ECEN 460
Power System Operation and Control
Spring 2025

Lecture 5: Transmission Line and Transformer Models

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Announcements

- Please read Chapters 3.3 (per unit); skim the rest of Chapter 3, and Chapters 4 and 5
  - In 460 we will be using the models developed in these chapters, but not deriving them since that is covered in 459
- By Feb 5 do book probs. 3.14, 3.19, 3.28, 4.18, 4.23, 4.39, 5.1, 5.12 (does not need to be turned in)
- The procedure for getting PowerWorld Simulator is now in Canvas

Power System Models, Cases and Islands

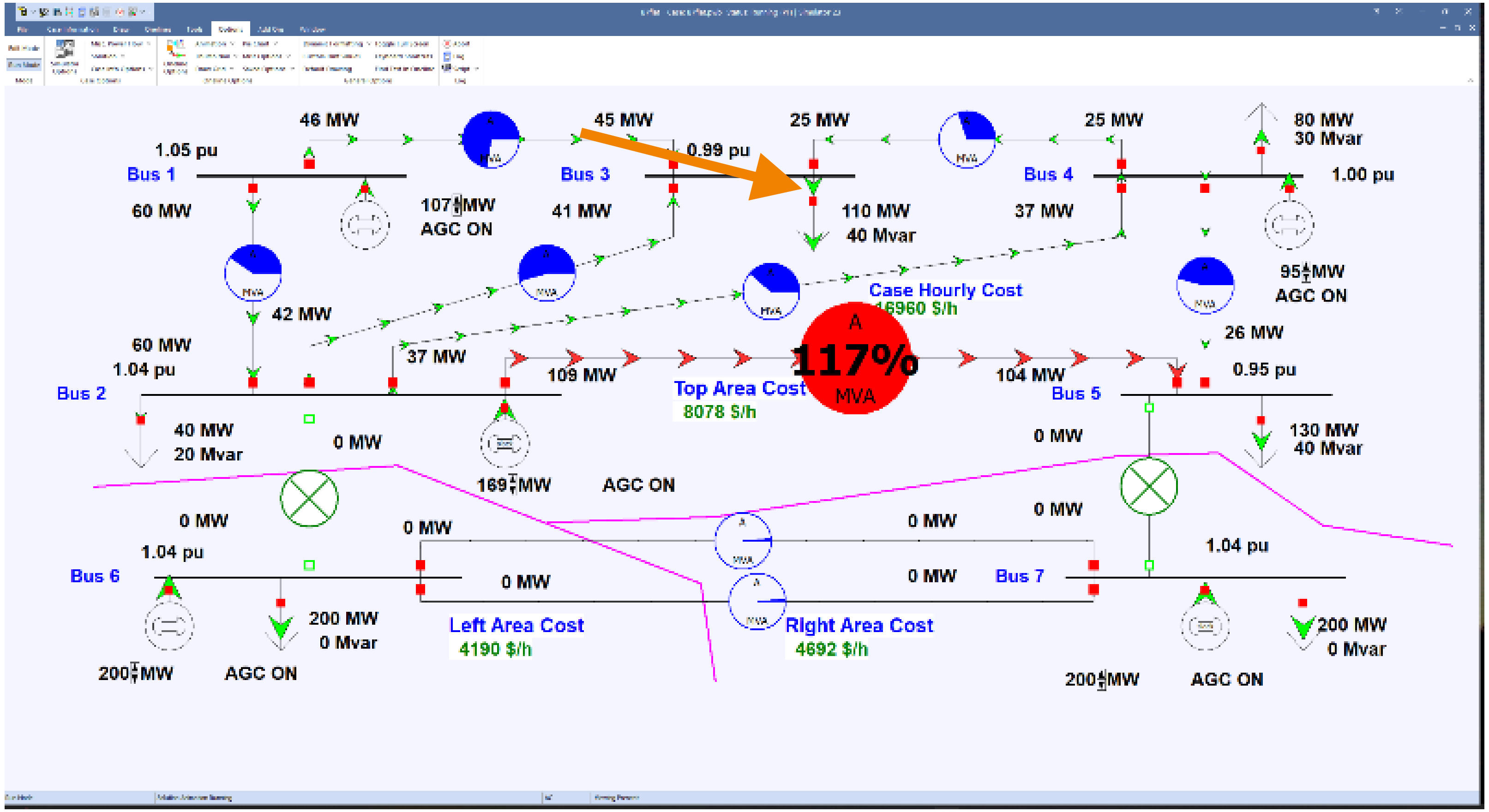
- Before moving forward it is helpful to clarify a three terms that commonly come up related to power flow
- In an engineering context the term "model", as defined by Merriam-Webster is, "A system of postulates, data, and inferences presented as a mathematical description of an entity or state of affairs."
  - Models for the same devices can vary in complexity, and (as noted by George Box), "All models are wrong but some models are useful."
  - For example models for a resistor are $V = R_0 I$ or $V = R_0(1 + 0.004*(T - T_0))\, I$
  - The values of the parameters (e.g. R_0 and T_0) then denote a particular model instance
- In power flow the term model if often used more generically to indicate a particular electric grid with the associated parameters for the more static electric grid components (e.g., the transmission lines and transformers)

Power System Models, Cases and Islands, cont.

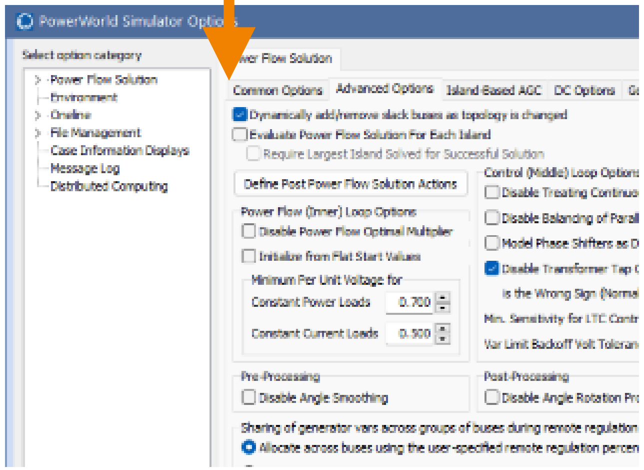
- The term "case" is usually used to a particular electric grid operating point including all values needed to solve the power flow
  - There is a fuzzy line between what is relatively constant (e.g., the impedance of a transformer) and what commonly changes (e.g., the amount of bus load, whether a particular transmission line is in-service)
- The term "base case" is often used to refer a starting case that is used for additional studies, such as with contingency analysis
- The term "island" refers to a set of ac-interconnected buses.
  - A small power flow case usually has just a single island, though they can certainly have multiple islands.
  - The number of islands can change as a result of topology changes
  - Large electric grid cases often have multiple islands; islands often connected via HVDC.

B7Flat Case Divided into Two Islands

When the two tielines joining Area Top to Areas Left and Right are opened, the grid becomes two electrical islands



Allowing new islands might require setting an option



PowerWorld Version, Build Date and Add-Ons

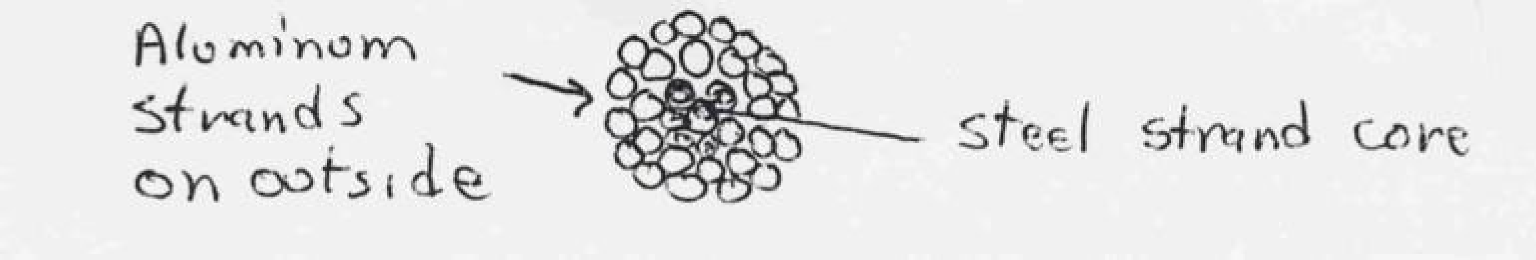
- Open PowerWorld Simulator; to see your version and build date select **Window, About**
  - Simulator stores information about the power system in *.pwb files and information about the onelines in *.pwd files
  - PowerWorld Simulator has the ability to read and write current and previous versions of its *.pwb and *.pwd files, but cannot read future versions

Development of Line and Transformer Models

- Goal of next few lectures is to introduce the models we'll be using in later parts of the class for power flow and transient stability analysis
  - More detailed coverage of some models is in ECEN 459
- Primary Methods for Long Distance Electric Power Transfer
  - overhead ac
  - underground ac
  - overhead dc
  - underground dc
  - other

Line Conductors

- Typical transmission lines use multi-strand conductors
- ACSR (aluminum conductor steel reinforced) conductors are most common. A typical Al. to St. ratio is about 4 to 1.



- AAC (all aluminum conductors) are lighter but have less strength; used in urban areas with shorter spans
- Copper is heavier, but has better conductance; used in cables where weight is not an issue

Image source: wikipedia

Line Conductors, cont'd

- Total conductor area is given in circular mils. One circular mil is the area of a circle with a diameter of $0.001 = \pi \times 0.0005^2$ square inches
- **Example:** what is the the area of a solid, 1" diameter circular wire? **Answer:** 1000 kcmil (kilo circular mils)
- Because conductors are stranded, the equivalent radius must be provided by the manufacturer. In tables this value is known as the GMR and is usually expressed in feet.

Line Resistance

Line resistance per unit length is given by

$$\mathrm{R} = \frac{\rho}{\mathrm{A}} \quad \text{where } \rho \text{ is the resistivity}$$

Resistivity of Copper = 1.68×10^{-8} Ω-m

Resistivity of Aluminum = 2.65×10^{-8} Ω-m

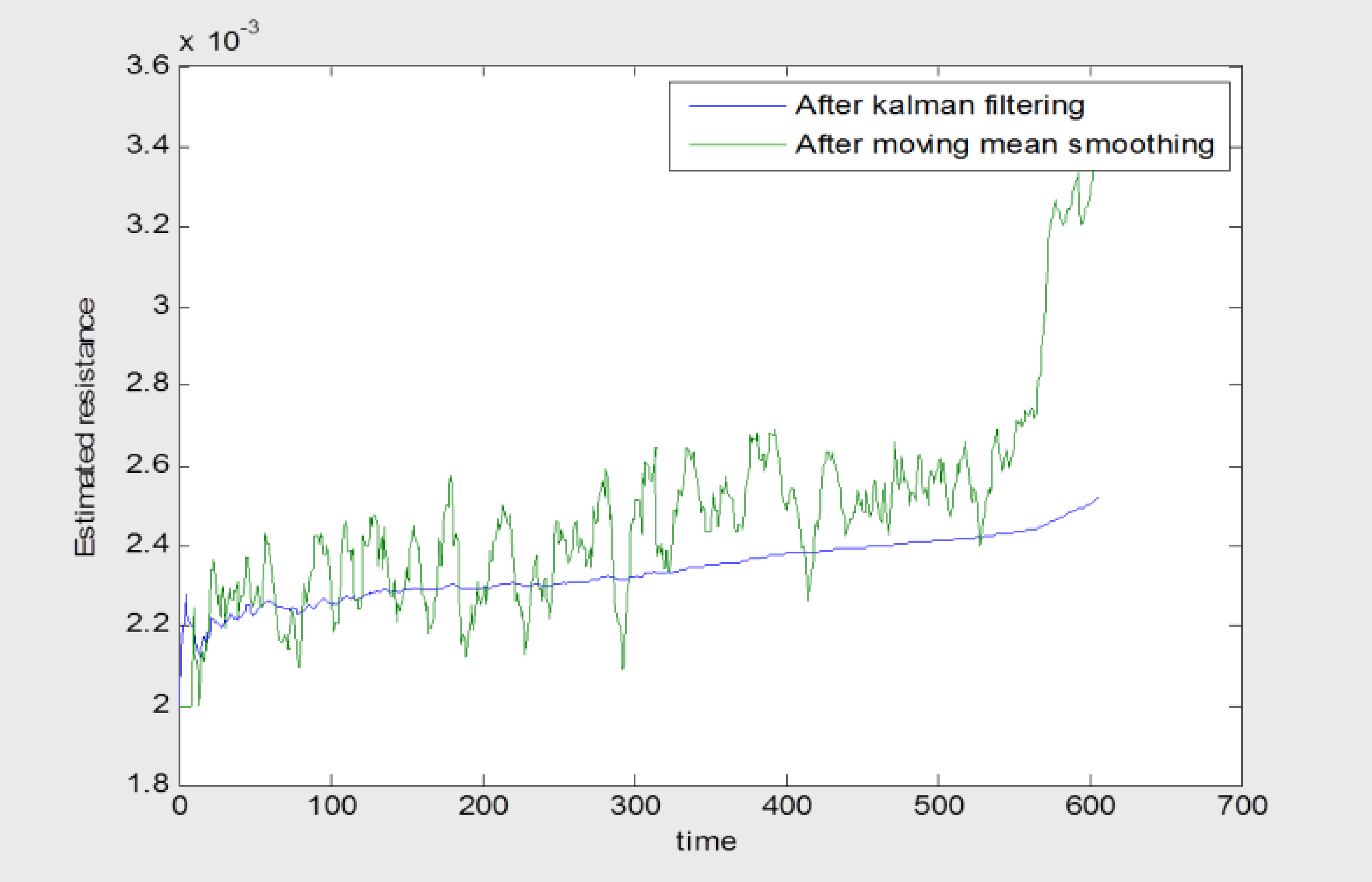
Example: What is the resistance in Ω / mile of a 1" diameter solid aluminum wire (at dc)?

$$R = \frac{2.65 \times 10^{-8} \ \Omega\text{-m}}{\pi \times 0.0127\text{m}^2} 1609 \frac{m}{mile} = 0.084 \frac{\Omega}{mile}$$

Line Resistance, cont'd

- Because ac current tends to flow towards the surface of a conductor, the resistance of a line at 60 Hz is slightly higher than at dc.
- Resistivity and hence line resistance increase linearly as conductor temperature increases (changes is about 0.4% per degree C)
  - In some locations conductor temperatures can vary by up to 100° C!
- Because ACSR conductors are stranded, actual resistance, inductance and capacitance needs to be determined from tables.

Variation in Line Resistance Example



Time is in Minutes. Input was 30 second ICCP data. Conductor resistance increase by about 50% indicating a 50/0.4 = 125 degree C rise in temperature!

Magnetics Review

- Ampere's circuital law:

$$F = \oint_{\Gamma} \mathbf{H} \bullet d\mathbf{l} = I_e$$

F = mmf = magnetomtive force (amp-turns)

H = magnetic field intensity (amp-turns/meter)

d**l** = Vector differential path length (meters)

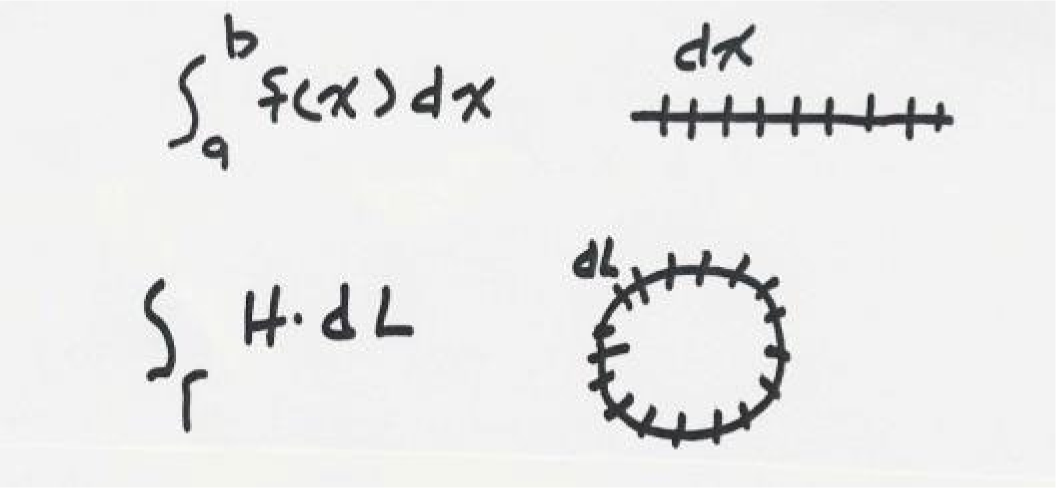
$\oint_{\Gamma}$ = Line integral about closed path Γ
(d**l** is tangent to path)

I_e = Algebraic sum of current linked by Γ

H is a vector

Line Integrals

- Line integrals are a generalization of traditional integration



Integration along the x-axis

Integration along a general path, which may be closed

Ampere’s law is most useful in cases of symmetry, such as with an infinitely long line

Magnetic Flux Density

Magnetic fields are usually measured in terms of flux density

$\mathbf{B}$ = flux density (Tesla [T] or Gauss [G]) (1T = 10,000G)

For a linear a linear magnetic material

$\mathbf{B} = \mu \mathbf{H}$ where μ is the called the permeability

B is also a vector

$\mu = \mu_0 \mu_r$ $\mu_0 =$ permeability of freespace $= 4\pi \times 10^{-7}\, H/m$

$\mu_r =$ relative permeability ≈ 1 for air

Total flux passing through a surface A is

$$\phi = \int_A \mathbf{B} \bullet d\mathbf{a}$$

$d\mathbf{a} =$ vector with direction normal to the surface

If flux density B is uniform and perpendicular to an area A then

$$\phi = BA$$

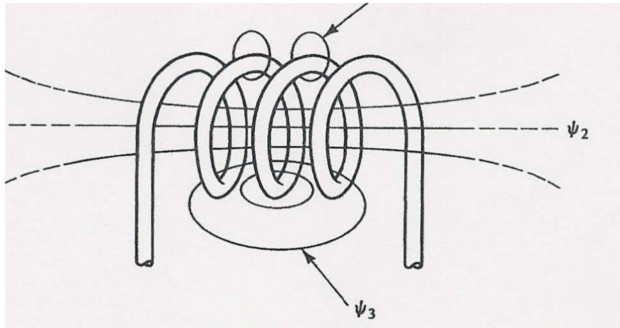
Flux Linkages, Faraday's Law and Inductance

- Flux linkages are defined from Faraday's law

$$V = \frac{d\lambda}{dt} \text{ where } V = \text{voltage}, \lambda = \text{flux linkages}$$

- The flux linkages tell how much flux is linking an N turn coil

$$\lambda = \sum_{i=1}^{N} \phi_i$$



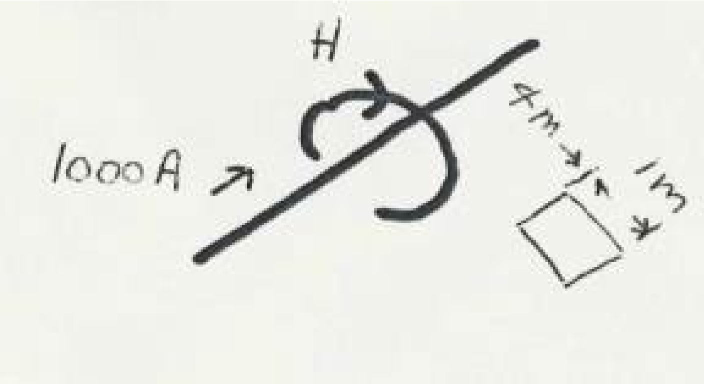
If all flux links every coil then $\lambda = N\,\phi$

If there is a single wire then $\lambda = \phi$

- For a linear magnetic system (where $\mathbf{B} = \mu\,\mathbf{H}$) then inductance, L, is defined at $\lambda = L\,I$
- L has units of Henrys (H)

Magnetic Fields from Single Wire

Assume we have an infinitely long wire with current of 1000A. How much magnetic flux passes through a 1 meter square, located between 4 and 5 meters from the wire?



Direction of **H** is given by the "Right-hand" Rule

Easiest way to solve the problem is to take advantage of symmetry. For an integration path we'll choose a circle with a radius of x.

Single Wire Example, cont'd

$$2\pi x H = I \quad \rightarrow H = \frac{I}{2\pi x} \qquad\qquad B = \mu_0 H$$

$$\phi = \int_A \mu_0 H \bullet dA = \int_4^5 \frac{\mu_0 I}{2\pi x} dx = \mu_0 \frac{I}{2\pi} \ln\frac{5}{4} = 2\times 10^{-7} I \ln\frac{5}{4}$$

$$\phi = 4.46\times 10^{-5}\,\mathrm{Wb}$$

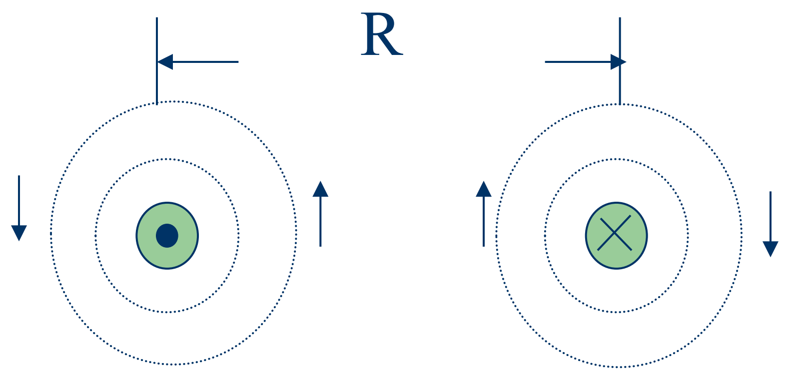
$$\mathrm{B} = \frac{2\times 10^{-4}}{x}\mathrm{T} = \frac{2}{\mathrm{x}}\mathrm{Gauss}$$

As x increases the total flux goes to infinity

Units of x are meters; for reference, the earth's magnetic field is about 0.6 Gauss (Central US)

Two Conductor Line Inductance

Key problem with the previous derivation is we assumed no return path for the current. Now consider the case of two wires, each carrying the same current I, but in opposite directions; assume the wires are separated by distance R.



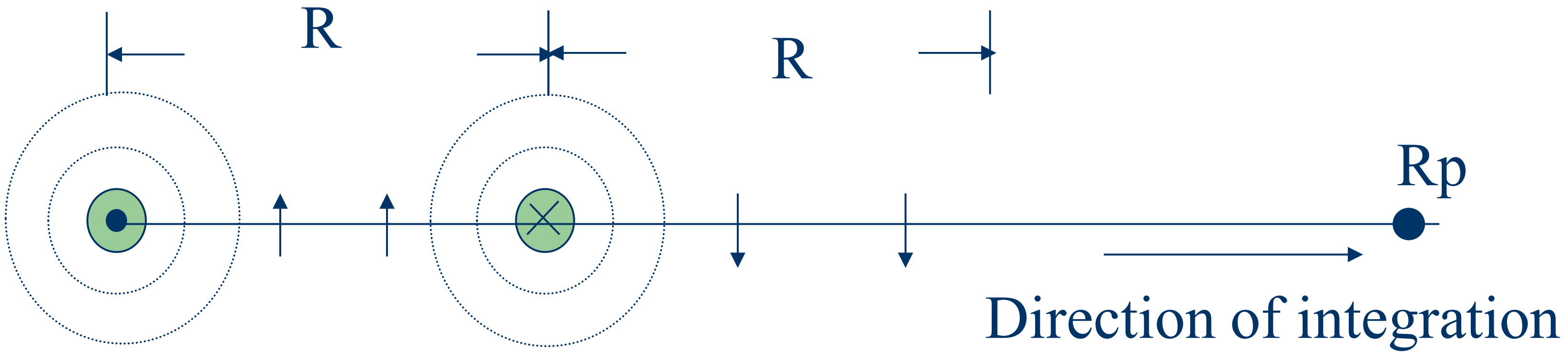
Creates counter-clockwise field

Creates a clockwise field

To determine the inductance of each conductor we integrate as before; however, now we get some field cancellation

We also need to consider what occurs inside the wire

Two Conductor Case, cont'd



Key Point: As we integrate for the left line, at distance 2R from the left line the net flux linked due to the Right line is zero! Use superposition to get total flux linkage.

For distance Rp, greater than 2R, from left line

$$\lambda_{\text{left}} = \underbrace{\frac{\mu_0}{2\pi} I \ln \frac{Rp}{r'}}_{\text{Left Current}} - \underbrace{\frac{\mu_0}{2\pi} I \ln\left(\frac{Rp - R}{R}\right)}_{\text{Right Current}}$$

Flux Linkages Inside the Wire

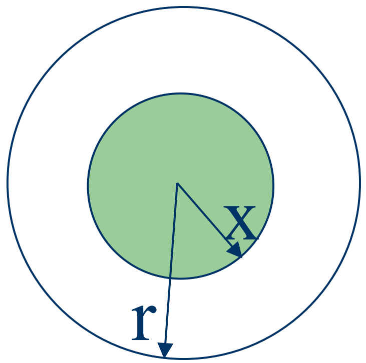
- Current inside a conductor tends to travel on the outside due to the skin effect; the penetration of current into a conductor is approximated using the skin depth

$$\text{skin depth} = \frac{1}{\sqrt{\pi \mathrm{f} \mu \sigma}}$$

For copper at 60 Hz the skin depth is about 0.33 inches

- Assuming uniform current density the flux linkage inside a wire is

$$\lambda_{\text{inside}} = \int_0^r \mu \frac{Ix}{2\pi r^2} \frac{x^2}{r^2} dx = \frac{\mu}{2\pi} \int_0^r \frac{Ix^3}{r^4} dx = \frac{\mu_0 \mu_r}{8\pi} I$$



- For a nonmagnetic wire (μ_r=1) this accounted for in the formula by using r′ ≈ 0.78 r

Two Conductor Inductance

Simplifying (with equal and opposite currents)

$$\lambda_{\text{left}} = \frac{\mu_0}{2\pi} I \left(\ln \frac{Rp}{r'} - \ln\left(\frac{Rp - R}{R} \right) \right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln Rp - \ln r' - \ln(Rp - R) + \ln R \right)$$

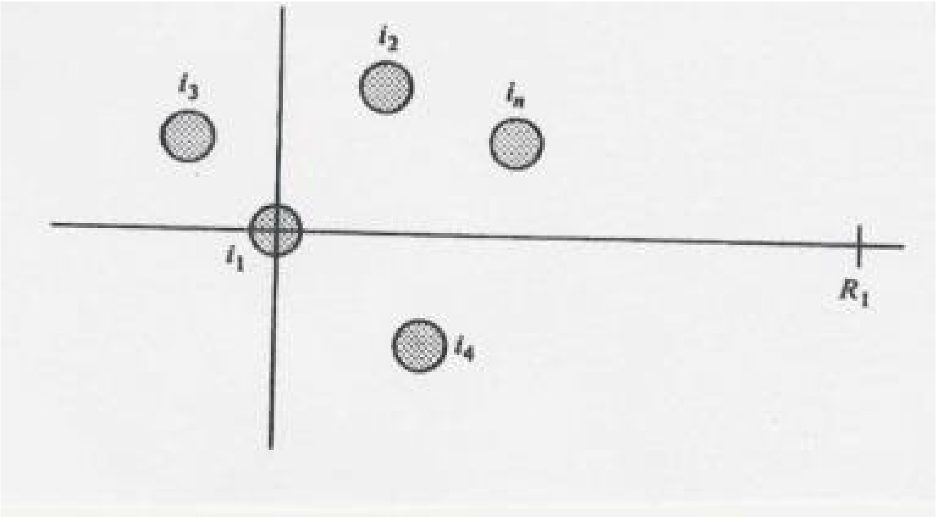
$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} + \ln \frac{Rp}{Rp - R} \right)$$

$$= \frac{\mu_0}{2\pi} I \left(\ln \frac{R}{r'} \right) \quad \text{as Rp} \rightarrow \infty$$

$$L_{left} = \frac{\mu_0}{2\pi} \left(\ln \frac{R}{r'} \right) \ \text{H/m}$$

Many-Conductor Case

Now assume we now have k conductors, each with current i_k, arranged in some specified geometry. We'd like to find flux linkages of each conductor.



Each conductor's flux linkage, λ_k, depends upon its own current and the current in all the other conductors.

To derive λ_1 we'll be integrating from conductor 1 (at origin) to the right along the x-axis.

Many-Conductor Case, cont'd

Therefore if $\sum_{j=1}^{n} i_j = 0$, which is true in a balanced three-phase system,

then the second term is zero and

$$\lambda_1 = \frac{\mu_0}{2\pi}\left[i_1 \ln \frac{1}{r_1^{'}} + i_2 \ln \frac{1}{d_{12}} + \cdots + i_n \ln \frac{1}{d_{1n}} \right]$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \cdots + L_{1n} i_n$$

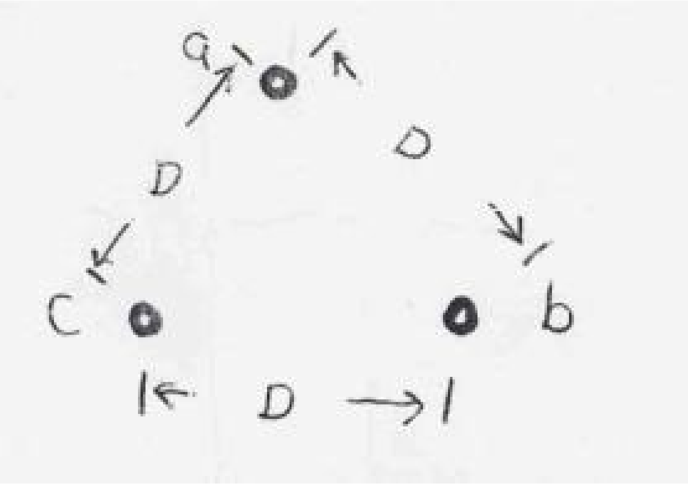
System has self and mutual inductance. However the mutual inductance

can be canceled for balanced three-phase systems with symmetry.

Symmetric Line Spacing – 69 kV

Line Inductance Example

Calculate the reactance for a balanced 3ϕ, 60Hz transmission line with a conductor geometry of an equilateral triangle with D = 5m, r = 1.24 cm (Rook conductor) and a length of 5 miles.



Since system is assumed balanced $i_a = -i_b - i_c$

$$\lambda_a = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{r'}\right) + i_b \ln\left(\frac{1}{D}\right) + i_c \ln\left(\frac{1}{D}\right)\right]$$

Line Inductance Example, cont'd

Substituting

$$\mathrm{i_a} = -i_b - i_c$$

Hence

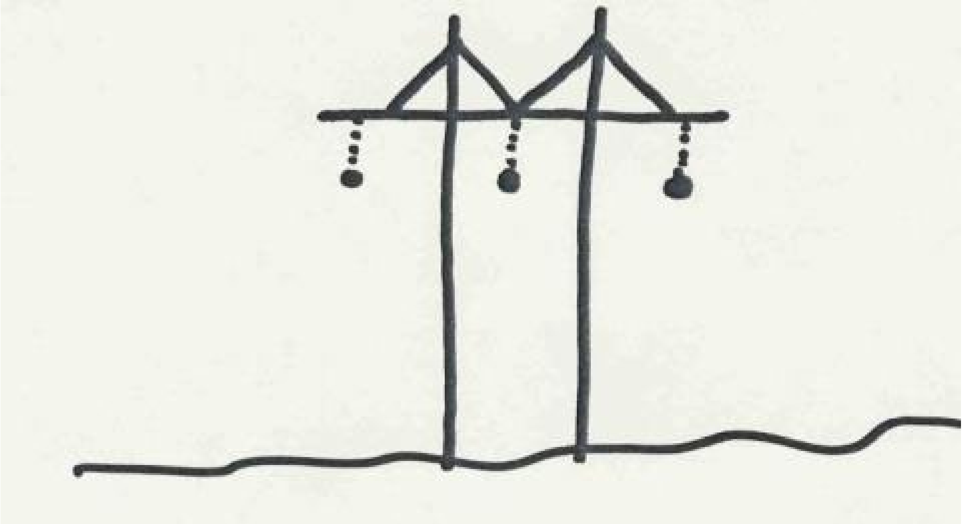
$$\lambda_{\mathrm{a}} = \frac{\mu_0}{2\pi}\left[i_a \ln\left(\frac{1}{r'}\right) - i_a \ln\left(\frac{1}{D}\right)\right] = \frac{\mu_0}{2\pi} i_a \ln\left(\frac{D}{r'}\right)$$

$$L_a = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right) = \frac{4\pi\times 10^{-7}}{2\pi}\ln\left(\frac{5}{9.67\times 10^{-3}}\right) = 1.25\times 10^{-6} \text{ H/m}$$

This is the equation we need for ECEN 460 (almost!)

Transmission Tower Configurations

- The problem with the line analysis we've done so far is we have assumed a symmetrical tower configuration. Such a tower figuration is seldom practical.



Typical Transmission Tower Configuration

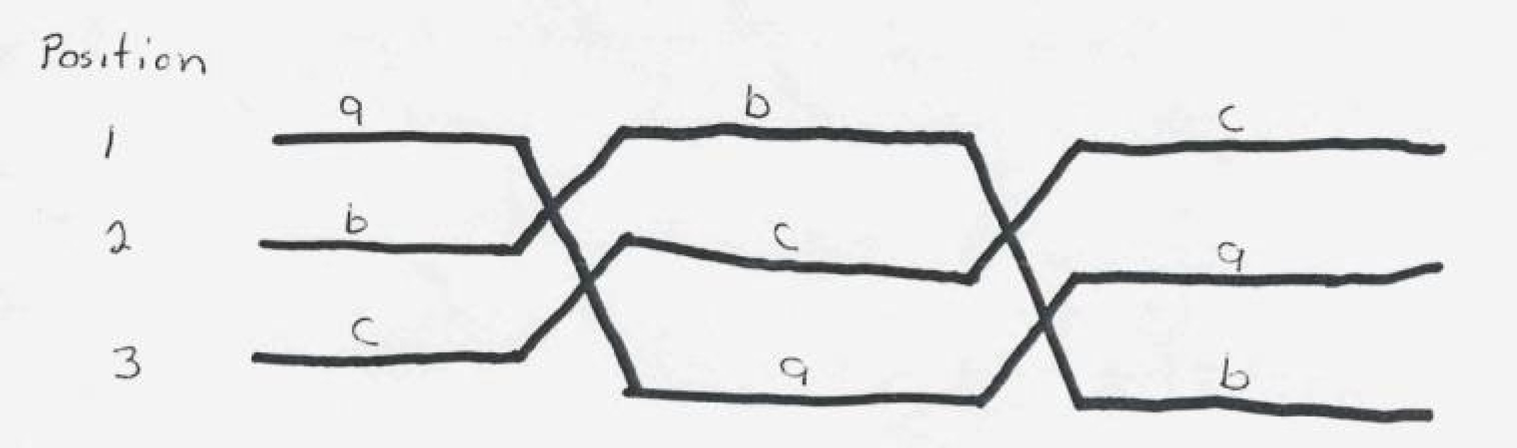
Therefore in general

$D_{ab} \neq D_{ac} \neq D_{bc}$

Unless something was done this would result in unbalanced phases

Transposition

- To keep system balanced, over the length of a transmission line the conductors are rotated so each phase occupies each position on tower for an equal distance. This is known as transposition.



Aerial or side view of conductor positions over the length of the transmission line.

Inductance of Transposed Line

Define the geometric mean distance (GMD)

$$D_m \triangleq (d_{12} d_{13} d_{23})^{1/3}$$

Then for a balanced 3ϕ system $(I_a = -I_b - I_c)$

$$\lambda_a = \frac{\mu_0}{2\pi}\left[I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D_m}\right] = \frac{\mu_0}{2\pi} I_a \ln\frac{D_m}{r'}$$

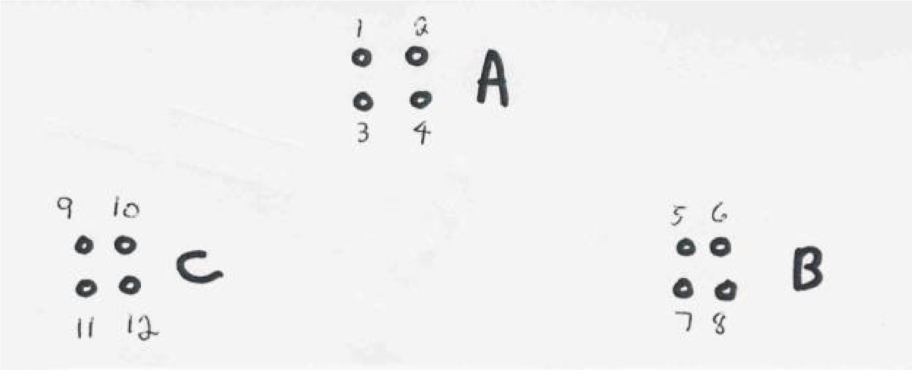
Hence

$$L_a = \frac{\mu_0}{2\pi}\ln\frac{D_m}{r'} = 2\times 10^{-7}\ln\frac{D_m}{r'} \ \text{H/m}$$

This is the equation we need for ECEN 460 (almost!)

Conductor Bundling

- To increase the capacity of high voltage transmission lines it is very common to use a number of conductors per phase. This is known as conductor bundling. Typical values are two conductors for 345 kV lines, three for 500 kV and four for 765 kV.



Conductors in the bundle have the same voltage

Photo Sources: BPA (three conductor 500 kV bundling) and American Electric Power (six conductor 765 kV)

Inductance of Lines with Bundled Conductors

- The per phase inductance is

$$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{R_b}\right)$$

This is the equation we need for ECEN 460

where

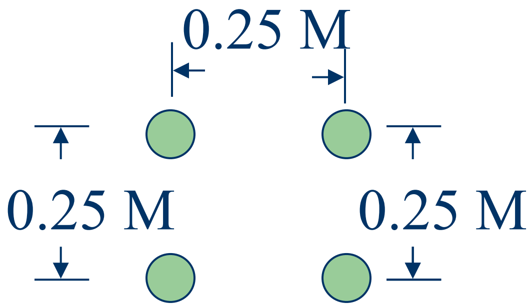
$$R_b \triangleq \text{ geometric mean radius (GMR) of bundle}$$

$$= (r' d_{12} \ldots d_{1b})^{1/b} \text{ in general}$$

- When calculating the per phase resistance of bundled lines, the total resistance is R per conductor divided by b, where b is the number of conductors in the bundle

Bundled Conductor Inductance Example

- Consider the previous example of the three phases symmetrically spaced 5 meters apart using wire with a radius of r = 1.24 cm. Except now assume each phase has 4 conductors in a square bundle, spaced 0.25 meters apart. What is the new inductance per meter?



$$r = 1.24\times 10^{-2}\ \text{m} \quad r' = 9.67\times 10^{-3}\ \text{m}$$

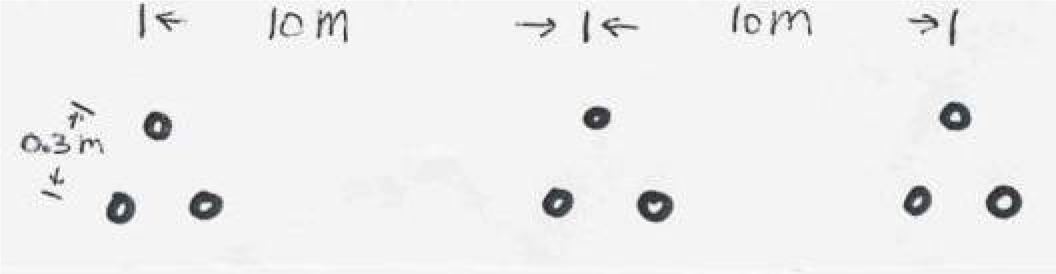
$$\text{R}_\text{b} = \left(9.67\times 10^{-3}\times 0.25\times 0.25\times\sqrt{2}\times 0.25\right)^{1/4}$$

$$= 0.12\ \text{m}\ \ \text{(ten times bigger!)}$$

$$\text{L}_\text{a} = \frac{\mu_0}{2\pi}\ln\frac{5}{0.12} = 7.46\times 10^{-7}\ \text{H/m}$$

Second Inductance Example

- Calculate the per phase inductance and reactance of a balanced 3ϕ, 60 Hz, line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1cm radius.



Answer: D_m = 12.6 m, R_b= 0.0889 m
Inductance = 9.9 x 10^{-7} H/m, Reactance = 0.6 Ω/Mile

Line Capacitance

- High voltage transmission lines and cables can have significant capacitance; capacitance increases as the lines get closer (e.g., in cables)
- For the case of uniformly transposed lines we use the same Dm and a similar GMR as with inductance

$$C = \frac{2\pi\varepsilon}{\ln \frac{D_m}{R_b^c}}$$

where $\mathrm{D_m} = \left[d_{ab} d_{ac} d_{bc}\right]^{1/3}$

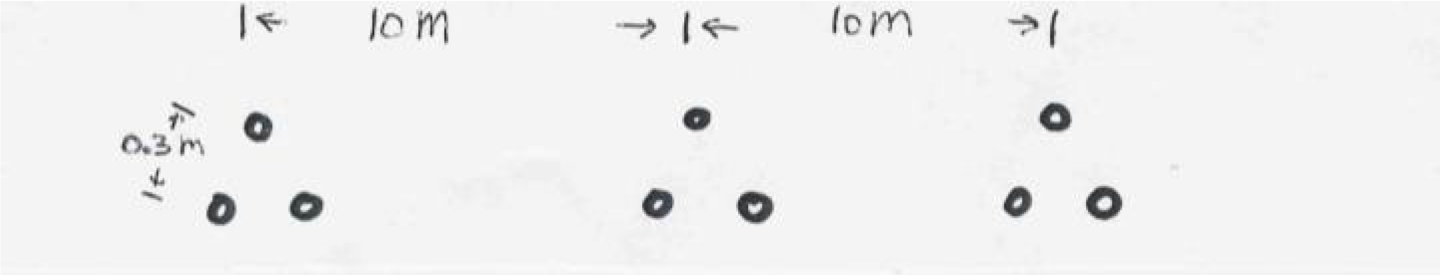
$$\mathrm{R_b^c} = (r d_{12} \cdots d_{1n})^{1/n} \quad \text{(note r NOT r')}$$

$$\varepsilon \text{ in air} = \varepsilon_o = 8.854 \times 10^{-12} \text{ F/m}$$

This is the equation we need for ECEN 460; if interested, see the book for the derivation

Line Capacitance Example

- Calculate the per phase capacitance and susceptance of a balanced 3ϕ, 60 Hz, transmission line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a a 1cm radius.



Line Capacitance Example, cont'd

$$R_b^c = (0.01\times 0.3\times 0.3)^{1/3} = 0.0963\text{ m}$$

$$\text{D}_\text{m} = (10\times 10\times 20)^{1/3} = 12.6\text{ m}$$

$$C = \frac{2\pi\times 8.854\times 10^{-12}}{\ln\frac{12.6}{0.0963}} = 1.141\times 10^{-11}\text{F/m}$$

$$\text{X}_\text{c} = \frac{1}{\omega C} = \frac{1}{2\pi 60\times 1.141\times 10^{-11}\text{F/m}}$$

$$= 2.33\times 10^{8}\ \Omega\text{-m (not }\Omega\text{ / m)}$$

ACSR Table Data (Similar to Table A.4)

TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES (*Cont'd*)

Code Word	Size (kcmil)	Stranding Al./St.	Number of Aluminum Layers	Resistance: dc 20°C (Ohms/ Mile)	Resistance: ac-60 Hz 25°C (Ohms/ Mile)	Resistance: ac-60 Hz 50°C (Ohms/ Mile)	Resistance: ac-60 Hz 75°C (Ohms/ Mile)	GMR (ft)	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Inductive Ohms/ Mile X_a	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Capacitive Megohm-Miles X'_a
Flicker	477	24/7	2	0.1889	0.194	0.213	0.232	0.0283	0.432	0.0992
Hawk	477	26/7	2	0.1883	0.193	0.212	0.231	0.0290	0.430	0.0988
Hen	477	30/7	2	0.1869	0.191	0.210	0.229	0.0304	0.424	0.0980
Osprey	556.5	18/1	2	0.1629	0.168	0.184	0.200	0.0284	0.432	0.0981
Parakeet	556.5	24/7	2	0.1620	0.166	0.183	0.199	0.0306	0.423	0.0969
Dove	556.5	26/7	2	0.1613	0.166	0.182	0.198	0.0313	0.420	0.0965
Eagle	556.5	30/7	2	0.1602	0.164	0.180	0.196	0.0328	0.415	0.0957
Peacock	605	24/7	2	0.1490	0.153	0.168	0.183	0.0319	0.418	0.0957
Squab	605	26/7	2	0.1485	0.153	0.167	0.182	0.0327	0.415	0.0953

GMR is equivalent to r'

Inductance and Capacitance assume a Dm of 1 ft.

Dove Example

$$GMR = 0.0313 \text{ feet}$$

$$\text{Outside Diameter} = 0.07725 \text{ feet (radius} = 0.03863)$$

Assuming a one foot spacing at 60 Hz

$$X_a = 2\pi 60 \times 2 \times 10^{-7} \times 1609 \times \ln \frac{1}{0.0313} \ \Omega/\text{mile}$$

$X_a = 0.420 \ \Omega/\text{mile}$, which matches the table

For the capacitance

$$X_C = \frac{1}{f} \times 1.779 \times 10^6 \ln \frac{1}{r} = 9.65 \times 10^4 \ \Omega\text{-mile}$$

Additional Transmission Topics

- **Multi-circuit lines**: Multiple lines often share a common transmission right-of-way. This does cause mutual inductance and capacitance, but is often ignored in system analysis.
- **Cables:** There are about 3000 miles of underground ac cables in U.S (out of a total of about 500,000 miles (100 kV and above). Cables are primarily used in urban areas. In a cable the conductors are tightly spaced, (< 1ft) with oil impregnated paper commonly used to provide insulation
  - inductance is lower
  - capacitance is higher, grealy limiting cable length

Total miles source is North American Reliability Corporation (NERC), *2024 State of Reliability*, June 2024

Additional Transmission Topics

- **Ground wires:** Transmission lines are usually protected from lightning strikes with a ground wire. This topmost wire (or wires) helps to attenuate the transient voltages/currents that arise during a lighting strike. The ground wire is typically grounded at each pole.
- **Corona discharge:** Due to high electric fields around lines, the air molecules become ionized. This causes a crackling sound and may cause the line to glow!
- **Shunt conductance:** Usually ignored. A small current may flow through contaminants on insulators.

Additional Transmission Topics

- **DC Transmission:** Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical for several specialized applications
  - long distance overhead power transfer (> 400 miles)
  - long cable power transfer such as underwater
  - providing an asynchronous means of joining different power systems (such as ERCOT to Eastern or Western grids)

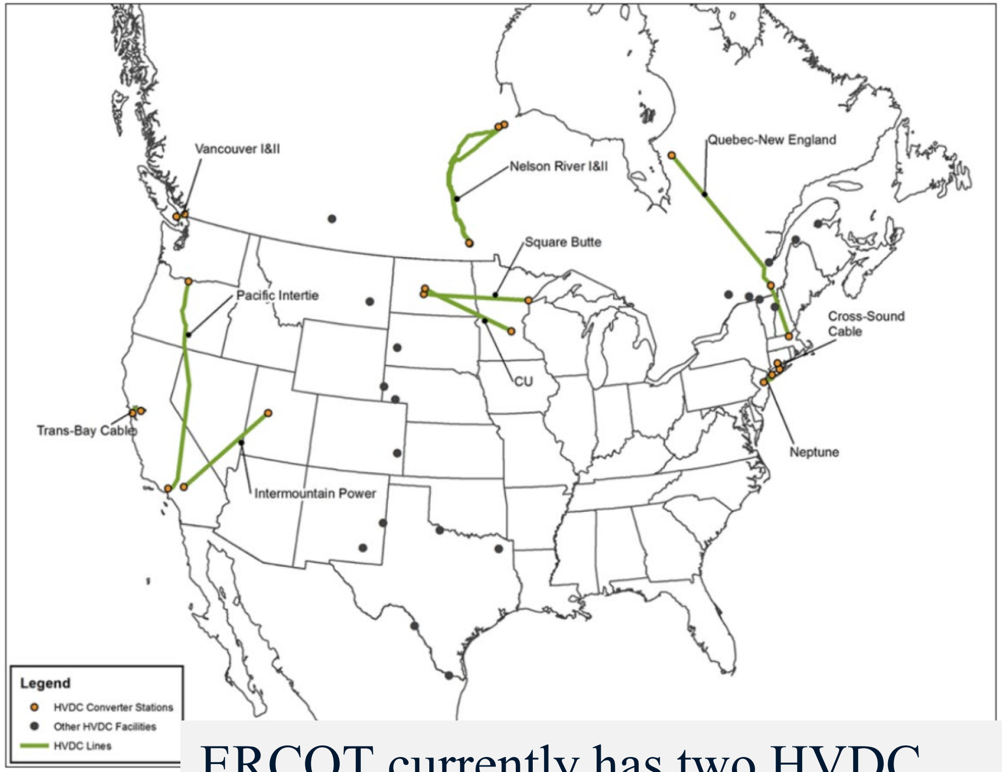


Image source (September 2023):
www.energy.gov/oe/articles/connecting-country-hvdc

ERCOT currently has two HVDC interconnections with the EI (820 MW total) and two with Mexico (400 MW)

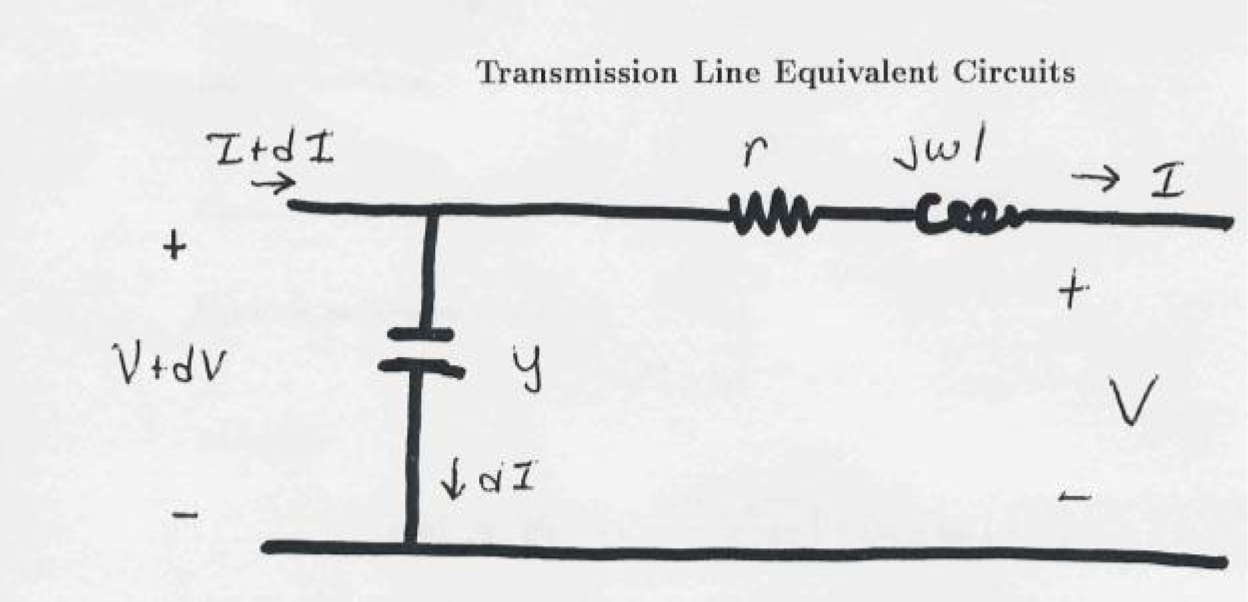
In the News: Southern Spirit HVDC

- There are plans to build a 3000 MW, 320 mile long HVDC transmission line (operating at about 500 kV) to transfer power between Texas [ERCOT] and Mississippi)
- This would more than triple the amount of transfer capacity between ERCOT and the Eastern Interconnect (EI)
- Such an HVDC line has been in the works for more than a decade; the current target is to begin construction by 2028

www.houstonchronicle.com/projects/2025/southern-spirit-transmission-line-texas/ (Claire Hao, Jan 2025)

Transmission Line Models: An Equivalent Circuit

- Our current model of a transmission line is shown below



Units on z and y are per unit length

For operation at frequency ω, let $z = r + j\omega L$ and $y = g + j\omega C$ (with g usually equal 0)

Propagation Constant, Characteristic and Surge Impedance

Define the propagation constant γ as

$$\gamma = \sqrt{yz} \quad = \quad \alpha + j\beta$$

where

$$\alpha \quad = \quad \text{the attenuation constant}$$

$$\beta \quad = \quad \text{the phase constant}$$

Define $Z_c \; = \; \sqrt{\dfrac{z}{y}} \; = \;$ characteristic impedance

Lossless Transmission Lines

For a lossless line the characteristic impedance, Z_c,

is known as the surge impedance.

$$Z_c = \sqrt{\frac{jwl}{jwc}} = \sqrt{\frac{l}{c}}\ \Omega \ \text{(a real value)}$$

If a lossless line is terminated at bus R in impedance

$$Z_c = \frac{V_R}{I_R}$$

Then $I_R Z_c = V_R$ so we get...