ECEN 460
Power System Operation and Control
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Lecture 11: Economic Dispatch and Optimal Power Flow (OPF)

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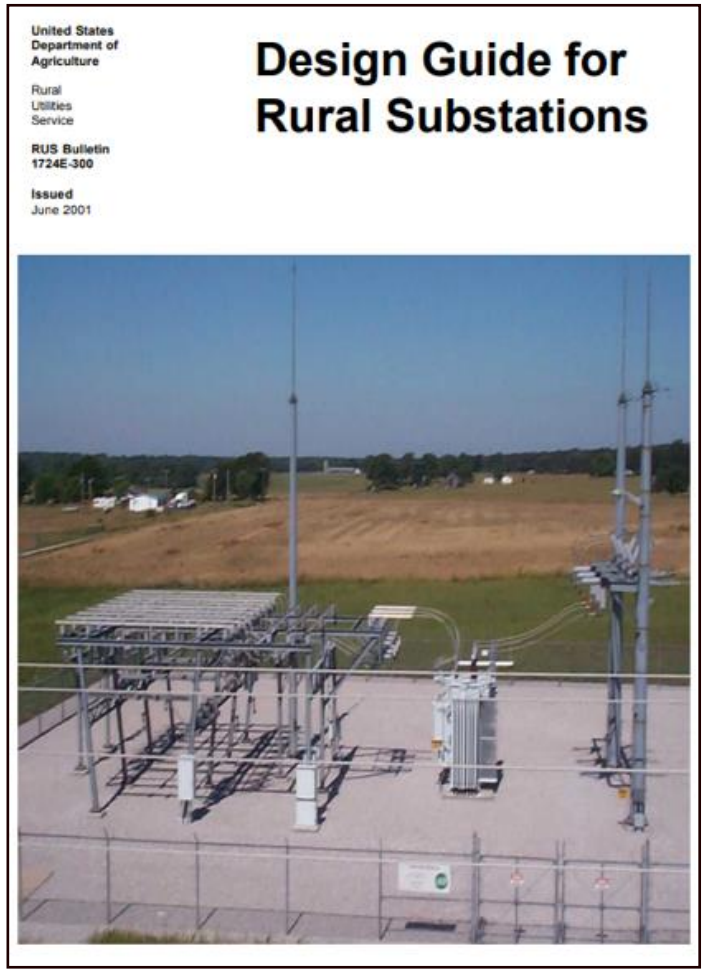
Breaker Configurations Relative Costs (from Tracy Rolstad)

What is best depends on application, with double breaker, double bus common

- Single Bus—Single Breaker
- Main and Transfer Bus
- Ring Bus
- Breaker and a half
- Double Bus—Double Breaker

Table 4-1: Switching Scheme Cost Comparison

Switching Scheme	Approximate Relative Cost Comparison
Single Bus	100%
Sectionalized Bus	122%
Main and Transfer Bus	143%
Ring Bus	114%
Breaker-and-a-Half	158%
Double Breaker–Double Bus	214%



Unit Commitment: Quick Coverage

- Before covering economic dispatch, we'll first touch on unit commitment
- Unit commitment is used to determine which generator units should be committed to meet the load
- The electric load varies substantially so there is almost always more generator capacity available than load
- Units have availability constraints
  - Minimum up time, time to start, cost to start
  - Minimum down time, time to shutdown, cost to shutdown
  - Ramp rates, minimum MW output
  - Scheduled and unscheduled outages
- System constraints including load, reserve, emissions, network

Solving Unit Commitment

- Unit commitment involves a potentially large number of integer and continuous variables
  - Not just the status of each unit, but also the amount of time it has been in a particular state (i.e., off or on)
- Solved for a set of discrete time periods, which at each time period there are lots of different potential states
- Solution approaches include
  - Dynamic programming
  - Lagrangian relaxation
  - Mixed Integer Programming (MIP) (currently state-of-the-art)
    - the idea is some or all of the variables are restricted to being integers; it is widely used in many applications, with a power application looking at generator statuses (on=1, off=0)

Longer Term Optimization: Quicker Coverage

- Longer term optimization is a key consideration in hydro systems with significant reservoir storage
  - Use the water when it is the most valuable taking into account potentially many other constraints
- Generator maintenance scheduling
- Building generation often involves large upfront capital costs to create an asset that will last 20 to 40 years; long-term contracts provide a way to share the risk
- Take-or-pay contracts obligate a purchaser to purchase so much of a product over a given time period

Example: Prairie State Energy Campus

- The Prairie State Energy Campus (PSEC) is a 1600 MW coal plant in Southern Illinois with its own coal mine that opened in 2012
  - It is owned by municipals and coops (my former coop got >60% of the energy from PSEC)
  - While relatively efficient, it is one of the US's largest sources of CO2 emissions
  - It cost an estimate $4 billion to build; if it sells its power at $30/MWh then maximum yearly income would be $30*1600*8760=$420 million

- Illinois's new clean energy law requires PSEC to reduce carbon emissions by 45% by 1/1/35 and be 100% carbon free by the end of 2045

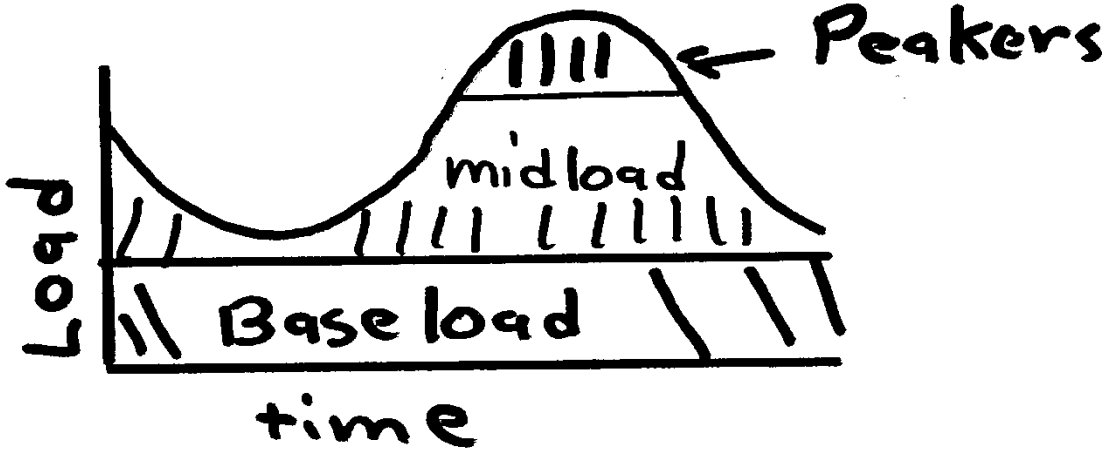
Image: Pantagraph.com

Generation Dispatch

- Since the load is variable and there must be enough generation to meet the load, almost always there is more generation capacity available than load
- Optimally determining which generators to use can be a complicated task due to many different constraints
  - For generators with low or no cost fuel (e.g., wind and solar PV) it is “use it or lose it”
  - For others like hydro there may be limited energy for the year
  - Some fossil has shut down and start times of many hours
- Economic dispatch looks at the best way to instantaneously dispatch the generation with the assumption of a fixed number of available generators
  - Unit commitment determines which generators are on

Generator Types (Traditional)

- Traditionally utilities have had three broad groups of generators
  - baseload units: large coal/nuclear; always on at max.
  - midload units: smaller coal that cycle on/off daily
  - peaker units: combustion turbines used only for several hours during periods of high demand



Wind and solar PV can be quite variable; usually they are operated at maximum available power

Generation Types (ERCOT)

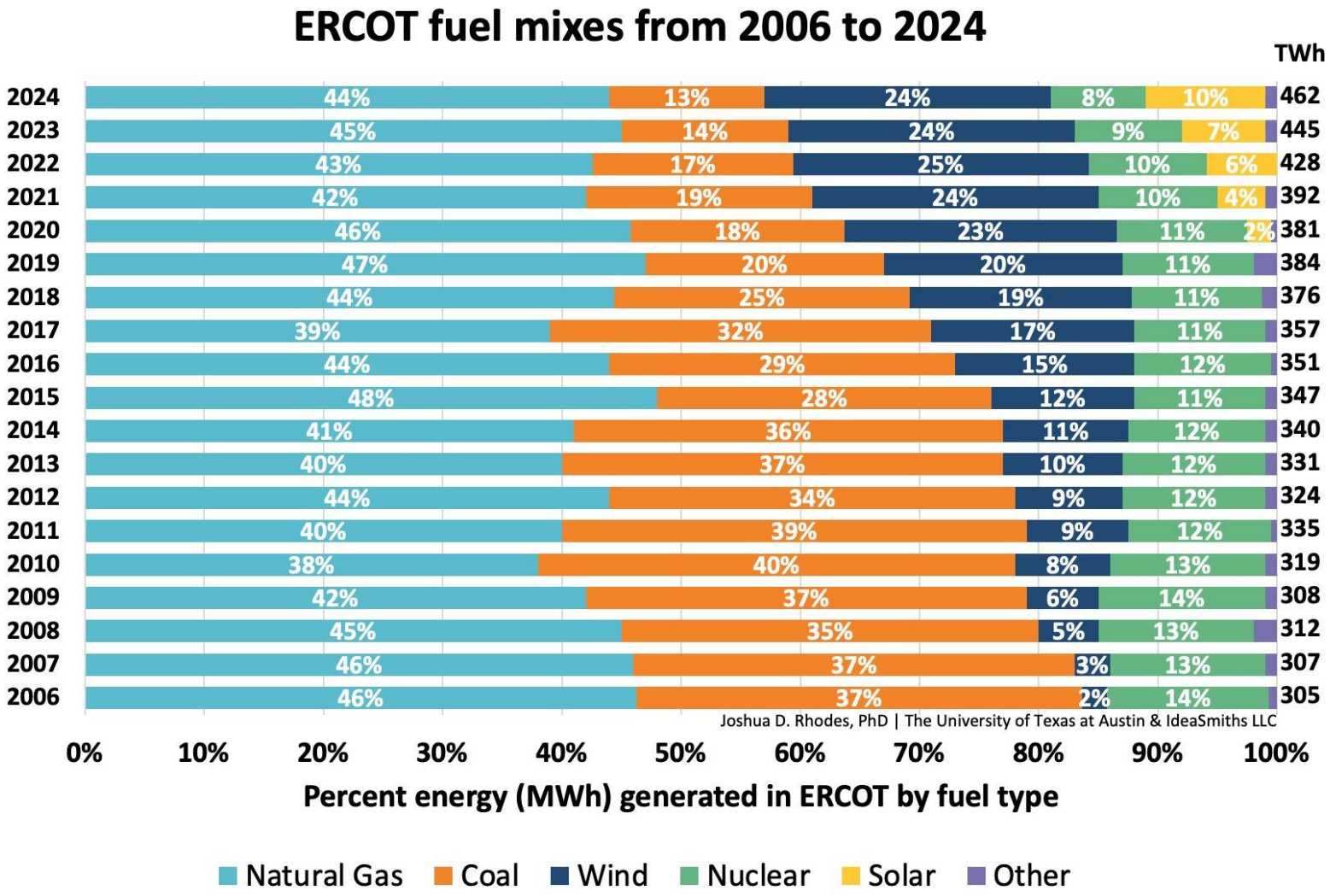
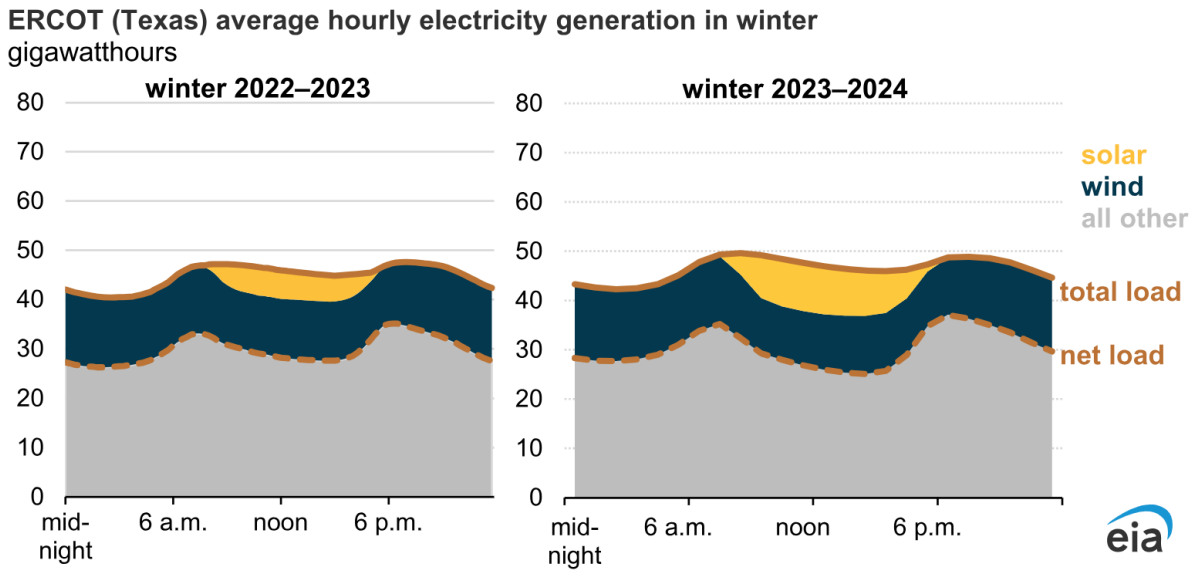
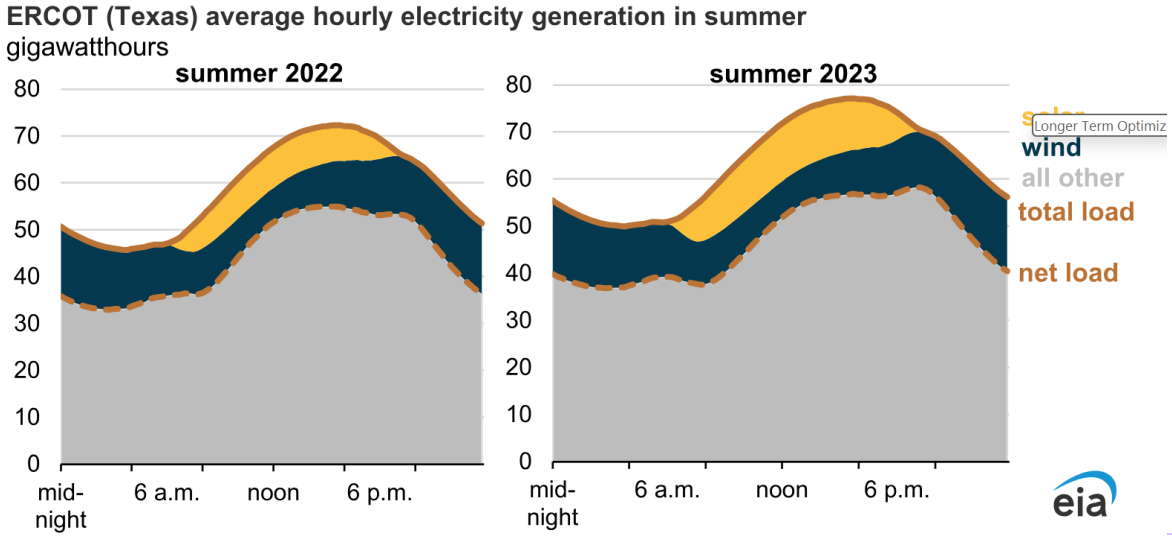


Image source: x.com/joshdr83/status/1882809452820332564?mx=2 (with the data ultimately from ERCOT)

ERCOT Typical Daily Curves

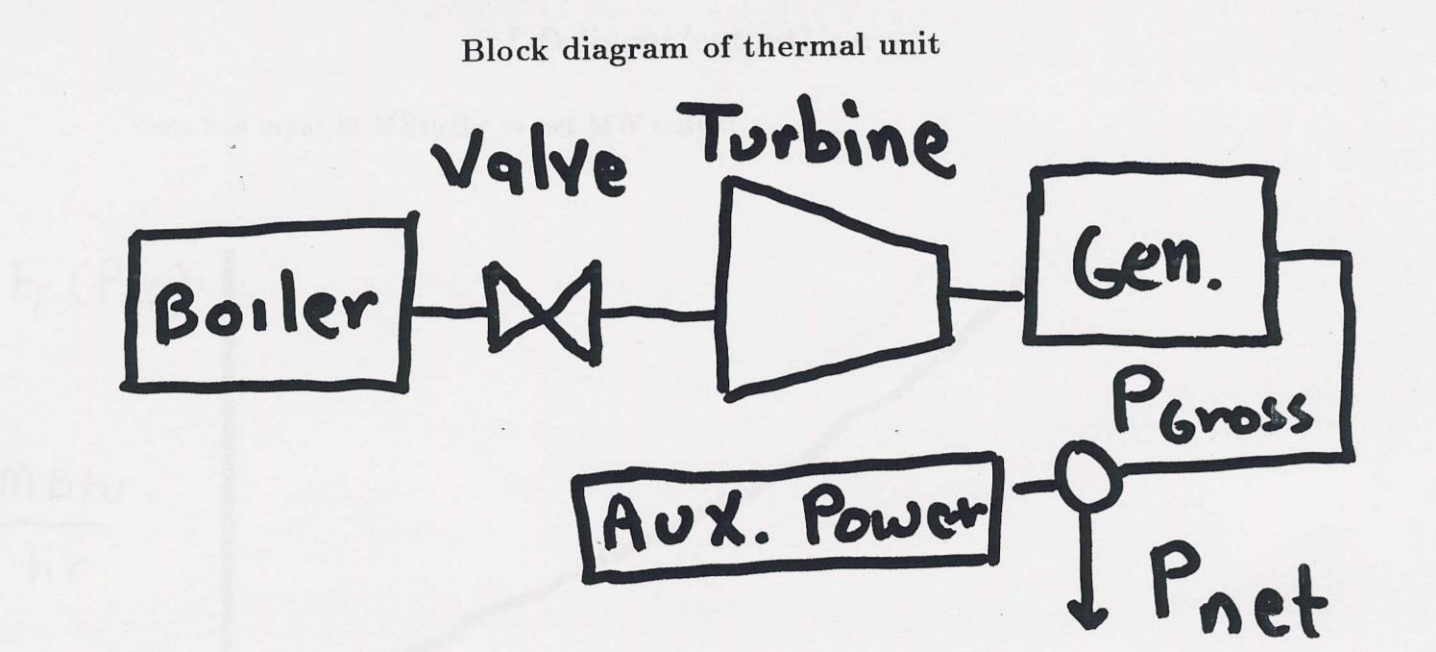




Thermal versus Hydro Generation

- The two main types of generating units are thermal and hydro, with wind and solar rapidly growing
- For hydro the fuel (water) is free but there may be many constraints on operation
  - fixed amounts of water available
  - reservoir levels must be managed and coordinated
  - downstream flow rates for fish and navigation
- Some countries get a large percentage of their generation from hydro including Brazil, Canada, Norway and Sweden
- Hydro optimization is typically longer term (many months or years)
- In 460 we will concentrate on thermal units and some wind and solar, looking at short-term optimization

Block Diagram of Thermal Unit



To optimize generation costs we need to develop cost relationships between net power out and operating costs. Between 2-6% of power is used within the generating plant; this is known as the auxiliary power

Coal Plant Diagram

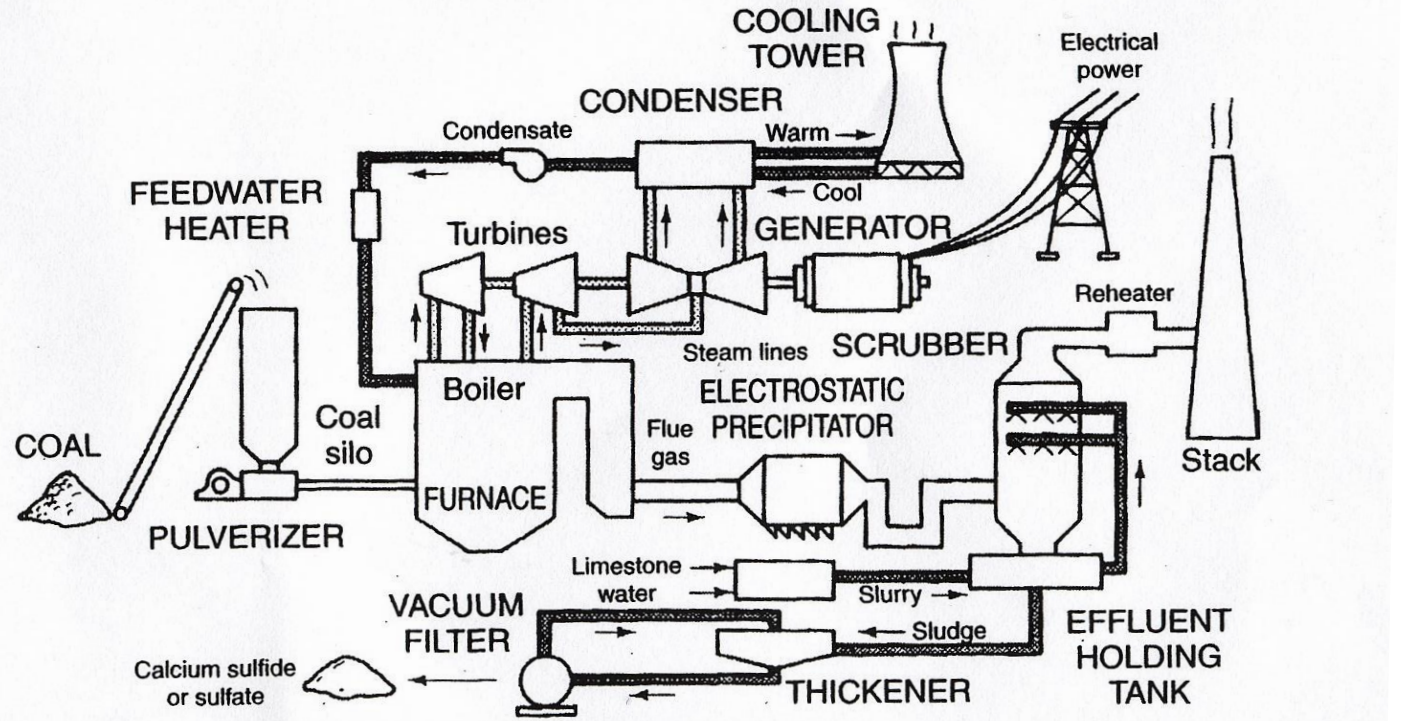


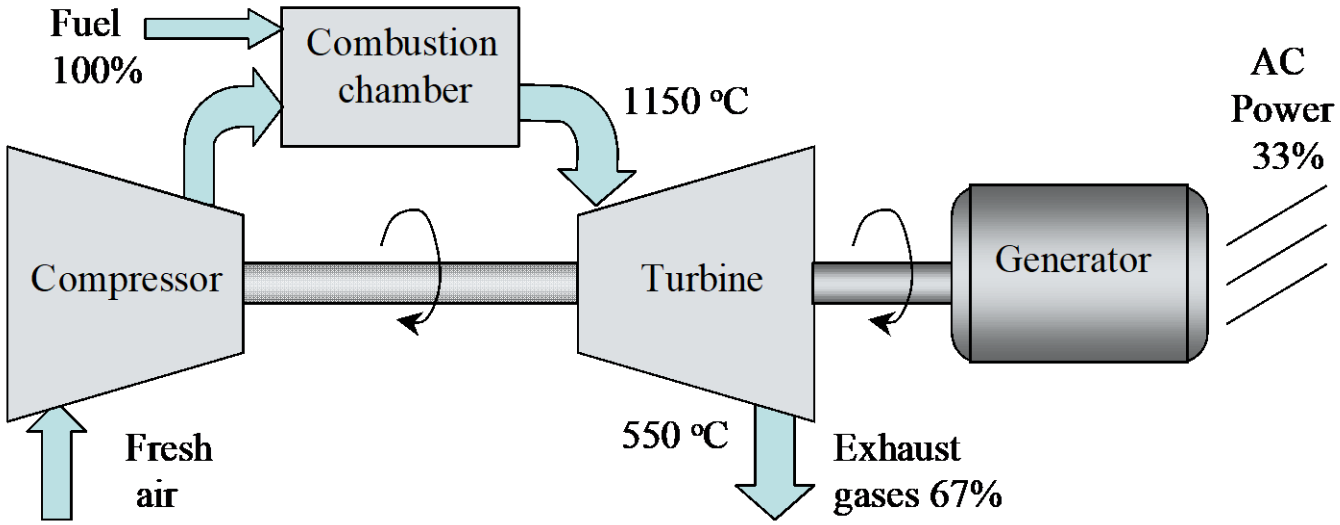
Figure 3.19 Typical modern coal-fired power plant using an electrostatic precipitator for particulate control and a limestone-based SO_2 scrubber. A cooling tower is shown for thermal pollution control. From Masters (1998).

Source: Masters, Renewable and Efficient Electric Power Systems, 2004

Turbine for Nuclear Power Plant

Source: images.pennnet.com/articles/pe/cap/cap_gephoto.jpg

Basic Gas Turbine



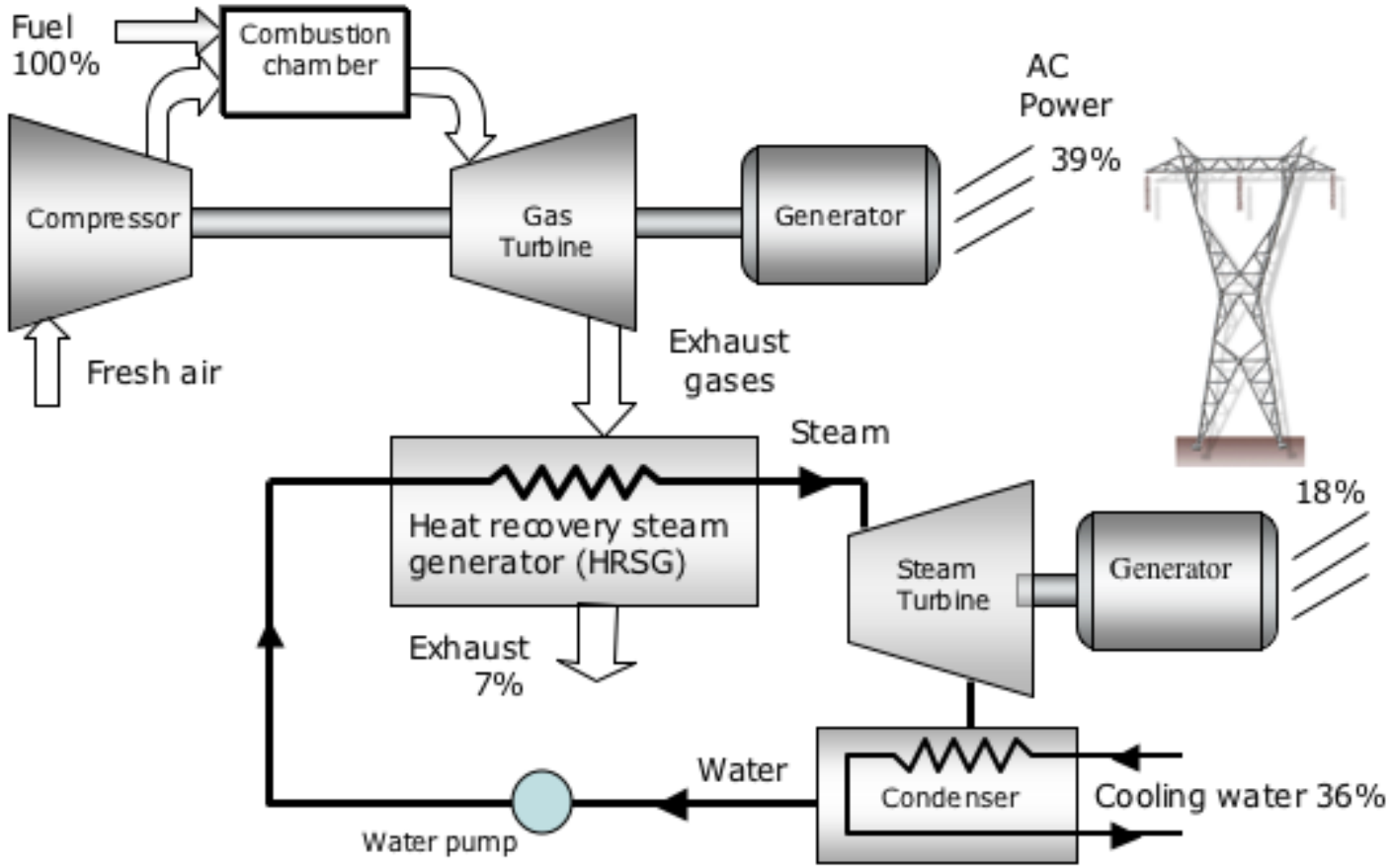
Most common fuel is natural gas

Typical efficiency is around 30 to 35%

Maximum Efficiency

$$= 1 - \frac{550 + 273}{1150 + 273} = 42\%$$

Combined Cycle Power Plant



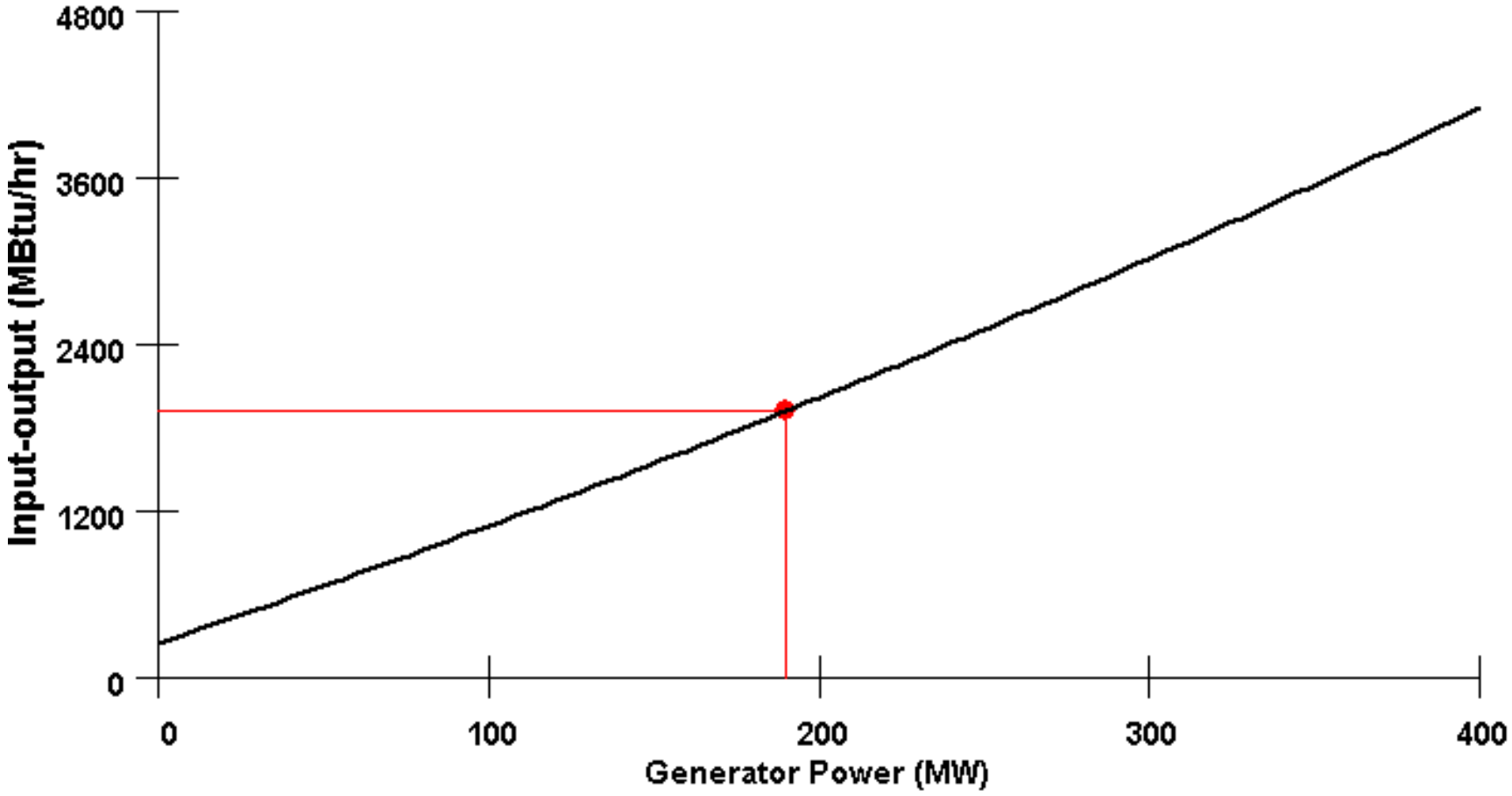
Efficiencies of up to 60% can be achieved, with even higher values when the steam is used for heating; fuel is usually natural gas

Generator Cost Curves

- Generator costs are typically represented by up to four different curves
  - input/output (I/O) curve
  - fuel-cost curve
  - heat-rate curve
  - incremental cost curve
- For reference
  - 1 Btu (British thermal unit) = 1054 J
  - 1 MBtu = $1x10^6$ Btu
  - 1 MBtu = 0.293 MWh
  - 3.41 Mbtu = 1 MWh

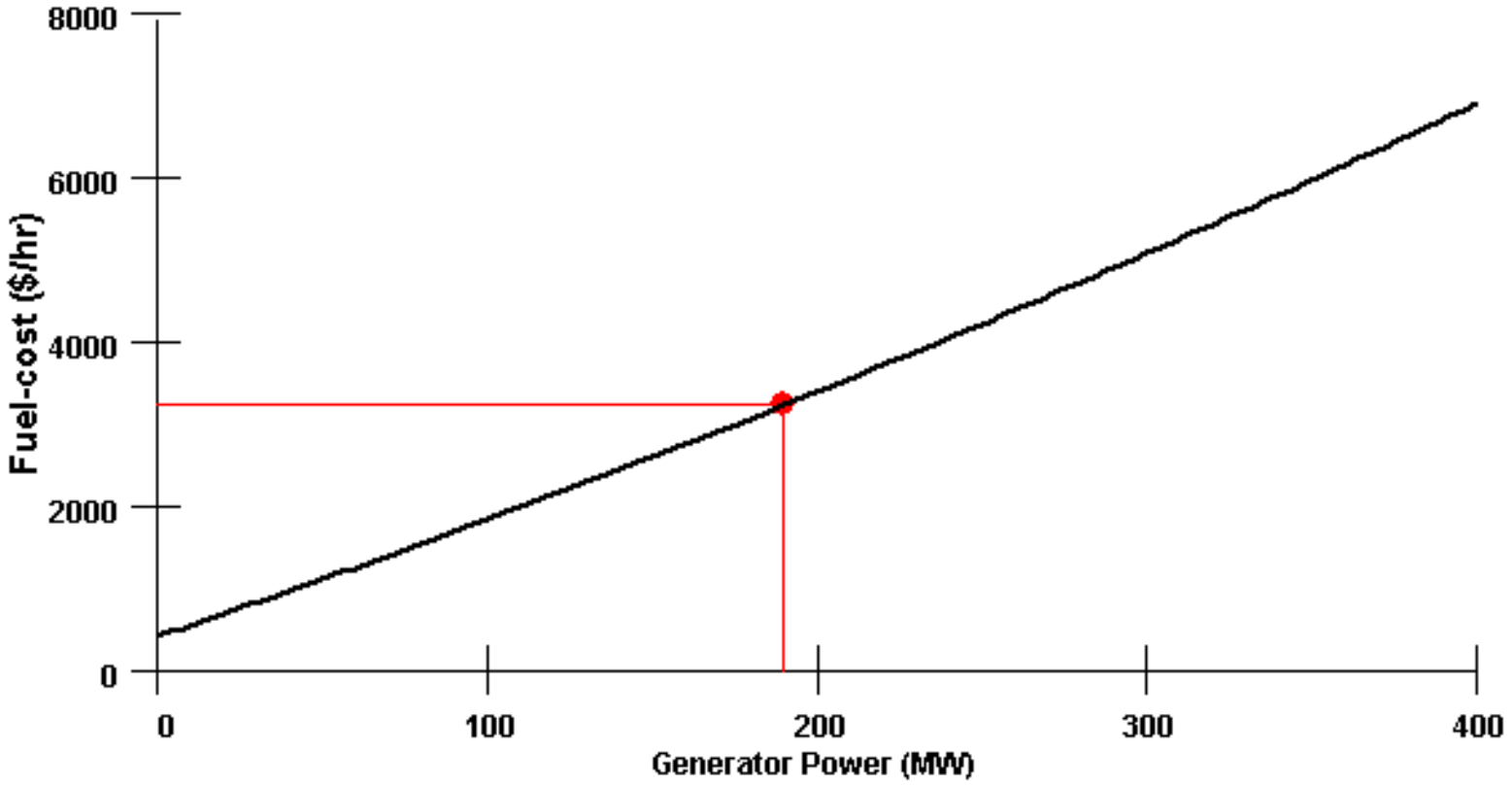
I/O Curve

- The IO curve plots fuel input (in MBtu/hr) versus net MW output.



Fuel-cost Curve

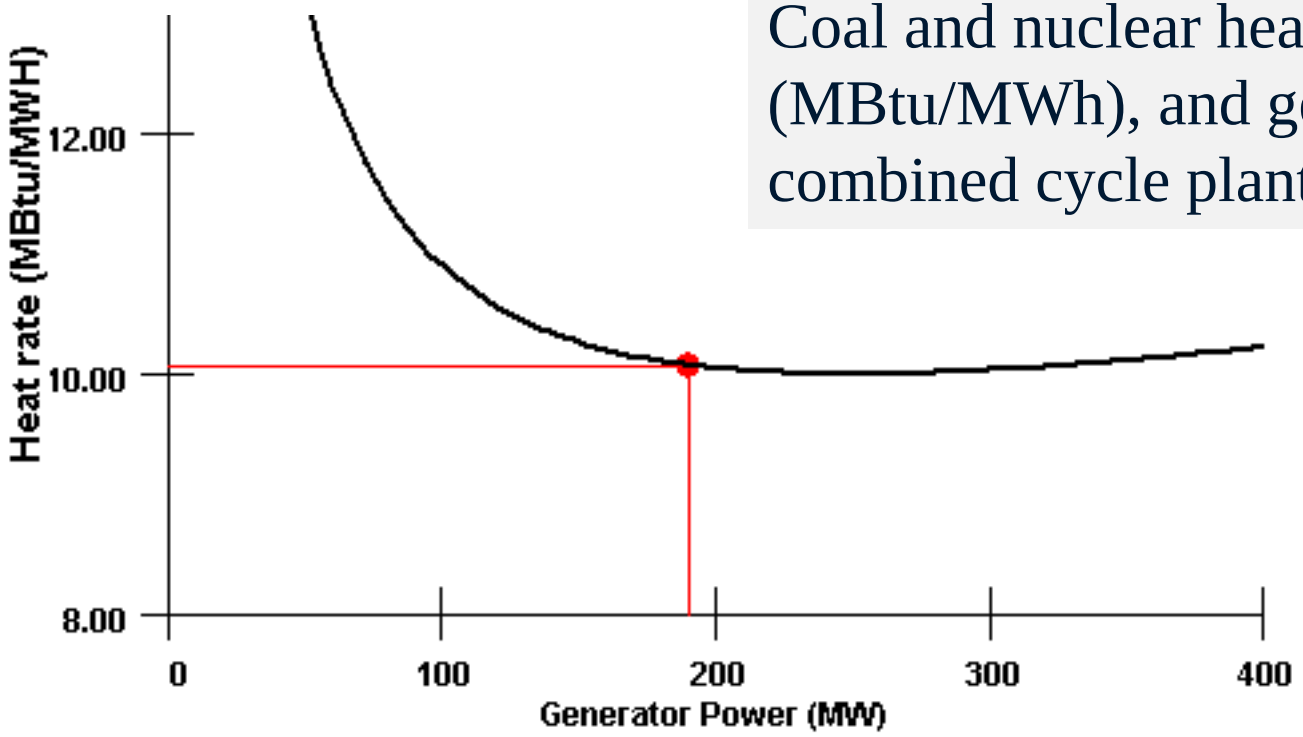
- The fuel-cost curve is the I/O curve scaled by fuel cost. Fuel costs vary (with the latest values in Chapter 4 of EIA's Electricity Monthly)



For 11/2024, coal in Texas was $1.80 per Mbtu, while natural gas was at $1.96; natural gas was about $4/Mbtu in California

Heat-rate Curve

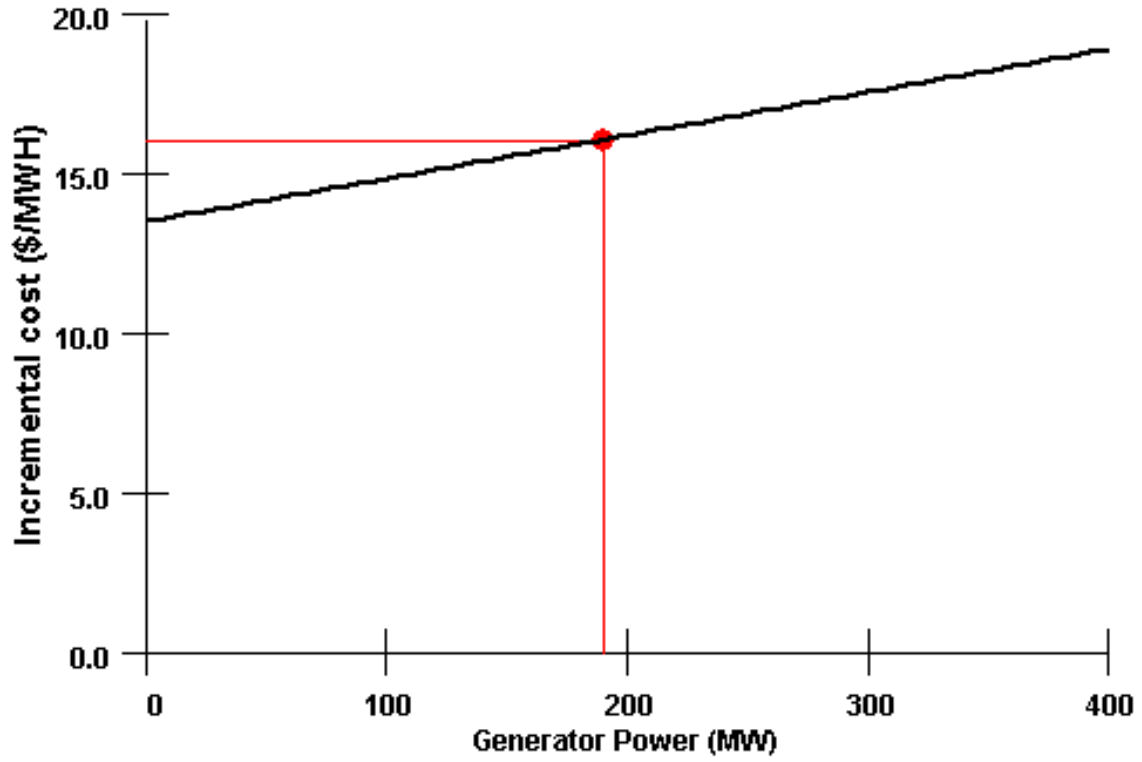
- Plots the average number of MBtu/hr of fuel input needed per MW of output.
- Heat-rate curve is the I/O curve scaled by MW; often heat rates are given in Btu/kWh, which is the Mbtu/MWh value multiplied by 1000



Coal and nuclear heat rates are around 10 (MBtu/MWh), and get down to 6.2 for highly efficient combined cycle plants

Incremental (Marginal) cost Curve

- Plots the incremental $/MWh as a function of MW.
- Found by differentiating the cost curve



Mathematical Formulation of Costs

- Generator cost curves are usually not smooth. However the curves can usually be adequately approximated using piece-wise smooth, functions.
- Two representations predominate
  - quadratic or cubic functions
  - piecewise linear functions
- In 460 we'll assume a quadratic representation

$$C_i(P_{Gi}) = \alpha_i + \beta P_{Gi} + \gamma P_{Gi}^2 \quad \$/\text{hr (fuel-cost)}$$

$$IC_i(P_{Gi}) = \frac{dC_i(P_{Gi})}{dP_{Gi}} = \beta + 2\gamma P_{Gi} \quad \$/\text{MWh}$$

Coal Usage Example 1

- A 500 MW (net) generator is 35% efficient. It is being supplied with Western grade coal, which costs $1.70 per MBtu and has 9000 Btu per pound. What is the coal usage in lbs/hr? What is the cost?

At 35% efficiency required fuel input per hour is

$$\frac{500\ \text{MWh}}{\text{hr}\times 0.35}=\frac{1428\ \text{MWh}}{\text{hr}}\times\frac{1\ \text{MBtu}}{0.29\ \text{MWh}}=\frac{4924\ \text{MBtu}}{\text{hr}}$$

$$\frac{4924\ \text{MBtu}}{\text{hr}}\times\frac{1\ \text{lb}}{0.009\text{MBtu}}=\frac{547{,}111\ \text{lbs}}{\text{hr}}$$

$$\text{Cost}=\frac{4924\ \text{MBtu}}{\text{hr}}\times\frac{\$1.70}{\text{MBtu}}=8370.8\ \$/\text{hr or }\$16.74/\text{MWh}$$

Coal Usage Example 2

- Assume a lights with a total power consumption of 100W are left on by mistake for 8 hours, and that the electricity is supplied by the previous coal plant and that transmission and distribution losses are 20%. How coal has been used?

  With 20% losses, a 100W load on for 8 hrs requires

  1 kWh of energy. With 35% gen. efficiency this requires

$$\frac{1\text{ kWh}}{0.35}\times\frac{1\text{ MWh}}{1000\text{ kWh}}\times\frac{1\text{ MBtu}}{0.29\text{ MWh}}\times\frac{1\text{ lb}}{0.009\text{MBtu}}=1.09\text{ lb}$$

Incremental Cost Example

For a two generator system assume

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \ \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \ \$/hr$$

Then

$$IC_1(P_{G1}) = \frac{dC_1(P_{G1})}{dP_{G1}} = 20 + 0.02P_{G1} \ \$/\text{MWh}$$

$$IC_2(P_{G2}) = \frac{dC_2(P_{G2})}{dP_{G2}} = 15 + 0.06P_{G2} \ \$/\text{MWh}$$

Incremental Cost Example, cont'd

If $P_{G1} = 250$ MW and $P_{G2} = 150$ MW Then

$$C_1(250) = 1000 + 20 \times 250 + 0.01 \times 250^2 = \$\ 6625/\text{hr}$$

$$C_2(150) = 400 + 15 \times 150 + 0.03 \times 150^2 = \$6025/\text{hr}$$

Then

$$IC_1(250) = 20 + 0.02 \times 250 = \$\ 25/\text{MWh}$$

$$IC_2(150) = 15 + 0.06 \times 150 = \$\ 24/\text{MWh}$$

Power System Economic Dispatch

- Economic dispatch is formulated as a constrained minimization
  - The cost function is often total generation cost in an area
  - Single equality constraint is the real power balance equation
- Solved by setting up the Lagrangian (with P_D the load and P_L the losses, which are a function the generation)

$$L(\mathbf{P}_G, \lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(\mathbf{P}_G) - \sum_{i=1}^{m} P_{Gi})$$

- A necessary condition for a minimum is that the gradient is zero. Without losses this occurs when all generators are dispatched at the same marginal cost (except when they hit a limit)

Power System Economic Dispatch

$$L(\mathbf{P}_G, \lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda(P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi})$$

$$\frac{\partial L(\mathbf{P}_G, \lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}) = 0$$

$$P_D + P_L(P_G) - \sum_{i=1}^{m} P_{Gi} = 0$$

- If losses are neglected then there is a single marginal cost (lambda); if losses are included then each bus could have a different marginal cost

Economic Dispatch Penalty Factors

Solving each equation for λ we get

$$\frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right) = 0$$

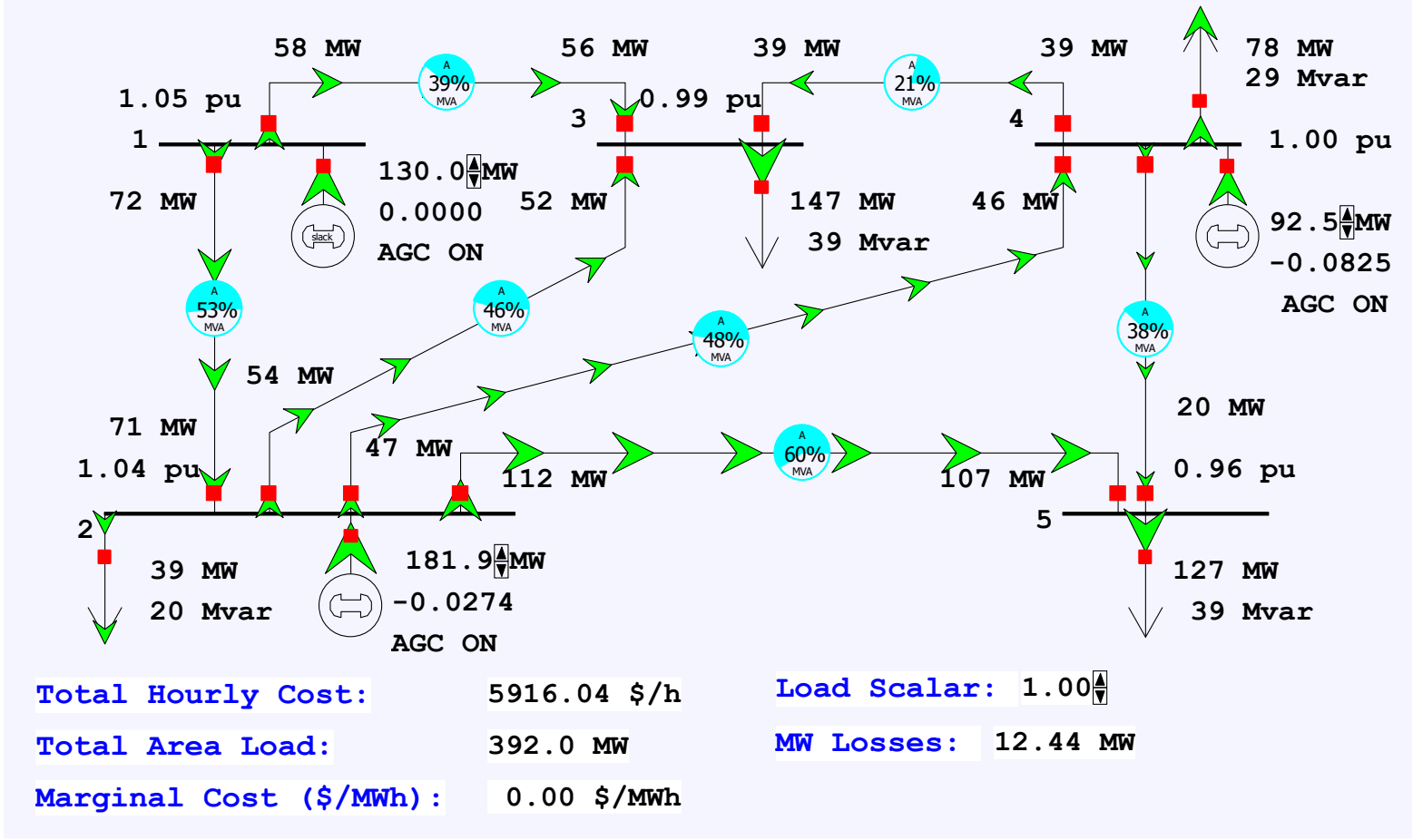
$$\lambda = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)} \frac{dC_i(P_{Gi})}{dP_{Gi}}$$

The penalty factor at the slack bus is always unity!

Define the penalty factor L_i for the i^{th} generator

$$L_i = \frac{1}{\left(1 - \frac{\partial P_L(P_G)}{\partial P_{Gi}}\right)}$$

Economic Dispatch Example



Total Hourly Cost:	5916.04 $/h	Load Scalar:	1.00
Total Area Load:	392.0 MW	MW Losses:	12.44 MW
Marginal Cost ($/MWh):	0.00 $/MWh		

Case is Example7_3 from the book; use **Power Flow Solution Options, Advanced Options** to set penalty factor options

Unconstrained Minimization

- Assume we have a function f(**x**) and wish to determine its minimum. This is an unconstrained minimization. A necessary (but not sufficient) condition for a minimum is the gradient of the function must be zero.
- The gradient generalizes the first derivative for multi-variable problems:

$$\nabla \mathbf{f}(\mathbf{x}) \equiv \left[\frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_1}, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_2}, \ldots, \frac{\partial \mathbf{f}(\mathbf{x})}{\partial x_n} \right] \qquad \nabla \mathbf{f}(\mathbf{x}) = \mathbf{0}$$

- This is not sufficient since the gradient is also zero at the maximum and at any saddle points

Minimization with Equality Constraint

- When the minimization is constrained with one or more equality constraints we can solve the problem using the method of Lagrange Multipliers
- Key idea is to modify a constrained minimization problem to be an unconstrained problem

That is, for the general problem

$$\text{minimize } \mathbf{f}(\mathbf{x}) \text{ s.t. } \mathbf{g}(\mathbf{x}) = \mathbf{0}$$

We define the Lagrangian $\quad L(\mathbf{x},\boldsymbol{\lambda}) = \mathbf{f}(\mathbf{x}) + \boldsymbol{\lambda}^T \mathbf{g}(\mathbf{x})$

Then a necessary condition for a minimum is the

$$\nabla L_{\mathbf{x}}(\mathbf{x},\boldsymbol{\lambda}) = 0 \quad \text{and} \quad \nabla L_{\boldsymbol{\lambda}}(\mathbf{x},\boldsymbol{\lambda}) = 0$$

Economic Dispatch Lagrangian

For the economic dispatch we have a minimization constrained with a single equality constraint

$$\mathrm{L}(\mathbf{P}_\mathrm{G},\lambda) = \sum_{i=1}^{m} C_i(P_{Gi}) + \lambda\left(P_D - \sum_{i=1}^{m} P_{Gi}\right) \quad \text{(no losses)}$$

The necessary conditions for a minimum are

$$\frac{\partial \mathrm{L}(\mathbf{P}_\mathrm{G},\lambda)}{\partial P_{Gi}} = \frac{dC_i(P_{Gi})}{dP_{Gi}} - \lambda = 0 \quad \text{(for i } = 1 \text{ to m)}$$

$$P_D - \sum_{i=1}^{m} P_{Gi} = 0$$

Economic Dispatch Example

What is economic dispatch for a two generator system $P_D = P_{G1} + P_{G2} = 500$ MW and

$$C_1(P_{G1}) = 1000 + 20P_{G1} + 0.01P_{G1}^2 \ \$/hr$$

$$C_2(P_{G2}) = 400 + 15P_{G2} + 0.03P_{G2}^2 \ \$/hr$$

Using the Largrange multiplier method we know

$$\frac{dC_1(P_{G1})}{dP_{G1}} - \lambda = 20 + 0.02P_{G1} - \lambda = 0$$

$$\frac{dC_2(P_{G2})}{dP_{G2}} - \lambda = 15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

Economic Dispatch Example, cont'd

We therefore need to solve three linear equations

$$20 + 0.02P_{G1} - \lambda = 0$$

$$15 + 0.06P_{G2} - \lambda = 0$$

$$500 - P_{G1} - P_{G2} = 0$$

$$\begin{bmatrix} 0.02 & 0 & -1 \\ 0 & 0.06 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -20 \\ -15 \\ -500 \end{bmatrix}$$

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 312.5 \text{ MW} \\ 187.5 \text{ MW} \\ 26.2 \text{ \$/MWh} \end{bmatrix}$$

Lambda-Iteration Solution Method

- The direct solution only works well if the incremental cost curves are linear and no generators are at their limits
- A more general method is known as the lambda-iteration
  - the method requires that there be a unique mapping between a value of lambda and each generator's MW output
  - the method then starts with values of lambda below and above the optimal value, and then iteratively brackets the optimal value

Lambda-Iteration Algorithm

Pick λ^{L} and λ^{H} such that

$$\sum_{i=1}^{m} P_{Gi}(\lambda^{L}) - P_D < 0 \qquad \sum_{i=1}^{m} P_{Gi}(\lambda^{H}) - P_D > 0$$

While $\left|\lambda^{H} - \lambda^{L}\right| > \varepsilon$ Do

$$\lambda^{M} = (\lambda^{H} + \lambda^{L}) / 2$$

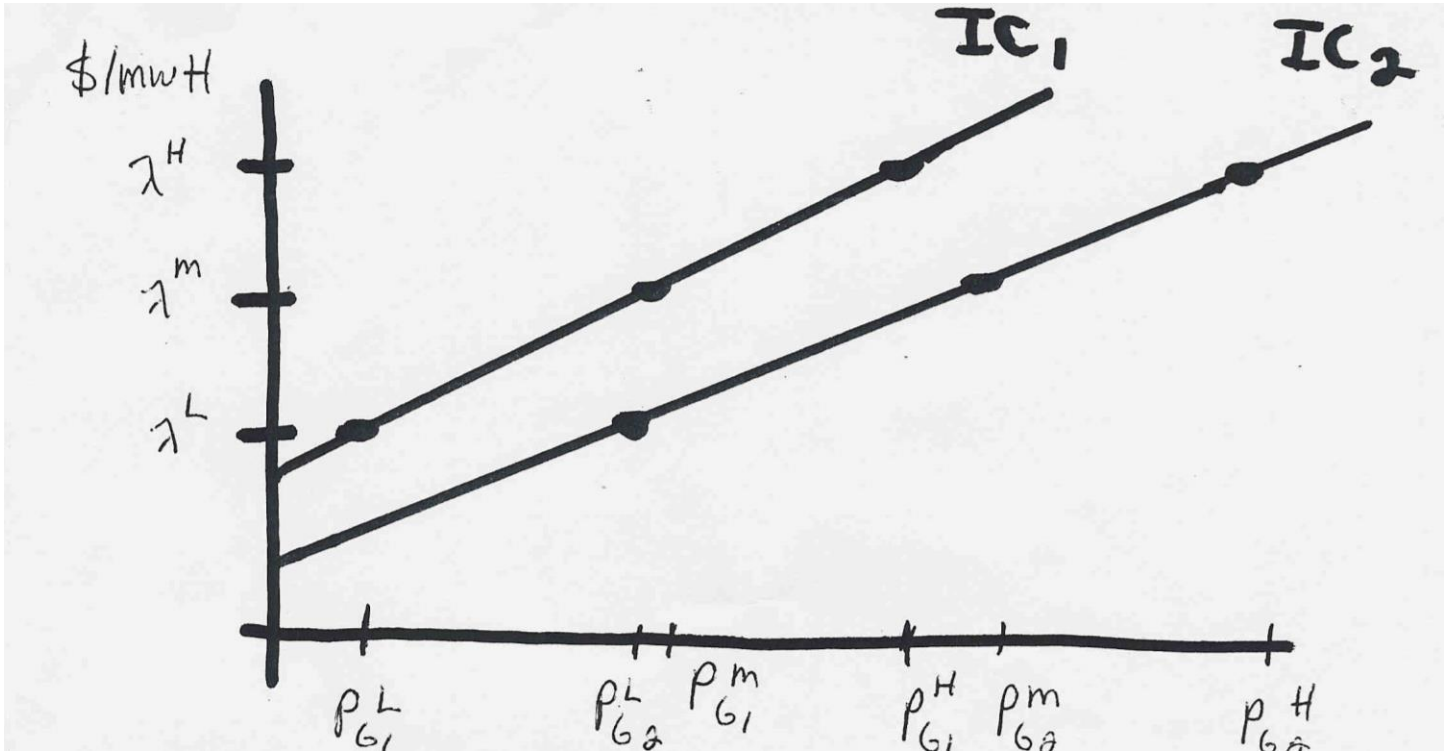
If $\sum_{i=1}^{m} P_{Gi}(\lambda^{M}) - P_D > 0$ Then $\lambda^{H} = \lambda^{M}$

Else $\lambda^{L} = \lambda^{M}$

End While

Lambda-Iteration: Graphical View

- In the graph shown below for each value of lambda there is a unique P_{Gi} for each generator. This relationship is the $P_{Gi}(\lambda)$ function.



Lambda-Iteration Example

Consider a three generator system with

$$IC_1(P_{G1}) = 15 + 0.02P_{G1} = \lambda \quad \$/\text{MWh}$$

$$IC_2(P_{G2}) = 20 + 0.01P_{G2} = \lambda \quad \$/\text{MWh}$$

$$IC_3(P_{G3}) = 18 + 0.025P_{G3} = \lambda \quad \$/\text{MWh}$$

and with constraint $P_{G1} + P_{G2} + P_{G3} = 1000\,\text{MW}$

Rewriting as a function of λ, $P_{Gi}(\lambda)$, we have

$$P_{G1}(\lambda) = \frac{\lambda - 15}{0.02} \qquad P_{G2}(\lambda) = \frac{\lambda - 20}{0.01}$$

$$P_{G3}(\lambda) = \frac{\lambda - 18}{0.025}$$

Lambda-Iteration Example, cont'd

Pick λ^L so $\sum_{i=1}^{m} P_{Gi}(\lambda^L) - 1000 < 0$ and

$$\sum_{i=1}^{m} P_{Gi}(\lambda^H) - 1000 > 0$$

Try $\lambda^L = 20$ then $\sum_{i=1}^{m} P_{Gi}(20) - 1000 =$

$$\frac{\lambda - 15}{0.02} + \frac{\lambda - 20}{0.01} + \frac{\lambda - 18}{0.025} - 1000 = -670 \text{ MW}$$

Try $\lambda^H = 30$ then $\sum_{i=1}^{m} P_{Gi}(30) - 1000 = 1230$ MW

Lambda-Iteration Example, cont'd

Pick convergence tolerance $\varepsilon = 0.05$ \$/MWh

Then iterate since $\left|\lambda^H - \lambda^L\right| > 0.05$

$$\lambda^M = (\lambda^H + \lambda^L)/2 = 25$$

Then since $\sum_{i=1}^{m} P_{Gi}(25) - 1000 = 280$ we set $\lambda^H = 25$

Since $|25 - 20| > 0.05$

$$\lambda^M = (25 + 20)/2 = 22.5$$

$$\sum_{i=1}^{m} P_{Gi}(22.5) - 1000 = -195 \text{ we set } \lambda^L = 22.5$$

Lambda-Iteration Example, cont'd

Continue iterating until $\left|\lambda^{\mathrm{H}} - \lambda^{L}\right| < 0.05$

The solution value of λ, λ^{*}, is 23.53 $/MWh

Once λ^{*} is known we can calculate the P_{Gi}

$$P_{G1}(23.5) = \frac{23.53 - 15}{0.02} = 426 \text{ MW}$$

$$P_{G2}(23.5) = \frac{23.53 - 20}{0.01} = 353 \text{ MW}$$

$$P_{G3}(23.5) = \frac{23.53 - 18}{0.025} = 221 \text{ MW}$$

Generator MW Limits

- Generators have limits on the minimum and maximum amount of power they can produce
- Often times the minimum limit is not zero. This represents a limit on the generator's operation with the desired fuel type
- Because of varying system economics usually many generators in a system are operated at their maximum MW limits.

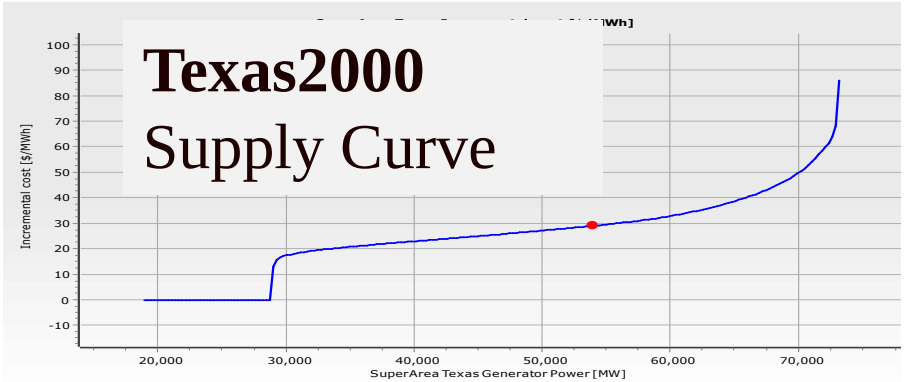
In the lambda-iteration method the limits are taken into account when calculating $P_{Gi}(\lambda)$:

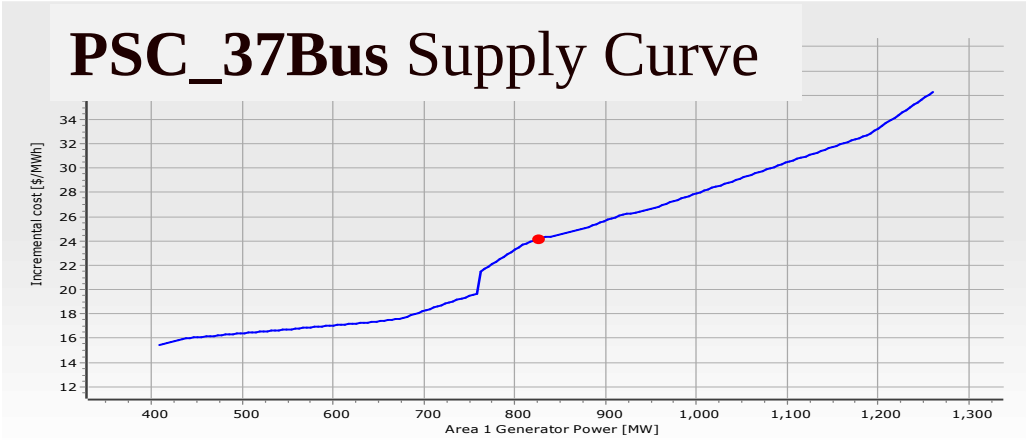
if $P_{Gi}(\lambda) > P_{Gi,\max}$ then $P_{Gi}(\lambda) = P_{Gi,\max}$

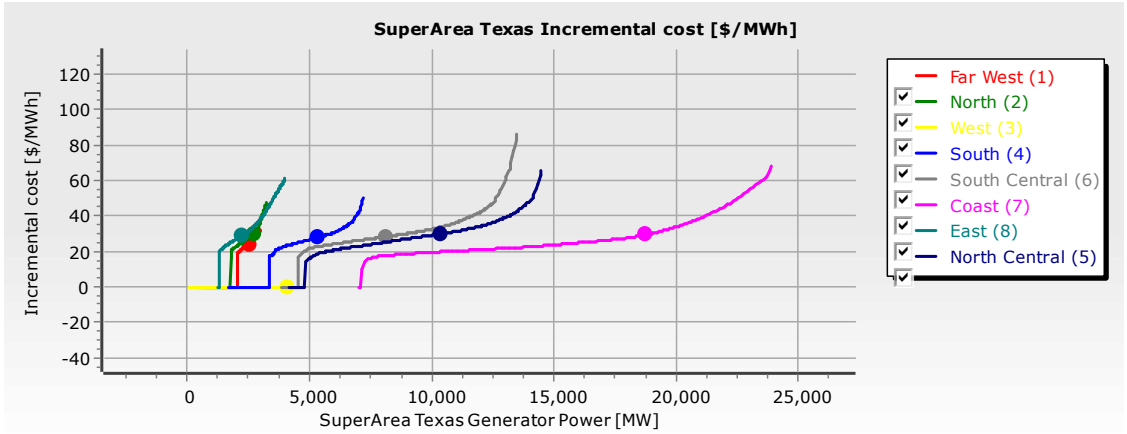
if $P_{Gi}(\lambda) < P_{Gi,\min}$ then $P_{Gi}(\lambda) = P_{Gi,\min}$

Area Supply Curve

- The area supply curve shows how the marginal cost for an area (or the entire case) varies assuming an economic dispatch
  - The curves for an area or super area are available in their case information displays by right-clicking and selecting **Plot** then one of its options
  - This is a quick way to get an overall feel for a case







Electric Grid Terminology: Security vs Reliability

- Historically electric grid design used the term security to mean the reliable operation of the electric grid. However, after Sept 11th, the term has been changed (mostly) to reliability
  - [From NERC History Book, page 62] The World Trade Center and the Pentagon were attacked by terrorists on September 11, 2001. Shortly after the attack, Mr. Richard Clarke, then national coordinator for Security, Infrastructure Protection, and Counterterrorism, met with NERC officials to discuss ways of protecting the electric system from acts of terrorism. After hearing NERC references to "security coordinators," Mr. Clarke interrupted to say, "Wait a minute— I'm the security coordinator." Shortly thereafter, NERC began to use the term "operating reliability" in place of "security," and "Reliability Coordinators" in place of "security coordinators."

Quote source: www.nerc.com/news/Documents/NERCHistoryBook.pdf

Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch
- Security Constrained OPF (SCOPF) adds in contingency analysis
- Goal of OPF and SCOPF is to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange