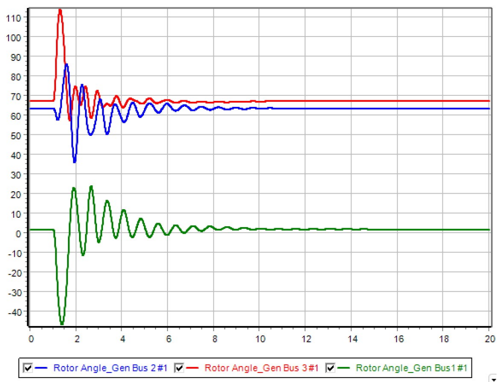
# Solution HW5

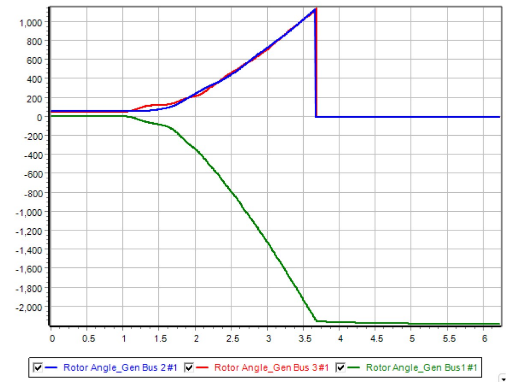
## Solution 1

Critical clearing time is 0.21 seconds.
Rotor angles diverge when clearing time is increased more than 1.21 seconds.

Rotor angles at $t_{clear} = 1.21s$




Rotor angles at $t_{clear} = 1.22s$

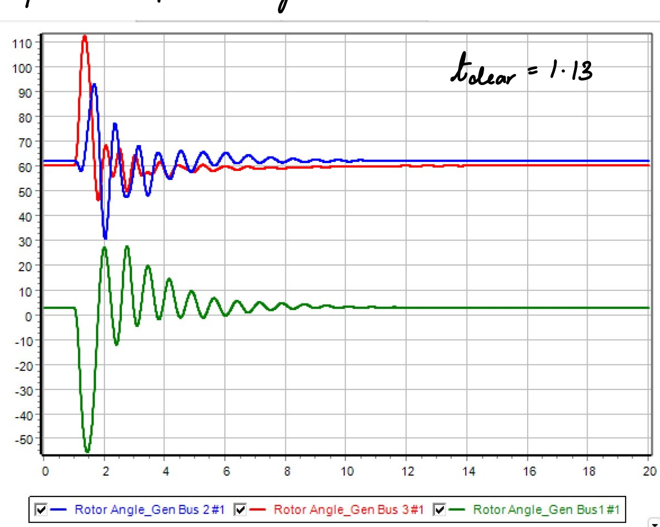


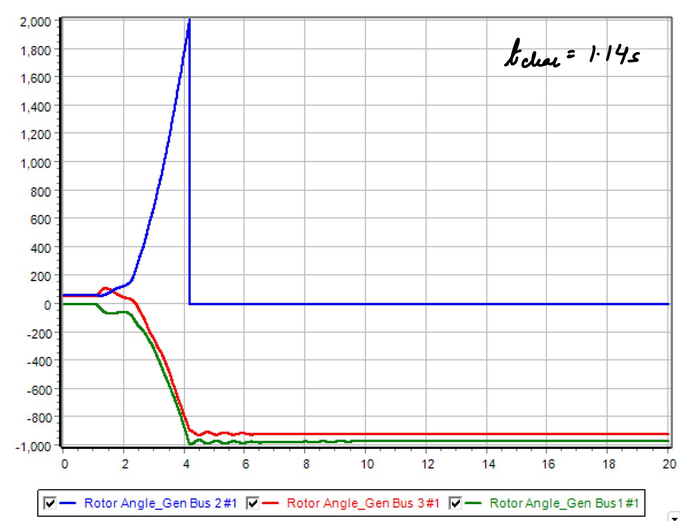

## Solution 2

| Sr. No. | KA | Real Power (MW) | Clearing time(s) |
|---|---|---|---|
| 1 | 0.5 | 85 | 0.20 |
| 2 | 10 | 85 | 0.21 |
| 3 | 20 | 85 | 0.21 |
| 4 | 40 | 85 | 0.21 |
| 5 | 60 | 85 | 0.21 |
| 6 | 20 | 100 | 0.15 |
| 7 | 40 | 120 | 0.13 |

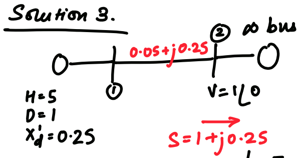
This could be because of the Voltage limits on $V_R$. (from block diag.)

Increasing $K_A$ does not have much impact on the clearing time. However, decreasing it to 0.5, changes the clearing time slightly.
Changing the MW output of the generator has a significant impact on the clearing time.
With Real power output changed to 100MW :-



Solution 3.




$\hookrightarrow I = \left(\frac{1+j0.25}{1\angle 0^\circ}\right)^* = 1-j0.25$ pu

$\Delta t = 0.01s$

Gen dynamics are:

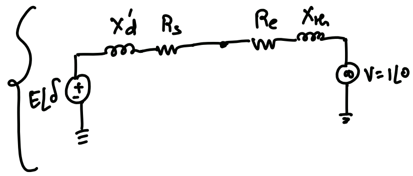
$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu} \cdot \omega_s \qquad (\omega_s = 2\pi f = 376.9)$$

$$\frac{d\Delta\omega_1}{dt} = \frac{1}{2H}\left(P_m - \frac{E_1 V_{inf}}{X_{th}} \sin\delta_1 - D_1(\Delta\omega_1)\right)$$

Gen internal voltage and $\delta$ can be found.

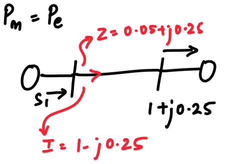
$t=0^+$ $(\Delta\omega = 0)$

$$E\angle\delta = 1\angle 0 + (0.05+j0.25+j0.25)(1-j0.25) = 1.175+j0.4875 = 1.27\angle \overset{\delta}{\uparrow}{22.53^\circ}$$




pre-fault (at steady state)

$P_m = P_e$




$$S_\infty + ((I)(Z))I^*$$

$$S_1 = (1+j0.25) + ((1-j0.25)(0.05+j0.25))(1+j0.25)$$

$$= \underbrace{1.0531}_{P_e} + j0.781$$

$P_m = 1.0531$ pu

during fault; $P_e = 0$ (since fault is at bus 1)

$$\dot{\delta} = (\Delta\omega_1)\omega_s$$

$$\Delta\dot{\omega}_1 = \frac{1}{2H}\left(P_m - \underset{0}{\cancel{P_e}} - D(\Delta\omega_1)\right)$$

initial conditions:

$$\begin{bmatrix}\delta \\ \Delta\omega\end{bmatrix} = \begin{bmatrix}0.393 \\ 0\end{bmatrix}$$

Euler's method $\rightarrow$ $x(t+\Delta t) = x(t) + \Delta t \cdot f(x(t))$

$\Delta t = 0.01$

$$x(0.01) = x(0) + (0.01)(f(x(0)))$$

$$= \begin{bmatrix}0.393 \\ 0\end{bmatrix} + 0.01\begin{bmatrix}0 \\ \frac{1}{2\times 5}(1.053 - 1(\cancel{0}))\end{bmatrix} = \begin{bmatrix}0.393 \\ 0.00105\end{bmatrix}$$

$$x(0.02) = x(0.01) + 0.01(f(x(0.01)))$$

$$= \begin{bmatrix}0.393 \\ 0.00105\end{bmatrix} + 0.01\begin{bmatrix}0.396 \\ \frac{1}{2\times 5}(1.053 - 1(0.00105))\end{bmatrix} = \begin{bmatrix}0.396 \\ 0.002102\end{bmatrix}$$

$$\delta(0.02) = 22.68^\circ$$

$$\omega(0.02) = \omega_s + \Delta\omega = (0.002)2\pi 60 + \omega_s = 377.78$$

Solution 4

For RK2; calculate $K_1$ and $K_2$.

$$K_1 = \Delta t\, f(x(t)) \quad \text{and} \quad K_2 = \Delta t \cdot f(x(t) + K_1)$$

$$x(t+\Delta t) = x(t) + \frac{1}{2}(K_1 + K_2)$$

initial conditions (remain same as ques. 3.)

$$\begin{bmatrix}\delta \\ \Delta\omega\end{bmatrix} = \begin{bmatrix}0.393 \\ 0\end{bmatrix} = x(0)$$

$$K_1 = 0.01\begin{bmatrix}0 \\ \frac{1}{10}(1.053)\end{bmatrix} = \begin{bmatrix}0 \\ 0.00105\end{bmatrix}$$

$$K_2 = 0.01\begin{bmatrix}0.396 \\ 0.105\end{bmatrix} = \begin{bmatrix}0.00396 \\ 0.00105\end{bmatrix}$$

$$x(0.01) = \begin{bmatrix}0.393 \\ 0\end{bmatrix} + \frac{1}{2}\begin{bmatrix}0.00396 \\ 0.0021\end{bmatrix} = \begin{bmatrix}0.394 \\ 0.00105\end{bmatrix}$$

$$k_1 = 0.01\begin{bmatrix}0.396\\0.105\end{bmatrix} = \begin{bmatrix}0.00396\\0.00105\end{bmatrix}$$

$$k_2 = 0.01\left[f\begin{pmatrix}0.397\\0.0021\end{pmatrix}\right] = 0.01\begin{bmatrix}0.7916\\0.1051\end{bmatrix} = \begin{bmatrix}0.00792\\0.00105\end{bmatrix}$$

$$x(0.02) = \begin{bmatrix}0.394\\0.00105\end{bmatrix} + \begin{bmatrix}0.00594\\0.00105\end{bmatrix}$$

$$= \begin{bmatrix}0.399\\0.0021\end{bmatrix}$$

$$\delta(0.02) = 22.86^\circ$$

$$\Delta\omega(0.02) = 377.78$$

<u>Solution.5</u>

<u>Implicit</u> Trapezoidal method;

$$x(t+\Delta t) = x(t) + \frac{\Delta t}{2}\left[f(x(t)) + f(x(t+\Delta t))\right]$$

initial conditions

$$\begin{bmatrix}\delta\\\Delta\omega\end{bmatrix} = \begin{bmatrix}0.393\\0\end{bmatrix}$$

Need an iterative approach to solve it. Thus, we need a jacobian

$$J(x(t+\Delta t)) = \frac{\Delta t}{2}\begin{bmatrix}0 & \omega_s\\0 & \frac{-D}{2H}\end{bmatrix} - I$$

w.r.t $\delta$ w.r.t $\Delta\omega$

from previous solutions $\longrightarrow$ $\dot{\delta} = (\Delta\omega)\,\omega_s$, $\Delta\dot{\omega} = \frac{1}{2H}\left(P_m - D(\Delta\omega)\right)$

$$\longrightarrow J = \frac{\Delta t}{2}\begin{bmatrix}\frac{d\dot{\delta}}{d\delta} & \frac{d\dot{\delta}}{d\Delta\omega}\\ \frac{d\dot{\delta}}{d\delta} & \frac{d\Delta\dot{\omega}}{d\Delta\omega}\end{bmatrix} - I$$

identity matrix

$$f(x) = \begin{bmatrix}\dot{\delta}\\\Delta\dot{\omega}\end{bmatrix}$$

and mismatch;

$$h(x(t+\Delta t)^k) = -x(t+\Delta t)^k + x(t) + \frac{\Delta t}{2}\left(f(x(t+\Delta t)^k) + f(x(t))\right)$$

then;

$$x(t+\Delta t)^{k+1} = x(t+\Delta t)^k - \left[J(x(t+\Delta t)^k)\right]^{-1} h\left(x(t+\Delta t)^k\right)$$

at $t=0$, $\Delta t = 0.01$

<u>iteration 1 $(k=0)$</u>

$$J(x(0.01)^0) = \frac{0.01}{2}\begin{bmatrix}0 & 2\pi\cdot 60\\0 & -0.1\end{bmatrix} - \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$= \begin{bmatrix}-1 & 1.885\\0 & -1.0005\end{bmatrix} \rightarrow \text{independent of } \delta, \Delta\omega \rightarrow \text{constant for all iterations}$$

$$x(0.01)^0 = x(0)\ [\text{initial guess is } x(0)] \longrightarrow x(0) = \begin{bmatrix}0.393\\0\end{bmatrix}$$

mismatch

$$h(x(0.01)^0) = -\cancel{x(0.01)^0} + \cancel{x(0)} + \frac{0.01}{2}\left(f(x(0.01)^0) + f(x(0))\right)$$

$$= \frac{0.01}{2}\left(f(x(0.01)^0) + f(x(0))\right)$$

$$= \frac{0.01}{2}\left(\begin{bmatrix}0\\0.10531\end{bmatrix} + \begin{bmatrix}0\\0.10531\end{bmatrix}\right)$$

$$= \begin{bmatrix}0\\0.001053\end{bmatrix}$$

$$x(0.01)^1 = x(0.01)^0 - \left[J(x(0.01)^0)\right]^{-1} h(x(0.01)^0)$$

$$= \begin{bmatrix}0.393\\0\end{bmatrix} - \begin{bmatrix}-1 & 1.885\\0 & -1.005\end{bmatrix}^{-1}\cdot\begin{bmatrix}0\\0.001053\end{bmatrix} = \begin{bmatrix}0.395\\0.001\end{bmatrix}$$

iteration 2 $(k=1)$

$$h\left(x(0.01)'\right) = -x(0.01)' + x(0) + \frac{0.01}{2}\left(f(x(0.01)') + f(x(0))\right)$$

$$= -\begin{bmatrix}0.395\\0.001\end{bmatrix} + \begin{bmatrix}0.393\\0\end{bmatrix} + \frac{0.01}{2}\left(\begin{bmatrix}0.376\\0.1052\end{bmatrix} + \begin{bmatrix}0\\0.1053\end{bmatrix}\right)$$

$$= \begin{bmatrix}-0.0001\\0.000053\end{bmatrix} \approx 0$$

$\therefore$ we have converged with $x(0.01) = \begin{bmatrix}0.395\\0.001\end{bmatrix}$

at $t = 0.01$

$\Delta t = 0.01$

iteration one $(k=0)$ $\left\{ x(0.02)^0 = x(0.01) \right.$ [initial guess.]

$$h\left(x(0.02)^0\right) = \cancel{-x(0.02)^0} + \cancel{x(0.01)} + \frac{0.01}{2}\left(f(x(0.02)^0) + f(x(0.01))\right)$$

$$= \frac{0.01}{2}\left(\begin{bmatrix}0.376\\0.1052\end{bmatrix} + \begin{bmatrix}0.376\\0.1052\end{bmatrix}\right)$$

$$= \begin{bmatrix}0.00376\\0.001052\end{bmatrix}$$

$$x(0.02)' = x(0.02)^0 - \left[J(x(0.02)^0)\right]^{-1} \cdot h\left(x(0.02)^0\right)$$

$$= \begin{bmatrix}0.395\\0.001\end{bmatrix} - \begin{bmatrix}-1 & 1.885\\0 & -1\end{bmatrix}^{-1} \cdot \begin{bmatrix}0.00376\\0.001052\end{bmatrix}$$

$$= \begin{bmatrix}0.4007\\0.00205\end{bmatrix}$$

$$h\left(x(0.02)'\right) = -x(0.02)' + x(0.01) + \frac{0.01}{2}\left(f(x(0.02)') + f(x(0.01))\right)$$

$$= -\begin{bmatrix}0.4007\\0.00205\end{bmatrix} + \begin{bmatrix}0.395\\0.001\end{bmatrix} + \frac{0.01}{2}\left(\begin{bmatrix}0.773\\0.1051\end{bmatrix} + \begin{bmatrix}0.376\\0.001052\end{bmatrix}\right)$$

$$= \begin{bmatrix}0.000045\\-0.000052\end{bmatrix} \approx 0$$

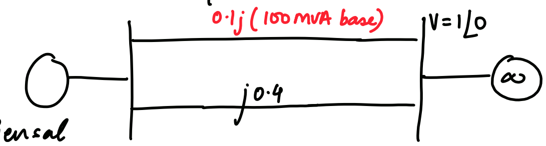
$\therefore$ we have converged with $x(0.02) = \begin{bmatrix}0.4007\\0.00205\end{bmatrix}$

value of $\delta$ at $t = 0.02$ is $22.95°$

value of $\omega$ at $t = 0.02$ is $377.764$

Solution 6

on a 400 MVA base
$j0.1 \times \frac{400}{100} = j0.4$




Gensal

400 MVA Base

$H = 5$

$P_{mech} = 1\,pu$

→ $t=0$ : fault occurs

→ $t=0.02$ : fault cleared

→ $t=0.04$

current into infinite bus ⇒ $S = VI^*$

$I^* = \frac{1}{1} = 1$

## pre-fault

$E = V + I\cdot(R_s + jX_q + jX_{line})$ [steady state representation of a synchronous machine]

$= 1\angle 0 + 1\cdot(0 + j1.5 + j0.2)$

$= 1 + j1.7 \longrightarrow 1.97\angle 59.53$

$\hookrightarrow \delta = 59.53 \longrightarrow 1.039$ radians

## during fault

$\frac{d\delta}{dt} = (\Delta\omega_{pu})\,\omega_s$

$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{2H}\left(P_m - P_e - \cancel{D(\Delta\omega)}\right) = \frac{1}{2H}(P_m - P_e)$

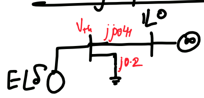
$H = 5;\quad P_m = 1$

$P_e = \frac{E\cdot V_{th}}{X_{th} + X_q}\sin\delta$

$= \frac{(1.97)(0.33)}{}\sin\delta$

$= 0.398\sin\delta$

using fault circuit




{ since fault occurs midway → $Z = 0.4 \| 0.2$

$V_{th} = \frac{1 \times 0.2}{0.6}$

$V_{th} = 0.33$

$X_{th} = 0.133$

$\frac{d\delta}{dt} = (\Delta\omega_{pu})377$

$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{10}(1 - 0.398\sin\delta)$

initial values

$\begin{bmatrix}\delta \\ \Delta\omega\end{bmatrix} = \begin{bmatrix}1.039 \\ 0\end{bmatrix} = x(0)$

$f(x) = \begin{bmatrix}\dot{\delta} \\ \Delta\dot{\omega}\end{bmatrix} = \begin{bmatrix}(\Delta\omega)\,377 \\ \frac{1}{10}(1 - 0.398\sin\delta)\end{bmatrix}$

Using Euler's method;

$x(t+\Delta t) = x(t) + \Delta t\cdot f(x(t))$

## at $t=0$

$x(0.01) = x(0) + 0.01\cdot f(x(0))$

$= \begin{bmatrix}1.039 \\ 0\end{bmatrix} + 0.01\begin{bmatrix}0 \\ 0.0656\end{bmatrix}$

$= \begin{bmatrix}1.039 \\ 0.00066\end{bmatrix}$

at $t = 0.01$

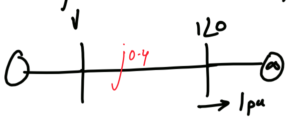
$$x(0.02) = x(0.01) + 0.01 \times f(x(0.01))$$

$$= \begin{bmatrix} 1.039 \\ 0.00066 \end{bmatrix} + 0.01 \begin{bmatrix} 0.248 \\ 0.0656 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0414 \\ 0.00132 \end{bmatrix}$$

at $t = 0.02$ (fault cleared)

after fault is cleared, the circuit will be




$P_e$ will change since V at gen. bus has changed.

$$V_{inf} = 1\angle 0$$

$$P_e = \frac{E \cdot V_{inf}}{X_q + X_{line}} \sin\delta$$

$$P_e = \frac{(1.97)(1)}{1.5 + 0.4} \sin\delta$$

$$= 1.036 \sin\delta$$

$$f(x) = \begin{bmatrix} (\Delta\omega_{pu})\, 377 \\ \frac{1}{10}(1 - 1.036 \sin\delta) \end{bmatrix} \qquad \text{initial value} = x(0.02) = \begin{bmatrix} 1.041 \\ 0.00132 \end{bmatrix}$$

$$x(0.03) = x(0.02) + (0.01) \cdot f(x(0.02))$$

$$= \begin{bmatrix} 1.041 \\ 0.00132 \end{bmatrix} + 0.01 \begin{bmatrix} 0.497 \\ 0.0106 \end{bmatrix}$$

$$= \begin{bmatrix} 1.046 \\ 0.00142 \end{bmatrix}$$

$$x(0.04) = x(0.03) + (0.01) \cdot f(x(0.03))$$

$$= \begin{bmatrix} 1.046 \\ 0.00142 \end{bmatrix} + 0.01 \begin{bmatrix} 0.497 \\ 0.0103 \end{bmatrix}$$

$$= \begin{bmatrix} 1.0509 \\ 0.00152 \end{bmatrix}$$

at $t = 0.04$

$\delta = 1.0509 \rightarrow 60.21°$

$\omega = 2\pi 60 + (\Delta\omega_{pu}) \cdot 2\pi 60 \longrightarrow 377.564$