A Simulation Based Approach to Pricing Reactive Power

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Abstract

In this paper we investigate the simulation of real and reactive power spot markets. While spot pricing of real power remains a viable option for the creation of a power system market, the future of a reactive power spot market remains cloudy. The large capital investment portion needed in pricing reactive power as well as the highly volatile nature of reactive power spot prices makes the creation of such a market difficult. In spite of this, a portion of the pricing scheme that is used for reactive power will likely be based on the spot pricing approach as this provides the most accurate signal for near real-time system operation. This paper will build on a simple modification to the standard optimal power flow (OPF) in order to simulate the spot markets for real and reactive power. To achieve this, price-dependent load models are introduced for both real and reactive power.

Key Words

Real and Reactive Pricing, Price-Dependent Loads, Real-Time Pricing, Optimal Power Flow (OPF), Load Models

1. Introduction

Throughout much of the history of electric power, electricity providers have treated consumer load as a variable that they have no direct control or indirect control over. The ability for the consumer to exercise price-based control has also been limited due to the flat and time-of-use rate structure presently used.

Over the past ten to fifteen years, however, there has been a lot of discussion regarding the possible implementation of a spot market for electricity. This would give consumers price signals allowing them to adjust and modify their loads in order to get the most utility out of their consumption of electricity. Much of the theory for this market is described in [1].

While many agree that the implementation of a spot market for real power could be an effective way to increase the economic efficiency of the electric power market, the viability of a spot market for reactive power remains cloudy. In [2], the creation of a full spot market for reactive power is put forth. While this does help create an efficient market for allocating the operational costs of the reactive power supply, it does not overcome two large issues in the reactive power market: the capital cost of reactive power equipment (such as capacitor banks and LTC's) are large compared to operational costs and reactive power spot prices are extremely volatile. For example, in both [3] and [4], reactive power spot prices are shown to vary by orders of magnitude when voltage limits are encountered in the power system.

In order to overcome these issues, both [4] and [5] propose the development of pricing schemes which take into account the capital investment required to install reactive power equipment along with alternatives which try to overcome some of the price volatility in reactive power spot prices.

Although there is still some debate regarding the viability of reactive power spot prices, a portion of any pricing scheme will likely be based on the spot pricing approach. This paper addresses the application of the optimal power flow (OPF) to the simulation of a spot market in an electric power system. It builds on the theory introduced in [6] and further developed in [7]. The OPF in this paper uses the Newton's method algorithm for its solution technique [8].

2. Notation

General conventions on notation for this paper

- All vector and matrix variables are in bold.
- All vectors are column vectors.
- Subscripts *p* and *q* signify a relation to real and reactive power respectively.

Variable Definitions

$\mathbf{x}$ = state variables and other controls (e.g. tap ratios)

$\mathbf{s} = \left[\mathbf{s}_p^T \quad \mathbf{s}_q^T\right]^T =$ the supply vector

$\mathbf{d} = \left[\mathbf{d}_p^T \quad \mathbf{d}_q^T\right]^T =$ the demand vector

$\hat{\mathbf{s}} = \left[\hat{\mathbf{s}}_p^T \quad \hat{\mathbf{s}}_q^T\right]^T =$ augmented supply vector including zeros where no suppliers exists

$\hat{\mathbf{d}} = \left[\hat{\mathbf{d}}_p^T \quad \hat{\mathbf{d}}_q^T\right]^T =$ augmented demand vector including zeros where no loads exist

$C(\mathbf{s}) = C(\mathbf{s}_p, \mathbf{s}_q) = \sum_{\text{all suppliers}} C_k(\mathbf{s}_p, \mathbf{s}_q)$ = Suppliers' Cost

$B(\mathbf{d}) = B(\mathbf{d}_p, \mathbf{d}_q) = \sum_{\text{all consumers}} B_k(\mathbf{d}_p, \mathbf{d}_q)$ = Consumers' Benefit

$\mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}} \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathbf{h}_p(\mathbf{x},\mathbf{s}_p,\mathbf{d}_p) \\ \mathbf{h}_q(\mathbf{x},\mathbf{s}_q,\mathbf{d}_q) \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix} =$ equality constraints

$\mathbf{h}_p(\mathbf{x},\mathbf{s}_p,\mathbf{d}_p) = \hat{\mathbf{h}}_p(\mathbf{x}) - \hat{\mathbf{s}}_p + \hat{\mathbf{d}}_p$ = real power flow equations. See Appendix for more detail.

$\mathbf{h}_q(\mathbf{x},\mathbf{s}_q,\mathbf{d}_q) = \hat{\mathbf{h}}_q(\mathbf{x}) - \hat{\mathbf{s}}_q + \hat{\mathbf{d}}_q$ = reactive power flow equations. See Appendix for more detail.

$\mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \mathbf{s}_{\min} - \mathbf{s} \\ \mathbf{s} - \mathbf{s}_{\max} \\ \mathbf{d}_{\min} - \mathbf{d} \\ \mathbf{d} - \mathbf{d}_{\max} \\ \overline{\mathbf{g}}(\mathbf{x}) \end{bmatrix}$ = inequality constraints

$\mathbf{f}(\mathbf{d},\mathbf{p}_d)$ = additional equation for consumer demand.

L, $\overline{L}$, $\tilde{L}$ = Lagrange functions

$\boldsymbol{\lambda} = \left[\boldsymbol{\lambda}_h^T \quad \boldsymbol{\lambda}_g^T \quad \boldsymbol{\lambda}_f^T\right]^T$ = Lagrange multiplier vector

$\boldsymbol{\lambda}_h = \left[\boldsymbol{\lambda}_{\hat{h}}^T \quad \boldsymbol{\lambda}_{\bar{h}}^T\right]^T = \left[\boldsymbol{\lambda}_{\hat{h}p}^T \quad \boldsymbol{\lambda}_{\hat{h}q}^T \quad \boldsymbol{\lambda}_{\bar{h}}^T\right]^T$ = Lagrange multiplier vector for power flow equations and other equality constraints.

$\boldsymbol{\lambda}_g = \left[\boldsymbol{\lambda}_{gs\min}^T \quad \boldsymbol{\lambda}_{gs\max}^T \quad \boldsymbol{\lambda}_{gd\min}^T \quad \boldsymbol{\lambda}_{gd\max}^T \quad \boldsymbol{\lambda}_{\bar{g}}^T\right]^T$ = Lagrange multiplier vector for inequality constraints

$\tilde{\boldsymbol{\lambda}}_{\hat{h}s} = \left[\tilde{\boldsymbol{\lambda}}_{\hat{h}sp}^T \quad \tilde{\boldsymbol{\lambda}}_{\hat{h}sq}^T\right]^T$ = reduced Lagrange multiplier vector including only entries for power flow equations which include a supply of real or reactive power.

$\tilde{\boldsymbol{\lambda}}_{\hat{h}d} = \left[\tilde{\boldsymbol{\lambda}}_{\hat{h}dp}^T \quad \tilde{\boldsymbol{\lambda}}_{\hat{h}dq}^T\right]^T$ = reduced Lagrange multiplier vector including only entries for power flow equations which include a demand of real or reactive power

$\boldsymbol{\lambda}_f = \left[\boldsymbol{\lambda}_{fp}^T \quad \boldsymbol{\lambda}_{fq}^T\right]^T$ = Lagrange multiplier vector for additional constraints

$\mathbf{p} = \left[\mathbf{p}_s^T \;\vdots\; \mathbf{p}_d^T\right]^T = \left[\mathbf{p}_{sp}^T \quad \mathbf{p}_{sq}^T \;\vdots\; \mathbf{p}_{dp}^T \quad \mathbf{p}_{dq}^T\right]^T$ = price vector for variable suppliers and variable consumers. (includes real and reactive prices)

$\mathbf{D}(\bullet)$ = is the functional inverse of $\frac{\partial B(\bullet)}{\partial \mathbf{d}}$. At optimal solution this is the consumer demand function.

$\mathbf{S}(\bullet)$ = is the functional inverse of $\frac{\partial C(\bullet)}{\partial \mathbf{s}}$. At optimal solution this is the supplier supply function.

3. Price-Dependent Load Theory

The theory needed for this paper was introduced in [6] and further solidified and developed in [7]. These results show that the addition of price-dependent loads to the standard OPF algorithm that minimizes supplier costs will result in an OPF that maximizes the social welfare of a system. While the full theory is not repeated here, a synopsis of the results is provided for background.

The standard OPF algorithm solves the non-linear program in equation (3.1).

$$\max_{\mathbf{s},\mathbf{x}} \; -C(\mathbf{s})$$
$$\text{s.t.} \quad \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}} \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix} = \mathbf{0}, \quad \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \mathbf{s}_{\min} - \mathbf{s} \\ \mathbf{s} - \mathbf{s}_{\max} \\ \mathbf{d}_{\min} - \mathbf{d} \\ \mathbf{d} - \mathbf{d}_{\max} \\ \overline{\mathbf{g}}(\mathbf{x}) \end{bmatrix} \le \mathbf{0} \tag{3.1}$$

After creating a Lagrange function and deriving the Kuhn-Tucker conditions for this program, the necessary condition of an optimal solution are found to be equation (3.2).

$$\begin{aligned} \boldsymbol{\lambda}_h{}^T \frac{\partial \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{d})}{\partial \mathbf{x}} + \boldsymbol{\lambda}_g{}^T \frac{\partial \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{d})}{\partial \mathbf{x}} &= \mathbf{0} \\ -\frac{\partial C(\mathbf{s})}{\partial \mathbf{s}} - \tilde{\boldsymbol{\lambda}}_{\hat{h}s} - \boldsymbol{\lambda}_{gs\min} + \boldsymbol{\lambda}_{gs\max} &= \mathbf{0} \\ \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{d}) &= \mathbf{0} \\ \boldsymbol{\lambda}_g^T \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{d}) &= \mathbf{0} \\ \boldsymbol{\lambda}_g &\ge \mathbf{0} \end{aligned} \tag{3.2}$$

Reference [7] investigates the solution of the similar non-linear program with the objective maximizing the social welfare as shown in equation (3.3).

$$\max_{\mathbf{x},\mathbf{s},\mathbf{d}} \; B(\mathbf{d}) - C(\mathbf{s})$$
$$\text{s.t.} \quad \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}} \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix} = \mathbf{0}, \quad \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{d}) = \begin{bmatrix} \mathbf{s}_{\min} - \mathbf{s} \\ \mathbf{s} - \mathbf{s}_{\max} \\ \mathbf{d}_{\min} - \mathbf{d} \\ \mathbf{d} - \mathbf{d}_{\max} \\ \overline{\mathbf{g}}(\mathbf{x}) \end{bmatrix} \le \mathbf{0} \tag{3.3}$$

Necessary conditions for the solution of this problem are found to be those of equation (3.4).

$$
\begin{aligned}
&\begin{pmatrix} \boldsymbol{\lambda}_h{}^T \dfrac{\partial \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d}+\boldsymbol{\lambda}_{gd\min}-\boldsymbol{\lambda}_{gd\max}))}{\partial \mathbf{x}} \\ +\boldsymbol{\lambda}_g{}^T \dfrac{\partial \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d}+\boldsymbol{\lambda}_{gd\min}-\boldsymbol{\lambda}_{gd\max}))}{\partial \mathbf{x}} \end{pmatrix} = \mathbf{0} \\
&-\frac{\partial C(\mathbf{s})}{\partial \mathbf{s}} - \tilde{\boldsymbol{\lambda}}_{\hat{h}s} - \boldsymbol{\lambda}_{gs\min} + \boldsymbol{\lambda}_{gs\max} = \mathbf{0} \\
&\mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d}+\boldsymbol{\lambda}_{gd\min}-\boldsymbol{\lambda}_{gd\max})) = \mathbf{0} \quad (3.4) \\
&\boldsymbol{\lambda}_g^T \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d}+\boldsymbol{\lambda}_{gd\min}-\boldsymbol{\lambda}_{gd\max})) = \mathbf{0} \\
&\boldsymbol{\lambda}_g \geq \mathbf{0}
\end{aligned}
$$

What is important to notice about equation (3.4) is, it is the same equation as equation (3.2) with equation (3.5) substituted in for the demand vector **d**.

$$\mathbf{d} = \mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d} + \boldsymbol{\lambda}_{gd\min} - \boldsymbol{\lambda}_{gd\max}) \qquad (3.5)$$

Where the function $\mathbf{D}(\bullet)$ is the functional inverse of $\dfrac{\partial B(\bullet)}{\partial \mathbf{d}}$ and is called the consumer demand function. Reference [7] demonstrates the implementation of a consumer demand function for real power, as a function of the real power spot price, into the OPF algorithm.

Thus, in order to take an existing OPF algorithm that minimizes the cost of generation, one only needs to add equation (3.5) to the necessary conditions.

4. Creation of a Demand Function for Reactive Power

As mentioned in the previous section the price-dependent load model is based on the existence of a consumer benefit function, B(**d**), where **d** includes both the real and reactive power demand: $\mathbf{d} = \begin{bmatrix}\mathbf{d}_p^T & \mathbf{d}_q^T\end{bmatrix}^T$. In [7] the consumer demand function for real power was assumed to be a linear function of the real power spot price as in Figure 1.

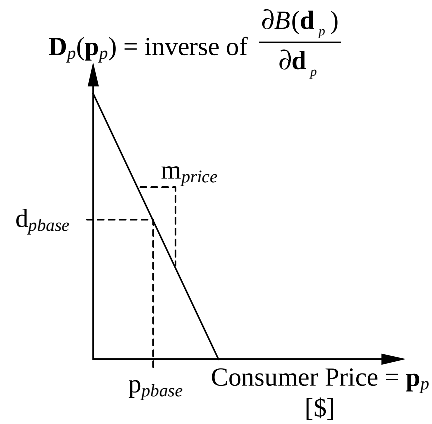

Figure 1 Real Power Demand Function

Equation (4.1) shows the matrix-vector representation of this for an entire power system.

$$\mathbf{D}_p(\mathbf{p}_p) = (\mathbf{d}_{p\,base} + \mathbf{M}_{price}\mathbf{p}_{p\,base}) - \mathbf{M}_{price}\mathbf{p}_p \qquad (4.1)$$

These demand functions correspond to a quadratic consumer benefit function that is only a function of the real power spot price as shown in equation (4.2).

$$B_p(\mathbf{d}_p) = \mathbf{d}_p^T\left(\mathbf{M}_{price}^{-1}\mathbf{d}_{pbase} + \mathbf{p}_{pbase}\right) - \frac{1}{2}\mathbf{d}_p^T\mathbf{M}_{price}^{-1}\mathbf{d}_p \qquad (4.2)$$

In this development, the consumer demand is assumed to follow a constant power factor, i.e. the demand for reactive power is always be equal to some constant times the real power. Essentially this corresponds to a benefit function for reactive power which is some constant when operating at constant power factor and equal to negative infinity when not operating at constant power factor.

In order to incorporate the simulation of a reactive power market through price-dependent reactive loads, it is first necessary to determine a benefit function that befits the benefits gained by reactive power consumption. This reactive benefit function should not follow the same mold as the real power benefit equation because reactive power really serves more as a service which enables the consumption of real power. Using this point of view, consider the benefit of the reactive power as the avoidance of moving the reactive power from some desired level for a given power consumption. Define a desired reactive power demand as a function of the real power demand: $d_{qdesired} = f(d_p)$. This desired reactive demand will be the demand which the load will naturally require at the given load level. Also assume that the magnitude of the function increases with d_p. Now consider a concave function, *k(x)*, which has a maximum value of zero at zero such as in Figure 2.

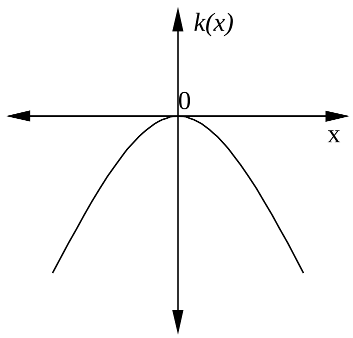

Figure 2 Concave *k(x)*

Then using $d_{qdesired} = f(d_p)$ along with the function *k(x)*, construct the reactive power benefit function for an individual load is as in equation (4.3).

$$B_q(d_p, d_q) = B_{qo}k\big(d_q - f(d_p)\big) \tag{4.3}$$

Due to the properties specified for *k(x)*, this benefit has a maximum value of zero which is achieved when the reactive power demand is equal to the desired reactive demand, $f(d_p)$. The benefit decreases on both sides of this value because the consumer must provide their own reactive power support using power electronics, capacitive/inductor support, etc. One can envision a system load with a filtering device such as that seen in Figure 3.

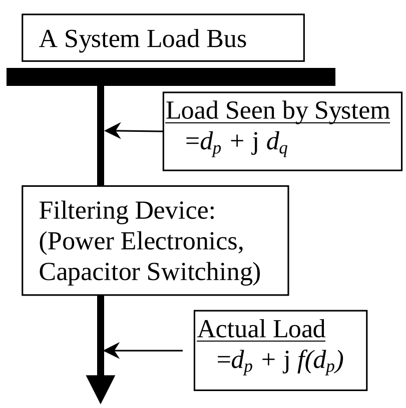

Figure 3 The Load Model with Reactive Control

The model here assumes that the cost of operating this filtering device is equal to zero when it is performing no filtering, i.e. when $d_q = f(d_p)$ and increases, according to *k(x)*, as it moves away from that point.

Incorporating equation (4.3) into the total consumer benefit results in a function of both the real and reactive demand.

$$B(\mathbf{d}_p, \mathbf{d}_q) = \sum_{\text{all consumers}} \big(B_p(d_p) + B_q(d_p, d_q)\big) \tag{4.4}$$

The consumer demand function can then be calculated from equation (4.4) by determining the functional inverse of the derivative of this consumer benefit function.

5. Example Demand Function for Reactive Power

As an example consider using the quadratic real power benefit function for each consumer of the form $B_p(d_p) = ad_p - bd_p^2$. This is the same benefit as that seen in equation (4.2). For the reactive power benefit function, use $d_{qdesired} = f(d_p) = \gamma d_p$ and $k(x) = -x^2$. This corresponds to a constant power factor load where $\gamma = \frac{\sqrt{1-\text{pf}^2}}{\text{pf}}$ and the cost of compensating for reactive power by the consumer increases quadratically as it moves away from $d_q = f(d_p)$. Thus the consumer benefit function is equation (5.1).

$$\begin{aligned} B(d_p, d_q) &= ad_p - bd_p^2 - B_{qo}\left[d_q - \gamma d_p\right]^2 \\ &= ad_p + (-b - B_{qo}\gamma^2)d_p^2 - B_{qo}d_q^2 + 2B_{qo}\gamma d_p d_q \end{aligned} \tag{5.1}$$

It should be noted that equation (5.1) is simply the summation of two concave functions and is therefore itself concave.

Now it is of interest to determine the consumer demand functions that result by calculating the functional inverse of the derivative of the consumer demand function. The derivative with respect to the real power demand is shown in equation (5.2).

$$\frac{\partial B(d_p, d_q)}{\partial d_p} = \underbrace{\left[a - 2bd_p\right]}_{\frac{\partial B_p(d_p)}{\partial d_p}} + \underbrace{\left[2B_{qo}\gamma(-\gamma d_p + d_q)\right]}_{\frac{\partial B_q(d_p, d_q)}{\partial d_p}} \tag{5.2}$$

In looking at the derivative of the consumer benefit function with respect to real power demand, it is normally expected that this derivative always be positive because some marginal benefit is expected from increased consumption. However, because of the model that is being proposed this will not always be true.

One possibility is that $\frac{\partial B_p(d_p)}{\partial d_p}$ is negative. This is an artifact of using the model design the range intended. The benefit model for real power should be concave and <u>increasing</u>. The quadratic function is concave, but will begin decreasing once the maximum point is reaching. It is therefore important that the real power load for a consumer be limited so that the function $a - 2bd_p$ is always positive. Otherwise the consumer gets more benefit by decreasing power consumption regardless of any other costs. This would not make sense as the consumer could always resell the power and thereby get some use.

The other possibility is that $\frac{\partial B_q(d_p, d_q)}{\partial d_p}$ be negative and begin to dominate the first term. This is not unrealistic. It would merely mean that further increases in real power would result in large costs for reactive power consumption and therefore reduce the consumer benefit.

Now consider taking the derivative with respect to reactive power demand.

$$\frac{\partial B(d_p, d_q)}{\partial d_q} = 2B_{qo}(-d_q + \gamma d_p) \tag{5.3}$$

Equation (5.3) shows that in order to increase benefit, the reactive power demand is always pushed toward the desired level γd_p.

In order to determine the consumer demand function, equate equations (5.2) and (5.3) to the price of real and reactive power respectively.

$$\frac{\partial B(d_p, d_q)}{\partial \mathbf{d}} = \begin{bmatrix} a \\ 0 \end{bmatrix} + \begin{bmatrix} -2b - 2B_{qo}\gamma^2 & 2B_{qo}\gamma \\ 2B_{qo}\gamma & -2B_{qo} \end{bmatrix} \begin{bmatrix} d_p \\ d_q \end{bmatrix} = \begin{bmatrix} p_p \\ p_q \end{bmatrix} \tag{5.4}$$

Then solve for real and reactive demand in terms of these prices, i.e. determine the functional inverse.

$$\mathbf{D}(\bullet) = \begin{bmatrix} -\frac{1}{2b}(p_p + \gamma p_q) + \frac{a}{2b} \\ \gamma\left(-\frac{1}{2b}(p_p + \gamma p_q) + \frac{a}{2b}\right) - \frac{1}{2B_{qo}} p_q \end{bmatrix} \tag{5.5}$$

$$\text{NOTE: } d_q = \gamma d_p - \frac{1}{2B_{qo}} p_q$$

As in [7], it is helpful to rewrite this additional equation in a more meaningful way. Equation (5.5) can be re-written as follows:

$$\mathbf{D}(\bullet) = \begin{bmatrix} d_{pbase} + m_p p_{pbase} \\ d_{qbase} + \gamma m_p p_{pbase} \end{bmatrix} - \begin{bmatrix} m_p & \gamma m_p \\ \gamma m_p & \left(\gamma^2 m_p + \frac{d_{qbase}}{2\overline{B_{qo}}}\right) \end{bmatrix} \begin{bmatrix} p_p \\ p_q \end{bmatrix} \tag{5.6}$$

with the following definitions:

$$m_p = \frac{1}{2b} \qquad \gamma = \frac{d_{qbase}}{d_{pbase}} \qquad B_{qo} = \frac{\overline{B_{qo}}}{d_{qbase}}$$

and one demand point of $d_p = d_{pbase}$ and $d_q = d_{qbase}$ at prices $p_p = p_{pbase}$ and $p_q = 0$ given.

Note that as $B_{qo} \to \infty$, this consumer demand function will behave very similar the consumer demand function of equation (4.1) with $d_q = \gamma d_p - \frac{1}{2B_{qo}} p_q \underset{B_{qo} \leftrightarrow \infty}{=} \gamma d_p$. There will be some small difference because the real power consumption will now also be based on both the reactive power price and the real power spot price as in equation (5.7).

$$\begin{aligned} d_p &= \left(d_{pbase} + m_p p_{pbase}\right) - m_p(p_p + \gamma p_q) \\ d_q &= \gamma d_p \end{aligned} \tag{5.7}$$

However, this difference will be small as long as the reactive power price is relatively small, as it normally is.

Now, at each demand bus specify a value of m_p, p_{pbase}, $\overline{B_{qo}}$, d_{pbase}, and d_{qbase} and substitute

$$\begin{aligned} \mathbf{D}_{busk}\left(\begin{bmatrix} \lambda_{\hat{h}dp} + \lambda_{gdp\min} - \lambda_{gdp\max} \\ \lambda_{\hat{h}dq} + \lambda_{gdq\min} - \lambda_{gdq\max} \end{bmatrix}\right) &= \begin{bmatrix} d_{pbase} + m_p p_{pbase} \\ d_{qbase} + \gamma m_p p_{pbase} \end{bmatrix} \\ &- \begin{bmatrix} m_p & \gamma m_p \\ \gamma m_p & \left(\gamma^2 m_p + \frac{d_{qbase}}{2\overline{B_{qo}}}\right) \end{bmatrix} \begin{bmatrix} \lambda_{\hat{h}dp} + \lambda_{gdp\min} - \lambda_{gdp\max} \\ \lambda_{\hat{h}dq} + \lambda_{gdq\min} - \lambda_{gdq\max} \end{bmatrix} \end{aligned} \tag{5.8}$$

into the optimal power flow necessary conditions shown in equation (3.4). This will result in the social welfare maximum for the economic load model that has been described.

6. Implementation of Real and Reactive Price-Dependent Loads into the OPF

This section will only study how the consumer demand function of equation (5.6) effects the calculations of the Newton's method algorithm. Equation (3.4) will be repeated here to further study how the choice consumer demand function will effect the solution of these equations.

$$\begin{aligned} &\left(\begin{aligned} &\boldsymbol{\lambda}_h{}^T \frac{\partial \mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d} + \boldsymbol{\lambda}_{gd\min} - \boldsymbol{\lambda}_{gd\max}))}{\partial \mathbf{x}} \\ &+ \boldsymbol{\lambda}_g{}^T \frac{\partial \mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d} + \boldsymbol{\lambda}_{gd\min} - \boldsymbol{\lambda}_{gd\max}))}{\partial \mathbf{x}} \end{aligned}\right) = \mathbf{0} \\ &-\frac{\partial C(\mathbf{s})}{\partial \mathbf{s}} - \tilde{\boldsymbol{\lambda}}_{\hat{h}s} - \boldsymbol{\lambda}_{gs\min} + \boldsymbol{\lambda}_{gs\max} = \mathbf{0} \\ &\mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d} + \boldsymbol{\lambda}_{gd\min} - \boldsymbol{\lambda}_{gd\max})) = \mathbf{0} \\ &\boldsymbol{\lambda}_g^T \mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{D}(-\tilde{\boldsymbol{\lambda}}_{\hat{h}d} + \boldsymbol{\lambda}_{gd\min} - \boldsymbol{\lambda}_{gd\max})) = \mathbf{0} \\ &\boldsymbol{\lambda}_g \geq \mathbf{0} \end{aligned} \tag{6.1}$$

It is first noted that after taking derivatives of **h** and **g** with respect to **x**, no dependence on **s** or **d** is found (as long as **s** and **d** are not functions of **x**, which we have assumed). Therefore, the choice of the consumer demand function has no effect on the first equation. The only influence comes in the third and fourth equations. The consumer demand function has only changed the demand function from a constant to one dependent on the Lagrange multipliers $\tilde{\boldsymbol{\lambda}}_{\hat{h}d}$, $\boldsymbol{\lambda}_{gd\,\min}$ and $\boldsymbol{\lambda}_{gd\,\max}$. This will not impede the OPF algorithm as it will only require a simple function evaluation.

In using Newton's method to solve these nonlinear equations, derivatives of the equations must be determined in order to calculate a Hessian matrix. In order to evaluate how the consumer demand function will effect these equations take the derivatives of the third and fourth equations with respect to $\tilde{\boldsymbol{\lambda}}_{\hat{h}d}$, $\boldsymbol{\lambda}_{gd\,\min}$ and $\boldsymbol{\lambda}_{gd\,\max}$.

$$\frac{\partial \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \tilde{\boldsymbol{\lambda}}_{\hat{h}d}} = \frac{\partial \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}}) \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix}}{\partial \mathbf{d}} \frac{\partial \mathbf{D}}{\partial \tilde{\boldsymbol{\lambda}}_{\hat{h}d}} = -[\hat{\mathbf{I}}_{PQ}][-\hat{\mathbf{M}}_{PQprice}]$$

$$\frac{\partial \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \boldsymbol{\lambda}_{gd\,\min}} = \frac{\partial \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}}) \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix}}{\partial \mathbf{d}} \frac{\partial \mathbf{D}}{\partial \boldsymbol{\lambda}_{gd\,\min}} = +[\hat{\mathbf{I}}_{PQ}][-\tilde{\mathbf{M}}_{PQprice}]$$

$$\frac{\partial \mathbf{h}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \boldsymbol{\lambda}_{gd\,\max}} = \frac{\partial \begin{bmatrix} \hat{\mathbf{h}}(\mathbf{x}) - \hat{\mathbf{s}} + \hat{\mathbf{d}}) \\ \overline{\mathbf{h}}(\mathbf{x}) \end{bmatrix}}{\partial \mathbf{d}} \frac{\partial \mathbf{D}}{\partial \boldsymbol{\lambda}_{gd\,\max}} = -[\hat{\mathbf{I}}_{PQ}][-\overline{\mathbf{M}}_{PQprice}]$$

$$\frac{\partial \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \tilde{\boldsymbol{\lambda}}_{\hat{h}d}} = \frac{\partial \begin{bmatrix} \mathbf{s}_{\min} - \mathbf{s} \\ \mathbf{s} - \mathbf{s}_{\max} \\ \mathbf{d}_{\min} - \mathbf{d} \\ \mathbf{d} - \mathbf{d}_{\max} \\ \overline{\mathbf{g}}(\mathbf{x}) \end{bmatrix}}{\partial \mathbf{D}} \frac{\partial \mathbf{D}}{\partial \tilde{\boldsymbol{\lambda}}_{\hat{h}d}} = +[\tilde{\mathbf{I}}_{PQ}][-\hat{\mathbf{M}}_{PQprice}]$$

$$\frac{\partial \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \boldsymbol{\lambda}_{gd\,\min}} = +[\tilde{\mathbf{I}}_{PQ}][\tilde{\mathbf{M}}_{PQprice}]$$

$$\frac{\partial \mathbf{g}(\mathbf{x},\mathbf{s},\mathbf{D})}{\partial \boldsymbol{\lambda}_{gd\,\max}} = +[\tilde{\mathbf{I}}_{PQ}][-\overline{\mathbf{M}}_{PQprice}]$$

where the variables are defined as

$\hat{\mathbf{I}}_{PQ} =$ matrix with diagonal entries of 1 corresponding to consumer demand variables $\mathbf{d}$

$\tilde{\mathbf{I}}_{PQ} =$ matrix with diagonal entries of 1 corresponding to consumer demands $\mathbf{d}$ which are at a limit

$$\hat{\mathbf{M}}_{PQ\,price} = \begin{bmatrix} m_p & \gamma m_p \\ \gamma m_p & \left(\gamma^2 m_p + \dfrac{d_{qbase}}{2B_{qo}}\right) \end{bmatrix}$$

a block diagonal matrix with 2X2 entries of corresponding to variables $\mathbf{d}$

$$\tilde{\mathbf{M}}_{PQprice} = \begin{bmatrix} m_p & \gamma m_p \\ \gamma m_p & \left(\gamma^2 m_p + \dfrac{d_{qbase}}{2B_{qo}}\right) \end{bmatrix}$$

a block diagonal matrix with 2X2 entries of corresponding to variables $\boldsymbol{\lambda}_{gd\,\min}$ related to $\mathbf{d}$

$$\overline{\mathbf{M}}_{PQprice} = \begin{bmatrix} m_p & \gamma m_p \\ \gamma m_p & \left(\gamma^2 m_p + \dfrac{d_{qbase}}{2B_{qo}}\right) \end{bmatrix}$$

a block diagonal matrix with 2X2 entries of corresponding to variables $\boldsymbol{\lambda}_{gd\,\max}$ related to $\mathbf{d}$

The key point to recognize is that the effect of the additional price-dependent equations on the Hessian matrix is limited to small 2X2 block diagonal entries. From equation (7) of reference [8], the Hessian matrix for the coupled OPF formulation is shown to have the structure in equation (6.2).

$$\mathbf{W} = \begin{bmatrix} \mathbf{H} & -\mathbf{J}^{\mathbf{T}} \\ -\mathbf{J} & \mathbf{0} \end{bmatrix} \tag{6.2}$$

The block diagonal entries which are added to the Hessian by the price dependent loads will be in the zero matrix in the lower right partition. Because some entries are added on the off-diagonals in this zero matrix, it is possible that some degradation of sparsity may occur, however it will be minor. Figure 4 shows this addition of 2X2 blocks along the diagonal to the lower right partition of the Hessian matrix for the IEEE 118-bus sample system. [7] shows the Hessian matrix for the same 118-bus system that maximizes social welfare including real power price dependence only as well as the Hessian matrix for minimizing generator costs only.

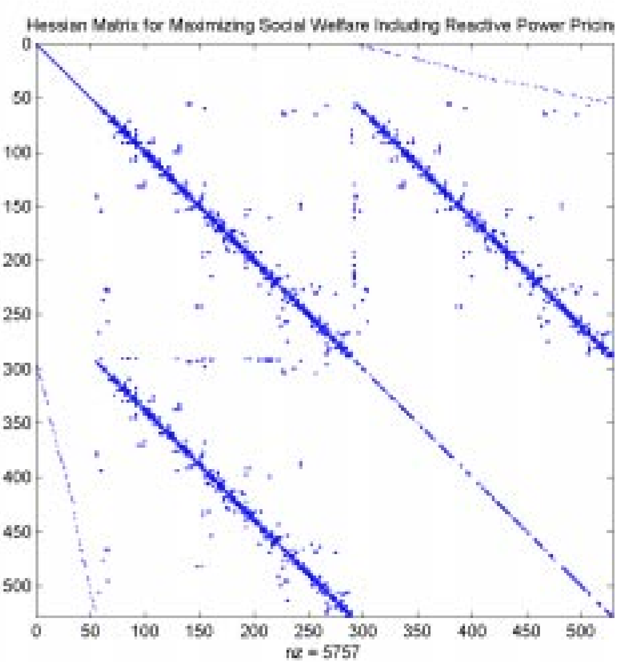

Figure 4 Hessian Matrix for IEEE 118-Bus System after Adding Real and Reactive Price Dependence

7. Sample Case

The OPF modifications as discussed in the previous section were implemented into a standard OPF that minimizes fuel costs [9]. As a case for comparison purposes, Figure 5 shows a small six-bus system that was optimized to maximize social welfare using only real power price dependence as in reference [7].

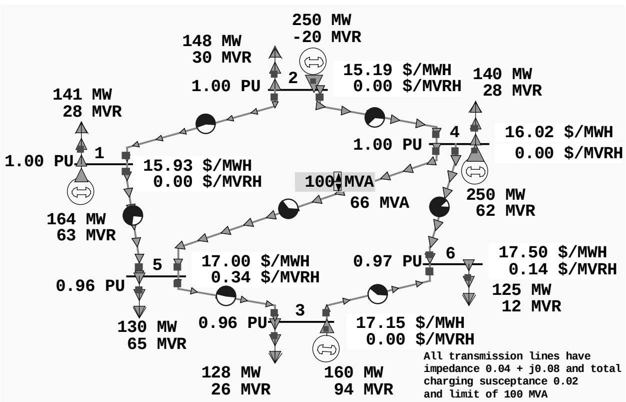

Figure 5 Maximizing Social Welfare Using Only Real Power Price Dependence

In Figure 5, the real power loads followed the following price dependent model while the reactive power load maintained constant power factor regardless of marginal cost.

$$d_p(p_p) = d_{pbase}\left(1+\frac{m_{price}}{d_{pbase}}(p_{pbase}-p_p)\right) = d_{pbase}\left(1+10(20-p_p)\right)$$

with d_{pbase} equal to the base demand for the bus. In this example d_{pbase} = 100 MW for each load. This price model means the loads will consume their d_{pbase} if the spot price is p_p = 20 $/MWH. If the spot price falls below 20 $/MWH the loads will begin to consume more power, while if the spot price increases the loads will respond by consuming less. This sensitivity to price is encapsulated in the term $\frac{m_{price}}{d_{pbase}}$ that determines the slope of demand function given in Figure 1. In Figure 5, the spot prices are all below 20 $/MWH causing the loads to all converge to more than their d_{pbase}.

By modeling the loads with reactive power price dependence, it is expected that the reactive power loads at buses 5 and 6 may be reduced, as their price is positive. The variable $\overline{B_{qo}}$ was chosen to be 0.5 and the OPF results are shown in Figure 6.

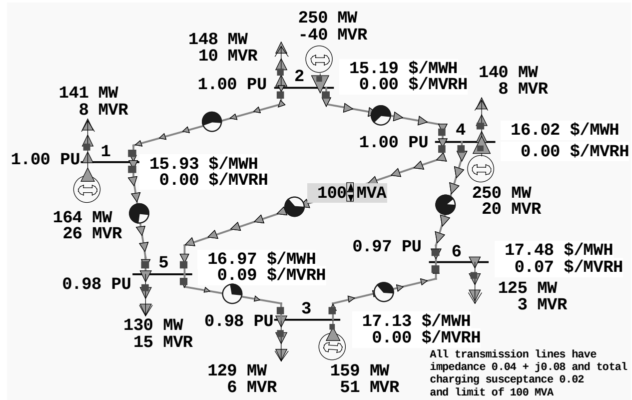

Figure 6 Reactive Power Price Dependence, $\overline{B_{qo}}$ = 0.5

As mentioned previously, as $\overline{B_{qo}}$ becomes large, the system should converge to a solution near the solution which ignored reactive power price dependence. This same six-bus solution with $\overline{B_{qo}}$ = 100 is shown in Figure 7.

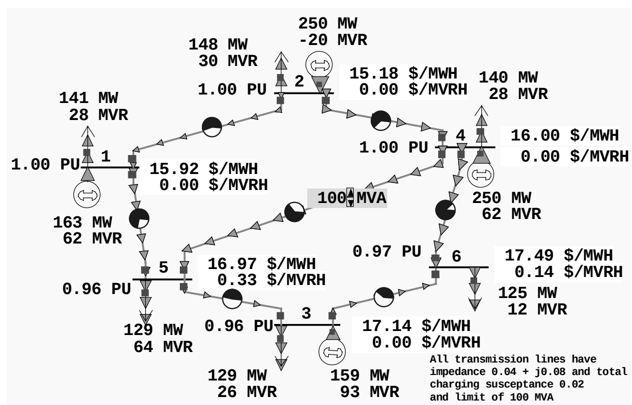

Figure 7 Reactive Power Price Dependence, $\overline{B_{qo}}$ = 100

Comparing Figure 5 and Figure 7, one sees that there is very little difference in the solutions. Some difference does exist, but this is small as it is due to the real power demand dependence on the reactive power price. Since the real power price is much larger than the reactive price, very little change in real power is seen.

As a power system approaches a voltage limit, however, the reactive power marginal costs increase rapidly. At these points, the influence of reactive power price dependence will make a large difference in the OPF solutions. Figure 8 shows a sample three-bus power system taken from [3] simulated using the real power price dependent model $d_p(p_p) = d_{pbase}\left(1+10(35-p_p)\right)$ and reactive power demand maintaining constant power factor.

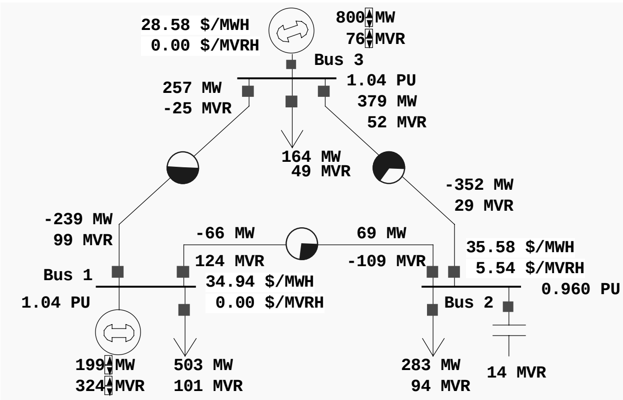

Figure 8 Three-Bus System with Real Power Price Dependence Only

Bus 2 in Figure 8 is at the voltage limit specified of 0.96 pu and as a result the reactive power marginal cost at this bus is a very large 5.54 $/MVRH. Applying the reactive power price dependence model in this situation will not only decrease the reactive power demand, but may also enable the bus to increase its real power demand as the voltage limit is removed. The result of simulating this system with $\overline{B_{qo}}$ = 1.0 is shown in Figure 9. The results shown in Figure 9 verify that the real power demand is able to increase as expected.

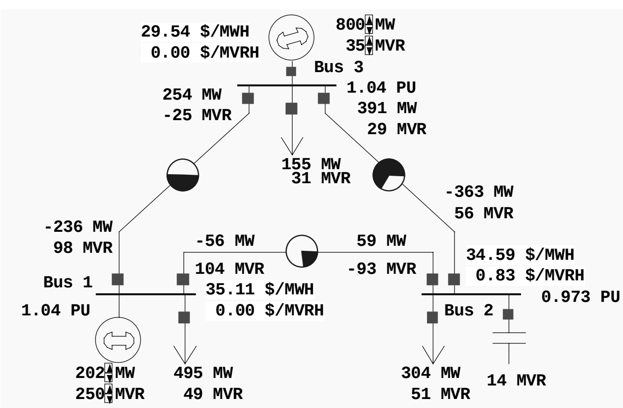

Figure 9 Reactive Power Price Dependence, $\overline{B_{qo}}$ =1.0

8. Conclusion

The results shown in this paper illustrate that the implementation of price dependent load models into the optimal power flow is an effective way to simulate both a real and reactive power spot markets. As market rules are created, these load models may be a valuable tool for modeling potential behavior. These simulations could aid in creating rules that encourage the market participants to find the social welfare maximum. On the other side of the coin, the simulations may be helpful to the market participants in modeling the behavior of other players in the market.

These methods could also be used to actually implement the market mechanisms. Participants could be required to submit consumer demand functions and supplier supply functions. These functions could then be fed into an OPF solution engine and determine the "optimal" point assuming that the participants were bidding their true marginal behavior.

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