ECEN 667
Power System Stability

Lecture 15: Time-Domain Simulation Solutions (Transient Stability)

Prof. Tom Overbye
Dept. of Electrical and Computer Engineering
Texas A&M University
overbye@tamu.edu

TEXAS A&M UNIVERSITY

Announcements

- Read Chapters 4 and 7
- Homework 4 is due today
- Homework 5 is due on Tuesday Oct 31
- IEEE Spectrum did have a nice biographical article on Charlie Concordia in 1999 (when he won the IEEE Medal of Honor at age 91)
  - He joined GE in 1926; his best contribution (he noted) was, “to increase the understanding of the dynamics of power systems”

Energy and Power Group Seminar on Oct 20

ENERGY & POWER SEMINAR

A Day in the Life of a Power Engineer

Abstract

This presentation will go over what an electric cooperative is and the role of a Systems Engineer at Rayburn Electric Cooperative. Some responsibilities of the Rayburn engineering team include metering, SCADA, protective relaying, event analysis, and transmission planning.

Katherine Garcia
Systems Engineer
Rayburn Electric Cooperative

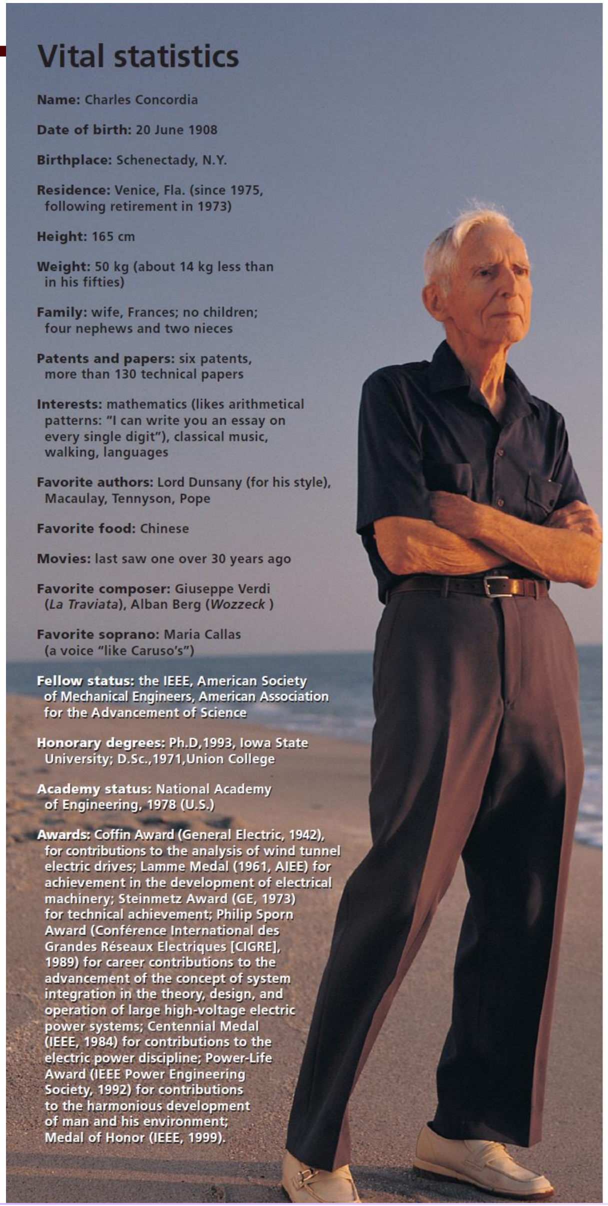
Friday, October 20
11:30 am - 12:20 pm
244 ZACH

Biography

Katherine received her Bachelor of Science in Electrical Engineering from Texas A&M University in December 2019. She has worked as a Systems Engineer at Rayburn Electric Cooperative for 3 years. Her main focus is on transmission planning and coordinating the interconnection of new solar and battery storage facilities to Rayburn's system.

A Power System Dynamics Giant: Charlie Concordia

- IEEE Spectrum did have a nice biographical article on Charlie Concordia in 1999 (when he won the IEEE Medal of Honor at age 91)
  - He joined GE in 1926; his best contribution (he noted) was, "to increase the understanding of the dynamics of power systems"
  - Image at right is from the IEEE Spectrum article; he passed away in 2003 at age 95
  - Thomas Edison, who founded a company that became part of GE at its founding, died in 1931



bility, speed governing and tie-line power and frequency control, design of power systems for maximum service reliability, computing machines, centrifugal compressors, and wind tunnel fan drives.

Less known but at least as important was Concordia's advancement of the application of digital computers to power engineering and other engineering disciplines. He was among the dozen or so founders of the Association for Computing Machinery in 1947, and the first chairman of the American Institute of Electrical Engineers' Computer Committee, a forerunner of the IEEE Computer Society. (AIEE was one of the IEEE's two founding organizations.)

Last February, Concordia attended the 1999 IEEE Power Engineering Society Winter Meeting in New York City. *Spectrum* interviewed him there, and also talked to his friends and colleagues.

Early days

Concordia's pride in his professional accomplishments reaches back to when he joined GE, at age 18. That same year, 1926, he joined the Schenectady Section of the AIEE and, he told *Spectrum*., "I still keep a pin as a token of my belonging to that section. I could not join the AIEE [proper] then as…no one under age 21 could join. This is the only pin…of this kind—it's most precious to me."

His originality and self-confidence had shown up early. As a boy of 12, he invented a salad in response to an advertisement and won US $25 for the recipe, though he never actually made the salad, wrote Philip L. Alger, a GE motor expert and Concordia contemporary, in his book, *The Human Side of Engineering* (Mohawk Development Services Inc., Schenectady, N.Y., 1972).

The first of his many technical inventions applied to railways—a detector of cracks in rails, which he developed in GE's General Engineering Laboratory. Being based on magnetic field measurements, his technique did away with the need to clean the rail beforehand—a prerequisite of the prevailing technique, which employed a Kelvin bridge for measuring the rail's relatively low electrical resistance. Further, GE was paying royalties for the other technique at the time and was eager to stop those payments.

An unwavering focus

Untiring in his search for solutions, he maintains an unwavering focus on the problem at hand, whatever the cost. Frank Maginniss, his friend and colleague of about 60 years' standing at GE, described him as "a very dedicated worker," who frequently spent Saturdays, Sundays, and evenings at his office desk, working out complicated engineering problems. "If, as

Classical Swing Equation

- Often in an introductory coverage of transient stability with the classical model the assumption is $\omega \approx \omega_s$ so the swing equation for the classical model is given as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = P_{Mi} - P_{Ei} - D_i\left(\Delta\omega_i\right)$$

with $P_{Ei} = \left(E_i'\angle\delta_i\right)\left(E_i'\angle\delta_i - \bar{V}_i\right)Y_i$

- We'll use this simplification for our initial example

As an example of this initial approach see Anderson and Fouad, *Power System Control and Stability*, 2nd Edition, Chapter 2 (with a newer version third edition of this book now available adding Vijay Vittal and Jim McCalley as authors).

Numerical Solution

- There are two main approaches for solving

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$
$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

  - Partitioned-explicit: Solve the differential and algebraic equations separately (alternating between the two) using an explicit integration approach
  - Simultaneous-implicit: Solve the differential and algebraic equations together using an implicit integration approach

Outline of the Solution Process

- The next group of slides will provide basic coverage of the solution process, partitioned explicit, then the simultaneous-implicit approach
- We'll start out with a classical model supplying an infinite bus, which can be solved by embedded the algebraic constraint into the differential equations

We'll start out just solving $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x})$

and then will extend to solving the full problem of

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}, \mathbf{u})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

Classical Swing Equation with Power Balance

- With a classical generator at bus i supplying an infinite bus with voltage magnitude V_{inf}, we can write the problem without algebraic constraints as

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left(P_{Mi} - \frac{E_i' V_{\inf}}{X_{th}} \sin\delta_i - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

with $P_{Ei} = \frac{E_i' V_{\inf}}{X_{th}} \sin\delta_i$

Note we are using the per unit speed approach

Explicit Integration Methods

- As covered during the first week of class, there are a wide variety of explicit integration methods
  - We considered Forward Euler, Runge-Kutta, Adams-Bashforth
- Here we will just consider Euler's, which is easy to explain but not too useful, and a second order Runge-Kutta, which is commonly used

Forward Euler

- Recall the Forward Euler approach is approximate

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}(t)) = \frac{d\mathbf{x}}{dt} \text{ as } \frac{\Delta\mathbf{x}}{\Delta t}$$

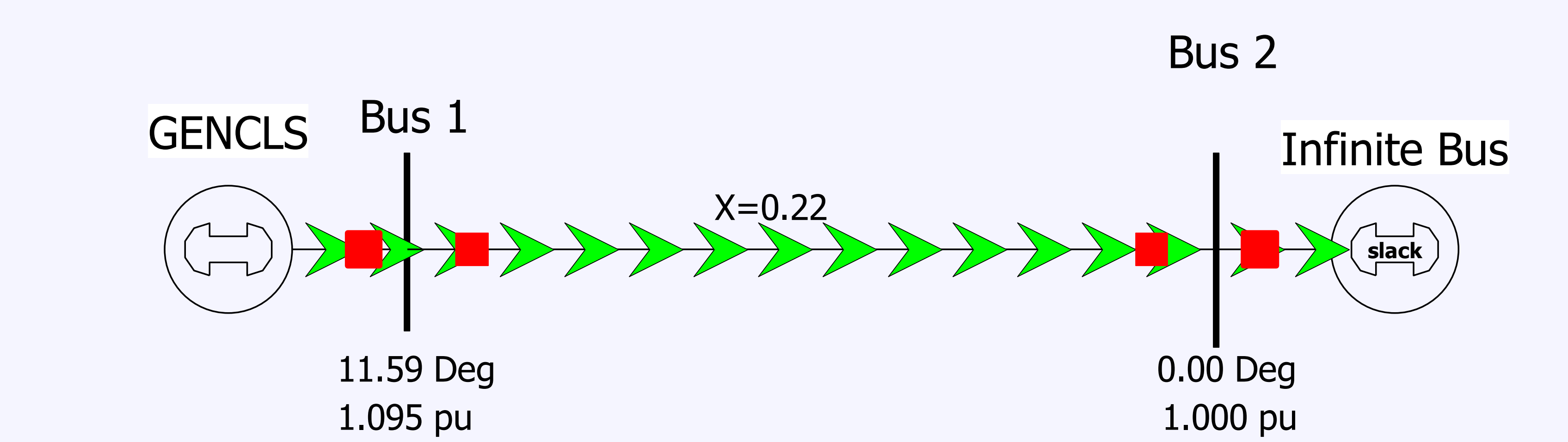
Then

$$\mathbf{x}(t+\Delta t) \approx \mathbf{x}(t) + \Delta t\, \mathbf{f}(\mathbf{x}(t))$$

- Error with Euler's varies with the square of the time step

Infinite Bus GENCLS Example using the Forward Euler's Method

- Use the four bus system from before, except now gen 4 is modeled with a classical model with X_d'=0.3, H=3 and D=0; also we'll reduce to two buses with equivalent line reactance, moving the gen from bus 4 to 1



In this example X_{th} = (0.22 + 0.3), with the internal voltage $\bar{E}'_1 = 1.281\angle 23.95°$ giving E'_1=1.281 and δ_1= 23.95°

Infinite Bus GENCLS Example

- The associated differential equations for the bus 1 generator are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}\left(1 - \frac{1.281}{0.52}\sin\delta_1\right)$$

- The value of $P_{M1} = 1$ is determined from the initial conditions, and would stay constant in this simple example without a governor
- The value $\delta_1 = 23.95°$ is readily verified as an equilibrium point (which is 0.418 radians)

Infinite Bus GENCLS Example

- Assume a solid three phase fault is applied at the generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear}, resulting in the post-fault system being identical to the pre-fault system
  - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu}\omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2\times 3}(1-0)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Euler's Solution

- The initial value of **x** is

$$\mathbf{x}(0) = \begin{bmatrix} \delta_1(0) \\ \Delta\omega_{1,pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Assuming a time step Δt = 0.02 seconds, and a T^{clear} of 0.1 seconds, then using Euler's

$$\mathbf{x}(0.02) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

Note Euler's assumes δ stays constant during the first time step

- Iteration continues until t = T^{clear}

Euler's Solution

- At t = T^{clear} the fault is self-cleared, with the equations changing to

$$\frac{d\delta}{dt} = \Delta\omega_{pu}\omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6}\left(1 - \frac{1.281}{0.52}\sin\delta\right)$$

- The integration continues using the new equations

Euler's Solution Results (Δt=0.02)

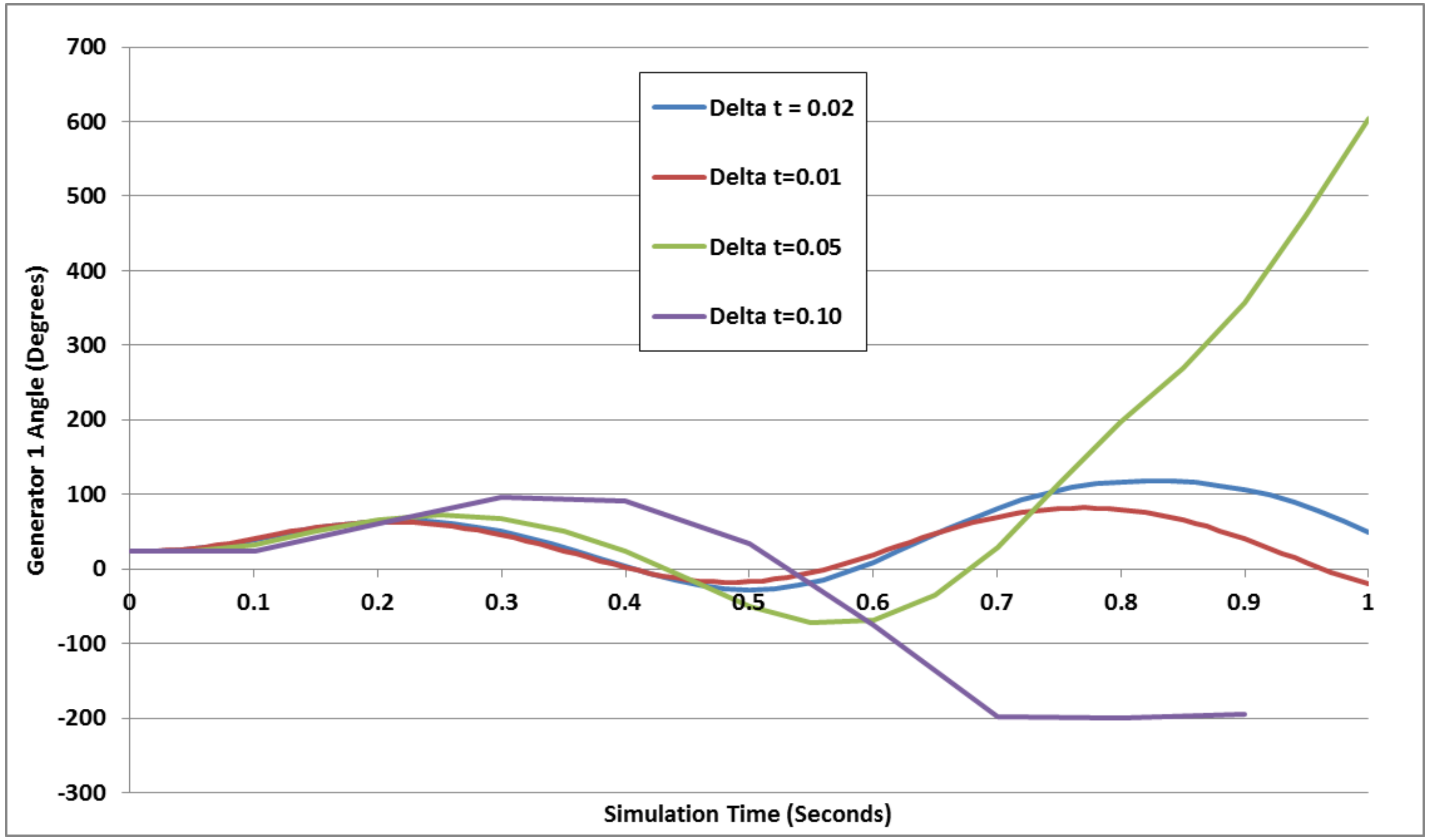
- The below table gives the results using Δt = 0.02 for the beginning time steps

Time	Gen 1 Rotor Angle, Degrees	Gen 1 Speed (Hz)
0	23.9462	60
0.02	23.9462	60.2
0.04	25.3862	60.4
0.06	28.2662	60.6
0.08	32.5862	60.8
0.1	38.3462	61
0.1	38.3462	61
0.12	45.5462	60.8943
0.14	51.9851	60.7425
0.16	57.3314	60.5543
0.18	61.3226	60.3395
0.2	63.7672	60.1072
0.22	64.5391	59.8652
0.24	63.5686	59.6203
0.26	60.8348	59.3791
0.28	56.3641	59.1488

This is saved as PowerWorld case **B2_CLS_Infinite**. The integration method is set to Euler's on the **Transient Stability, Options, Power System Model** page

Generator 1 Delta: Euler's

- The below graph shows the generator angle for varying values of Δt; numerical instability is clearly seen



Second Order Runge-Kutta

- Runge-Kutta methods improve on Euler's method by evaluating **f**(**x**) at selected points over the time step
- One approach is a second order method (RK2) in which

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{2}(\mathbf{k}_1 + \mathbf{k}_2)$$

where

$$\mathbf{k}_1 = \Delta t\ \mathbf{f}(\mathbf{x}(t))$$

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}(\mathbf{x}(t) + \mathbf{k}_1)$$

This is also known as Heun's method or as the Improved Euler's or Modified Euler's Method

- That is, $\mathbf{k}_1$ is what we get from Euler's; $\mathbf{k}_2$ improves on this by reevaluating at the estimated end of the time step
- Error varies with the cubic of the time step

Second Order Runge-Kutta (RK2)

- Again assuming a time step Δt = 0.02 seconds, and a T^{clear} of 0.1 seconds, then using Heun's approach

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \Delta\omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

$$\mathbf{k}_1 = 0.02 \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.00333 \end{bmatrix}, \quad \mathbf{x}(0) + \mathbf{k}_1 = \begin{bmatrix} 0.418 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{k}_2 = 0.02 \begin{bmatrix} 1.257 \\ 0.1667 \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \end{bmatrix}$$

$$\mathbf{x}(0.020) = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{1}{2}\left(\mathbf{k}_1 + \mathbf{k}_2\right) = \begin{bmatrix} 0.431 \\ 0.00333 \end{bmatrix}$$

RK2 Solution Results (Δt=0.02)

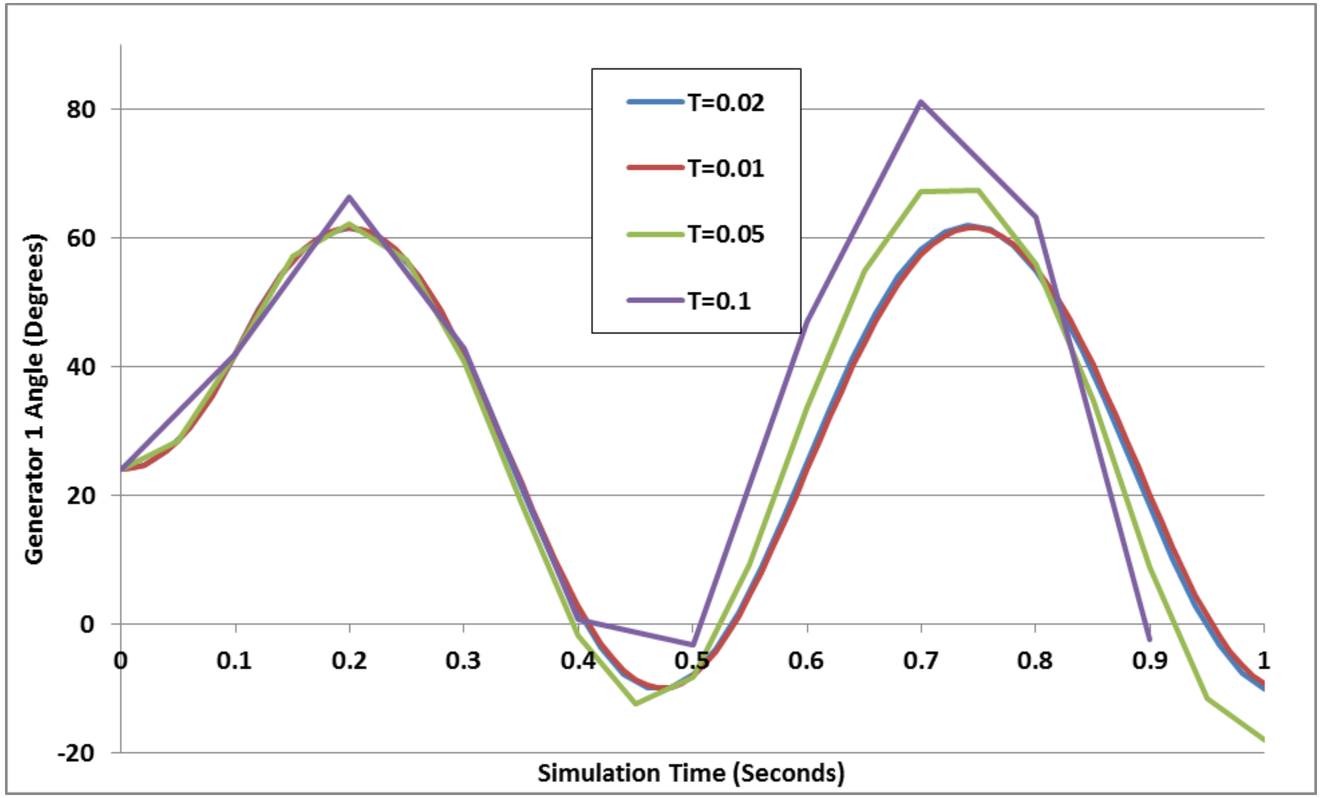
- The below table gives the results using Δt = 0.02 for the beginning time steps

Time	Gen 1 Rotor Angle, Degrees	Gen 1 Speed (Hz)
0	23.9462	60
0.02	24.6662	60.2
0.04	26.8262	60.4
0.06	30.4262	60.6
0.08	35.4662	60.8
0.1	41.9462	61
0.1	41.9462	61
0.12	48.6805	60.849
0.14	54.1807	60.6626
0.16	58.233	60.4517
0.18	60.6974	60.2258
0.2	61.4961	59.9927
0.22	60.605	59.7598
0.24	58.0502	59.5343
0.26	53.9116	59.3241
0.28	48.3318	59.139

This is saved as PowerWorld case B2_CLS_Infinite. The integration method should be changed to Second Order Runge-Kutta on the **Transient Stability, Options, Power System Model** page

Generator 1 Delta: RK2

- The below graph shows the generator angle for varying values of Δt; much better than Euler's but still the beginning of numerical instability with larger values of Δt



Adding Network Equations

- Previous slides with the network equations embedded in the differential equations were a special case
- In general with the explicit approach we'll be alternating between solving the differential equations and solving the algebraic equations
- Voltages and currents in the network reference frame can be expressed using either polar or rectangular coordinates
- In rectangular with the book's notation we have

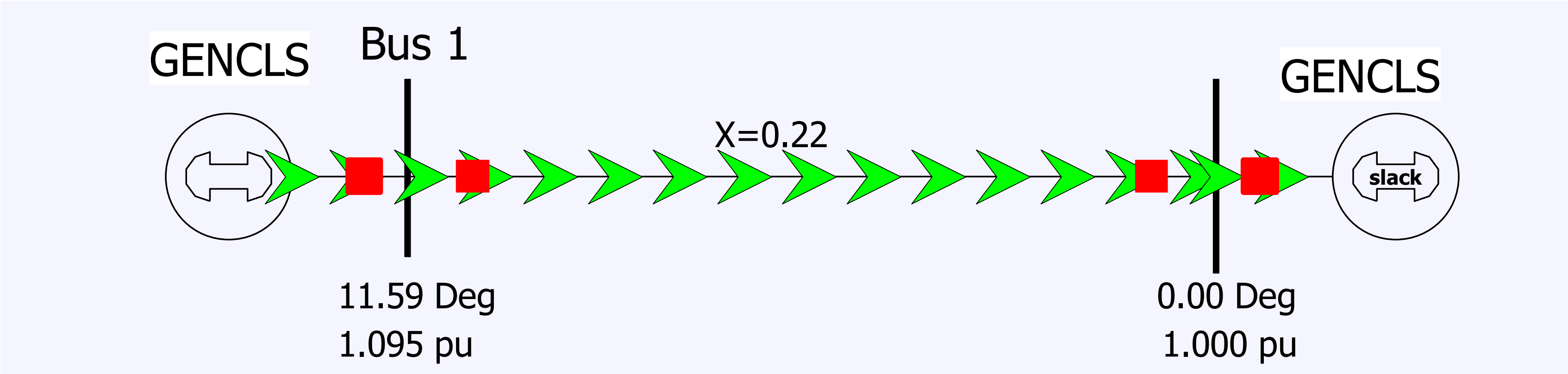
$$\bar{V}_i = V_{Di} + jV_{Qi}, \quad \bar{I}_i = I_{Di} + jI_{Qi}$$

Adding Network Equations

- Network equations will be written as $\mathbf{Y}\,\mathbf{V} - \mathbf{I}(\mathbf{x},\mathbf{V}) = \mathbf{0}$
  - Here **Y** is as from the power flow, except augmented to include the impact of the generator's internal impedance
  - Constant impedance loads are also embedded in **Y**; non-constant impedance loads are included in **I(x,V)**
- If **I** is independent of **V** then this can be solved directly: $\mathbf{V} = \mathbf{Y}^{-1}\mathbf{I}(\mathbf{x})$
- In general an iterative solution is required, which we'll cover shortly, but initially we'll go with just the direct solution

Two Bus Example, Except with No Infinite Bus

- To introduce the inclusion of the network equations, the previous example is extended by replacing the infinite bus at bus 2 with a classical model with $X_{d2}'=0.2$, $H_2=6.0$



PowerWorld Case is **B2_CLS_2Gen**

Bus Admittance Matrix

- The network admittance matrix is

$$\mathbf{Y}_N = \begin{bmatrix} -j4.545 & j4.545 \\ j4.545 & -j4.545 \end{bmatrix}$$

- This is augmented to represent the Norton admittances associated with the generator models (X_{d1}'=0.3, X_{d2}'=0.2)

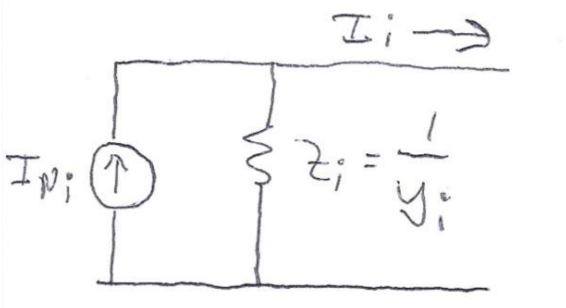
$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.3} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

In PowerWorld you can see this matrix by selecting **Transient Stability, States/Manual Control, Transient Stability Ybus**

Current Vector

- For the classical model the Norton currents are given by

$$\bar{I}_{Ni} = \frac{E_i' \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad Y_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$



- The initial values of the currents come from the power flow solution
- As the states change (δ_i for the classical model), the Norton current injections also change

B2_CLS_Gen Initial Values

- The internal voltage for generator 1 is as before

$$\bar{I} = 1 - j0.3286$$

$$\bar{E}_1 = 1.0 + (j0.22 + j0.3)\bar{I} = 1.1709 + j0.52 = 1.281\angle 23.95^\circ$$

0.4179 radians

- We likewise solve for the generator 2 internal voltage

$$\bar{E}_2 = 1.0 - (j0.2)\bar{I} = 0.9343 - j0.2 = 0.9554\angle -12.08$$

0.2108 radians

- The Norton current injections are then

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

Keep in mind the Norton current injections are not the current out of the generator

B2_CLS_Gen Initial Values

- To check the values, solve for the voltages, with the values matching the power flow values

$$\mathbf{V} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Swing Equations

- With the network constraints modeled, the swing equations are modified to represent the electrical power in terms of the generator's state and current values

$$\mathrm{P}_{Ei} = E_{Di} I_{Di} + E_{Qi} I_{Qi}$$

$I_{Di}+jI_{Qi}$ is the current being injected into the network by the generator

- Then swing equation is then

$$\frac{d\delta_i}{dt} = \Delta\omega_{i.pu}\omega_s$$

$$\frac{d\Delta\omega_{i,pu}}{dt} = \frac{1}{2H_i}\left(P_{Mi} - \left(E_{Di} I_{Di} + E_{Qi} I_{Qi}\right) - D_i\left(\Delta\omega_{i,pu}\right)\right)$$

Two Bus, Two Generator Differential Equations

- The differential equations for the two generators are

$$\frac{d\delta_1}{dt} = \Delta\omega_{1.pu}\omega_s$$

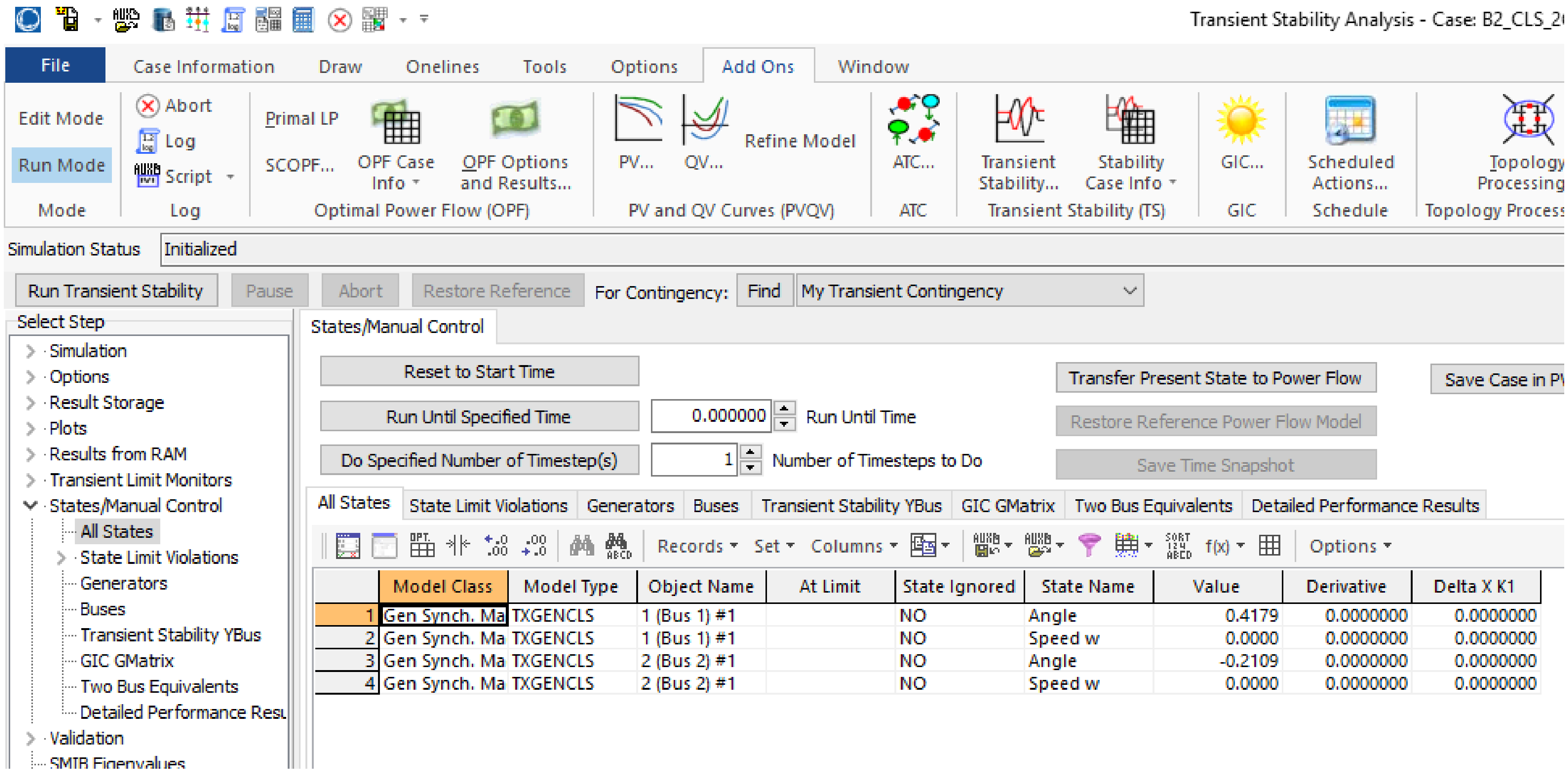
$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2H_1}\left(P_{M1} - \left(E_{D1}I_{D1} + E_{Q1}I_{Q1}\right)\right)$$

$$\frac{d\delta_2}{dt} = \Delta\omega_{2.pu}\omega_s$$

$$\frac{d\Delta\omega_{2,pu}}{dt} = \frac{1}{2H_2}\left(P_{M2} - \left(E_{D2}I_{D2} + E_{Q2}I_{Q2}\right)\right)$$

In this example
P_{M1} = 1 and P_{M2} = -1

PowerWorld GENCLS Initial States



Solution at t=0.02

- Usually a time step begins by solving the differential equations. However, in the case of an event, such as the solid fault at the terminal of bus 1, the network equations need to be first solved
- Solid faults can be simulated by adding a large shunt at the fault location
  - Amount is somewhat arbitrary, it just needs to be large enough to drive the faulted bus voltage to zero
- With Euler's the solution after the first time step is found by first solving the differential equations, then resolving the network equations

Solution at t=0.02

- Using Y_{fault} = -j1000, the fault-on conditions become

$$\mathbf{V} = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.733 - j3.903 \\ -1 - j4.671 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.1053 \end{bmatrix}$$

Solving for the currents into the network

$$I_1 = \frac{(1.1702 + j0.52) - V_1}{j0.3} = 1.733 - j3.900$$

$$I_2 = \frac{(0.9343 - j0.2) - (0.486 - j0.1053)}{j0.2} = -0.473 - j2.240$$

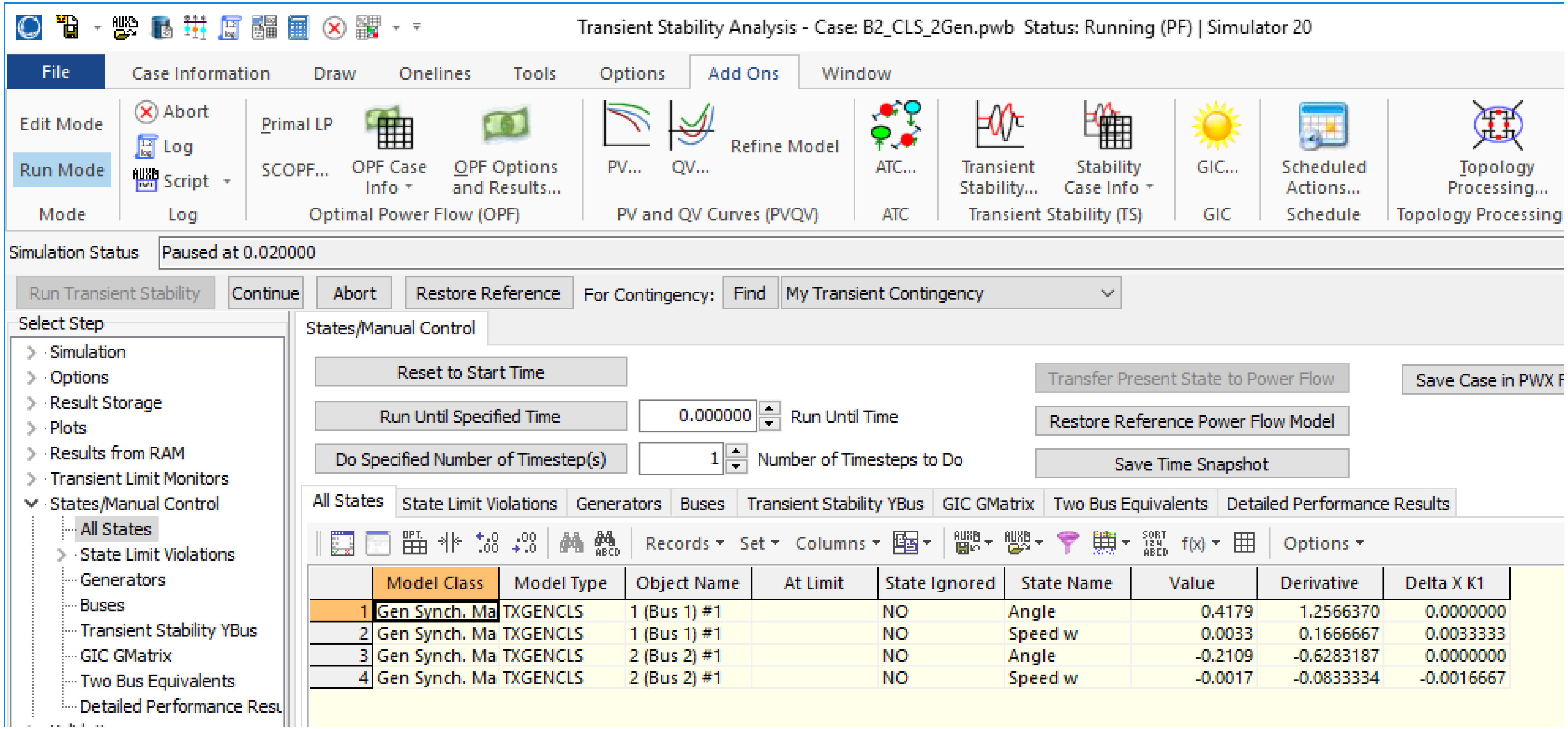
Solution at t=0.02

- Then the differential equations are evaluated, using the new voltages and currents
  - These impact the calculation of P_{Ei} with $P_{E1}=0$, $P_{E2}=0$

$$\begin{bmatrix} \delta_1(0.02) \\ \Delta\omega_1(0.02) \\ \delta_2(0.02) \\ \Delta\omega_1(0.02) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.0 \\ -0.211 \\ 0 \end{bmatrix} + 0.02 \begin{bmatrix} 0 \\ \frac{1}{6}(1-0) \\ 0 \\ \frac{1}{12}(-1-0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- If solving with Euler's this is the final state value; using these state values the network equations are resolved, with the solution the same here since the δ's didn't vary

PowerWorld GENCLS at t=0.02



Solution Values Using Euler's

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	23.9462	60.2	-12.0829	59.9
0.04	25.3862	60.4	-12.8029	59.8
0.06	28.2662	60.6	-14.2429	59.7
0.08	32.5862	60.8	-16.4029	59.6
0.1	38.3462	61	-19.2829	59.5
0.1	38.3462	61	-19.2829	59.5
0.12	45.5462	60.9128	-22.8829	59.5436
0.14	52.1185	60.7966	-26.169	59.6017
0.16	57.8541	60.6637	-29.0368	59.6682
0.18	62.6325	60.5241	-31.426	59.7379
0.2	66.4064	60.385	-33.3129	59.8075
0.22	69.1782	60.2498	-34.6988	59.8751
0.24	70.9771	60.1197	-35.5982	59.9401
0.26	71.8392	59.9938	-36.0292	60.0031
0.28	71.7949	59.8702	-36.0071	60.0649

Solution at t=0.02 with RK2

- With RK2 the first part of the time step is the same as Euler's, that is solving the network equations with

$$\mathbf{x}(t+\Delta t)^{(1)} = \mathbf{x}(t) + \mathbf{k}_1 = \mathbf{x}(t) + \Delta T \mathbf{f}(\mathbf{x}(t))$$

- Then calculate k2 and get a final value for **x**(t+Δt)

$$\mathbf{k}_2 = \Delta t\ \mathbf{f}\left(\mathbf{x}(t) + \mathbf{k}_1\right)$$

$$\mathbf{x}(t+\Delta t) = \mathbf{x}(t) + \frac{1}{2}\left(\mathbf{k}_1 + \mathbf{k}_2\right)$$

- Finally solve the network equations using the final value for **x**(t+Δt)

Solution at t=0.02 with RK2

- From the first half of the time step

$$x\left(0.02\right)^{(1)} = \begin{bmatrix} 0.418 \\ 0.00333 \\ -0.211 \\ -0.00167 \end{bmatrix}$$

- Then

$$\mathbf{k}_2 = \Delta t \ \mathbf{f}\left(\mathbf{x}\left(t\right) + \mathbf{k}_1\right) = 0.02 \begin{bmatrix} 1.256 \\ \frac{1}{6}\left(1-0\right) \\ -0.628 \\ \frac{1}{12}\left(-1-0\right) \end{bmatrix} = \begin{bmatrix} 0.0251 \\ 0.00333 \\ -0.0126 \\ -0.00167 \end{bmatrix}$$

Solution at t=0.02 with RK2

- The new values for the Norton currents are

$$\bar{I}_{N1} = \frac{1.281\angle 24.69^\circ}{j0.3} = 1.851 - j3.880$$

$$\bar{I}_{N2} = \frac{0.9554\angle -12.43^\circ}{j0.2} = -1.028 - j4.665$$

$$\mathbf{V}(0.02) = \begin{bmatrix} -j1007.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}^{-1} \begin{bmatrix} 1.851 - j3.880 \\ -1.028 - j4.665 \end{bmatrix}$$

$$= \begin{bmatrix} -0.006 - j0.001 \\ 0.486 - j0.108 \end{bmatrix}$$

Solution Values Using RK2

- The below table gives the results using $\Delta t = 0.02$ for the beginning time steps

Time (Sec)	Gen 1 Rotor Angle	Gen 1 Speed (Hz)	Gen 2 Rotor Angle	Gen2 Speed (Hz)
0	23.9462	60	-12.0829	60
0.02	24.6662	60.2	-12.4429	59.9
0.04	26.8262	60.4	-13.5229	59.8
0.06	30.4262	60.6	-15.3175	59.7008
0.08	35.4662	60.8	-17.8321	59.6008
0.1	41.9462	61	-21.0667	59.5008
0.1	41.9462	61	-21.0667	59.5008
0.12	48.7754	60.8852	-24.4759	59.5581
0.14	54.697	60.7538	-27.4312	59.6239
0.16	59.6315	60.6153	-29.8931	59.6931
0.18	63.558	60.4763	-31.8509	59.7626
0.2	66.4888	60.3399	-33.3109	59.8308
0.22	68.4501	60.2071	-34.286	59.8972
0.24	69.4669	60.077	-34.789	59.9623
0.26	69.5548	59.9481	-34.8275	60.0267
0.28	68.7151	59.8183	-34.4022	60.0916

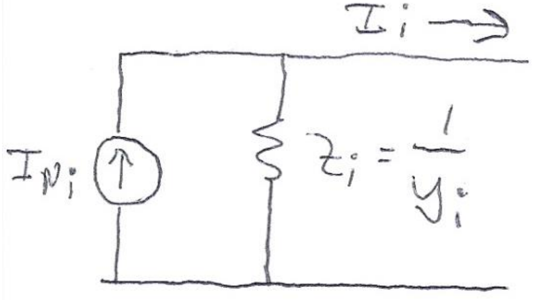
Angle Reference

- The initial angles are given by the angles from the power flow, which are based on the slack bus's angle
- As presented the transient stability angles are with respect to a synchronous reference frame
  - Sometimes this is fine, such as for either shorter studies, or ones in which there is little speed variation
  - Oftentimes this is not best since the when the frequencies are not nominal, the angles shift from the reference frame
- Other reference frames can be used, such as with respect to a particular generator's value, which mimics the power flow approach; the selected reference has no impact on the solution

Subtransient Models

- The Norton current injection approach is what is commonly used with subtransient models in industry
- If subtransient saliency is neglected (as is the case with GENROU and GENSAL in which $X''_d = X''_q$) then the current injection is

$$I_{Nd} + jI_{Nq} = \frac{E''_d + jE''_q}{R_s + jX''} = \frac{\left(-\psi''_q + j\psi''_d\right)\omega}{R_s + jX''}$$



  – Subtransient saliency can be handled with this approach, but it is more involved (see Arrillaga, *Computer Analysis of Power Systems*, section 6.6.3)

Subtransient Models

- Note, the values here are on the dq reference frame
- We can now extend the approach introduced for the classical machine model to subtransient models
- Initialization is as before, which gives the δ's and other state values
- Each time step is as before, except we use the δ's for each generator to transfer values between the network reference frame and each machine's dq reference frame
  - The currents provide the coupling

Two Bus Example with Two GENROU Models

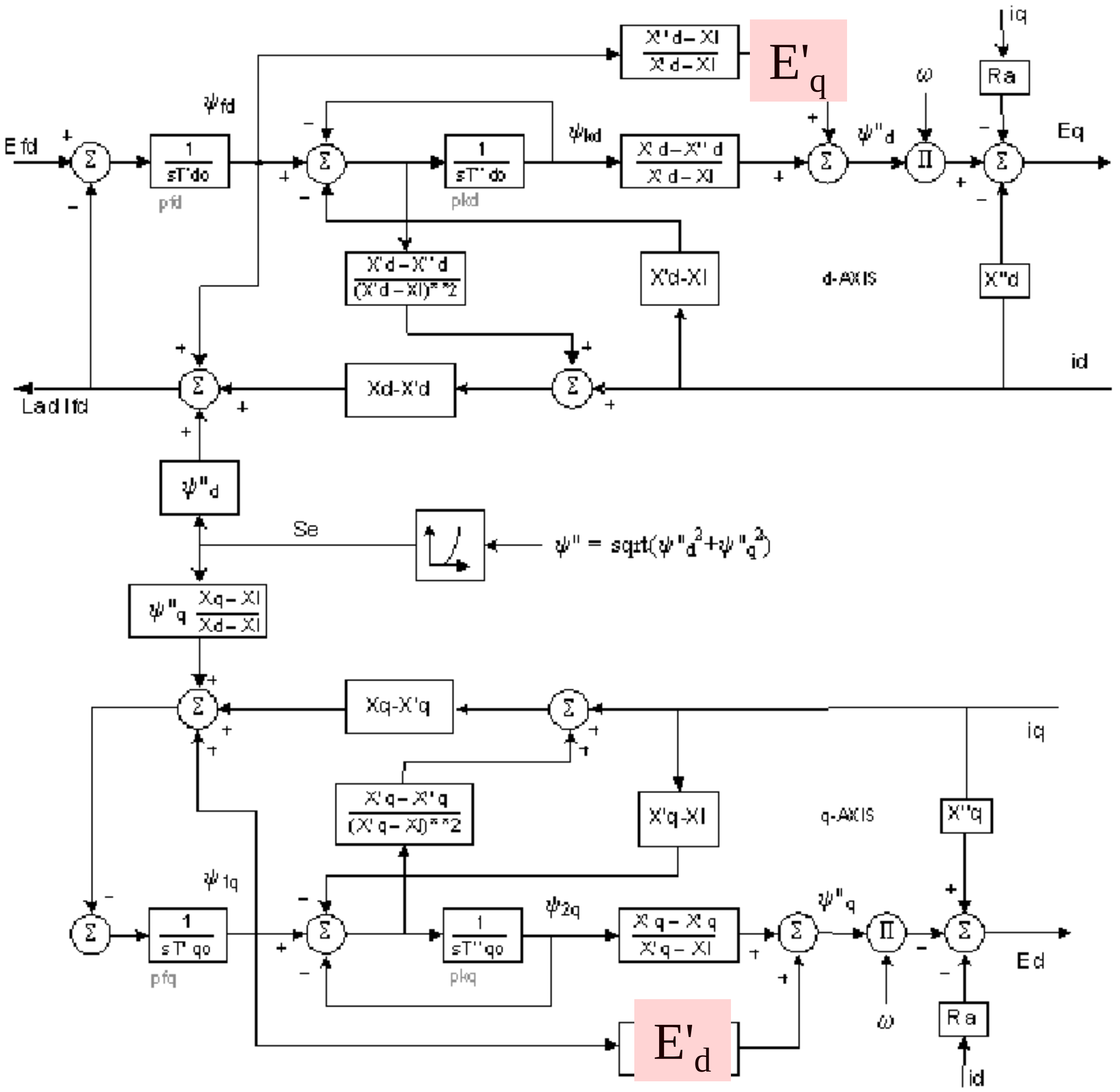
- Use the same system as before, except with we'll model both generators using GENROUs
  - For simplicity we'll make both generators identical except set H_1=3, H_2=6; other values are X_d=2.1, X_q=0.5, X'_d=0.2, X'_q=0.5, X''_q=X''_d=0.18, X_l=0.15, T'_{do} = 7.0, T'_{qo}=0.75, T''_{do}=0.035, T''_{qo}=0.05; no saturation
  - With no saturation the value of the δ's are determined (as per the earlier lectures) by solving

$$\left|E\right|\angle\delta = \bar{V} + \left(R_s + jX_q\right)\bar{I}$$

  - Hence for generator 1

$$\left|E_1\right|\angle\delta_1 = 1.0946\angle 11.59° + \left(j0.5\right)\left(1.052\angle -18.2°\right) = 1.431\angle 30.2°$$

GENROU Block Diagram



Two Bus Example with Two GENROU Models

- Using the early approach the initial state vector is

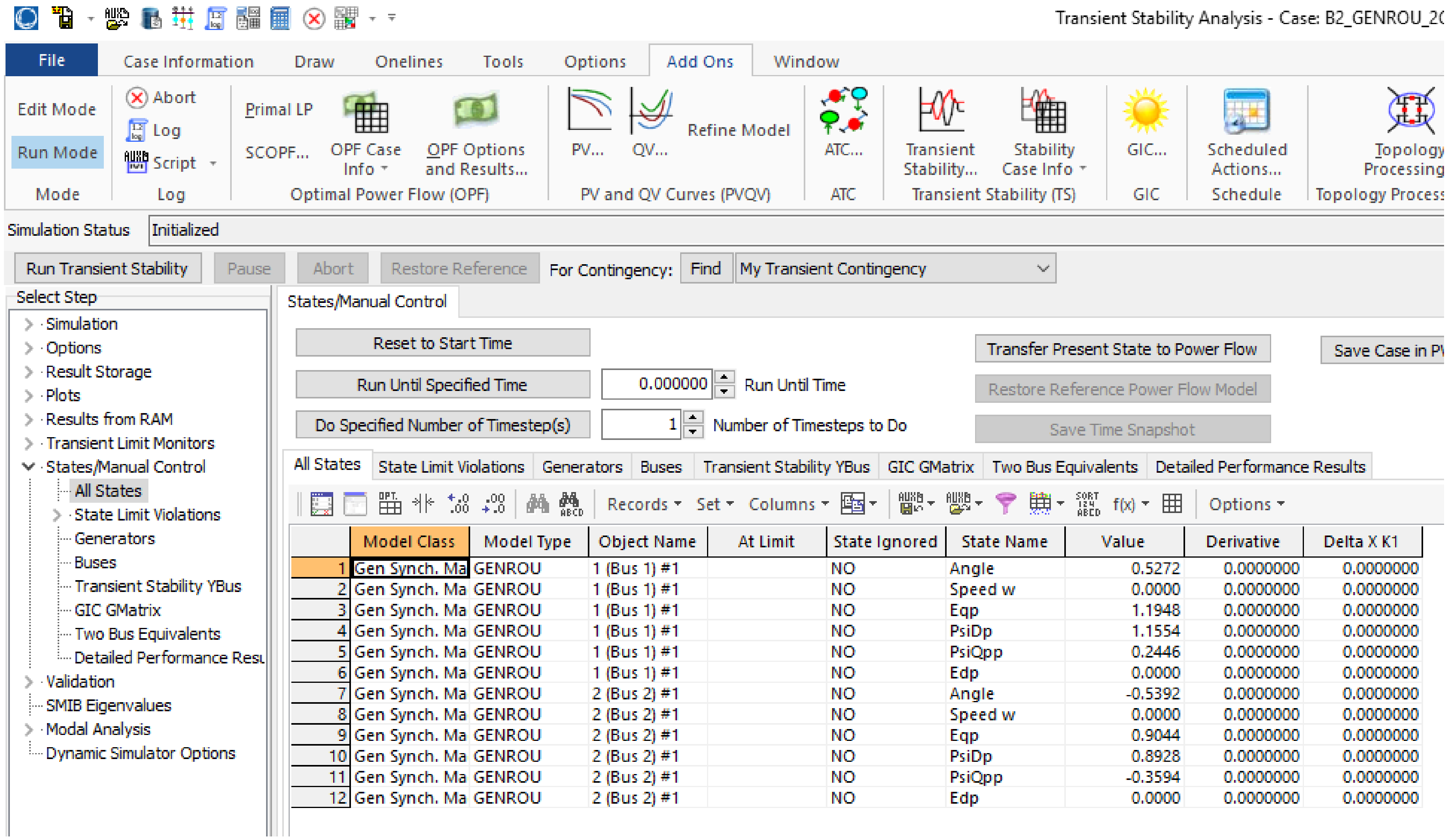
$$\mathbf{x}(0) = \begin{bmatrix} \delta_1 \\ \Delta\omega_1 \\ E'_{q1} \\ \psi_{1d1} \\ \psi_{2q1} \\ E'_{d1} \\ \delta_2 \\ \Delta\omega_2 \\ E'_{q2} \\ \psi_{1d2} \\ \psi_{2q2} \\ E'_{d2} \end{bmatrix} = \begin{bmatrix} 0.5273 \\ 0.0 \\ 1.1948 \\ 1.1554 \\ 0.2446 \\ 0 \\ -0.5392 \\ 0 \\ 0.9044 \\ 0.8928 \\ -0.3594 \\ 0 \end{bmatrix}$$

Note that this is a salient pole machine with $X'_q = X_q$; hence E'_d will always be zero

The initial currents in the dq reference frame are $I_{d1}=0.7872$, $I_{q1}=0.6988$, $I_{d2}=0.2314$, $I_{q2}=-1.0269$

Initial values of $\psi''_{q1} = -0.2236$, and $\psi''_{d1} = 1.179$

PowerWorld GENROU Initial States



	Model Class	Model Type	Object Name	At Limit	State Ignored	State Name	Value	Derivative	Delta X K1
1	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Angle	0.5272	0.0000000	0.0000000
2	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Speed w	0.0000	0.0000000	0.0000000
3	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Eqp	1.1948	0.0000000	0.0000000
4	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiDp	1.1554	0.0000000	0.0000000
5	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	PsiQpp	0.2446	0.0000000	0.0000000
6	Gen Synch. Ma	GENROU	1 (Bus 1) #1		NO	Edp	0.0000	0.0000000	0.0000000
7	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Angle	-0.5392	0.0000000	0.0000000
8	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Speed w	0.0000	0.0000000	0.0000000
9	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Eqp	0.9044	0.0000000	0.0000000
10	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiDp	0.8928	0.0000000	0.0000000
11	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	PsiQpp	-0.3594	0.0000000	0.0000000
12	Gen Synch. Ma	GENROU	2 (Bus 2) #1		NO	Edp	0.0000	0.0000000	0.0000000

Solving with Euler's

- We'll again solve with Euler's, except with Δt set now to 0.01 seconds (because now we have a subtransient model with faster dynamics)
  - We'll also clear the fault at t=0.05 seconds
- For the more accurate subtransient models the swing equation is written in terms of the torques

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s = \Delta\omega_i$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = \frac{2H_i}{\omega_s}\frac{d\Delta\omega_i}{dt} = T_{Mi} - T_{Ei} - D_i\left(\Delta\omega_i\right)$$

$$\text{with } T_{Ei} = \psi''_{d,i} i_{qi} - \psi''_{q,i} i_{di}$$

Other equations are solved based upon the block diagram

Norton Equivalent Current Injections

- The initial Norton equivalent current injections on the dq base for each machine are

$$I_{Nd1} + jI_{Nq1} = \frac{\left(-\psi''_{q1} + j\psi''_{d1}\right)\omega_1}{jX''_1} = \frac{\left(-0.2236 + j1.179\right)(1.0)}{j0.18}$$

$$= 6.55 + j1.242$$

$$I_{ND1} + jI_{NQ1} = 2.222 - j6.286$$

$$I_{Nd2} + jI_{Nq2} = 4.999 + j1.826$$

$$I_{ND2} + jI_{NQ2} = -1 - j5.227$$

Recall the dq values are on the machine's reference frame and the DQ values are on the system reference frame

Moving between DQ and dq

- Recall

$$\begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & -\cos\delta \\ \cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix}$$

- And

$$\begin{bmatrix} I_{Di} \\ I_{Qi} \end{bmatrix} = \begin{bmatrix} \sin\delta & \cos\delta \\ -\cos\delta & \sin\delta \end{bmatrix} \begin{bmatrix} I_{di} \\ I_{qi} \end{bmatrix}$$

The currents provide the key coupling between the two reference frames

Bus Admittance Matrix

- The bus admittance matrix is as from before for the classical models, except the diagonal elements are augmented using

$$Y_i = \frac{1}{R_{s,i} + jX''_{d,i}}$$

$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.18} & 0 \\ 0 & \frac{1}{j0.18} \end{bmatrix} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}$$

Algebraic Solution Verification

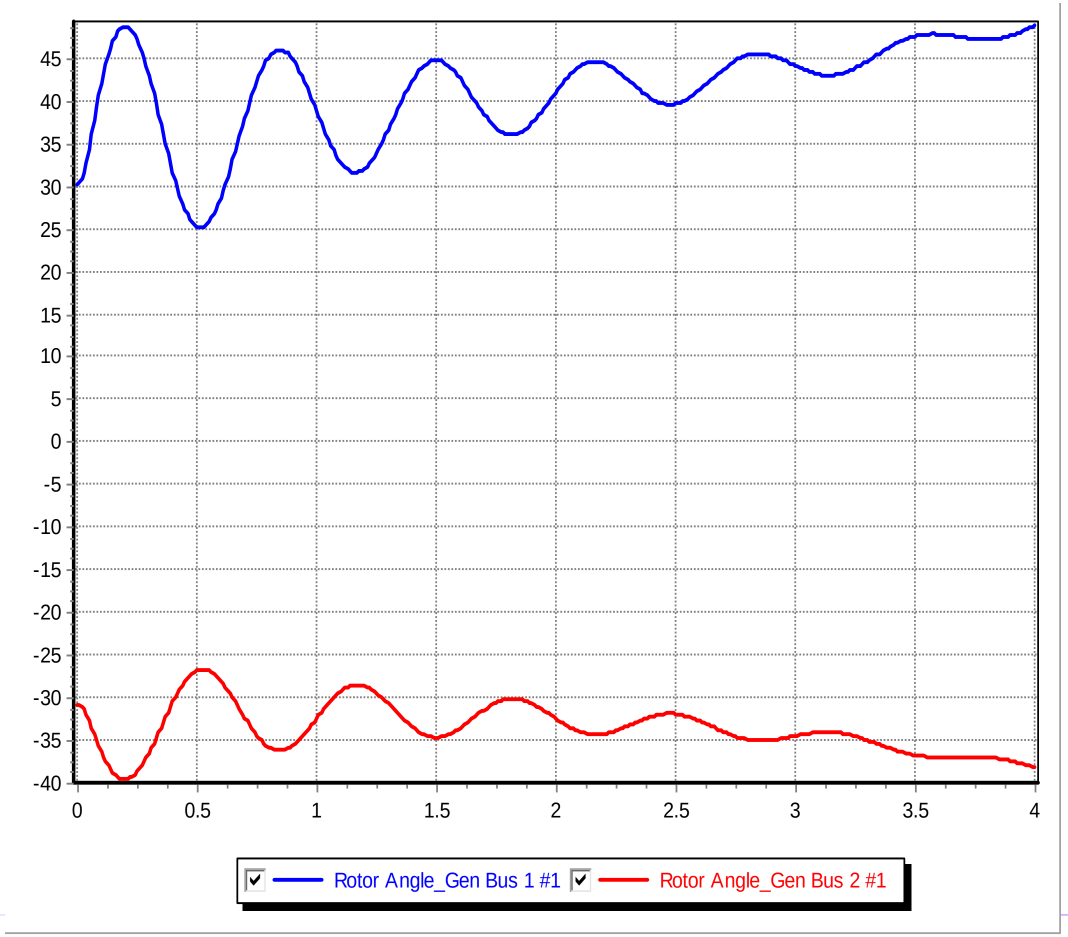
- To check the values solve (in the network reference frame)

$$\mathbf{V} = \begin{bmatrix} -j10.101 & j4.545 \\ j4.545 & -j10.101 \end{bmatrix}^{-1} \begin{bmatrix} 2.222 - j6.286 \\ -1 - j5.227 \end{bmatrix}$$

$$= \begin{bmatrix} 1.072 + j0.22 \\ 1.0 \end{bmatrix}$$

Results

- The below graph shows the results for four seconds of simulation, using Euler's with Δt=0.01 seconds

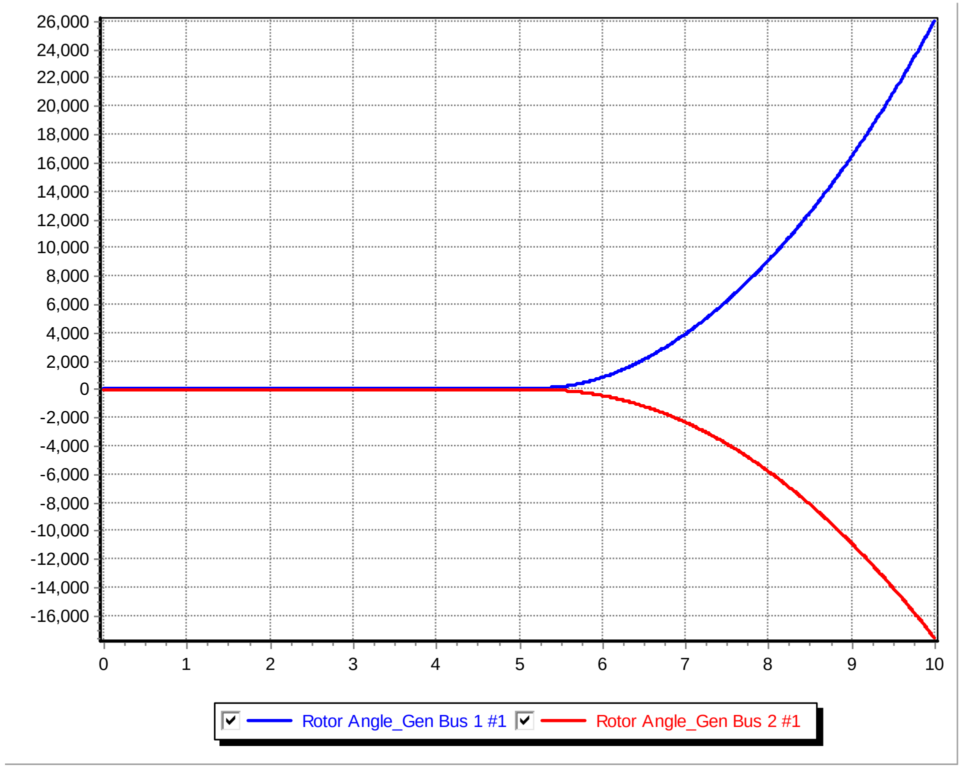


PowerWorld case is

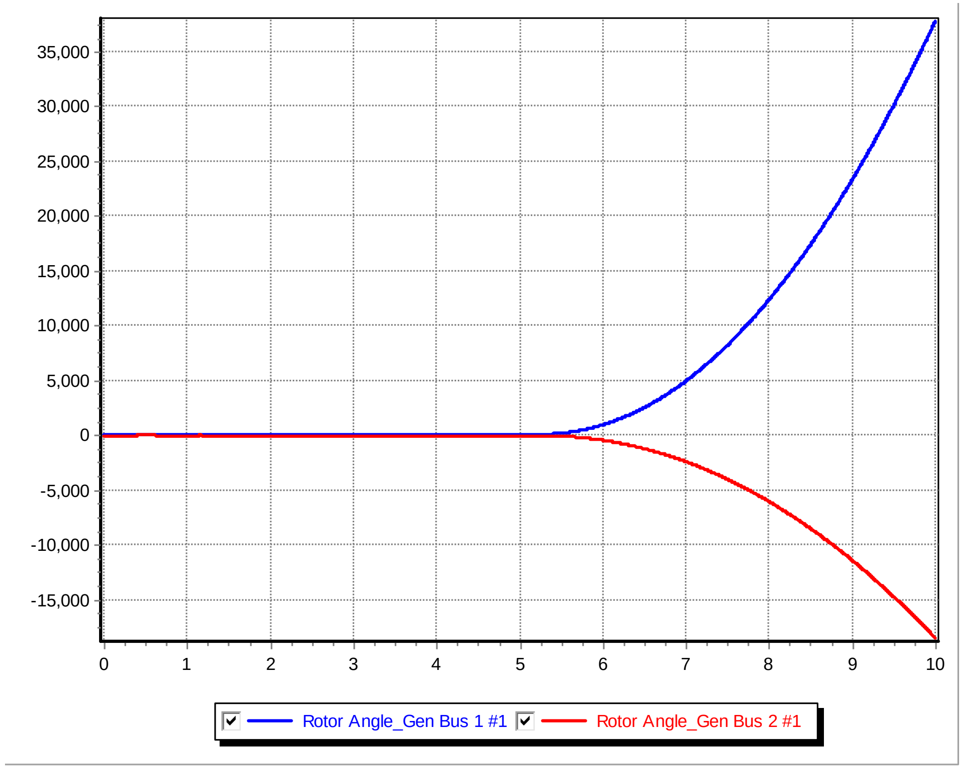
B2_GENROU_2GEN_EULER

Results for Longer Time

- Simulating out 10 seconds indicates an unstable solution, both using Euler's and RK2 with Δt=0.005, so it is really unstable!



Euler's with Δt=0.01



RK2 with Δt=0.005