

ECEN 667

Power System Stability

Lecture 17: Time-Domain Simulation Solutions, Load Modeling

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Announcements



- Read Chapters 4 and 7
- Homework 5 is due on Thursday Nov 2



Simultaneous Implicit



- The other major solution approach is the simultaneous implicit in which the algebraic and differential equations are solved simultaneously
- This method has the advantage of being numerically stable
- In the last lecture we considered the trapezoidal approach

$$\mathbf{x}(t + \Delta t) = \mathbf{x}(t) + \frac{\Delta t}{2} \left[\mathbf{f}(\mathbf{x}(t)) + \mathbf{f}(\mathbf{x}(t + \Delta t)) \right]$$

For a linear system we have

$$\mathbf{x}(t + \Delta t) = \left[I - \Delta t \mathbf{A} \right]^{-1} \left[I + \frac{\Delta t}{2} \mathbf{A} \right] \mathbf{x}(t)$$

Infinite Bus GENCLS Implicit Solution



- Assume a solid three phase fault is applied at the bus 1 generator terminal, reducing P_{E1} to zero during the fault, and then the fault is self-cleared at time T^{clear} , resulting in the post-fault system being identical to the pre-fault system
 - During the fault-on time the equations reduce to

$$\frac{d\delta_1}{dt} = \Delta\omega_{1,pu} \omega_s$$

$$\frac{d\Delta\omega_{1,pu}}{dt} = \frac{1}{2 \times 3} (1 - 0)$$

That is, with a solid fault on the terminal of the generator, during the fault $P_{E1} = 0$

Infinite Bus GENCLS Implicit Solution



- The initial conditions are

$$\mathbf{x}(0) = \begin{bmatrix} \delta(0) \\ \omega_{pu}(0) \end{bmatrix} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix}$$

- Let $\Delta t = 0.02$ seconds
- During the fault the Jacobian is

$$\mathbf{J}(\mathbf{x}(t + \Delta t)) = \frac{0.02}{2} \begin{bmatrix} 0 & \omega_s \\ 0 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}$$

- Set the initial guess for $\mathbf{x}(0.02)$ as $\mathbf{x}(0)$, and

$$\mathbf{f}(\mathbf{x}(0)) = \begin{bmatrix} 0 \\ 0.1667 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution



- Then calculate the initial mismatch

$$\mathbf{h}(\mathbf{x}(0.02)^{(0)}) = -\mathbf{x}(0.02)^{(0)} + \mathbf{x}(0) + \frac{0.02}{2} (\mathbf{f}(\mathbf{x}(0.02)^{(0)}) + \mathbf{f}(\mathbf{x}(0)))$$

- With $\mathbf{x}(0.02)^{(0)} = \mathbf{x}(0)$ this becomes

$$\mathbf{h}(\mathbf{x}(0.02)^{(0)}) = -\begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2} \left(\begin{bmatrix} 0 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix} \right) = \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix}$$

- Then
$$\mathbf{x}(0.02)^{(1)} = \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ 0 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0.00334 \end{bmatrix} = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution



- Repeating for the next iteration

$$\mathbf{f}(\mathbf{x}(0.02)^{(1)}) = \begin{bmatrix} 1.259 \\ 0.1667 \end{bmatrix}$$

$$\begin{aligned} \mathbf{h}(\mathbf{x}(0.02)^{(1)}) &= -\begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix} + \begin{bmatrix} 0.418 \\ 0 \end{bmatrix} + \frac{0.02}{2} \left(\begin{bmatrix} 1.259 \\ 0.167 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.167 \end{bmatrix} \right) \\ &= \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix} \end{aligned}$$

- Hence we have converged with $\mathbf{x}(0.02) = \begin{bmatrix} 0.4306 \\ 0.00334 \end{bmatrix}$

Infinite Bus GENCLS Implicit Solution



- Iteration continues until $t = T^{\text{clear}}$, assumed to be 0.1 seconds in this example

$$\mathbf{x}(0.10) = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix}$$

- At this point, when the fault is self-cleared, the equations change, requiring a re-evaluation of $\mathbf{f}(\mathbf{x}(T^{\text{clear}}))$

$$\frac{d\delta}{dt} = \Delta\omega_{pu} \omega_s$$

$$\frac{d\Delta\omega_{pu}}{dt} = \frac{1}{6} \left(1 - \frac{1.281}{0.52} \sin \delta \right)$$

$$\mathbf{f}(\mathbf{x}(0.1^+)) = \begin{bmatrix} 6.30 \\ -0.1078 \end{bmatrix}$$

Infinite Bus GENCLS Implicit Solution



- With the change in $\mathbf{f}(\mathbf{x})$ the Jacobian also changes

$$\mathbf{J}(\mathbf{x}(0.12^{(0)})) = \frac{0.02}{2} \begin{bmatrix} 0 & \omega_s \\ -0.305 & 0 \end{bmatrix} - \mathbf{I} = \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}$$

- Iteration for $\mathbf{x}(0.12)$ is as before, except using the new function and the new Jacobian

This also converges quickly, with one or two iterations

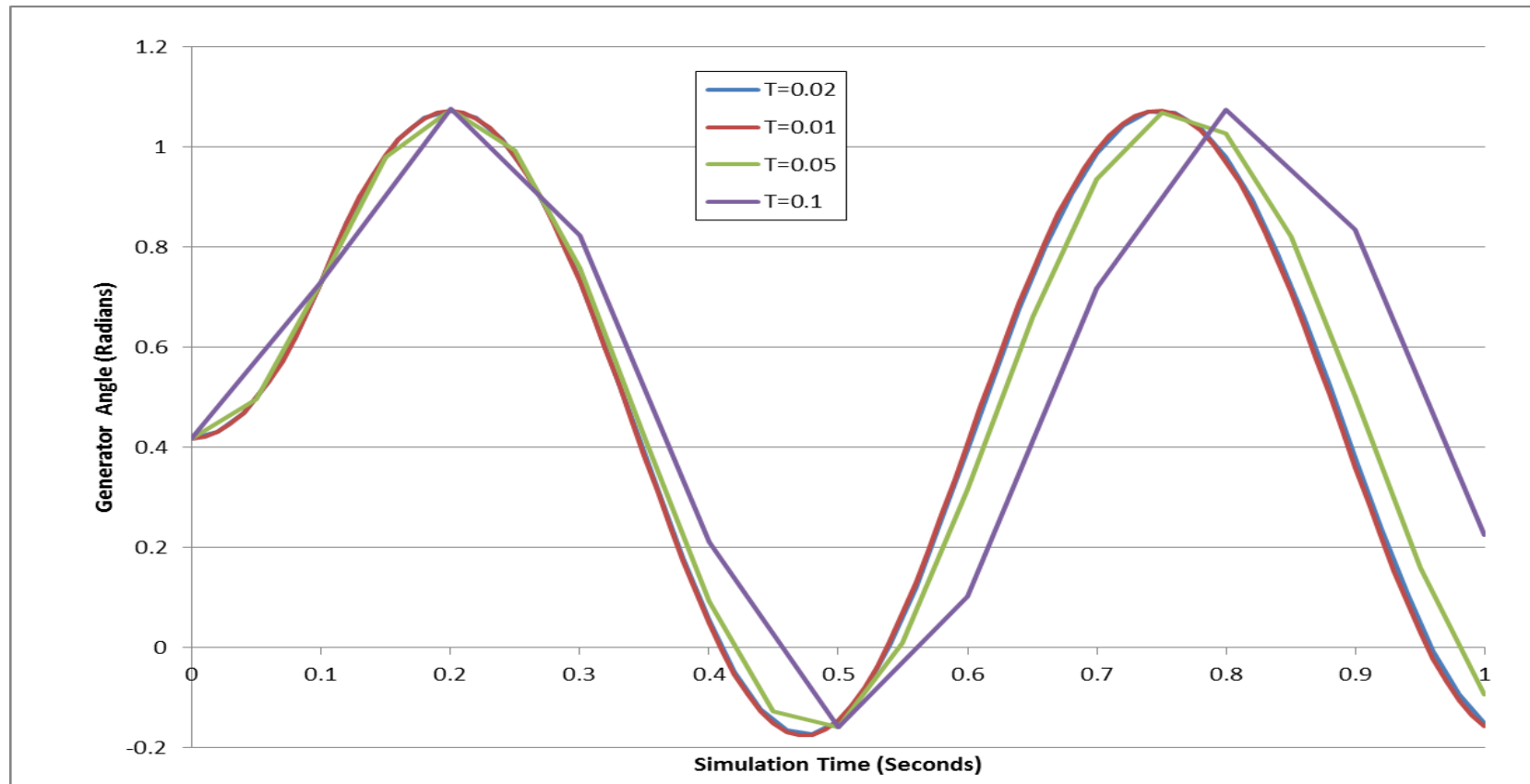
$$\mathbf{h}(\mathbf{x}(0.12)^{(0)}) \square -\mathbf{x}(0.12)^{(0)} + \mathbf{x}(0.01) + \frac{0.02}{2} (\mathbf{f}(\mathbf{x}(0.12)^{(0)}) + \mathbf{f}(\mathbf{x}(0.10^+)))$$

$$\mathbf{x}(0.12)^{(1)} = \begin{bmatrix} 0.7321 \\ 0.0167 \end{bmatrix} - \begin{bmatrix} -1 & 3.77 \\ -0.00305 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 0.1257 \\ -0.00216 \end{bmatrix} = \begin{bmatrix} 0.848 \\ 0.0142 \end{bmatrix}$$

Two Bus System Results



- The below graph shows the generator angle for varying values of Δt ; recall the implicit method is numerically stable



Adding the Algebraic Constraints



- Since the classical model can be formulated with all the values on the network reference frame, initially we just need to add the network equations
- We'll again formulate the network equations using the form
$$\mathbf{I}(\mathbf{x}, \mathbf{y}) = \mathbf{Y} \mathbf{V} \quad \text{or} \quad \mathbf{Y} \mathbf{V} - \mathbf{I}(\mathbf{x}, \mathbf{y}) = \mathbf{0}$$
- As before the complex equations will be expressed using two real equations, with voltages and currents expressed in rectangular coordinates

Adding the Algebraic Constraints



- The network equations are as before

$$\mathbf{y} = \begin{bmatrix} V_{D1} \\ V_{Q1} \\ V_{D2} \\ \vdots \\ V_{Dn} \\ V_{Qn} \end{bmatrix} \quad \mathbf{g}(\mathbf{x}, \mathbf{y}) = \begin{bmatrix} \sum_{k=1}^n (G_{1k} V_{Dk} - B_{1k} V_{QK}) - I_{ND1}(\mathbf{x}, \mathbf{y}) = 0 \\ \sum_{k=1}^n (G_{ik} V_{Qk} + B_{ik} V_{DK}) - I_{NQ1}(\mathbf{x}, \mathbf{y}) = 0 \\ \sum_{k=1}^n (G_{2k} V_{Dk} - B_{2k} V_{QK}) - I_{ND2}(\mathbf{x}, \mathbf{y}) = 0 \\ \vdots \\ \sum_{k=1}^n (G_{nk} V_{Dk} - B_{nk} V_{QK}) - I_{NDn}(\mathbf{x}, \mathbf{y}) = 0 \\ \sum_{k=1}^n (G_{nk} V_{Qk} + B_{nk} V_{DK}) - I_{NQn}(\mathbf{x}, \mathbf{y}) = 0 \end{bmatrix}$$

Coupling of \mathbf{x} and \mathbf{y} with the Classical Model



- In the simultaneous implicit method \mathbf{x} and \mathbf{y} are determined simultaneously; hence in the Jacobian we need to determine the dependence of the network equations on \mathbf{x} , and the state equations on \mathbf{y}
- With the classical model the Norton current depends on \mathbf{x} as

$$\bar{I}_{Ni} = \frac{E'_i \angle \delta_i}{R_{s,i} + jX'_{d,i}}, \quad G_i + jB_i = \frac{1}{R_{s,i} + jX'_{d,i}}$$

$$\bar{I}_{Ni} = I_{DNi} + jI_{QNi} = E'_i (\cos \delta_i + j \sin \delta_i) (G_i + jB_i)$$

$$E_{Di} + jE_{Qi} = E'_i (\cos \delta_i + j \sin \delta_i)$$

$$I_{DNi} = E_{Di} G_i - E_{Qi} B_i$$

$$I_{QNi} = E_{Di} B_i + E_{Qi} G_i$$

Recall with the classical model E_i' is constant

Coupling of x and y with the Classical Model



- In the state equations the coupling with y is recognized by noting

$$P_{Ei} = E_{Di}I_{Di} + E_{Qi}I_{Qi}$$

$$I_{Di} + jI_{Qi} = \left((E_{Di} - V_{Di}) + j(E_{Qi} - V_{Qi}) \right) (G_i + jB_i)$$

$$I_{Di} = (E_{Di} - V_{Di})G_i - (E_{Qi} - V_{Qi})B_i$$

These are the algebraic equations

$$I_{Qi} = (E_{Di} - V_{Di})B_i + (E_{Qi} - V_{Qi})G_i$$

$$P_{Ei} = E_{Di} \left((E_{Di} - V_{Di})G_i - (E_{Qi} - V_{Qi})B_i \right) + E_{Qi} \left((E_{Di} - V_{Di})B_i + (E_{Qi} - V_{Qi})G_i \right)$$

$$P_{Ei} = \left(E_{Di}^2 - E_{Di}V_{Di} \right) G_i + \left(E_{Qi}^2 - E_{Qi}V_{Qi} \right) G_i + \left(E_{Di}V_{Qi} - E_{Qi}V_{Di} \right) B_i$$

Hence we have P_{Ei} written in terms of the voltages (y)

Variables and Mismatch Equations



- In solving the Newton algorithm the variables now include \mathbf{x} and \mathbf{y} (recalling that here \mathbf{y} is just the vector of the real and imaginary bus voltages)
- The mismatch equations now include the state integration equations

$$\mathbf{h}\left(\mathbf{x}(t + \Delta t)^{(k)}\right) = -\mathbf{x}(t + \Delta t)^{(k)} + \mathbf{x}(t) + \frac{\Delta t}{2} \left(\mathbf{f}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right) + \mathbf{f}\left(\mathbf{x}(t), \mathbf{y}(t)\right) \right)$$

- And the algebraic equations

$$\mathbf{g}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right)$$

Jacobian Matrix



- Since the $\mathbf{h}(\mathbf{x}, \mathbf{y})$ and $\mathbf{g}(\mathbf{x}, \mathbf{y})$ are coupled, the Jacobian is

$$J\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right) = \begin{bmatrix} \frac{\partial \mathbf{h}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \frac{\partial \mathbf{h}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right)}{\partial \mathbf{y}} \\ \frac{\partial \mathbf{g}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right)}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}\left(\mathbf{x}(t + \Delta t)^{(k)}, \mathbf{y}(t + \Delta t)^{(k)}\right)}{\partial \mathbf{y}} \end{bmatrix}$$

- With the classical model the coupling is the Norton current at bus i depends on δ_i (i.e., \mathbf{x}) and the electrical power (P_{Ei}) in the swing equation depends on V_{Di} and V_{Qi} (i.e., \mathbf{y})

Jacobian Matrix Entries



- The dependence of the Norton current injections on δ is

$$I_{DNi} = E'_i \cos \delta_i G_i - E'_i \sin \delta_i B_i$$

$$I_{QNi} = E'_i \cos \delta_i B_i + E'_i \sin \delta_i G_i$$

$$\frac{\partial I_{DNi}}{\partial \delta_i} = -E'_i \sin \delta_i G_i - E'_i \cos \delta_i B_i$$

$$\frac{\partial I_{QNi}}{\partial \delta_i} = -E'_i \sin \delta_i B_i + E'_i \cos \delta_i G_i$$

- In the Jacobian the sign is flipped because we defined

$$\mathbf{g}(\mathbf{x}, \mathbf{y}) = \mathbf{Y} \mathbf{V} - \mathbf{I}(\mathbf{x}, \mathbf{y})$$

Jacobian Matrix Entries



- The dependence of the swing equation on the generator terminal voltage is

$$\dot{\delta}_i = \Delta \omega_{i,pu} \omega_s$$

$$\Delta \dot{\omega}_{i,pu} = \frac{1}{2H_i} \left(P_{Mi} - P_{Ei} - D_i (\Delta \omega_{i,pu}) \right)$$

$$P_{Ei} = \left(E_{Di}^2 - E_{Di} V_{Di} \right) G_i + \left(E_{Qi}^2 - E_{Qi} V_{Qi} \right) G_i + \left(E_{Di} V_{Qi} - E_{Qi} V_{Di} \right) B_i$$

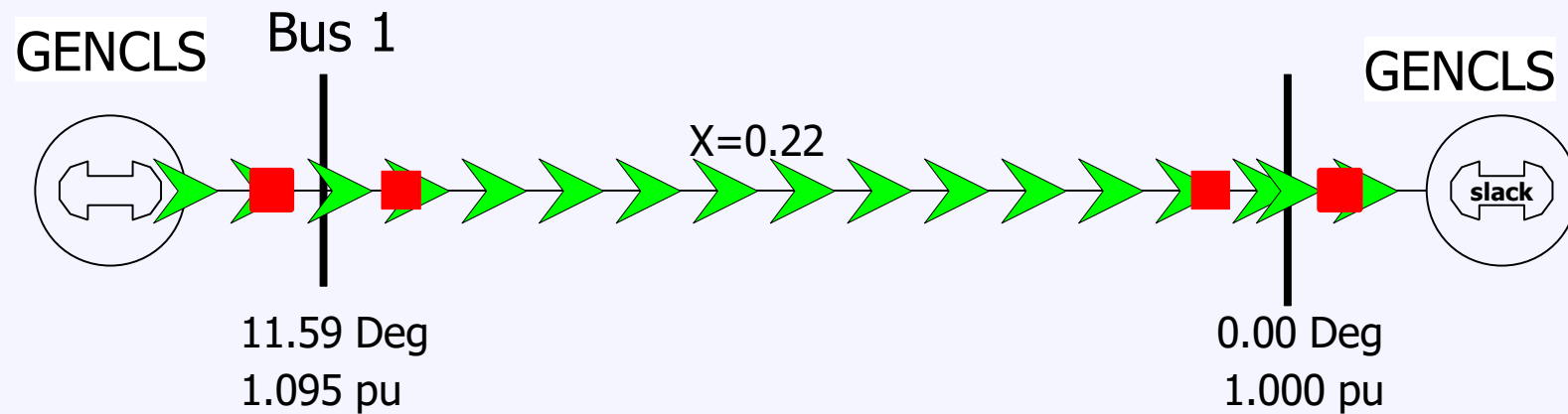
$$\frac{\partial \Delta \dot{\omega}_{i,pu}}{\partial V_{Di}} = \frac{1}{2H_i} \left(E_{Di} G_i + E_{Qi} B_i \right)$$

$$\frac{\partial \Delta \dot{\omega}_{i,pu}}{\partial V_{Qi}} = \frac{1}{2H_i} \left(E_{Qi} G_i - E_{Di} B_i \right)$$

Two Bus, Two Gen GENCLS Example



- We'll reconsider the two bus, two generator case from the previous lecture ; fault at Bus 1, cleared after 0.06 seconds
 - Initial conditions and \mathbf{Y}_{bus} are as covered in Lecture 16



PowerWorld Case **B2_CLS_2Gen**

Two Bus, Two Gen GENCLS Example



- Initial terminal voltages are

$$V_{D1} + jV_{Q1} = 1.0726 + j0.22, \quad V_{D2} + jV_{Q2} = 1.0$$

$$\bar{E}_1 = 1.281 \angle 23.95^\circ, \quad \bar{E}_2 = 0.955 \angle -12.08$$

$$\bar{I}_{N1} = \frac{1.1709 + j0.52}{j0.3} = 1.733 - j3.903$$

$$\bar{I}_{N2} = \frac{0.9343 - j0.2}{j0.2} = -1 - j4.6714$$

$$\mathbf{Y} = \mathbf{Y}_N + \begin{bmatrix} \frac{1}{j0.333} & 0 \\ 0 & \frac{1}{j0.2} \end{bmatrix} = \begin{bmatrix} -j7.879 & j4.545 \\ j4.545 & -j9.545 \end{bmatrix}$$

Two Bus, Two Gen Initial Jacobian

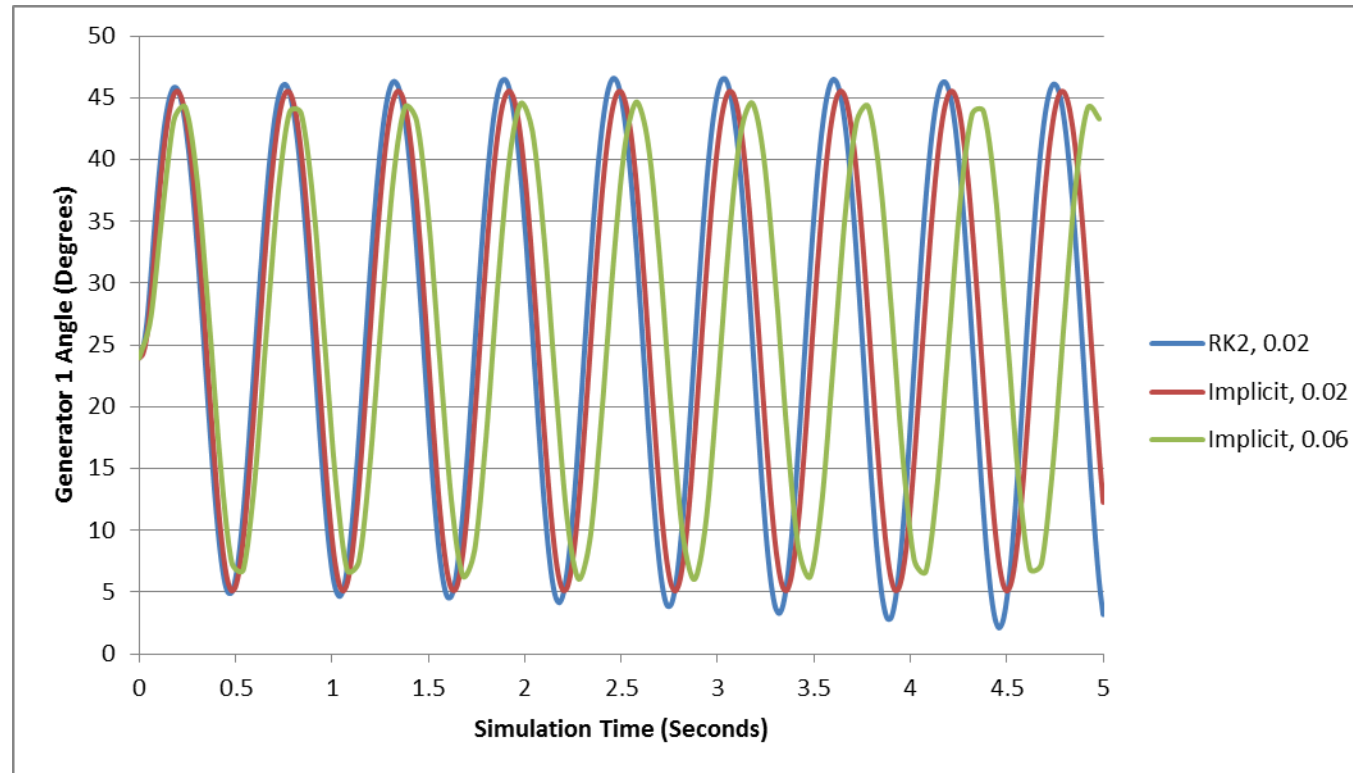


$$\begin{bmatrix} \delta_1 & \Delta\omega_1 & \delta_2 & \Delta\omega_2 & V_{D1} & V_{Q1} & V_{D2} & V_{Q2} \\ \dot{\delta}_1 & -1 & 3.77 & 0 & 0 & 0 & 0 & 0 \\ \Delta\dot{\omega}_1 & -0.0076 & -1 & 0 & 0 & -0.0029 & 0.0065 & 0 \\ \dot{\delta}_2 & 0 & 0 & -1 & 3.77 & 0 & 0 & 0 \\ \Delta\dot{\omega}_2 & 0 & 0 & -0.0039 & -1 & 0 & 0.0008 & 0.0039 \\ I_{D1} & -3.90 & 0 & 0 & 0 & 0 & 7.879 & -4.545 \\ I_{Q1} & -1.73 & 0 & 0 & 0 & -7.879 & 0 & 4.545 \\ I_{D2} & 0 & 0 & -4.67 & 0 & 0 & -4.545 & 9.545 \\ I_{Q2} & 0 & 0 & 1.00 & 0 & 4.545 & 0 & -9.545 \end{bmatrix}$$

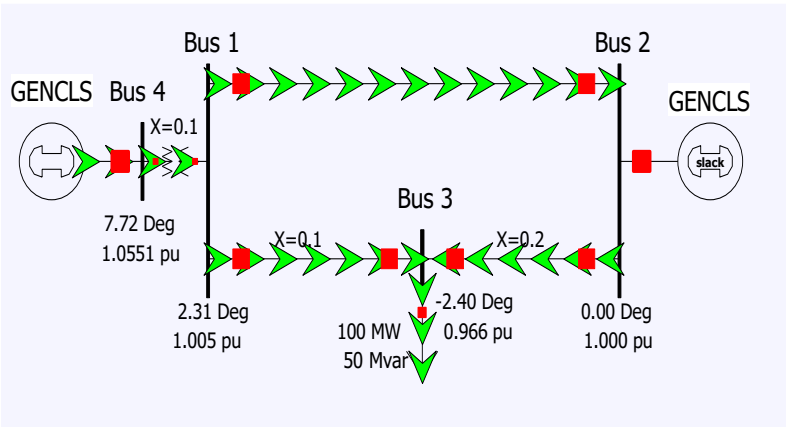
Results Comparison



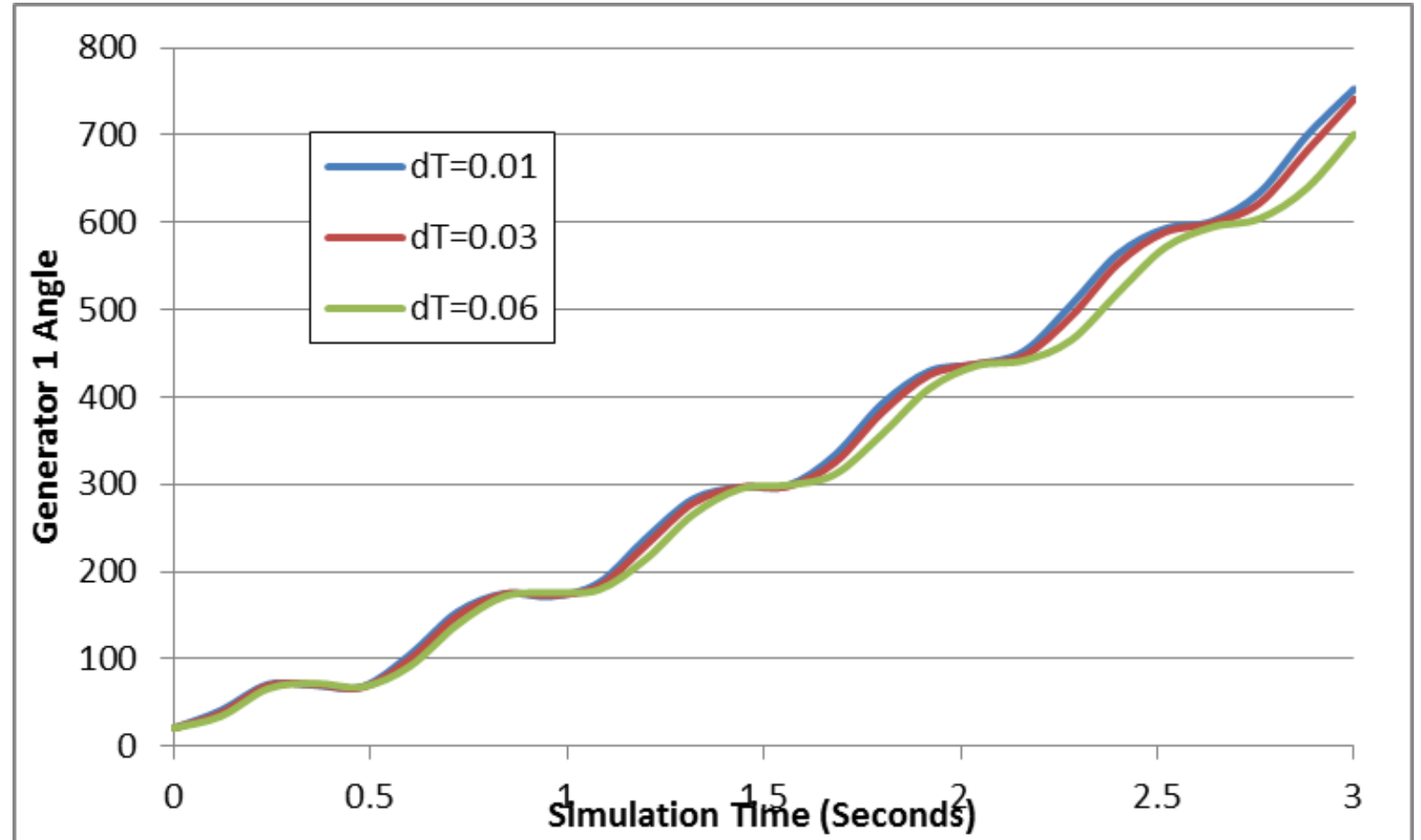
- The below graph compares the angle for the generator at bus 1 using $\Delta t=0.02$ between RK2 and the Implicit Trapezoidal; also Implicit with $\Delta t=0.06$



Four Bus Comparison



Fault at Bus 3 for 0.12 seconds; self-cleared



Done with Stability Solutions: On to Load Modeling



- Load modeling is certainly challenging!
- For large system models an aggregate load can consist of many thousands of individual devices
- The load is constantly changing, with key diurnal and temperature variations
 - For example, a higher percentage of lighting load at night, more air conditioner load on hot days
- Load model behavior can be quite complex during the low voltages that may occur in transient stability
- Testing aggregate load models for extreme conditions is not feasible
 - we need to wait for disturbances!

Load Modeling



- Traditionally load models have been divided into two groups
 - Static: load is a algebraic function of bus voltage and sometimes frequency
 - Dynamic: load is represented with a dynamic model, with induction motor models the most common
- The simplest load model is a static constant impedance
 - Has been widely used
 - Allowed the \mathbf{Y}_{bus} to be reduced, eliminating essentially all non-generator buses
 - Presents no issues as voltage falls to zero
 - No longer commonly used

Load Modeling References



- Many papers and reports are available!
- A classic reference on load modeling is by the IEEE Task Force on Load Representation for Dynamic Performance, "Load Representation for Dynamic Performance Analysis," IEEE Trans. on Power Systems, May 1993, pp. 472-48
- NERC 2016, “Dynamic Load Modeling”; available at <https://www.nerc.com/comm/PC/LoadModelingTaskForceDL/Dynamic%20Load%20Modeling%20Tech%20Ref%202016-11-14%20-%20FINAL.PDF>
- NERC Reliability Guideline, “Developing Load Model Composition Data, 2017”
- EPRI Technical Guide to Composite Load Modeling, Sept. 2020
 - <https://www.epri.com/research/programs/027570/results/3002019209>

ZIP Load Model (Covered Last Lecture)



- Another common static load model is the ZIP, in which the load is represented as

$$P_{Load,k} = P_{BaseLoad,k} \left(P_{z,k} |\bar{V}_k|^2 + P_{i,k} |\bar{V}_k| + P_{p,k} \right)$$

$$Q_{Load,k} = Q_{BaseLoad,k} \left(Q_{z,k} |\bar{V}_k|^2 + Q_{i,k} |\bar{V}_k| + Q_{p,k} \right)$$

- Some models allow more general voltage dependence

$$P_{Load,k} = P_{BaseLoad,k} \left(a_{1,k} |\bar{V}_k|^{n1} + a_{2,k} |\bar{V}_k|^{n2} + a_{3,k} |\bar{V}_k|^{n3} \right)$$

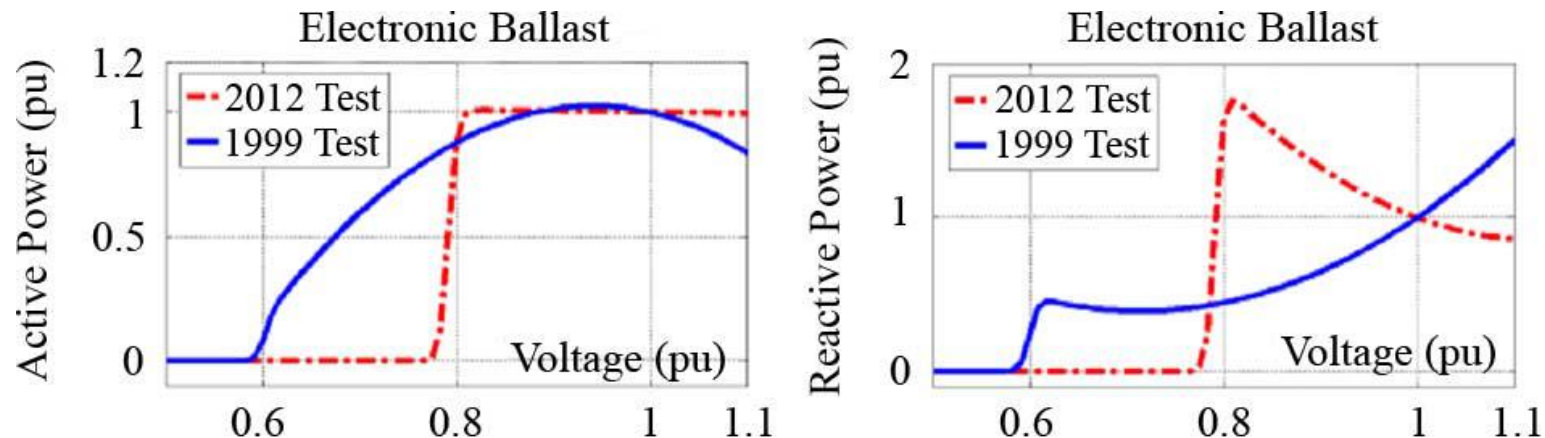
$$Q_{Load,k} = Q_{BaseLoad,k} \left(a_{4,k} |\bar{V}_k|^{n4} + a_{5,k} |\bar{V}_k|^{n5} + a_{6,k} |\bar{V}_k|^{n6} \right)$$

The voltage exponent for reactive power is often > 2

ZIP Model Coefficients



- An interesting paper on the experimental determination of the ZIP parameters is A. Bokhari, et. al., "Experimental Determination of the ZIP Coefficients for Modern Residential and Commercial Loads, and Industrial Loads," *IEEE Trans. Power Delivery*, 2014
 - Presents test results for loads as voltage is varied; also highlights that load behavior changes with newer technologies
 - Below figure (part of fig 4 of paper), compares real and reactive behavior of light ballast



ZIP Model Coefficients



TABLE VII
ACTIVE AND REACTIVE ZIP MODEL. FIRST HALF OF THE ZIPS
WITH 100-V CUTOFF VOLTAGE. SECOND HALF REPORTS THE ZIPS WITH ACTUAL CUTOFF VOLTAGE

Equipment/ component	No. tested	V_{cut}	V_o	P_o	Q_o	Z_p	I_p	P_p	Z_q	I_q	P_q
Air compressor 1 Ph	1	100	120	1109.01	487.08	0.71	0.46	-0.17	-1.33	4.04	-1.71
Air compressor 3 Ph	1	174	208	1168.54	844.71	0.24	-0.23	0.99	4.79	-7.61	3.82
Air conditioner	2	100	120	496.33	125.94	1.17	-1.83	1.66	15.68	-27.15	12.47
CFL bulb	2	100	120	25.65	37.52	0.81	-1.03	1.22	0.86	-0.82	0.96
Coffeemaker	1	100	120	1413.04	13.32	0.13	1.62	-0.75	3.89	-6	3.11
Copier	1	100	120	944.23	84.57	0.87	-0.21	0.34	2.14	-3.67	2.53
Electronic ballast	3	100	120	59.02	5.06	0.22	-0.5	1.28	9.64	-21.59	12.95
Elevator	3	174	208	1381.17	1008.3	0.4	-0.72	1.32	3.76	-5.74	2.98
Fan	2	100	120	163.25	83.28	-0.47	1.71	-0.24	2.34	-3.12	1.78
Game consol	3	100	120	60.65	67.61	-0.63	1.23	0.4	0.76	-0.93	1.17
Halogen	3	100	120	97.36	0.84	0.46	0.64	-0.1	4.26	-6.62	3.36
High pressure sodium HID	4	100	120	276.09	52.65	0.09	0.7	0.21	16.6	-28.77	13.17
Incandescent light	2	100	120	87.16	0.85	0.47	0.63	-0.1	0.55	0.38	0.07
Induction light	1	100	120	44.5	4.8	2.96	-6.04	4.08	1.48	-1.29	0.81
Laptop charger	1	100	120	35.94	71.64	-0.28	0.5	0.78	-0.37	1.24	0.13
LCD Television	1	100	120	208.03	-20.58	0.11	-0.17	1.06	1.58	-1.72	1.14
LED light	1	100	120	3.38	5.85	0.58	1.13	-0.71	1.78	-0.8	0.02
Magnetic ballast	1	100	120	81.23	8.2	-1.58	3.79	-1.21	36.18	-67.78	32.6
Mercury vapor HID light	2	100	120	268.27	77.66	0.52	1.02	-0.54	-1.33	2.4	-0.07
Metal halide HID electronic ballast	2	100	120	113.7	26.37	1	-2.02	2.02	8.8	-18.64	10.84
Metal halide HID magnetic ballast	2	100	120	450	102.94	0.86	-0.66	0.8	32.54	-59.83	28.29
Microwave	2	100	120	1365.53	451.02	1.39	-1.96	1.57	50.07	-93.55	44.48
Minibar	1	100	120	90.65	126.94	2.5	-4.1	2.6	2.56	-2.76	1.2
PC (Monitor & CPU)	1	100	120	118.9	172.79	0.2	-0.3	1.1	0	0.6	0.4

The Z,I,P coefficients sum to zero; note that for some models the absolute values of the parameters are quite large, indicating a difficult fit

A portion of Table VII from the Bokhari 2014 paper

Discharge Lighting Models



- Discharge lighting (such as fluorescent lamps) have been at major portion of the load (10-15%) though now it is getting increasingly replaced by LEDs
- Discharge lighting has been modeled for sufficiently high voltage with a real power as constant current and reactive power with a high voltage dependence
 - Linear reduction for voltage between 0.65 and 0.75 pu
 - Extinguished (i.e., no load) for voltages below

$$P_{DischargeLighting} = P_{Base} \left(|\bar{V}_k| \right)$$

$$Q_{DischargeLighting} = Q_{Base} \left(|\bar{V}_k|^{4.5} \right)$$

May need to change with newer electronic ballasts – e.g., reactive power increasing as the voltage drops!

Static Load Model Frequency Dependence



- Frequency dependence is sometimes included, to recognize that the load could change with the frequency

$$P_{Load,k} = P_{BaseLoad,k} \left(P_{z,k} |\bar{V}_k|^2 + P_{i,k} |\bar{V}_k| + P_{p,k} \right) \left(1 + P_{f,k} (f_k - 1) \right)$$

$$Q_{Load,k} = Q_{BaseLoad,k} \left(Q_{z,k} |\bar{V}_k|^2 + Q_{i,k} |\bar{V}_k| + Q_{p,k} \right) \left(1 + Q_{f,k} (f_k - 1) \right)$$

- Here f_k is the per unit bus frequency, which is calculated as

$$\theta_k \rightarrow \boxed{\frac{s}{1+sT}} \rightarrow f_k$$

A typical value for T is about 0.02 seconds. Some models just have frequency dependence on the constant power load

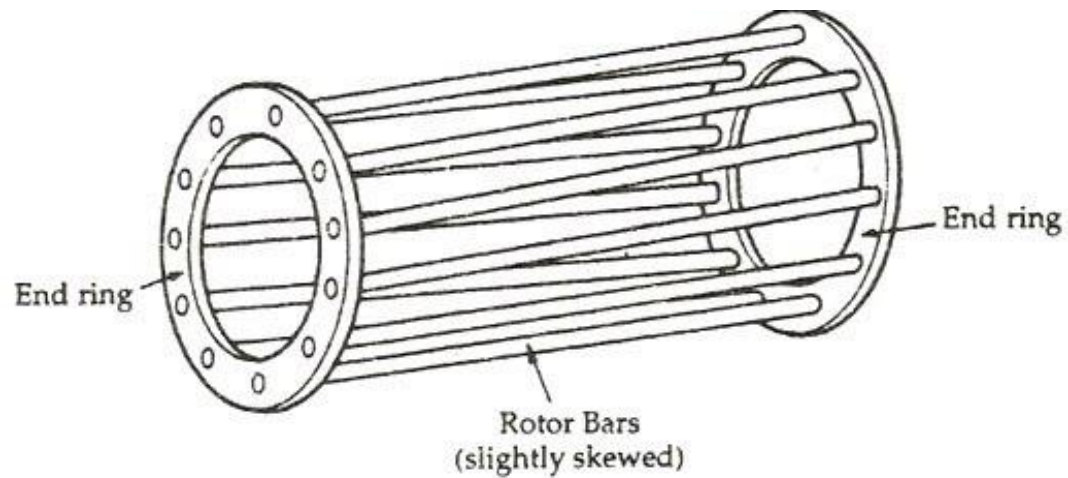
- Typical values for P_f and Q_f are 1 and -1 respectively

Induction Machines



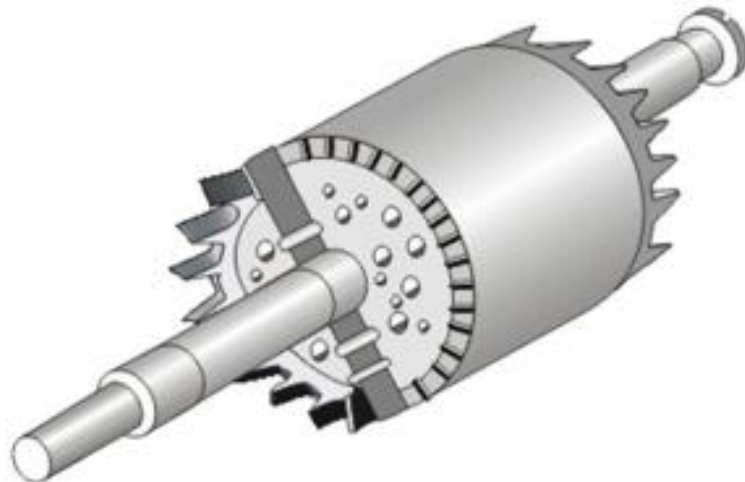
- Term induction machine is used to indicate either generator or motor; most uses are as motors
- Induction machines have two major components
 - A stationary stator, which is supplied with an ac voltage; windings in stator create a rotating magnetic field
 - A rotating rotor, in which an ac current is induced (hence the name)
- Two basic design types based on rotor design
 - Squirrel-cage: rotor consists of shorted conducting bars laid into magnetic material in a cage structure
 - Wound-rotor: rotor has windings similar to stator, with slip rings used to provide external access to the rotor windings

Squirrel Cage Rotor Picture



[Image 1 Source: www.quora.com/What-will-happen-If-the-Squirrel-cage-motor-rotor-conductors-are-not-skewed](http://www.quora.com/What-will-happen-If-the-Squirrel-cage-motor-rotor-conductors-are-not-skewed)

Embedded in laminated magnetic material



[Image 2 Source: www.polytechnichub.com/squirrel-cage-rotor/](http://www.polytechnichub.com/squirrel-cage-rotor/)

Induction Machine Overview



- Speed of rotating magnetic field (synchronous speed) depends on number of poles

$$N_s = f_s \frac{120}{p} \quad \text{where } N_s \text{ is the synchronous speed in RPM, } f_s \text{ is}$$

the stator electrical frequency (e.g., 60 or 50Hz) and p is the number of poles

- Frequency of induced currents in rotor depends on frequency difference between the rotating magnetic field and the rotor

$$\omega_r = \omega_s - \left(\frac{p}{2} \right) \omega_m$$

where ω_s is the stator electrical frequency, ω_m is mechanical speed,

and ω_r is the rotor electrical frequency

Induction Machine Slip



- Key value is slip, s , defined as

$$s = \frac{N_s - N_{act}}{N_s}$$

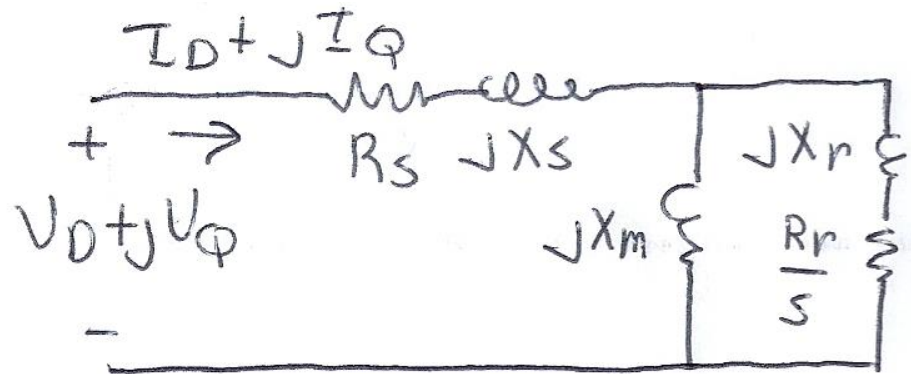
where N_s is the synchronous speed, and

N_{act} is the actual speed (in RPM)

- As defined, when operating as a motor an induction machine will have a positive slip, slip is negative when operating as a generator
 - Slip is zero at synchronous speed, a speed at which no rotor current is induced;
 $s=1$ at stand still

Basic Induction Machine Model

- A basic (single cage) induction machine circuit model is given below
 - Model is derived in an undergraduate machines class



$$\frac{R_r}{s} = R_r + \frac{(1-s)}{s} R_r$$

- Circuit is useful for understanding the static behavior of the machine
- Effective rotor resistance (R_r/s) models the rotor electrical losses (R_r) and the mechanical power $R_r(1-s)/s$

Induction Machine Dynamics



- Expressing all values in per unit (with the base covered later), the mechanical equation for a machine is

$$\frac{ds}{dt} = \frac{1}{2H} (T_M - T_E)$$

where H is the inertia constant, T_M is the mechanical torque and T_E is the electrical torque (to be defined)

- Similar to what was done for a synchronous machine, the induction machine can be modeled as an equivalent voltage behind a stator resistance and transient reactance (later we'll introduce, but not derive, the subtransient model)

Induction Machine Dynamics



- Define

$$X' = X_s + \frac{X_r X_m}{X_r + X_m}$$

$$X = X_s + X_m$$

where X' is the apparent reactance seen when the rotor is locked ($s=1$) and X is the synchronous reactance

- Also define the open circuit time constant

$$T'_o = \frac{(X_r + X_m)}{\omega_s R_r}$$

Induction Machine Dynamics



- Electrically the induction machine is modeled similar to the classical generator model, except here we use the "motor convention" in which $I_D + jI_Q$ is assumed positive into the machine

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = \omega_s s E'_Q - \frac{1}{T'_o} (E'_D + (X - X') I_Q)$$

$$\frac{dE'_Q}{dt} = -\omega_s s E'_D - \frac{1}{T'_o} (E'_Q - (X - X') I_D)$$

All calculations are done on the network reference frame

Induction Machine Dynamics



- The induction machine electrical torque, T_E , and terminal electrical load, P_E , are then

$$T_E = \frac{(E'_D I_D + E'_Q I_Q)}{\omega_s}$$

$$P_E = V_D I_D + V_Q I_Q$$

Recall we are using the motor convention so positive P_E represents load

- Similar to a synchronous machine, once the initial values are determined the differential equations are fairly easy to simulate
 - Key initial value needed is the slip

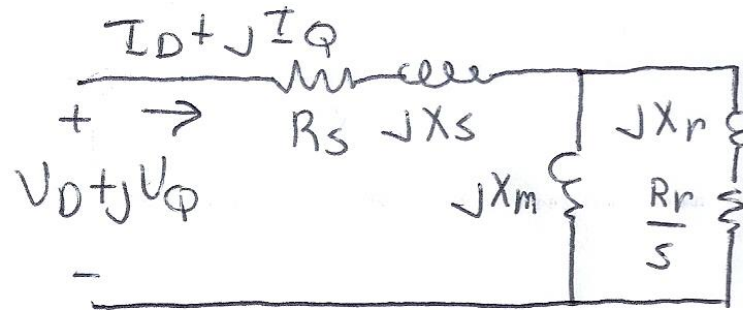
Specifying Induction Machine Parameters



- In transient stability packages induction machine parameters are specified in per unit
 - If unit is modeled as a generator in the power flow (such as CIMTR1 or GENWRI) then use the generator's MVA base (as with synchronous machines)
 - With loads it is more complicated.
 - Sometimes an explicit MVA base is specified. If so, then use this value. But this can be cumbersome since often the same per unit machine values are used for many loads
 - The default is to use the MW value for the load, often scaled by a multiplier (say 1.25)

Determining the Initial Values

- To determine the initial values, it is important to recognize that for a fixed terminal voltage there is only one independent value: the slip, s
 - For a fixed slip, the model is just a simple circuit with resistances and reactances



- The initial slip is chosen to match the power flow real power value. Then to match the reactive power value (for either a load or a generator), the approach is to add a shunt capacitor in parallel with the induction machine
- We'll first consider torque-speed curves, then return to determining the initial slip

Torque-Speed Curves



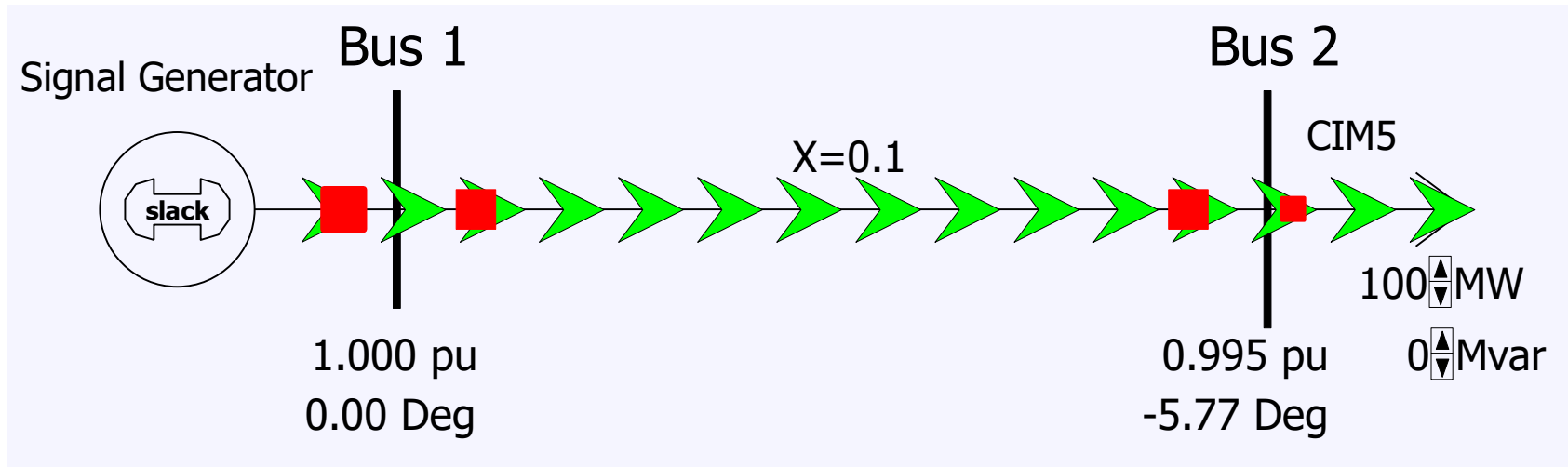
- To help understand the behavior of an induction machine it is useful to plot various values as a function of speed (or equivalently, slip)
 - Solve the equivalent circuit for a specified terminal voltage, and varying values of slip
 - Plot results
 - Recall torque times speed = power
 - Here speed is the rotor speed
 - When using per unit, the per unit speed is just $1-s$

$$P_E = T_E (1 - s)$$

Induction Motor Example



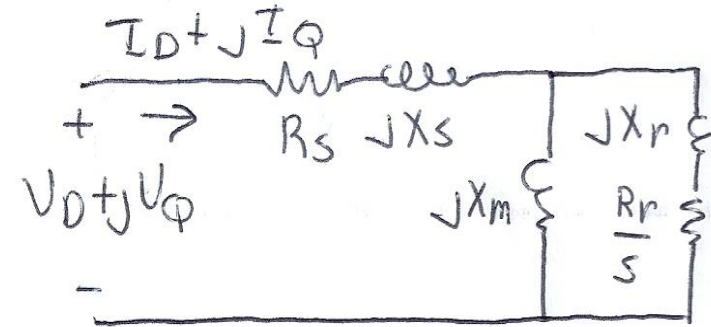
- Assume the below 60 Hz system, with the entire load modeled as a single cage induction motor with per unit values on a 125 MVA base of $H=1.0$, $R_s=0.01$, $X_s=0.06$, $X_m=4.0$, $R_r=0.03$, $X_r=0.04$
 - In the CIM5 model $R_1=R_r$ and $X_1=X_r$



PowerWorld case **B2_IndMotor**

Induction Motor Example

- With a terminal voltage of $0.995 \angle 0^\circ$ we can solve the circuit for specified values of s
- The input impedance and current are



$$Z_{in} = (R_s + jX_s) + \frac{jX_m \left(\frac{R_r}{s} + jX_r \right)}{\frac{R_r}{s} + j(X_r + X_m)}, \quad \bar{I} = \frac{\bar{V}}{Z_{in}} = \frac{0.995 \angle 0^\circ}{Z_{in}}$$

- Then with $s=1$ we get

Note, values are per unit on a 125 MVA base

$$\bar{I} = \frac{0.995}{0.0394 + j0.0998} = 3.404 - j8.624 \rightarrow S = 3.387 + j8.581$$

Induction Motor Example

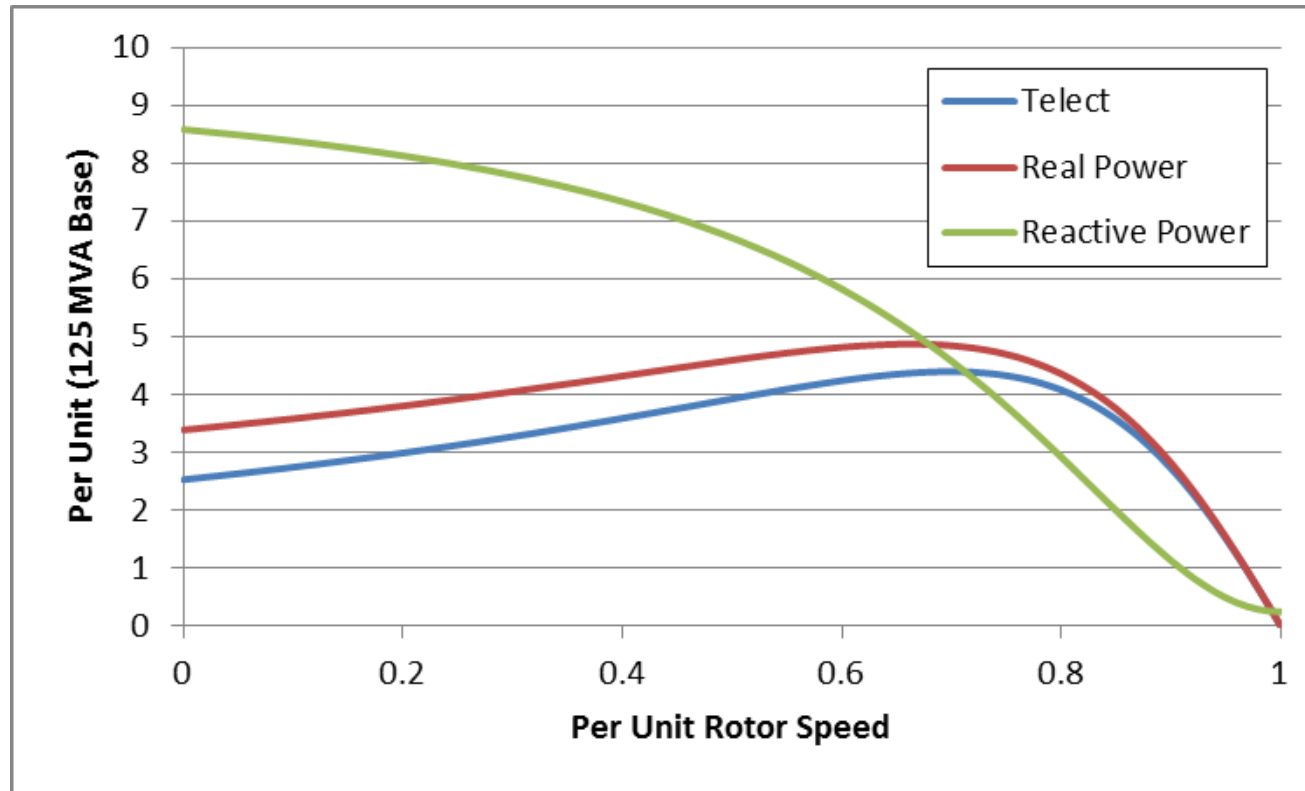


- PowerWorld allows for display of the variation in various induction machine values with respect to speed
 - Right click on load, select Load Information Dialog, Stability
 - On bottom of display click Show Torque Speed Dialog
 - Adjust the terminal voltage and pu scalar as desired; set $v=0.995$ and the pu scalar to 1.0 to show values on the 125 MVA base used in the previous solution
 - Right click on column and select Set/Toggle/Columns, Plot Column to plot the column

Induction Motor Example Torque-Speed Curves



- The below graph shows the torque-speed curve for this induction machine; note the high reactive power consumption on starting (which is why the lights may dim when starting a cloth dryer!)



From the graph you can see with a 100 MW load (0.8 pu on the 125 MW base), the slip is about 0.025

Calculating the Initial Slip



- One way to calculate the initial slip is to just solve the below five equations for five unknowns (s, I_D, I_Q, E'_D, E'_Q) with P_E, V_D and V_Q inputs

$$P_E = V_D I_D + V_Q I_Q$$

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = 0 = \omega_s s E'_Q - \frac{1}{T_o'} (E'_D + (X - X') I_Q)$$

$$\frac{dE'_Q}{dt} = 0 = \omega_s s E'_D - \frac{1}{T_o'} (E'_Q - (X - X') I_D)$$

These are nonlinear equations that can have multiple solutions so use Newton's method, with an initial guess of s small (say 0.01)

Initial slip in example is 0.0251

Double Cage Induction Machines



- In the design of induction machines, there are various tradeoffs, such as between starting torque (obviously one needs enough to start) and operating efficiency
 - The highest efficiency possible is 1-slip, so operating at low slip is desirable
- A common way to achieve high starting torque with good operating efficiency is to use a double cage design
 - E.g., the rotor has two embedded squirrel cages, one with a high R and lower X for starting, and one with lower R and higher X for running
 - Modeled by extending our model by having two rotor circuits in parallel; add subtransient values X'' and T''_o

Example Double Cage Model

- Double cage rotors are modeled by adding two additional differential equations

Some models also include saturation, a topic that we will skip

PowerWorld case
B2_IndMotor_DoubleCage

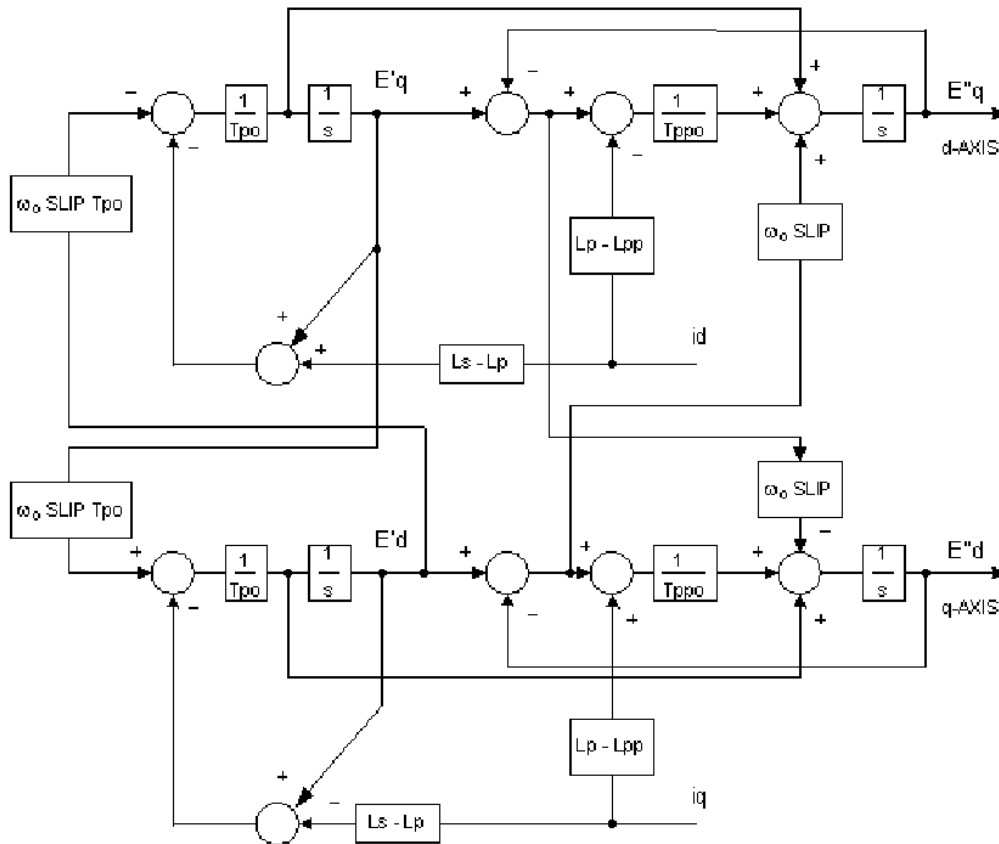
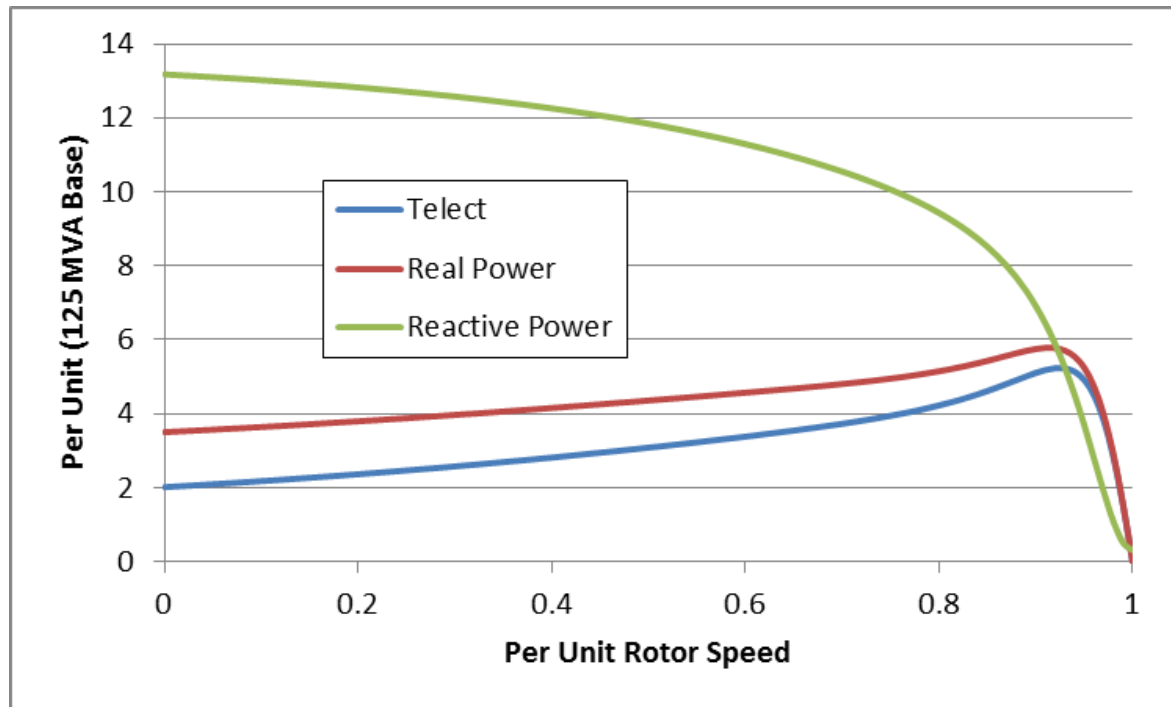


Image source: PSLF Manual, version 18.1_02; MotorW

Double Cage Induction Motor Model



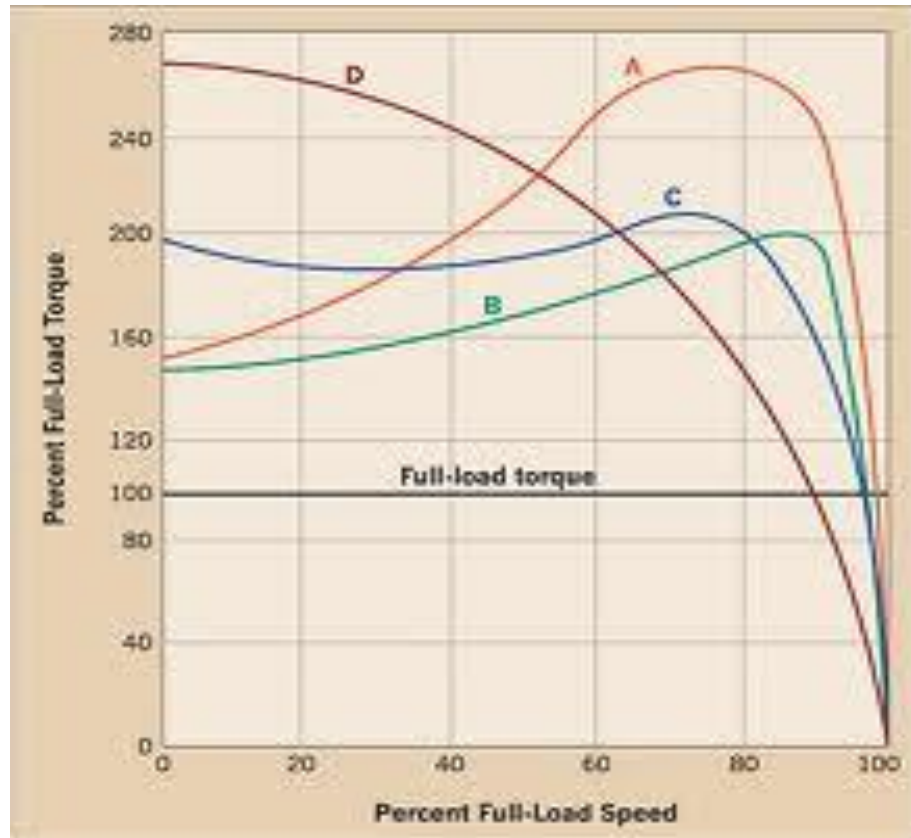
- The previous example can be extended to model a double cage rotor by setting $R_2=0.01$, $X_2=0.08$
 - The below graph shows the modified curves, notice the increase in the slope by $s=0$, meaning it is operating with higher efficiency ($s=0.0063$ now!)



The additional winding does result in lower initial impedance and hence a higher starting reactive power

Induction Motor Classes

- Four major classes of induction motors, based on application. Key values are starting torque, pull-out torque, full-load torque, and starting current



In steady-state the motor will operate on the right side of the curve at the point at which the electrical torque matches the mechanical torque

- A: Fans, pumps machine tools
- B: Similar to A
- C: Compressors, conveyors
- D: High inertia such as hoists

Induction Motor Stalling

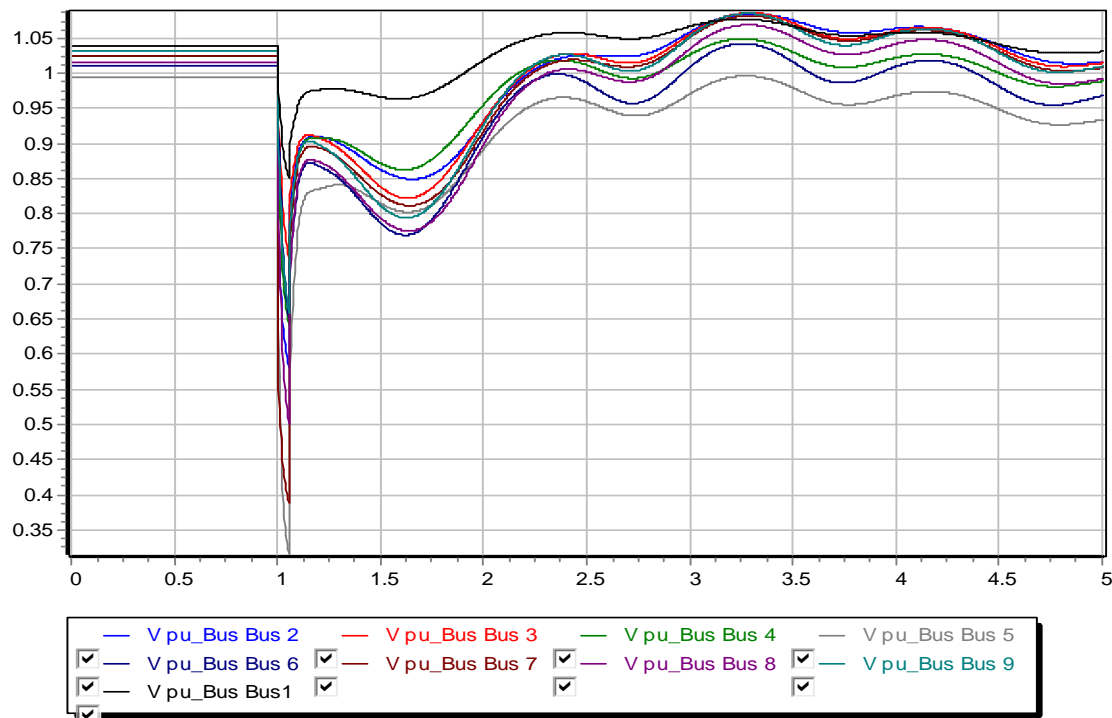


- Height of the torque-speed curve varies with the square of the terminal voltage
- When the terminal voltage decreases, such as during a fault, the mechanical torque can exceed the electrical torque
 - This causes the motor to decelerate, perhaps quite quickly, with the rate proportional to its inertia
 - This deceleration causing the slip to increase, perhaps causing the motor to stall with $s=1$, resulting in a high reactive current draw
 - Too many stalled motors can prevent the voltage from recovering

Motor Stalling Example



- Using case WSCC_CIM5, which models the WSCC 9 bus case with 100% induction motor load
- Change the fault scenario to say a fault midway between buses 5 and 7, cleared by opening the line

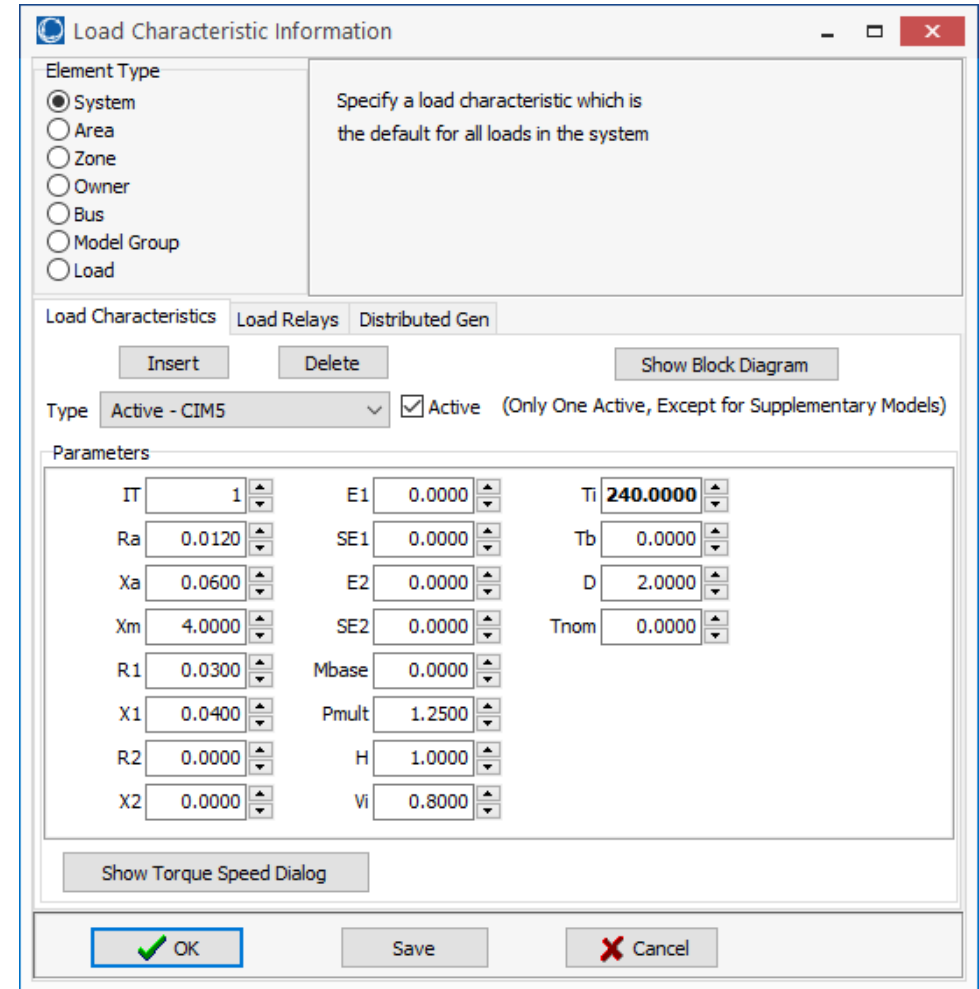


Results are for a 0.05 second fault

Usually motor load is much less than 100%

Impact of Model Protection Parameters

- Some load models, such as the CIM5, have built-in protection system models. For CIM5 the V_i and T_i fields are used to disconnect the load when its voltage is less than V_i for T_i cycles
 - When running simulations you need to check for such events



Load Characteristic Information

Element Type

- System
- Area
- Zone
- Owner
- Bus
- Model Group
- Load

Specify a load characteristic which is the default for all loads in the system

Load Characteristics | Load Relays | Distributed Gen

Insert Delete Show Block Diagram

Type: Active - CIM5 Active (Only One Active, Except for Supplementary Models)

Parameters

IT	1	E1	0.0000	Ti	240.0000
Ra	0.0120	SE1	0.0000	Tb	0.0000
Xa	0.0600	E2	0.0000	D	2.0000
Xm	4.0000	SE2	0.0000	Tnom	0.0000
R1	0.0300	Mbase	0.0000		
X1	0.0400	Pmult	1.2500		
R2	0.0000	H	1.0000		
X2	0.0000	Vi	0.8000		

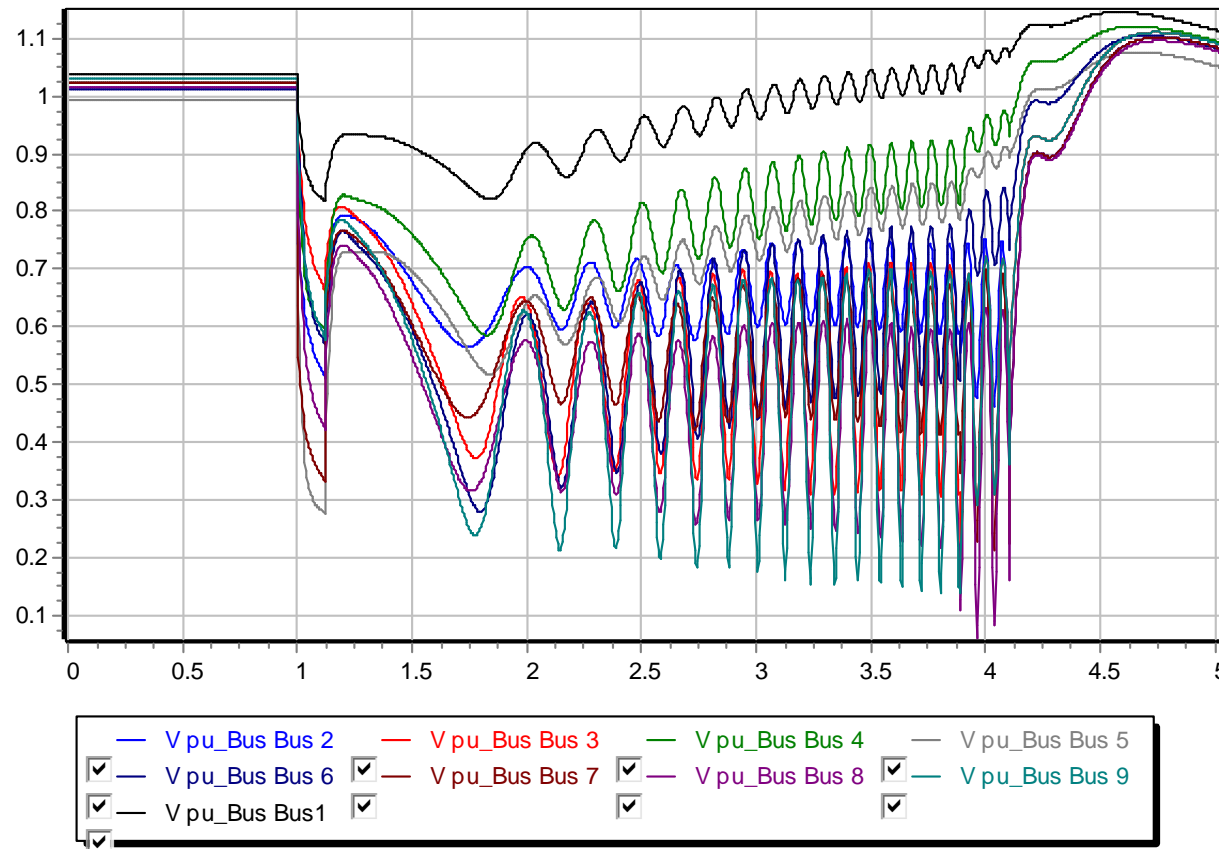
Show Torque Speed Dialog

OK Save Cancel

Motor Stalling With Longer Fault



- The below image shows the WECC_CIM5 system with the fault clearing extended to 0.12 seconds



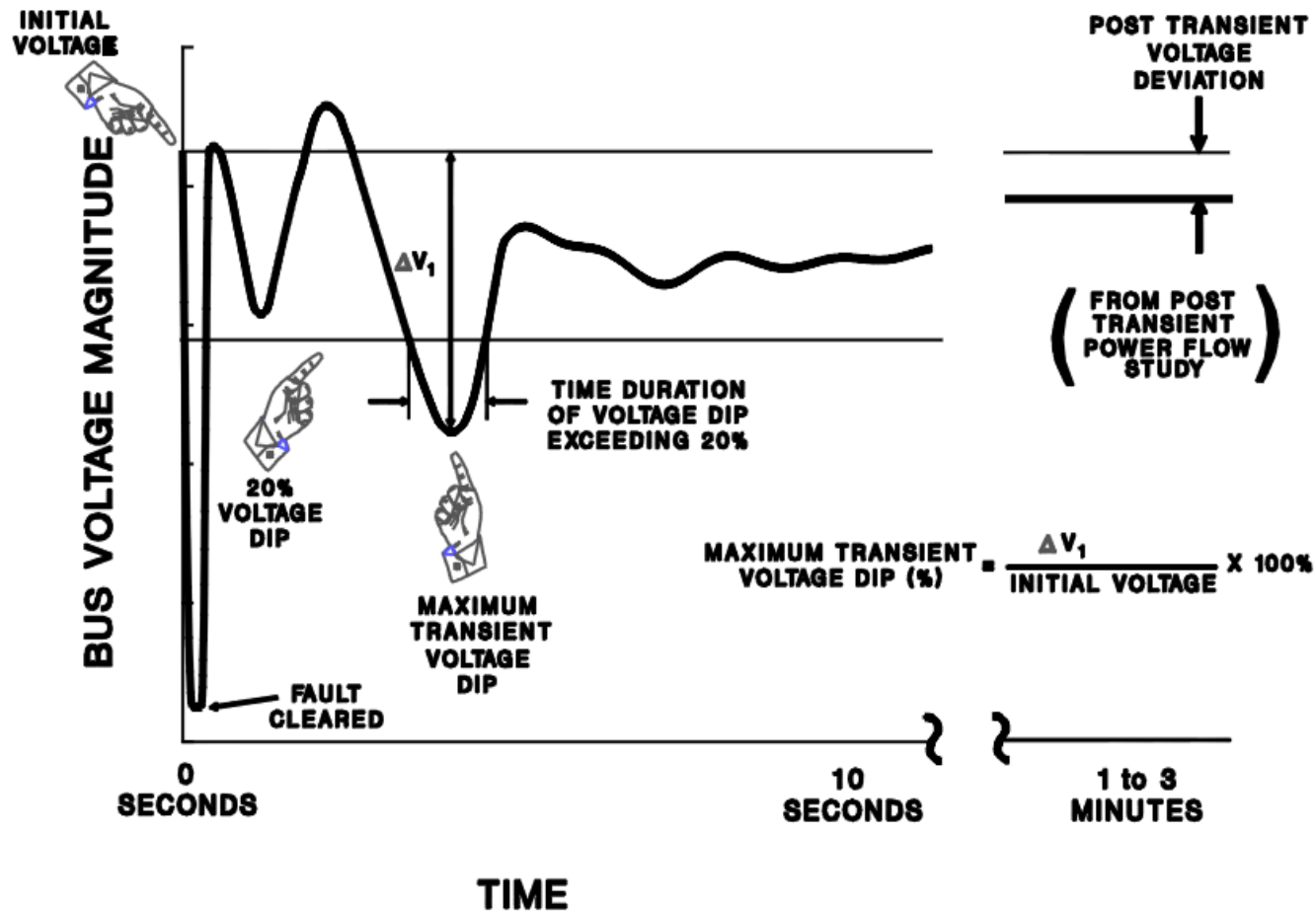
The models are no longer giving realistic results; two generators trip on over speed; then the load trips after 4 seconds.

Transient Limit Monitors



- There are different performance criteria that need to be met for a scenario

VOLTAGE PERFORMANCE PARAMETERS



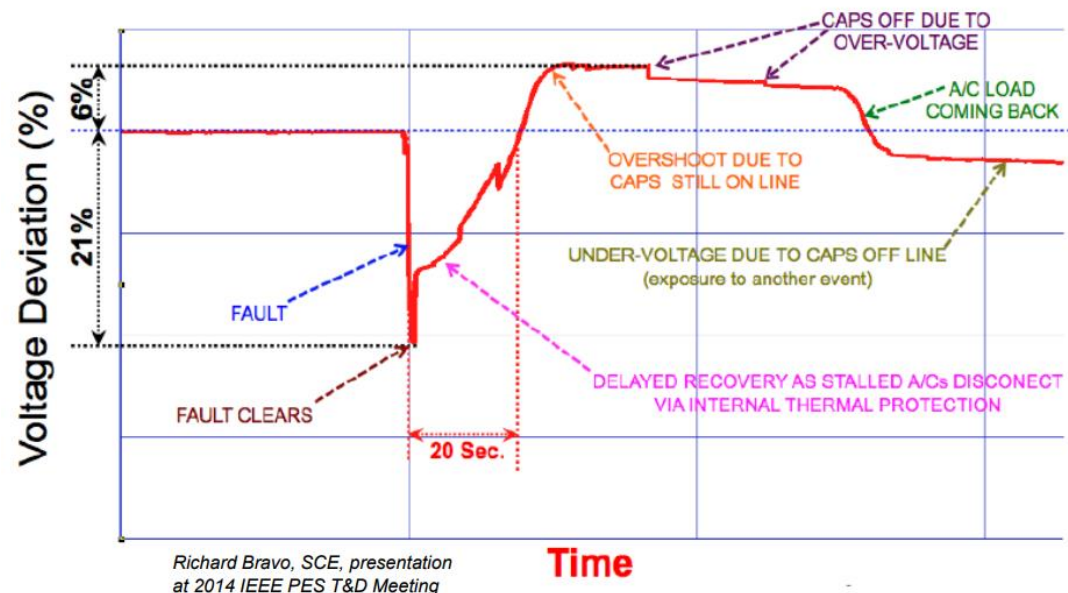
Similar performance criteria exist for frequency deviations

Image from WECC Planning and Operating Criteria

A Concern: Fault Induced Delayed Voltage Recovery (FIDVR)



- FIDVR is a situation in which the system voltage remains significantly reduced for at least several seconds following a fault (at either the transmission or distribution level)
 - It is most concerning in the high voltage grid, but found to be unexpectedly prevalent in the distribution system
- Stalled residential air conditioning units are a key cause of FIDVR – they can stall within the three cycles needed to clear a fault



Motor Starting



- Motor starting analysis looks at the impacts of starting a motor or a series of motors (usually quite large motors) on the power grid
 - Examples are new load or black start plans
- While not all transient stability motor load models allow the motor to start, some do
- When energized, the initial condition for the motor is slip of 1.0
- Motor starting can generate very small time constants

Motor Starting Example

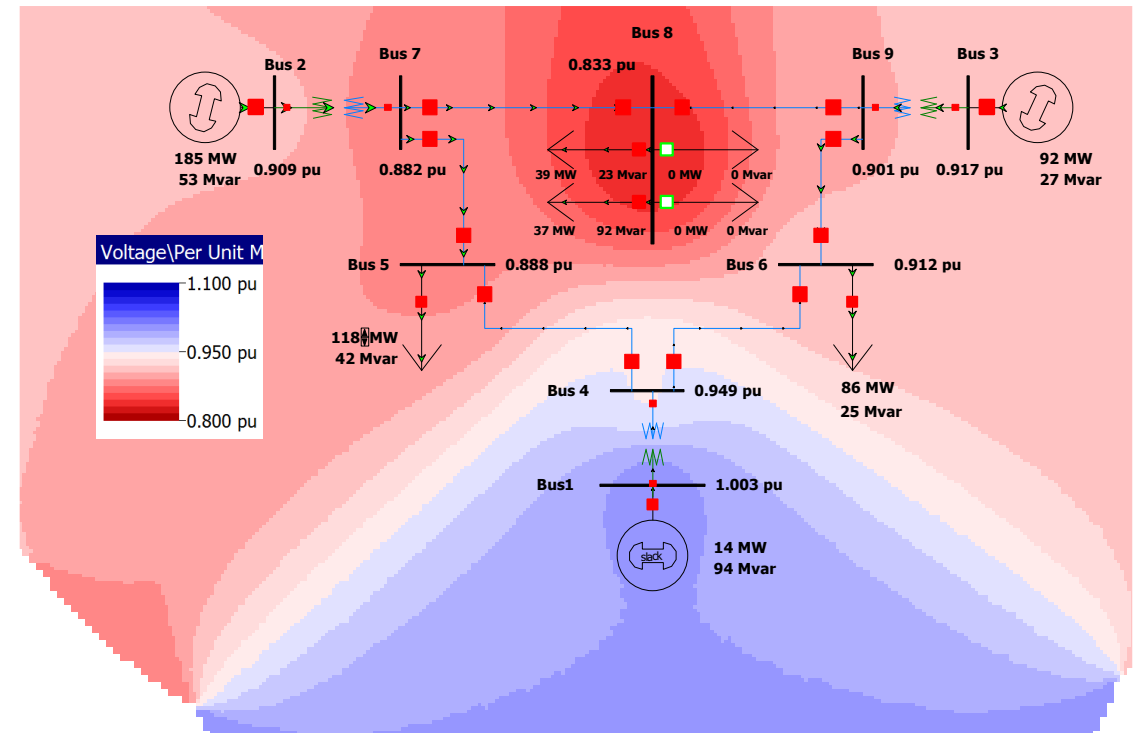
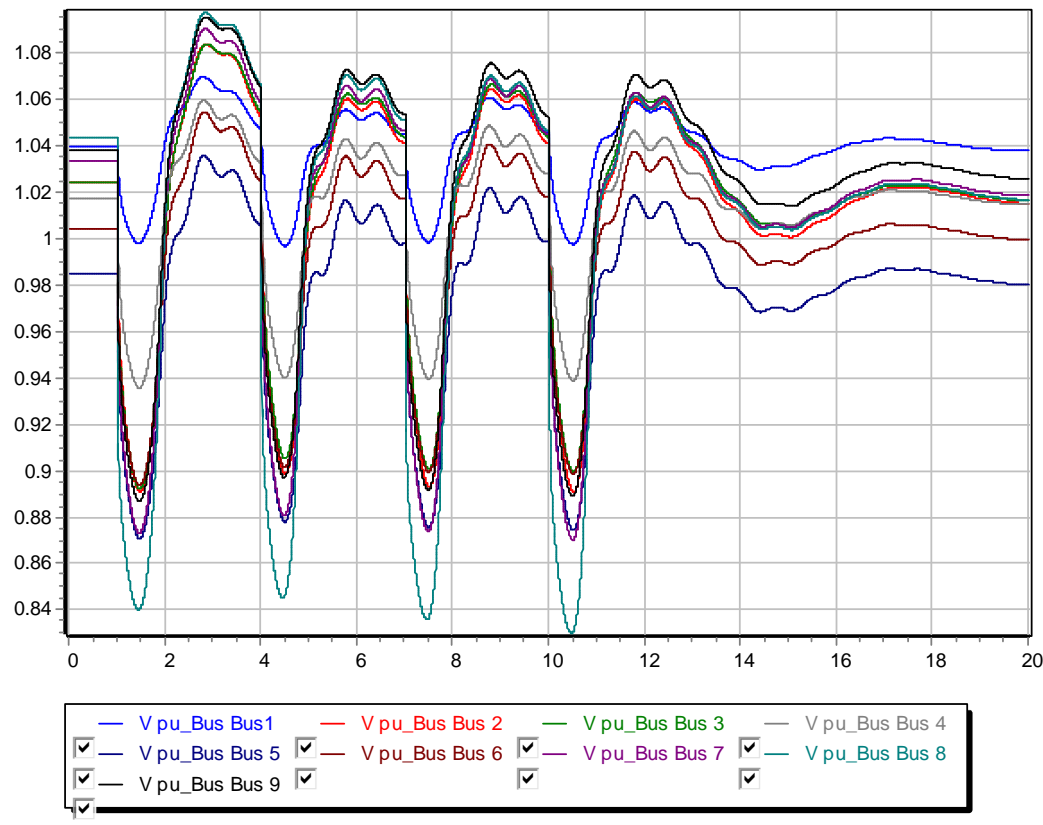


- Case WSCC_MotorStarting takes the previous WSCC case with 100% motor load, and considers starting the motor at bus 8
- In the power flow the load at bus 8 is modeled as zero (open) with a CIM5
- The contingency is closing the load
 - Divided into four loads to stagger the start (we can't start it all at once)
- Since power flow load is zero, the CIM5 load must also specify the size of the motor
 - This is done in the Tnom field and by setting an MVA base value

Motor Starting Example



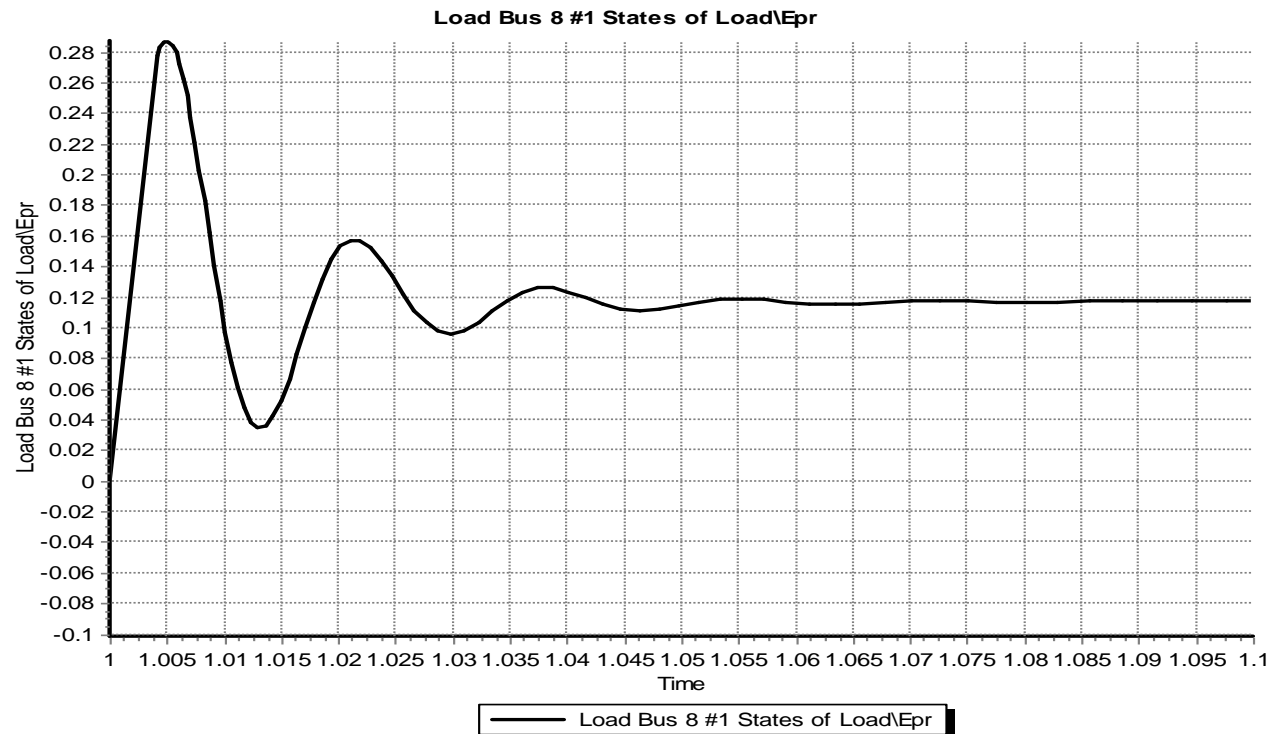
- Below graph shows the bus voltages for starting the four motors three seconds apart



Motor Starting: Fast Dynamics



- One issue with the starting of induction motors is the need to model relatively fast initial electrical dynamics
 - Below graph shows E'r for a motor at bus 8 as it is starting



Time scale is from 1.0 to 1.1 seconds

Motor Starting: Fast Dynamics



- These fast dynamics can be seen to vary with slip in the $\omega_s s$ term

$$V_D = E'_D + R_s I_D - X' I_Q$$

$$V_Q = E'_Q + R_s I_Q + X' I_D$$

$$\frac{dE'_D}{dt} = \omega_s s E'_Q - \frac{1}{T'_o} \left(E'_D + (X - X') I_Q \right)$$

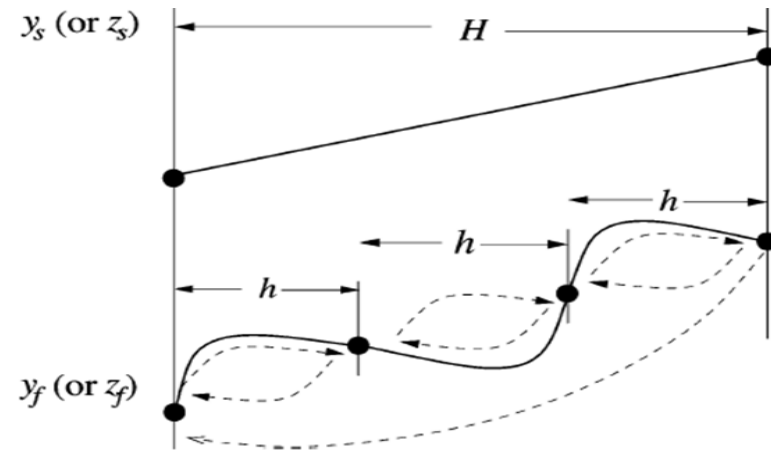
$$\frac{dE'_Q}{dt} = -\omega_s s E'_D - \frac{1}{T'_o} \left(E'_Q - (X - X') I_D \right)$$

- Simulating with the explicit method either requires a small overall Δt or the use of multi-rate methods

Multi-Rate Explicit Integration



- Key idea is to integrate some differential equations with a potentially much faster time step than others



- Faster variables are integrated with time step h , slower variable with time step H
 - Slower variables assumed fixed or interpolated during the faster time step integration

Multi-Rate Explicit Integration



- First proposed by C. Gear in 1974
- Power systems use first presented by M Crow in 1994
- In power systems usually applied to some exciters, stabilizers, and to induction motors when their slip is high
- Subinterval length can be customized for each model based on its parameters (in range of 4 to 128 times the regular time step)
- Tradeoff in computation

C. Gear, Multirate Methods for Ordinary Differential Equations, Univ. Illinois at Urbana-Champaign, Tech. Rep., 1974.

M. Crow and J. G. Chen, “The multirate method for simulation of power system dynamics,” *IEEE Trans. Power Syst.*, vol. 9, no. 3, pp.1684–1690, Aug. 1994.