ECEN 460
Power System Operation and Control
Spring 2025

Lecture 6: Transmission Lines, Transformers, Per Unit

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Transmission Line Insulators

- The overhead transmission distribution lines are bare conductors, so they need to be connected to the grounded towers using insulators
- Different types of insulators are used depending on the application. Examples include
  - Pin insulators for low voltage (up to 33 kV) applications
  - Suspension insulators, which have a number of porcelain disks connected together, with each having about 11 kV of insulation
  - Strain insulators, which are designed for corners, in which they have to handle the line strain; they are an assembly of suspension insulators

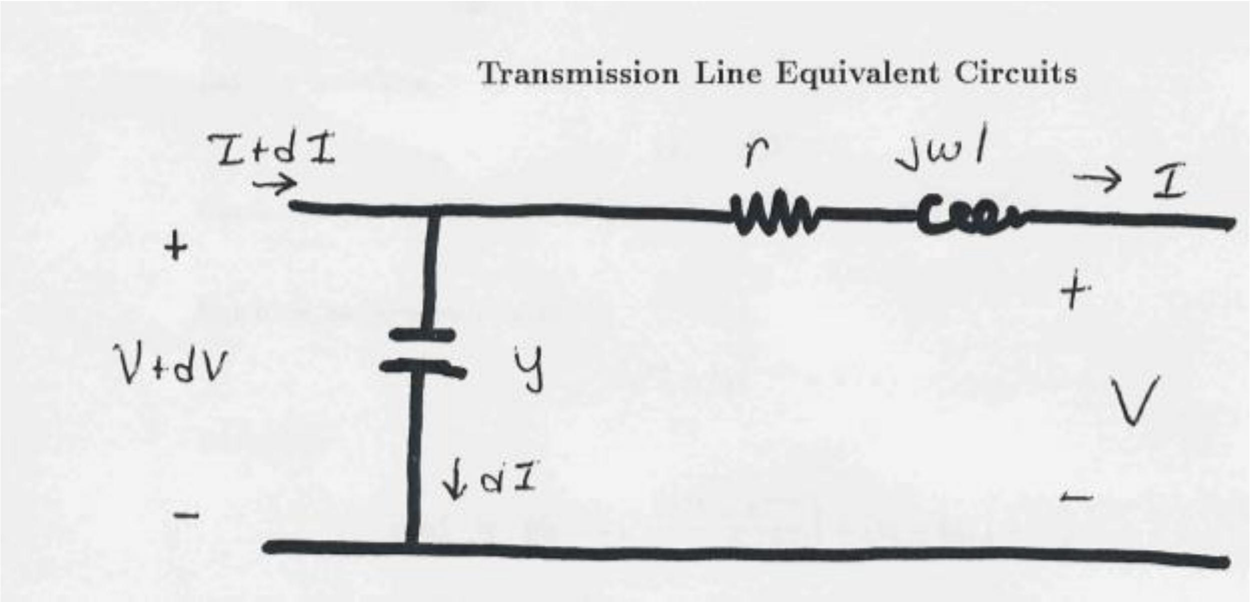
Pin Insulator

Suspension and strain insulators

Image source: www.electricaleasy.com/2016/10/insulators-used-in-overhead-power-lines.html

Transmission Line Models: An Equivalent Circuit

- Our current model of a transmission line is shown below



Units on z and y are per unit length

For operation at frequency ω, let $z = r + j\omega L$

and $y = g + j\omega C$ (with g usually equal 0)

Propagation Constant, Characteristic and Surge Impedance

Define the propagation constant γ as

$$\gamma = \sqrt{yz} \quad = \quad \alpha + j\beta$$

where

α = the attenuation constant

β = the phase constant

Define $Z_c = \sqrt{\dfrac{z}{y}}$

= characteristic impedance

Lossless Transmission Lines

For a lossless line the characteristic impedance, Z_c, is known as the surge impedance.

$$Z_c = \sqrt{\frac{jwl}{jwc}} = \sqrt{\frac{l}{c}}\ \Omega \quad \text{(a real value)}$$

If a lossless line is terminated at bus R in impedance

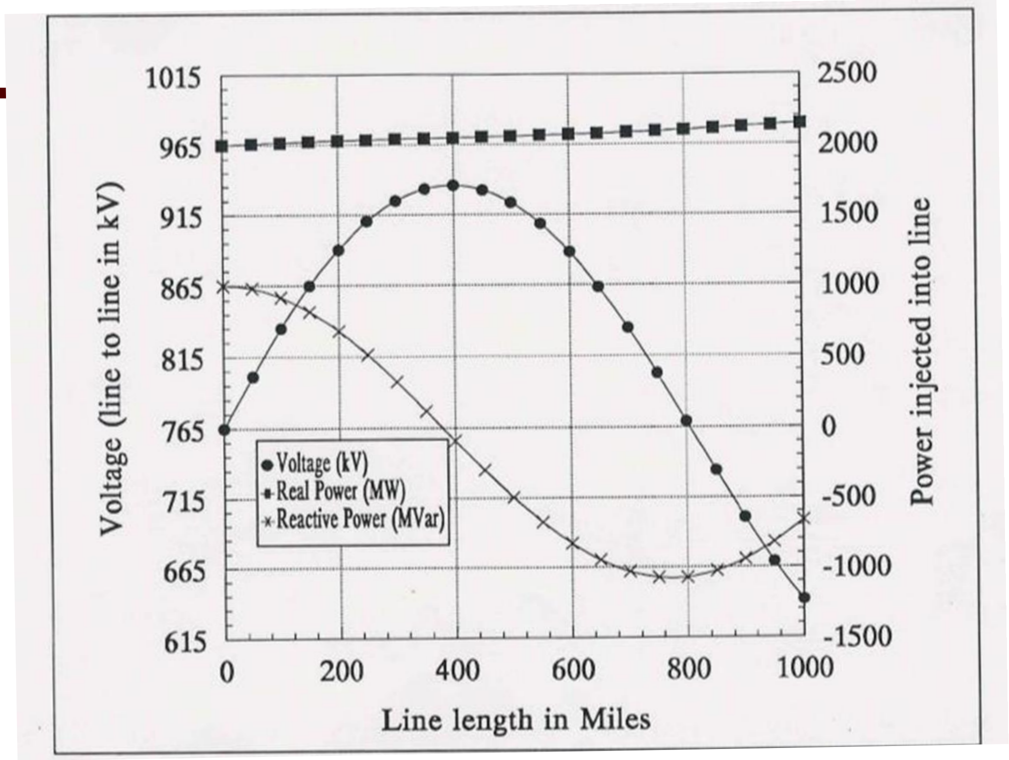
$$Z_c = \frac{V_R}{I_R}$$

Then $I_R Z_c = V_R$ so we get...

Lossless Transmission Lines

$$V(x) = V_R \cosh \gamma x + V_R \sinh \gamma x$$
$$I(x) = I_R \cosh \gamma x + I_R \sinh \gamma x$$
$$\frac{V(x)}{I(x)} = Z_c$$

Derived in Chapter 5,
but this derivation
is not part of ECEN 460



Define $\frac{|\mathrm{V(x)}|^2}{Z_c}$ as the surge impedance load (SIL). Since the line is lossless this implies

$$|V(x)| = |V_R|$$
$$|I(x)| = |I_R|$$

If P > SIL then line consumes vars; otherwise line generates vars.

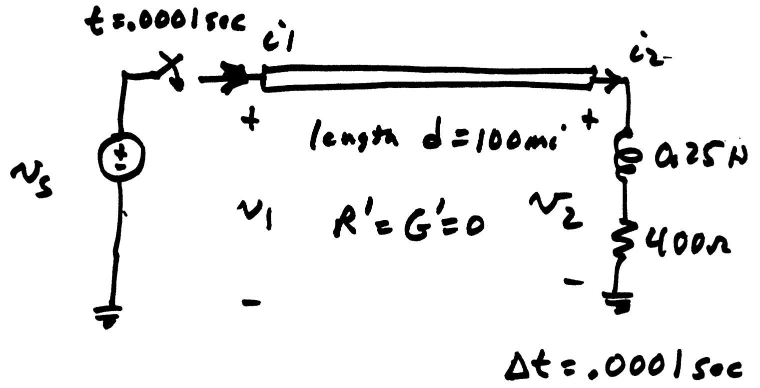
Velocity of Propagation in a Lossless line

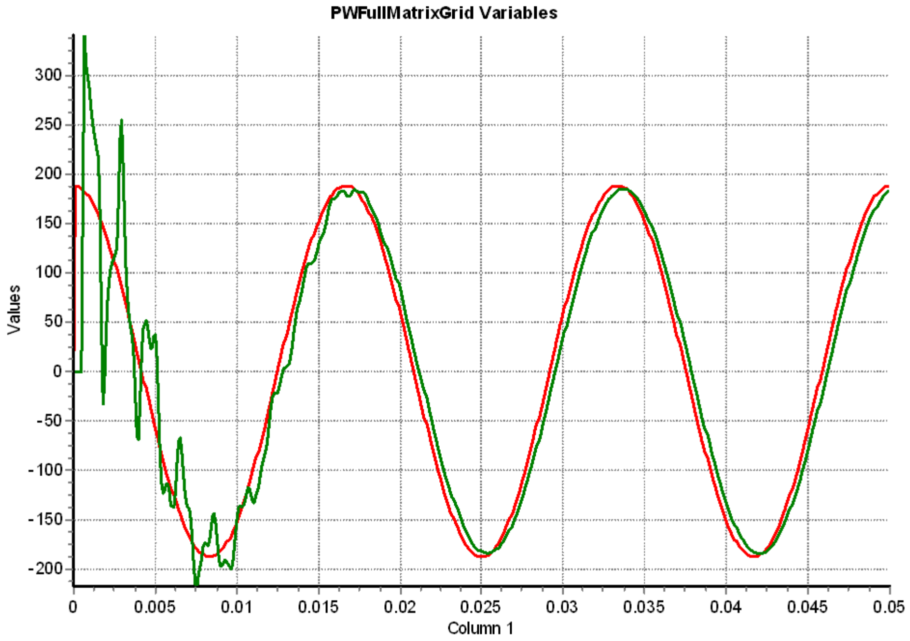
Velocity of propagation is close to the speed of light, but slightly less

$$L' = \frac{\mu_0}{2\pi}\ln\left(\frac{D}{r'}\right), \quad C' = \frac{2\pi\varepsilon_0}{\ln D/r}$$

$$v_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu_0\varepsilon_0 \frac{\ln D/r'}{\ln D/r}}} = c\frac{1}{\sqrt{\frac{\ln D/r'}{\ln D/r}}}$$

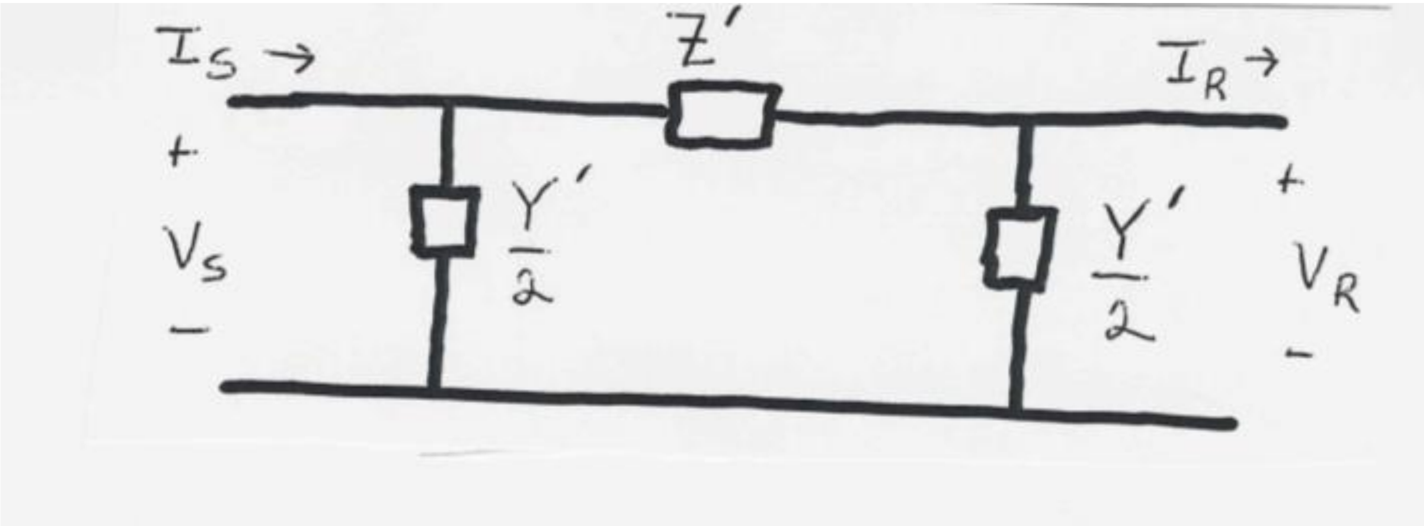
Switching dynamics (outside of 460, more 616)





Equivalent Circuit Model

The common representation is the π equivalent circuit



Next, we'll quickly cover simplified ways to determine the parameters Z' and Y'

Simplified Parameters

These values can be derived to be

$$Z' = Z_C \sinh \gamma l \quad = \sqrt{\frac{z\,l\,z}{y\,l\,z}} \sinh \gamma l$$

$$= Z \frac{\sinh \gamma l}{\gamma l} \quad \text{with Z □ zl} \quad (\text{recalling } \gamma = \sqrt{zy})$$

$$\frac{Y'}{2} = \frac{1}{Z_c} \tanh \frac{\gamma l}{2} \quad = \sqrt{\frac{y\,l\,y}{z\,l\,y}} \tanh \frac{\gamma l}{2}$$

$$= \frac{Y}{2} \frac{\tanh {}^{\gamma l}\!/_{2}}{{}^{\gamma l}\!/_{2}} \quad \text{with Y □ } yl$$

Simplified Parameters, cont.

For most lines make the following approximations:

$$Z' = Z \qquad \left(\text{assumes } \frac{\sinh \gamma l}{\gamma l} \approx 1\right)$$

$$\frac{Y'}{2} = \frac{Y}{2} \qquad \left(\text{assumes } \frac{\tanh(\gamma l/2)}{\gamma l/2} \approx 1\right)$$

Length	$\mathbf{\frac{\sinh \gamma l}{\gamma l}}$	$\mathbf{\frac{\tanh(\gamma l/2)}{\gamma l/2}}$
50 miles	$0.998\angle 0.02^\circ$	$1.001\angle -0.01^\circ$
100 miles	$0.993\angle 0.09^\circ$	$1.004\angle -0.04^\circ$
200 miles	$0.972\angle 0.35^\circ$	$1.014\angle -0.18^\circ$

Three Line Models

Long Line Model (longer than 250 miles)

use $Z' = Z\dfrac{\sinh \gamma l}{\gamma l}, \quad \dfrac{Y'}{2} = \dfrac{Y}{2}\dfrac{\tanh \gamma l/2}{\gamma l/2}$

Medium Line Model (less than 250 miles)

use Z and $\dfrac{Y}{2}$

Short Line Model (not used much but useful for insight)

use Z (i.e., assume Y is zero; it small for short or low voltage lines)

Once the parameters have been derived, the Long and Medium models are the same; on ECEN 460 we will almost exclusively be using the Medium Line Model

Power Transfer in Short Lines

Often we'd like to know the maximum power that could be transferred through a short transmission line

$$S_{12} = V_1 I_1^* = V_1 \left(\frac{V_1 - V_2}{Z} \right)^*$$

with $V_1 = |V_1| \angle \theta_1, \quad V_2 = |V_2| \angle \theta_2 \quad Z = |Z| \angle \theta_Z$

$$S_{12} = \frac{|V_1|^2}{|Z|} \angle \theta_Z - \frac{|V_1||V_2|}{|Z|} \angle \theta_Z + \theta_{12}$$

Power Transfer in Lossless Lines

If we assume a line is lossless with impedance jX and are just interested in the real power transfer then:

$$P_{12} + jQ_{12} = \frac{|V_1|^2}{|Z|}\angle 90° - \frac{|V_1||V_2|}{|Z|}\angle 90° + \theta_{12}$$

Since $-\cos(90° + \theta_{12}) = \sin\theta_{12}$, we get

$$P_{12} = \frac{|V_1||V_2|}{X}\sin\theta_{12}$$

Hence the maximum power transfer is

$$P_{12}^{Max} = \frac{|V_1||V_2|}{X}$$

Limits Affecting Max. Power Transfer

- Thermal limits
  - limit is due to heating of conductor and hence depends heavily on ambient conditions.
  - For many lines, sagging is the limiting constraint.
  - Newer conductors limit can limit sag, carrying more current and hence increasing the ratings for existing lines (once they are reconductored)
    - for example, 3M has ACCR (Aluminum Conductor Composite Reinforced) with the outer wires composed of a hardened aluminum-zirconium alloy, and the core having fiber-reinforced metal; they have half the thermal expansion and can operate continuously above 200° C!
    - Higher currents and operating temperatures mean higher losses!
  - Trees grow, and many will eventually hit lines if they are planted under the line.

Image source: www.3m.com/3M/en_US/p/d/b00014620/

Other Limits Affecting Power Transfer

- Angle limits
  - while the maximum power transfer occurs when line angle difference is 90 degrees, actual limit is substantially less due to multiple lines in the system
- Voltage stability limits
  - as power transfers increases, reactive losses increase as I^2X. As reactive power increases the voltage falls, resulting in a potentially cascading voltage collapse.
- In cables the higher charging capacitance, which increases with the length of the line, limits the amount of current that can be transferred through the line since some of the current is associated with the charging capacitance
  - Cables can be very sensitive to overloads since it is more of a challenge to dissipate heat

Transmission Line Series Compensation

- One way to increase the transmission capacity of a transmission line that is limited by its reactance is to add series compensation
  - Capacitors are placed in series with the transmission line

Image shows BPA series capacitors in a 500 kV line

Subsynchronous oscillations can be a concern; the natural frequency of a series LC circuit is given below; in the right equation where f_0 is the system frequency

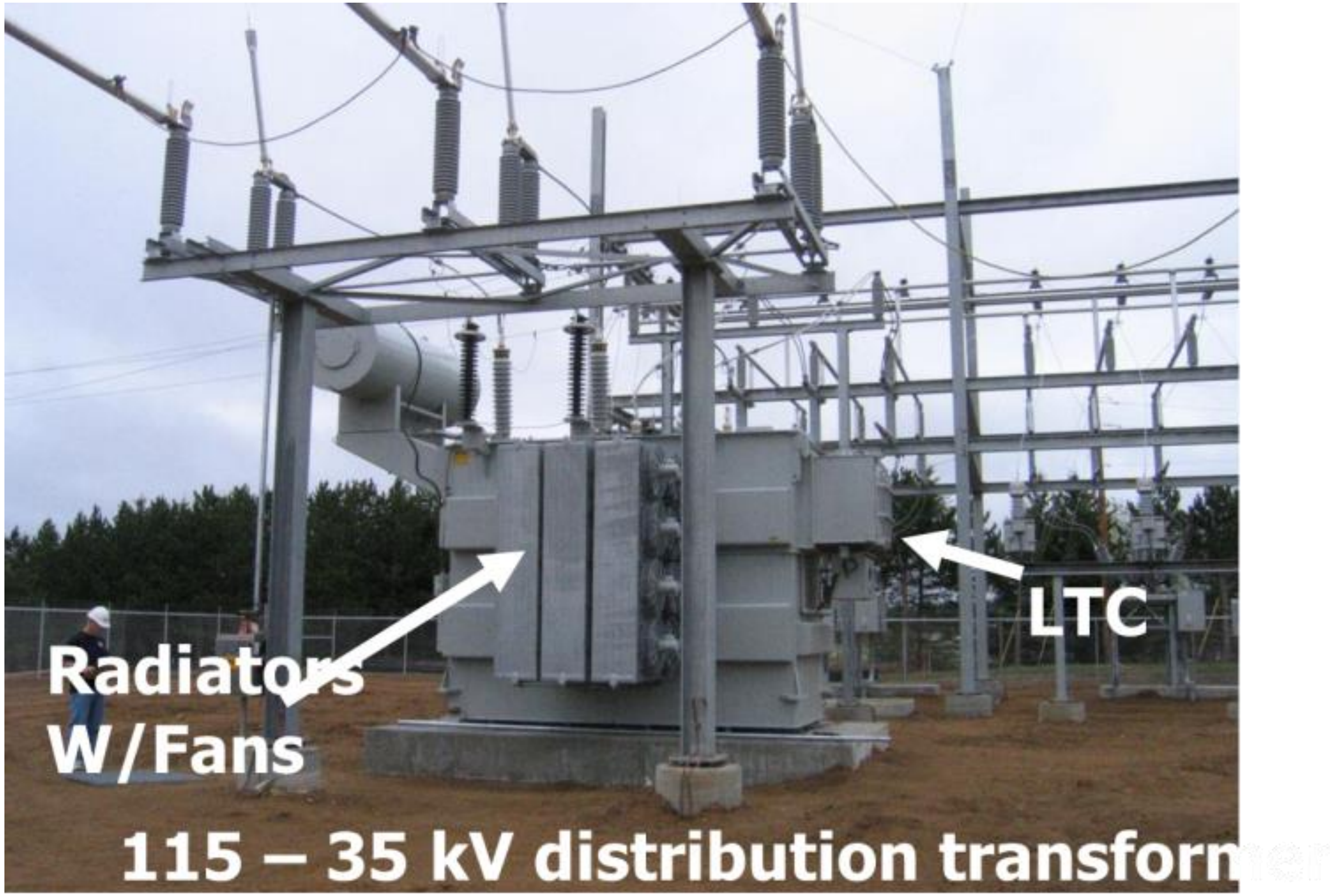
$$f_n = \frac{1}{2\pi\sqrt{LC}} = f_0\sqrt{\frac{X_C}{X_L}}$$

Image: https://www.bpa.gov/news/newsroom/Pages/Chief-engineers-reunite-reminisce-for-BPAs-75th.aspx

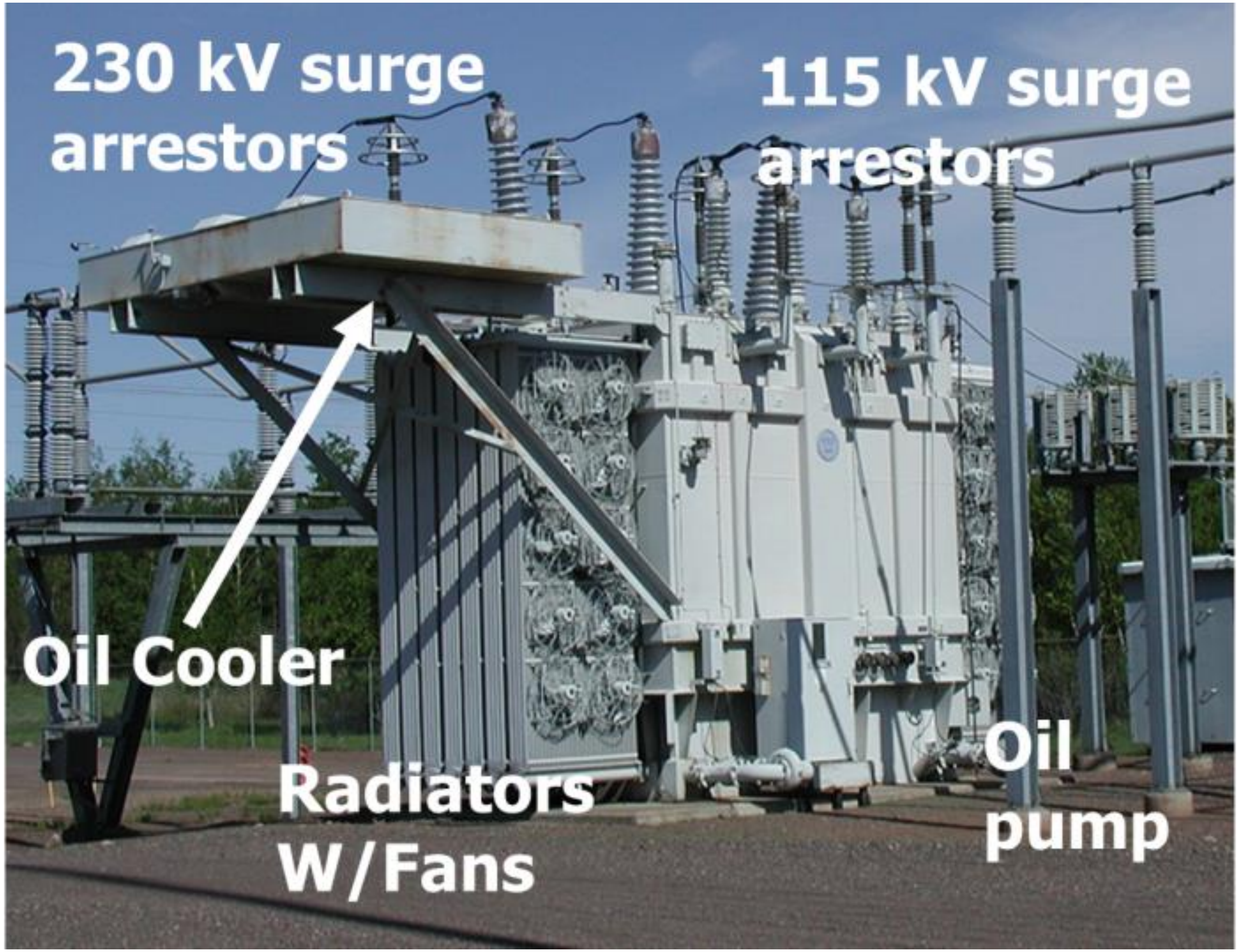
Transformers Overview

- Power systems are characterized by many different voltage levels, ranging from 765 kV down to 240/120 volts.
- Transformers are used to transfer power between different voltage levels.
- The ability to inexpensively change voltage levels is a key advantage of ac systems over dc systems.
- In this section we'll development models for the transformer and discuss various ways of connecting three phase transformers.

Transmission to Distribution Transformer



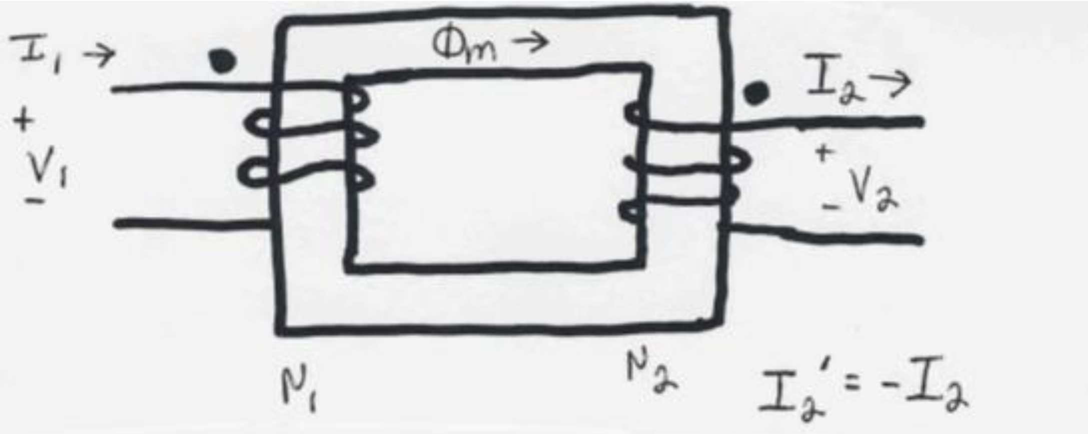
Transmission Level Transformer



Ideal Transformer

- First we review the voltage/current relationships for an ideal transformer
  - no real power losses
  - magnetic core has infinite permeability
  - no leakage flux
- We’ll define the “primary” side of the transformer as the side that usually takes power, and the secondary as the side that usually delivers power.
  - primary is usually the side with the higher voltage, but may be the low voltage side on a generator step-up transformer.

Ideal Transformer Relationships



Assume we have flux ϕ_m in magnetic material. Then

$$\lambda_1 = N_1\phi_m \qquad \lambda_2 = N_2\phi_m$$

$$v_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\phi_m}{dt} \qquad v_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\phi_m}{dt}$$

$$\frac{d\phi_m}{dt} = \frac{v_1}{N_1} = \frac{v_2}{N_2} \rightarrow \frac{v_1}{v_2} = \frac{N_1}{N_2} = a = \text{turns ratio}$$

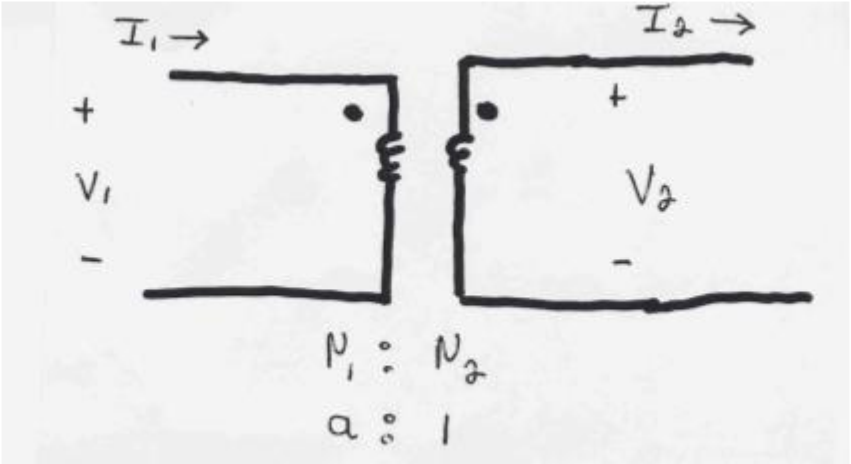
Current/Voltage Relationships

If μ is infinite then $0 = N_1 i_1 + N_2 i_2^{'}$. Hence

$$\frac{i_1}{i_2^{'}} = -\frac{N_2}{N_1} \quad \text{or} \quad \frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

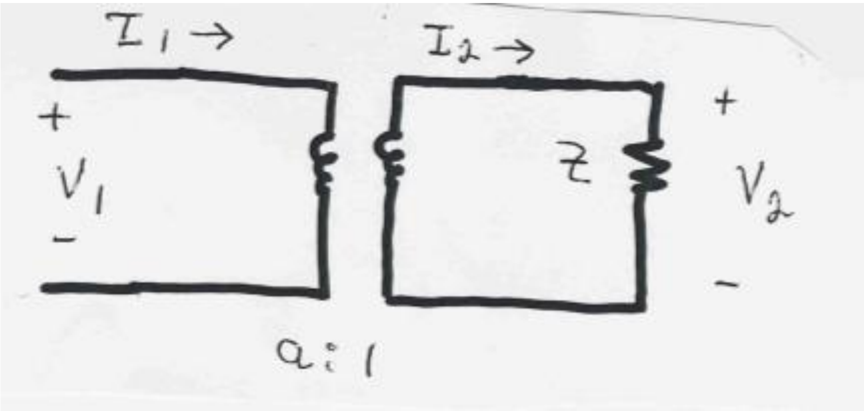
Then

$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ i_2 \end{bmatrix}$$



Impedance Transformation Example

- **Example:** Calculate the primary voltage and current for an impedance load on the secondary



$$\begin{bmatrix} v_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & \frac{1}{a} \end{bmatrix} \begin{bmatrix} v_2 \\ v_2/Z \end{bmatrix}$$

$$v_1 = a v_2 \qquad i_1 = \frac{1}{a}\frac{v_2}{Z}$$

$$\frac{v_1}{i_1} = a^2 Z$$

Real Transformers

- Real transformers
  - have losses
  - have leakage flux
  - have finite permeability of magnetic core

- Real power losses

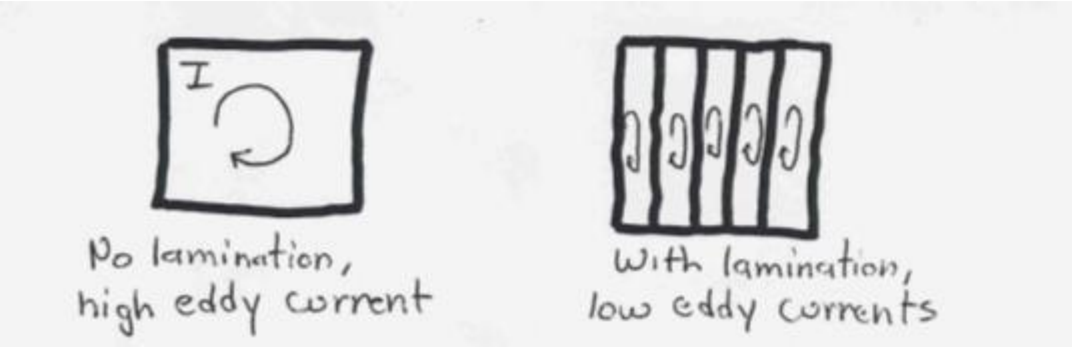
  resistance in windings ($i^2 R$)

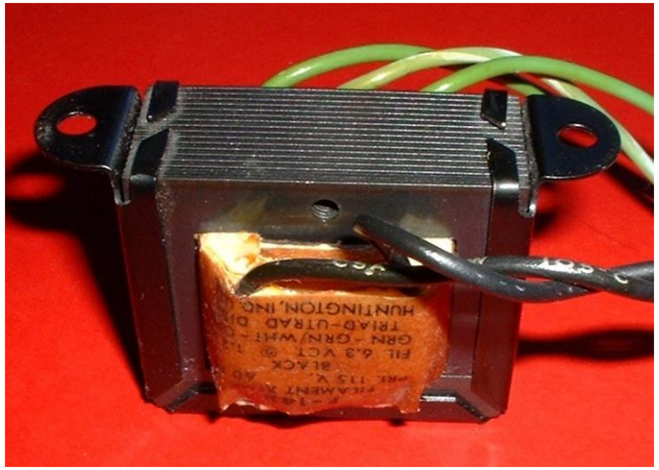
  core losses due to eddy currents and hysteresis

Transformer Core losses

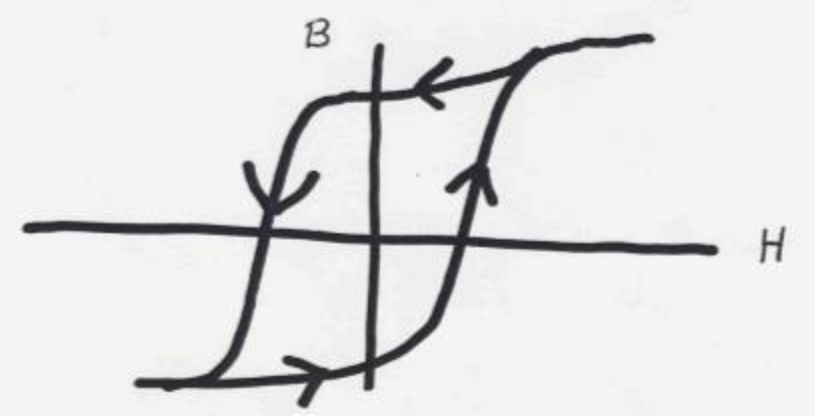
Eddy currents arise because of changing flux in core.

Eddy currents are reduced by laminating the core





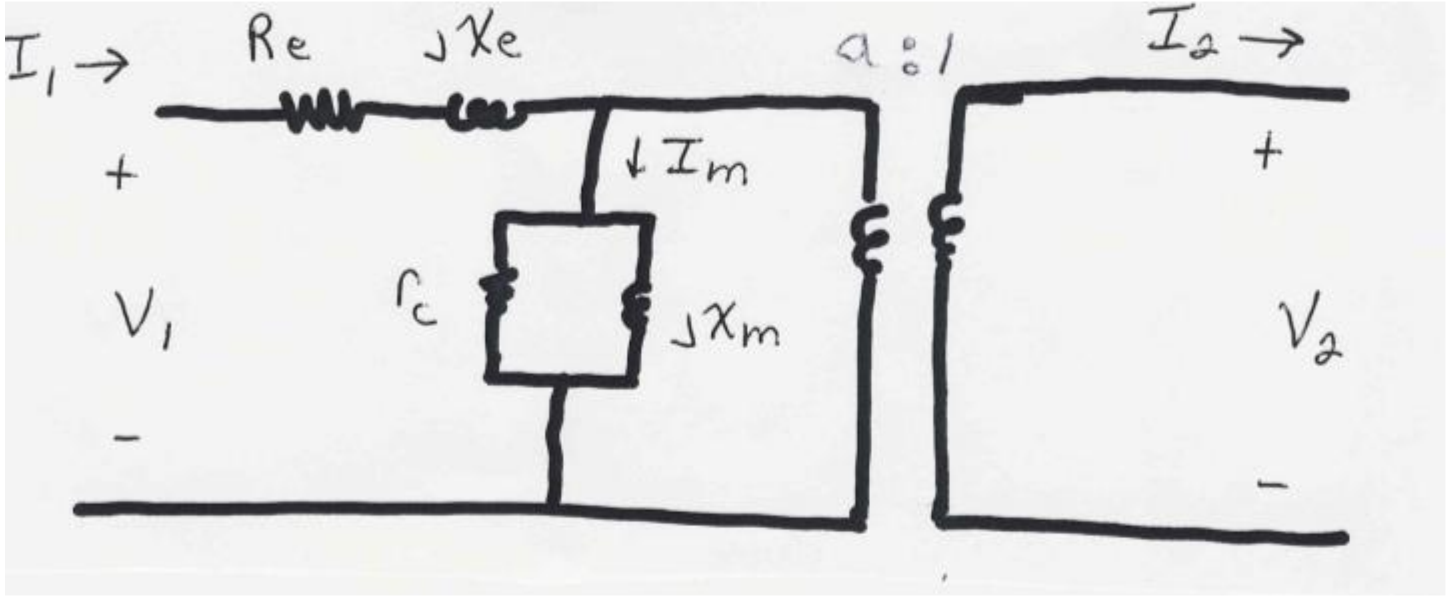
Hysteresis losses are proportional to area of BH curve and the frequency



These losses are reduced by using material with a thin BH curve

Image: en.wikipedia.org/wiki/Transformer_types#/media/File:Transformer.filament.agr.jpg

Simplified Equivalent Circuit



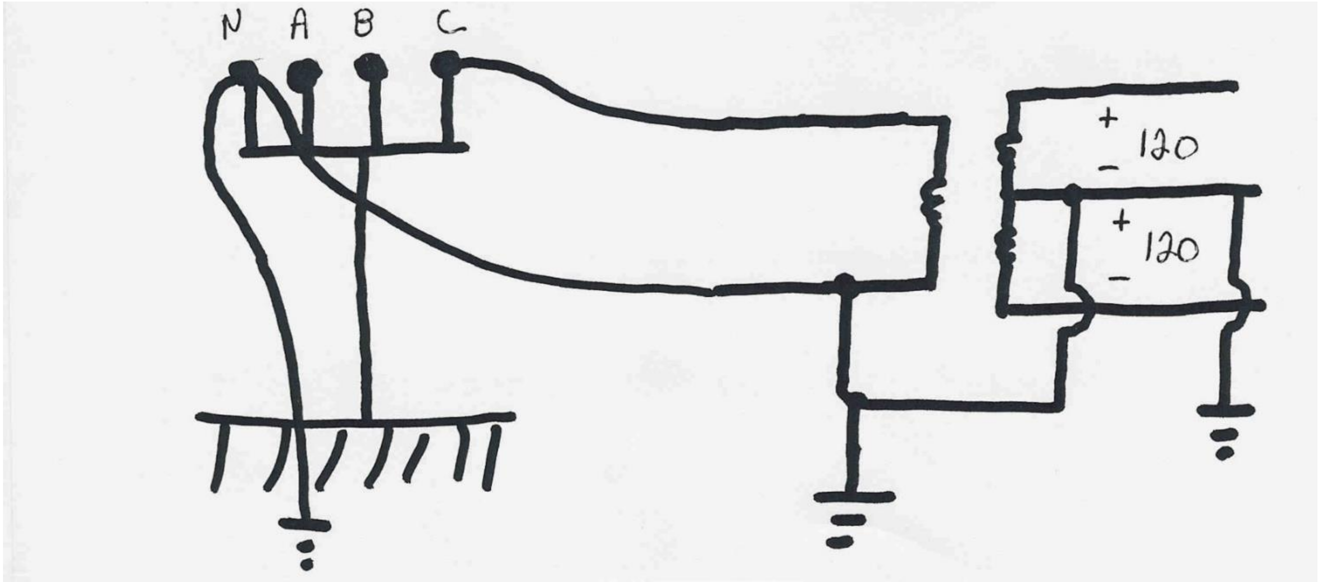
Often the shunt elements are assumed to have such high impedance that they are ignored

Calculation of Model Parameters

- The parameters of the model are determined based upon
  - nameplate data: gives the rated voltages and power
  - open circuit test: rated voltage is applied to primary with secondary open; measure the primary current and losses (the test may also be done applying the voltage to the secondary, calculating the values, then referring the values back to the primary side).
  - short circuit test: with secondary shorted, apply voltage to primary to get rated current to flow; measure voltage and losses.
- This topic is mostly outside the scope of ECEN 460, covered in ECEN 459

Residential Distribution Transformers

- Single phase transformers are commonly used in residential distribution systems. Most distribution systems are 4 wire, with a multi-grounded, common neutral.



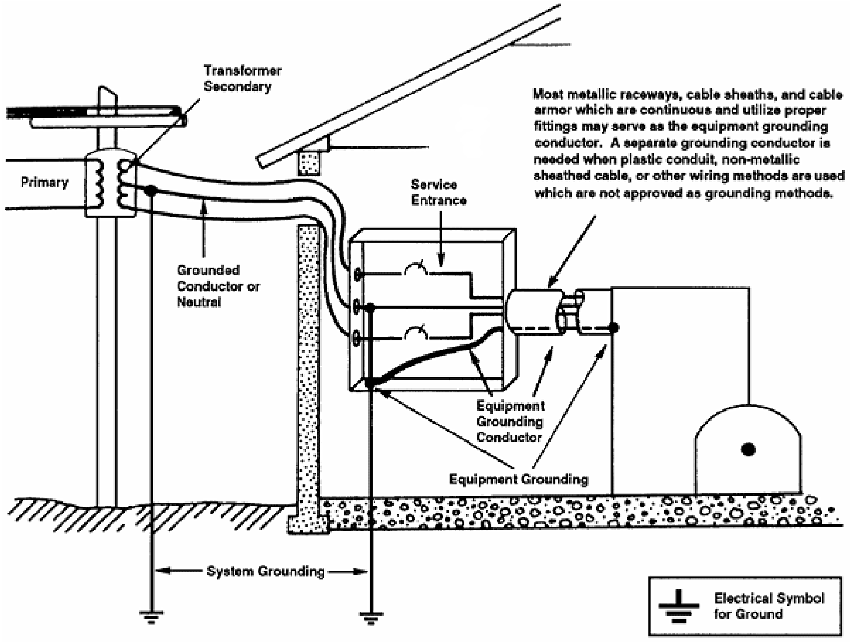
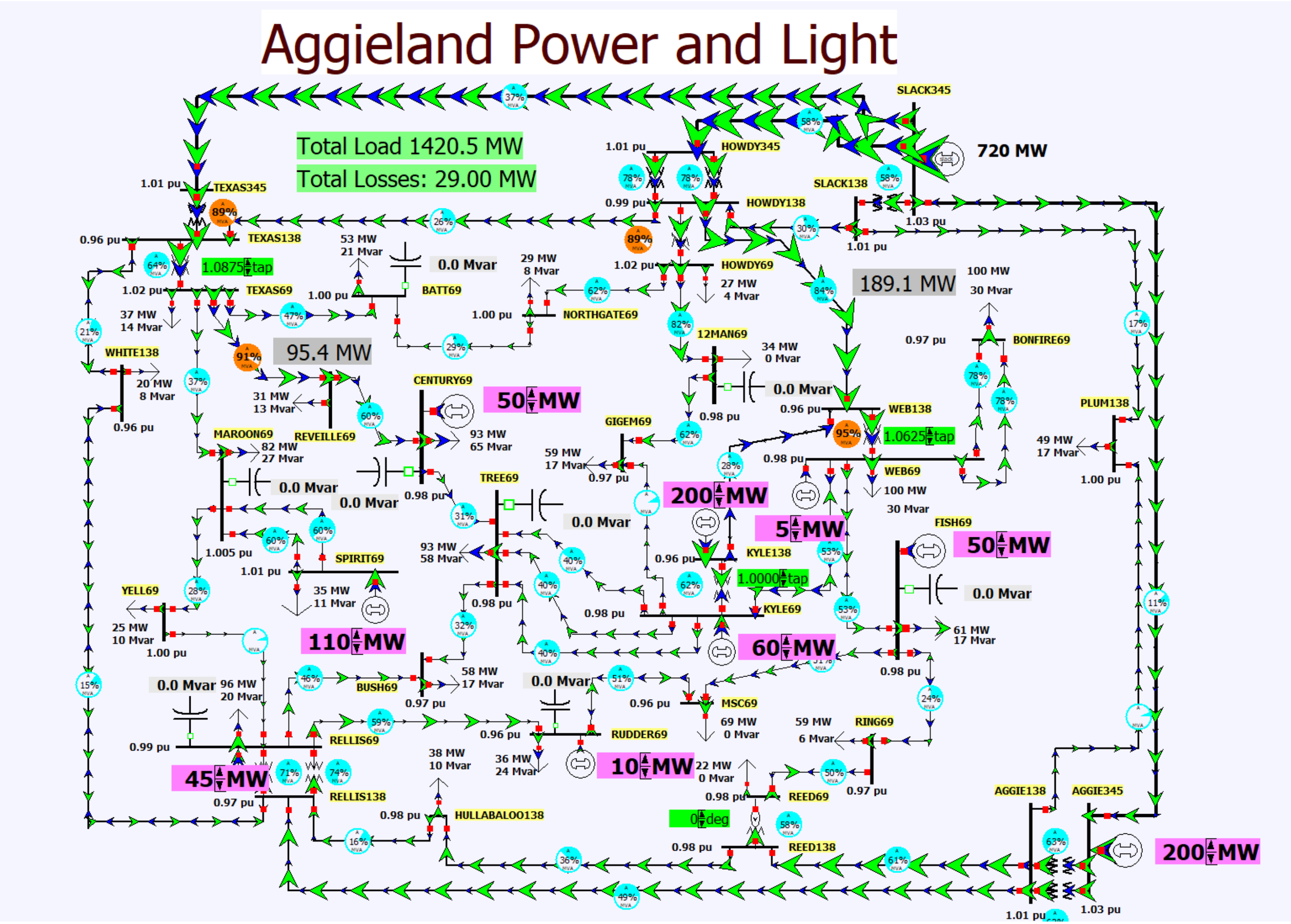


Image: www.osha.gov/dte/library/electrical/electrical_10.gif

Load Tap-Changing Transformers

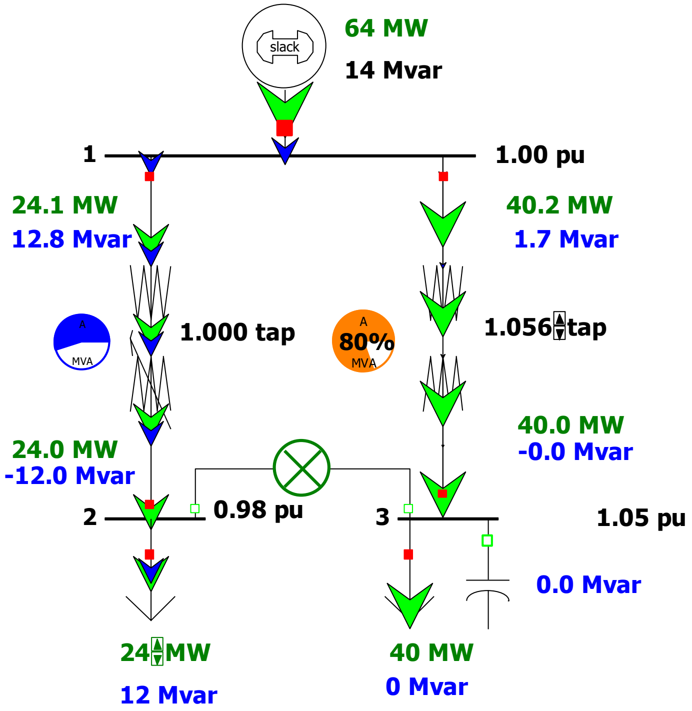
- LTC transformers have tap ratios that can be varied to regulate bus voltages
  - Sometimes called on-load tap-changing transformers (OLTCs) or under-load tap-changing transformers (ULTCs); on the distribution system called voltage regulators
- The typical range of variation is ±10% from the nominal values, usually in 33 discrete steps (0.0625% per step).
- Because tap changing is a mechanical process, LTC transformers usually have a time delay deadband to avoid repeated changes (e.g., 30 seconds)

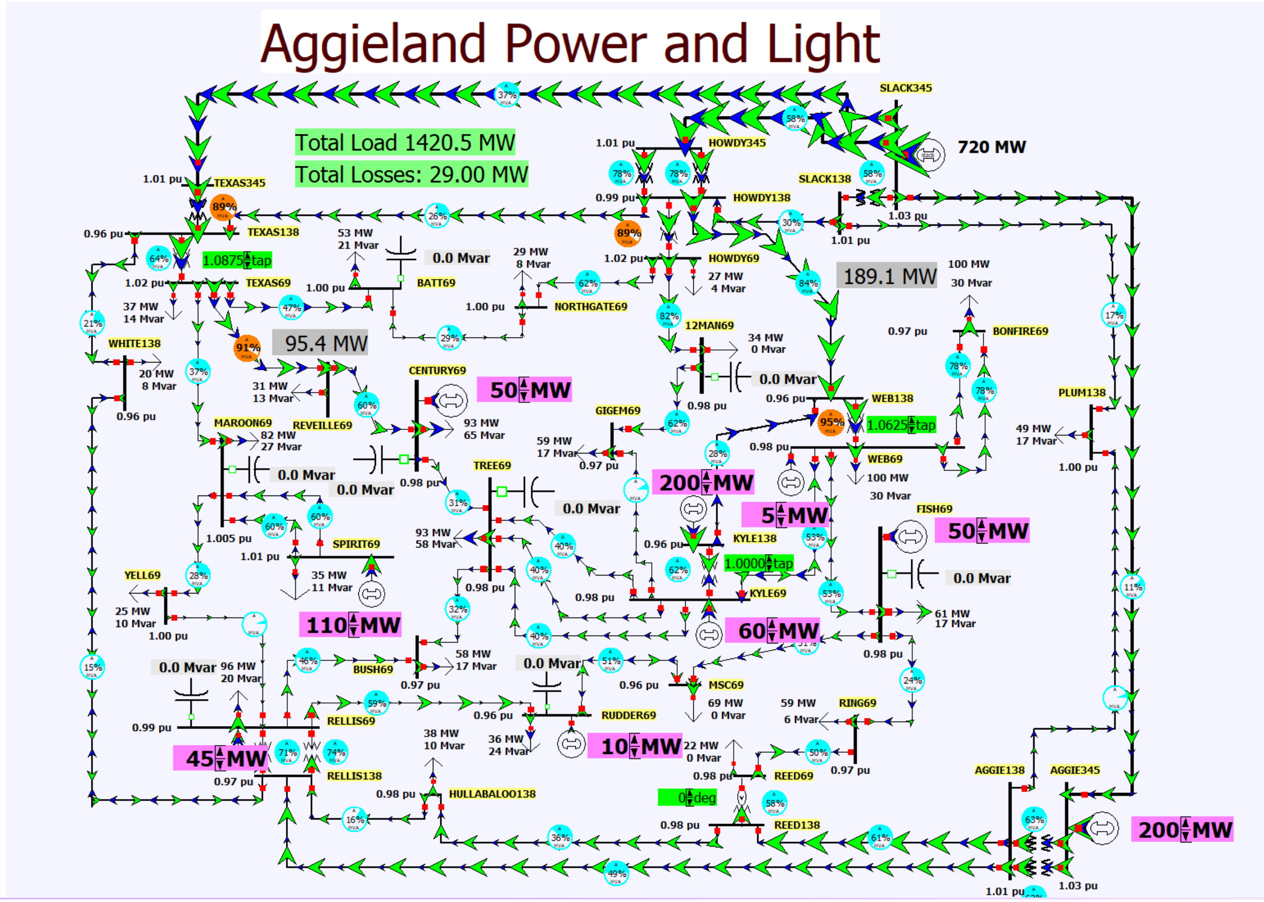
Lab Three LTC Example



LTCs and Circulating Vars

- Unbalanced transformer taps can cause large amounts of reactive power to circulate, increasing power system losses and overloading transformers





Phase Shifting Transformers (PSTs)

- Phase shifting transformers are used to control the phase angle across the transformer
  - Also called phase angle regulators (PARs) or quadrature booster transformers (British usage)
- Since power flow through the transformer depends upon phase angle, this allows the transformer to regulate the power flow through the transformer
- Phase shifters can be used to prevent inadvertent "loop flow" and to prevent line overloads.
- I believe there are nine PSTs in ERCOT

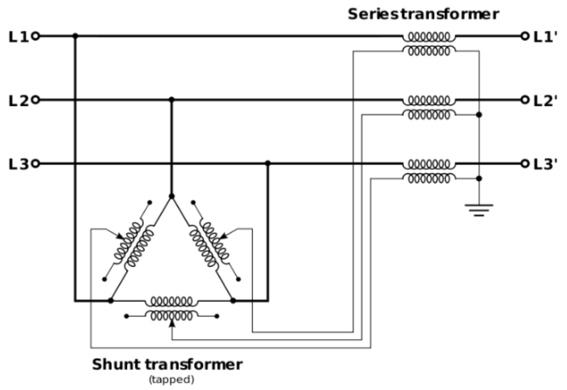
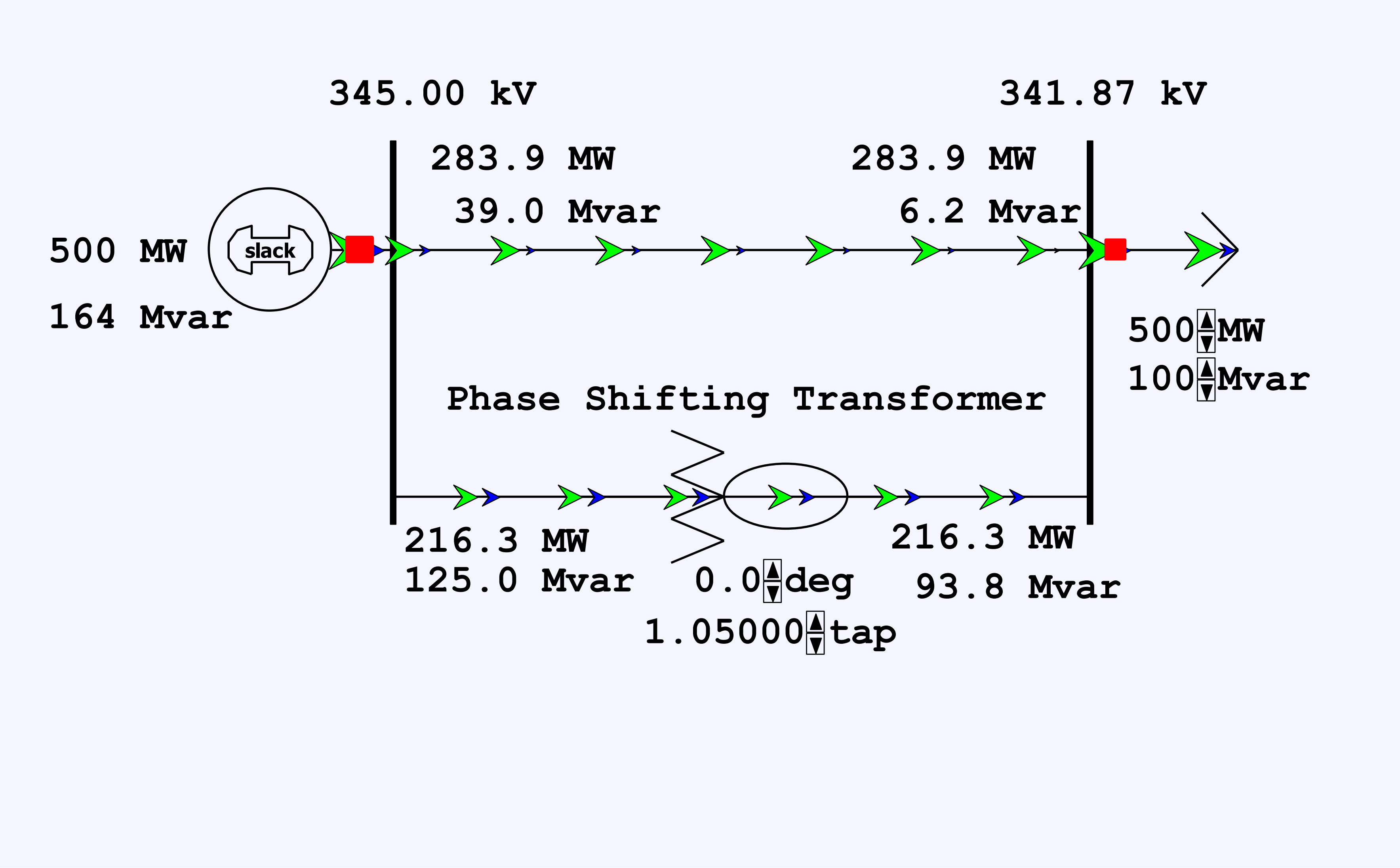


Image Source: en.wikipedia.org/wiki/Quadrature_booster#/media/File:Qb-3ph.svg

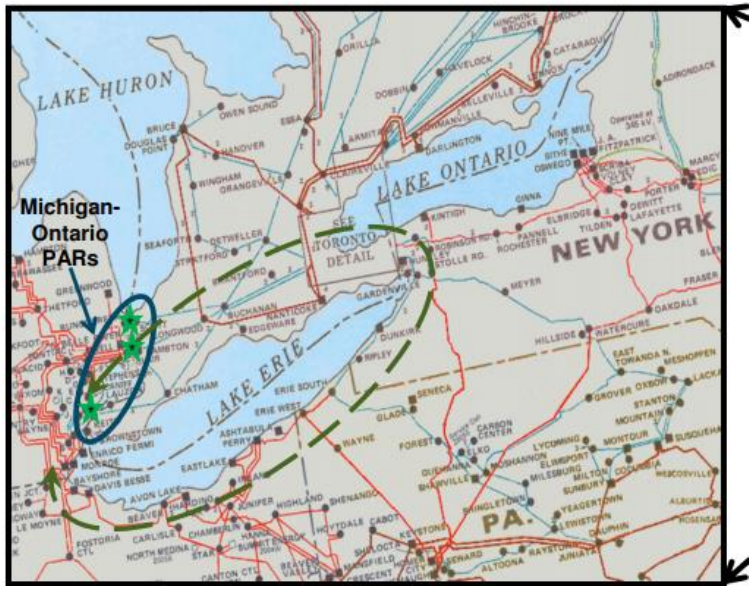
Phase Shifter Example 3.13



Phase Shifter Example: Lake Erie Loop Flow

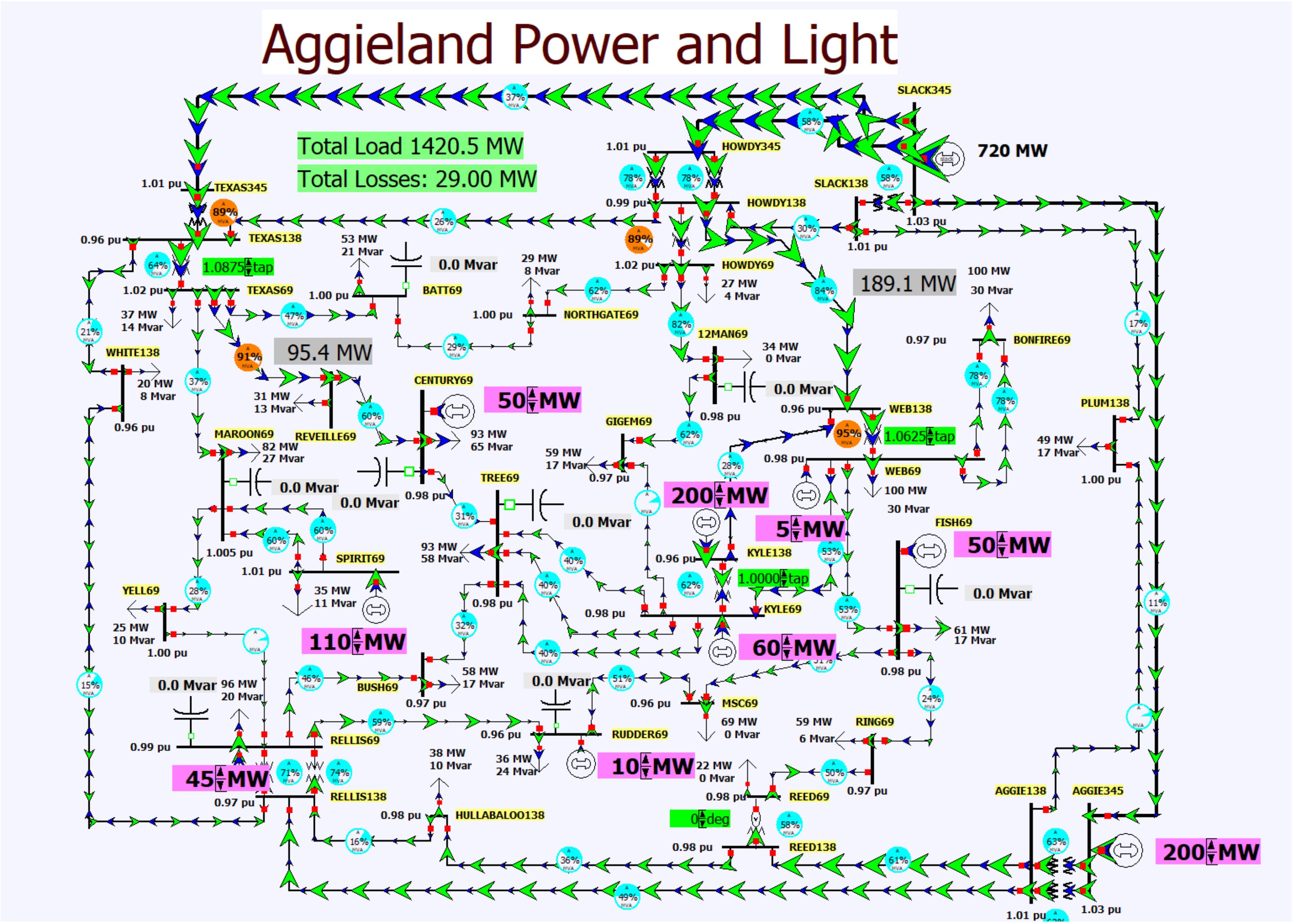
- There are five phase shifters (called phase angle regulators [PARs] here) at the border between Michigan and Ontario. They are used to control how much electricity travels between the US and Canada
- By adjusting the PARs the flow of electricity that “loops” around Lake Erie can be controlled

Michigan Ontario PARs



Source: MISO Board of Directors Market Committee Update, 9/30/13

Lab Three Phase Shifter Example



Per Unit Calculations

- A key problem in analyzing power systems is the large number of transformers.
  - It would be very difficult to continually have to refer impedances to the different sides of the transformers
- This problem is avoided by a normalization of all variables.
- This normalization is known as per unit analysis.

$$\text{quantity in per unit} = \frac{\text{actual quantity}}{\text{base value of quantity}}$$

Per Unit Conversion Procedure, Single-Phase

1. Pick a single-phase VA base for the entire system, S_B
2. Pick a voltage base for each different voltage level, V_B. Voltage bases are related by transformer turns ratios. Voltages are line to neutral.
3. Calculate the impedance base, $Z_B = (V_B)^2/S_B$
4. Calculate the current base, $I_B = V_B/Z_B$
5. Convert actual values to per unit

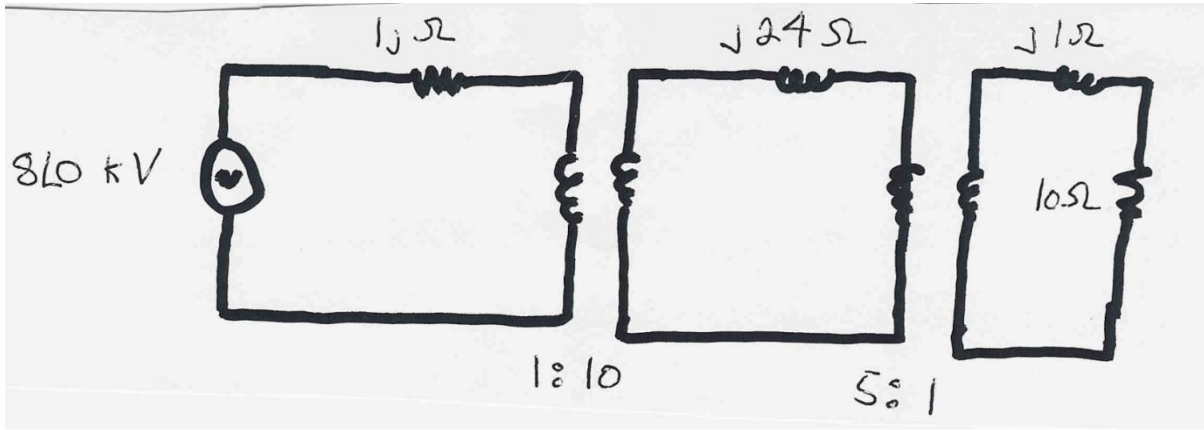
Note, per unit conversion on affects magnitudes, not the angles. Also, per unit quantities no longer have units (i.e., a voltage is 1.0 p.u., not 1 p.u. volts)

Per Unit Solution Procedure

1. Convert to per unit (p.u.) (many problems are already in per unit)
2. Solve
3. Convert back to actual as necessary

Per Unit Example

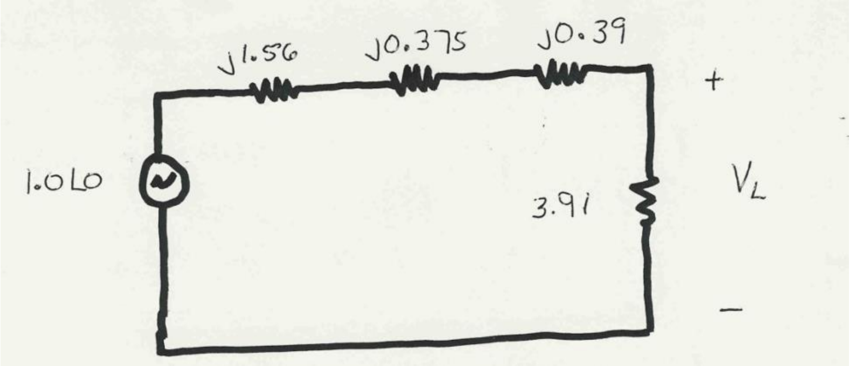
- Solve for the current, load voltage and load power in the circuit shown below using per unit analysis with an S_B of 100 MVA, and voltage bases of 8 kV, 80 kV and 16 kV



$$Z_B^{Left} = \frac{8kV^2}{100MVA} = 0.64\Omega$$

$$Z_B^{Middle} = \frac{80kV^2}{100MVA} = 64\Omega$$

$$Z_B^{Right} = \frac{16kV^2}{100MVA} = 2.56\Omega$$



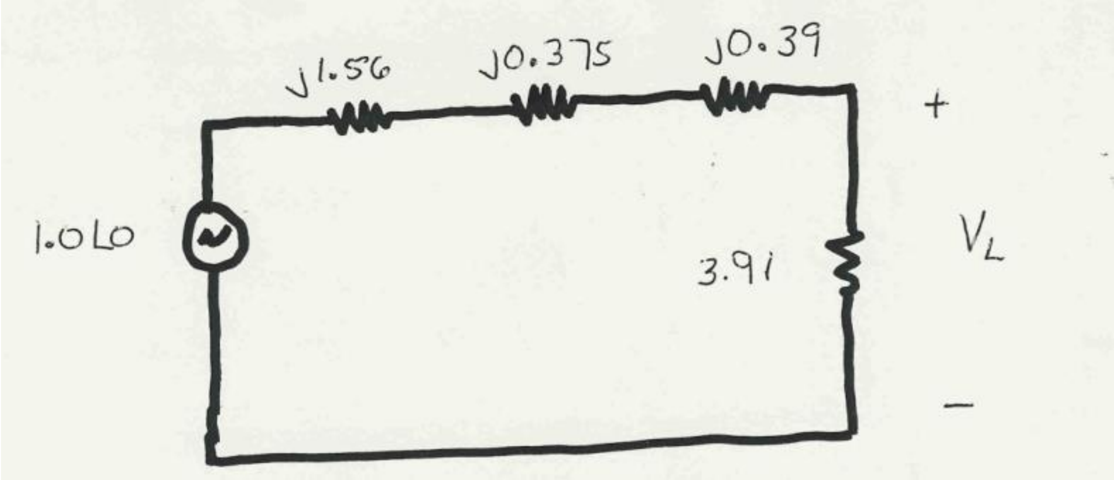
Same circuit, with values expressed in per unit.

Per Unit Example, cont'd

$$I = \frac{1.0\angle 0^\circ}{3.91 + j2.327} = 0.22\angle -30.8^\circ \text{ p.u. (not amps)}$$

$$V_L = 1.0\angle 0^\circ - 0.22\angle -30.8^\circ \times 2.327\angle 90^\circ$$

$$= 0.859\angle -30.8^\circ \text{ p.u.}$$



$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0^\circ \times 0.22\angle 30.8^\circ = 0.22\angle 30.8^\circ \text{ p.u.}$$

Per Unit Example, cont'd

- To convert back to actual values just multiply the per unit values by their per unit base

$$V_L^{\text{Actual}} = 0.859\angle -30.8^\circ \times 16 \text{ kV} = 13.7\angle -30.8^\circ \text{ kV}$$

$$S_L^{\text{Actual}} = 0.189\angle 0^\circ \times 100 \text{ MVA} = 18.9\angle 0^\circ \text{ MVA}$$

$$S_G^{\text{Actual}} = 0.22\angle 30.8^\circ \times 100 \text{ MVA} = 22.0\angle 30.8^\circ \text{ MVA}$$

$$\mathrm{I}_{\mathrm{B}}^{\text{Middle}} = \frac{100 \text{ MVA}}{80 \text{ kV}} = 1250 \text{ Amps}$$

$$\mathrm{I}_{\text{Middle}}^{\text{Actual}} = 0.22\angle -30.8^\circ \times 1250 \text{ Amps} = 275\angle -30.8^\circ \text{ A}$$

Procedure for Balanced Three-Phase Circuits

Procedure is very similar to single-phase except we use a three-phase VA base, and use line to line voltage bases

1. Pick a 3ϕ VA base for the entire system,
2. Pick a voltage base for each different voltage level, V_B. Voltages are line to line.
3. Calculate the impedance base

$$Z_B = \frac{V_{B,LL}^2}{S_B^{3\phi}} = \frac{(\sqrt{3}\,V_{B,LN})^2}{3S_B^{1\phi}} = \frac{V_{B,LN}^2}{S_B^{1\phi}}$$

Exactly the same impedance bases as with single phase!

Three-Phase Per Unit, cont'd

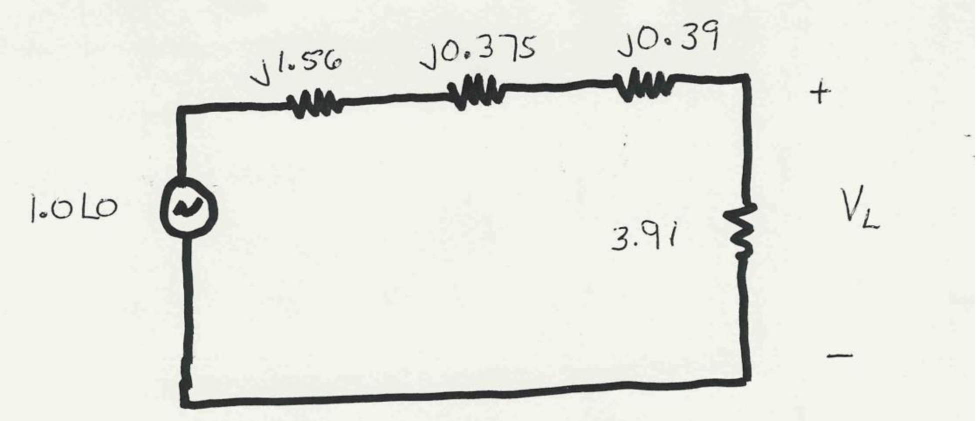
4. Calculate the current base, I_B

$$I_B^{3\phi} = \frac{S_B^{3\phi}}{\sqrt{3}\,V_{B,LL}} = \frac{3\,S_B^{1\phi}}{\sqrt{3}\,\sqrt{3}\,V_{B,LN}} = \frac{S_B^{1\phi}}{V_{B,LN}} = I_B^{1\phi}$$

5. Convert actual values to per unit

Three-Phase Per Unit Example

Solve for the current, load voltage and load power in the previous circuit, assuming a 3ϕ power base of **300 MVA**, and line to line voltage bases of 13.8 kV, 138 kV and 27.6 kV (square root of 3 larger than the 1ϕ example voltages). Also assume the generator is Y-connected so its line to line voltage is 13.8 kV.



Convert to per unit as before. Note the system is exactly the same!

Three-Phase Per Unit Example, cont.

$$I = \frac{1.0\angle 0°}{3.91 + j2.327} = 0.22\angle -30.8° \text{ p.u. (not amps)}$$

$$\mathrm{V_L} = 1.0\angle 0° - 0.22\angle -30.8° \times 2.327\angle 90°$$

$$= 0.859\angle -30.8° \text{ p.u.}$$

Again, analysis is exactly the same!

$$S_L = V_L I_L^* = \frac{|V_L|^2}{Z} = 0.189 \text{ p.u.}$$

$$S_G = 1.0\angle 0° \times 0.22\angle 30.8° = 0.22\angle 30.8° \text{ p.u.}$$

Three-Phase Per Unit Example, cont'd

- Differences appear when we convert back to actual values

$$V_L^{Actual} = 0.859\angle -30.8° \times 27.6\text{ kV} = 23.8\angle -30.8°\text{ kV}$$

$$S_L^{Actual} = 0.189\angle 0° \times 300\text{ MVA} = 56.7\angle 0°\text{ MVA}$$

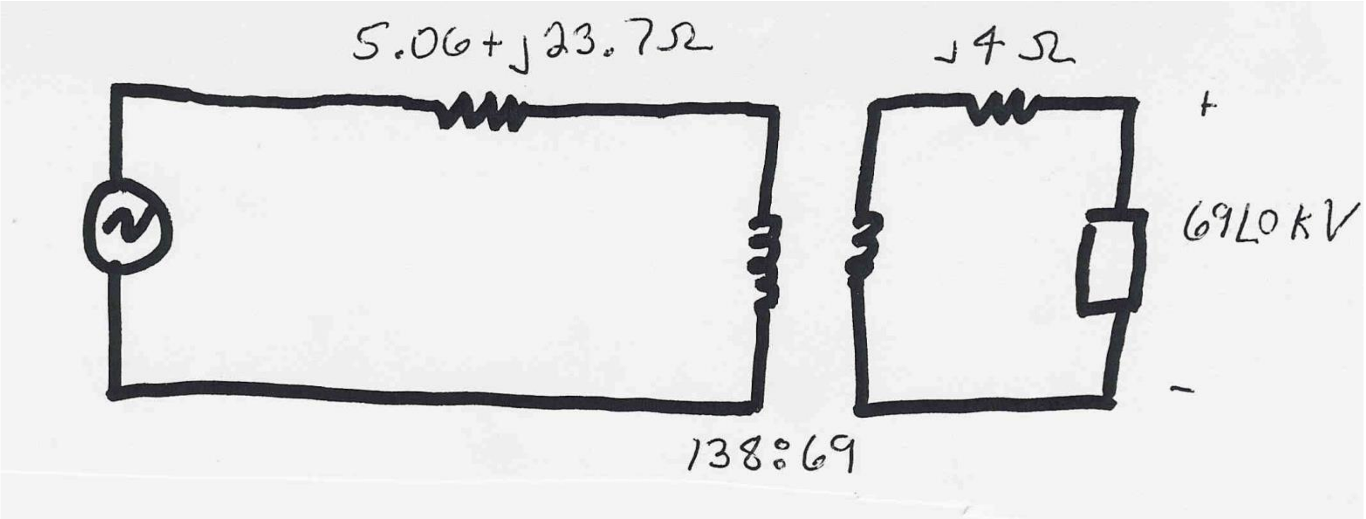
$$S_G^{Actual} = 0.22\angle 30.8° \times 300\text{ MVA} = 66.0\angle 30.8°\text{ MVA}$$

$$I_B^{Middle} = \frac{300\text{ MVA}}{\sqrt{3}\,138\text{ kV}} = 1250\text{ Amps} \quad \text{(same current!)}$$

$$I_{Middle}^{Actual} = 0.22\angle -30.8° \times 1250\text{ Amps} = 275\angle -30.8°\text{ A}$$

Three-Phase Per Unit Example 2

- Assume a three-phase load of 100+j50 MVA with V_{LL} of 69 kV is connected to a source through the below network:



What is the supply current and complex power?

Answer: I=467 amps, S = 103.3 + j76.0 MVA

Per Unit Change of MVA Base

- Parameters for equipment are often given using power rating of equipment as the MVA base
- To analyze a system all per unit data must be on a common power base

$$Z_{pu}^{OriginalBase} \rightarrow Z_{actual} \rightarrow Z_{pu}^{NewBase}$$

$$\text{Hence } \; Z_{pu}^{OriginalBase} \times \frac{V_{base}^2}{S_{Base}^{OriginalBase}} \Big/ \frac{V_{base}^2}{S_{Base}^{NewBase}} = Z_{pu}^{NewBase}$$

$$Z_{pu}^{OriginalBase} \times \frac{S_{Base}^{NewBase}}{S_{Base}^{OriginalBase}} = Z_{pu}^{NewBase}$$

Transformer Reactance

- Transformer reactance is often specified as a percentage, say 10%. This is a per unit value (divide by 100) on the power base of the transformer.
- Example: A 350 MVA, 230/20 kV transformer has leakage reactance of 10%. What is p.u. value on 100 MVA base? What is value in ohms (230 kV)?

$$X_e = 0.10 \times \frac{100}{350} = 0.0286 \text{ p.u.}$$

$$0.0286 \times \frac{230^2}{100} = 15.1\,\Omega$$