ECEN 460
Power System Operation and Control
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Lecture 7: Generator and Load Models, Bus Admittance Matrix

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Book Chapter 2 Picture

Per Unit Change of MVA Base

- Parameters for equipment are often given using power rating of equipment as the MVA base
- To analyze a system all per unit data must be on a common power base

$$Z_{pu}^{OriginalBase} \rightarrow Z_{actual} \rightarrow Z_{pu}^{NewBase}$$

$$\text{Hence} \;\; Z_{pu}^{OriginalBase} \times \frac{V_{base}^{2}}{S_{Base}^{OriginalBase}} \Big/ \frac{V_{base}^{2}}{S_{Base}^{NewBase}} = Z_{pu}^{NewBase}$$

$$Z_{pu}^{OriginalBase} \times \frac{S_{Base}^{NewBase}}{S_{Base}^{OriginalBase}} = Z_{pu}^{NewBase}$$

Transformer Reactance

- Transformer reactance is often specified as a percentage, say 10%. This is a per unit value (divide by 100) on the power base of the transformer.
- Example: A 350 MVA, 230/20 kV transformer has leakage reactance of 10%. What is p.u. value on 100 MVA base? What is value in ohms (230 kV)?

$$X_e = 0.10 \times \frac{100}{350} = 0.0286 \text{ p.u.}$$

$$0.0286 \times \frac{230^2}{100} = 15.1\,\Omega$$

Three Phase Transformers

- There are 4 different ways to connect 3ϕ transformers: Y-Y, Δ-Δ, Y-Δ, Δ-Y
  - The reasons have to do with grounding and harmonics, which are outside of the ECEN 460 scope
  - Only Y connections can be grounded
  - Mixing Y and Δ introduces a 30 degree phase shift
- Most high voltage generator step-up transformers (GSUs) are Δ on the generator side, grounded Y on the transmission side
- Most transmission to distribution is Δ on the transmission side, grounded Y on the distribution side

Autotransformers

- Autotransformers are transformers in which the primary and secondary windings are coupled magnetically and electrically.
- This results in lower cost, and smaller size and weight.
- Most transmission level transformers are autotransformers, connected Y-Y with the low side grounded
- The key disadvantage is loss of electrical isolation between the voltage levels; not used when a is large. For example in stepping down 7160/240 V we do not ever want 7160 on the low side!

Three Winding Transformers

- Many high voltage transformers have a third winding, called the tertiary winding; called three winding transformers
- There are a number of benefits in having 3 windings
  - Tertiary can be used to provide lower voltage electric service, including providing substation service for remote transmission substations; sometimes capacitors are connected to the tertiary
  - Helps with fault protection by reducing the zero sequence current providing higher zero sequence currents (beyond ECEN 460 scope)
  - When Δ-connected helps to reduce unbalanced and third harmonic issues (again beyond ECEN 460 scope)

Load Modeling

- We need a model for the aggregate electric load. Challenges are 1) the heterogeneity of the load (i.e., there are lots of different devices), and 2) the load keeps changing
- Traditionally load models have been divided into two groups
  - Static: load is a algebraic function of bus voltage and sometimes frequency; covered now
  - Dynamic: load is represented with a dynamic model, with induction motor models the most common; covered when we get to stability
- The simplest load model is a static constant impedance, but it is not commonly used; what is commonly used is a constant power load

Justification for the Common Power Flow Constant Power Model

- While many loads do exhibit voltage dependence, there is good justification for using a constant power model in the power flow
- A major justification is since the power flow is a steady-state analysis tool and is usually focused on representing load at the transmission system level, the assumption is the distribution tap-changing transformers have had enough time to respond so that in steady-state the voltage magnitude seen by the load is mostly independent of the transmission level voltage
- Another justification is that in the longer term power flow time frame since many loads have external controllers (i.e., thermostats for heating) when their behavior is aggregated over time they tend to look like constant power

Generator Models

- Engineering models depend upon application
- Generators have traditionally been synchronous machines, but they increasingly include inverter-based resources (e.g., solar PV and wind) in which the power electronics are used to connect the generator to the system
- For generators we will use two different models:
  - a steady-state model, treating the generator as a constant power source operating at a fixed voltage; this model will be used for power flow and economic analysis; we’ll also consider reactive power limits
  - a short term model treating the generator with its dynamics, covered later when we get to stability

Power Flow Analysis

- We now have the necessary models to start to develop the power system analysis tools
- The most common power system analysis tool is the power flow (also known sometimes as the load flow, terms that have been used interchangeably for at least 60 years!)
  - power flow determines how the power flows in a network
  - also used to determine all bus voltages and all currents
  - because of constant power models, power flow is a nonlinear analysis technique
  - power flow is a quasi steady-state analysis tool

Book Chapter 6 Photo

Linear versus Nonlinear Systems

A function **H** is linear if

$$\mathbf{H}(\alpha_1\boldsymbol{\mu}_1 + \alpha_2\boldsymbol{\mu}_2) = \alpha_1\mathbf{H}(\boldsymbol{\mu}_1) + \alpha_2\mathbf{H}(\boldsymbol{\mu}_2)$$

That is

1) the output is proportional to the input
2) the principle of superposition holds

Linear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}$

$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2) = c\mathbf{x}_1 + c\,\mathbf{x}_2$$

Nonlinear Example: $\mathbf{y} = \mathbf{H}(\mathbf{x}) = c\,\mathbf{x}^2$

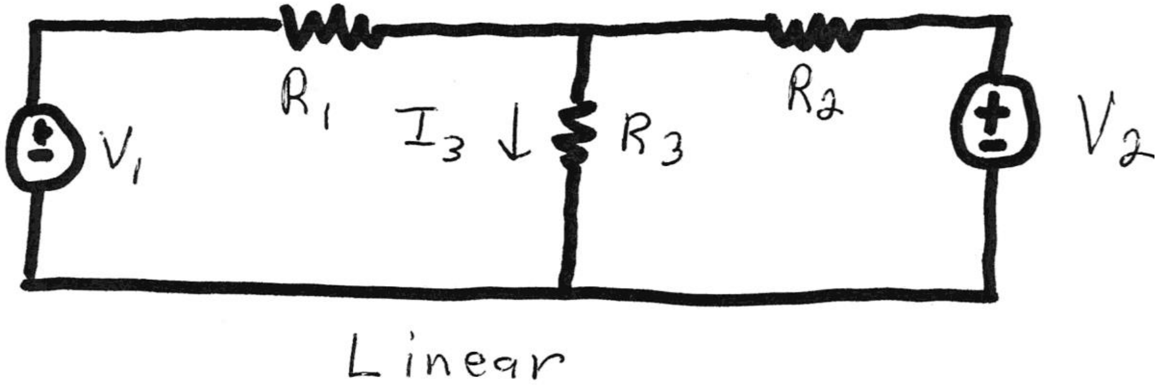
$$\mathbf{y} = c(\mathbf{x}_1+\mathbf{x}_2)^2 \neq (c\mathbf{x}_1)^2 + (c\,\mathbf{x}_2)^2$$

Linear Power System Elements

Resistors, inductors, capacitors, independent voltage sources and current sources are linear circuit elements

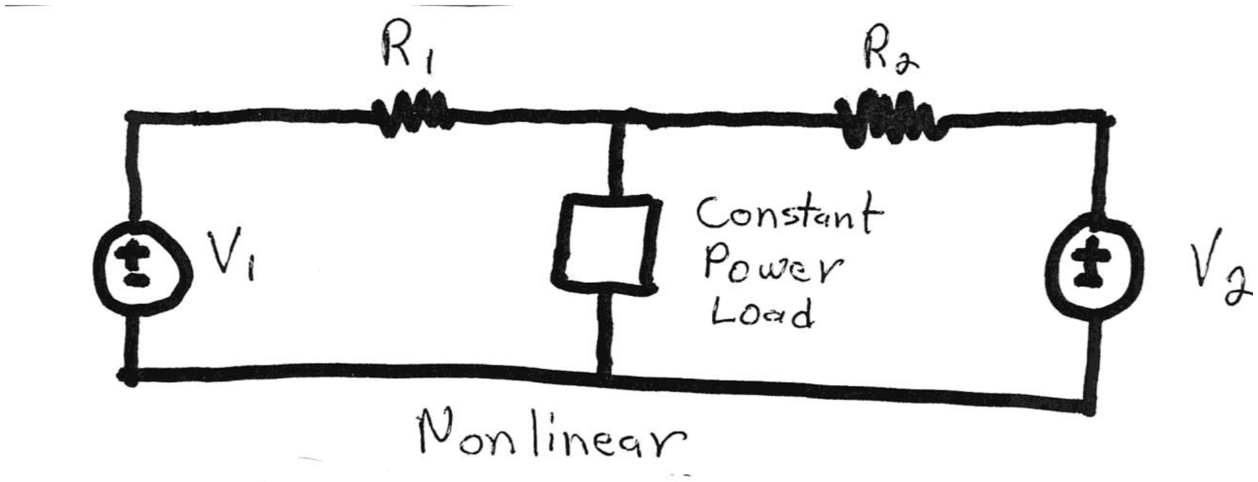
$$V = R I \qquad V = j\omega L\, I \qquad V = \frac{1}{j\omega C} I$$

Such systems may be analyzed by superposition



Nonlinear System Example

- Constant power loads and generator injections are nonlinear and hence systems with these elements can not be analyzed by superposition



Nonlinear problems can be very difficult to solve, and usually require an iterative approach

Nonlinear Systems May Have Multiple Solutions or No Solution

Example 1: $x^2 - 2 = 0$ has solutions $x = \pm 1.414\ldots$

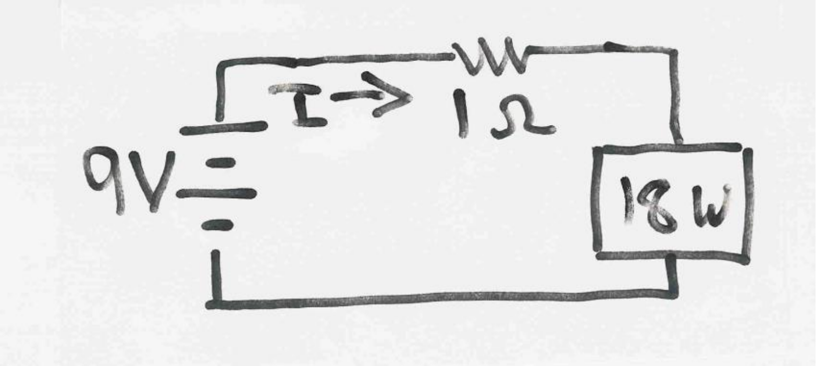
Example 2: $x^2 + 2 = 0$ has no real solution

$f(x) = x^2 - 2$ $\qquad\qquad$ $f(x) = x^2 + 2$

two solutions where $f(x) = 0$ $\qquad$ no solution $f(x) = 0$

Multiple Solution Example

- The dc system shown below has two solutions:



where the 18 watt load is a resistive load

What is the maximum P_{Load}?

The equation we're solving is

$$I^2 R_{Load} = \left(\frac{9 \text{ volts}}{1\Omega + \mathrm{R}_{\mathrm{Load}}}\right)^2 R_{Load} = 18 \text{ watts}$$

One solution is $\mathrm{R}_{\mathrm{Load}} = 2\Omega$

Other solution is $\mathrm{R}_{\mathrm{Load}} = 0.5\Omega$

Bus Admittance Matrix or Y_{bus}

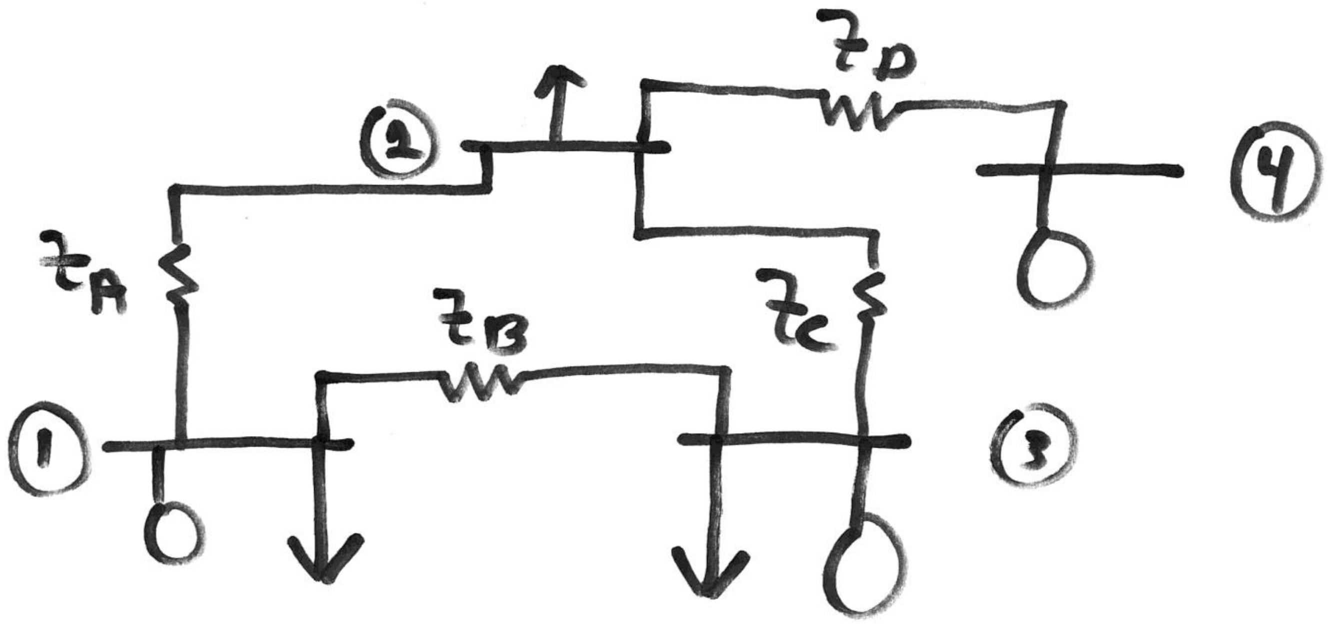
- First step in solving the power flow is to create what is known as the bus admittance matrix, often call the $\mathbf{Y}_{bus}$.
- The $\mathbf{Y}_{bus}$ gives the relationships between all the bus current injections, **I**, and all the bus voltages, **V**,

$$\mathbf{I} = \mathbf{Y}_{bus}\,\mathbf{V}$$

- The $\mathbf{Y}_{bus}$ is developed by applying KCL at each bus in the system to relate the bus current injections, the bus voltages, and the branch impedances and admittances

Y_{bus} Example

Determine the bus admittance matrix for the network shown below, assuming the current injection at each bus i is $I_i = I_{Gi} - I_{Di}$ where I_{Gi} is the current injection into the bus from the generator and I_{Di} is the current flowing into the load



Y_{bus} Example, cont'd

By KCL at bus 1 we have

$$I_1 = I_{G1} - I_{D1}$$

$$I_1 = I_{12} + I_{13} = \frac{V_1 - V_2}{Z_A} + \frac{V_1 - V_3}{Z_B}$$

$$I_1 = (V_1 - V_2)Y_A + (V_1 - V_3)Y_B \qquad \text{(with } \mathrm{Y_j} = \frac{1}{Z_j}\text{)}$$

$$= (Y_A + Y_B)V_1 - Y_A V_2 - Y_B V_3$$

Similarly

$$I_2 = I_{21} + I_{23} + I_{24}$$

$$= -Y_A V_1 + (Y_A + Y_C + Y_D)V_2 - Y_C V_3 - Y_D V_4$$

Y_{bus} Example, cont'd

We can get similar relationships for buses 3 and 4

The results can then be expressed in matrix form

$$\mathbf{I} = \mathbf{Y}_{bus}\mathbf{V}$$

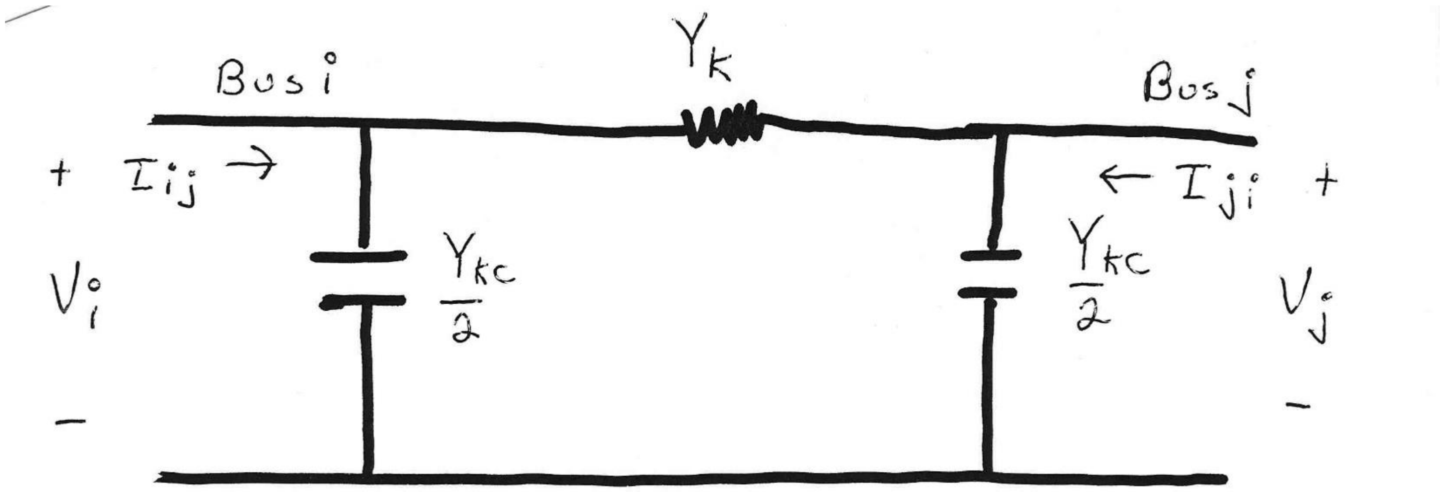
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_A + Y_B & -Y_A & -Y_B & 0 \\ -Y_A & Y_A + Y_C + Y_D & -Y_C & -Y_D \\ -Y_B & -Y_C & Y_B + Y_C & 0 \\ 0 & -Y_D & 0 & Y_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

For a system with n buses the Y_{bus} is an n by n symmetric matrix (i.e., one where $A_{ij} = A_{ji}$); however this will not be true in general when we consider phase shifting transformers

Y_{bus} General Form

- The diagonal terms, Y_{ii}, are the self admittance terms, equal to the sum of the admittances of all devices incident to bus i.
- The off-diagonal terms, Y_{ij}, are equal to the negative of the sum of the admittances joining the two buses.
- With large systems Y_{bus} is a sparse matrix (that is, most entries are zero)
- Shunt terms, such as with the π line model, only affect the diagonal terms.

Modeling Shunts in the Y_{bus}

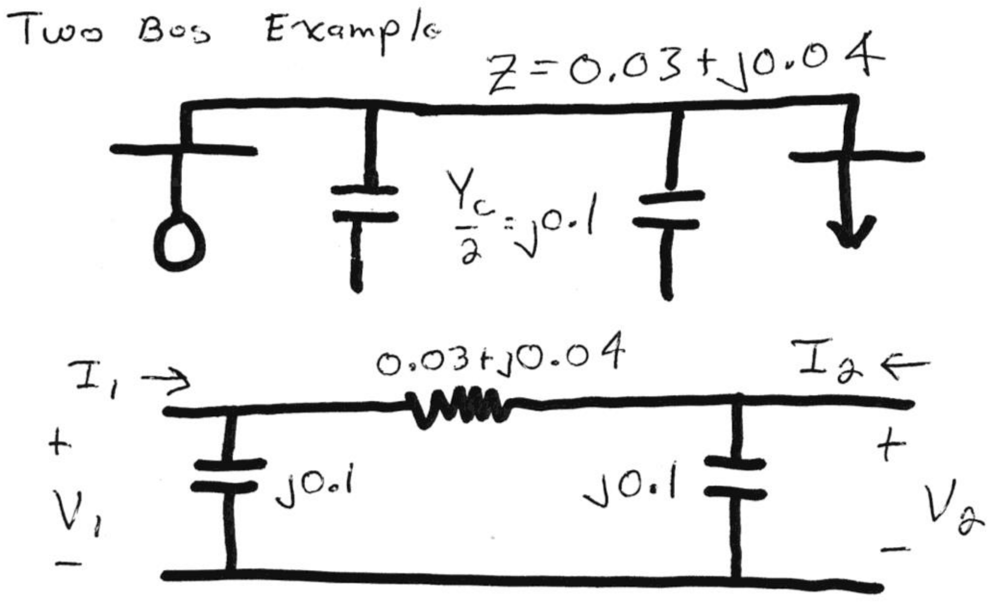


Since $I_{ij} = (V_i - V_j)Y_k + V_i \frac{Y_{kc}}{2}$

$$Y_{ii} = Y_{ii}^{\text{from other lines}} + Y_k + \frac{Y_{kc}}{2}$$

Note $Y_k = \frac{1}{Z_k} = \frac{1}{R_k + jX_k} \frac{R_k - jX_k}{R_k - jX_k} = \frac{R_k - jX_k}{R_k^2 + X_k^2}$

Two Bus System Example



$$I_1 = \frac{(V_1 - V_2)}{Z} + V_1 \frac{Y_c}{2} \qquad \frac{1}{0.03 + j0.04} = 12 - j16$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Using the Y_{bus}

If the voltages are known then we can solve for the current injections:

$$\mathbf{Y}_{bus}\mathbf{V} = \mathbf{I}$$

If the current injections are known then we can solve for the voltages:

$$\mathbf{Y}_{bus}^{-1}\mathbf{I} = \mathbf{V} = \mathbf{Z}_{bus}\mathbf{I}$$

where $\mathbf{Z}_{\text{bus}}$ is the bus impedance matrix

However, this requires that $\mathbf{Y}_{\text{bus}}$ not be singular; note it will be singular if there are no shunt connections!

Solving for Bus Currents

For example, in previous case assume

$$\mathbf{V} = \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix} \begin{bmatrix} 1.0 \\ 0.8 - j0.2 \end{bmatrix} = \begin{bmatrix} 5.60 - j0.70 \\ -5.58 + j0.88 \end{bmatrix}$$

Therefore the power injected at bus 1 is

$$\mathrm{S}_1 = V_1 I_1^* = 1.0 \times (5.60 + j0.70) = 5.60 + j0.70$$

$$S_2 = V_2 I_2^* = (0.8 - j0.2) \times (-5.58 - j0.88) = -4.64 + j0.41$$

Solving for Bus Voltages

For example, in previous case assume

$$\mathbf{I} = \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix}$$

Then

$$\begin{bmatrix} 12 - j15.9 & -12 + j16 \\ -12 + j16 & 12 - j15.9 \end{bmatrix}^{-1} \begin{bmatrix} 5.0 \\ -4.8 \end{bmatrix} = \begin{bmatrix} 0.0738 - j0.902 \\ -0.0738 - j1.098 \end{bmatrix}$$

Therefore the power injected is

$$\mathrm{S}_1 = V_1 I_1^* = (0.0738 - j0.902) \times 5 = 0.37 - j4.51$$

$$S_2 = V_2 I_2^* = (-0.0738 - j1.098) \times (-4.8) = 0.35 + j5.27$$

Power Flow Analysis

- When analyzing power systems we know neither the complex bus voltages nor the complex current injections
- Rather, we know the complex power being consumed by the load, and the power being injected by the generators plus their voltage magnitudes
- Therefore we can not directly use the Y_{bus} equations, but rather must use the power balance equations

Power Balance Equations

From KCL we know at each bus i in an n bus system the current injection, I_i, must be equal to the current that flows into the network

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} I_{ik}$$

Since $\mathbf{I} = \mathbf{Y}_{\text{bus}}\mathbf{V}$ we also know

$$I_i = I_{Gi} - I_{Di} = \sum_{k=1}^{n} Y_{ik} V_k$$

The network power injection is then $\mathrm{S_i} = V_i I_i^*$

Power Balance Equations, cont'd

$$S_i = V_i I_i^* = V_i \left(\sum_{k=1}^{n} Y_{ik} V_k \right)^* = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^*$$

This is an equation with complex numbers.

Sometimes we would like an equivalent set of real power equations. These can be derived by defining

$$Y_{ik} = G_{ik} + jB_{ik}$$

$$V_i = |V_i| e^{j\theta_i} = |V_i| \angle \theta_i$$

$$\theta_{ik} = \theta_i - \theta_k$$

Recall $e^{j\theta} = \cos\theta + j\sin\theta$

Real Power Balance Equations

$$\mathrm{S_i} = P_i + jQ_i = V_i \sum_{k=1}^{n} Y_{ik}^* V_k^* = \sum_{k=1}^{n} |V_i||V_k| e^{j\theta_{ik}} (G_{ik} - jB_{ik})$$

$$= \sum_{k=1}^{n} |V_i||V_k| (\cos\theta_{ik} + j\sin\theta_{ik})(G_{ik} - jB_{ik})$$

These are probably the most important power system analysis equations!!!

Resolving into the real and imaginary parts

$$\mathrm{P_i} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik}) = P_{Gi} - P_{Di}$$

$$\mathrm{Q_i} = \sum_{k=1}^{n} |V_i||V_k| (G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik}) = Q_{Gi} - Q_{Di}$$

There are different ways to show them (e.g., 6.4.10 and 6.4.11, and 6.4.12 and 6.4.13)

Slack Bus

- We can not arbitrarily specify S at all buses because total generation must equal total load + total losses
- We also need an angle reference bus.
- To solve these problems we define one bus as the "slack" bus. This bus has a fixed voltage magnitude and angle, and a varying real/reactive power injection.
- In an actual power system the slack bus does not really exist; frequency changes locally when the power supplied does not match the power consumed

Three Types of Power Flow Buses

- There are three main types of power flow buses
  - Load (PQ) at which P/Q are fixed; iteration solves for voltage magnitude and angle.
  - Slack at which the voltage magnitude and angle are fixed; iteration solves for P/Q injections
  - Generator (PV) at which P and |V| are fixed; iteration solves for voltage angle and Q injection

Newton-Raphson Algorithm

- The book gives two approaches for solving the power flow: Gauss-Seidel (Section 6.5) and Newton-Raphson (Section 6.6)
- Gauss-Seidel is included because it is easy to explain, but it is no longer used commercially and won't be covered here
- Most common technique for solving the power flow problem is to use the Newton-Raphson algorithm
- Key idea behind Newton-Raphson is to use sequential linearization

General form of problem: Find an $\mathbf{x}$ such that

$$\mathbf{f}(\hat{\mathbf{x}}) = 0$$

Newton-Raphson Method (scalar)

1. For each guess of $\hat{x}$, $x^{(v)}$, define

   $\Delta x^{(v)} = \hat{x} - x^{(v)}$

2. Represent $f(\hat{x})$ by a Taylor series about $f(x)$

$$
\begin{aligned}
f(\hat{x}) \;=\; & f(x^{(v)}) + \frac{df(x^{(v)})}{dx}\Delta x^{(v)} + \\
& + \frac{1}{2}\frac{d^2 f(x^{(v)})}{dx^2}\left(\Delta x^{(v)}\right)^2 + \text{higher order terms}
\end{aligned}
$$

Newton-Raphson Method, cont'd

3. Approximate $f(\hat{x})$ by neglecting all terms except the first two

$$f(\hat{x}) = 0 \approx f(x^{(v)}) + \frac{df(x^{(v)})}{dx}\Delta x^{(v)}$$

4. Use this linear approximation to solve for $\Delta x^{(v)}$

$$\Delta x^{(v)} = -\left[\frac{df(x^{(v)})}{dx}\right]^{-1} f(x^{(v)})$$

5. Solve for a new estimate of x̂

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$$

Newton-Raphson Example

Use Newton-Raphson to solve $f(x) = x^2 - 2 = 0$

The equation we must iteratively solve is

$$\Delta x^{(v)} = -\left[\frac{df(x^{(v)})}{dx}\right]^{-1} f(x^{(v)})$$

$$\Delta x^{(v)} = -\left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

$$x^{(v+1)} = x^{(v)} + \Delta x^{(v)}$$

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

Newton-Raphson Example, cont'd

$$x^{(v+1)} = x^{(v)} - \left[\frac{1}{2x^{(v)}}\right]((x^{(v)})^2 - 2)$$

Guess $x^{(0)} = 1$. Iteratively solving we get

v	$x^{(v)}$	$f(x^{(v)})$	$\Delta x^{(v)}$
0	1	–1	0.5
1	1.5	0.25	–0.08333
2	1.41667	6.953×10^{-3}	-2.454×10^{-3}
3	1.41422	6.024×10^{-6}	