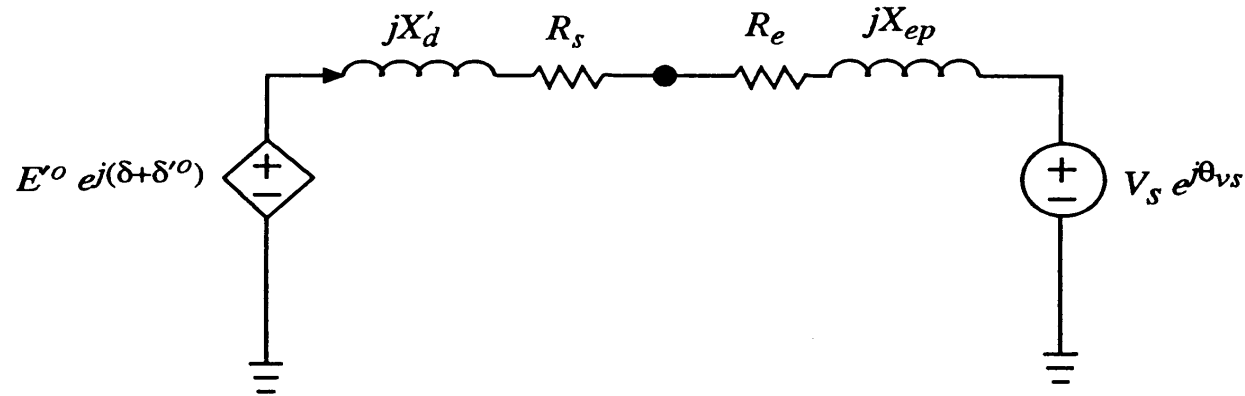
Rotor Angle Sensitivity to Tqop



- Graph shows variation in the rotor angle as Tqop is varied, showing the flux decay is the same as Tqop = 0



Classical Model



The classical model had been widely used because it is simple. At best it can only approximate a very short term response. It is no longer common.

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_0} \frac{d\omega}{dt} = T_M^0 - \frac{E'^0 V_s}{X'_d + X_{ep}} \sin(\delta - \theta_{vs}) - T_{FW}$$

This is a pendulum model

Classical Model Justification



- It is difficult to justify. One approach would be to go from the flux decay model and assume

$$X_q = X'_d \quad T'_{do} = \infty$$

$$E' = E'_q \quad \delta'^0 = 0$$

- Or go back to the two-axis model and assume

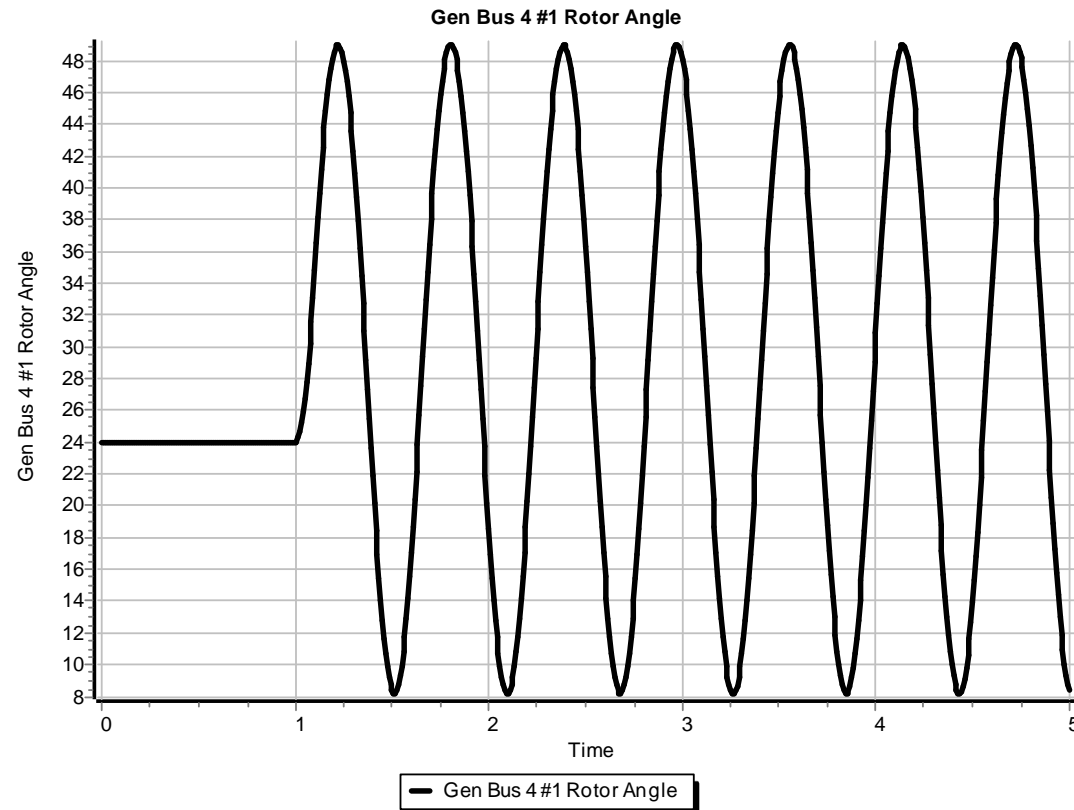
$$X'_q = X'_d \quad T'_{do} = \infty \quad T'_{qo} = \infty$$
$$(E'_q = \text{const} \quad E'_d = \text{const})$$

$$E' = \sqrt{E_q'^{02} + E_d'^{02}}$$
$$\delta'^0 = \tan^{-1} \left(\frac{E_q'^0}{E_d'^0} \right) - \pi/2$$

Classical Model Response



- Rotor angle variation for same fault as before



Notice that even though the rotor angle is quite different, its initial increase (of about 24 degrees) is similar. However, there is no damping.

Saved as case **B4_GENCLS**

Subtransient Models



- The two-axis model is a transient model
- Essentially all commercial studies now use subtransient models
- First models considered are GENSAL and GENROU, which require $X''_d = X''_q$
- This allows the internal, subtransient voltage to be represented as

$$\bar{E}'' = \bar{V} + (R_s + jX'')\bar{I}$$

$$E''_d + jE''_q = (-\psi''_q + j\psi''_d)\omega$$

Subtransient Models



- Usually represented by a Norton Injection with

$$I_d + jI_q = \frac{E_d'' + jE_q''}{R_s + jX''} = \frac{(-\psi_q'' + j\psi_d'')\omega}{R_s + jX''}$$

- May also be shown as

$$-j(I_d + jI_q) = I_q - jI_d = \frac{-j(-\psi_q'' + j\psi_d'')\omega}{R_s + jX''} = \frac{(\psi_d'' + j\psi_q'')\omega}{R_s + jX''}$$

In steady-state $\omega = 1.0$

Standards



- Standards play important roles in many aspects of engineering analysis by providing public (though often not free) access to standard models and other things (such as data formats)
 - Standards can be updated (e.g., 1110-2019 updated 1110-2002)
- The standards are then used by manufacturers (and others) to create compatible products
- However, manufacturers do not need to always follow the standards

IEEE SA
STANDARDS
ASSOCIATION

**IEEE Guide for Synchronous Generator
Modeling Practices and Parameter
Verification with Applications in Power
System Stability Analyses**

IEEE Power and Energy Society

Developed by the
Electric Machinery Committee

IEEE Std 1110™-2019
(Revision of IEEE Std 1110-2002)



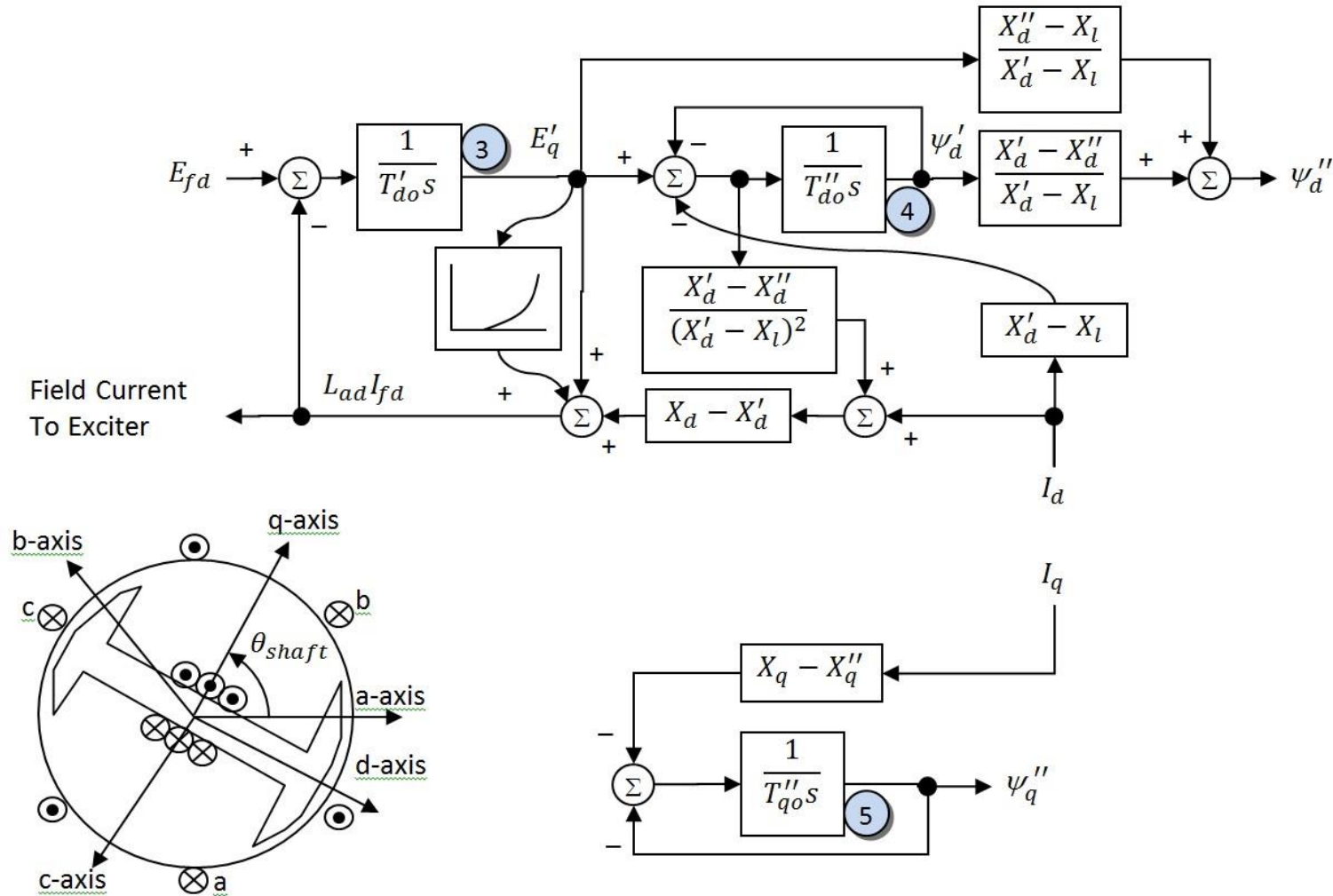
STANDARDS

GENSAL



- The GENSAL model had been widely used to model salient pole synchronous generators
 - In salient pole models saturation is only assumed to affect the d-axis
 - In the 2010 WECC cases about 1/3 of machine models were GENSAL; in 2013 essentially none are, being replaced by GENTPF or GENTPJ
 - A 2014 series EI model had about 1/3 of its machines models set as GENSAL
 - In November 2016 NERC issued a recommendation to use GENTPJ rather than GENSAL for new models. See www.nerc.com/comm/PC/NERCModelingNotifications/Use%20of%20GENTPJ%20Generator%20Model.pdf
- GENSAL is now considered obsolete, but is still a useful introduction to the subtransient models

GENSAL Block Diagram



A quadratic saturation function is used; for initialization it only impacts the E_{fd} value

GENSAL Example



- Assume same system as before with same common generator parameters:
 $H=3.0$, $D=0$, $R_a = 0$, $X_d = 2.1$, $X_q = 2.0$, $X'_d = 0.3$, $X''_d=X''_q=0.2$, $X_l = 0.13$,
 $T'_{do} = 7.0$, $T''_{do} = 0.07$, $T''_{qo} = 0.07$, $S(1.0) = 0$, and $S(1.2) = 0$.
- Same terminal conditions as before
 - Current of $1.0-j0.3286$ and generator terminal voltage of $1.072+j0.22 = 1.0946 \angle 11.59^\circ$
- Use same equation to get initial δ

$$\begin{aligned} |E| \angle \delta &= \bar{V} + (R_s + jX_q) \bar{I} \\ &= 1.072 + j0.22 + (0.0 + j2)(1.0 - j0.3286) \\ &= 1.729 + j2.22 = 2.81 \angle 52.1^\circ \end{aligned}$$

Same delta as with
the other models

Saved as case
B4_GENSAL

GENSAL Example



- Then as before

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.0723 \\ 0.220 \end{bmatrix} = \begin{bmatrix} 0.7107 \\ 0.8326 \end{bmatrix}$$

and

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.000 \\ -0.3287 \end{bmatrix} = \begin{bmatrix} 0.9909 \\ 0.3553 \end{bmatrix}$$

$$\begin{aligned} \bar{E}'' &= \bar{V} + (R_s + jX'')\bar{I} = E_r'' + E_i'' \leftarrow \\ &= 1.072 + j0.22 + (0 + j0.2)(1.0 - j0.3286) \\ &= 1.138 + j0.42 \end{aligned}$$

Needs to be converted to dq

GENSAL Example



- Giving the initial fluxes (with $\omega = 1.0$) of

$$\begin{bmatrix} -\psi_q'' \\ \psi_d'' \end{bmatrix} = \begin{bmatrix} 0.7889 & -0.6146 \\ 0.6146 & 0.7889 \end{bmatrix} \begin{bmatrix} 1.138 \\ 0.420 \end{bmatrix} = \begin{bmatrix} 0.6396 \\ 1.031 \end{bmatrix}$$

Recall $E_d'' + jE_q'' = (-\psi_q'' + j\psi_d'')\omega$

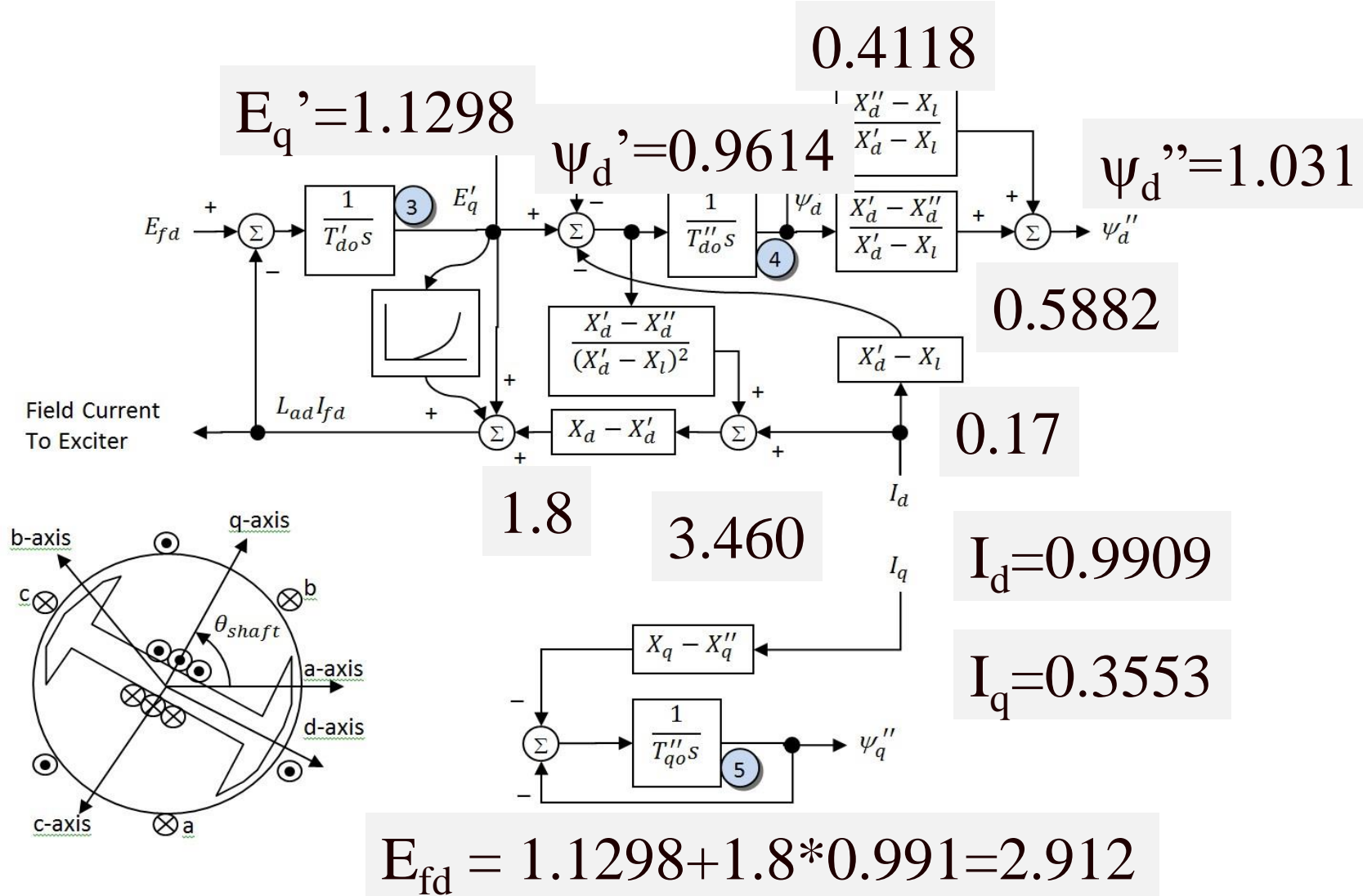
- To get the remaining variables set the differential equations to zero,

$$\psi_q'' = -(X_q - X_q'')I_q = -(2 - 0.2)(0.3553) = -0.6396$$

$$E_q' = 1.1298, \quad \psi_d' = 0.9614$$

Solving the d-axis requires solving two linear equations for two unknowns

GENSAL Example



Comparison Between Gensal and Flux Decay

