ECEN 667 Power System Stability

Lecture 21: Oscillations, Measurement-Based Modal Analysis

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Announcements

- Read Chapter 8
- Homework 6 is due on Tuesday Nov 21



Small Signal Stability Analysis

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- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
 - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) obviously must be stable
- Small system stability analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
 - There must be positive damping

Model Based SSA

- The system can be linearized about an equilibrium point

 $\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \mathbf{y}$ $\mathbf{0} = \mathbf{C} \Delta \mathbf{x} + \mathbf{D} \Delta \mathbf{y}$

• Eliminating Δy gives

If there are just classical generator models then **D** is the power flow Jacobian; otherwise it also includes the stator algebraic equations

 $\Delta \dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\Delta \mathbf{x} = \mathbf{A}_{sys}\Delta \mathbf{x}$

Modal Analysis - Comments

- The matrix A_{sys} can be calculated doing a partial factorization, just like what is done with Kron reduction in creating power system equivalents
- SSA is done by looking at the eigenvalues (and other properties) of A_{sys}
- Modal analysis (analysis of small signal stability through eigenvalue analysis) is at the core of SSA software
- In Modal Analysis one looks at:
 - Eigenvalues, Eigenvectors (left or right)
 - Participation factors
 - Mode shape
- Power System Stabilizer (PSS) design in a multi-machine context can be done using the modal analysis method.

Goal is to determine how the various parameters affect the response of the system

Eigenvalues, Right Eigenvectors

• For an n by n matrix A the eigenvalues of A are the roots of the characteristic equation:

 $\det[\mathbf{A} - \lambda \mathbf{I}] = |\mathbf{A} - \lambda \mathbf{I}| = 0$

- Assume $\lambda_1 \dots \lambda_n$ as distinct (no repeated eigenvalues).
- For each eigenvalue λ_i there exists an eigenvector such that:

 $\mathbf{A}\mathbf{v}_i = \lambda_i \mathbf{v}_i$

- \mathbf{v}_i is called a right eigenvector
- If λ_i is complex, then \mathbf{v}_i has complex entries



Left Eigenvectors, Eigenvector Properties

- For each eigenvalue λ_i there exists a left eigenvector \mathbf{w}_i such that: $\mathbf{w}_i^t \mathbf{A} = \mathbf{w}_i^t \lambda_i$
- Equivalently, the left eigenvector is the right eigenvector of \mathbf{A}^{T} ; that is, $\mathbf{A}^{t}\mathbf{w}_{i} = \lambda_{i}\mathbf{w}_{i}$
- The right and left eigenvectors are orthogonal i.e. $\mathbf{w}_{i}^{t}\mathbf{v}_{i} \neq 0$, $\mathbf{w}_{i}^{t}\mathbf{v}_{j} = 0$ $(i \neq j)$
- We can normalize the eigenvectors so that:

$$\mathbf{w}_i^t \mathbf{v}_i = 1$$
, $\mathbf{w}_i^t \mathbf{v}_j = 0$ $(i \neq j)$



Eigenvector Example

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \ \left| \mathbf{A} - \lambda \mathbf{I} \right| = 0 \implies \begin{vmatrix} 1 - \lambda & 4 \\ 3 & 2 - \lambda \end{vmatrix} = 0$$
$$\lambda^2 - 3\lambda - 10 = 0 \implies \lambda_{1,2} = \frac{3 \pm \sqrt{(3)^2 + 4(10)}}{2} = \frac{3 \pm \sqrt{49}}{2} = 5, -2$$

Right Eigenvectors $\lambda_1 = 5$

$$\mathbf{A}\mathbf{v}_{1} = 5\mathbf{v}_{1} \Longrightarrow \mathbf{v}_{1} = \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} \Longrightarrow \frac{v_{11} + 4v_{21} = 5v_{11}}{3v_{11} + 2v_{21} = 5v_{21}}$$

Similarly,

$$\lambda_2 = -2 \implies \mathbf{v}_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$$

choose
$$\mathbf{v}_{2l} = l \Rightarrow \mathbf{v}_{1l} = l$$

 $\mathbf{v}_{l} = \begin{bmatrix} l \\ l \end{bmatrix}$



Eigenvector Example

• Left eigenvectors

$$\lambda_{1} = 5 \mathbf{w}_{1}^{t} \mathbf{A} = \mathbf{w}_{1}^{t} 5 \Rightarrow [w_{11} \ w_{21}] \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} = 5[w_{11} \ w_{21}]$$

$$w_{11} + 3w_{21} = 5w_{11}$$

$$4w_{11} + 2w_{21} = 5w_{21} \Rightarrow Let \ w_{21} = 4, then \ w_{11} = 3$$

$$\mathbf{w}_{1} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \qquad \lambda_{2} = -2 \Rightarrow \mathbf{w}_{2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\mathbf{v}_{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \mathbf{v}_{2} = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \mathbf{w}_{1} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \mathbf{w}_{2} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Verify \ \mathbf{w}_{1}^{t}\mathbf{v}_{1} = 7, \ \mathbf{w}_{2}^{t}\mathbf{v}_{2} = 7, \ \mathbf{w}_{2}^{t}\mathbf{v}_{1} = 0, \ \mathbf{w}_{1}^{t}\mathbf{v}_{2} = 0$$
We would like to make $w_{i}^{t}v_{i} = 1$.
This can be done in many ways.



Eigenvector Example

Let
$$\mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

Then $\mathbf{W}^T \mathbf{V} = \mathbf{I}$

 $Verify \quad \frac{1}{7} \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- It can be verified that $\mathbf{W}^{\mathrm{T}}=\mathbf{V}^{-1}$
- The left and right eigenvectors are used in computing the participation factor matrix.



- The deviation away from an equilibrium point can be defined as $\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x}$
- From this equation it is difficult to determine how parameters in A affect a particular x because of the variable coupling
- To decouple the problem first define the matrices of the right and left eigenvectors (the modal matrices)

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n] \& \mathbf{W} = [\mathbf{w}_1, \mathbf{w}_2, \dots, \mathbf{w}_n]$$
$$\mathbf{AV} = \mathbf{V}\mathbf{\Lambda} \quad \text{when } \mathbf{\Lambda} = Diag(\lambda_i)$$

V represents the right eigenvectors

• It follows that

 $\mathbf{V}^{-1}\mathbf{A}\mathbf{V}=\boldsymbol{\Lambda}$

- To decouple the variables define z so $\Delta x = Vz \rightarrow \Delta \dot{x} = V\dot{z} = A\Delta x = AVz$
- Then

 $\dot{\mathbf{z}} = \mathbf{V}^{-1}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{W}\mathbf{A}\mathbf{V}\mathbf{z} = \mathbf{\Lambda}\mathbf{z}$

- Since Λ is diagonal, the equations are now uncoupled with $\dot{z}_i = \lambda_i z_i$
- So $\Delta \mathbf{x}(t) = \mathbf{V}\mathbf{z}(t)$







• Thus the response can be written in terms of the individual eigenvalues and right eigenvectors as

$$\Delta \mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_{i} z_{i}(0) e^{\lambda_{i} t}$$

• Furthermore with

 $\Delta \mathbf{x} = \mathbf{V} \mathbf{Z} \implies \mathbf{z} = \mathbf{V}^{-1} \mathbf{x} = \mathbf{W}^{T} \mathbf{x}$

that the eigenvalues be distinct!

Note, we are requiring

• So z(t) can be written as using the left eigenvectors as

$$\mathbf{z}(t) = \mathbf{W}^{t} \mathbf{x}(t) = [\mathbf{w}_{1} \ \mathbf{w}_{2} \dots \mathbf{w}_{n}]^{t} \begin{bmatrix} x_{1}(t) \\ \vdots \\ x_{n}(t) \end{bmatrix}$$



- We can then write the response x(t) in terms of the modes of the system
 - $z_{i}(t) = w_{i}^{t} x(t)$ $z_{i}(0) = w_{i}^{t} x(0) \underline{\Delta} c_{i}$ so $\mathbf{x}(t) = \sum_{i=1}^{n} \mathbf{v}_{i} c_{i} e^{\lambda_{i} t}$ Expanding $\Delta x_{i}(t) = v_{i1} c_{1} e^{\lambda_{1} t} + v_{i2} c_{2} e^{\lambda_{2} t} + \dots v_{in} c_{n} e^{\lambda_{n} t}$
- So c_i is a scalar that represents the magnitude of excitation of the ith mode from the initial conditions

Numerical example

$$\begin{bmatrix} \Delta \dot{x}_{1} \\ \Delta \dot{x}_{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 8 & -2 \end{bmatrix} \begin{bmatrix} \Delta x_{1} \\ \Delta x_{2} \end{bmatrix}, \Delta \mathbf{x}(0) = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$$

Eigenvalues are $\lambda_{1} = -4, \lambda_{2} = 2$
Eigenvectors are $\mathbf{v}_{1} = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \mathbf{v}_{2} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
Modal matrix $\mathbf{V} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$
Normalize so $\mathbf{V} = \begin{bmatrix} 0.2425 & 0.4472 \\ -0.9701 & 0.8944 \end{bmatrix}$

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Left eigenvector matrix is: $\mathbf{W}^{\mathrm{T}} = \mathbf{V}^{-1} = \begin{bmatrix} 1.3745 & -0.6872 \\ 1.4908 & 0.3727 \end{bmatrix}$ $\dot{\mathbf{z}} = \mathbf{W}^{\mathrm{T}} \mathbf{A} \mathbf{V} \mathbf{z}$ $\begin{bmatrix} \dot{z}_{1} \\ \dot{z}_{2} \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} z_{1} \\ z_{2} \end{bmatrix}$

Numerical example (contd)

$$\dot{z}_{1} = -4z_{1} , \mathbf{z}(0) = V^{-1}\mathbf{x}(0)$$

$$\dot{z}_{2} = 2z_{2} , \begin{bmatrix} z_{1}(0) \\ z_{2}(0) \end{bmatrix} = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

$$z_{1}(t) = z_{1}(0)e^{-4t} ; z_{2}(t) = z_{2}(0)e^{2t}, \mathbf{C} = \mathbf{W}^{T}\mathbf{x}(0) = \begin{bmatrix} 4.123 \\ 0 \end{bmatrix}$$

 $\mathbf{x} = \mathbf{V}\mathbf{z}$ $\begin{bmatrix} x_{1}(t) \\ x_{2}(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} z_{1}(t) \\ z_{2}(t) \end{bmatrix}$ Because of the initial condition, the 2nd mode does not get excited $= c_{1} \begin{bmatrix} 0.2425 \\ -0.9701 \end{bmatrix} z_{1}(t) + c_{2} \begin{bmatrix} 0.4472 \\ 0.8944 \end{bmatrix} z_{2}(t) = \sum_{i=1}^{2} c_{i}\mathbf{v}_{i}z_{i}(0)e^{\lambda_{i}t}$



Mode Shape, Sensitivity and Participation Factors

• So we have

 $\mathbf{x}(t) = \mathbf{V}\mathbf{z}(t), \quad \mathbf{z}(t) = \mathbf{W}^{t}\mathbf{x}(t)$

- **x**(t) are the original state variables, **z**(t) are the transformed variables so that each variable is associated with only one mode.
- From the first equation the right eigenvector gives the "mode shape" i.e. relative activity of state variables when a particular mode is excited.
- For example the degree of activity of the state variable x_k in the v_i mode is given by the element V_{ki} of the right eigenvector matrix V

Mode Shape, Sensitivity and Participation Factors

- The magnitude of elements of v_i give the extent of activities of *n* state variables in the ith mode and angles of elements (if complex) give phase displacements of the state variables with regard to the mode.
- The left eigenvector \mathbf{w}_i identifies which combination of original state variables display only the ith mode.

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Eigenvalue Parameter Sensitivity

• To derive the sensitivity of the eigenvalues to the parameters recall $Av_i = \lambda_i v_i$; take the partial derivative with respect to A_{ki} by using the chain rule

$$\frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}} \mathbf{v}_i + \mathbf{A} \frac{\partial \mathbf{v}_i}{\partial A_{kj}} = \frac{\partial \lambda_i}{\partial A_{kj}} \mathbf{v}_i + \lambda_i \frac{\partial \mathbf{v}_i}{\partial A_{kj}}$$

Multiply by \mathbf{w}_i^t

$$\mathbf{w}_{i}^{t} \frac{\partial \mathbf{A}}{\partial \mathbf{A}_{kj}} \mathbf{v}_{i} + \mathbf{w}_{i}^{t} \mathbf{A} \frac{\partial \mathbf{v}_{i}}{\partial A_{kj}} = \mathbf{w}_{i}^{t} \frac{\partial \lambda_{i}}{\partial A_{kj}} \mathbf{v}_{i} + \mathbf{w}_{i}^{t} \lambda_{i} \frac{\partial \mathbf{v}_{i}}{\partial A_{kj}}$$
$$\mathbf{w}_{i}^{t} \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_{i} + \mathbf{w}_{i}^{t} [\mathbf{A} - \lambda_{i} \mathbf{I}] \frac{\partial \mathbf{v}_{i}}{\partial A_{kj}} = \mathbf{w}_{i}^{t} \frac{\partial \lambda_{i}}{\partial A_{kj}} \mathbf{v}_{i}$$

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Eigenvalue Parameter Sensitivity

- This is simplified by noting that $\mathbf{w}_i^t (\mathbf{A} \lambda_i \mathbf{I}) = 0$ by the definition of \mathbf{w}_i being a left eigenvector
- Therefore

$$\mathbf{w}_{i}^{t} \frac{\partial \mathbf{A}}{\partial A_{kj}} \mathbf{v}_{i} = \frac{\partial \lambda_{i}}{\partial A_{kj}}$$

- Since all elements of $\frac{\partial \mathbf{A}}{\partial A_{kj}}$ are zero, except the kth row, jth column is 1
- Thus

$$\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki} V_{ji}$$

Sensitivity Example

• In the previous example we had

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \lambda_{1,2} = 5, -2, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix}$$

• Then the sensitivity of λ_1 and λ_2 to changes in A are

$$\frac{\partial \lambda_i}{\partial A_{kj}} = W_{ki}V_{ji} \longrightarrow \frac{\partial \lambda_1}{\partial \mathbf{A}} = \frac{1}{7} \begin{bmatrix} 3 & 3\\ 4 & 4 \end{bmatrix}, \quad \frac{\partial \lambda_2}{\partial \mathbf{A}} = \frac{1}{7} \begin{bmatrix} 4 & -3\\ -4 & 3 \end{bmatrix}$$

• For example with $\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 2.1 \end{bmatrix}$, $\hat{\lambda}_{1,2} = 5.057, -1.957$

• Or if
$$\hat{\mathbf{A}} = \begin{bmatrix} 1 & 4 \\ 3 & 3 \end{bmatrix}$$
, $\hat{\lambda}_{1,2} = 5.61, -1.61$,



Participation Factors

• The participation factors, P_{ki}, are used to determine how much the kth state variable participates in the ith mode

 $P_{ki} = V_{ki} W_{ki}$

- The sum of the participation factors for any mode or any variable sum to 1
- The participation factors are quite useful in relating the eigenvalues to portions of a model
- For the previous example with $P_{ki} = V_{ki}W_{ik}$ and

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}, \quad \mathbf{W} = \frac{1}{7} \begin{bmatrix} 3 & 1 \\ 4 & -1 \end{bmatrix} \longrightarrow \mathbf{P} = \frac{1}{7} \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}$$

• Consider the two bus, two classical generator system from lectures 18 and 20 with X_{d1} '=0.3, H_1 =3.0, X_{d2} '=0.2, H_2 =6.0



• Essentially everything needed to calculate the **A**, **B**, **C** and **D** matrices was covered in lecture 15

• The A matrix is calculated differentiating f(x,y) with respect to x (where x is δ_1 , $\Delta \omega_1$, δ_2 , $\Delta \omega_2$)

$$\begin{aligned} \frac{d\delta_{I}}{dt} &= \Delta \omega_{I.pu} \omega_{s} \\ \frac{d\Delta \omega_{I.pu}}{dt} &= \frac{1}{2H_{I}} \left(P_{MI} - P_{EI} - D_{I} \Delta \omega_{I.pu} \right) \\ \frac{d\delta_{2}}{dt} &= \Delta \omega_{2.pu} \omega_{s} \\ \frac{d\Delta \omega_{2.pu}}{dt} &= \frac{1}{2H_{2}} \left(P_{M2} - P_{E2} - D_{2} \Delta \omega_{I.pu} \right) \\ P_{Ei} &= \left(E_{Di}^{2} - E_{Di} V_{Di} \right) G_{i} + \left(E_{Qi}^{2} - E_{Qi} V_{Qi} \right) G_{i} + \left(E_{Di} V_{Qi} - E_{Qi} V_{Di} \right) B_{i} \\ E_{Di} + jE_{Qi} &= E_{i}' \left(\cos \delta_{i} + j \sin \delta_{i} \right) \end{aligned}$$



- Giving $\mathbf{A} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.761 & 0 & 0 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0 & 0 & -0.389 & 0 \end{bmatrix}$
- **B**, **C** and **D** are as calculated previously for the implicit integration, except the elements in B are not multiplied by $\Delta t/2$

	0	0	0	0]	
D _	-0.2889	0.6505	0	0	
D =	0	0	0	0	
	0	0	0.0833	0.3893	



• The C and D matrices are

$$\mathbf{C} = \begin{bmatrix} -3.903 & 0 & 0 & 0 \\ -1.733 & 0 & 0 & 0 \\ 0 & 0 & -4.671 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 7.88 & 0 & -4.54 \\ -7.88 & 0 & 4.54 & 0 \\ 0 & -4.54 & 0 & 9.54 \\ 4.54 & 0 & -9.54 & 0 \end{bmatrix}$$

• Giving

$$\mathbf{A}_{sys} = \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} = \begin{bmatrix} 0 & 376.99 & 0 & 0 \\ -0.229 & 0 & 0.229 & 0 \\ 0 & 0 & 0 & 376.99 \\ 0.114 & 0 & -0.114 & 0 \end{bmatrix}$$



SSA Two Generator

- Calculating the eigenvalues gives a complex pair and two zero eigenvalues
- The complex pair, with values of +/- j11.39 corresponds to the generators oscillating against each other at 1.81 Hz
- One of the zero eigenvalues corresponds to the lack of an angle reference
 - Could be rectified by redefining angles to be with respect to a reference angle (see book 226) or we just live with the zero
- Other zero is associated with lack of speed dependence in the generator torques

SSA Two Generator Speeds

• The two generator system response is shown below for a small disturbance

60.5 60.45 60.4 60.35 60.3 60.25 60.2 60.15 60.1 60.05 60 59.95 59.9 59.85 59.8 59.75 59.7 59.65 59.6 59.55 59.5 0.5 1.5 2 2.5 3 3.5 4.5 0 4 5 - Speed, Gen Bus 1 #1 🔽 - Speed, Gen Bus 2 #1 \checkmark

Notice the actual response closely matches the calculated frequency

SSA Three Generator Example

• The two generator system is extended to three generators with the third generator having H_3 of 8 and X_{d3} '=0.3



SSA Three Generator Example

• Using SSA, two frequencies are identified: one at 2.02 Hz and one at 1.51 Hz The oscillation is star



The oscillation is started with a short, self-clearing fault

Shortly we'll discuss modal analysis to determine the contribution of each mode to each signal

PowerWorld case
B2_CLS_3Gen_SSA

Large System Studies

- The challenge with large systems, which could have more than 100,000 states, is the shear size
 - Most eigenvalues are associated with the local plants
 - Computing all the eigenvalues is computationally challenging, order n^3
- Specialized approaches can be used to calculate particular eigenvalues of large matrices
 - See Kundur, Section 12.8 and associated references

Single Machine Infinite Bus

- A quite useful analysis technique is to consider the small signal stability associated with a single generator connected to the rest of the system through an equivalent transmission line
- Driving point impedance looking into the system is used to calculate the equivalent line's impedance
 - The Z_{ii} value can be calculated quite quickly using sparse vector methods
- Rest of the system is assumed to be an infinite bus with its voltage set to match the generator's real and reactive power injection and voltage

Small SMIB Example

• As a small example, consider the 4 bus system shown below, in which bus 2 really is an infinite bus



 Z_{44} is Z_{th} in parallel with jX'_{d,4} (which is j0.3) so Z_{th} is j0.22

• To get the SMIB for bus 4, first calculate Z_{44}

$$Y_{bus} = j \begin{bmatrix} -25 & 0 & 10 & 10 \\ 0 & 1 & 0 & 0 \\ 10 & 0 & -15 & 0 \\ 10 & 0 & 0 & -13.33 \end{bmatrix} \rightarrow Z_{44} = j0.1269$$

Small SMIB Example

• The infinite bus voltage is then calculated so as to match the bus i terminal voltage and current

$$\overline{V}_{inf} = \overline{V}_i - Z_i \overline{I}_i$$

where $\left(\frac{P_i + jQ_i}{\overline{V}_i}\right)^* = \overline{I}_i$

While this was demonstrated on an extremely small system for clarity, the approach works the same for any size system

• In the example we have

$$\begin{split} \left(\frac{P_4 + jQ_4}{\overline{V}_4}\right)^* &= \left(\frac{1 + j0.572}{1.072 + j0.220}\right)^* = 1 - j0.328\\ \overline{V}_{\text{inf}} &= \left(1.072 + j0.220\right) - (j0.22)\left(1 - j0.328\right)\\ \overline{V}_{\text{inf}} &= 1.0 \end{split}$$

Calculating the A Matrix

- The SMIB model **A** matrix can then be calculated either analytically or numerically
 - The equivalent line's impedance can be embedded in the generator model so the infinite bus looks like the "terminal"
- This matrix is calculated in PowerWorld by selecting Transient Stability, SMIB Eigenvalues
 - Select Run SMIB to perform an SMIB analysis for all the generators in a case
 - Right click on a generator on the SMIB form and select Show SMIB to see the Generator SMIB Eigenvalue Dialog
 - These two bus equivalent networks can also be saved, which can be quite useful for understanding the behavior of individual generators



Example: Bus 4 SMIB Dialog

• On the SMIB dialog, the General Information tab shows information about the two bus equivalent

Bus Number	4			Find By Number	Status			
Bus Name	Bus 4		•	Find By Name	Open	Oclosed		
ID	1			Find	Area Name Ho	me (1)		
Generator Inf	formation (on	Generator	MVA Base)					
General Info	A Matrix	Eigenvalues]					
Generator N	IVA Base	100.000						
Infinite Bus	Voltage Magn	nitude (pu)	1.0000	Infinite Bus An	gle (deg)	-0.0000		
Terminal Cu	rrent Magnitu	ide (pu)	1.0526	Terminal Curre	nt Angle (deg)			
Terminal Vol	ltage Magnitu	ide (pu)	1.0946	Terminal Volta	ge Angle (deg)	11.5942	1	
Network I	mpedance on	Generator	MVA Base	Network Imped	ance on System M	1VA Base		
Network F	R (Gen Base)	0.0000	00	Network R (Sy	stem Base)	0.00000		
Network)	K (Gen Base)	0.220	00	Network X (Sys	stem Base)	0.22000		

PowerWorld case **B4_SMIB**

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Example: Bus 4 SMIB Dialog

• On the SMIB dialog, the A Matrix tab shows the A_{sys} matrix for the SMIB generator

Bus Number	4	3	• 😤 (F	ind By Number	Status						
Bus Name	Bus 4			Find By Name	Open	0) Closed				
ID	1			Find	Area Name	Home (1)	1				
General Info	A Matrix	n Generator M Eigenvalues	IVA Base)								
General Info	A Matrix	Eigenvalues	IVA Base) Records • Se	et • Columns •		- AUXD -	🕈 ∰+	SORT 124 ABED	f(x) -	=	
General Info	A Matrix A Matrix	n Generator M Eigenvalues	IVA Base) Records × Se Machine Angle	et • Columns • Machine Speed		. × AUXB +	❣ 閧+	SORT 124 ABED	f(x) 🔻	⊞	
General Info	formation (or A Matrix k tot ÷00 Row Na hine Angle	n Generator M Eigenvalues	Records - Se Machine Angle 0.0000	t Columns Machine Speed W 376.9911	B * ∰		❣ 曲•	SORT 12N ABED	f(x) ~	=	

• In this example A_{21} is showing

$$\frac{\partial \Delta \omega_{4,pu}}{\partial \delta_4} = \frac{1}{2H_4} \left(\frac{-\partial P_{E,4}}{\partial \delta_4} \right) = -\left(\frac{1}{6} \right) \left(\left(\frac{-1}{0.3 + 0.22} \right) \left(-1.2812 \cos\left(23.94^\circ\right) \right) \right)$$
$$= -0.3753$$

Example: Bus 4 with GENROU

- The eigenvalues can be calculated for any set of generator models
- This example replaces the bus 4 generator classical machine with a GENROU model
 - There are now six eigenvalues, with the dominate response coming from the electro-mechanical mode with a frequency of 1.84 Hz, and damping of 6.9%

Generator	Information (on	Generator MVA Bas	e)				
General In	fo A Matrix B	Eigenvalues					
: 🖾 🖸	86. 세 🏥 [.00 +.0 # ABCD R	ecords 👻 Set 👻	Columns 🔻 📴 🔻		$\underset{\substack{\blacksquare\\ ABED}}{} \bullet \underset{\substack{12}\\ ABED}{\overset{SORT}{}} f(x) \bullet$	Options *
	Real Part	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)
1	-21.2472	0.0000	21.2472	1.0000	0.0000	-Def March	3,3816
2	-0.8040	11.5563	11.5842	0.0694	1.8392	0.5437	1.8437
3	-0.8040	-11.5563	11.5842	0.0694	-1.8392	-0.5437	1,8437
4	-14.2256	0.0000	14.2256	1.0000	0.0000		2.2641
5	-3.7087	0.0000	3.7087	1.0000	0.0000		0.5903
6	-0.4248	0.0000	0.4248	1.0000	0.0000		0.0676

PowerWorld case B4_GENROU_Sat_SMIB



Example: Bus 4 with GENROU Model and Exciter

- Adding an relatively slow EXST1 exciter adds additional states (with $K_A=200, T_A=0.2$)
 - As the initial reactive power output of the generator is decreased, the system becomes unstable (below example is with a generator reactive power output of 0 Mvar)

General	Info A Matrix	Eigenvalues					
: 🖂		.00 /A # Re	cords * Set *	Columns 👻 🔄 👻			⊞ Options ▼
	Real Part 🔻	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq Ma (Hz)
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1.5179
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796
5	-3.6849	-6.4281	7.4094	0.4973	-1.0231	-0.9775	1.1792
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533

PowerWorld case B4_GENROU_Sat_SMIB_QZero

Example: Bus 4 with GENROU Model and Exciter

• The below image shows the system response to a brief bus 4 selfclearing fault



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Example: Bus 4 with GENROU Model and Exciter

• The remainder of the Eigenvalues page shows the participation factors for the various states in the modes

💽 Gener	ator SMIB Eige	nvalue Informa	tion												- 0
Bus Numbe Bus Name	r 4 Bus 4	~	Find By N	umber Status	en 💿 Clos	sed									
II	D 1		Find .	Area Na	me Home (1)										
Generator	Information (on G	Generator MVA Bas	e)												
General In	nfo A Matrix E	genvalues		- FC		Fidu SORT	m								
		•.0 10 ABCD Re	ecords * Set *	Columns ▼ 增雪 ▼		₩ * 1250 f(x) *	Options *	1	1	(-			
	Real Part V	Imag Part	Magnitude	Damping Ratio	Damped Freq (Hz)	Damped Period (Sec)	Undamped Freq (Hz)	Machine Angle	Machine Speed w	Machine Eqp	Machine PsiDp	Machine PsiQpp	Machine Edp	Exciter EField before limit	Exciter VF
1	0.2704	-9.5336	9.5374	-0.0283	-1.5173	-0.6591	1,5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
2	0.2704	9.5336	9.5374	-0.0283	1.5173	0.6591	1.5179	0.6920	0.6810	0.1642	0.0250	0.0137	0.0139	0.1714	0.0000
3	-1.0000	0.0000	1.0000	1.0000	0.0000		0.1592	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000
4	-3.0137	0.0000	3.0137	1.0000	0.0000		0.4796	0.0071	0.0098	0.0573	0.0011	0.1263	0.9865	0.0865	0.0000
5	-3.6849	-6.4281	7,4094	0.4973	-1.0231	-0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
6	-3.6849	6.4281	7.4094	0.4973	1.0231	0.9775	1.1792	0.1643	0.1764	0.6494	0.0534	0.0350	0.0964	0.7120	0.0000
7	-14.4234	0.0000	14.4234	1.0000	0.0000		2.2956	0.0054	0.0049	0.0219	0.9995	0.0013	0.0028	0.0226	0.0000
8	-21.6978	0.0000	21.6978	1.0000	0.0000		3.4533	0.0030	0.0037	0.0009	0.0006	0.9971	0.0762	0.0011	0.0000
v	ж	ave	X Cancel	? Help	Print										



SMIB Eigenvalues for TSGC_2000 Case

• All the SMIB eigenvalues can be calculated quickly even for relatively large grids

un Transient Stability Pause	Abo	rt Restore Re	ference For Con	tingency: F	ind Tornad	0		~										
ect Step	SMIB E	genvalues																
Simulation				1. 100			Contraction of the local data											
Options	Run	SMIB Eigen Analys	is Re-Initializ	e Eigen	value Analysi	s Last Run:	11/8/2021 4:58:	07 PM										
Result Storage	: 171	- Rh ++ **	.02 44 44 3	H Records	* Set * C	olumns -	· · · · · · · · · · · · · · · · · · ·	😪 🖽 - 1057	f(v) + III Onti	nns 🕶								
Plots	* 1220		+.0 arm apop	11 1000103	Jec e		Car. Cha	B REAL ARED	ind III obru		Lance acted							
Result Analyzer - Damping		Number of Bus	Name of Bus	ID N	VA Base	Area Name of	Machine	Exciter	Governor	Stabilizer	Calculate	Number of	Number of Zero	Min Eigenvalue	Max Eigenvalue	Swing Equation	Swing Equation	Swing Equation
esults from RAM		1004	O DOMNELL 1 1		252.2.5	Gen	WITHO	IAT AF			VEC	Eigenvalues	Eigenvalues	0.2250	40.0323	rreq. (n2)	Damping 0.0000	D Equivalent
ransient Limit Monitors	2	1004	BIG SPRING 5.1		200.2 F	ar West	WT4G	WT4C			VES	9	0	-0.2250	49,9525	0.0000	0.0000	0.000
tates/Manual Control		1000	IDAAN 2 1		99.0 F	ar West	WTAG	WTAE			VES	9	0	-0.1042	49 9203	0.0000	0.0000	0.00
alidation	4	1011	PRESIDIO 1 1		12.0 E	ar West					YES	0	0	0.0000	0.0000	0.0000	0.0000	0.00
MIB Eigenvalues	5	1021	BIG SPRING 1 1		239.4 F	ar West	WT4G	WT4E			YES	9	0	-0.2693	-49,8987	0.0000	0.0000	0.000
Iodal Analysis	6	1023	O DONNELL 2 1		216.0 F	ar West	WT4G	WT4E			YES	9	0	-0.2793	-49.8853	0.0000	0.0000	0.00
ynamic Simulator Options	7	1026	BIG SPRINGS 1	1	149.0 F	ar West	WT4G	WT4E	1	3	YES	9	0	-0.2736	-49.8928	0.0000	0.0000	0.00
	8	1033	MCCAMEY 1 1		333.6 F	ar West	WT4G	WT4E		P	YES	9	0	-0.2110	-49.9407	0.0000	0.0000	0.00
	9	1035	BIG SPRING 4 1	1	108.0 F	ar West	WT4G	WT4E			YES	9	0	-0.2462	-49.9185	0.0000	0.0000	0.00
	10	1039	FORT STOCKTOI	1	177.0 F	ar West	WT4G	WT4E			YES	9	0	-0.2692	-49.8992	0.0000	0.0000	0.00
	- 11	1042			146.3 F	ar West	WT4G	WT4E										
	12	1043	FORSAN 2		70.6 F	ar West	WT4G	WT4E			YES	9	0	-0.2235	-49.9335	0.0000	0.0000	0.00
	13	1048				ar West	GENROU	ESST4B	GGOV1	IEEEST								
		1049	MONAHANS 1.2		390.2 F	ar West	GENROU	EXAC2	GGOV1	IEEEST	NO	G						
	15	1050	MONAHANS 1 3		107.3 F	ar West	GENROU	ESST48	GGOV1	IEEEST	YES	20	1	-0.0015	-77.1569	1.8102	0.0643	8.43
	16	1051	MONAHANS 14		107.3 F	ar West	GENROU	EXPICT	GGOVI	IEEEST	YES	21		-0.0818	-76.9490	0.9537	0.0733	13.68
		1052	MONAHANS 15		107.3 F	ar west	GENROU	ESSI4B	GGOVI	IEEESI	YES	20		-0.0830	-76.9891	1.1103	0.0620	15.83
	10	1055	LENORALIA .		144.0 5		MILAC	LOST 4D	66011	ICCCSI								
	26	1057			575	ar Wart	GENROLL	EVAC2	GGOVA	IFFECT								
	21	1050			2.4 F	ar West	GENROU	ESST4B	660V1	IFFEST								
	22	1052				ar West	Sentres	230110	00011	TEECOT								
	23	1063				ar West												
	24	1066	BIG SPRING 3 1		171.0 F	ar West	WT4G	WT4E			YES	9	0	-0.2728	-49.8939	0.0000	0.0000	0.00
	25	1070	IRAAN 1 1		192.6 F	ar West	WT4G	WT4E			YES	9	0	-0.2670	-49.8965	0.0000	0.0000	0.00
	26	1072	ODESSA 1 1		230.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1053	-64.1893	0.6171	0.6281	69.84
	27	1073	ODESSA 1 2		230.6 F	ar West	GENROU	ESST48	GGOV1	IEEEST	YES	20	1	-0.1350	-63.4621	8.5123	0.6088	799.24
	28	1074	ODESSA 1 3		230.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0812	-48.4635	0.5445	0.7105	66.84
	29	1075	ODESSA 1 4		230.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0479	-73.9551	0.5988	0.7503	83.56
	30	1076	ODESSA 1 5		230.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0315	-54.6018	13.1315	0.4923	942.29
	31	1077	ODESSA 1 6		230.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.1454	-56.7124	0.9590	0.5193	61.62
	32	1078	ODESSA 1 7		114.6 F	ar West	GENROU	ESST4B	GGOV1	IEEEST	YES	20	1	-0.0932	-53.9516	0.6988	0.6303	99.78
	33	1079	ODESSA 1 8		114.6 F	ar West	GENROU	EXAC2	GGOV1	IEEEST	YES	23	1	-0.0081	-72.9229	1.9024	0.3274	87.75
	34	1080	ODESSA 1 9		114.6 F	ar vvest	GENROU	E5514B	GGOVI	IEEESI	NO	0	0	0.2000	77.0007	0.2200	0.5051	015.00
		1081	OUESSA 1 10		343.8 F	ar west	GENROU	E33148	660/1	100001	TES	20	1	-0.2000	-//.260/	9.2208	0.5061	815.99
	27	1082	BIG SPRING 2.0		138 E E	ar west	WT4G	WINE			VES	9	0	-0.2455	49.9189	0.0000	0.0000	0.00
s Contingencies	20	1004	MCCAMEV 20		90.0 F	ar West	WT4G	WTAE			VES	9	0	-0.1092	-49,9505	0.0000	0.0000	0.00
e Contingency at a time	30	1000	COLDSMITHO		183 A F	ar West	MITAG	MATAE			VEC	9	0	0.2263	40 0700	0.0000	0.0000	0.000
Itiple Contingencies	<																	
																	1773-140	1 1 10000

Saving a Two Bus Equivalent

- PowerWorld makes it easy to save a two bus equivalent from the SMIB Eigenvalues page
 - Right-click and select Save Two Bus Equivalent
- As the name implies, the two bus equivalent is the generator connected to an infinite bus through its driving point impedance
- Two bus equivalents provide a convenient way to track down at least some causes of instability issues

Small Signal Analysis and Measurement-Based Modal Analysis

- Small signal analysis has been used for decades to determine power system frequency response
 - It is a model-based approach that considers the properties of a power system, linearized about an operating point
- Measurement-based modal analysis determines the observed dynamic properties of a system
 - Input can either be measurements from devices (such as PMUs) or dynamic simulation results
 - The same approach can be used regardless of the measurement source
- Focus in this section is on the measurement-based approach

Ring-down Modal Analysis

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- Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance
- There are several different techniques, with the Prony approach the oldest (from 1795); introduced into power in 1990 by Hauer, Demeure and Scharf
- Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

$$y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos\left(\omega_i t + \phi_i\right) \quad \text{Damping (\%)} = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100$$

Goal: Extracting Modes from the Signals

- The goal is to gain information about the electric grid by extracting modal information from its signals
 - The frequency and damping of the modes is key
- The premise is we'll be able to reproduce a complex signal, over a period of time, as a set a of sinusoidal modes
 - We'll also allow for linear detrending $0.1t + \cos(2\pi 2t)$

 $\begin{array}{c} 2.0 \\ 1.5 \\ 1.0 \\ 0.5 \\ -0.5 \\ -1.0 \end{array}$



Example: Summation of Two Damped Exponentials

- This example was created by going from the modes to a signal
- We'll be going in the opposite direction (i.e., from a measured signal to the modes)



Some Reasonable Expectations

- Verifiable to show how well the modes match the original signal(s)
 - We'll show this
- **Flexible** to handle between one and many signals – We'll go up to simultaneously considering 40,000 signals
- Fast
 - What is presented will be, with a discussion of the computational scaling
- Easy to use
 - This is software implementation specific; results shown here were done using the modal analysis tool integrated into PowerWorld Simulator (version 22)



Example: One Signal

This could be any signal; image shows the result of the original signal (blue) and the reproduced signal (red)





Verification: Linear Trend Line Only

			-		_				
art Time Use	ed End Time Use	2d Time Windov	N Gen Spee	ed 3 - 10					
3.00000	10.00000	0 Contingenc	y My Trans	sient Contir	ngency				
		Objec	t Gen 'Bus	1_16.50''	1'				
		Field	d TSSpeed	1					
istics Mod	des and Damping	Object Fields							
ndamped M	lodes	A (constant	t) B	(linear)	C (quadra	tic)			
98	0 Trend	1 1.00	0.00	000617	C	.0			
	00. 0.* %k ⊞	🖗 🚜 Recor	ds 🕶 Set 🕶	Column	s + Es +		₩ - SORT 124 ABED f(• III Options •	
des for Sele	ected Signal					Upda	e 🗹 Auto	1.0012	
Mode	e Magnitude de	Magnitude A End	ngle	Rank	Mode Frequency	Mode Damping %	Mode Lambda	1.001	_
Repr	oduce 0.00166	0.0000643	111.15	41.13	0 171	39.67	0.465		
2 NO	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543	1.0006	
3 NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427		
4 NO	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383		
5 NO	0.0000223)000000445	-59.64	0.553	2.017	6.99	-0.888	1.0002	-
								0.9998	-
								0.9996	
								0 9994	
								3 4 5 6 7 8 9	10
								Raw Signal Reproduced Signal	

49

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Verification: Linear Trend Line + One Mode

art time Used E	to oppose	time window	Gen Speed 3 - 1												
3.000000	10.000000	Contingency	My Transient Co	ntingency											
		Object	Gen 'Bus1_16.50	' '1'											
		Field	TSSpeed												
atistics Modes an	d Damping O	bject Fields													
Indamped Modes	-	A (constant)	B (linear)	C (quadra	atic)										
0	Trend	1.00	0.0000617		0.0										
□ □ □ □ **	*.0 :00 M	ARCO Record	s ▼ Set ▼ Colur	nns 🔻 📴 🔻		₩ - SORT f(() ▼ ⊞ Opt	ons 🕶							
odes for Selected S	Signal				Upda	te 🗹 Auto	1.0012 -			-	-			_	
Mode Include Reproduce	Magnitude M 0.00166 (0.00014 (0.00097 (0.0000167 (0.0000223)0	lagnitude An nd 0.0000643	gje Rank 111.15 41.1 0.00 28.2 69.05 24.0 68.00 6.0 59.64 0.55	Mode Frequency 3 0.171 8 0.000 1 1.364 2 0.000 3 2.017	Mode Damping % 39,67 100,00 4,98 -100,00 6,99	Mode Lambda -0.465 -0.543 -0.543 -0.427 0.383 -0.888	1.001 1.0008 1.0006 1.0004 1.0002 1 0.9998 0.9996 0.9994		\bigwedge	N		A			
							3	4		5 — Raw Signa	6 I 🔽 Repr	7 Dduced Signal	8	9	10

50

Verification: Linear Trend Line + Two Modes

Result Analysi	is Signal						- 0
tart Time Used	End Time Used	Time Window	Gen Speed 3 - 1	0			
3.000000	10.000000	Contingency	My Transient Co	ntingency			
		Object	Gen 'Bus1_16.5)' '1'			
		Field	TSSpeed				
tistics Modes a	and Damping C	bject Fields					
ndamped Modes		A (constant)	B (linear)	C (quadr	atic)		
~	ireno ای ۰۰۰ یا	1.00	0.00061/		80%6 80%6 -	SORT -	A THE Options -
odes for Selected	l° .oo ∔10 l® dSignal	B ABCD Record	s + Ser + Colu		Upda		
Mode Include	Magnitude M	lagnitude An nd	igle Rank	Mode Frequency	Mode Damping %	Mode Lambda	1.001
1 YES	0.00166	0.0000643	111.15 41.1	3 0.171	39.67	-0.465	
2 YES	0.00114	0.0000256	0.00 28.2	8 0.000	100.00	-0.543	1.0006
3 NO	0.00097	0.0000488	69.05 24.0	1 1.364	4.98	-0.427	1.0004
5 NO	0.0000223 30	0.000245 -	-59.64 0.55	3 2.017	6.99	-0.888	
	0.000022227						1.002
							0.9998
							0.0005
							0.0000
							0.9994 - V
							3 4 5 6 7 8 9 10
							V - Raw Signal V Reproduced Signal



Verification: Linear Trend Line + Three Modes

t Time Used	End Time Used	Time Window	Gen Speed 3 - 10												
3.000000	10.000000	Contingency	My Transient Con	tingency											
		Object	Gen 'Bus1_16.50'	'1'											
		Field	TSSpeed												
stics Modes	and Damping Obje	ect Fields													
damped Mode	5 G 10 G 1	A (constant)	B (linear)	C (guadra	itic)										
0	Trend	1.00	0.0000617		0.0										
	* ^{*.0} .00 /	Records	▼ Set ▼ Colum	ns 🕶 📴 🕶		T SORT f(0 ≠ ⊞ 0	ptions *							
es for Selecte	d Signal				Upda	e 🗹 Auto	1.0012 -	<u> </u>	_						
Mode Include Reprodu	Magnitude Mag	gnitude Ang J	gle Rank	Mode Frequency	Mode Damping %	Mode Lambda	1.001	~		/ 20	At	Af	57.051		
1 YES	0.00166 0.0	000643 1	11.15 41.13	0.171	39.67	-0.465	1.0008			Λ	FV	VI	A	~	
2 YES 3 NO	0.00114 0.0	000256	0.00 28.28 69.05 24.01	0.000	4.98	-0.543 -0.427	1.0006	Λ		11	I F				2
4 YES	0.0000167 0.	.000243 -1	80.00 6.02 59.64 0.553	0.000	-100.00	0.383	1.0004			1					
3 110	0.00002237000		JJ.04 0.555	2.017	0.55	-0.000	1.0002								
							1		T						
							0.9998			V					
							0.9996	v	V						
							0.9994 -		V,						
								3	4	5	6	7	8	9	10
										Raw S	ignal 🔽 F	eproduced Si	ginal		

Verification: Linear Trend Line + Four Modes

tart lime Used	End Time Used Time	e Window Gen S	peed 3 - 10									
3.000000	10.000000 Co	ntingency My Tra	ansient Contingen	ю								
		Object Gen 'B	us1_16.50''1'									
		Field TSSpe	ed									
atistics Modes	and Damping Object F	Fields										
Undamped Mode 0	s A (Trend	(constant) 1.00 0 Records * Se	B (linear) C (.0000617	(quadratic) 0.0 편국 - 웹방 - 웹방 -	₩ • SORT f(x	* III Option	+					
odes for Selecte	ed Signal	1		Upda	e 🗹 Auto	1.0012			2 🗡			1
Mode Include Reprodu	Magnitude Magnit End	tude Angle	Rank M Freq	ode Mode uency Damping %	Mode Lambda	1.001	۸	^	$\Lambda /$	M		
2 YES 3 YES	0.00116 0.0000	0256 0.00 0488 69.05	28.28 24.01	0.171 59.67 0.000 100.00 1.364 4.98	-0.465 -0.543 -0.427	1.0006	Λ	Λ $()$		V	\sim	\sim
4 YES 5 NO	0.0000167 0.000	0243 -180.00 0445 -59.64	6.02 0.553	0.000 -100.00 2.017 6.99	0.383 -0.888	1.0002			V			
						1						
						0.9998		V				
						0.9996	V					
						3	4	5	6	7	8 9	10
								Raw		roduced Signal		

Verification: Linear Trend Line + Five Modes

Start Time Used End Time Used Time Window	Gen Speed 3 - 10	
3.000000 10.000000 Contingency	My Transient Contingency	
Object	t Gen 'Bus1_16.50' '1'	
Field	d TSSpeed	
tatistics Modes and Damping Object Fields		
Undamped Modes A (constant) 0 Trend 1.00) B (linear) C (quadratic)] 0.0000617 0.0 ds + Set + Columns + 国 + 職 + 職 + 職 1	f(x) • III Options •
Nodes for Selected Signal	Update Auto	1.0012
Mode Include Reproduce Magnitude End Magnitude End A 1 YES 0.00166 0.0000643 3 2 YES 0.00114 0.0000256 3 3 YES 0.000097 0.0000488 4 YES 0.000167 0.000243 5 YES 0.0000223 000000445	ngle Rank Mode Frequency Mode Damping % Mode Lambda 111.15 41.13 0.171 39.67 -0.465 0.00 28.28 0.000 100.00 -0.543 69.05 24.01 1.364 4.98 -0.427 -180.00 6.02 0.000 -100.00 0.383 -59.64 0.553 2.017 6.99 -0.888	1.001 1.0008 1.0006 1.0004 1.0002 1 0.9998
t is hard to tell	a difference	
on this one, illu	istrating that	Raw Signal V Reproduced Signal
nodes manifest	t differently in	OK Cancel
11.00	-	

A Larger Example We'll Finish With

Applying the developed techniques to the response of all 43,400 substation frequencies from an 110,000 bus electric grid(20 million plus values)





Measurement-Based Modal Analysis

- There are a number of different approaches
- The idea of all techniques is to approximate a signal, y_{org}(t), by the sum of other, simpler signals (basis functions)
 - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
 - Properties of the original signal can be quantified from basis function properties
 - Examples are frequency and damping
 - Signal is considered over time with t=0 as the start
- Approaches sample the original signal $y_{org}(t)$

Measurement-Based Modal Analysis

- Vector y consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of ΔT , starting with t=0, with values y_j for j=1...m
 - Times are then $t_j = (j-1)\Delta T$
 - At each time point j, the approximation of y_j is

$$\hat{y}_j(\mathbf{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^n b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where α is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j, \mathbf{\alpha}) = e^{\alpha_i t_j}$ for α_i corresponding to a real eigenvalue, and $\phi_i(t_j, \mathbf{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$ and $\phi_{i+1}(\mathbf{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$

for a complex eigenvector value



Measurement-Based Modal Analysis

- Error (residual) value at each point j is $r_j(t_j, \mathbf{a}) = y_j - \hat{y}_j(t_j, \mathbf{a})$
- The closeness of the fit can be quantified using the Euclidean norm of the residuals

$$\frac{1}{2}\sum_{j=1}^{m}(y_{j}-\hat{y}_{j}(t_{j},\boldsymbol{\alpha}))^{2}=\frac{1}{2}\|\mathbf{r}(\boldsymbol{\alpha})\|_{2}^{2}$$

• Hence we need to determine α and \mathbf{b}

$$\hat{y}_j(\mathbf{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^n b_i \phi_i(t_j, \boldsymbol{\alpha})$$



Sampling Rate and Aliasing

- The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
 - For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz
- Sampling shifts the frequency spectrum by 1/T (where T is the sample time), which causes frequency overlap
- This is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal



Aliasing can be reduced by fast sampling and/or low pass filters

Image: upload.wikimedia.org/wikipedia/commons/thumb/2/28/AliasingSines.svg/2000px-AliasingSines.svg.png

One Solution Approach: The Matrix Pencil Method

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60

- There are several algorithms for finding the modes. We'll use the Matrix Pencil Method
 - This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method
 - The Matrix Pencil Method is useful when there is signal noise
- Given m samples, with L=m/2, the first step is to form the Hankel Matrix, Y such that

		$\begin{array}{c} \vdots \\ \mathcal{Y}_{m-L} \end{array}$: ${\cal Y}_{m-L+1}$	•••	: ${\cal Y}_m$
	$\mathbf{Y} =$		•	•	• L+2
This not a sparse matrix		v_{2}	v_{2}	•••	V
T I: ()		$\int y_1$	${\mathcal Y}_2$	•••	\mathcal{Y}_{L+1}

Reference: A. Singh and M. Crow, "The Matrix Pencil for Power System Modal Extraction," IEEE Transactions on Power Systems, vol. 20, no. 1, pp. 501-502, Institute of Electrical and Electronics Engineers (IEEE), Feb 2005.

Algorithm Details, cont.

• Then calculate **Y**'s singular values using an economy singular value decomposition (SVD)

 $\mathbf{Y} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^{\mathrm{T}}$

- The ratio of each singular value is then compared to the largest singular value σ_c ; retain the ones with a ratio > than a threshold
 - This determines the modal order, M
 - Assuming V is ordered by singular values (highest to lowest), let V_p be then matrix with the first M columns of V

The computational complexity increases with the cube of the number of measurements!

This threshold is a value that can be changed; decrease it to get more modes.



Aside: The Matrix Singular Value Decomposition (SVD)

• The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce

 $\mathbf{Y} = \mathbf{U} \boldsymbol{\Sigma} \mathbf{V}^{\mathrm{T}}$

The original concept is more than 100 years old, but has found lots of recent applications

62

where Σ is a diagonal matrix of the singular values

• The singular values are non-negative, real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix)

Aside: SVD Image Compression Example



Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

Image Source: www.math.utah.edu/~goller/F15_M2270/BradyMathews _SVDImage.pdf

Figure 3.1: Image size 250x236 - modes used {{1,2,4,6},{8,10,12,14},{16,18,20,25},{50,75,100,original image}}

63

Ā M

Matrix Pencil Algorithm Details, cont.

- Then form the matrices \mathbf{V}_1 and \mathbf{V}_2 such that
 - V_1 is the matrix consisting of all but the last row of V_p
 - V_2 is the matrix consisting of all but the first row of V_p
- Discrete-time poles are found as the generalized eigenvalues of the pair $(\mathbf{V}_2^T \mathbf{V}_1, \mathbf{V}_1^T \mathbf{V}_1) = (\mathbf{A}, \mathbf{B})$ If **B** is popular (the situat
- These eigenvalues are the discrete-time poles, z_i with the modal eigenvalues then

If **B** is nonsingular (the situation here) then the generalized eigenvalues are the eigenvalues of $B^{-1}A$

 $\lambda_i = \frac{\ln(z_i)}{\Delta T}$

The log of a complex number $z=r \angle \theta$ is $\ln(r) + j\theta$

Matrix Pencil Method with Many Signals

- The Matrix Pencil approach can be used with one signal or with multiple signals
- Multiple signals are handled by forming a Y_k matrix for each signal k using the measurements for that signal and then combining the matrices

$$\mathbf{Y}_{k} = \begin{bmatrix} y_{1,k} & y_{2,k} & \cdots & y_{L+1,k} \\ y_{2,k} & y_{3,k} & \cdots & y_{L+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L,k} & y_{m-L+1,k} & \cdots & y_{m,k} \end{bmatrix}$$
$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{I} \\ \vdots \\ \mathbf{Y}_{N} \end{bmatrix}$$

The required computation scales linearly with the number of signals



Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately we are finding

$$y_j(\mathbf{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^n b_i \phi_i(t_j, \boldsymbol{\alpha})$$

• The α is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Quickly Determining the b Vectors

• A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal k

 $\mathbf{y}_k = \mathbf{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k$

And then the residual is minimized by selecting $\mathbf{b}_k = \mathbf{\Phi}(\mathbf{\alpha})^+ \mathbf{y}_k$ where $\mathbf{\Phi}(\mathbf{\alpha})$ is the m by n matrix with values

 $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_i corresponds to a real eigenvalue,

and
$$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \cos(\alpha_{i+1} t_j)$$
 and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j} \sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = (j-1)\Delta T$

Finally, $\Phi(\alpha)^+$ is the pseudoinverse of $\Phi(\alpha)$

Where m is the number of measurements and n is the number of modes

A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013

