ECEN 460
Power System Operation and Control
Spring 2025

Lecture 12: Optimal Power Flow (OPF) and Security-Constrained OPF (SCOPF)

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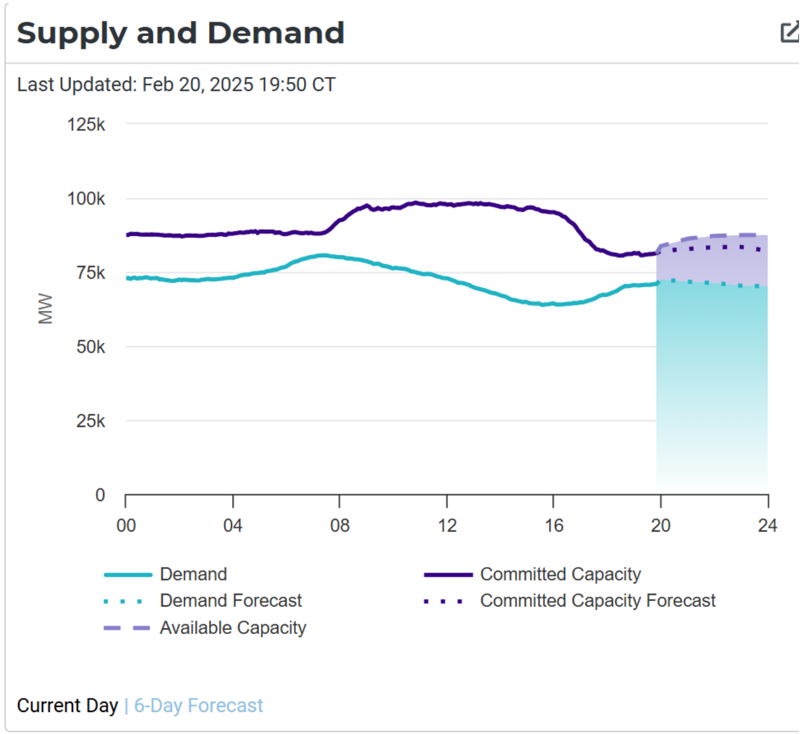
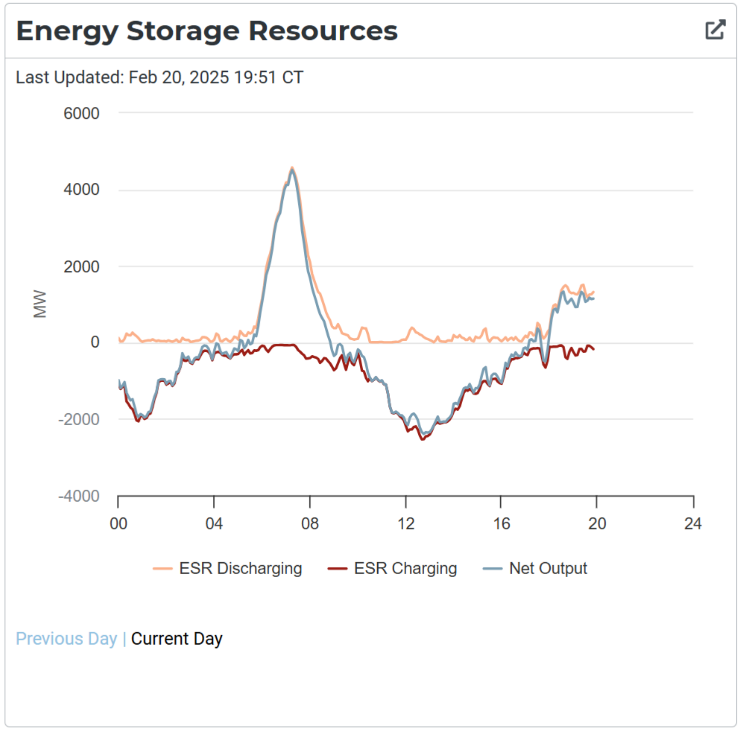
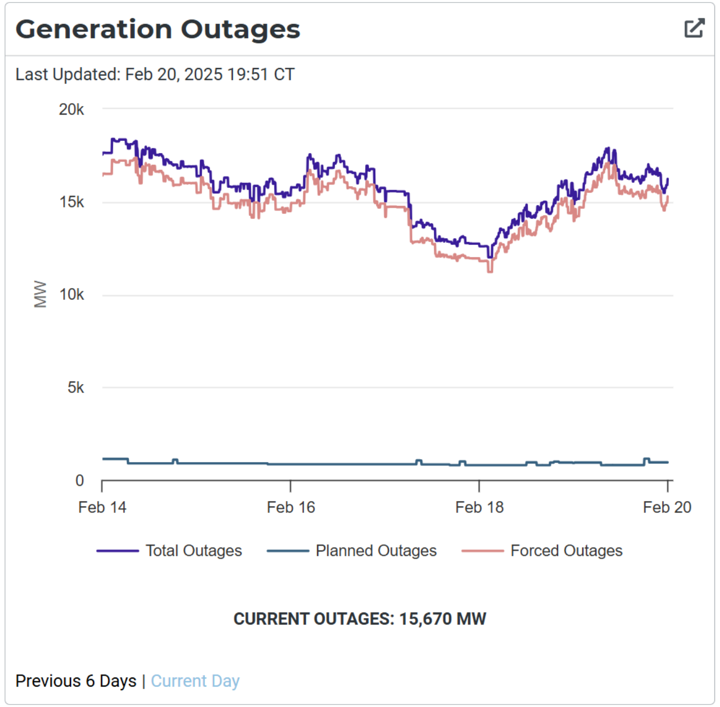
TEXAS A&M UNIVERSITY

ERCOT, Feb 20 2025 (New Winter Peak Load)

- Maximum load of 80.6 GW, a new winter peak for ERCOT







Optimal Power Flow (OPF)

- OPF functionally combines the power flow with economic dispatch
- Security Constrained OPF (SCOPF) adds in contingency analysis
- Goal of OPF and SCOPF is to minimize a cost function, such as operating cost, taking into account realistic equality and inequality constraints
- Equality constraints
  - bus real and reactive power balance
  - generator voltage setpoints
  - area MW interchange

OPF, cont.

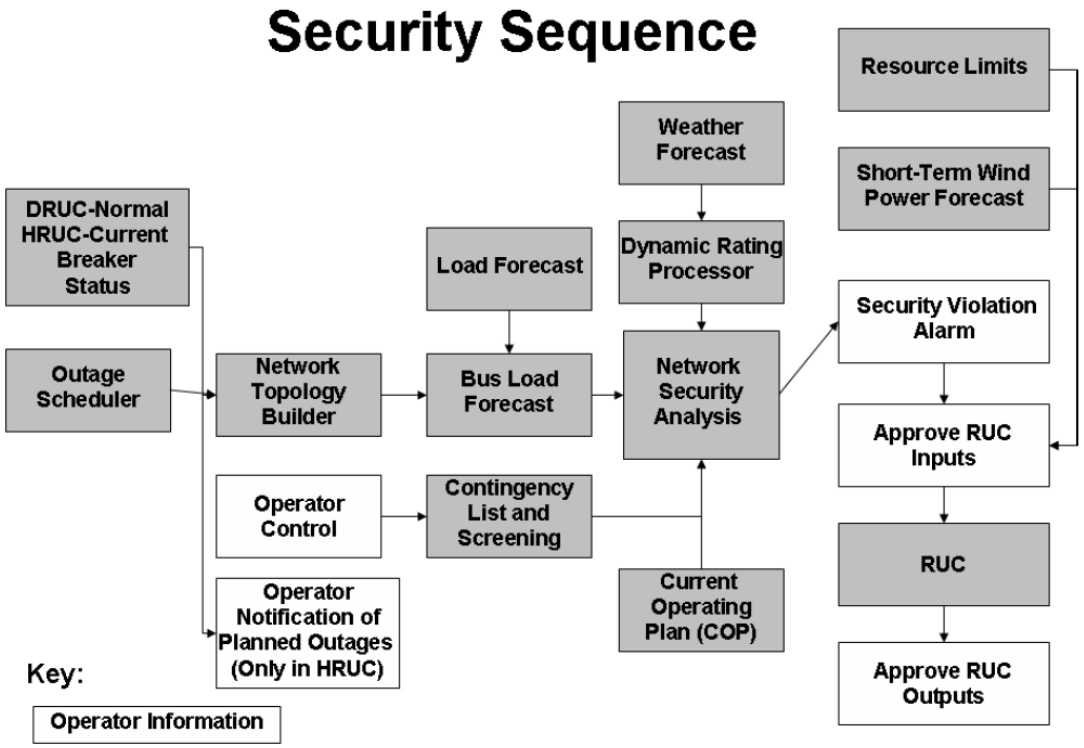
- Inequality constraints
  - transmission line/transformer/interface flow limits
  - generator MW limits
  - generator reactive power capability curves
  - bus voltage magnitudes (not yet implemented in Simulator OPF)
- Available Controls
  - generator MW outputs
  - transformer taps and phase angles
  - reactive power controls

Key SCOPF Application: Locational Marginal Prices (LMPs)

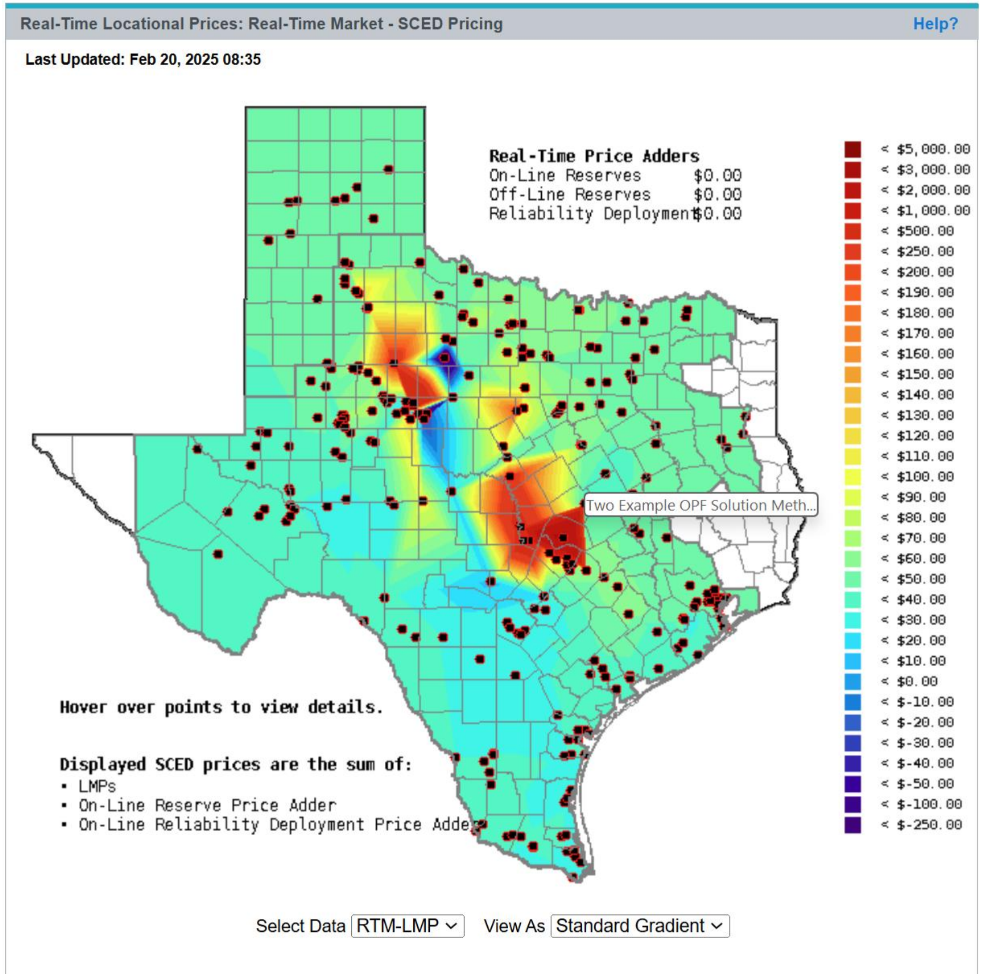
- When OPF includes contingency analysis it is known as the Security-Constrained OPF (SCOPF)
- OPF dates back to 1960's with thousands of papers
- The locational marginal price (LMP) tells the cost of providing electricity to a given location (bus) in the system
- Concept introduced by Schweppe in 1985
  - F.C. Schweppe, M. Caramanis, R. Tabors, "Evaluation of Spot Price Based Electricity Rates," *IEEE Trans. Power App and Syst*., July 1985
- LMPs are a direct result of an SCOPF, and are widely used in many electricity markets worldwide both ahead and in real-time
- The exact calculations are market specific

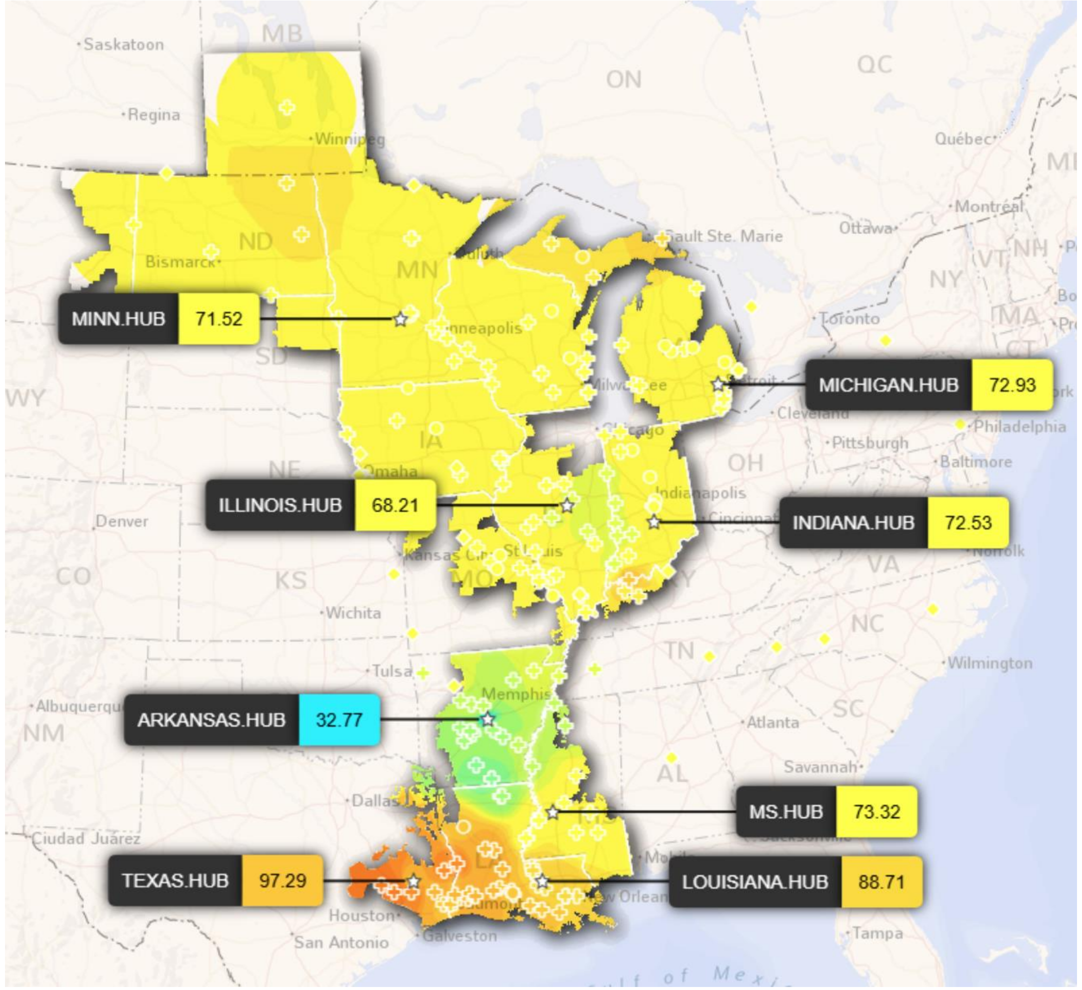
Example: ERCOT Security Sequence

- The ERCOT Nodal Protocols document details the process used by ERCOT
  - RUC is Reliability Unit Commitment
  - DRUC is the Day-Ahead Reliability Unit Commitment
  - HRUC is the Hourly Reliability Unit Commitment
- The most recent documents are at www.ercot.com/mktrules/nprotocols/current



ERCOT and MISO LMPs, Feb 20, 2025 at about 9am





Images: www.ercot.com/content/cdr/contours/rtmLmp.html, api.misoenergy.org/misortwd/lmpcontourmap.html

OPF Problem Formulation

- The OPF is usually formulated as a minimization with equality and inequality constraints

$$\text{Minimize } F(\mathbf{x},\mathbf{u})$$

$$\mathbf{g}(\mathbf{x},\mathbf{u}) = \mathbf{0}$$

$$\mathbf{h}_{\min} \le \mathbf{h}(\mathbf{x},\mathbf{u}) \le \mathbf{h}_{\max}$$

$$\mathbf{u}_{\min} \le \mathbf{u} \le \mathbf{u}_{\max}$$

where **x** is a vector of dependent variables (such as the bus voltage magnitudes and angles), **u** is a vector of the control variables, F(**x**,**u**) is the scalar objective function, **g** is a set of equality constraints (e.g., the power balance equations) and **h** is a set of inequality constraints (such as line flows)

Two Example OPF Solution Methods

- Non-linear approach using Newton's method
  - handles marginal losses well, but is relatively slow and has problems determining binding constraints
  - Generation costs (and other costs) represented by quadratic or cubic functions
- Linear Programming
  - fast and efficient in determining binding constraints, but can have difficulty with marginal losses.
  - used in PowerWorld Simulator
  - generation costs (and other costs) represented by piecewise linear functions
- Both can be implemented using an ac or dc power flow

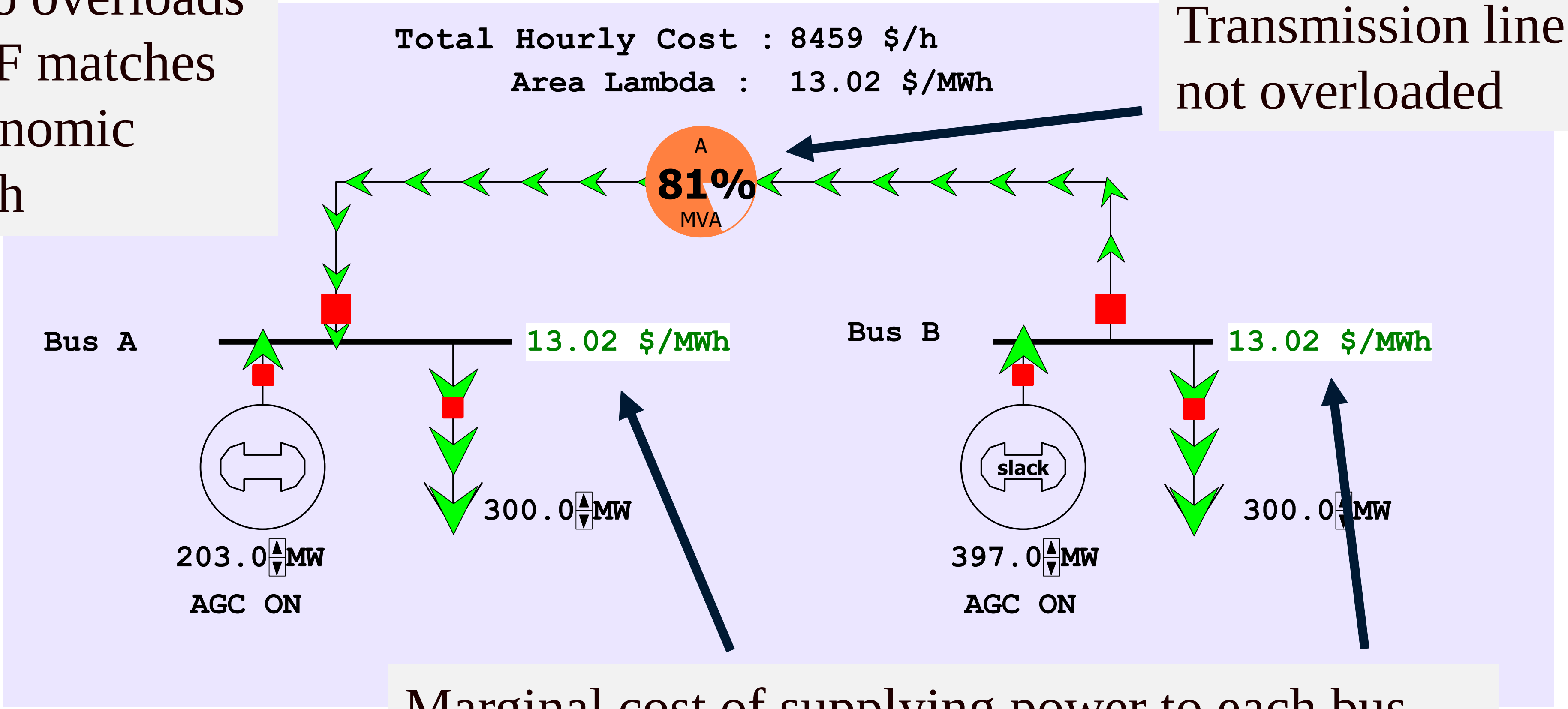
LP OPF Solution Method

- Solution iterates between
  - solving a full ac or dc power flow solution
    - enforces real/reactive power balance at each bus
    - enforces generator reactive limits
    - system controls are assumed fixed
    - takes into account non-linearities
  - solving a primal LP
    - changes system controls to enforce linearized constraints while minimizing cost

Two Bus with Unconstrained Line

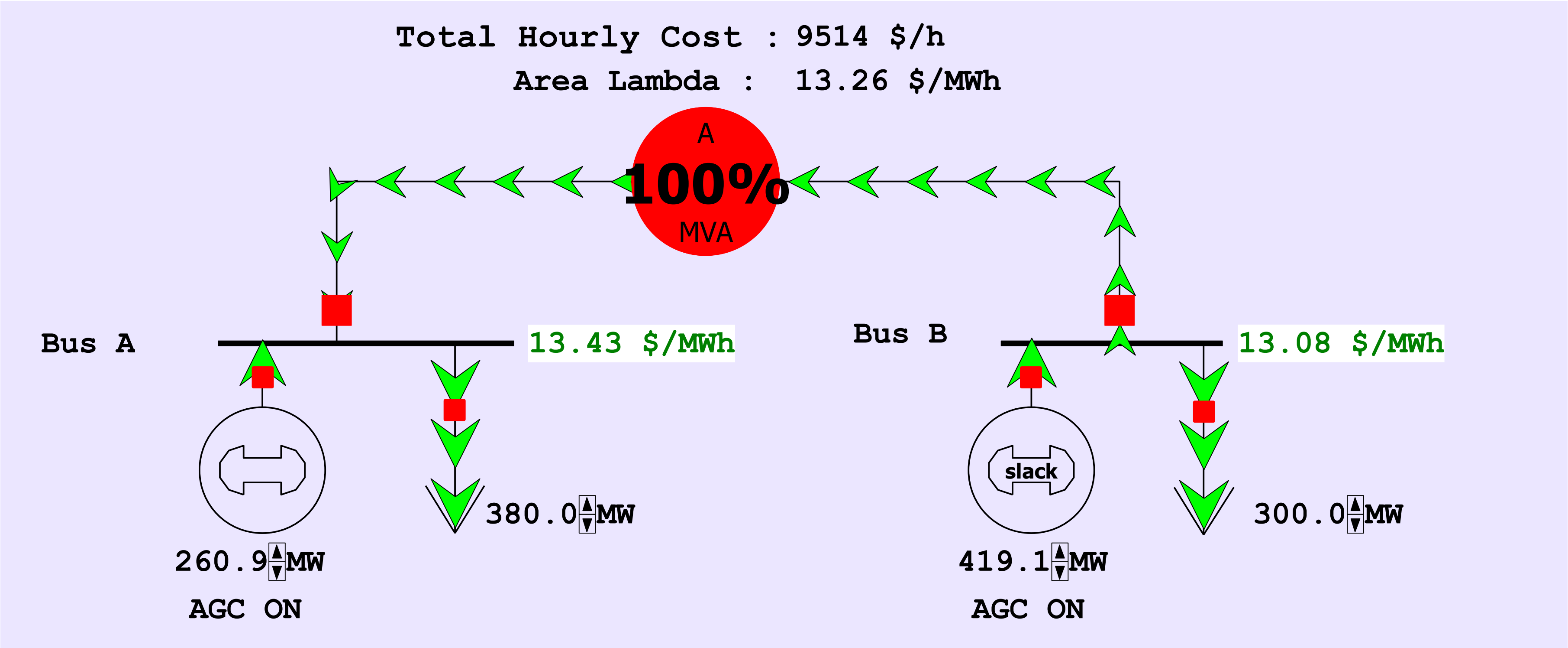
With no overloads the OPF matches the economic dispatch

Transmission line is not overloaded



Marginal cost of supplying power to each bus (locational marginal costs)

Two Bus with Constrained Line

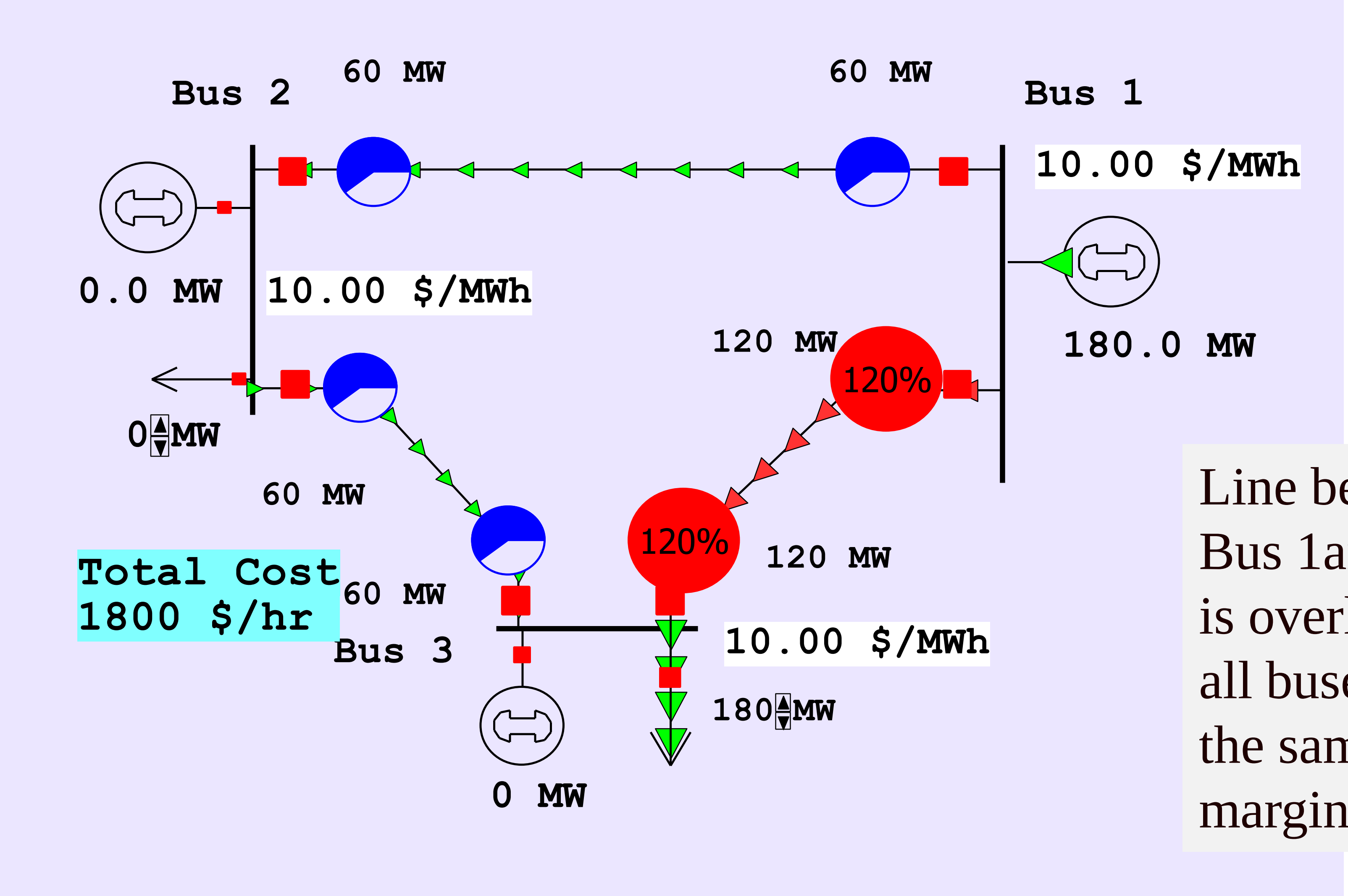


With the line loaded to its limit, additional load at Bus A must be supplied locally, causing the marginal costs to diverge.

Three Bus (B3) Example

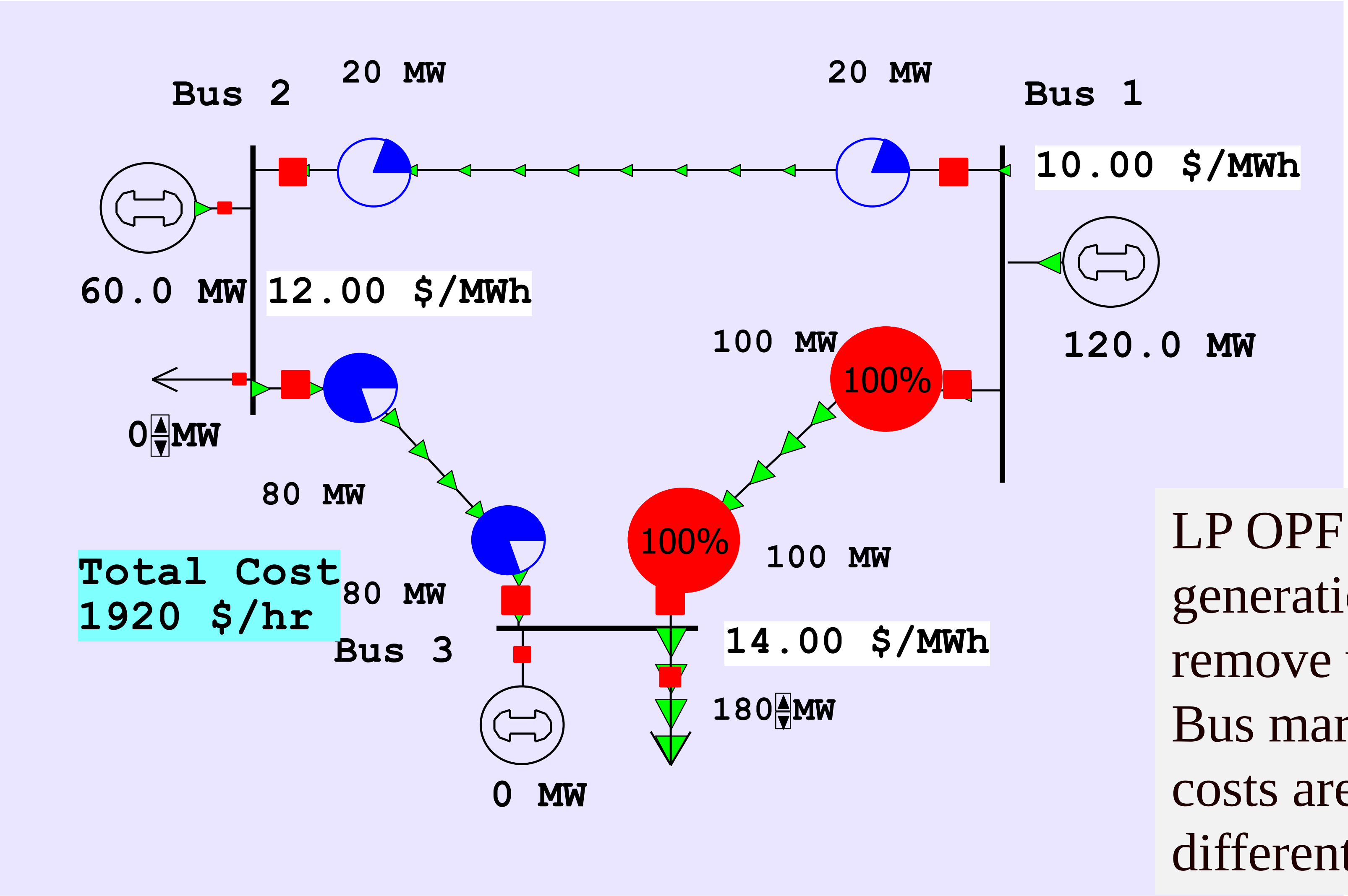
- Consider a three bus case (Bus 1 is system slack), with all buses connected through 0.1 pu reactance lines, each with a 100 MVA limit
- Let the generator marginal costs be
  - Bus 1: 10 $ / MWhr; Range = 0 to 400 MW
  - Bus 2: 12 $ / MWhr; Range = 0 to 400 MW
  - Bus 3: 20 $ / MWhr; Range = 0 to 400 MW
- Assume a single 180 MW load at bus 2

B3 with Line Limits NOT Enforced



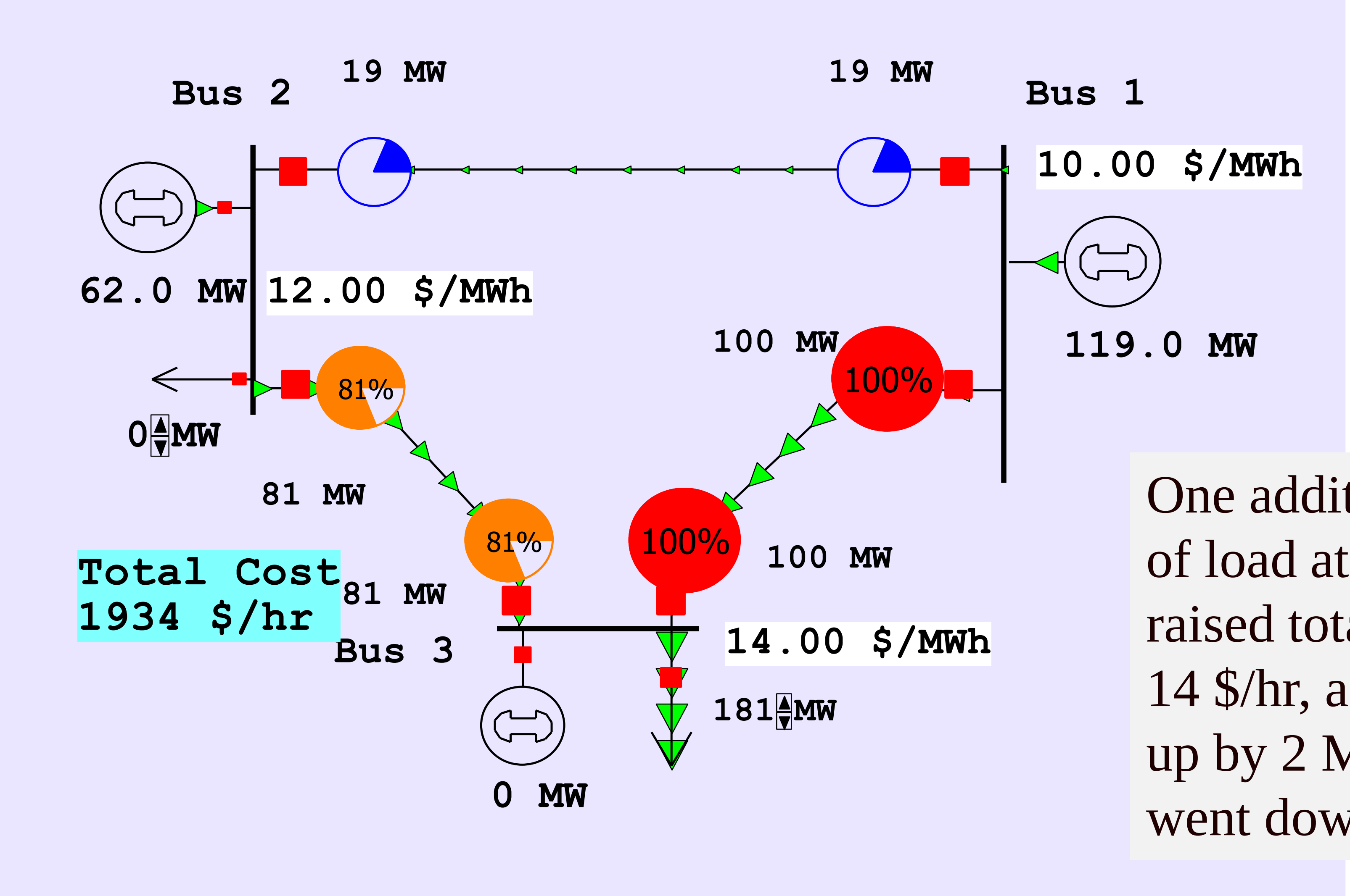
Line between Bus 1and Bus 3 is overloaded; all buses have the same marginal cost

B3 with Line Limits Enforced



LP OPF changes generation to remove violation. Bus marginal costs are now different.

Verify Bus 3 Marginal Cost



One additional MW of load at bus 3 raised total cost by 14 $/hr, as G2 went up by 2 MW and G1 went down by 1MW

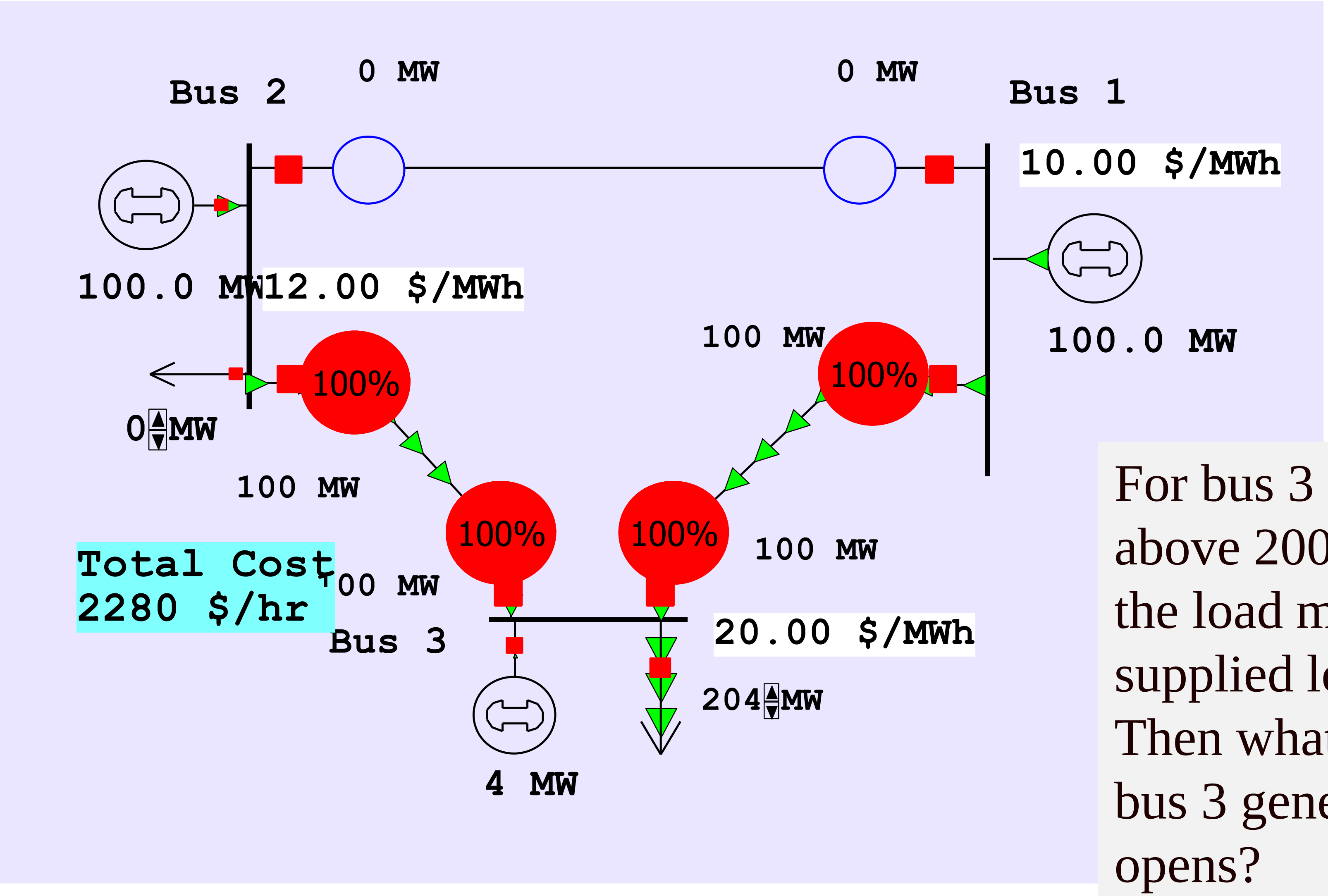
Why is bus 3 LMP = $14 /MWh

- All lines have equal impedance. Power flow in a simple network distributes inversely to impedance of path.
  - For bus 1 to supply 1 MW to bus 3, 2/3 MW would take direct path from 1 to 3, while 1/3 MW would “loop around” from 1 to 2 to 3.
  - Likewise, for bus 2 to supply 1 MW to bus 3, 2/3MW would go from 2 to 3, while 1/3 MW would go from 2 to 1to 3.

Why is bus 3 LMP $ 14 / MWh, cont'd

- With the line from 1 to 3 limited, no additional power flows are allowed on it.
- To supply 1 more MW to bus 3 we need
  - $\Delta P_{G1} + \Delta P_{G2} = 1$ MW
  - $2/3\, \Delta P_{G1} + 1/3\, \Delta P_{G2} = 0$; (no more flow on 1-3)
- Solving requires we up P_{G2} by 2 MW and drop P_{G1} by 1 MW -- a net increase of \$24 – \$10 = \$14.

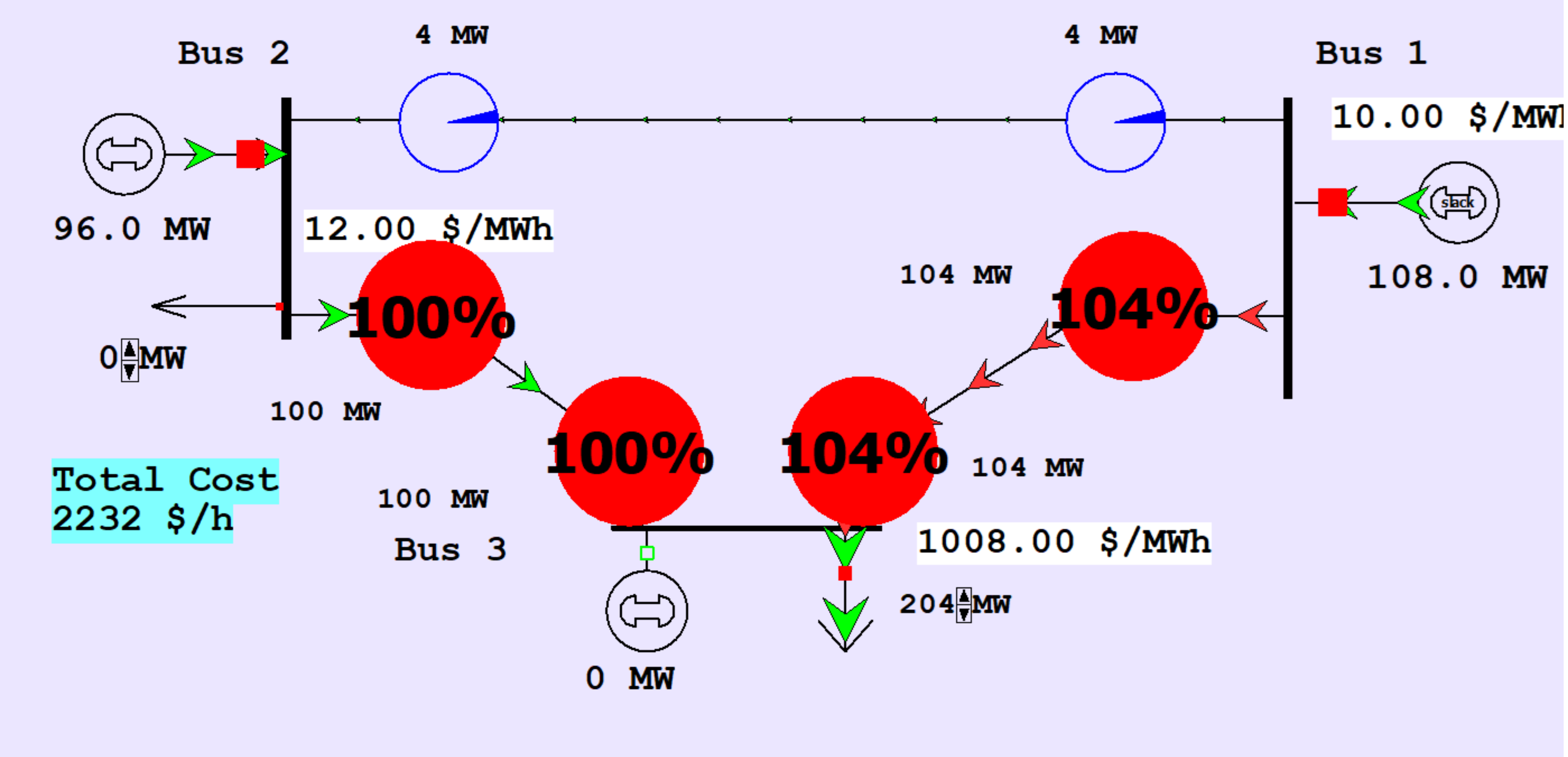
Both lines into Bus 3 Congested



For bus 3 loads above 200 MW, the load must be supplied locally. Then what if the bus 3 generator opens?

Both lines into Bus 3 Congested

An infeasible example can be created by opening the generator at Bus 3 with the Bus 3 load above 200 MW. There is no way to serve the load without overloading a transmission line.

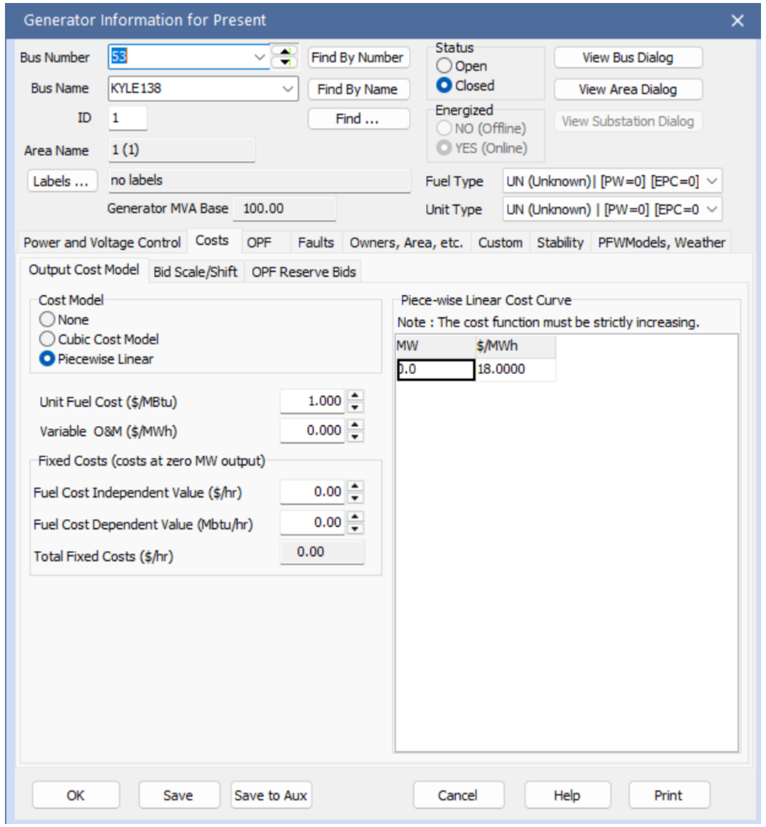


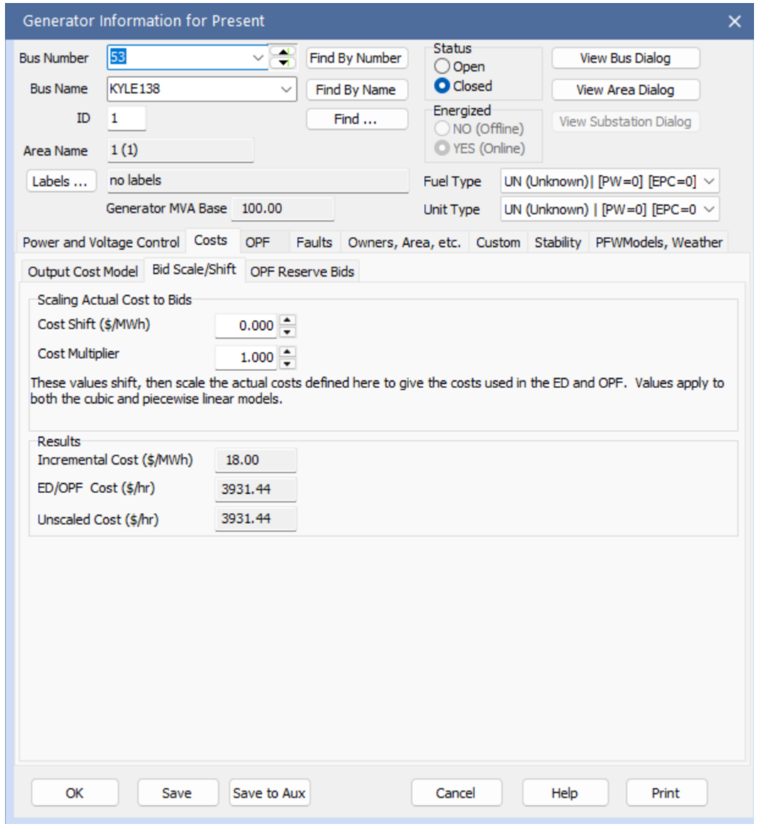
Lab 6 Comments

- Key inputs for the OPF and the SCOPF are the assumed generator costs. In PowerWorld these values can be specified either using a 4th order polynomial (with the cubic function from economic dispatch a special case), or as a piecewise linear curve (which is more common in power markets)
- In Lab 6, which at OPF and SCOPF, you’ll be using and modifying these values; in the lab you’ll just be scaling the provided cost curves
- OPF and SCOPF can be solved using either an ac power flow or a dc power flow; in the lab you’ll be using both

Lab 6 Comments, cont.

- Below example is from the lab's 37 bus case





Total Load 994.8 MW Load Multiplier 0.70

Cost: 16331 $/h Total Losses: 22.98 MW

Average LMP: 17.94 $/MWh

Generator Values

Gen Name	MW	Cost ($/MWh)	Cost Multiplier	LMP ($/MWh)	Profit $/hr
RUDDER69	0.0	42.0	1.00	18.64	0 $/h
CENTURY69	0.0	25.0	1.00	18.24	0 $/h
FISH69	0.0	40.0	1.00	18.54	0 $/h
AGGIE345	400.0	15.0	1.00	17.19	878 $/h
SLACK345	400.0	16.0	1.00	17.34	538 $/h
SPIRIT69	0.0	25.0	1.00	18.34	0 $/h
RELLIS69	0.0	25.0	1.00	18.02	0 $/h
WEB69	0.0	28.0	1.00	18.16	0 $/h
KYLE138	218.4	18.0	1.00	18.00	0 $/h
KYLE69	0.0	32.0	1.00	18.10	0 $/h

Total Profit: 1416 $/h

Quick Coverage of Linear Programming (LP)

- LP is probably the most widely used mathematical programming technique
- It is used to solve linear, constrained minimization (or maximization) problems in which the objective function and the constraints can be written as linear functions
- Linear programming got its start during WWII but it was secret throughout the war
- George Dantzig published the simplex method in 1947, and John von Neuman developed the theory of duality around the same time; it became widely used

Example Problem 1

- Assume that you operate a lumber mill which makes both construction-grade and finish-grade boards from the logs it receives. Suppose it takes 2 hours to rough-saw and 3 hours to plane each 1000 board feet of construction-grade boards. Finish-grade boards take 2 hours to rough-saw and 5 hours to plane for each 1000 board feet. Assume that the saw is available 8 hours per day, while the plane is available 15 hours per day. If the profit per 1000 board feet is $100 for construction-grade and $120 for finish-grade, how many board feet of each should you make per day to maximize your profit?

Problem 1 Setup

Let x_1 =amount of cg, x_2 = amount of fg

Maximize $\quad 100x_1 + 120x_2$

s.t. $\quad 2x_1 + 2x_2 \le 8$

$$3x_1 + 5x_2 \le 15$$

$$x_1, x_2 \ge 0$$

Notice that all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are seeking to determine the values of x_1 and x_2

Example Problem 2 (Nutritionist Problem)

- A nutritionist is planning a meal with 2 foods: A and B. Each ounce of A costs \$ 0.20, and has 2 units of fat, 1 of carbohydrate, and 4 of protein. Each ounce of B costs \$0.25, and has 3 units of fat, 3 of carbohydrate, and 3 of protein. Provide the least cost meal which has no more than 20 units of fat, but with at least 12 units of carbohydrates and 24 units of protein.

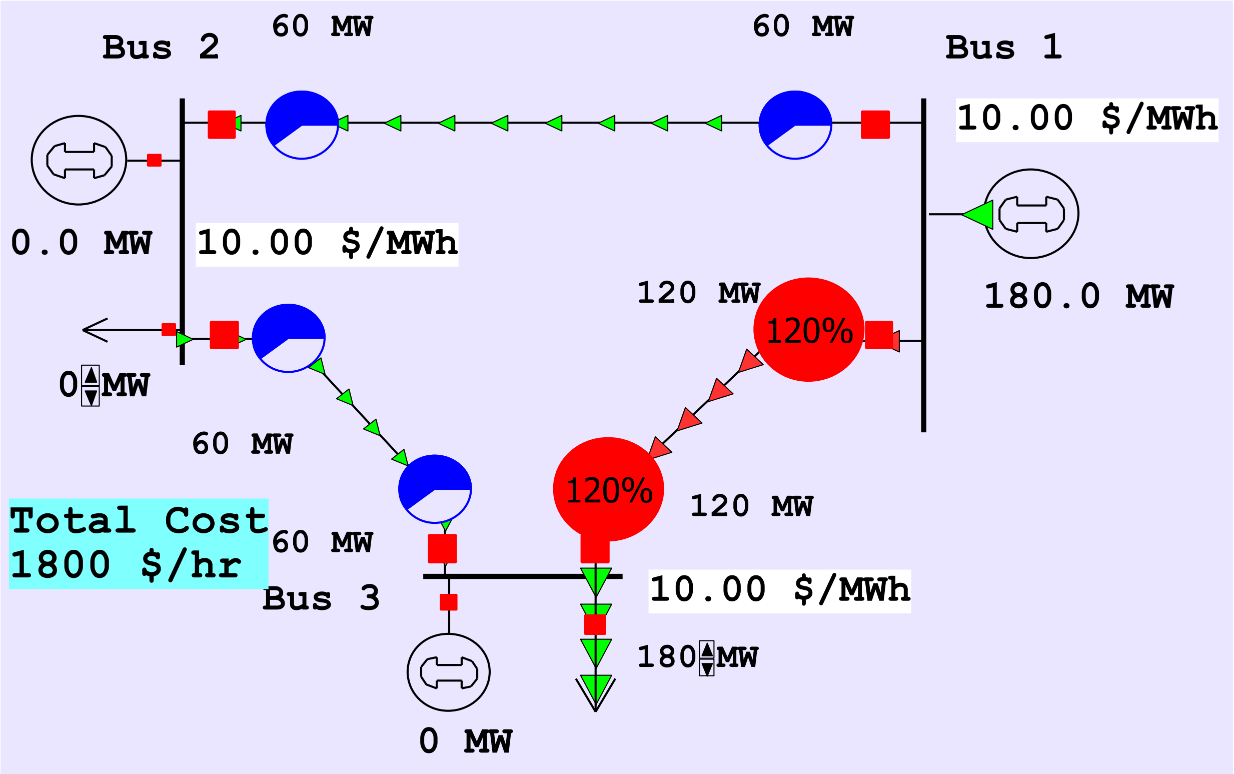
Let x_1=ounces of A, x_2= ounces of B

$$
\begin{aligned}
\text{Minimize} \quad & 0.20x_1 + 0.25x_2 \\
\text{s.t.} \quad & 2x_1 + 3x_2 \leq 20 \\
& x_1 + 3x_2 \geq 12 \\
& 4x_1 + 3x_2 \geq 24 \\
& x_1, x_2 \geq 0
\end{aligned}
$$

Again all of the equations are linear, but they are inequality, as opposed to equality, constraints; we are again seeking to determine the values of x_1 and x_2; notice there are also more constraints than solution variables

Three Bus Case Formulation

- For the earlier three bus system given the initial condition of an overloaded transmission line, minimize the cost of generation such that the change in generation is zero, and the flow on the line between buses 1 and 3 is not violating its limit
- Can be setup considering the change in generation, (ΔP_{G1}, ΔP_{G2}, ΔP_{G3})



Three Bus Case Problem Setup

Let $x_1 = \Delta P_{G1}$, $x_2 = \Delta P_{G2}$, $x_3 = \Delta P_{G3}$

Minimize $\quad 10x_1 + 12x_2 + 20x_3$

s.t. $\quad \frac{2}{3}x_1 + \frac{1}{3}x_2 \leq -20$ Line flow constraint

$x_1 + x_2 + x_3 = 0$ Power balance constraint

enforcing limits on $x_1,\ x_2,\ x_3$

LP Standard Form

The standard form of the LP problem is

Minimize $\mathbf{c}\mathbf{x}$

s.t. $\mathbf{A}\mathbf{x} = \mathbf{b}$

$\mathbf{x} \geq \mathbf{0}$

Maximum problems can be treated as minimizing the negative

where $\mathbf{x}$ = n-dimensional column vector

$\mathbf{c}$ = n-dimensional row vector

$\mathbf{b}$ = m-dimensional column vector

$\mathbf{A}$ = m×n matrix

For the LP problem usually n>> m

The previous examples were not in this form!

Replacing Inequality Constraints with Equality Constraints

- The LP standard form does not allow inequality constraints
- Inequality constraints can be replaced with equality constraints through the introduction of slack variables, each of which must be greater than or equal to zero

$$\ldots \le b_i \rightarrow \ldots + y_i = b_i \quad \text{with } y_i \ge 0$$

$$\ldots \ge b_i \rightarrow \ldots - y_i = b_i \quad \text{with } y_i \ge 0$$

- Slack variables have no cost associated with them; they merely tell how far a constraint is from being binding, which will occur when its slack variable is zero

Lumber Mill Example with Slack Variables

- Let the slack variables be x_3 and x_4, so

Minimize the negative

$$\begin{aligned} \text{Minimize} \quad & -(100x_1 + 120x_2) \\ \text{s.t.} \quad & 2x_1 + 2x_2 + x_3 = 8 \\ & 3x_1 + 5x_2 + x_4 = 15 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

LP Definitions

A vector $\mathbf{x}$ is said to be basic if

This is a key LP concept!

1. $\mathbf{Ax} = \mathbf{b}$
2. At most m components of $\mathbf{x}$ are non-zero; these are called the basic variables; the rest are non basic variables; if there are less than m non-zeros then $\mathbf{x}$ is called degenerate

$\mathbf{A}_{\mathrm{B}}$ is called the basis matrix

Define $\mathbf{x} = \begin{bmatrix} \mathbf{x}_{\mathrm{B}} \\ \mathbf{x}_{\mathrm{N}} \end{bmatrix}$ (with $\mathbf{x}_{\mathrm{B}}$ basic) and $\mathbf{A} = \begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix}$

With $\begin{bmatrix} \mathbf{A}_B & \mathbf{A}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{B}} \\ \mathbf{x}_{\mathrm{N}} \end{bmatrix} = \mathbf{b}$ so $\mathbf{x}_{\mathrm{B}} = \mathbf{A}_B^{-1}(\mathbf{b} - \mathbf{A}_N \mathbf{x}_{\mathrm{N}})$

Fundamental LP Theorem

- Given an LP in standard form with **A** of rank m then
  - If there is a feasible solution, there is a basic feasible solution
  - If there is an optimal, feasible solution, then there is an optimal, basic feasible solution
- Note, there could be a LARGE number of basic, feasible solutions
  - Simplex algorithm determines the optimal, basic feasible solution usually very quickly

Simplex Algorithm

- The key is to move intelligently from one basic feasible solution (i.e., a vertex) to another, with the goal of continually decreasing the cost function
- The algorithm does this by determining the “best” variable to bring into the basis; this requires that another variable exit the basis, while always retaining a basic, feasible solution
- This is called pivoting
- For more details on the solution process take an optimization classes, or for those continuing on ECEN 615 next semester (which I’ll be teaching)

Marginal Costs of Constraint Enforcement in LP

If we would like to determine how the cost function will change for changes in **b**, assuming the set of basic variables does not change then we need to calculate

$$\frac{\partial z}{\partial \mathbf{b}} = \frac{\partial(\mathbf{c}_B \mathbf{x}_B)}{\partial \mathbf{b}} = \frac{\partial(\mathbf{c}_B \mathbf{A}_B^{-1} \mathbf{b})}{\partial \mathbf{b}} = \mathbf{c}_B \mathbf{A}_B^{-1} = \boldsymbol{\lambda}$$

So the values of $\boldsymbol{\lambda}$ tell the marginal cost of enforcing each constraint.

The marginal costs will be used to determine the OPF locational marginal costs (LMPs)

Nutrition Problem Marginal Costs

- In this problem we had basic variables 1, 2, 3; nonbasic variables of 4 and 5

$$\mathbf{x}_B = \mathbf{A}_B^{-1}(\mathbf{b} - \mathbf{A}_N\mathbf{x}_N) = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 0 \\ 4 & 3 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 12 \\ 24 \end{bmatrix} = \begin{bmatrix} 4 \\ 2.67 \\ 4 \end{bmatrix}$$

$$\boldsymbol{\lambda} = \mathbf{c}_B\mathbf{A}_B^{-1} = \begin{bmatrix} 0.2 & 0.25 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 0 \\ 4 & 3 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 \\ 0.044 \\ 0.039 \end{bmatrix}$$

There is no marginal cost with the first constraint since it is not binding; values tell how cost changes if the **b** values were changed

Lumber Mill Example Solution

Minimize $-(100x_1 + 120x_2)$

s.t. $2x_1 + 2x_2 + x_3 = 8$

$3x_1 + 5x_2 + x_4 = 15$

$x_1, x_2, x_3, x_4 \geq 0$

An initial basic feasible solution is $x_1 = 0, x_2 = 0, x_3 = 8, x_4 = 15$

The solution is $x_1 = 2.5, x_2 = 1.5, x_3 = 0, x_4 = 0$

Then $\boldsymbol{\lambda} = \begin{bmatrix} 100 & 120 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} 35 \\ 10 \end{bmatrix}$

Economic interpretation of $\boldsymbol{\lambda}$ is the profit is increased by 35 for every hour we up the first constraint (the saw) and by 10 for every hour we up the second constraint (plane)

Marginal Cost of Constraint Enforcement

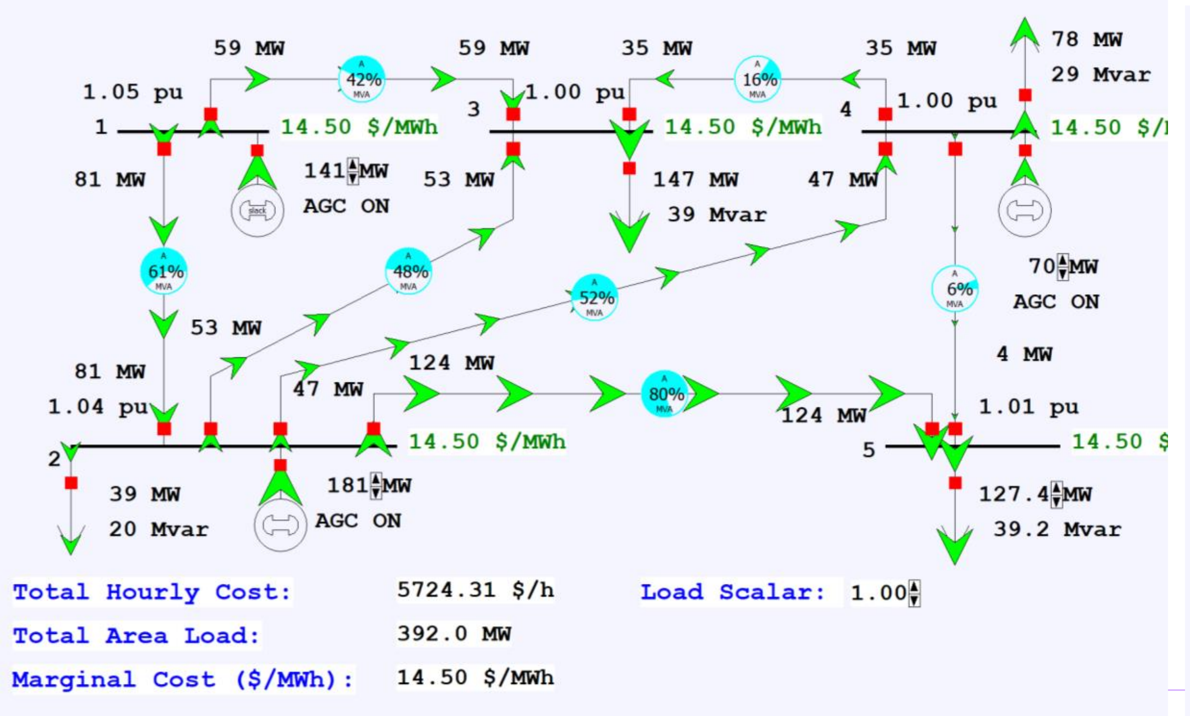
- In an LP solution the marginal costs of enforcing each constraint are provided by the λ vector
- Marginal costs are only associated with enforcing binding constraints; inequality constraints that are not binding have no associated cost
- If there are no binding limit constraints, then the only constraint is associated with the power balance for each area (or the whole system)
  - The bus costs might be different because of the impact of marginal losses

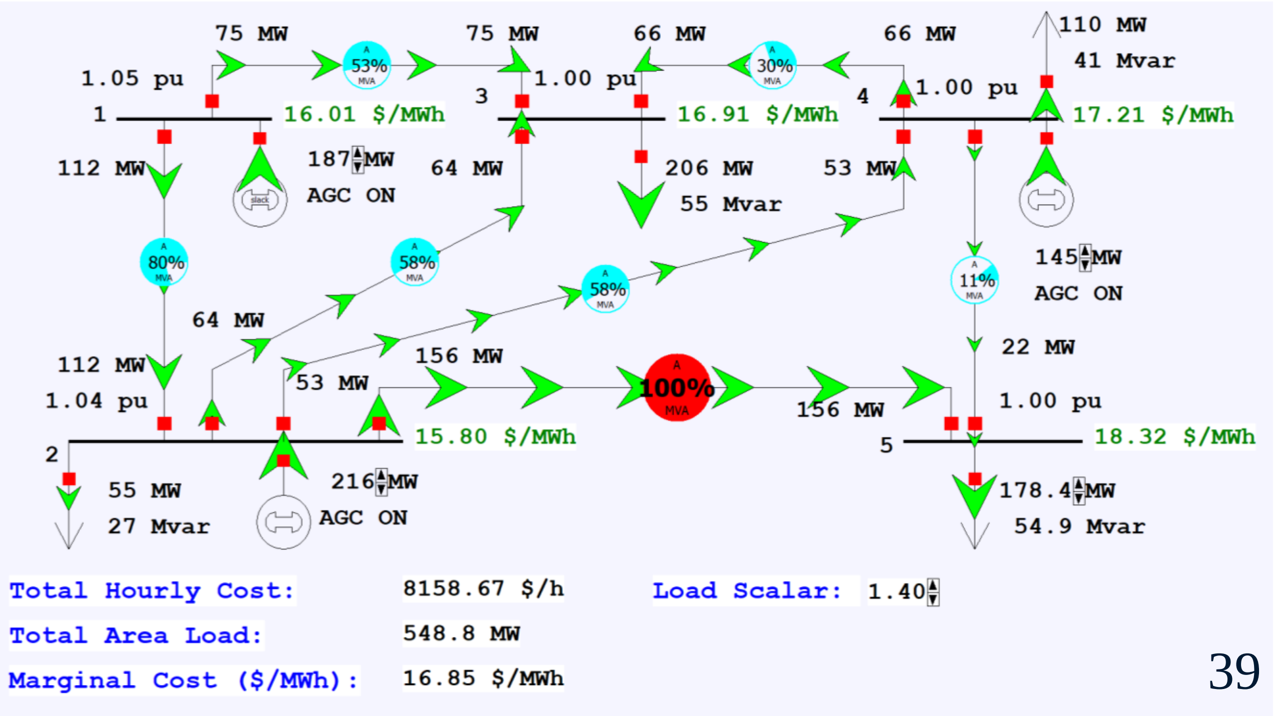
Locational Marginal Prices (LMPs)

- In an OPF solution, the bus LMPs tell the marginal cost of supplying electricity to that bus
- The term “congestion” is used to indicate when there are elements (such as transmission lines or transformers) that are at their limits; that is, the constraint is binding
- Without losses and without congestion, all the LMPs would be the same
- Congestion or losses causes unequal LMPs
- The LMPs are calculated using the marginal costs of enforcing each constraint

Five Bus Case Optimal Power Flow Example

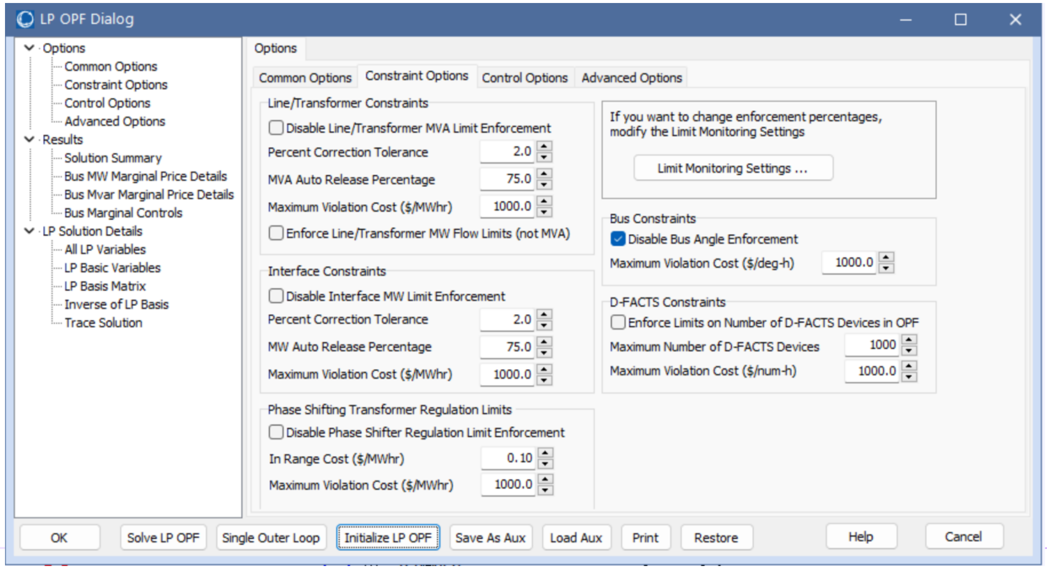
- Load the case **Example7_7**. Start the simulation and gradually increase the load; watch the variation in the LMP values
  - In Simulator a case can be automatically solved using the OPF by 1) setting the area AGC Status to **OPF**, and 2) in **Simulator Options, Environment** page setting the **Play Animation/Solution Method** to Optimal Power Flow

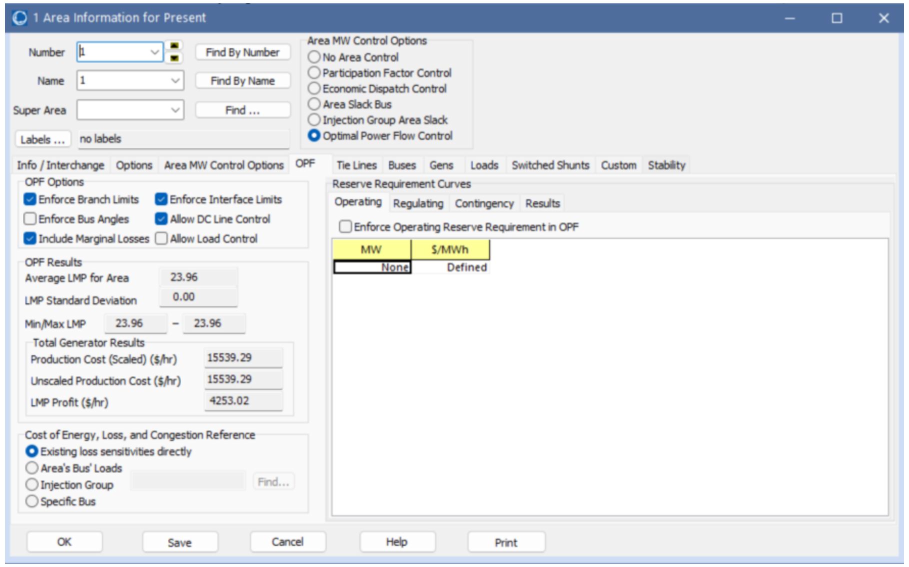




Solving the OPF in Planning Software

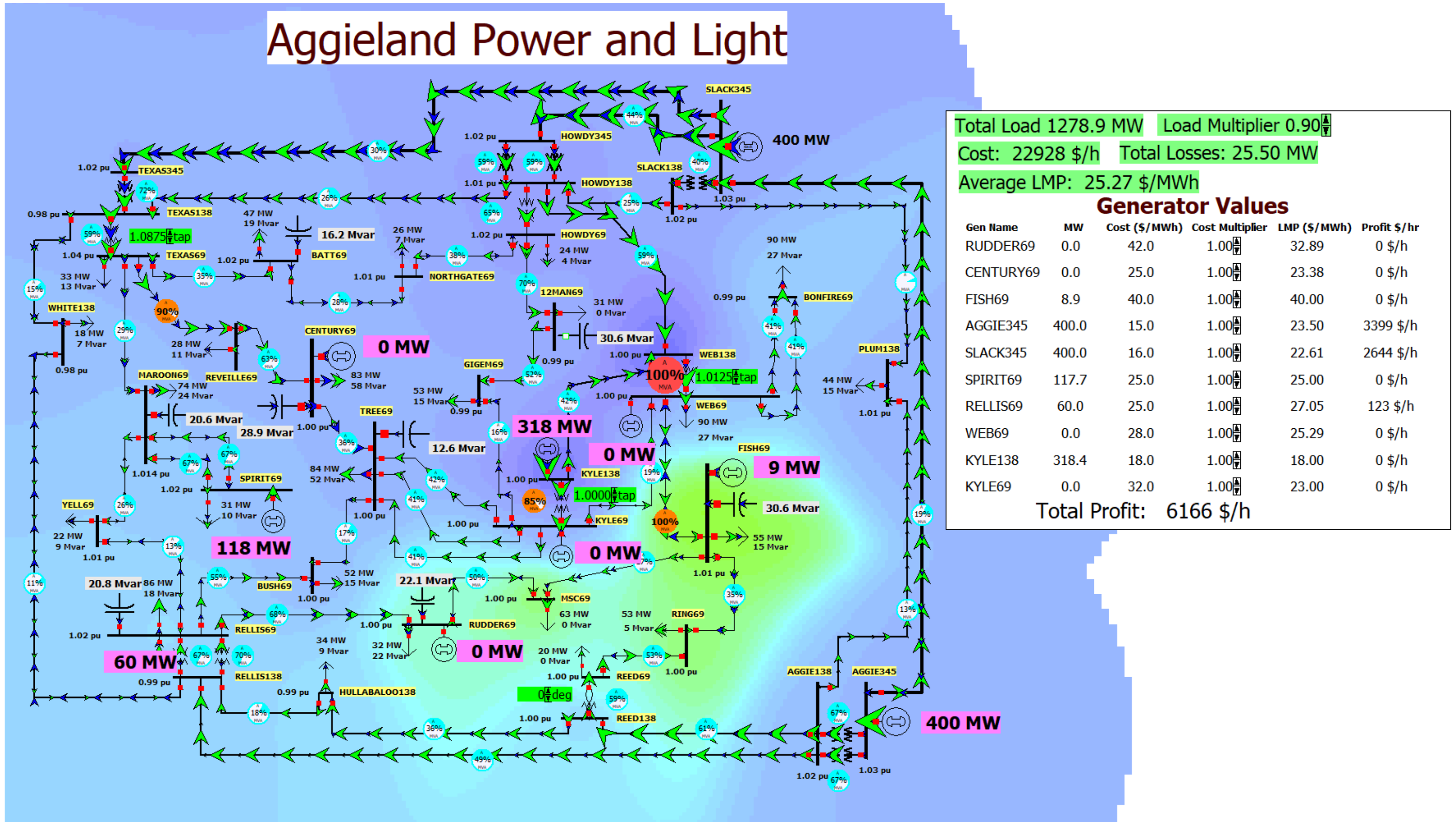
- When solving an OPF in any planning software there will likely be lots of options; key values control 1) which constraints to enforce, and 2) which controls to use to enforce those constraints
- In Simulator some options are available by selecting **Add Ons, OPF Case Info, OPF Options and Results**; other options are available on the **OPF** page of the Area Information dialog





37 Bus Example

- Repeat the previous example with the 37 bus case used in Lab6 (Lab_AGLOPF); if desired display the contour to show the LMPs; try opening some of the transmission lines



Security Constrained OPF

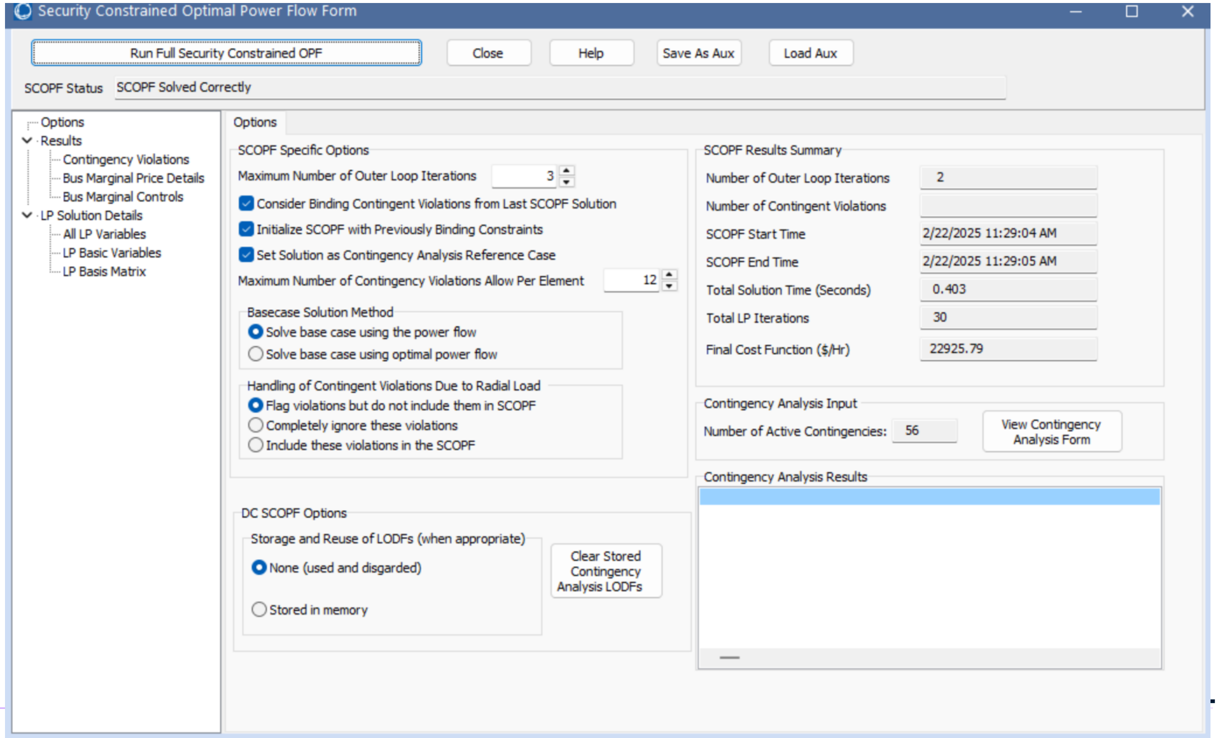
- Security constrained optimal power flow (SCOPF) is similar to OPF except it also includes contingency constraints
  - Again the goal is to minimize some objective function, usually the current system cost, subject to a variety of equality and inequality constraints
  - This adds significantly more computation, but is required to simulate how the system is actually operated (with N-1 reliability)
- A common solution is to alternate between solving a power flow and contingency analysis, and an LP

Security Constrained OPF, cont.

- With the inclusion of contingencies, there needs to be a distinction between what control actions must be done pre-contingent, and which ones can be done post-contingent
  - The advantage of post-contingent control actions is they would only need to be done in the unlikely event the contingency actually occurs
- Pre-contingent control actions are usually done for line overloads, while post-contingent control actions are done for most reactive power control and generator outage re-dispatch

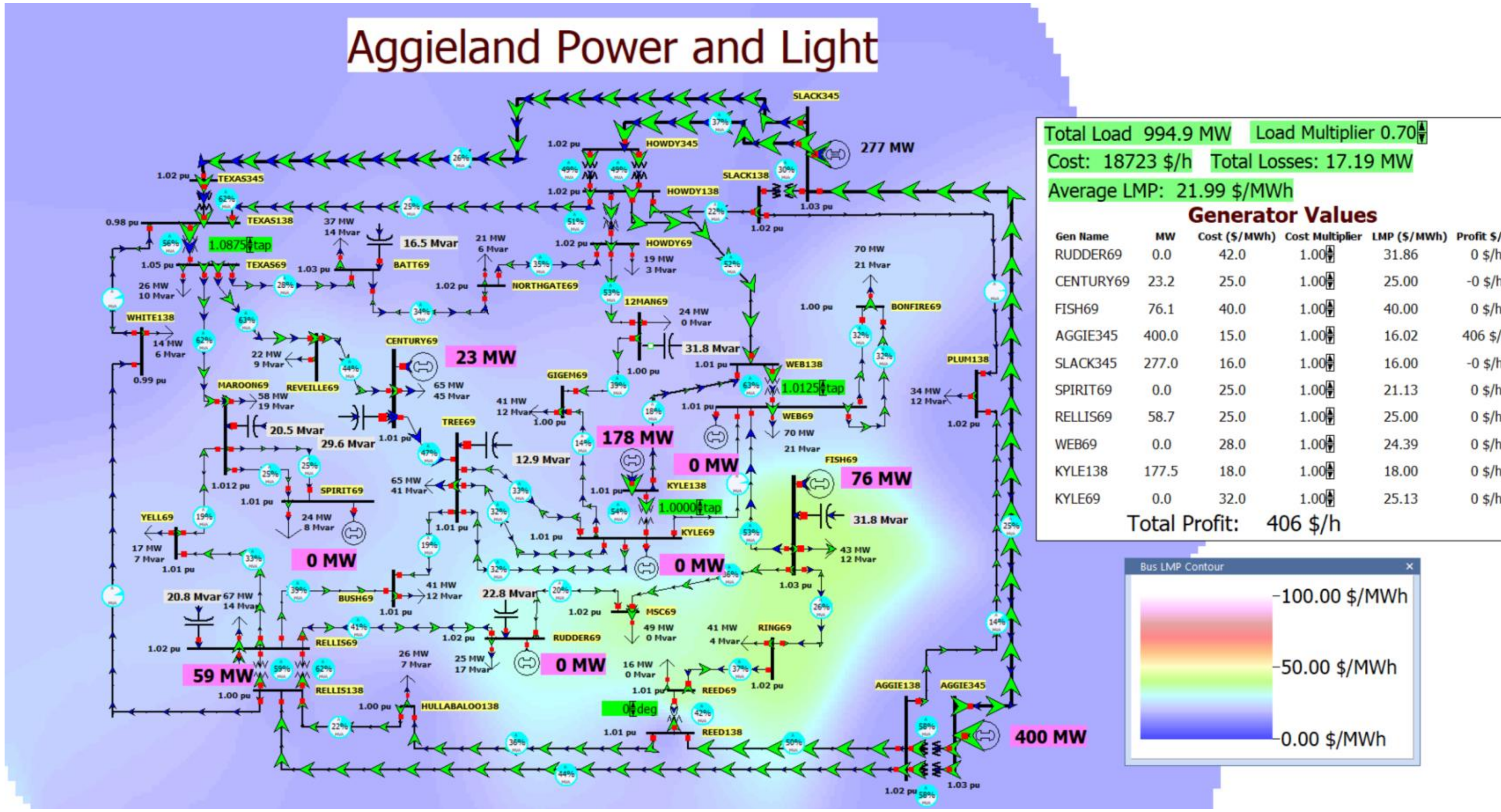
PowerWorld SCOPF Application

- To see the PowerWorld SCOPF application, first open the Lab_AGCSCOPF case and set the load multiplier to 0.9 and solve the case with the OPF; look at the results
- Then select **Tools, Contingency Analysis** to verify that some contingencies have been defined
  - On the **Contingency Analysis** form click **Start Run** to do the contingency analysis; note the violations
- Select **Add Ons, SCOPF** to open the SCOPF
- Click **Run Full Security Constrained OPF**



37 Bus Case SCOPF Results

- Keeping the SCOPF form open, contour the bus LMPs
- What had been a relatively boring OPF solution indicates some major issues
- Looking at the SCOPF form **Results, Contingency Violations** indicates there are some contingencies with unenforceable constraints



LP OPF and SCOPF Issues

- The LP approach is widely used for the OPF and SCOPF, particularly when implementing a dc power flow approach
- A key issue is determining the number of binding constraints to enforce in the LP tableau
  - Enforcing too many is time-consuming, enforcing too few results in excessive iterations
- The LP approach is limited by the degree of linearity in the power system
  - Real power constraints are fairly linear, reactive power constraints much less so

Additional OPF and SCOPF Solution Methods

- There are several additional approaches for solving the OPF and SCOPF
- It continues to be an area of active research
- More general commercial optimization packages are being applied to the problem, including Gurobi and CPLEX
  - Over the years there has been great progress in this area, including with the solution of mixed-integer programming problems (speedups of up to 1 million times have been reported since 1991 with new algorithms and faster computers)