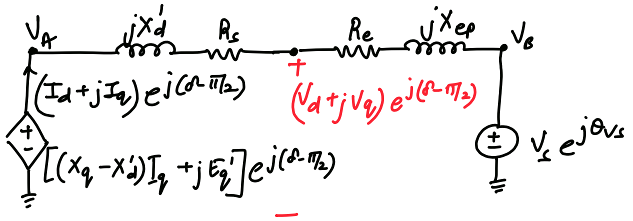
Solution 1

Book 5.2

one-axis dynamic circuit - fig. 5.3.

Get Id and Iq in terms of circuit parameters $+\delta$, E_q', V_s, θ_{vs}



Applying circuit theory,

$$V_A - V_B = (I_d + jI_q)e^{j(\delta-\pi/2)} \times \underbrace{(R_s + jX_d') + (R_e + jX_{ep})}_{Z}$$

We know;

$$V_A = [(X_q - X_d')I_q + jE_q']e^{j(\delta-\pi/2)}$$

and

$$V_B = V_s e^{j\theta_{V_s}}$$

dividing by $e^{j(\delta-\pi/2)}$ on both sides;

$$\underbrace{(X_q - X_d')I_q + jE_q'}_{LHS} = \underbrace{(I_d + jI_q)Z + V_s e^{j(\theta_{vs} - \delta + \pi/2)}}_{RHS}$$

solving LHS,

$$(X_q - X_d')I_q + E_q' = I_q X_q - I_q X_d' + jE_q' \quad \text{—} (1)$$

Solving RHS,

$$(I_d + jI_q)\left((R_s + R_e) + j(X_{ep} + X_d')\right) + V_s\left(\sin(\delta - \theta_{vs}) + j\cos(\delta - \theta_{vs})\right)$$

$$(I_d + jI_q)(R_s + R_e) + j(X_{ep} + X_d')(I_d + jI_q) + V_s(\sin(\delta - \theta_{vs})) + jV_s(\cos(\delta - \theta_{vs})) \quad \text{—} (2)$$

on solving further you can equate the real and imaginary coefficients for equation 1 and 2;

$$0 = (R_s + R_e)I_d - (X_q + X_{ep})I_q + V_s(\sin(\delta - \theta_{vs}))$$

$$E_q' = (R_s + R_e)I_q + (X_d' + X_{ep})I_d + V_s\cos(\delta - \theta_{vs})$$

since we want to find Id and Iq, we bring those terms to one side;

$$-V_s \sin(\delta-\theta_{vs}) = (R_s+R_e)\,I_d - (X_q + X_{ep})\,I_q$$

$$E_q' - V_s\cos(\delta-\theta_{vs}) = (R_s+R_e)\,I_q - (X_d' + X_{ep})\,I_d$$

Above equations can be written in matrix format;

$$\underbrace{\begin{bmatrix} (R_s+R_e) & -(X_q+X_{ep}) \\ (X_d'+X_{ep}) & (R_s+R_e) \end{bmatrix}}_{A} \begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} -V_s \sin(\delta-\theta_{vs}) \\ E_q' - V_s\cos(\delta-\theta_{vs}) \end{bmatrix}$$

solving for I_d & I_q;

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = A^{-1} \begin{bmatrix} -V_s \sin(\delta-\theta_{vs}) \\ E_q' - V_s\cos(\delta-\theta_{vs}) \end{bmatrix}$$

$$\therefore I_q = \frac{(X_d'+X_{ep})\,V_s \sin(\delta-\theta_{vs}) + (R_s+R_e)\left[E_q' - V_s\cos(\delta-\theta_{vs})\right]}{(R_s+R_e)^2 + (X_q+X_{ep})(X_d'+X_{ep})}$$

and

$$I_d = \frac{(X_q+X_{ep})\left[E_q' - V_s\cos(\delta-\theta_{vs})\right] - (R_s+R_e)\,V_s \sin(\delta-\theta_{vs})}{(R_s+R_e)^2 + (X_d'+X_{ep})(X_{ep}+X_q)}$$

Solution 2

Book 6.5

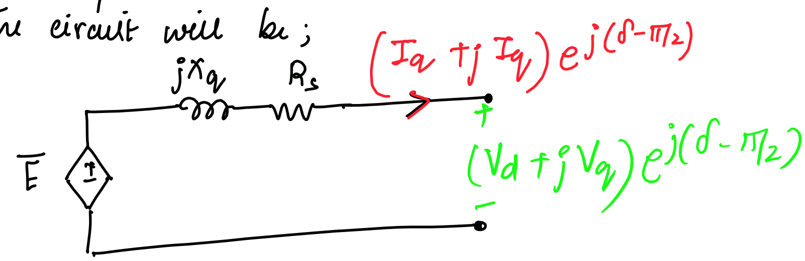
→ syn. generator with two axis model

$$(V_d + jV_q)\,e^{j(\delta-\pi/2)} = 1\angle 0 \text{ pu}$$

$$(I_d + jI_q)\,e^{j(\delta-\pi/2)} = 0.5\angle 30^\circ \text{ pu}$$

$R_s = 0$
$X_d = 1.4$
$X_q = 1.1$
$X_d' = 0.2\text{pu} = X_q'$

At steady state
the circuit will be;



applying KVL;

$$\bar{E} = (V_d+jV_q)\,e^{j(\delta-\pi/2)} + (I_d+jI_q)\,e^{j(\delta-\pi/2)}\left[R_s + jX_q\right]$$

$$= 1\angle 0 + 0.5\angle 30^\circ\,[j1.1]$$

$$= 0.725 + j0.476 \rightarrow 0.867\angle 33.3^\circ$$

$$\therefore \boxed{\delta = 33.3^\circ}$$

use δ to get I_d, I_q, V_d, V_q;

$$I_d + jI_q = \frac{0.5\angle 30^\circ}{e^{j(33.3-90^\circ)}} = 0.5\angle 30^\circ - 33.3^\circ + 90^\circ$$

$$= 0.5\angle 86.67^\circ$$

$$= \underbrace{0.029}_{I_d} + j\,\underbrace{0.49}_{I_q}$$

$$V_d + jV_q = \frac{1\angle 0}{e^{j(33.3-90)}} = 1\angle +56.7^\circ$$

$$= \underbrace{0.549}_{V_d} + j\,\underbrace{0.835}_{V_q}$$

you can also use the transformation matrix to get these values!

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \underbrace{\begin{bmatrix} V_r \\ V_i \end{bmatrix}}_{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \underbrace{\begin{bmatrix} I_r \\ I_i \end{bmatrix}}_{\begin{bmatrix} 0.433 \\ 0.25 \end{bmatrix}}$$

$$E_q' = V_q + X_d' I_d$$

$$= 0.835 + 0.2(0.0287)$$

$$\boxed{E_q' = 0.8407 \text{ pu}}$$

$$E_d' = V_d - X_q' I_q$$

$$= 0.549 - (0.2)(0.499)$$

$$\boxed{E_d' = 0.449 \text{ pu}}$$

(b) $E_{fd} = E_q' + (X_d - X_d') I_d$

$$= 0.8292 + (1.4 - 0.2)(0.0287)$$

$$\boxed{E_{fd} = 0.875 \text{ pu}}$$

$$T_m = \psi_d I_q - \psi_q I_d + T_{FW}$$

we know that;

$$\psi_d = E_q' - X_d' I_d$$

$$\psi_q = -X_q' I_q - E_d'$$

$$T_{FW} = 0 \quad \because \frac{d\omega}{dt} = 0$$

Also from the slides,

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - T_{FW}$$

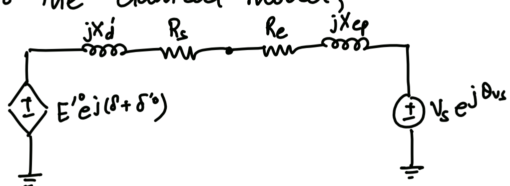
$$\frac{2H}{\omega_s}\underbrace{\cancel{\frac{d\omega}{dt}}}_{0} = T_m - E_d' I_q - E_q' I_q - (X_q' - X_d') I_d I_q - \overbrace{\cancel{T_{FW}}}^{0}$$

$$T_m = E_d' I_q + E_q' I_q + \overbrace{\cancel{(X_q' - X_d')}}^{0} I_q I_d$$

$$= (0.449)(0.0287) + (0.8407)(0.499) + 0$$

$$\boxed{T_m = 0.4323 \text{ pu}}$$

(c) For the classical model;



Using circuit theory;

$$E'^o e^{j(\delta+\delta'^o)} = jX_d'(0.5\angle 30^\circ) + (1\angle 0)$$

$$= \underbrace{0.954}_{E'^o}\angle \underbrace{5.209^\circ}_{\delta+\delta'^o} \text{ pu}$$

$$\therefore \delta + \delta'^o = 5.204^\circ$$

$$\delta'^o = -28.096^\circ \quad (\because \underbrace{\delta = 33.3^\circ}_{\text{from steady state.}})$$

Solution 3

Two-axis model

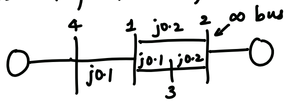
current into infinite bus is $1\angle 0^\circ$ pu.

Find $\delta, V_d, V_q, I_d, I_q, E_q', E_d'$

From slides, we know

Gen o/p is 1 pu real power @ 0.95 pf lagging.

system from slide;



equivalent impedance $= j0.22$

$$V_s = V_4 = V_2 + I(Z)$$

$$= 1\angle 0 + (1\angle 0)\,j0.22$$

$$= 1 + j0.22 \text{ pu}$$

$$E\angle\delta = V_s + (R_s + jX_q)I$$

$$= (1 + j0.22) + j2(1\angle 0)$$

$$= 2.435\angle 65.75^\circ \text{ pu}$$

$$\boxed{\delta = 65.75^\circ}$$

Using transformation matrix;

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix} \quad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.821 \\ 0.611 \end{bmatrix} \text{pu}$$

similarly using the transformation matrix,

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} 0.912 \\ 0.411 \end{bmatrix} \text{pu.}$$

$$E_q' = V_q + R_s I_q + X_d' I_d$$

$$= V_q + X_d' I_d = 0.864 \text{ pu}$$

$$E_d' = V_d - R_s I_d - X_q' I_q$$

$$= V_d - X_q' I_q = 0.616 \text{ pu.}$$

Solution 4

per unit power to ∞ bus $= 2 + j0.2$

$S(1.0) = 0.02$

$S(1.2) = 0.1$

$S = VI^*$

$I = 2 - j0.2$

$= 2\angle -5.71°$

Gen. terminal voltage $= I(j0.22) + 1.0$ [Same circuit as previous solution]

$= 1.044 + j0.44$

$E\angle\delta = V + (R_s + jX_q) I_d = 4.68 \angle 71.98°$

$\boxed{\delta = 71.98°}$

Using the transformation matrix;

$$\begin{bmatrix} V_d \\ V_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} V_r \\ V_i \end{bmatrix}$$

$$= \begin{bmatrix} 0.8554 \\ 0.741 \end{bmatrix}$$

$$\begin{bmatrix} I_d \\ I_q \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} 2 \\ -0.2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.963 \\ 0.427 \end{bmatrix}$$

$\overline{E''} = \overline{V} + (R_s + jX'')\overline{I}$

$= 1.044 + j0.44 + (j0.2)(2 - j0.2)$

$= 1.084 + j0.84$

$$\begin{bmatrix} -\psi_q'' \\ \psi_d'' \end{bmatrix} = \begin{bmatrix} 0.95 & -0.309 \\ 0.309 & 0.95 \end{bmatrix} \begin{bmatrix} 1.084 \\ 0.84 \end{bmatrix} = \begin{bmatrix} 0.771 \\ 1.1337 \end{bmatrix}$$

$E_q' = V_q + X_d' I_d + \cancel{R_s I_q}^{\,0}$

$= 1.3302$ pu

$E'' = \overline{V} + (R_s + jX'') I$

$E_d'' + jE_q'' = -(\psi_q'' + j\psi_d'')\omega$ $\quad \omega = 1$

$E_q'' = V_q + X_d I_d = 0.741 + 0.3(1.963) = 1.329$ pu

$\psi_d' = E_q' - (X_d' - X_L) I_d = 1.33 - ((0.3 - 0.18) \times 1.962) = 0.9964$ pu

finding values of A and B of saturation function;

$$B = \frac{S(1.2)}{(1.2 - A)^2}$$

$$S(1.0) = \frac{S(1.2)}{(1.2 - A)^2}(1 - A)^2$$

$$0.2 = \frac{(1-A)^2}{(1.2-A)^2}$$

$$0.288 + 0.2A^2 - 0.48A = 1 + A^2 - 2A$$

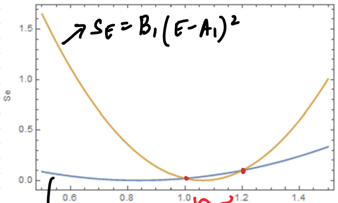
$\therefore\ A_1 = 1.0618$ OR $A_2 = 0.8381$
$B_1 = 5.236$ $\quad B_2 = 0.763$

with $A = 0.838$, the min. of $S(E)$ does not lie between $S(1)$ and $S(1.2)$.

$\therefore\ A = 0.838$
$B = 0.763$

$$E_{fd} = E_q'(1 + S(E_q)) + (X_d - X_d')I_d$$

$$\boxed{E_{fd} = 5.112\ pu}$$



$S_E = B_2(E-A_2)^2$

we don't want min. value of E to occur here!

you can also find minimum by differentiating the saturation function.

Solution 5

The original parameters for the generator model (GENROU_Sat) connected to Bus 4 are as indicated in the below table.

H	D	Ra	Xd	Xq	Xdp	Xqp	Xdpp	Xl	Td0p	Tq0p	Tdopp	Tq0pp	S1	S12	Rcomp	Xcomp
3	0	0	2.1	2	0.3	0.5	0.28	0.13	7	0.75	0.073	0.07	0.05	0.2	0	0

The damping ratio and critical clearing time for the above starting parameters are as below:

Damping Ratio	0.28
Critical Clearing time	1.17

This is considered as case 0.
The parameters are changed one by one and labelled as separate cases. Observation for each case is listed.

Case 1,2 and 3:
The parameter **H** (shaft inertia constant) is changed keeping the remaining parameters same as case 0.
Observation:

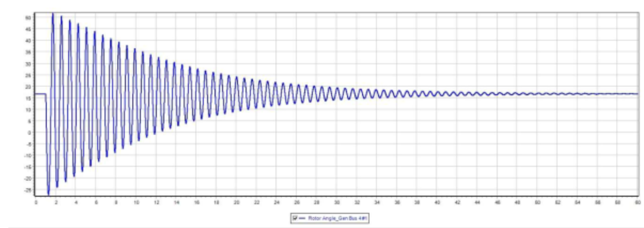
H	Damping Ratio	Critical Clearing time
4	0.25	1.19
5	0.16	1.21
2	0.29	1.14

Case 2,3 and 4:
The parameter **D** is changed keeping the remaining parameters same as case 0. I'm not sure if D can be negative. But the system gives a result with increased damping and reduced clearing time.

D	Damping Ratio	Critical Clearing time
-1	0.31	1.16
1	0.23	1.17
2	0.22	1.17

Case 5 and 6:
The parameter **Ra** (Armature Resistance) is changed keeping the remaining parameters same as case 0. The damping ratio varies significantly and the clearing time also reduces. A graph is added for this case to indicate the amount of oscillations before finally settling.

Ra	Damping Ratio	Critical Clearing time
1	0.82	1.04
2	0.47	1.01



Case 7,8 and 9:
The parameter **Xd** (Reactance of D-axis) is changed keeping the remaining parameters same as case 0.

Xd	Damping Ratio	Critical Clearing time
3.1	0.27	1.16
5.1	0.25	1.15
6.1	0.27	1.15

Case 10,11 and 12:
The parameter **Xq** (Reactance of Q-axis) is changed keeping the remaining parameters same as case 0.
Observation: The damping ratio generally decreases except for Xq=5 and the clearing time decreases.

Xq	Damping Ratio	Critical Clearing time
3	0.27	1.16
5	0.31	1.16
6	0.26	1.15

Case 13 and 14:
The parameter **Xd'** (Transient Reactance of D-axis) is changed keeping the remaining parameters same as case 0. The damping ratio and clearing time, both decrease significantly.

Xd'	Damping Ratio	Critical Clearing time
0.6	0.10	1.14
1.3	0.04	1.10

Case 15 and 16:
The parameter **Xq'** (Transient Reactance of D-axis) is changed keeping the remaining parameters same as case 0.
Observation: The damping ratio does not increase as significantly as it does when Xd' is changed.

Xq'	Damping Ratio	Critical Clearing time
1	0.26	1.17
1.5	0.28	1.17

Case 17 and 18:
The parameter **Xd', Xq', Xd''** (Transient and Subtransient Reactance of D-axis, transient reactance of Q-axis) is changed keeping the remaining parameters same as case 0. The damping ratio increases and the critical clearing time decreases significantly.

Solution 6

Book 4.1

$$(4.20) \quad 0 = -(K_E + S_E(E_{fd}))E_{fd} + V_R$$

$$(4.22) \quad S_E(E_{fd}) = A_x e^{B_x E_{fd}}$$

find E_{fdmax}, A_x & B_x.

using 4.20;

$$0 = -(1+0.9)E_{fd} + 8 \Rightarrow E_{fdmax} = 4.211 \text{ pu}$$

using 4.22;

$$0.9 = A_x e^{B_x E_{fdmax}}$$

$$0.5 = A_x e^{0.75 \times E_{fdmax}}$$

solving above eqns simultaneously;

$$A_x = 0.0857$$

$$B_x = 0.5584$$

Solution 7

Book 4.3

$$(4.18) \quad T_E \frac{dE_{fd}}{dt} = -(K_{E_{self}} + S_E(E_{fd}))E_{fd} + V_R$$

$$(4.22) \quad S_E(E_{fd}) = A_x e^{B_x E_{fd}}.$$

At steady state, eq 4.18 becomes;

$$0 = -(K_{E_{self}} + A_x e^{B_x E_{fd}})E_{fd} + V_R$$

$$K_{E_{self}} = -0.132$$

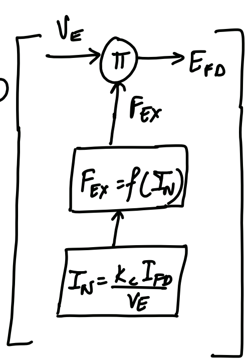
Solution 8

$$E_{fd} = I_{fd} = 3.1866 \text{ pu}$$

find I_N first;

$$V_E \cdot F_{EX} = E_{FD} \quad -(1)$$

$$I_N = \frac{k_c \cdot I_{fd}}{V_E} \quad -(2)$$



using (1) and (2);

$$\frac{K_c \cdot \cancel{I_{fd}} \cdot F_{EX}}{I_N} = \cancel{E_{FD}}$$

$$I_N = k_c \cdot F_{EX}$$

$$I_N = 0.3 F_{EX} \quad -(3)$$

IN → FEX

If $I_N \le 0.$	$F_{EX} = 1$ ✗
If $I_N \le 0.433$	$F_{EX} = 1 - 0.577\, I_N$
If $0.433 < I_N < 0.75$	$F_{EX} = \sqrt{0.75 - I_N^2}$ ✗
If $I_N \ge 0.75$	$F_{EX} = 1.732(1 - I_N)$ ✗
If $I_N > 1$	$F_{EX} = 0$ ✗

only condition that satisfies (3)

$$\therefore F_{EX} = 1 - 0.577 I_N \quad -(4)$$

Using above equations;

$$I_N = 0.256 \quad \text{and} \quad F_{EX} = 0.852$$

$E_1 = 3.3$ $\quad$ $E_2 = 4.4$

$SE_1 = 0.03$ $\quad$ $SE_2 = 0.08$

$$S_E = B(E - A)^2$$

$$\therefore\ 0.03 = B(3.3 - A)^2$$

$$0.08 = B(4.4 - A)^2$$

solving above equations we get;

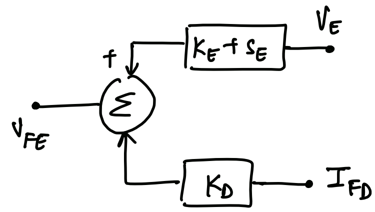
$A = 1.562$ and $B = 0.0099$

$$\therefore S_E = 0.0099(E - 1.562)^2$$

$$V_{FE} = \cancel{I_{FD}\cdot K_D}^{\,0} + K_E\cdot V_E + S_E(V_E)$$

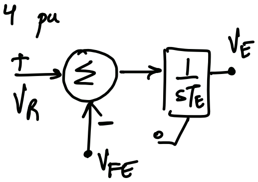
$$= \frac{E_{FD}}{F_{EX}}$$

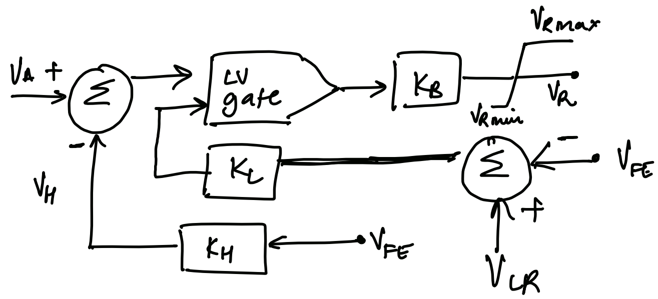
$$= K_E\left(\frac{E_{FD}}{F_{EX}}\right) + S_E\left(\frac{E_{FD}}{F_{EX}}\right)$$



$V_{FE} = 3.914$ pu

$V_R = V_{FE}$





$$V_R = K_B \min\left(V_L,\ V_A - \cancel{K_H V_{FE}}^{\,0}\right)$$

where :- $V_L = K_L(V_{LR} - V_{FE})$

$$0.196 = \min(4.859, V_A)$$

$$\therefore V_A = 0.196$$

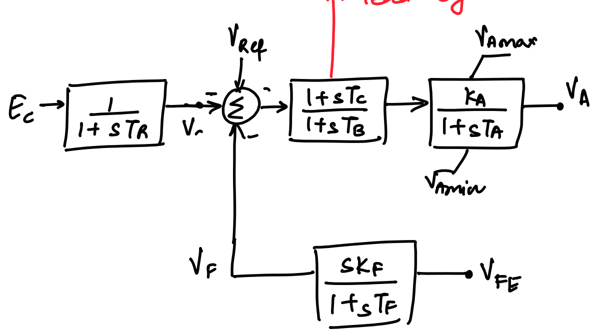
Correction in question

- Don't ignore LV gate
- assume $V_s = 0$.

Now, to find V_{ref}.

Lec 10 (last slide)

in steady input = output for lead lag block



$$V_{ref} - V_c - \cancel{V_F}^{\,0} + \cancel{V_s}^{\,0} = \frac{V_A}{K_A}$$

$$V_{ref} = \frac{V_A}{K_A} + V_c$$

$$V_{ref} = 1.0953 \text{ pu.}$$