ECEN 667 Homework 7 (Fall 2023)
Does Not Need to be Turned In, But Should be Done Before the Second Exam

1. Book 8.8

Problem #1 Book Problem 8.8

Find the participation factors of the eigenvalues for the following system, where $\dot{x} = Ax$.

Part A

$$A = \begin{bmatrix} 3 & 8 \\ 2 & 3 \end{bmatrix}$$

First, we need to get the eigenvalues.

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0 \implies \begin{vmatrix} 3-\lambda & 8 \\ 2 & 3-\lambda \end{vmatrix} = 0$$
$$\implies \lambda_1 = 7 \qquad \lambda_2 = -1$$

Then, we can calculate the right eigenvectors:

$$\boldsymbol{A}\boldsymbol{v}_1 = \lambda_1 \boldsymbol{v}_1 \implies \boldsymbol{v_1} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \boldsymbol{A}\boldsymbol{v}_2 = \lambda_2 \boldsymbol{v}_2 \implies \boldsymbol{v_2} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Then the left eigenvectors:

$$\boldsymbol{w}_1^t \boldsymbol{A} = \boldsymbol{w}_1^t \lambda_1 \implies \boldsymbol{w}_1^t = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \boldsymbol{w}_2^t \boldsymbol{A} = \boldsymbol{w}_2^t \lambda_1 \implies \boldsymbol{w}_2^t = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

However, we need $w_i^t v_i = 1$, so we scale every element of w_i by $\frac{1}{4}$.

$$\implies \boldsymbol{p} = \boldsymbol{v}\left(\frac{1}{4}\boldsymbol{w}\right) = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$$

Part B

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 3 & 1 \\ 0 & 5 & -1 \end{bmatrix}$$

We apply the same procedure as before, and get the following results:

$$\lambda_1 = -2,\ \boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -5 \end{bmatrix},\ \boldsymbol{w}_1 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \qquad \lambda_2 = 1,\ \boldsymbol{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \boldsymbol{w}_2 = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} \qquad \lambda_3 = 4,\ \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ \boldsymbol{w}_3 = \begin{bmatrix} 0 \\ 5 \\ 1 \end{bmatrix}$$

Scaling $\boldsymbol{w}$ to get $w_i^t v_i = 1$, we get:

$$\implies \boldsymbol{p} = \boldsymbol{v}\boldsymbol{w} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{5}{6} & \frac{1}{6} & 0 \\ \frac{1}{6} & \frac{5}{6} & 0 \end{bmatrix}$$

2. In the **HW7_2023_Prob2** case the generator 2 exciter Ka value has been modified to give an unstable response. Using the SMIB tool, plot the eigenvalues with positive real parts as the value of Ka is decreased. Give several points in your plot. At what value does SMIB indicate the system becomes stable? Be sure to **Re-Initialize** the case between each SMIB solution with different parameters. How closely does the response expected from the SMIB match the actual system response? Comment on why they might be different.

Figure 3: Plot of Eigenvalues vs Ka value

When Ka become 125, the SMIB indicates that the system becomes stable. When comparing the dominant mode from SMIB analysis, we see that there is a mode at 1.927 Hz with damping of 9.7%. Comparing this to the modal analysis of Generator 2's rotor angle, we see somewhat close results, where we have a mode with 2.054 Hz frequency, and a damping of 9.3%.

3. The case **HW7_2023_Prob3** case models a four bus, four generator system with GENROU models. Using the tool or method of your choice, determine the frequency and damping for the key bus frequency modes. Analyze the data from 0.1 to 5 seconds, and you can ignore modes with frequencies below 0.2 Hz. To provide each student with a customized result, change the H value for the i^{th} generator to equal the i^{th} **last** digit in your student ID (where i goes from 1 to 4 so i=1 is the last digit); treat a zero as 10.

(a) Inertia Values for Generators

Gen #	UIN Digit	H Value
1	8	8
2	0	10
3	5	5
4	0	10

Freq(Hz)	Damping(%)
1.471	8.287
1.182	7.166
1.052	10.636
0.185	29.063

4. Using the **WSCC_9Bus_Stab** case from class first use the same procedure from problem 4 to customize the problem by changing the inertia of the generators based on the last three digits in your student ID. Then repeat the tuning procedure presented in class for a PSS1A stabilizer at the one of the generators (your choice which one!). Assume there are no stabilizers at the other two generators.

First, the inertia of the generators is changed as below:

Gen at Bus 1 → 10

Gen at Bus 2 → 5

Gen at Bus 3 → 10

The goal is to tune the generator at bus 2 (my choice).

We begin by manipulating a case in which there are no stabilizers in the system, and changing the generator inertia constants based on UIN ending 050. If we look at the modes of the rotor angles using the Matrix Pencil method, we get the modes seen in Table (1). From this, we select the 1.2126 Hz mode, with 19.82% damping. Then, we applying a SIGNALSTAB stabilizer to Generator 2, and set the input signal to have a magnitude of 0.05 pu, at a frequency of 1.126 Hz. Applying this to the case, we perform modal analysis on the statistics for Generator 2 and get the angles seen in Table 2b. From here, we need to get the angle necessary to damp out that signal and use it to calculate α.

Table (1)

Freq. (Hz)	Damping(%)
1.126	19.82
0.922	18.99
0.141	92.91
0.036	43.55

Table (2)

	Angle
Gen Speed	37.597
Gen MW	-52.683
V pu	-160.724
V stab	-56.683

From Table (2), we get that we need to have approximately -105.43 degree of compensation. Split between two lead-lag blocks, we get that ϕ = -52.57 degree.

$$\alpha = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$\alpha = 8.713$

$$T_1 = \frac{1}{2\pi f \sqrt{\alpha}}$$

$T_1 = 0.00089$

$$T_2 = \alpha T_1$$

$T_2 = 0.00775$

Finally, we also have to tune Ks, which we get to be about 40. This is from the fact that a Ks value of 120 results in almost instability (see plot), and we divide that by 3. Taking all these values into account, we get the results in Table (3), which shows our frequency now has a damping of about 21.717%.

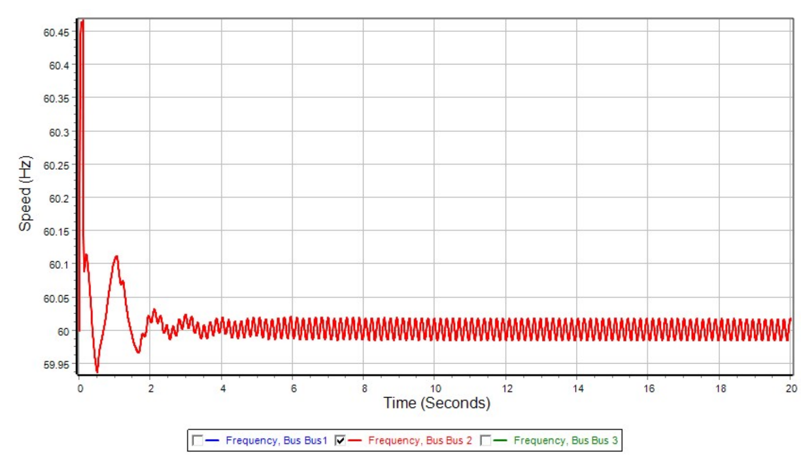

Table (3)

Freq. (Hz)	Damping(%)
1.341	21.717
1.00	9.675
0.182	89.519