ECEN 460
Power System Operation and Control
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Lecture 3: Complex Power, Three-Phase

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TEXAS A&M UNIVERSITY

Transformer Fire Video

After class the video will be available in Canvas

Complex Power

Power

$$
\begin{aligned}
p(t) &= v(t)\, i(t) \\
\mathrm{v(t)} &= V_{\max} \cos(\omega t + \theta_V) \\
i(\mathrm{t}) &= I_{\max} \cos(\omega t + \theta_I) \\
\cos\alpha \cos\beta &= \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)] \\
p(t) &= \frac{1}{2} V_{\max} I_{\max} [\cos(\theta_V - \theta_I) + \\
&\quad \cos(2\omega t + \theta_V + \theta_I)]
\end{aligned}
$$

Complex Power, cont'd

Average Power

$$p(t) = \frac{1}{2}V_{\max}I_{\max}[\cos(\theta_V - \theta_I) + \cos(2\omega t + \theta_V + \theta_I)]$$

$$P_{avg} = \frac{1}{T}\int_0^T p(t)dt$$

$$= \frac{1}{2}V_{\max}I_{\max}\cos(\theta_V - \theta_I)$$

$$= |V||I|\cos(\theta_V - \theta_I)$$

Power Factor Angle = $\phi = \theta_V - \theta_I$

Complex Power

$$S = |V||I|[\cos(\theta_V - \theta_I) + j\sin(\theta_V - \theta_I)]$$
$$= P + jQ$$
$$= V\,I^*$$

(Note: S is a complex number but not a phasor)

P = Real Power (W, kW, MW)

Q = Reactive Power (var, kvar, Mvar)

S = Complex power (VA, kVA, MVA)

Power Factor (pf) = $\cos\phi$

If current leads voltage then pf is leading

If current lags voltage then pf is lagging

Complex Power, cont'd

Relationships between real, reactive and complex power

$$P = |S|\cos\phi$$

$$Q = |S|\sin\phi = \pm|S|\sqrt{1-pf^2}$$

Example: A load draws 100 kW with a leading pf of 0.85. What are ϕ (power factor angle), Q and $|S|$?

$$\phi = -\cos^{-1}0.85 = -31.8°$$

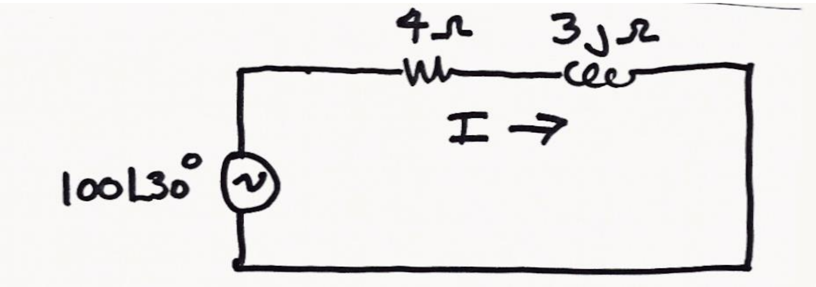
$$|S| = \frac{100kW}{0.85} = 117.6 \text{ kVA}$$

$$Q = 117.6\sin(-31.8°) = -62.0 \text{ kVar}$$

Conservation of Power

- At every node (bus) in the system
  - Sum of real power into node must equal zero
  - Sum of reactive power into node must equal zero
- This is a direct consequence of Kirchhoff's current law, which states that the total current into each node must equal zero.
  - Conservation of power follows since $S = VI^*$

Conversation of Power Example



Earlier we found $I = 20\angle\text{-}6.9°$ amps

$$S = V\,I^{*} = 100\angle 30° \times 20\angle 6.9° = 2000\angle 36.9° \;\; \text{VA}$$

$$\phi = 36.9° \qquad \text{pf} = 0.8 \text{ lagging}$$

$$\mathrm{S_R} = V_R\,I^{*} = 4 \times 20\angle -6.9° \times 20\angle 6.9°$$

$$\mathrm{P_R} = 1600W = |I|^2 R \qquad (\mathrm{Q_R} = 0)$$

$$\mathrm{S_L} = V_L\,I^{*} = 3j \times 20\angle -6.9° \times 20\angle 6.9°$$

$$\mathrm{Q_L} = 1200\,\text{var} = |I|^2 X \qquad (\mathrm{P_L} = 0)$$

Power Consumption in Devices

Resistors only consume real power

$$P_{\text{Resistor}} = \left|I_{\text{Resistor}}\right|^2 R$$

Inductors only consume reactive power

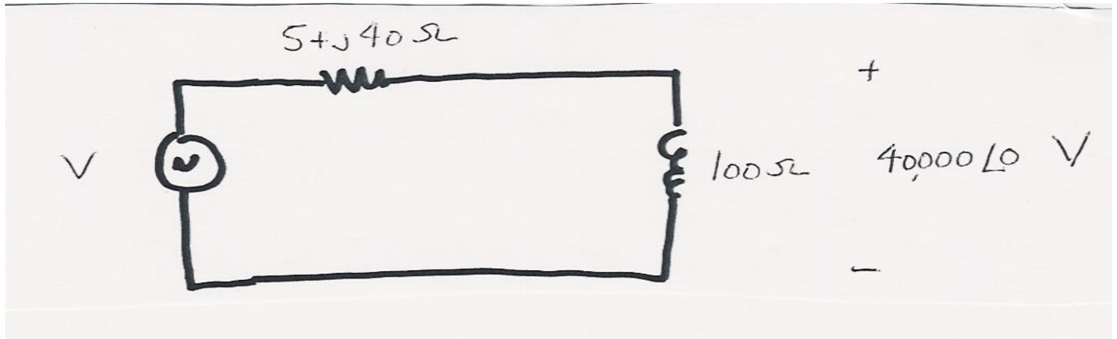
$$Q_{\text{Inductor}} = \left|I_{\text{Inductor}}\right|^2 X_{\text{L}}$$

Capacitors only generate reactive power

$$Q_{\text{Capacitor}} = -\left|I_{\text{Capacitor}}\right|^2 X_{\text{C}} \qquad X_C = \frac{1}{\omega C}$$

$$Q_{\text{Capacitor}} = -\frac{\left|V_{\text{Capacitor}}\right|^2}{X_{\text{C}}}$$ (Note-some define X_C negative)

Example



First solve the basic circuit

$$I = \frac{40000\angle 0^\circ\, V}{100\angle 0^\circ\, \Omega} = 400\angle 0^\circ \text{ Amps}$$

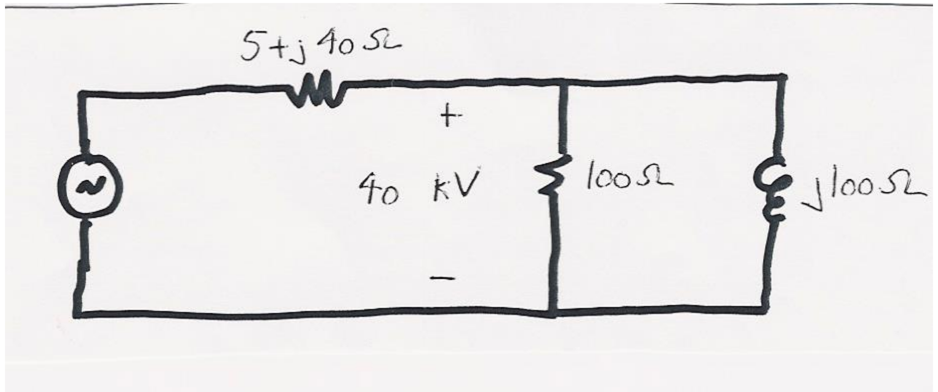
$$V = 40000\angle 0^\circ + (5 + j40)\, 400\angle 0^\circ$$

$$= 42000 + j16000 = 44.9\angle 20.8^\circ \text{ kV}$$

$$S = VI^* = 44.9\text{k}\angle 20.8^\circ \times 400\angle 0^\circ$$

$$= 17.98\angle 20.8^\circ \text{ MVA} \quad = 16.8 + j6.4 \text{ MVA}$$

Example, cont'd

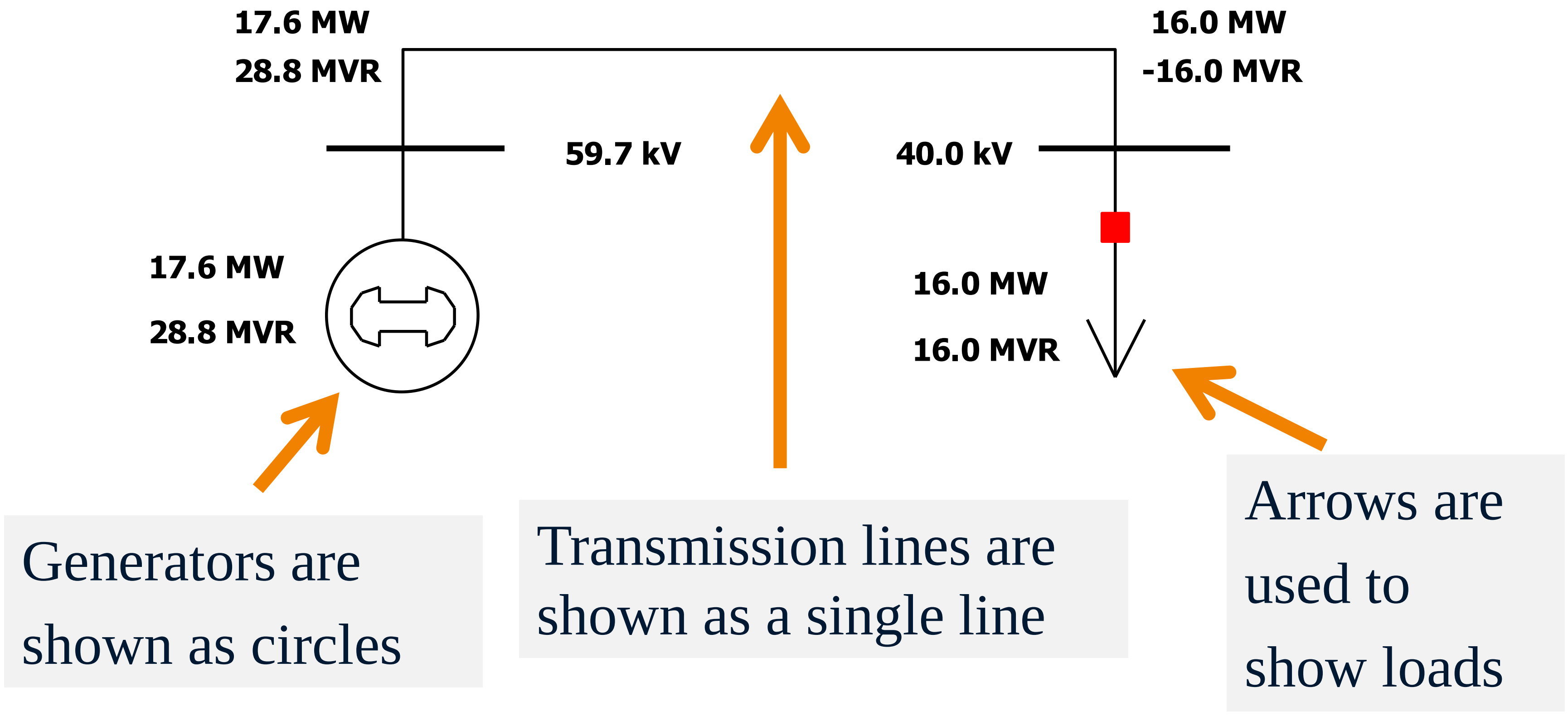


Now add additional reactive power load and resolve

$$
\begin{aligned}
Z_{Load} &= 70.7\angle 45^\circ \quad pf = 0.7 \text{ lagging} \\
I &= 564\angle -45^\circ \text{ Amps} \\
V &= 59.7\angle 13.6^\circ \text{ kV} \\
S &= 33.7\angle 58.6^\circ \text{ MVA} = 17.6 + j28.8 \text{ MVA}
\end{aligned}
$$

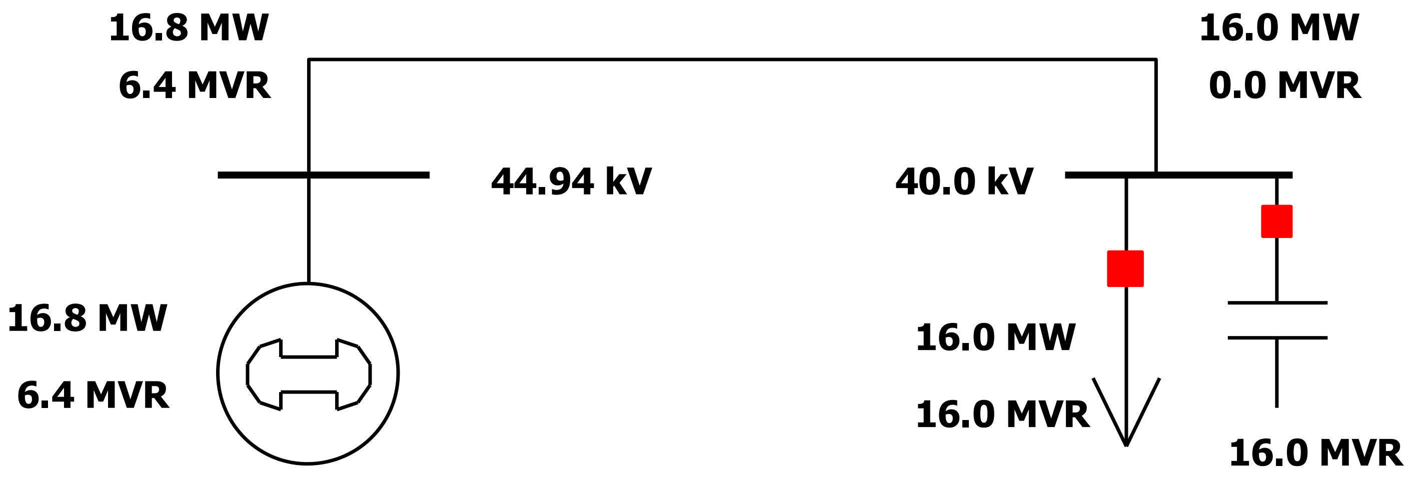
Power System Notation

- Power system components are usually shown as oneline diagrams
  Previous circuit redrawn



Reactive Compensation

- Key idea of reactive compensation is to supply reactive power locally. In the previous example this can be done by adding a 16 Mvar capacitor at the load



Power engineers think of capacitors in terms of how much reactive power they can supply at a specified (rated) voltage; not in terms of Farads

Compensated circuit is identical to first example with just real power load

Reactive Compensation, cont'd

- Reactive compensation decreased the line flow from 564 Amps to 400 Amps. This has advantages
  - Lines losses, which are equal to $I^2 R$ decrease
  - Lower current allows utility to use small wires, or alternatively, supply more load over the same wires
  - Voltage drop on the line is less
- Reactive compensation is used extensively by utilities
- Capacitors can be used to "correct" a load's power factor to an arbitrary value.

Power Factor Correction Example

Assume we have 100 kVA load with pf=0.8 lagging, and would like to correct the pf to 0.95 lagging

$$S = 80 + j60 \text{ kVA} \qquad \phi = \cos^{-1} 0.8 = 36.9°$$

$$\text{PF of 0.95 requires } \phi_{\text{desired}} = \cos^{-1} 0.95 = 18.2°$$

$$S_{\text{new}} = 80 + j(60 - Q_{\text{cap}})$$

$$\frac{60 - Q_{\text{cap}}}{80} = \tan 18.2° \quad \Rightarrow \quad 60 - Q_{\text{cap}} = 26.3 \text{ kvar}$$

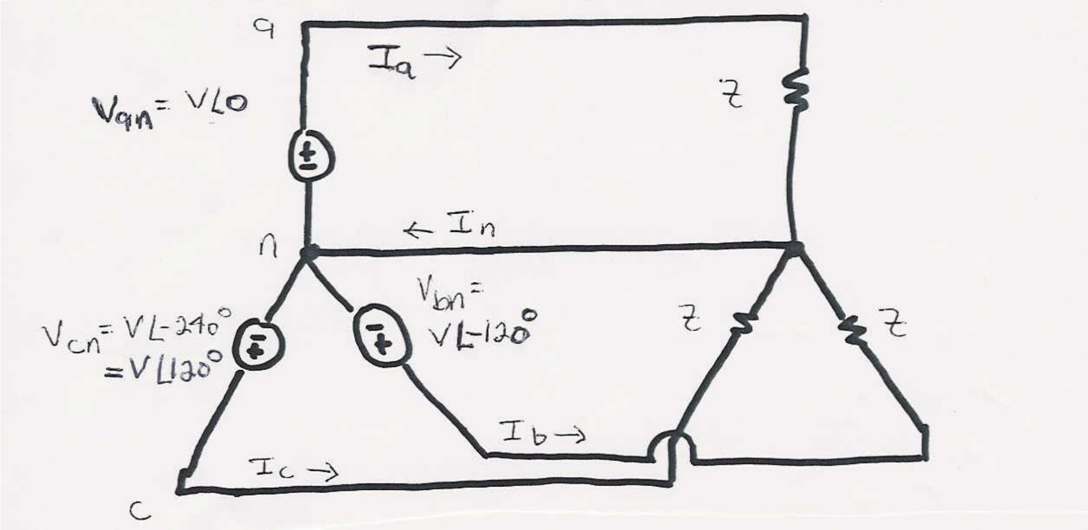
$$Q_{\text{cap}} = 33.7 \text{ kvar}$$

Distribution System Capacitors

Balanced Three-Phase (ϕ) Systems

- A balanced three-phase (ϕ) system has
  - three voltage sources with equal magnitude, but with an angle shift of 120°
  - equal loads on each phase
  - equal impedance on the lines connecting the generators to the loads
- Bulk power systems are almost exclusively 3ϕ
- Single-phase is used primarily only in low voltage, low power settings, such as residential and some commercial

Balanced Three-Phase: No Neutral Current



$$I_n = I_a + I_b + I_c$$

$$I_n = \frac{V}{Z}(1\angle 0° + 1\angle -120° + 1\angle 120°) = 0$$

$$S = V_{an}I_{an}^* + V_{bn}I_{bn}^* + V_{cn}I_{cn}^* = 3V_{an}I_{an}^*$$

Advantages of Three-Phase Power

- Can transmit more power for same amount of wire (twice as much as single phase)
- Torque produced by three-phase machines is constant
- Three-phase machines use less material for same power rating
- Three-phase machines start more easily than single-phase machines

Three-Phase Wye Connection

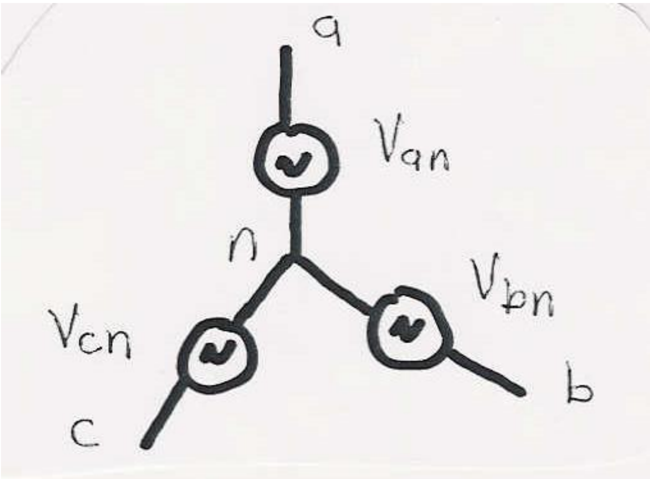
- There are two ways to connect three-phase systems
  - Wye (Y)
  - Delta (Δ)

Wye Connection Voltages

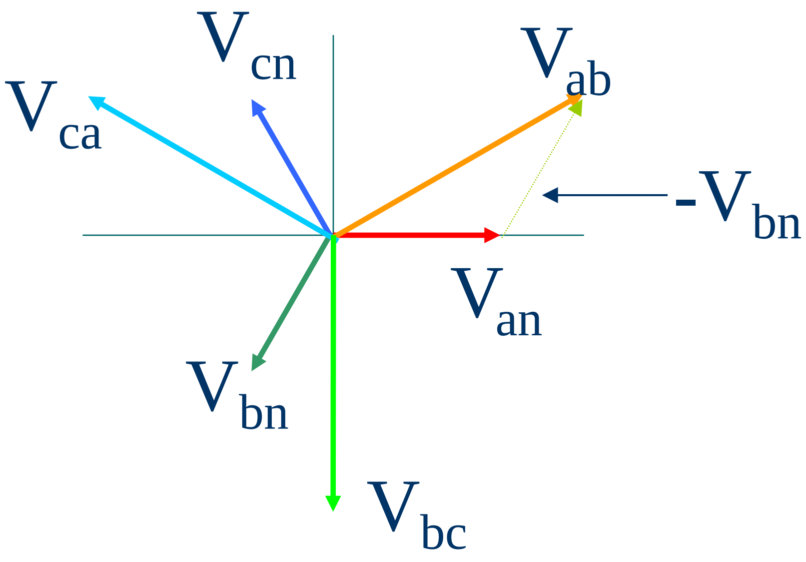
$$V_{an} = |V| \angle \alpha^\circ$$

$$V_{bn} = |V| \angle \alpha^\circ - 120^\circ$$

$$V_{cn} = |V| \angle \alpha^\circ + 120^\circ$$



Wye Connection Line Voltages



($\alpha = 0$ in this case)

$$V_{ab} = V_{an} - V_{bn} = |V|(1\angle\alpha - 1\angle\alpha + 120^\circ)$$
$$= \sqrt{3}\,|V|\angle\alpha + 30^\circ$$
$$V_{bc} = \sqrt{3}\,|V|\angle\alpha - 90^\circ$$
$$V_{ca} = \sqrt{3}\,|V|\angle\alpha + 150^\circ$$

The line to line voltages are also balanced

Wye Connection, cont'd

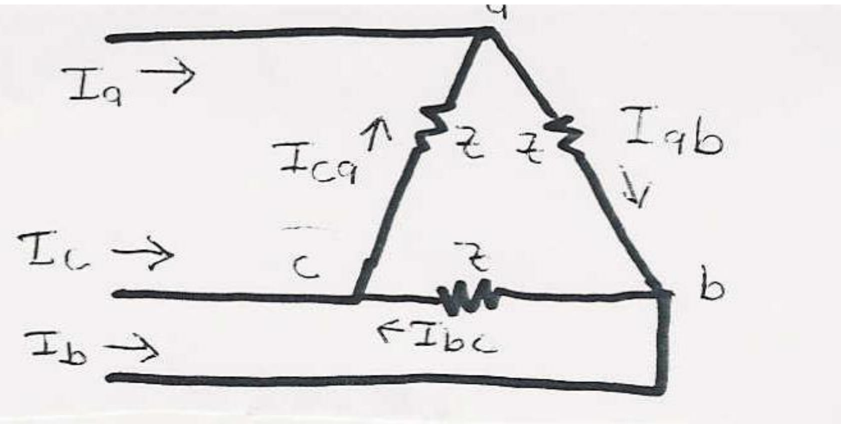
- Define voltage/current across/through device to be phase voltage/current
- Define voltage/current across/through lines to be line voltage/current

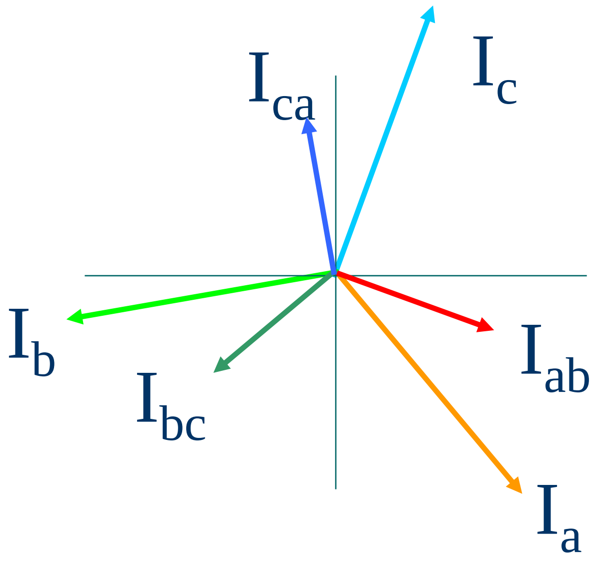
$$V_{Line} = \sqrt{3}\,V_{Phase}\,1\angle 30^\circ = \sqrt{3}\,V_{Phase}\,e^{j\pi/6}$$

$$I_{Line} = I_{Phase}$$

$$S_{3\phi} = 3\,V_{Phase}\,I^*_{Phase}$$

Delta Connection





For the Delta

phase voltages equal

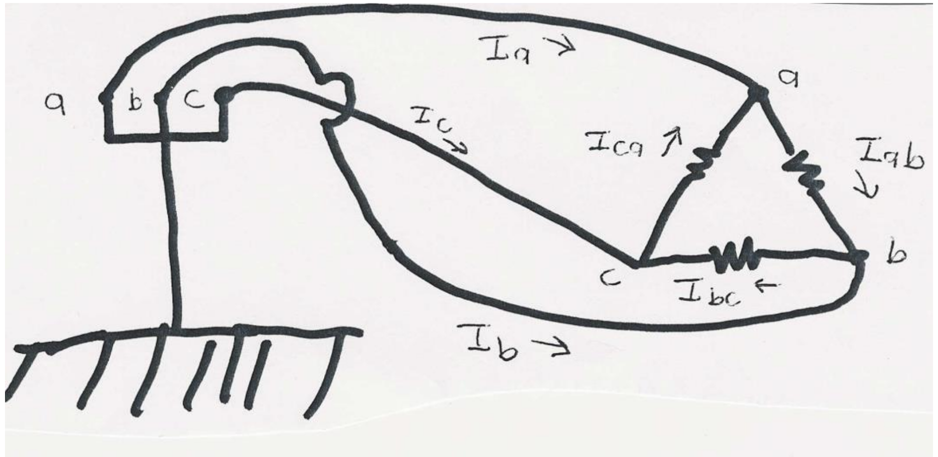
line voltages

For currents

$$\mathrm{I_a} = I_{ab} - I_{ca}$$
$$= \sqrt{3}\, I_{ab} \angle -30°$$
$$\mathrm{I_b} = I_{bc} - I_{ab}$$
$$\mathrm{I}_c = I_{ca} - I_{bc}$$
$$S_{3\phi} = 3\, V_{Phase}\, I^*_{Phase}$$

Three-Phase Example

Assume a Δ-connected load is supplied from a 3ϕ 13.8 kV (L-L) source with Z = 100∠20°Ω



$$V_{ab} = 13.8\angle 0^\circ\ kV$$
$$V_{bc} = 13.8\angle -120^\circ\ kV$$
$$V_{ca} = 13.8\angle 120^\circ\ kV$$

$$I_{ab} = \frac{13.8\angle 0^\circ\ kV}{100\angle 20^\circ\ \Omega} = 138\angle -20^\circ\ amps$$

$$I_{bc} = 138\angle -140^\circ\ amps \qquad I_{ca} = 138\angle 100^\circ\ amps$$

Three-Phase Example, cont'd

$$I_a = I_{ab} - I_{ca} = 138\angle -20^\circ - 138\angle 100^\circ$$
$$= 239\angle -50^\circ \text{ amps}$$
$$I_b = 239\angle -170^\circ \text{ amps} \quad I_c = 239\angle 70^\circ \text{ amps}$$

$$S = 3 \times V_{ab} I_{ab}^{*} = 3 \times 13.8\angle 0^\circ \text{kV} \times 138\angle 20^\circ \text{ amps}$$
$$= 5.7\angle 20^\circ \text{ MVA}$$
$$= 5.37 + j1.95 \text{ MVA}$$
$$\text{pf} = \cos 20^\circ = 0.94 \text{ lagging}$$

Delta-Wye Transformation

To simplify analysis of balanced 3ϕ systems:

1) Δ-connected loads can be replaced by

   Y-connected loads with $Z_Y = \frac{1}{3} Z_\Delta$

2) Δ-connected sources can be replaced by

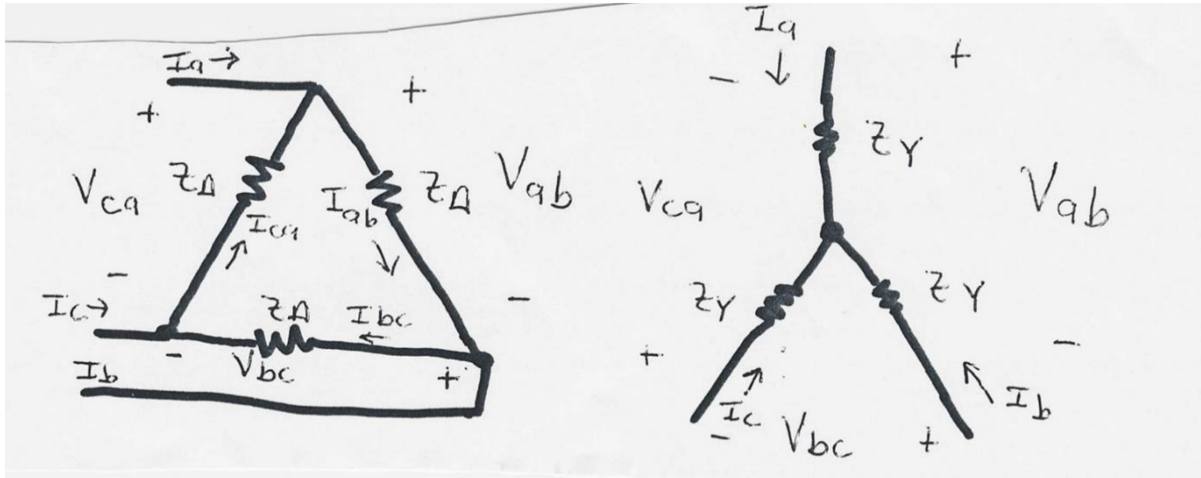
   Y-connected sources with $V_{phase} = \frac{V_{Line}}{\sqrt{3}\angle 30°}$

Delta-Wye Transformation Proof

From the Δ side we get

$$I_a = \frac{V_{ab}}{Z_\Delta} - \frac{V_{ca}}{Z_\Delta} = \frac{V_{ab} - V_{ca}}{Z_\Delta}$$

Hence $$Z_\Delta = \frac{V_{ab} - V_{ca}}{I_a}$$



From the Y side we get

$$V_{ab} = Z_Y(I_a - I_b) \qquad V_{ca} = Z_Y(I_c - I_a)$$

$$V_{ab} - V_{ca} = Z_Y(2I_a - I_b - I_c)$$

Since $\mathrm{I_a} + I_b + I_c = 0 \Rightarrow I_a = -I_b - I_c$

Hence $V_{ab} - V_{ca} = 3\,Z_Y I_a$

$$3\,Z_Y = \frac{V_{ab} - V_{ca}}{I_a} = Z_\Delta$$

Therefore $$Z_Y = \frac{1}{3} Z_\Delta$$

Three-Phase Transmission Line

Ground wire for lightning protection

138 kV transmission line

Distribution circuit sharing same poles

Transmission Arcing Video

After class the video will be available in Canvas

Modeling Cautions

- "All models are wrong but some are useful," George Box, *Empirical Model-Building and Response Surfaces*, (1987, p. 424)
  - Models are an approximation to reality, not reality, so they always have some degree of approximation
  - Box went on to say that the practical question is how wrong to they have to be to not be useful
- A good part of engineering is deciding what is the appropriate level of modeling, and knowing under what conditions the model will fail
- Of course engineering often involves using widely accepted modeling and analysis approaches for common problems. Increasingly this requires adhering to particular standards
  - Still, the engineer needs to know the conditions under which these approaches fail!

Per Phase Analysis

- Per phase analysis allows analysis of balanced three-phase systems with the same effort as for a single-phase system
- Balanced Three-Phase Theorem: For a balanced three-phase system with
  - All loads and sources Y connected
  - No mutual Inductance between phases
- Then
  - All neutrals are at the same potential
  - All phases are COMPLETELY decoupled
  - All system values are the same sequence as sources. The sequence order we've been using (phase b lags phase a and phase c lags phase a) is known as "positive" sequence (negative and zero sequence systems are mostly covered in ECEN 459)

Per Phase Analysis Procedure

To do per phase analysis

1. Convert all Δ load/sources to equivalent Y's
2. Solve phase "a" independent of the other phases
3. Total system power $S = 3 V_a I_a^*$
4. If desired, phase "b" and "c" values can be determined by inspection (i.e., ±120° degree phase shifts)
5. If necessary, go back to original circuit to determine line-line values or internal Δ values.

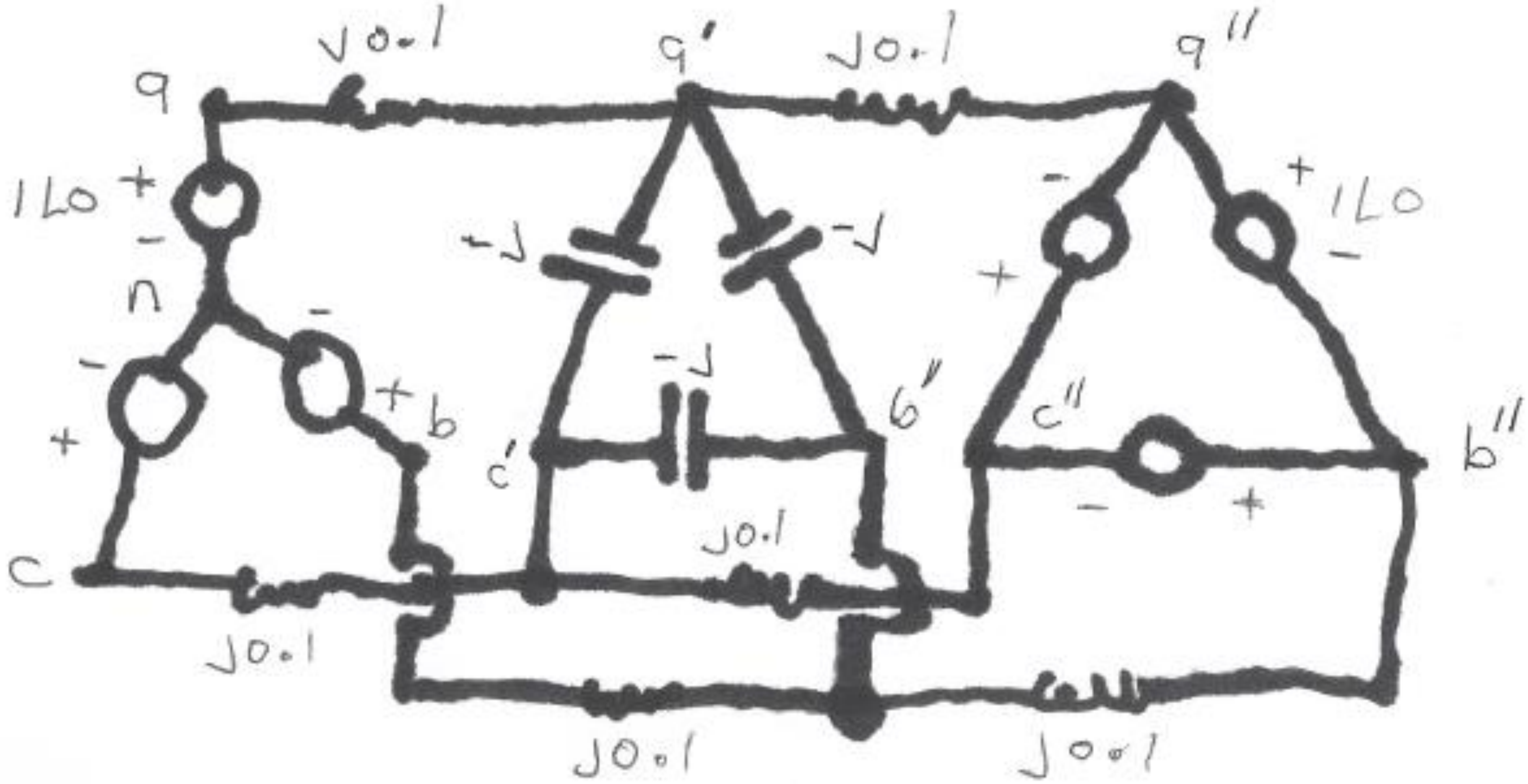
Per Phase Example

Assume a three-phase, Y-connected generator with $V_{an} = 1\angle 0°$ volts supplies a Δ-connected load with $Z_\Delta = -j\Omega$ through a transmission line with impedance of $j0.1\Omega$ per phase. The load is also connected to a Δ-connected generator with $V_{a''b''} = 1\angle 0°$ through a second transmission line which also has an impedance of $j0.1\Omega$ per phase.

Find

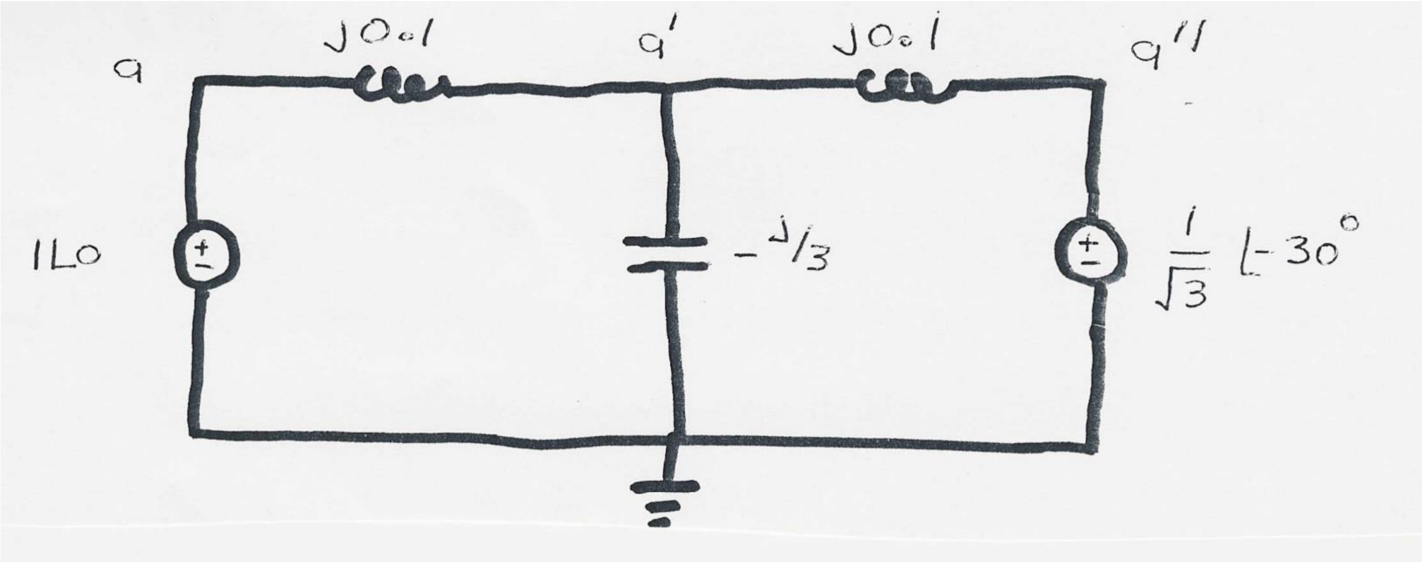
1. The load voltage $V_{a'b'}$
2. The total power supplied by each generator, S_Y and S_Δ

Per Phase Example, cont'd



First convert the delta load and source to equivalent Y values and draw just the "a" phase circuit

Per Phase Example, cont'd



To solve the circuit, write the KCL equation at a'

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + (V_a' - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

Per Phase Example, cont'd

To solve the circuit, write the KCL equation at a'

$$(V_a' - 1\angle 0)(-10j) + V_a'(3j) + (V_a' - \frac{1}{\sqrt{3}}\angle -30°)(-10j) = 0$$

$$(10j + \frac{10}{\sqrt{3}}\angle 60°) = V_a'(10j - 3j + 10j)$$

$$V_a' = 0.9\angle -10.9° \text{ volts} \qquad V_b' = 0.9\angle -130.9° \text{ volts}$$

$$V_c' = 0.9\angle 109.1° \text{ volts} \qquad V_{ab}' = 1.56\angle 19.1° \text{ volts}$$

$$S_{ygen} = 3V_a I_a^* = 3V_a\left(\frac{V_a - V_a'}{j0.1}\right)^* = 5.1 + j3.5 \text{ VA}$$

$$S_{\Delta gen} = 3V_a''\left(\frac{V_a'' - V_a'}{j0.1}\right)^* = -5.1 - j4.7 \text{ VA}$$

Power System Operations Overview

- Goal is to provide an intuitive feel for power system operation
- Emphasis will be on the impact of the transmission system
- Introduce basic power flow concepts through small system examples

PowerWorld Simulator

- Commercial tool in actual use by utilities and others (1000+ customers in 70 countries)
- Originally designed to be well-suited to teaching students about power systems. Still has that capability while being full-featured for industry use
- Strengths are user-friendliness and advanced data visualization

- The computers here have the full (unlimited bus) version. There is a free student 42-bus version available at powerworld.com/gloveroverbyesarma

Per Unit

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer transformer values (like their impedances) to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

- Engineers commonly talk about voltages in per unit. A 138 kV (base) transmission line operating at 140 kV is 140/138 = 1.022 p.u.