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# Low Voltage Power Flow Solutions and Their Role in Exit Time Based Security Measures for Voltage Collapse

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## ABSTRACT

This paper expands on previous works that consider security measures for judging vulnerability to voltage collapse. The security measure is based on expected exit time from the region of attraction for the stable operating point. This expected exit time can be related to a natural Lyapunov or energy function for the power system, evaluated at a "low voltage magnitude" solution of the powerflow equations. This paper examines the practical calculation of such measures in a multimachine power system. Heuristics for initializating for a Newton-Raphson(N-R) iteration are suggested to locate relevant low voltage solutions. Regions of convergence (with respect to the N-R iteration) are examined for various sample power systems that exhibit multiple low voltage power flow solutions.

## I. Introduction and Background

Many works that attempt to characterize voltage collapse in electric power systems have observed that under conditions associated with collapse, the power flow equations may display multiple solutions [1,2]. Indeed, most analysis methods for predicting the critical point of collapse are either explicitly or implicitly based on identification of points where the Jacobian of the power flow equations becomes singular. Standard results from bifurcation theory [3] indicate that singularity of the Jacobian is a necessary condition for a change in the number of solutions of the power flow; i.e. at the singular point, multiple solutions may coalesce into a single solution. Further changes in the "bifurcation parameter" (often the reactive load level in power system studies) may cause these multiple solutions to disappear entirely.

In attempt to link the appearance of closely grouped multiple power flow solutions to dynamics of the network, a stochastic exit formulation has previously been proposed [4]. An intuitive viewpoint on this approach may be described as follows. In an appropriate dynamic model for the power system that includes relevant voltage effects, the solutions of the power flow equations represent equilibria of the system. If the system meets the minimal requirement of being small disturbance stable (though perhaps only marginally), the solution representing the operating point will have an associated region of attraction. Intuitively, one may think of this region (or a subset thereof) as a "potential well," with the stable equilibrium forming the lowest point in this well. This potential well can be rigorously defined if an appropriate Lyapunov or energy function can be found for the dynamic model. Other powerflow solutions represent other equilibria, many of which will be unstable with respect to the dynamics. For the class of dynamic models to be considered here, one or more unstable equilibrium points must lie on the boundary of the region of attraction associated with the stable equilibrium point. Clearly, if one has a situation where gradual increases in reactive load are causing the unstable equilibria to move closer to the operating point, the potential well associated with the operating point must be shrinking. The critical collapse point occurs when this well disappears entirely.

The stochastic exit approach attempts to identify the vulnerability of the system before stability is completely lost. It is well accepted that customer loads have a small magnitude randomly varying component that is typically very broad spectrum [5]. The test of vulnerability proposed in the stochastic exit problem identifies how "easily" random load variations can drive the system state out of its potential well. Formally, one computes an asymptotic approximation to the expected time required for the randomly perturbed state to exit the well. The expression is obtained as the solution to an optimal control problem. In essence, on computes the minimum energy that the load variations must inject to drive the state out of the well. This is related to the height of the potential boundary to be overcome, which in turn is determined by the location of the lowest energy unstable equilibrium point on the boundary. This is an intuitively appealing measure of the security of an operating point, and proves to be computationally tractable in a significant class of models. However, a key point in this analysis is identification of the unstable equilibrium point of interest. This paper will explore that issue.

## II. Lyapunov Functions and the Expected Exit Time Expression

A detailed discussion of the dynamic models of interest may be found in [4]. For the purposes of this paper, it is sufficient to note that the dynamic models used will be "strueture preserving" in the sense described in [6]; i.e. both generator and load buses will be explicitly represented in the network model. In the idealized model, active loads will be assumed to be affine functions of bus frequency; that is, a constant active power part plus a term proportional to deviation from synchronous frequency. Reactive power loads may be modelled as arbitrary functions of bus voltage magnitude. Note that this type of modeling implies that equilibria are determined by solutions to a fairly standard formulation of the power flow equations (note that the active power term dependent on frequency deviation must go to zero at equilibrium).

Subject to the assumptions above, the expression for the expected exit time is based on the following "energy function" for the power system:

$$\vartheta(\mathbf{x}^0, \mathbf{x}) := \int_{(0,\alpha^0,V^0)}^{(\omega,\alpha,V)} \left[(M\lambda)^T, \mathbf{f}^T(\xi,\mu), \mathbf{g}^T(\xi,\mu)\right]\left[d\lambda^T, d\xi^T, d\mu^T\right]^T \quad (1)$$

where:

$$\mathbf{f}(\alpha, \mathbf{V}) := \tilde{\mathbf{f}}(\alpha, \mathbf{V}) + \mathbf{P}^0$$

$\tilde{f}_i(\alpha, \mathbf{V})$ := active power absorbed by network at bus i

$P_i^0$ := nominal active power demand at bus i (sign convention: + for load, - for gen)

$$\mathbf{g}(\alpha, \mathbf{V}) := \{\mathrm{diag}[\mathbf{V}]\}^{-1}\{\tilde{\mathbf{g}}(\alpha, \mathbf{V}) + \mathbf{Q}_D(\mathbf{V})\}$$

$\tilde{g}_i(\alpha, \mathbf{V})$ := reactive power absorbed by network at bus i

$Q_{D,i}(V_i)$ := reactive power demand at bus i (sign convention: + for load, - for gen)

and M, $D_1$, $D_V$ are constant diagonal matrices describing system parameters, and $T_1$ and $T_2$ are constant matrices describing network topology.

For $\mathbf{f}(\alpha,\mathbf{V})$ and $\mathbf{g}(\alpha,\mathbf{V})$ as defined, the composite vector function $[(\mathbf{M}\omega)^T, \mathbf{f}^T(\alpha,\mathbf{V}), \mathbf{g}^T(\alpha,\mathbf{V})]^T$ is exact. Hence the value of a path integral of this function from $(\mathbf{0}, \alpha^0, \mathbf{V}^0)$ to $(\omega, \alpha, \mathbf{V})$ will depend only on the endpoints; i.e., it is path independent. For purposes of computation, it is useful to note that the integral may be expressed in closed form as:

$$\frac{1}{2}\omega^T\mathbf{M}\omega - \frac{1}{2}\sum_{i=0}^{n}\sum_{k=0}^{n} B_{ik}V_i V_k \cos(\alpha_i - \alpha_k) + \frac{1}{2}\sum_{i=0}^{n}\sum_{k=0}^{n} B_{ik}V_i^0 V_k^0 \cos(\alpha_i^0 - \alpha_k^0) + \sum_{k=0}^{n}\int_{V_k^0}^{V_k}\frac{Q_{D,k}(\mu)}{\mu}d\mu - [\mathbf{P}^0]^T(\alpha - \alpha^0) \tag{2}$$

As indicated in section I, the goal is to use the expected exit time to rank the vulnerability of various operating points to voltage collapse. For a set $\Omega$ contained in the region of attraction surrounding a stable equilibrium point $\mathbf{x}^0 = (\mathbf{0}, \alpha^0, \mathbf{V}^0)$, the expected time required to exit $\Omega$ is shown in [4] to be proportional to an exponential evaluated at the minimum value of $\vartheta(\mathbf{0}, \alpha, \mathbf{V})$ on the boundary of $\Omega$. In the limit as $\Omega$ is expended to "fill" the whole region of attraction, this becomes the value of $\vartheta(\mathbf{0}, \alpha, \mathbf{V})$ at the closest unstable equilibrium point. The "closest point" is here interpreted to be that u.e.p. first encountered by expanding contours of $\vartheta(\mathbf{0}, \alpha, \mathbf{V})$ = constant away from $(\mathbf{0}, \alpha^0, \mathbf{V}^0)$. The energy function values to be used in the following section will be obtained by evaluating (3) at various power flow solutions

## III. Low Voltage Powerflow Solutions

The powerflow solutions often display more than one possible solution. This phenomenon is easily verified in simple radial line configurations, as have been examined in some of the earliest works on voltage collapse [7,8]. Consider a system with a single lossless line connecting two buses, number 1 and 2. Bus 1 is treated as the generator bus with its voltage magnitude fixed at 1.0 pu. Since the line is lossless, the real power injection at bus 1 must equal the real power consumed at bus 2. We will assume the load at bus 2 is represented as a constant P-Q demand. The following analysis can easily be extended into the case of P and Q specified as functions of bus voltage. The resulting power balance equations at bus 2 are:

$$P_L - B_{12}V\sin(\alpha) = 0$$
$$Q_L - B_{22}V^2 - B_{12}V\cos(\alpha) = 0$$

where:

V := bus voltage magnitude at bus 2

$\alpha := \delta_1 - \delta_2$ = phase angle voltage difference from bus 1 to bus 2.

For $B_{12} = -B_{22} = 10.0$, the locus of the points in the V-P space satisfying these two constraints for a range of load power factor values is shown in figure 1. As illustrated, a radial line with a fixed sending voltage typically has two solutions. This is represented in figure 1 by the dual voltage solutions for any given P value and power factor. These solution values will be referred to throughout this report as the "high voltage solution" and the "low voltage solution," distinguished by their relative values of voltage magnitude. As shown in the figures, for certain critical values of load level the constraints have only one solution. If load is increased further, the powerflow has no solution. At this bifurcation point (i.e., the point where the two solutions coalesce into one), the Jacobian of the two power balance equations is singular. As noted above, this observation has been used by some authors as a method of predicting proximity to voltage collapse. In particular, [2] recommends the use of the smallest singular value of the Jacobian of the powerflow equations, evaluated at the normal operating point of the system.

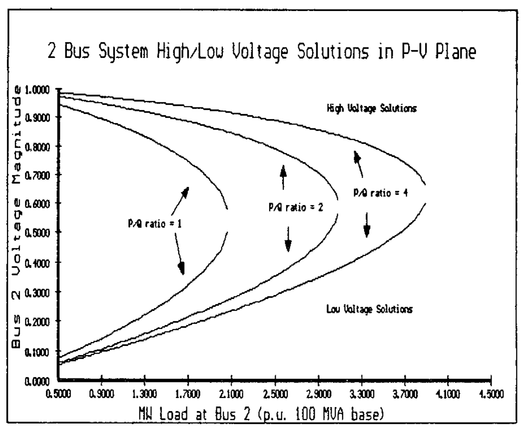

Figure 1

As a "next step" in complexity over the two bus system, consider the case of a three bus system with two radial lines connected to a single slack bus. Because of the slack bus between them, the two halves of the system are essentially isolated from one another. This system could be likened to a larger system with two potentially weak areas, isolated from one another by a relatively strong portion of the system. For this case the maximum number of solutions is seen by inspection to be four (both loads at their high bus solutions, both at their low voltage solution, and the two combinations of one load high and one load low), with each side reaching its bifurcation point independently as its load is increased. The next logical extension of this system is to couple the two loads by adding a third line between them; this system results in a much more interesting set of solutions.

Consider the three bus system with bus 1 as the slack and buses 2 and 3 as load buses with constant P/Q loads. Each bus is connected to the other two with lossless lines of 0.2 per unit impedance. With an initial load of 50 MW and 25 MVAR at each of the load buses, 4 solutions are possible. Figure 2 shows the solution trajectories in the $V_2$-$V_3$ plane as the load at both buses is increased at the same rate, maintaining a constant powerfactor. The initial starting voltage points are labeled 1, 2, 3, and 4. Point 1 corresponds to the normal operating point of the powerflow. As the load is uniformly increased at buses 2 and 3, trajectory 1 moves downward to the left, indicating that the voltages at both buses are falling. This is the expected power system behavior. Eventually the voltage collapse point is reached (labeled point 5); at this point the Jacobian becomes singular and no further increase in load is possible. The other three points correspond to the three other initial powerflow solutions (the 'low voltage' solutions). At point 2 the voltage at bus 3 is higher than that at bus 2; at point 3 both voltages are the same; and at point 4 the voltage at bus 2 is higher than the voltage at bus 3. As the load at both buses is increased, the three trajectories converge, coalescing into a single trajectory at point 6. By the implicit function theorem [3] it is clear that at this point of coalescing, the Jacobian must also be singular. As the load is further increased, the trajectory continues to the upper right, eventually reaching the voltage collapse point 5, where the Jacobian again becomes singular.

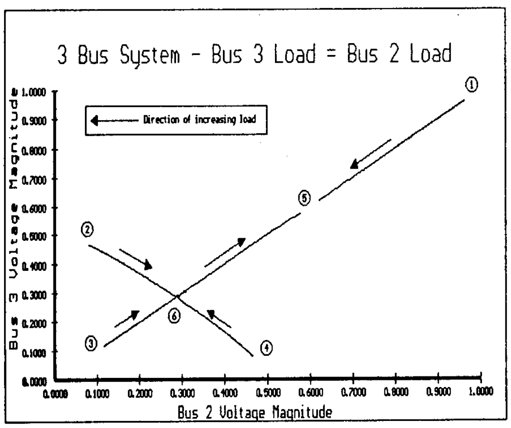


Figure 2

If the loads and their participation factors are changed so that the load at bus 2 is no longer equal to the load at bus 3, the three trajectories no longer join together. Figure 3 shows the results for the same system examined previously except that the initial loads were changed to with 50 MW and 25 MVAR at bus 2 and 45 MW and 22.5 MVAR at bus 3. As the load in the system was increased, the load participation at bus 3 was such that it remained 90% of the load at bus 2. In this example, trajectory 1 remained relatively unchanged, with the voltages dropping as the load is increased. Because of its larger load, bus 2's voltage was always slightly below that of bus 3. The low voltage trajectories changed substantially. Trajectory 2 now no longer joins those of 3 and 4, but rather moves towards an intersection with trajectory 1 at the point of voltage collapse. Trajectories 3 and 4 also move towards a point of intersection, however the load value associated with this intersection point is substantially below the value associated with the point of voltage collapse for the system. As would be expected, the Jacobian of the system at the intersection point of trajectories 3 and 4 is singular.

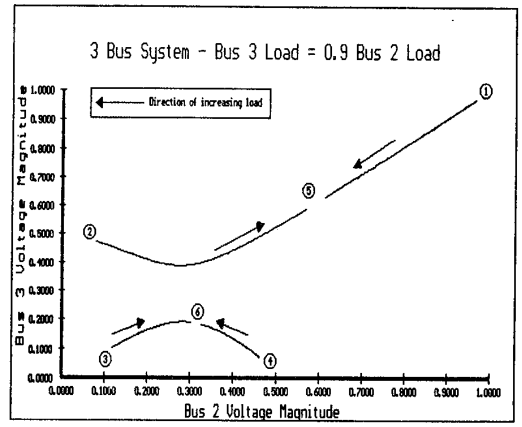


Figure 3

As the previous examples have shown, the power system equations often have multiple 'low voltage' solutions. In order to use the exit time based ranking of security, it is necessary to determine which of these solutions is relevant; i.e., which represents the closest u.e.p. An intuitive interpretation of the of the role of the lowest energy low voltage solution may be found in the simple three bus system. Figure 4 plots the energy difference between each of the low voltage solutions relative to the operating point as a function of the load at bus 2 (the loads at bus 2 and 3 are held equal). As can be seen, the energy value associated with the case where the voltages at both 2 and 3 are low is significantly higher than for the cases where either 2 or 3 is high. Intuitively, the energy value in the former case would correspond to a "path of exit" in which both areas experiencing voltage collapse simultaneously (an unlikely event if they are truly independent), while in the later case the energy value would correspond to one of the buses collapsing independent of the other (a much more likely event, and hence a lower energy difference).

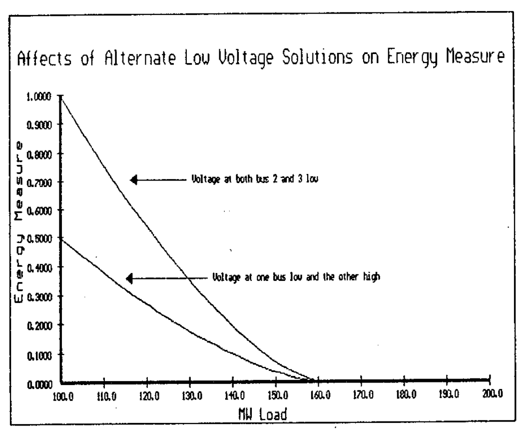


Figure 4

For the three bus system, identification of the low voltage solution with the lowest energy is straightforward. However, in larger networks, closed form identification of the relevant low voltage solution is unlikely. As in the case of locating u.e.p.'s for swing dynamic models, a more practical approach is to develop good heuristics for initializing a Newton-Raphson iteration in a region that is expected to converge to the low voltage solution yielding lowest energy. To illustrate these types of heuristics, a sixteen bus system was considered. Data for this system is found in appendix A.

The common feature of all initial guesses used in our method was the identification of a "weakest bus" within a weak area. At the "weakest bus," the initial guess for voltage phasors was taken to be 0.1 at an angle of zero, with all other voltages initialized at 1.0. Clearly, the identification of this bus is key to the initialization. Whenever possible, it was taken to be a bus within an area having no local reactive support, and weak ties to other areas with reactive sources. The first case examined identified a weak area containing buses 13, 14, 15, and 16; this will be denoted Area 1. Within this set, identification of bus 16 as the weakest bus from line data was straightforward (all lines incident on bus 16 have large $B_{ij}$ values). Figure 5 shows the voltage magnitude profile for a low voltage solution using bus 16 as the low initial guess, along with the profile of the corresponding "high voltage" operating point..

The second case examined uses a weak area containing buses 9, 10, 11, and 12; this will be denoted Area 2. Identifying a weakest bus in Area 2 was more difficult. Any of the buses in the area could have been classified as weakest under the criterion established above. The heuristic used was to select the bus which had the lowest voltage magnitude in the high voltage solution. Figure 6 shows the voltage magnitude profiles for the high voltage solution along with the two low voltage solutions; one using bus 9, the other using bus 12 as the low initial guess.

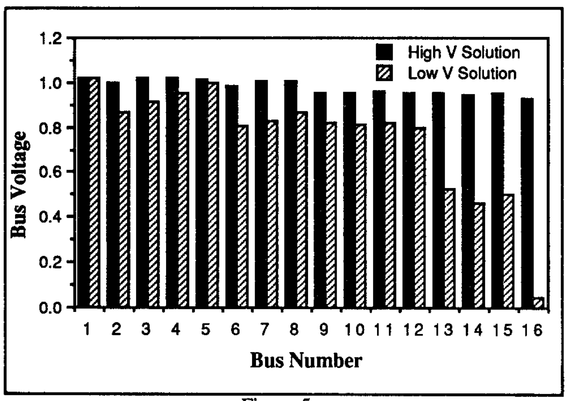


Figure 5

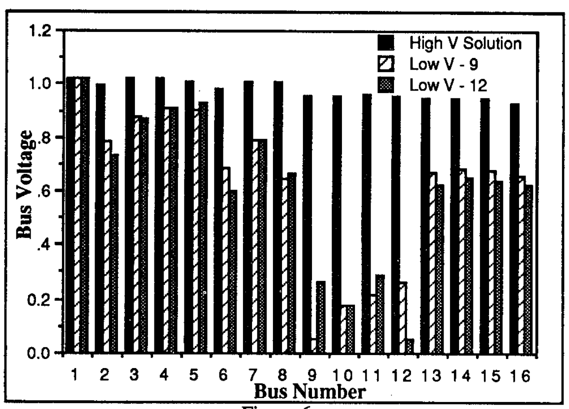


Figure 6

It is interesting to note that as the load in the 16 bus system was uniformly increased, areas 1 and 2 became less and less independent. Eventually it was no longer possible to obtain the low voltage solution for the low initial guess at bus 12. This was similar to what was observed in the three bus system eventually some solution trajectories terminate at load levels below the final loss of solution point. As the load was increased further, the solution trajectory that was followed from initialization of the Newton-Raphson iteration with bus 9 low also terminated.

Identifying low voltage powerflow solutions via iterative methods requires some prediction of the region of attraction of of a low voltage solution with respect to the Newton Raphson iteration. A heuristic for determining the initial guess of the voltage magnitudes and angles in order to arrive at a desired solution is suggested above, but clearly much further examination of this issue is necessary. The difficulty can be illustrated by considering the three bus system considered previously. Figure 7 shows the "regions of attractions" with respect to the N-R iteration for various power flow solutions in the three system for a range of initial voltage magnitude guesses at buses 2 and 3, and with constant initial angle guesses of 0. In this problem, with load values of 50 MW and 25 MVAR at bus 2 and 45 MW and 25 MVAR at bus 3, the four solutions are shown in Table 1.

Table 1

| | \|V1\| | Angle 1 | \|V2\| | Angle 2 | \|V3\| | Angle 3 |
|---|---|---|---|---|---|---|
| 1) | 1.0 | 0.0° | 0.944 | -5.88° | 0.946 | -5.66° |
| 2) | 1.0 | 0.0° | 0.082 | -62.33° | 0.462 | -14.76° |
| 3) | 1.0 | 0.0° | 0.450 | -16.03° | 0.074 | -63.14° |
| 4) | 1.0 | 0.0° | 0.120 | -56.54° | 0.105 | -58.75° |

These four solutions have associated four regions of attraction shown in figure 7. The regions appear to be contiguous, but detailed examination shows their boundaries are not smooth. Note that the white areas represent initial conditions for which a solution was not found.

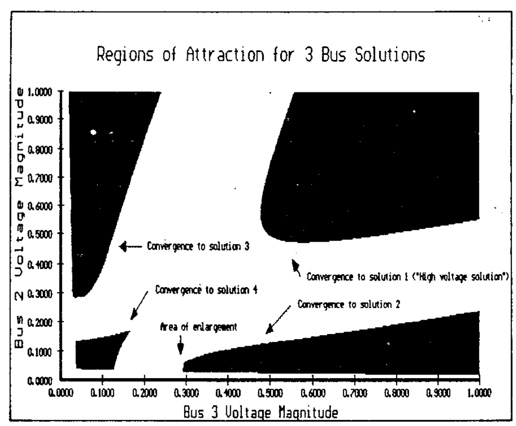


Figure 7

Note that the regions shown above are cross-sections contained in the subspace where initial angles are all zero. The cross-sections become more complex in the case where the initial angle guesses were no longer zero. Figure 8 shows the case with initial angles at bus 2 and 3 of -5.73° -11.46° degrees respectively. The shapes of the four large regions from the previous problem appear relatively unchanged; however a new region has formed immediately to the left of area 1. Surprisingly, initial guesses in this area result in convergence to solution 2. Clearly, convergence of one initial condition to a particular solution does not allow one to draw very strong conclusions regarding other initial conditions in a neighborhood of the known point.

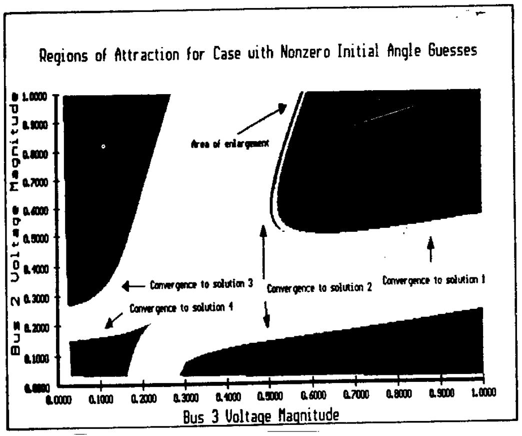


Figure 8

## IV. CONCLUSIONS

This paper has examined the nature of low voltage power flow solution in power system models that involve voltage magnitude dependent reactive loads. The role of these solutions in exit time based measures of security with respect to voltage collapse was considered, and the special role of a "lowest energy" solution highlighted. Heuristics for initializing the Newton-Raphson iteration for finding this low voltage solution were proposed and illustrated in sample networks. Finally, the convergence properties of the Newton-Raphson algorithm were illustrated by examining the region of attraction associated with various low voltage power flow solution with respect to the N-R iteration in sample networks.

## Appendix A

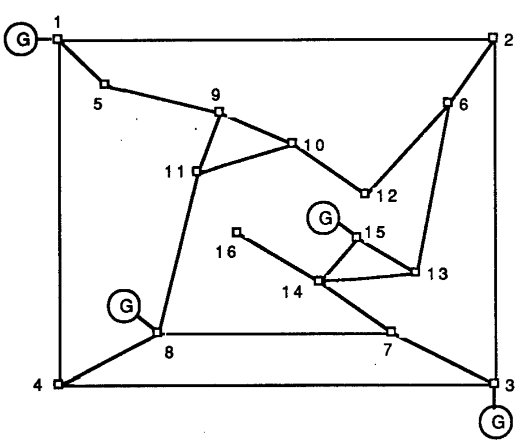


Table A1: Bus Data

| Bus | Type | Load: | | Generation: | | |
|---|---|---|---|---|---|---|
| | | MW | Mvar | MW | Voltage | Var Limit |
| 1 | 3 | 0.0 | 0.0 | 100.0 | 1.02 | ±9999 |
| 2 | 0 | 0.0 | 0.0 | | | |
| 3 | 2 | 0.0 | 0.0 | 200.0 | 1.02 | ±150 |
| 4 | 0 | 0.0 | 0.0 | | | |
| 5 | 0 | 20.0 | 5.0 | | | |
| 6 | 0 | 0.0 | 0.0 | | | |
| 7 | 0 | 30.0 | 12.0 | | | |
| 8 | 2 | 60.0 | 18.0 | 80.0 | 1.01 | ±50 |
| 9 | 0 | 30.0 | 15.0 | | | |
| 10 | 0 | 20.0 | 8.0 | | | |
| 11 | 0 | 20.0 | 4.0 | | | |
| 12 | 0 | 30.0 | 13.0 | | | |
| 13 | 0 | 40.0 | 12.0 | | | |
| 14 | 0 | 25.0 | 15.0 | | | |
| 15 | 2 | 20.0 | 8.0 | 20.0 | 1.00 | ±10 |
| 16 | 0 | 15.0 | 10.0 | | | |

Table A2: Branch Data

| From | To | R Value | X Value | C Value |
|---|---|---|---|---|
| 1 | 2 | 0.01 | 0.1 | 0.05 |
| 1 | 4 | 0.01 | 0.05 | 0.1 |
| 1 | 5 | 0.001 | 0.05 | 0.0 |
| 2 | 3 | 0.03 | 0.3 | 0.1 |
| 2 | 6 | 0.0 | 0.04 | 0.0 |
| 3 | 4 | 0.01 | 0.04 | 0.2 |
| 3 | 7 | 0.005 | 0.03 | 0.0 |
| 4 | 8 | 0.03 | 0.2 | 0.0 |
| 5 | 9 | 0.14 | 0.32 | 0.0 |
| 6 | 12 | 0.05 | 0.15 | 0.0 |
| 6 | 13 | 0.05 | 0.15 | 0.0 |
| 7 | 8 | 0.02 | 0.08 | 0.0 |
| 7 | 14 | 0.06 | 0.08 | 0.0 |
| 8 | 11 | 0.04 | 0.11 | 0.0 |
| 9 | 10 | 0.02 | 0.08 | 0.0 |
| 9 | 11 | 0.02 | 0.08 | 0.0 |
| 10 | 11 | 0.01 | 0.05 | 0.0 |
| 10 | 12 | 0.01 | 0.05 | 0.0 |
| 13 | 14 | 0.02 | 0.08 | 0.0 |
| 13 | 15 | 0.01 | 0.06 | 0.0 |
| 14 | 15 | 0.01 | 0.08 | 0.0 |
| 14 | 16 | 0.06 | 0.08 | 0.0 |

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