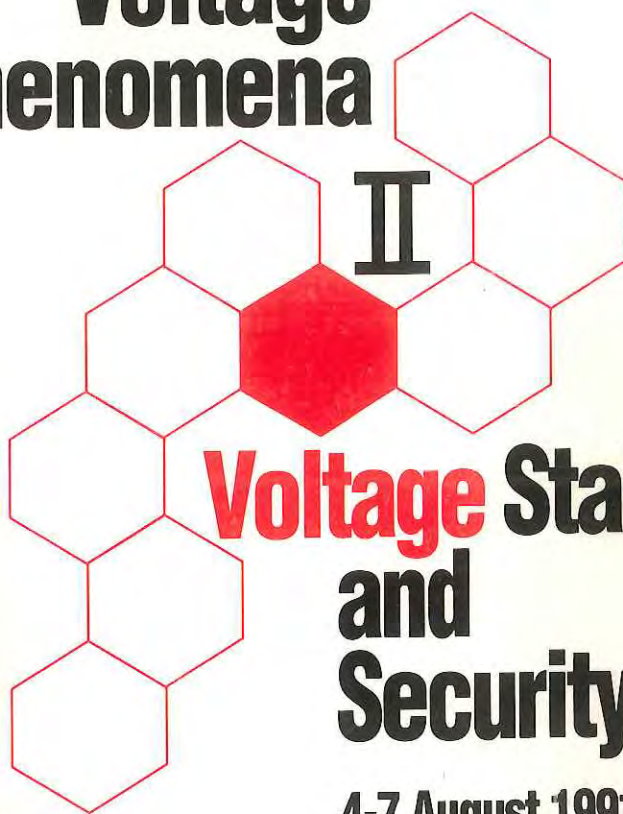




Bulk Power System Voltage Phenomena



Voltage Stability and Security

4-7 August 1991

**Deep Creek Lake
Maryland**



PROCEEDINGS
BULK POWER SYSTEM VOLTAGE PHENOMENA II
VOLTAGE STABILITY AND SECURITY

An International Seminar
Deep Creek Lake, Maryland
4-7 August 1991

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Potosi, Missouri

19-24 September 1988

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PREFACE

A first International Seminar on Bulk Power System Phenomena was held under the auspices of the Engineering Foundation, the National Science Foundation, and the Electric Power Research Institute in Potosi, Missouri during 19-24 September 1988. The Proceedings of that Seminar were published by the Electric Power Research Institute in January 1989, as EL-6183.

Concern with voltage related operational problems on power systems, as potential causes of system disruption, has continued unabated since that first Seminar, and research activity has increased. The present Seminar was planned to present and examine significant advances in understanding of large scale power system voltage phenomena that have been achieved since 1988, and to stimulate further research and development on algorithmic tools adequate to the burgeoning needs of the industry.

The Seminar was organized into sessions addressing recent results in the analysis of voltage collapse phenomena, applications of bifurcation analysis, aspects of dynamic and steady state security analysis and control, of voltage control generally, and voltage related problems in system component modeling.

As in the case of the first Seminar, organization and participation has been a delight. Responses to the announcement of the Seminar and to the call for papers was enthusiastic and gratifying. The support of Dr. Kevin A Clements, during his tenure at the National Science Foundation, was critical to the success of the Seminar. Special thanks are due to the authors who agreed to serve as session chairmen, and as editors of the discussion periods. Most important of all, of course, is the contribution by the authors of the results of their investigations.

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SESSION 1

Introduction

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Analysis of Collapse Phenomena

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V. Venkatasubramanian, H. Schättler, J. Zaborszky (Washington University)

Failure Modes of a Collapsing Power System

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Voltage Collapse Mechanisms - A Case Study

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Computational Experience with the Point of Collapse Method on Very Large AC/DC Systems

C. A. Cañizares, F. L. Alvarado (University of Wisconsin)

Discussion

INTRODUCTORY OVERVIEW OF VOLTAGE PROBLEMS

Lester H. Fink, ECC, Inc.

Within this generation, there have been profound and continuing dislocations within the structure of the electric energy utility industry. One manifestation has been loss of congruency between the nation's generation and transmission plants, which has put severe and increasing strain on the latter. Beginning with the oil embargo of the early 1970s, the shift away from, and in many cases the subsequent retirement of oil fired plants led to transmission of energy from hydro, mine-mouth coal, and nuclear plants over long distances, in patterns that the transmission plant had not been designed to support. A subsequent prolonged lack of growth together with heightened environmental constraints precluded any significant rebuilding of transmission to accord with the new operating requirements. More recently, significant shifts in regulatory philosophy have led to wild card non-utility generation over which the operating utilities have little or no control, and increasing pressure for open access to transmission. A recent additional development has been a resumption of significant growth in demand for electric energy.

A significant effect of such changes has been pervasive and increasing problems associated with maintaining acceptable voltage profiles on bulk power electrical energy systems. Major system blackouts in this country and abroad have been ascribed to deficiencies in voltage support [1,2,3]. One major industry journal has recently labeled voltage security "the hidden threat to transmission networks" [4]. The economic effects of the situation have become very severe in some parts of the country [4,5]. In order to preserve system security, affected utilities are observing increasingly costly constraints on energy transfers. As long ago as 1980, one system was incurring extra production costs of up to \$10M per month.

There are many reasons why these effects have been so severe. Difficulties with system voltage control are a result of inadequate supply of reactive energy, either globally or locally. The shortage is in part structural, and in part due to inadequate coordination of available sources. Earlier technological changes, namely the adoption of higher transmission voltages and an increase in the power factor of large generating units, decreased the availability of reactive energy. On the other hand, increased loading of transmission plant, a virtual moratorium on new construction, and recently renewed growth in demand have resulted in increased reactive losses.

Voltage control practices were developed during a century when demands on them were minimal due to the absence of conditions that foster voltage problems. In most U.S.

utilities, overall voltage control is still manual; the system dispatcher is responsible for maintaining an acceptable system voltage profile using a variety of potential control means, each of which has complex non-linear effects on both the economy and the security of system operation. Due to a general lack of tools for adequate voltage security assessment, he must work with only limited information about the system state. Such automatic controls as exist usually are local in nature, uncoordinated, and often have counterproductive effects during system disturbances [6].

Persistence of these problems has been due in part to a lack of adequate understanding of voltage dynamics. Voltage stability and collapse are phenomena that are not yet fully understood by the academic community [5], and many responsible engineers are unsure whether their operating constraints are too conservative or not conservative enough [4]. The problem of voltage control is of increasing urgency, and major benefits, in terms of improved system security and lower operating costs, could be obtained by its proper solution.

It is the purpose of this paper to set forth an overview of the evolution and current understanding of the causes and effects of voltage phenomena.

EMERGENCE OF VOLTAGE PROBLEMS

One of the challenges currently facing power system engineering is that of bringing the control of system voltage, not just to the same level of adequacy as that of system power generation, but to a considerably higher level. Until relatively recently, power systems had evolved in the absence of conditions that foster voltage control problems; the early challenges were for the effective and economic distribution of power while maintaining system stability, and system control structures were developed accordingly. It is only within the past generation, as environmental, economic, and regulatory changes have forced the modern large, complex systems into operating heavily loaded, closer to physical limits, that voltage phenomena have emerged as problematic, severely impacting system security and economy. Much of the problem arises from the fact that voltage control means (having, as mentioned, developed during a century of conditions under which the demands on them were minimal) are diverse, uncoordinated, and generally ad hoc in nature.

In the mid 70's a major blackout which was a direct consequence of inadequate voltage control occurred in the Italian ENEL system. According to the late Dr. Quazza of ENEL,

with a proper voltage profile generation would have been adequate to serve the load, and the regional blackout would have been avoided. Similarly, poor VAR control was a major contributor to the collapse of the French system in December of 1978, resulting in a major blackout [7]. Similar incidents have occurred in Britain, Sweden [8], Japan, Canada [9] and the United States, leading to regional power disruptions of varying severity. Other consequences of poor voltage control, less drastic than a blackout, but still very costly to the system, are increased vulnerability to system cascading outages, increased VAR output from generators resulting in reduced real power output, and increased losses due to increased VAR flow. Perhaps the most immediately pressing consequence, however, is reduced power transfer capability across the transmission system, which is resulting in dramatic increases in system running costs.

Recently the demand for electric power has begun to increase at a more rapid rate (4.5% in 1987, 4.7% in 1988 [10]) than that experienced since the oil embargo of the early seventies (roughly 2%). Even though not yet as rapid as in the preceding decades (around 7%), this increase will soon make controlling voltage an even more formidable task for system dispatchers without the help of a direct on-line VAR control system. Such a tool is required if present standards of reliability are to be preserved. Reconsideration of conventional practice could very well lead to new control regimes that would avoid many of the current problems. A thorough understanding is needed of both the static and the dynamic functional relationships of system bus voltage magnitudes to other system variables, and based on such understanding a restructuring of voltage control means into a coordinated regime.

MEGAWATTS AND REACTIVE VOLT-AMPERES

Proper understanding of the voltage control problem is hindered by tantalizing similarities between it and the more familiar MW control problem, similarities that cloud underlying differences. There is something of a dualism in the manner in which the two problems are commonly perceived and formulated. The system state vector is the vector of bus voltages. The bus voltages are expressed in terms of their magnitudes and angles, which respectively are closely coupled to the flows of reactive (VAR) and active (MW) energy. However, in the MW control problem as commonly conceived, attention is focused on control of the supply of active energy in order to track the load demand, with the maintenance of the angle elements of the state vector within limits being a condition in the background, while in the voltage control problem attention is focused on control of the magnitudes of bus voltages, which are elements of the state vector, with the necessary supply of reactive energy being a condition in the background.

Another similarity arises from the characteristic of weak coupling, which results in the localization of many disturbances. A significant feature of power system topology is a granularity in the association of buses, i.e., the system consists of (relatively) weakly coupled groups of (relatively) tightly coupled buses. This is reflected in the phenomenon of coherency: the association of generators in nonoverlapping groups such that those within a group are more tightly interconnected among themselves than are the groups with each other. In responding to severe disturbances, machines within these groups tend to maintain mutual synchronism while oscillating as groups against each other. This concept has found widespread use in the development of efficient programs for the analysis of system stability. Similarly, a significant feature of the voltage phenomena is a restricted range of effect of changes in reactive energy injections, which often results in a localization of voltage disturbances.

For our purposes we may term the resulting groups of affected buses *voltage zones*, which we define as *a group of one or more "tightly" coupled generator buses, together with the union of the sets of load buses that they (severally) support*. Some authors have observed [11] that sensitivity to changes of reactive power injection at a given bus generally drops sharply beyond a closed cut set through the nearest PV (voltage controlled) buses, and have defined voltage "areas" whose boundaries pass through nodes containing strong VAR support [11,12]. Adjacent areas, under this definition, will merge when one or more PV generators reach their reactive limits. We suggest that "zones" are more robust and more useful [13].

Each of these groupings of buses (coherent areas and voltage zones) is a reflection of the granularity of power system networks. The two groupings should be distinguished, since the sensitivity of bus voltage magnitudes to a change in reactive power injection at a given bus may not follow the same pattern as that of bus voltage angles to a change in active power injection at the same bus. Voltage zones can draw to only a limited extent on external supplies of reactive power for support of internal voltage levels, due in large part to the non-linearly increasing burden of reactive losses as transmission line loadings increase.

A third similarity that obscures a difference is that both the MW and the voltage control problems deal with the maintenance of a necessary balance between supply and demand - namely that of active and of reactive energy respectively. Equality constraints represent a (physically) necessary balance between energy production and consumption, both real and reactive. Fluctuations in this balance are accommodated to the extent that is physically possible through certain relevant system characteristics. The fundamental consideration with regard to both voltage angle and magnitude behavior is how the physical variables of the system respond to imbalances. The major differences are imposed

(i) by the predominantly reactive nature of line impedance, so that localized reactive imbalances can have serious consequences even in the context of sufficient global reserves, and (ii) by the tight coupling of real power to the inertial masses of the physical system, which entails not only longer time constants, but also mechanical consequences of real power imbalances in contrast to the essentially electromagnetic consequences of reactive imbalances.

The comparison in Table 1, between classical angle stability and the more recently discussed voltage stability, may clarify some of the issues. (See also [14]).

VARIETIES OF VOLTAGE PROBLEMS

Voltage problems manifest themselves under a variety of conditions and at a variety of system locations [6]. For our purposes, we may distinguish between three categories:

- primary: phenomena related to system structure - these reflect the autonomous response of the system to reactive supply/demand imbalances;
- secondary: phenomena related to control actions - these reflect the counterproductive nature of some manual or automatic control actions;
- tertiary: phenomena resulting from interaction of those above.

TABLE 1

<u>(Voltage) Angle Stability</u>	<u>Voltage (Magnitude) Stability</u>
Fluctuations in the active energy balance are accommodated by variations in angular momentum of rotating equipment connected to the system, evidenced by changes in machine speeds and (average) system frequency.	Fluctuations in the reactive energy balance are accommodated by variations in bus voltages, which affect the energy stored in shunt capacitors and inductors.
Flow of active energy, between sources and loads and among sources, is lightly damped because of low resistance in branches.	The flow of reactive energy, between sources and loads and among sources, is highly damped because of the (relatively) high inductance of branches over which the energy must flow.
Stability is studied in the context of (active) potential energy contours in (voltage) angle space.	Stability is currently being studied by means of an energy function defined on the system state space [17].
Loss of stability follows upon a drastic/sudden change in (active) energy supply or demand, beyond the ability of the system to accommodate it by changes in speed (rotational inertia) without loss of synchronism among units or groups of units, and operation of protective relays.	Loss of stability follows upon a change in either (reactive) energy supply or demand beyond the ability of the system to accommodate it by changes in voltage magnitude levels without stalling of induction motors and/or disconnection of equipment due to protective relay operation.
<u>Loss of System Integrity</u>	<u>Voltage Collapse</u>
Operation of protective relays leads to disconnection of loads and/or system breakup into separate subsystems, some or all of which may be characterized by such imbalance between (active) energy supply and demand as to be irrecoverable.	Operation of protective relays leads to disconnection of loads and/or system breakup into separate subsystems, some or all of which may be characterized by such imbalance between (reactive) supply and demand as to be irrecoverable.
(Voltage) angle system instability is often obviated by past developments in machine design and by functioning of controls such as power system stabilizers that have been developed for that purpose on the basis of experience.	Voltage (magnitude) system instability has not yet had the same impact on equipment design. Machine power factors have actually been increased. Existing controls may actually function counterproductively during some circumstances.

Primary, structure-related phenomena

These phenomena usually involve distortions in local reactive supply/demand balances across the system due to inability of the system to support large flows of reactive power without counterproductively large reactive losses. Thus, even though the overall supply of reactive energy on the system may be adequate, there may be areas of the system where local shortages result in unacceptably low bus voltages. Reactive imbalance may become manifest as system load (or as load within a local area) grows beyond the point where bus voltages can be supported by available reactive supply. Alternatively, imbalances may result from a disturbance, the effects of which may include: on loss of one or more ties, increased loading and hence increased reactive requirements of remaining ties; on loss of a generator, loss of its reactive capability, coupled with increased loading and hence, again, increased reactive requirements of ties. Global shortages of reactive, while not, of course, "structure-related," result in the same phenomena as do local shortages.

These reactive imbalance phenomena are governed by system parameters, and are studied by bifurcation theory [15]. They may involve loss of conditions necessary for stable operation as system load increases or as some system parameter exceeds a critical value; in such case the system will be subject to one or another type of (very rapid) voltage instability leading to voltage collapse. The fine-detail taxonomy of these phenomena can be very complex [16], but from an operator's standpoint the immediate result, voltage collapse, is clear and unambiguous.

Another phenomenon is the occurrence of high voltages. These may occur at the end of long, lightly-loaded radial transmission lines, e.g. during system restoration following a blackout, or may result from self-excitation (due to resonance between system capacitive reactance and generator quadrature axis reactance following load rejection. Such voltage excursions may result in catastrophic breaker or surge arrester failures.

Secondary, control-related phenomena.

Because the dynamics of these phenomena are much slower, being governed by the time delays of the various control loops involved or by manual actions, they are observable to the operator, and are to some extent open to heuristic analysis [1] as well as to more rigorous analysis by conventional nonlinear control theory. They can, however be very complex in their interaction, and some of their manifestations can be counterintuitive.

Clark [18] has given a clear description of the circumstance known by operators as "failing to get under the morning

load." Briefly, it results from the reactive-supply vs. voltage curve of static capacitors intersecting at two voltage levels with the U-shaped Q-V relationship at a load bus; once buses get trapped at the lower of these points, it is difficult to extricate them. Thus, if voltages are allowed to sag, as might occur during a rapid morning load build-up with reactive sources not brought on-line somewhat ahead of the load build-up, the result will be a higher burden on generator reactive output, which may reach thermal limits. At this point, capacitors brought on-line will provide less than rated reactive support due to the prevailing low voltage, and some bus voltages may be trapped at the lower operating point.

Many authors have noted the counterproductive behaviour of voltage-regulating devices [19,20]: tap changers or voltage regulators may act to support distribution or subtransmission voltages when transmission voltages are low but tolerable. This action provides no additional reactive, but does result in higher currents and therefore higher reactive losses on the transmission system, potentially leading to voltage collapse.

Tertiary, interactive phenomena

As would be expected of systems as complex as power systems, related phenomena are not likely to be observed in pure form. Either a primary or a secondary phenomenon may trigger the other. Clark [21] describes the limit-cycle-like behavior of induction motors, which may stall under low voltage (primary category) induced in part by their own reactive demand, then restart when voltage recovers (secondary category) and again depress the voltage. Another, inverse interaction may occur when exciters that have been operating at (overexcitation) overload (in which condition, so far as physical relationships on the network are concerned, there is no reactive shortage) time out and reduce their reactive input, thus precipitating a collapse; in this case, operation of a local control loop (secondary category) induces a reactive imbalance (primary category).

CONCLUSIONS

If this description of voltage phenomena is correct, certain implications for power system research and power system engineering and operation emerge.

- For the research community, it is clear that the instability mechanisms thus far explored via bifurcation theory, while important, represent but a small part of the overall picture. Moreover, dynamic as well as static phenomena are involved [15,16]. We can expect that our understanding of these phenomena will be broadened and deepened for many years to come as their study continues.

- o For engineers and operators, a clear lesson seems to be that many of the problems encountered in the field, especially those we have categorized as secondary and tertiary, are severely aggravated, if not completely caused, by the action - and interactions - of ad hoc, uncoordinated controls. Many of these are survivors from an earlier, simpler era, when operators kept awake during their spare time by "chasing VARs around the system." We now know enough about voltage phenomena, and the problems they induce have become severe enough, and costly enough, that progress toward practicable and effective coordinated controls should be pursued vigorously.

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A TAXONOMY OF THE DYNAMICS OF THE LARGE POWER SYSTEM WITH EMPHASIS ON ITS VOLTAGE STABILITY

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ABSTRACT

This work develops a taxonomy of voltage stability for the large power system and analyzes it mathematically, operationally, and gives an initial survey of computational features.

In modelling some open questions are examined. It is shown that the *singularity or the singular surface*, is far from being just a quirk of the algebraically constrained model. The singularity pinpoints an area of very different and important physical events, which are very fast and random. The quasi stationarity assumption is grossly violated near the singular surface. Hence, the entire methodology of AC dynamic analysis loses validity along with its basic concepts such as RMS voltage, phase shift, reactive power, equivalent π circuit etc. A complete, circuit theory type nonmodulated model would have to be used. This also raises serious questions about the *validity of some singular perturbation* type approaches near the singularity. Models are developed for locally controlled tap changers (under load, of course) which fit in as integer parts of the analytical models for studying stability, feasibility and viability of the large power system. No more does tap changing require a special disjoint study. Matters like cascading voltage drop problems become integrated directly into the analysis.

This analysis of the taxonomy reveals a two level cellular structure in both the parameter and state spaces. The *parameter space* (which contains structural parameters like system composition, reactances, etc. and operating parameters like loads, generation, etc.) is composed of *typal regions*, bounded by bifurcation surfaces. Typal regions are characterized by different types of operations as defined by the number and stability of equilibria in the state space and the arrangement of the trajectories. Typal regions are grouped into *feasibility regions* within which a specific operating point (a stable equilibrium or load flow) can be shifted around smoothly by operator action (such as in remedying an alert state) without danger of collapse (roughly, feasibility could be thought of as a much broader and precise version of what is loosely referred to as static voltage stability). The boundary of this region can be calculated as zero sets of explicit analytic functions and is thus relatively accessible to computation. This makes the feasibility region a very useful and powerful potential guide for system operation, on or off line. The boundary of the feasibility region is composed of five different kinds of bifurcation surfaces corresponding to 1) the common (P-V curve type) saddle node bifurcation, 2) the Hopf bifurcation, and 3) the singularity induced bifurcation (which is rigorously analyzed here for the first time) and two bifurcation surfaces (also identified here the first time) caused by locally controlled

tap changers (if they are present on the system), namely 4) another saddle node type bifurcation connected with the cascading voltage drop disturbance and 5) potentially another Hopf bifurcation connected with periodic up and down motions of two or more tap changers.

The *state space* (which defines the dynamic stability of voltages, rotor angles, frequency etc.) is separated into connected components by singular surfaces. No trajectories pass through the singularity. In fact, trajectories entering a small vicinity of the singularity, physically generate a very fast random process, so that events after the entrance into the singular region are unpredictable except possibly, statistically (but not using a $\epsilon\dot{E} = g$ approximation). Within each component stable equilibria have their regions of attractions, bounded by stability boundaries and other regions where no convergence or convergence to a singularity may occur. The stability boundary consists of segments associated with different kinds of anchor points including stable manifolds of unstable equilibria and periodic orbits, and of portions of trajectories anchored to the singular surface at what will be called pseudo equilibria or semi-singular points. Also certain sections of the singular surface where trajectories originate and the system is transversal to the singularity may lie in the stability boundary. To provide a practical tool for judging the stability of the operating point an energy function type method is developed for finding the sections of the stability boundary (which has multitudes of sections on a large system) where the boundary may be breached in an emergency.

A *taxonomy based classification of voltage collapse* is presented. The *principal contribution* of the paper is the presentation of the taxonomy for the large power system in a precise and complete body of mathematical analysis. This new knowledge can serve as the theoretical foundation for developing computational techniques and software, for informed system operation, planning and security. Some of the computational approaches for using the results of this research as practical tools of operation are briefly surveyed.

For gaining insight and to fully understand the implications of the taxonomy, rather extensive explanatory material using low order illustrative examples, is offered. It should be emphasized, however, that all results in this paper are directly applicable to the large system and only software development is required. Mathematically, the analysis is complete.

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I. INTRODUCTION

The work reported in this paper pursues two objectives:

1. To analyse the taxonomy that defines the structural architecture which differentiates and at the same time intertwines the various component features of the large power system in the state space and the parameter space (i.e. operating conditions and topography of the system).
2. To develop new or improved practical tools for power system operation made possible by the new understanding of its taxonomy.

Certainly the results of the paper cover the general stability and feasibility of the power system electromagnetically (voltage) as well as electromechanically (rotor phase, frequency). In fact most of the results are applicable even more generally to other large systems which share some of the mathematical features of the power system. The implications of this general taxonomy are however developed in this paper with concentration on the problems of voltage stability and voltage feasibility.

A *Proper Operating Condition* for a power system (large or small) must have

1. *Feasibility*, i.e. the operating point exists as a stable equilibrium which is continuously moveable as such over an open

— *feasibility region, F ,*

consisting entirely of feasible operating points such that any point in F can be moved within F to any other point in F through (operator induced) change of (operating or system) parameters.

2. *Viability*, i.e. all voltages, currents, frequency etc. are within the applicable tolerances. This occurs over an open

— *viability region, V ,*

within the feasibility region in the parameter space.

3. *Transient stability* i.e. trajectories converge back to the operating point from any state within an open

— *region of attraction, W^s ,*

in the state space.

4. *Security*, i.e. the system has regions of feasibility, viability and attraction of adequate shape and size either 1) to as-

sure continued system operation or 2) return of the system to the operating point, or 3) (in case of equipment loss) the shift to an alternate feasible, viable and transiently stable equilibrium point following any "first contingency".

A *Desirable Operating Condition* will in addition satisfy conditions such as to distribute generation over the system (including trading of power by traditional or by currently developing on line trading methods) with maximum economy, i.e. to minimize production and transmission costs.

A clear understanding of the position of the operating point within these regions, and particularly, relative to their boundaries, can guide the operator and give him confidence and competence in making the many decisions for planning and choosing operating conditions and in instituting major changes in the operating conditions, decisions which are the backbone of power system operation. Transient stability and security are traditional, standard concepts and are part of the current state of art. The concept of viability was introduced earlier [40, 45] and its use is spreading in the field. Feasibility is a new concept, introduced in a recent paper [36] for a rudimentary power system, and it is being developed here fully for a power system of any size. The feasibility can be roughly viewed as a much broader and precise version of what is loosely referred to as the static voltage stability, however feasibility is not restricted to static events.

It should be noted that feasibility and transient stability are both predicated on the existence of local stability at the equilibrium point. Feasibility is concerned with the persistence of local stability while changes take place in the parameter space, connected with operation and operator action. Transient stability is concerned with the persistence of local stability at the operating point in the state space following dislocation of it by disturbances.

A true and precise understanding of this complex of practically useful tools of power system operation is only possible, however, through the knowledge of the taxonomy of the large power system which is developed in this paper. It will be shown that very practical and applicable results emerge. Matters of practical use will be elaborated in the conclusion in section IX. The main features of the new taxonomy and new concepts are introduced in Sections II and III. Modelling issues, including the validity of conventional AC concepts and techniques near singularities and dynamic models for locally controlled tap changers are discussed in section IV. Precise mathematical statements of the results are given in section V and the proofs will be given in section VIII. Computational aspects are discussed in section VI while numerical examples are given in section VII. Accordingly, less mathematically inclined readers may wish to concentrate on sections I-IV, VI-VII and IX. Mathematically inclined readers may start reading section V directly.

II. A GENERAL OUTLINE OF THE TAXONOMY

Models involved in the analysis of power systems of all sizes have the structure

$$\dot{x} = f(x, y, p, u) \quad (1)$$

$$0 = g(x, y, p, u) \quad (2)$$

$$x \in X \subset \mathbb{R}^n, \quad y \in Y \subset \mathbb{R}^m, \quad p \in P \subset \mathbb{R}^p, \quad u \in U$$

Here f and g are, respectively, n and m -vectors of smooth, for the power system actually real-analytic functions; x and y are the dynamic and instantaneous state variables, p denotes parameters and u (secondary) controls.

The following analysis reveals a two level cellular structure – the backbone of the taxonomy – for both the parameter space and the state space. To develop insight into this structure, illustrations will be quoted from a rudimentary example described in section VII, selected to facilitate graphic representations. General and precise mathematical results for the large system follow in section V.

II.1 THE PARAMETER SPACE

The parameter space P contains system parameters P_S (such as the systems topography, i.e. which lines, buses etc. are energized, and equipment constants such as inductances, transformation ratios etc.) and operating parameters P_O (such as loads, generation, voltage setpoints) which determine the current stationary operating condition, i.e. the stable operating

equilibrium, more commonly known as the load flow or power flow. A certain type of phase portrait in the state space will persist within open regions of the parameter space. These regions, which are separated by bifurcation surfaces, will be called *typal regions*. Changes in the type of the phase portrait occur as parameters cross the bifurcation surfaces. Typal regions are, among other aspects, characterized by the number of stable and unstable equilibria and their relative arrangement. Each typal region has a typical phase portrait, that is a transient behavior. This is displayed in Figure 1, for each of the eight typal regions of the eight typal regions of the parameter space, separated by the different bifurcation surfaces shown in Figure 2. These illustrations are taken from a simple illustrative example selected for facilitating insight and understanding of the complex results of the paper. Some of the typal regions can be grouped together into a *feasibility region* where operation at a specific stable equilibrium point is feasible in the sense that the reference equilibrium can be shifted smoothly, retaining its stable character, from any point in the region to any other point through normal operating practices. The rest of the parameter space is then infeasible. There equilibria are unstable or nonviable. The boundary between these regions is called *feasibility boundary* (Figure

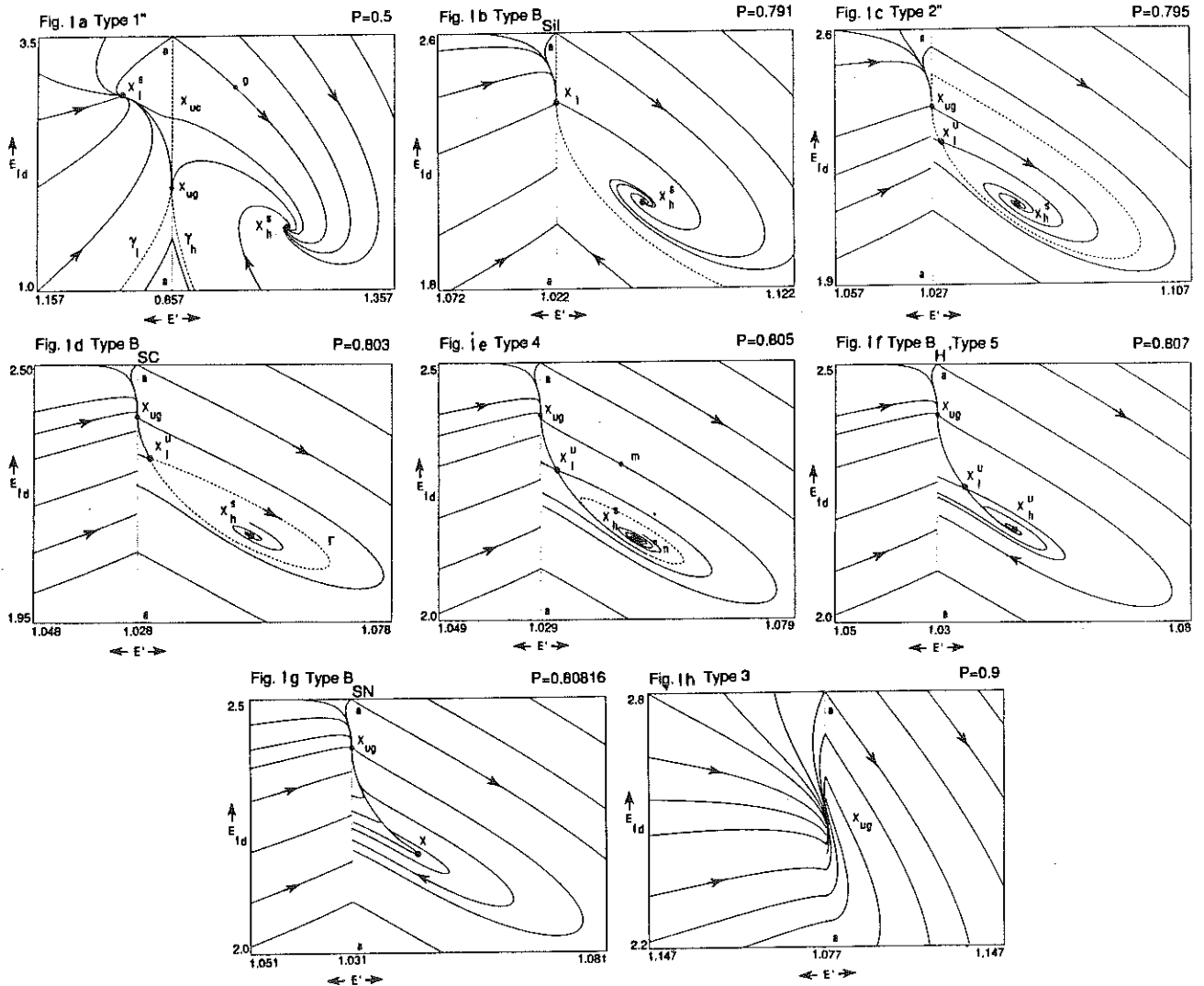


Figure 1. Phase portraits for a control gain in Level 2, $K=2.6$ in Figure 2.

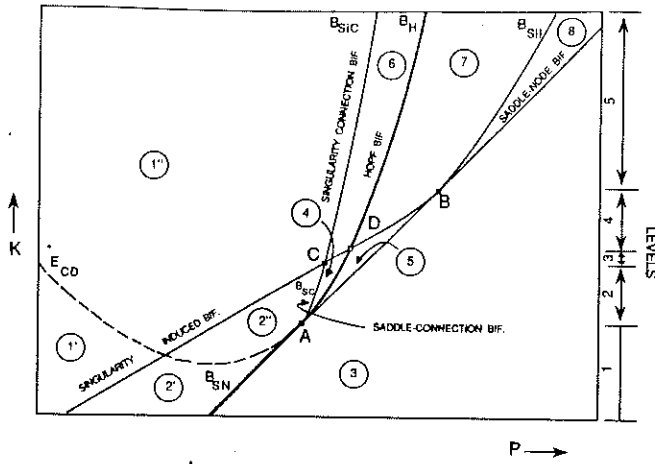


Figure 2: Qualitative representation of the bifurcation diagram for the rudimentary system in Figure 6 (A simulation plot is given in Figure 7). Heavy line is the feasibility boundary for the high equilibrium x_h^* without the tap changer control.

2). Only a part of this feasibility boundary (corresponding to lower control gains in the example) is the conventionally studied static saddle-node bifurcation on the P-V curve. At higher control gains feasibility ends at a Hopf bifurcation ahead of the tip of the P-V curve. Additional segments which may be present are systematically listed in section II.3. The feasibility boundary can relatively easily be computed even for large systems and is therefore of high practical value.

II.2 THE STATE SPACE

The typical state variables are the time dependent values of bus and internal voltages, generator rotor phases etc. These variables describe the time behavior of the system state, both the stationary one and the transients which may occur around it. The load flow constraint $g = 0$ limits the state to a load flow set $L \subset X \times Y$ at all times. This algebraic constraint $g = 0$ can sometimes be eliminated by inversion or reasonable approximations, but often this is not possible in voltage stability problems. Typically L is a stratified set, in simple cases a manifold with maximal dimension n . Through every point $q \in L$ with the exception of points q in a singular subset S , there exists a unique trajectory (i.e. transient) which passes through q and lies in L . The singular set S is defined as the set where the conditions of the implicit function theorem for eliminating y from the algebraic constraint are violated. Typically it is a stratified set of maximal dimension $n - 1$ embedded in L and L is separated by S into open regions, the components C of the state space. So the state space is confined to the complement of S in L . Typically no solution exists within the singular set S . The singular set S is an artifact of the model given by equations (1) and (2), particularly of the algebraic constraint (2); yet the singularity S implies the physical presence of a major fast, random event, actually the emergence of a random process, when the state reaches S . Details follow in section II.5. Within each component C of the state space the typical cellular structure corresponds to regions of attraction around stable equilibria or periodic orbits and regions where no equilibria exist. Since all dynamics are constrained to the individual open components, C the stability boundaries typically also contain segments of S .

Details follow in section II.6.

Thus a two level cellular structure exists in both spaces. In the parameter space there are feasibility regions defined by feasibility boundaries and subdivided by bifurcation boundaries into typical regions. In the state space the components are bounded by singular boundaries and subdivided by stability boundaries into regions of attraction, regions of no convergence, regions of divergence to the singular surface etc. This cellular structure is essential in understanding the dynamics of the power system.

II.3 THE FEASIBILITY REGION

The feasibility boundary is composed of segments of five different kinds of bifurcations.

(1) For systems without locally controlled tap changers (Figures 2 and 3) the three principal segments of the feasibility boundary are:

1. the saddle node bifurcation, as observed at the tip of the P-V curve. This phenomenon has been studied intensively, but usually not in any taxonomic context.
2. the Hopf bifurcation, also usually studied as an isolated phenomenon.
3. the singularity induced bifurcation. This phenomenon has only been touched lightly in the literature. Here a rigorous mathematical study will be given in sections V and VIII.

(2) For systems with locally controlled tap changers, (Figure 4) the feasibility region may be further constrained by boundary sections connected with one of two possible voltage collapse phenomena caused by uncoordinated local tap changing (under load) within a (possibly large) power system.

4. Cascading voltage collapse at one tap changer. This manifests itself in cascaded upward tap changes, each actually resulting in a drop of the voltage on the secondary, leading to voltage collapse unless the taps run out first. This type is well known, but has not been analysed as a boundary of feasible operations in a taxonomic context or for the large system with numerous tap changers using a realistic model. It is identified in this study as typically another saddle-node bifurcation.

5. Possibly interesting alternating up and down motion of several tap changers within a (large) system connected with wild, but slow up and down periodic voltage sweeps (a limit cycle). This phenomenon resembles another Hopf bifurcation.

A systematic analysis of feasibility (a parametric stability limit) connected with the tap changer for potentially large systems is presented here for the first time in sections IV, V and VII. The tap changer dynamics is formulated as a compatible integer part of the system dynamics (section IV.5.1). The restriction of the boundary caused by the tap changers can be computed with nonexcessive effort.

The feasibility boundary as defined in this work applies to one specific stable operating point. Existence in any form of this operating point, and hence feasibility, ends at the boundary. The simplest possible example would be the annihilation

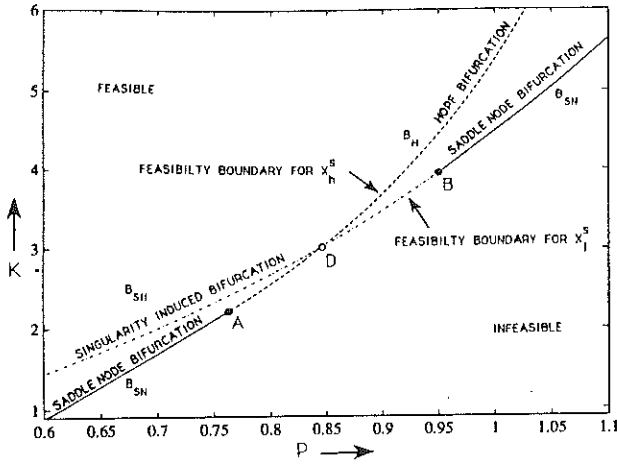


Figure 3. Feasibility regions for the two operating points, x_h^s (viable) and x_l^s (nonviable) without the tap changer control. (Parameter values are shown in Figure 7).

of the operating point of the upper branch of the P-V curve at its tip, that is, in the saddle node section of the feasibility boundary. Under this definition different operating points have different feasibility regions. See for instance, Figure 3, which shows such regions for two different stable equilibria. This definition seems more desirable in a practical sense than the alternative where no stable equilibria of any type exist beyond the feasibility boundary. The latter would define a larger feasibility region (the union of the two feasibility regions in Figure 3), but this would only add uncertainty since there is typically only one loadflow (operating point) with voltages around normal (i.e. viable). Hence additional stable equilibria in the expanded feasibility region would be without practical interest. The feasibility region with and without tap changer control is shown in Figure 4. For the feasibility boundary with the tap changer control, the middle segment of the boundary (dotted line in Figure 4) is a direct consequence of the tap changer control. Note that the presence of the tap changer has a restrictive effect on the size of the feasibility region even where the infeasibility is not directly due to the tap changer control. On the other hand, the feasibility region is also viable upto the feasibility boundary, when the tap changer is present.

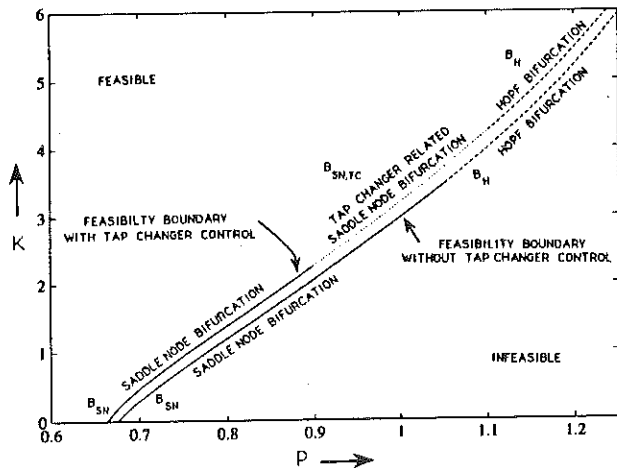


Figure 4. Feasibility region for the operating point x_h^s with and without the tap changer control. (Parameter values are as shown in Figure 7 with $Q_{Lo} = 0.05E_L^2$).

II.4 TYPAL REGIONS

Bifurcation surfaces in the parameter space are the loci of specific changes in the structure of the state space. The number of stable and unstable equilibria can change, they can be reversing stability or changing their order of instability at bifurcations. The trajectories connecting them may change their topology (at global bifurcations). Of course, in time domain terms, this corresponds to a variety of transient responses. Two examples are shown in Figure 5. All this is a very complex conglomeration of surfaces, but typically it will tend to establish cells, the typical regions, within the parameter space, some of which may be unbounded in certain directions others may be bounded. In all cases the boundaries would be the bifurcation surfaces. The operating type then will remain the same within one typical region. The number and character of stable and unstable equilibria will remain the same and the topology of the trajectories does not change as long as the operating point remains in the typical region. In the state space at the same time this means that the phase portrait does not change in its character and connections although it will warp, as the operating point moves around in the typical region. There is relatively little else that can be said in a general way. It is obviously important from an operating viewpoint to know where the operating point is, that is, what to expect in its behavior. The more common bifurcation surfaces, like the ones corresponding to the saddle node or Hopf bifurcations, are computed relatively easily. But to establish the complete typical structure would be a rather heroic task, although feasible for small size systems.

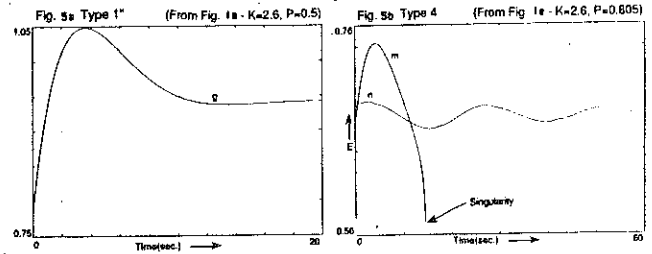


Figure 5. Sample time histories for the bus voltage E . Compare to the phase portraits shown in Figure 1.

An illustration of these structures is presented in Figure 2. A two dimensional crosssection of the parameter space displays six bifurcation surfaces separating eight typical regions. For the control gain values K in Level 2 the system may be in any of five typical regions (1, 2, 4, 5 or 3). The corresponding phase portraits are shown in Figure 1, as a two dimensional foldout of the equilibrium manifold. Typical region 1 has two stable equilibria; region 2 has one stable and one unstable equilibrium; an unstable limit cycle exists preparatory to the Hopf bifurcation in region 4; region 5 has two unstable equilibria, while region 3 has no equilibria at all. Phase portraits at the bifurcation boundaries are also given in Figure 1. Figure 5 shows a few examples of transients in the time domain corresponding to the phase portraits. These results are described in detail in [36].

II.5 THE SINGULAR SURFACE

Two assumptions underlie the use of the model (1) and (2).

1. The equations of the model (1) and (2) are written within the conventional methodology of AC circuit analysis which was developed based on the quasi stationarity assumption. The quasi stationarity assumption claims that the modulating envelopes (RMS voltages, currents, phases etc.) vary slowly enough to be effectively decoupled from the 60 Hz carrier. If so, then the AC dynamics can be written in terms of the envelopes.
2. The transmission system dynamics, which is dominated by the very fast and distributed constant dynamics of the transmission lines, is instantaneous that is, outputs such as RMS currents or power follow inputs like voltages, instantly and precisely.

These are good and justified approximation as long as the quasi stationary dynamics of (1) and (2) has a solution. Such solutions represent equilibrium points or points moving along trajectories, all on the load flow manifold, i.e. the stationary load flow solution $g = 0$ of the transmission network, for the slow moving quasi stationary inputs resulting from $\dot{x} = f$. Indeed under these circumstances the transients of the transmission system itself are small and fast enough to make no difference.

However, near the singular points, the scenario is different.

1. The variation of some RMS state variables becomes very fast near the singularity (e.g. See the trajectory of the voltage E in Figure 5b).
2. The nonlinear model (1) and (2) typically has no solution in the singular set S .

So, it is the quasi stationary AC component which is disappearing or losing validity in the vicinity of the singularity S and events are dominated by the fast dynamics of the transmission system for which also no quasi stationarity applies.

Thus the original state space is superseded by a multiplicity of new nonquasi stationary state variables and the actual state moves off the load flow manifold. For these fast events the entire methodology of AC dynamics vanishes. It is not meaningful to talk about modulating signals which are faster than the carrier. In fact, because the phase of the AC carrier is random (it is not synchronized to the modulation), every trajectory arrives at an arbitrary phase and it becomes a generator of a random process with unpredictable destination when it reaches the vicinity of the singularity. Thus no trajectory can cross or bypass the singularity in a predictable manner. Attempts to bridge the singularity in a meaningful way, e.g. by some singular perturbation type smooth approximation, must be carefully analysed for physical validity. Introducing such parasitic dynamics is mathematically precise for circuits with nonmodulated signals[32]. However, these results are not directly applicable to circumstances where the quasi stationary modulation of the power system is breaking up. More details follow in section IV.1.

II.6 REGIONS OF ATTRACTION AND STABILITY BOUNDARIES

There will be regions of attraction around each stable equilibrium (points or periodic orbits). As stated in section II.2,

the stability boundary around these regions may contain regular and singular segments. The regular sections consist of the stable manifolds of anchor points which consist of unstable equilibrium points, unstable periodic orbits, and special singular surfaces called the semi-singular surface and the pseudoequilibrium surface. The composition of the stability boundary with the unstable equilibrium points and unstable periodic orbits has been previously studied for systems without algebraic constraints [5, 42]. The singular sections contain singular points where the trajectories leave the singular surface transversally. Trajectories which reach the singular boundary transversally, terminate there from both sides or similar pairs may leave the singular surface. So the singular sections are the origins or endpoints of trajectories. However, only sections where trajectories originate can occur on a stability boundary for a region of attraction. Sections of these two types on the surface are separated by high dimensional surfaces of pseudo equilibria or semi-singular points. These are new mathematical concepts introduced and studied in sections V and VIII.

These state space events are illustrated with phase portraits for all typical regions in Figure 1 for a rudimentary system. Each of the phase portraits displayed shows the load-flow manifold L for a third order system folded out in a plane. The vertical line in the middle is the singular set S . The two half planes on either side of S are two (in this case unbounded) open sets within L , i.e. the two components C of the state space. Stability boundaries in Typical Region 1 (dashed lines in Figure 1a) is composed of a trajectory and a singular surface which are anchored at a special singular point x_{ug} , a pseudo equilibrium point. In Typical Region 2 (Figure 1c), the boundary also consists of another trajectory, anchored at the unstable equilibrium point x_u^* . In Typical Region 4 (Figure 1e), the stability boundary is an unstable limit cycle. There are no stable equilibria or regions of attractions in Typical Regions 5 and 3 (Figures 1f and 1h). More details on the behavior at the boundaries (Figures 1b, 1d, 1f and 1g) are explained in [36].

II.7 UNIFICATION OF THE STATE SPACE STRUCTURE THROUGH A SINGULAR TRANSFORMATION

The state space can be structurally unified in a mathematical sense by a singular transformation which is described precisely in sections V.3 and in VIII.1. This transformation gives rise to a smooth transformed system which has the same phase portrait, except for time rescaling along the trajectories and reversal of flow on some sections. All singular points either become regular or what will be called pseudo equilibria (which are equilibria only in the transformed state) or semi-singular points. This restores a complete mathematical unity to the state space and firmly establishes the structural relations between the various components of the state space. It facilitates the analysis and understanding of the ways in which changes in the parameter space will effect the feasibility and type of the operating points. It does not however alter the physical facts of the state space and it does not tell us what happens dynamically in the singular boundaries and in their vicinity.

II.8 SCOPE OF THE TAXONOMY

The taxonomy presented here is general in nature. The size of the system can range from one bus to thousands of them. Angle and voltage stability are covered in a combined manner.

Either can be emphasized (e.g. instability initiated in the angle subspace or in the voltage subspace.) The voltage side is emphasized in this paper. All customary power equipment is covered; load models covering static nonlinear dependence on voltage and/or frequency as well as load dynamics; customary generator models in any detail; 60 Hz quasi stationary transmission line models in any detail; control devices for excitation, frequency, in any detail as long as they can be modeled by smooth functions; selective protection; capacitors, reactors, thyristor controlled reactors (SVC), HV-DC converters, etc. Somewhat of a breakthrough is represented by the manner in which the tap changer problem is coopted into the taxonomy. So this will be summarized in the next section. Hard limits are included through the viability concept, but not discussed extensively in this paper.

II.9 COOPTING LOCALLY CONTROLLED TAP CHANGERS INTO THE TAXONOMY

The only type of tap changer of interest here is of course the type which can operate under load. The other kind is quite different in equipment and mode of activation, so misinterpretations are not to be reckoned with. Accordingly the awkward terms of "under load", ULTC or OLTC will be omitted in this paper. Tap changers are used in two different ways in the power transmission system:

1. Within the transmission system itself in a way where there are loops of transmission lines closed through them. The purpose is to control the distribution of reactive power flow within the transmission system (or active power flow with out of phase taps) and hence its voltage and stability. This application, normally under secondary control, has no conspicuous special role in voltage collapse. A detailed, discrete study of it has been published [26].
2. At subtransmission or distribution substations to keep the voltage on the secondary side within tolerances in face of voltage variations on the primary side. Under uncoordinated local control, combined with load characteristics and system conditions, this type can cause a puzzling cascading operation of rising taps and falling secondary voltage into collapse as well as other irregularities.

Condition 2 is the subject of much interest and intensive studies. Here it is directly coopted into the taxonomy as an integral part and treated within the same mathematical and computational framework. Both parameter space and state space aspects emerge.

(1) The cascading descent problem just mentioned is a parameter space event in the sense that the inception of this phenomenon is marked by a section of the feasibility boundary. Thus it fits directly into the security monitoring and operation planning process and can be very useful there. It is treated in detail in sections V.1.2, and VI-VIII.

(2) Questions of transient behavior involving tap changers seem to be less pressing, however it has a sizeable literature. It is directly accessible to methods derived from the taxonomy and will not be discussed in detail in this paper.

Modelling and problem formulation aspects of both (1) and (2) are treated in detail in section IV.5.1.

II.10 MATHEMATICAL FOUNDATION OF THE WORK

Equations (1) and (2) describe a parameter dependent dynamical system subject to algebraic constraints. The singular transformation mentioned in section II.7 allows, from a mathematical viewpoint, to eliminate the singular aspects of the problem completely. Therefore a complete set of techniques from the realm of dynamical systems is directly applicable to the transformed (and smooth) system. They include, for instance, λ -lemmas for stable and unstable manifolds in the investigation of regions of attraction, or standard bifurcation theorems in the determination of the feasibility regions. Since the phase portraits of the original and of the transformed system are identical (topologically equivalent, to be more precise), results about the transformed system translate directly to the original system under consideration. However, the singular surface plays no special role in the transformed equations and therefore an extra analysis of the system near the singularity is needed throughout. This is a novel contribution of this work also from a mathematical perspective. These more mathematical aspects connected with the taxonomy are described in section V and proofs are given (sometimes only modulo technical details which are simply too lengthy to be included here) in section VIII.

II.11 SUMMARY

Recognizing a two level cellular structure which partitions state and parameter space into sets of structurally different dynamic behaviors is essential in understanding the dynamics of the power system. This paper gives a first systematic study of the taxonomy of voltage stability of the large electric power system. Based on these results, numerous phenomena, loosely known as voltage collapse, are classified within one framework in the next section.

III. TAXONOMY BASED CLASSIFICATION OF VOLTAGE COLLAPSE SITUATIONS

III.1 VOLTAGE COLLAPSE RESULTING FROM RELATIVELY GRADUAL DISLOCATIONS IN THE PARAMETER SPACE

There are two types of parameters, operating parameters such as values of loads or generation, and system parameters such as values of reactances, capacitances and system geography. Either of these can change in such a manner, slowly or more abruptly, as to cause a voltage crisis. However, it is assumed that the change is gradual enough that no critical dynamic activity results in the state space. There are two main types.

(1) *A change to nonviable voltage and/or current conditions within the feasibility region:* Since the system is still within the feasibility region there is no immediate stability problem and corrective action by operator or emergency control within the feasibility region is still possible. Without such action, however, the protective relaying will break up the system and restoration becomes necessary. Note that nonviability here is caused by parameter change.

(2) *A shift of the operating point outside the feasibility region:* As has just been pointed out in section II.3, the parameter space

contains for each stable operating point (load flow) a feasibility boundary. When some parameters (e.g. the load at some buses or the tap changer setting) move outside this boundary, slowly or abruptly, the operating point ceases to exist in the parameter space as a stable equilibrium and the voltage collapses, the system breaks up and needs to be restored.

Since the feasibility boundary is computable even for large systems with reasonable effort, its potential usefulness for security and planning is very large. It makes it possible to delineate the system operating conditions where operation is feasible.

III.2 VOLTAGE COLLAPSE RESULTING FROM MAJOR DISLOCATIONS IN THE STATE SPACE SUCH AS MAY RESULT FROM FAULTS OR EQUIPMENT LOSS

Sudden changes of state caused by loss of equipment (parametric event) or faults (dynamic event) or being thrown on a random trajectory after reaching a singularity, are followed by transients in the state space which may take the system into states (voltage conditions) from where it cannot recover without prior system breakup (or possibly automatic emergency control). Typical situations of this type are

(1) *Convergence to a nonviable equilibrium*: The system ends up in a state space region of attraction where all trajectories converge to a stable equilibrium point which is grossly nonviable, usually too low in voltage and too high in current. The only possible escape is system breakup by selective protection. As an example, in typical region 1 in Figure 1a, a nonviable but stable equilibrium exists to the left of the singular line $a-a$ in the state space at all gain levels K illustrated in Figure 2. Hence, if the system parameters are in region 1 in Figure 2, then whenever the post fault transient leaves the system at an initial state (point) to the left of the stability boundary (composed of a trajectory below point x_{ug} and a segment of the vertical singular line $a-a$ above x_{ug}) then the initial state is on a trajectory which will converge to the low voltage equilibrium x_l^* on the left side of $a-a$, that is, on the low branch of the P-V curve. It then gets trapped there by the stability of this nonviable equilibrium until the system is broken up by the selective protection activated by nonviable voltage and current conditions. This type of entrapment can result from sudden parametric change or from state space transients. This form of voltage collapse is not prominent in the consciousness of the field, but it has been noticed for instance in the "failing to get under the morning load" [8].

(2) *Convergence to a stable limit cycle*: The system state reaches the region of attraction of a stable limit cycle. The state variables start fluctuating at a slow period. Part of the limit cycle may be nonviable and this would cause a system break-up by selective protection. This condition is known classically as hunting and it is studied extensively in current literature as cross system swings.

(3) *Divergence to a singularity or to infinity*: The system ends up in a state space region, where all trajectories reach the random events at a singular surface (as in the triangular area around the lower part of line $a-a$ in Figure 1a) or keep diverging. (In unlikely events the state may wind up on a stable trajectory converging to an unstable equilibrium, whence it would again diverge). This clearly is the complement of being trapped in a stable but nonviable equilibrium. It is equally destructive and it

also leads to system breakup. If the state reaches a singularity, it will continue from there on a particular sample function of a random process outside the load flow surface. This will eventually rejoin the load flow surface at some unpredictable point, possibly subject to some unknown probabilistic event.

Depending on the location of this initial state either the course of (1), (2) or (3) can be followed. In a specific situation (3) may apply and the state returns again to the singularity. This could result in a randomly repeated bounce which of course is broken up by the protection.

It can then be observed that transients which end up (upon clearing of the fault) outside the region of attraction of the stable operating point will almost always lead to fast dynamic voltage collapse, even when the system is well within the feasible region parametrically. Hence to judge the security of the system operating point it will be very important to be able to identify the location of the stability boundary. This is more difficult theoretically and computationally than finding the feasibility boundary in the parameter space. The results of this work analyse the theoretical structure of the stability boundary in detail and lay the basis for finding it under specific system conditions.

III.3 VOLTAGE COLLAPSE CAUSED BY SPECIFIC OPERATING OR CONTROL PRACTICES

The most common subcases of this kind of voltage collapse are:

(1) *Locally controlled uncoordinated tap changers* can cause either a cascaded sequence of upward tap changes accompanied by gradually reducing secondary voltage, or a periodic up and down motion of this type involving two or more tap changers. These phenomena were briefly described in section II.9 and will be studied in detail in sections IV.5.1, V.1.2, and illustrated in section VII.

(2) *Locally controlled load dropping* which can cause wild swings of voltage (a parametric control event) [35]. The effect of locally controlled load dropping or underfrequency load dropping on the system has much in common with locally controlled uncoordinated tap changing. It will not be explored here because of space limitations.

(3) *Relay operations during transients* which are not justified by fault clearing. Here the zone system of distance relaying is often the culprit. Note that normal relay operation which is within 1-2 cycles basically shows up in the stability calculations as a sudden parameter change and as such it is directly treatable by the results presented in this paper.

(4) *Short range adjustment of voltage controls after a fault* can often cause long range stability problems [22].

IV. DESCRIPTION OF THE POWER SYSTEM MODEL

The results of this paper are applicable to any power system model regardless of size (from 1 bus to thousands of buses) as long as the functions f and g in (1) and (2) satisfy minimal smoothness assumptions and a list of generic conditions to be

specified in the statements of the theorems in section V. For the power system, smoothness is not a problem normally. Models for most components are actually analytic, though nonlinear. Tap changers can be coopted as such in various ways depending on the problem. For using some of the results, however, computational aspects will become a limiting factor for large systems.

On the other hand, the "quasi stationary" assumption which underlies the entire methodology of AC analysis can be violated in the voltage stability problem. It requires great care to avoid unwarranted conclusions.

IV.1 THE PROBLEMS RELATED TO QUASI STATIONARITY

For the model proposed in (1) and (2) some of the state variables directly represent actual physical quantities, like rotor angles θ , rotor velocities $\omega = \dot{\theta}$ and magnetic fields E' , E_{fd} . In contrast the phasors (e.g. $E\angle\delta$ for bus voltage), which represents much of the electrical side, actually stand for the modulating envelope of a modulated signal with 60 Hz carrier i.e.

$$E\angle\delta \longleftrightarrow \frac{1}{\sqrt{2}}e \sin(\omega t + \delta)$$

$$E = \text{RMS of } e(t) = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} e^2 \sin^2 \omega t dt} = \frac{1}{\sqrt{2}}e$$

provided $e(t) = \text{const.}$, that is when the system is stationary. Equations (1) and (2) are exactly valid only for the stationary system, that is for $f = 0, g = 0$. Clearly, if e is time varying, then $E \neq \frac{1}{\sqrt{2}}e$. However, if the variation of e is small during one cycle of the carrier (60Hz), an approximation can be proposed as: $E \approx \frac{1}{\sqrt{2}}e$. Then the differential equations can be written in terms of the modulating signal envelope. This is the quasi stationary approximation. Equations (1) and (2) are commonly written in RMS phasor form under the quasi stationary assumption. This fact cannot be ignored with impunity when fast moving events are involved, if physically valid results are expected.

IV.1.1. Validity of the $\dot{x} = f, 0 = g$ model in the neighborhood of the singularity

When modulated signals are used in communications, the ratio of the frequency ranges of the carrier and the modulation is in the thousands or higher. No problem exists with quasi stationarity there. In power systems this ratio is much much smaller, more like ten. Consequently, the limits of validity for quasi stationarity must be carefully examined.

From earlier studies of one of the authors it appears [44] that the error of the quasi stationary approach is around 5% when the frequency of the modulating signal is 3 Hz. So the quasi stationary assumption should be acceptable for studying dynamic stability of swings (including voltage stability in most cases) which tend to be slower than 3Hz. The subtransient range time constant (0.05 sec.) would stretch quasi stationarity to the limit, except that by the Blondel transformation the subtransient damping field components are actually nonmodulated magnetic fields so that their dynamics is outside quasi stationary restrictions. The same applies to E' , E_{fd} . A much faster time scale applies to transients within the transmission network. These transients involve the distributed reactances, resistances and capacitances of the transmission lines. Basically the transients are composed of traveling waves bouncing back and forth between busses. It has been understood for a long time (e.g.

[43, chapter 6]) that a hundred mile long transmission line only covers a fraction of the 60 Hz wave length (3110 miles), so the effective frequency range of the composite transmission network transients is about an order higher than the 60 Hz carrier. Near the singular points, some of the RMS variables, such as voltages and angles, themselves are moving at a very high speed (See for instance the high speed variation of E in Figure 5). So, both components of $\dot{y}$ i.e. 1) the composite transmission network transient and 2) the "RMS dynamics" near the singularity, are moving at high speeds. These facts have profound consequences on the concepts customarily used in studying power system dynamics and so also on the existence of the models for f and g near the singular points.

1. There is no meaningful way to talk about modulation when the modulating signal is faster (much faster here) than the carrier. Consequently all modulation based concepts become nondefinable and meaningless. This includes, among others, RMS voltages and currents, phasors, impedances, reactances, and phase angles.
2. The concept of reactive power as a physical entity ceases to exist. (Note: $g = 0$ is typically the reactive power balance). The RMS form of active power also. All become meaningless for the transients within the transmission system as well as for modulated quantities subject to fast change.
3. "Equivalent" π circuits representing transmission lines lose all validity (they are about as valid as using a π representation for a 3000 mile long line at 60 Hz.)
4. All the customary network manipulations, like eliminating buses, lose meaning and validity. In fact the entire conventional AC dynamics must be discarded for the fast phenomena of transmission network dynamics.

The conclusion is that because the rate of change of network voltages, such as $\dot{E}$, becomes very fast in this neighborhood of the singular points, the function g itself as written in the conventional manner and any "dynamic" modification of it e.g. $\epsilon \dot{y} = g(x, y, p)$ or $\epsilon \dot{E} = g(x, y, p)$ lose their validity not only at the singularity, but in a local though not arbitrarily small neighborhood of it. $g = 0$ remains an excellent approximation away from the singularity, where quasi stationary transients prevail and transmission network transients play a negligible role. Singular perturbation models like $\epsilon \dot{E} = g$ in a quasi stationary sense should not be affected if they are not used in the neighborhood of a singularity and if they are otherwise used properly.

IV.1.2 Outline of a valid model in the vicinity of the singularity

At the singularity, and in a local, but not arbitrarily small neighborhood of it, the validity of the quasi stationary model $\dot{x} = f, 0 = g$ disappears and particularly g is annihilated. A fast model of an entirely different nature needs to be substituted for any study of events in the singular neighborhood. The format of such fast dynamics would still be $\dot{x} = f(x, y), \dot{y} = g(x, y)$ but x, y now represent a different set of state variables which are not transforms of the phasors in (1) and (2) and are not even related to them in most cases. Part of g would be distributed constant type.

The composition of this fast model is quite readily specified in principle.

1. *Internal dynamics of generators* are already modelled in the desired way in the customary two reaction style. The internal generator voltage $e(t)$ represents dynamic magnetic fields by the Blondel transformation so they are not modulated quantities.
2. *Accordingly, the generators still may produce 60 Hz driving voltage inputs* to the system initially which must be stated in this instantaneous form e.g. $v(t) = e(t) \sin(\omega t + \delta(t))$. Typically now these inputs to the system are not held together by the network in synchronism any more. Also note that $e(t) \sin \omega t$ is not a sine wave when e changes fast.
3. *Transmission lines* modeled in distributed constant i.e. in traveling wave fashion.
4. *Initial conditions* : Instantaneous voltages on capacitors, currents in inductances, initial wave forms of current and voltages on transmission lines at the time of arrival of the state to the singularity (or less crisply into the region of the singularity—see below).
5. *Inductances, capacitances, resistances* modeled dynamically, in circuit theory fashion. π circuits for transmission lines and any network reductions are not applicable.

Other items should be handled in matching ways. Clearly such a model can be composed by existing techniques of ordinary circuit theory, but it would be very complex, and getting the necessary numerical data, as a practical matter, would be quite hopeless. Nevertheless important observations can be made.

IV.1.3 The singularity as a source of a random process

One must remember that on any state space trajectory the modulation and the carrier are not synchronized in any way, hence repeated runs on the same trajectory will reach the singularity in totally arbitrary phases of carrier waves $\sin(\omega t + \delta)$. In fact, this phase will have a uniform probability distribution between 0 and 2π . All the initial charges, currents etc. found at arrival of the state at the singularity will be similarly random. This makes a very big difference indeed for the events developing in the area surrounding the singularity. Furthermore, the validity of the quasi-stationary model does not collapse just when the trajectory reaches the singularity, but already ahead of it as the speed of the movement along the trajectory increases. Where this occurs (in a somewhat fuzzy manner) will depend again on the exact 60 Hz phase where a certain point on the trajectory is reached near the singularity. Furthermore, even the details and manner of transition between the two models is fuzzy. All this means that virtually all repeated arrivals to the singularity on a specific $\dot{x} = f, 0 = g$ trajectory will generate a radically different, fast moving trajectory following the arrival, all will have to leave the load flow surface L . That is: *Dynamically the singularity acts as a generator of a random process. It is not possible to predict where the continuation of a trajectory after reaching the singularity will carry the system off the load flow surface and where it will again land on or off the load flow surface, except statistically - and that only in principle.* It would be extremely difficult practically to develop such a statistic.

IV.1.4 Validity Problems and Singular Perturbation as a way to bypass the singularity

In a carefully written paper by Sastry and Varaiya [33], it is

shown that singular perturbation is valid near equilibrium points and that the perturbed trajectory must move arbitrarily close to the residual one at the limit. Singular perturbation can also be used to analyse the jump phenomenon in nonlinear circuits by introducing parasitic dynamics of the form

$$\epsilon \dot{E} = g \quad \text{or} \quad \epsilon E \dot{E} = g \quad (3)$$

This is mathematically precise for systems with nonmodulated signals[32].

On the other hand, it must be emphasized that these results are not casually adaptable to the violent and random events connected with the singularity in power systems, as discussed above. Before one embarks on such a course, a very detailed and careful study must be made to verify the validity of the fast quasi stationary model of the form (3) near the singularity.

It needs to be emphasized here that reservations on the validity are based on the very fast and *random* events involved at the singularity and the *breakdown* of the quasi stationarity under such circumstances. No reference is made to the use of singular perturbation itself where these factors are absent.

IV.1.5 Interpretation of the Results in this Paper

Here stability boundaries are calculated for the mathematical model described by equations (1) and (2). The boundaries typically contain singular segments. Our analysis stops as trajectories reach singular points. As the results stand, near the singularity they provide an *approximation* of the true power system behavior with the singular sections signalling an unpredictable random behavior. For models based on quasi stationarity this is all what the modelling permits. In this sense, though seemingly less informative than results based on singular perturbations, nevertheless this interpretation gives a realistic rendering of the power system behavior near the singularity, avoiding unwarranted conclusions. Currently no attempt has been made to further analyse the system near the singularity as this would first require a precise mathematical modelling effort. But away from the singularity, our results are precise in the sense that the actual stability boundary of the system agrees with the boundary computed in our analysis. Even this may not be correct in a singular perturbation approach.

IV.2 THE LOAD MODEL

Modelling the composite system load may be the large unsolved problem of the power system. Many models have been proposed, but not verified. The results of this paper are applicable to any model which is mathematically smooth. Without excluding other possibilities, the following appears to be the best choice at the present state of the load modeling problem.

(1) Injection Component

$$P_I = P_{L0} + ME + GE^2 + A\omega + C\omega^2 \quad (4)$$

$$Q_I = Q_{L0} + HE + BE^2 \quad (5)$$

These equations represent the variation of load demand in response to the voltage E and frequency ω at the bus as commanded by the system (i.e. E, ω are inputs in the load equations). They can be viewed either as power series expansions of the actually nonlinear relationships or it can be pointed out that the first terms in each of (4) and (5) represent reactive power

injection, the second current injection and the third constant impedance loads. The last two terms in (4) similarly represent real power dependence on frequency. The coefficients need to be determined experimentally. But only limited results are available on this topic [31].

(2) Dynamic Component

Electromagnetic load dynamics:

$$a\ddot{P}_{DE} + b\dot{P}_{DE} = P_{DE} + K_P E \quad (6)$$

$$c\ddot{Q}_{DE} + d\dot{Q}_{DE} = Q_{DE} + K_Q E \quad (7)$$

These forms are based on some recent competent test results [13, 21, 28] which reveal that with a voltage change as the input, there is a transient adjustment of the outputs P_D or Q_D which are of the nature of a second order linear response, (first order if $a = c = 0$), at least approximately. In the literature often $\dot{E}$ is used instead of $\dot{Q}$ but this does not seem to jibe with the physical facts. Sometimes the argument is that the load acts as the driving input, that is, suppose the load is increased by a step, then the bus voltage will go down. But this argument overlooks the fact that this reduction comes from the system dynamics on the other side of the load bus, rather than the inherent dynamic response of the load itself.

Electromechanical load dynamics:

$$h\dot{P}_{DM} = P_{DM} + D_L \omega_L \quad (8)$$

Usually there is little synchronous machinery in the load, hence δ_L is not a state variable. But there is considerable mass and kinetic energy $\frac{1}{2}J_L\omega_L^2$ which can be involved in dynamic interactions P_D with the system.

(3) The total load is then

$$P_L = P_I + P_{DE} + P_{DM} \quad (9)$$

$$Q_L = Q_I + Q_{DE} \quad (10)$$

This model is based on power series expansions of assumed functional relations neglecting cross coupling terms. At the present state of load modeling this seems to be the most realistic assumption. The results of this paper however are applicable to any other load model possessing proper mathematical smoothness properties. Recently it has become fashionable to model the dynamic load as essentially an equivalent induction motor. But induction motors are just one of the dynamic components and their share would be hard to assess. So it seems more realistic to assess a dynamic model which would respond in a manner consistent with the observations and then find the coefficients from measurements as best as can be.

IV.3 THE TRANSMISSION SYSTEM MODEL

This would normally consist of the typical load flow equations describing the relations between active and reactive power flow and, respectively, bus voltage magnitude and phase. Resistance and shunt branches (line charging) can be considered optionally. The load flow equations can typically be treated as non-dynamic. Very fast transients (travelling waves) connected with line capacitances and inductances can be considered in special situations such as when studying the random behavior around singular points. The related problems were discussed in a general framework in section IV.1.

IV.4 THE SYNCHRONOUS MACHINE MODEL

The dynamics of synchronous machines has been studied long and thoroughly and is well understood. The results of the paper apply to any version of generator models from two axis with damper (subtransient) windings through one axis and down to the "classical" model with constant voltage behind the transient reactance.

IV.5 PRIMARY CONTROL DEVICES

Automatic tap changers (of course under load (ULTC)) on transformers, thyristor controlled reactors (SVC), excitation control of synchronous generators and synchronous condensers, and HV-DC converters form the major primary voltage control tools on the system. They all fit well within any power system model on which the results of the paper are to be applied. Capacitor and reactor banks can be switched and loads dropped in discrete events which represent dynamic disturbances in the sense of the results of the paper and are readily treated as such.

IV.5.1 Locally controlled automatic tap changers

(1) *Physical features:* Traditionally, to operate a tap changer under load, it will be necessary first to bridge two transformer taps through appropriate inductors or resistors with center taps, and then interrupt the circulating current at the old tap. This involves considerable mechanical operation of contacts and switches and the use of small interruptors (often vacuum type). Operating these mechanisms takes a considerable and fixed amount of time which is independent of the deviation of the bus voltage from the set point.

The point is that the system experiences one (or two) fixed magnitude step jumps of the transformation ratio per tap change (usually less than 1%) and a minimum time interval, also of fixed magnitude of many seconds between consecutive changes (depending on type, 40-100 sec., 3-8 seconds, or, in new thyristor devices, under 1 sec.). It is clear that the response to small step increases, placed many seconds apart will break down to two decoupled phenomena.

1. A transient response to the small step jump which is normally quite harmless unless the system is operating with inadequate security that is much too close to its feasibility or stability boundary. This transient decays during a small initial part of the minimum delay preceding the next tap change.
2. Thus the success of a tap change is expressed by the steady state voltage E_L^* reached after the last tap change and before the next one. This voltage is independent of the transient which has died out by that time. So typically only the steady state value is significant.

The traditional tap changer operating control is a simple device which switches one step up or down when the deviation between the steady state load voltage E_{Li}^* and the reference setting E_{ri} reaches or exceeds the tolerance value, ΔE .

$$n_{k+1} = \begin{cases} n_k + \Delta n & \text{if } E_{Li}^* - E_{ri} < -\Delta E \\ n_k & \text{if } |E_{Li}^* - E_{ri}| \leq \Delta E \\ n_k - \Delta n & \text{if } E_{Li}^* - E_{ri} > \Delta E \end{cases} \quad (11)$$

where Δn is the tap step size (usually equal or slightly less than

ΔE) and E_{Li}^s is the steady state voltage at the load i and as such the solution of

$$0 = f(x, y, n, p), \quad 0 = g(x, y, n, p) \quad (12)$$

where, for convenience, the vector n of tap positions is displayed separately from states and parameters. Its nature may vary depending on the problem. If we denote bus voltages by $x_1 = \{E_i^s\}$, the load voltages (secondary of tap changers) by $x_2 = \{E_{Li}^s\}$ and the remaining (dynamic and instantaneous) state variables by x_3 , and all the other parameters are grouped as p , then the system at steady state can be written comprehensively as

$$0 = h(x_1, x_2, x_3, n, p). \quad (13)$$

This model is analyzed in detail in section V.1.2. Note that this analysis (See also subsection (2) below) applies to the traditional type of tap changer. Thyristor controlled tap changers are simply part of the system dynamics covered by the taxonomy and is directly a part of it. So it actually requires no special solutions (See also subsection (3) below).

There exist a variety of problems connected with the tap changer that can be formulated. The most useful from an operating, security monitoring and planning angle is the following one.

(2) *Effect of the tap changer on the feasibility boundary in the parameter space* : The actual postchange stationary voltage will depend both on the tap size and the system composition. As long as there is a rise in the secondary voltage above the set-point, no critical problem arises. However, because of load characteristics and other system features, the stationary voltage can actually go down in response to an upward tap change. When that happens, the simple automatic control device (11) set for local control (and even an inexperienced operator) will order another upward step initiating a chain reaction of descending and ultimately collapsing voltage. When a number of tap changers are involved, this scenario generalizes to the positive definiteness of the sensitivity matrix, i.e. the sensitivity of the load voltages to the tap changer positions. When this matrix is positive definite, for any load bus, then if the tap ratio is raised, the load bus voltage also increases, which is the desired effect. But, if the sensitivity matrix is not positive definite, then there is at least one direction in which the raise of the tap ratio may lead to a cascaded voltage collapse. It is later shown that the tap changer control becomes unstable when the sensitivity matrix is not positive definite. In the parameter space the boundary of after switch tap positions where the cascading phenomenon commences is where the sensitivity matrix becomes indefinite. Thus the boundary can be pinpointed by postulating an arbitrarily small tap size. For a finite tap size the actual boundary for the initiating step would then be lower by one step size. Over a region of system conditions, this condition defines a section of the feasibility boundary in parameter space where the cascading phenomenon sets in, i.e. local stability of the tap changer control is lost beyond this boundary. This leaves the small step boundary as the core of the answer which can be readily adjusted to the actual step size (See the details of analysis in section V.1.2).

(3) *Transient phenomena connected with the tap changer in the state space* The traditional slow tap changer has no real dynamic characteristics beyond (10) and (11) in other words when the voltage is off the reference by the tap size a tap change is

implemented regardless of anything else. So on this type of device there is no $\dot{n}$ that is tap rate term—in fact the tap change is always strictly one tap at a time regardless of the size of the deviation. So this is a discrete element. For computing trajectories and analyzing stability by a discrete-continuous analysis [19, 26] it is usually assumed that the tap switching times are synchronized system wide however the tap changer has an option of not switching when the voltage is within the neutral band $|E_L^s - E_r| \leq \Delta E$. This is a quite realistic approximation but it still leaves a rather difficult hybrid analytical problem. So it is tempting to introduce a continuous approximation of the form

$$\dot{E}_L^s = \frac{1}{T}(E_L^s - E_r) \quad \text{or} \quad (14)$$

$$\dot{n} = \frac{1}{T}(E_L^s - E_r) \quad (15)$$

Justification of this latter step of approximation seems less convincing for the traditional slow tap changers in fact often it is simply stated as an unsupported assumption [1]. Since the normal dynamic elements (generator excitation control, load dynamics etc.) are much too fast to couple into the tap changing circle the practical meaning of this assumption is dubious unless a much slower f, g system is substituted, like boiler dynamics or some AGC action.

On the other hand, using very fast thyristor type tap changers, it would be quite easy to implement an actual controller with the control law of (14) or (15) in the usual transient stability range. In this latter case if such a control dynamics exists then the system itself would need to be modeled in its full dynamic form, that is $\dot{x} = f(x, y, p), 0 = g(x, y, p)$ where now $\dot{x}$ includes $\dot{E}_L^s$ or $\dot{n}$ and f includes equation (14) or (15). The results of this paper are directly applicable for such studies so nothing more needs to be said here.

The new introduction in subsection (2) above, of studying the effect of the tap changer on the feasibility boundary seems to be providing the most pertinent information connected with the special type of collapse (the cascading one) which is not quite dynamic in the ordinary sense and which is an important practical problem in operation. The more directly dynamic instabilities which would result from (14) or (15) do not seem to be typical in practice but they may become so if fast tap changing control of the type (15) becomes common. In that case, they can be studied routinely within the taxonomy.

IV.5.2 Excitation Control of Synchronous Machines

Typical excitation controls are presented with their models in an IEEE publication [49]. The literature of voltage stability uses mostly a simplification of type 1 which is a servomechanism age linear control and hence has all possible smoothness properties unless the hard limits are reached. A highly simplified version is being used in the illustrations in section VII. With increasing use of thyristor and other excitation controls, also described in [49], however, these should be given proper attention.

IV.5.3 SVC and other Thyristor Devices

SVC or static var control (static in the sense of no moving parts, somewhat misleadingly) is basically a thyristor controlled reactor. Along with HV-DC controls, also by thyristors, they are fast and effective primary control tools. Models are available [15], but they will not be further treated here.

Additional IEEE descriptions [48] are available for modelling governors, turbines, boilers etc. The results of this paper directly apply to this side of the power system, but the current paper concentrates on voltage stability for system models complete with their mechanical dynamics, but in an operating range where mechanical instability is not an issue.

V. MATHEMATICAL PROPERTIES AND RESULTS

Consider a differential-algebraic system defined by

$$\dot{x} = f(x, y, p) \quad , \quad f : \mathbb{R}^{n+m+p} \rightarrow \mathbb{R}^n \quad (16)$$

$$0 = g(x, y, p) \quad , \quad g : \mathbb{R}^{n+m+p} \rightarrow \mathbb{R}^m \quad (17)$$

$$x \in X \subset \mathbb{R}^n, \quad y \in Y \subset \mathbb{R}^m, \quad p \in P \subset \mathbb{R}^p$$

For the power system the state space $X \times Y$ consists of dynamic state variables x like generator voltages and rotor phases, and instantaneous state variables y like bus voltages and other load flow variables. The parameter space $P = P_S \times P_O$ is composed of system parameters (which describe for instance the system topography, i.e. which lines, buses etc. are energized, and equipment constants such as inductances, transformation ratios etc.), and operating parameters (such as loads, generation, voltage setpoints etc.) Both of these spaces have distinct structural features (introduced in sections II and III), sometimes intertwined, which will be described in this chapter. The presence or absence of automatic discrete controls, like tap changers, leads to a different mathematical analysis and will therefore be studied in separate sections. Also, here we only formulate the results. Proofs will be given in section VIII.

V.1 FEASIBILITY REGIONS AND FEASIBILITY BOUNDARIES

V.1.1 For Systems without Automatic Discrete Controls

Let

$$EQ = \{(x, y, p) \in X \times Y \times P : f(x, y, p) = 0, g(x, y, p) = 0\}$$

$$OP = \{(x, y, p) \in EQ : D_y g \text{ is nonsingular and}$$

$$J = D_x f - D_y f (D_y g)^{-1} D_x g$$

has eigenvalues with negative real part\};

Here $D_x f$ etc. denote the matrix of partial derivatives of the components of f with respect to x . EQ is the set of equilibrium points and OP is the subset of stable equilibria. Note that the full Jacobian in x and y is nonsingular for $(x, y, p) \in OP$ and therefore, by the implicit function theorem, the equations $f(x, y, p) = 0$ and $g(x, y, p) = 0$ can theoretically be solved uniquely for x and y as functions of the parameter p . Hence OP is a p -dimensional submanifold embedded in $EQ \subset X \times Y \times P$.

Definition 1 Given a stable equilibrium $z_0^o = (x_0, y_0)$ (a load flow solution) for parameter value p_0 , the connected component F of OP which contains (x_0, y_0, p_0) is called the feasibility region of z_0^o . Its boundary (relative to EQ) is the feasibility boundary.

Note that the feasibility region is defined as a subset of $X \times Y \times P$, not as a subset of the parameter space. This way, all possible operating conditions which can be reached from (x_0, y_0, p_0) by continuous variations of the parameter while main-

taining stability form the feasibility region. Typically there exist a number of different stable load flow solutions for a set of parameters p_1 . Therefore it is possible for a stable operating point (x_0, y_0, p_0) to change along different curves $\pi : [0, 1] \rightarrow P$ and $\tilde{\pi} : [0, 1] \rightarrow P$ with initial point $p_0 = \pi(0) = \tilde{\pi}(0)$ and equal terminal points $\pi(1) = \tilde{\pi}(1)$ into different stable equilibria (x_1, y_1, p_1) and $(\tilde{x}_1, \tilde{y}_1, p_1)$ over the same parameter set p_1 . The specific parameter variation may determine which equilibrium point is reached. In this case there does not exist a well-defined projection into the parameter space. The definition given avoids this ambiguity by considering all possible liftings of curves $\pi : [0, 1] \rightarrow P$, $\pi(0) = p_0$ into OP . However, as stated at the beginning of this paper, a proper operating point must also be viable in addition to being feasible. Typically there is not more than one viable and stable equilibrium point (one constrained load flow solution with voltages and currents within appropriate tolerances for a given set of parameters, i.e. a given assignment of PV and PQ bus values.) Excluding nonviable operating points, the remainder of the feasibility region has then a well-defined projection into the parameter space and can be considered a subset of P .

Theorem 1 (Feasibility Boundary Theorem) For a system defined in equations (16) and (17) without locally controlled tap changers, the feasibility boundary of a feasibility region F consists of three zero sets

$$\partial F = (\partial F \cap C_S) \cup (\partial F \cap C_Z) \cup (\partial F \cap C_H),$$

where

$$C_S = \{(x, y, p) \in EQ : \det D_y g = 0\},$$

$$C_Z = \{(x, y, p) \in EQ : \det D_y g \neq 0, \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} = 0\}$$

and

$$C_H = \{(x, y, p) \in EQ : \det D_y g \neq 0, \det \begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix} \neq 0, \det(H_{n-1}(J)) = 0\}$$

where

$$J = D_x f - D_y f (D_y g)^{-1} D_x g \quad (18)$$

and H_{n-1} is the Hurwitz matrix,

$$H_{n-1} = \begin{pmatrix} a_1 & a_3 & a_5 & * & * & a_{2n-3} \\ a_0 & a_2 & a_4 & * & * & a_{2n-4} \\ 0 & a_1 & a_3 & * & * & a_{2n-5} \\ 0 & a_2 & a_4 & * & * & a_{2n-6} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & * & a_{n-1} \end{pmatrix} \quad (19)$$

corresponding to the coefficients a_i of the characteristic polynomial of J

$$\det(sI - J) = a_0 s^n + a_1 s^{n-1} + \dots + a_n. \quad (20)$$

and $a_k = 0$ for $k > n$. $\square$

The three zero sets correspond to specific conditions on the eigenvalues of the system. As a point on EQ approaches C_S some eigenvalues diverge to infinity. The equilibrium has zero eigenvalues in the set C_Z and purely imaginary eigenvalues for points in the set C_H . Under generic conditions local bifurcations occur at all these points, namely a singularity induced bifurca-

tion in C_S , a saddle-node bifurcation in C_Z and a Hopf bifurcation in C_H . Moreover, generically these bifurcations constitute a dense subset of the feasibility boundary for equations (16) and (17). Saddle-node and Hopf bifurcations are wellknown in the literature and their definitions will be summarized here for the reader's convenience. The singularity induced bifurcation will be analyzed.

A *saddle-node bifurcation* occurs when a system has a non-hyperbolic equilibrium with a simple zero eigenvalue at the bifurcation point and additional transversality conditions are satisfied. Points in C_Z can locally be described as $\dot{x} = f_R(x, p)$ for some reduced function f_R . Then points in C_Z are saddle-node bifurcations if the following conditions are satisfied:

(SN1) The matrix $D_x f_R = D_x f - D_y f (D_y g)^{-1} D_x g$ has a simple zero eigenvalue with right eigenvector v and left eigenvector w .

(SN2) $w(D_p f_R) = w(D_p f - D_y f (D_y g)^{-1} D_p g) \neq 0$

(SN3) $w(D_x^2 f_R(v, v)) \neq 0$ $\square$

A *Hopf bifurcation* occurs at points where the system has a nonhyperbolic equilibrium connected with a pair of purely imaginary eigenvalues, but no zero eigenvalue, and additional transversality conditions are met. Specifically, for the system (16) and (17) it is shown in the proof of Theorem 1 (in section VIII.3) that the Hurwitz function H_{n-1} detects the presence of purely imaginary eigenvalues. The assumption $\det(J) \neq 0$ implies that locally there exists a smooth curve of equilibria, $(x(p), y(p), p)$, with eigenvalues $\lambda(p), \bar{\lambda}(p)$ of $D_x f_R$ which are imaginary and vary smoothly with p . A Hopf bifurcation occurs at any point in C_H where the following transversality conditions are satisfied:

(H1) $J = D_x f_R$ has a simple pair of purely imaginary eigenvalues.

(H2) $D_p \text{Real}(\lambda(p)) \neq 0$. $\square$

A *singularity induced bifurcation* occurs when an equilibrium is at the singular surface

$$S = \{(x, y, p) \in EQ : \Delta(x, y, p) := \det(D_y g(x, y, p)) = 0\}$$

and, again, certain additional transversality conditions are satisfied. To analyse this bifurcation, first assume that a single parameter μ is being varied and that the equilibrium is at the singularity for $\mu = \mu_0$. More precisely, let

$$f(0, 0, \mu_0) = 0, \quad g(0, 0, \mu_0) = 0 \quad \text{and} \quad \Delta(0, 0, \mu_0) = 0.$$

Suppose that for $\mu = \mu_0$, the Jacobian $D_y g$ has a simple zero eigenvalue and the full Jacobian

$$\begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix}$$

is nonsingular. Then, locally near $\mu = \mu_0$, by the implicit function theorem, there exists a unique equilibrium locus $(x(\mu), y(\mu), \mu)$. We also assume that the determinant Δ is dominated by the linear term along the equilibrium locus, i.e.

$$c := \frac{\partial}{\partial \mu} \Delta(x(\mu), y(\mu), \mu) \neq 0. \quad (21)$$

It can be seen that for $\mu = \mu_0$, as the equilibrium is at the singular surface, the system is not smooth. To facilitate the analysis a singular transformation will be introduced in the proof. Essentially, the vector field is scaled pointwise by $\Delta(x, y, \mu)$. The transformed smooth vector field has a unique nonzero eigenvalue at $\mu = \mu_0$ which will be denoted by b . For the true system, this eigenvalue blows up along

$$\frac{b}{c(\mu - \mu_0)} + \mathcal{O}(\mu - \mu_0).$$

The precise statement of the theorem is as follows:

Theorem 2 (Singularity Induced Bifurcation Theorem) Consider equations (16), (17) with a 1-dimensional parameter space. Suppose the following conditions are satisfied at $(0, 0, \mu_0)$:

(SI1) $f(0, 0, \mu_0) = 0, g(0, 0, \mu_0) = 0$ and $D_y g$ has a simple zero eigenvalue

(SI2) $\begin{pmatrix} D_x f & D_y f \\ D_x g & D_y g \end{pmatrix}$ is nonsingular

(SI3') $D_\mu \Delta(x(\mu), y(\mu), \mu) \neq 0$.

Then there exists a smooth curve of equilibria in $\mathbb{R}^{n+m+1}$ which passes through $(0, 0, \mu_0)$ and is transversal to the singular surface at $(0, 0, \mu_0)$. When μ increases through μ_0 , one eigenvalue of the system, (i.e. an eigenvalue of

$$J = D_x f - D_y f (D_y g)^{-1} D_x g$$

evaluated along the equilibrium locus), moves from $\mathbb{C}^-$ to $\mathbb{C}^+$ if $\frac{b}{c} > 0$ (respectively from $\mathbb{C}^+$ to $\mathbb{C}^-$ if $\frac{b}{c} < 0$) by diverging through ∞ . The other eigenvalues remain bounded. $\square$

Essentially, this bifurcation is induced when an equilibrium crosses the singularity. The number of stable eigenvalues of the system changes by one when the eigenvalue blows up. The computation of the eigenvalues which remain bounded at $\mu = \mu_0$ is given in the proof of the theorem in section VIII.2. For a two-dimensional system it can be shown that this bifurcation is equivalent to a transcritical bifurcation in the transformed system[37]. In the full system, a singularity induced bifurcation occurs if the gradient of Δ along the local equilibrium locus does not vanish,

(SI3) $D_p \Delta(x(p), y(p), p) \neq 0$.

It is clear that the three bifurcation types discussed above arise when additional independent inequality constraints are imposed on points in the sets C_S , C_Z and C_H . Generically these conditions will be satisfied on dense subsets. Henceforth it will be assumed that

(D1) The set of points where $\{(SN1), (SN2), (SN3)\}$ are satisfied, is dense in C_Z .

(D2) The set of points where $\{(H1), (H2)\}$ are satisfied, is dense in C_H .

(D3) The set of points where $\{(SI1), (SI2), (SI3)\}$ are satisfied, is dense in C_S .

The boundary of the closure of the feasibility region will be called the quasi-feasibility boundary.

Theorem 3 (Quasi Feasibility Boundary Theorem)

Suppose system (16), (17) satisfies assumptions (D1), (D2) and (D3). Then the feasibility boundary is also the quasi-feasibility boundary, i.e. the stability of the equilibrium is lost at most of the boundary points. The saddle-node bifurcation, the Hopf bifurcation and the singularity induced bifurcation occur on a dense subset of the boundary ∂F .

$$\partial F = \overline{\partial F} = (\overline{\partial F \cap C_H}) \cup (\overline{\partial F \cap C_S}) \cup (\overline{\partial F \cap C_Z}). \quad (22)$$

□

In short, the feasibility boundary can be computed as zero sets. Generically, for the system (16), (17) most of the boundary points lead to a saddle-node, Hopf or a singularity induced bifurcation.

V.1.2 Tap-Changer Induced Feasibility Boundary

As pointed out in section IV.5.1, the pertinent model for studying the difficulties resulting from locally controlled tap changers is the one describing the steady state reached after the tap change. This model (13) will now be analysed for the restriction of the feasibility region caused by these types of tap changers. Let d denote the number of buses with tap changers. If bus i is equipped with tap changer, then $x_{2i} = x_{1i}/n_i$, so that $d = \dim(x_1) = \dim(x_2) = \dim(n)$. Substituting $x_{1i} = x_{2i}/n_i$ in (13) gives

$$0 = h_1(x_2, x_3, n, p) \quad (23)$$

Here the number of equations is equal to the sum of the dimensions of x_2 and x_3 . Suppose n and p are within the feasibility region and away from the feasibility boundary as introduced in section V.1.1. Then, since $(D_{x_2}h_1, D_{x_3}h_1)$ is nonsingular, locally near any such point the solutions to equation (23) can be represented as

$$x_2 = x_2(n, p), \quad x_3 = x_3(n, p). \quad (24)$$

The tap-changer continues switching until $x_{2i} = E_{ri}$ for all load buses. Hence, for the system with automatic tap changer control, the solution of (23) with $x_{2i} = E_{ri}$ defines the equilibrium state. The stability of the tap-changer control is determined by the Jacobian

$$J_d := -D_n x_2 = (I_d, 0)(D_{x_2}h_1, D_{x_3}h_1)^{-1} D_n h_1. \quad (25)$$

The equilibrium is stable if $-D_n x_2$ is stable (negative definite) and stability is lost when it has eigenvalues on the imaginary axis. This can be demonstrated by approximating the discrete dynamics of the tap-changer control with a continuous model as follows: The tap-changer changes the tap size by a small step if the voltage differs from the set point voltage by more than its allowed tolerance value. Usually, the tap-step size and the tolerance values are small, about 1% in p.u. Hence, we can approximate the discrete model by a continuous model of the form

$$\dot{n}_i = f_i(x_{2i}) = \begin{cases} c & \text{if } x_{2i} < E_{ri} \\ -c & \text{if } x_{2i} > E_{ri} \end{cases} \quad (26)$$

where c is a suitable positive constant. The system described by equation (26) is not smooth at $x_{2i} = E_{ri}$, but the 'slope' $D_{x_{2i}}f_i$ can be taken arbitrarily large negative. To analyze the local stability, choose a suitable smooth function $\tilde{f}_i$ which approximates f_i to the desired accuracy and, in addition is such that $D_{x_2}\tilde{f} = -kJ_d$, where k is a positive constant. The Jacobian of the system $\dot{n}_i = \tilde{f}_i(x_{2i})$ is then given by kJ_d . Hence, the approximated system is stable if all the eigenvalues of J_d are in

$\mathbb{C}^-$, and is unstable if there exists at least one eigenvalue in $\mathbb{C}^+$. Considering the limiting case for $f_i(x_{2i})$, as the slope goes to $-\infty$, the result follows. Hence, the segment(s) of the feasibility boundary induced by the tap changer are pinpointed (at least after taking the closure, if necessary) by the parameters where the sensitivity matrix $(-J_d)$ has eigenvalues on the imaginary axis.

The new boundary is then where J_d loses stability, i.e. where the eigenvalues of J_d cross the imaginary axis (as we stay away from ∂F , eigenvalues do not blow up), i.e. for either $\det(J_d) = 0$ (zero eigenvalue) or $\det(H_{n-1}(J_d)) = 0$ (purely imaginary eigenvalues) [47]. The Jacobian, J_d can be simplified as

$$J_d(x_2, x_3, n, p) = J_d(E_{ri}, x_3, \left(\frac{E_r}{x_{1i}}\right)_i, p) = \tilde{J}_d(x_1, x_3, p).$$

Essentially, the tap-changer tries to track the solutions of

$$0 = h(x_1, x_2, x_3, n, p) \quad \text{where } x_{2i} = E_{ri}, \quad n_i = \frac{E_{ri}}{x_{1i}}.$$

Rewriting these equations, the steady-state solutions with the tap-changer control are the solutions of

$$0 = h_2(x_1, x_3, p) \quad (27)$$

and the new feasibility boundaries are defined by the conditions

$$0 = \det(\tilde{J}_d) \quad \text{and} \quad 0 = \det(H_{n-1}(\tilde{J}_d)).$$

This discussion can now be summarized as follows: The set of steady-state solutions with tap-changer control is defined by

$$EQ_{TC} = \{(x_1, x_2, x_3, n, p) : h(x_1, x_2, x_3, n, p) = 0, \\ x_{2i} = E_{ri}, \quad n_i = \frac{E_{ri}}{x_{1i}}\}$$

The set of stable solutions OP_{TC} within set EQ_{TC} is defined by

$$OP_{TC} = \{(x_1, x_2, x_3, n, p) \in EQ_{TC} : J_d \text{ is negative definite}\}.$$

The connected component of the stable equilibria OP_{TC} which contains a specific solution (x_s, n_s, p_0) is defined as the *feasibility region with tap-changer control* and is denoted by F_{TC} .

Theorem 4 (Feasibility Boundary Theorem with Tap Changer)

For a system containing locally controlled automatic tap changers, the feasibility boundary ∂F_{TC} of a stable equilibrium point (x_s, n_s, p_0) consists of five zero sets

$$\partial F_{TC} = \partial F_{TC} \cap \{C_S \cup C_Z \cup C_H \cup C_{Z,TC} \cup C_{H,TC}\}.$$

The sets C_S , C_Z and C_H are as defined in Theorem 1; $C_{Z,TC}$ and $C_{H,TC}$ are defined as

$$C_{Z,TC} = \{(x_1, x_3, p) : h_2(x_1, x_3, p) = 0, \det(\tilde{J}_d)(x_1, x_3, p) = 0, \\ \det(D_{x_2}h_1(x_2, x_3, p), D_{x_3}h_1(x_2, x_3, p)) \neq 0\}$$

and

$$C_{H,TC} = \{(x_1, x_3, p) : h_2(x_1, x_3, p) = 0, \det(\tilde{J}_d)(x_1, x_3, p) \neq 0, \\ \det(D_{x_2}h_1(x_2, x_3, p), D_{x_3}h_1(x_2, x_3, p)) \neq 0, \\ \det H_{n-1}(\tilde{J}_d)(x_1, x_3, p) = 0\}$$

where $H_{n-1}(\tilde{J}_d)$ is the Hurwitz determinant of the Jacobian $\tilde{J}_d$ and is defined exactly as in the definition of C_H . □

V.2 TYPAL REGIONS

A feasibility region consists of possibly several open subregions, called *typal regions* in this paper, and some of the respective boundaries. Typal regions are defined by the property that the phase-portraits of the system are topologically equivalent (i.e. structurally stable) within one subregion. The boundaries between these regions are determined by various bifurcation surfaces. For a rudimentary power systems model, a complete classification of the typal regions has been given in [36] for a realistic range of power and control as parameters (see Figure 2). On the level of the general power system, for reasons of dimensionality, a similar complete description is unrealistic. But the more important situations like saddle-node, Hopf, or singularity induced bifurcations can be analyzed with existing methods. Here no results in this direction will be given. From a security point of view, typal regions are of less importance than the feasibility regions, but some of the bifurcations typically signal the onset of dynamic phenomena which can lead to voltage collapse (see [36]).

V.3 REGIONS OF ATTRACTION

Now a fixed parameter set p_0 is considered in equations (16) and (17). For notational simplicity the parameter is dropped and the equations are written in the form

$$\dot{x} = f(x, y), \quad f: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^n \quad (28)$$

$$0 = g(x, y), \quad g: \mathbb{R}^{n+m} \rightarrow \mathbb{R}^m \quad (29)$$

The following sets to a large extent determine the structure of the state space.

$$\begin{aligned} L &= \{(x, y) \in \mathbb{R}^{n+m} : g(x, y) = 0\} \\ M &= \{(x, y) \in L : \text{rank}(D_x g(x, y), D_y g(x, y)) = m\} \\ S &= \{(x, y) \in L : \Delta(x, y) = \det D_y g(x, y) = 0\} \\ \Psi &= \{(x, y) \in S : \kappa(x, y) = \\ &\quad \text{adj}(D_y g(x, y)) D_x g(x, y) f(x, y) = 0\} \\ \Xi &= \{(x, y) \in S \setminus \Psi : D_y \Delta(x, y) \kappa(x, y) = 0\} \\ R &= S \setminus (\Psi \cup \Xi) \end{aligned}$$

L is the load flow surface determined by the algebraic constraint. The system must stay on L and henceforth all topological definitions are relative to L . M is a regular subset of L which, by the implicit function theorem, is an n -dimensional manifold. S is called the singular surface or the singularity. Typically no load flow solution exists for points in S . Ψ corresponds to a subset of positive codimension where the system formally has equilibria on S and is called the *pseudo equilibrium surface* (In the definition, $\text{adj}(D_y g)$ denotes the classical matrix adjoint of the Jacobian $D_y g$). This typically high dimensional subset displays all the behavior of an equilibrium surface and indeed becomes an equilibrium surface if the vector field is singularly rescaled as follows: Let

$$\begin{aligned} Z &= (f(x, y), -(D_y g(x, y))^{-1} D_x g(x, y) f(x, y)) \\ Z^T &= (f(x, y) \Delta(x, y), -\text{adj}(D_y g(x, y)) D_x g(x, y) f(x, y)) \end{aligned}$$

The flow of the vector field Z , denoted by Φ_t , describes the dynamics of the system. The vector field Z^T , whose flow is denoted by Φ_t^T , is obtained from Z by rescaling it at every point by multiplying with the determinant Δ . The transformed vector field is smooth and leaves the load flow surface invariant. It will be used in the proofs, but the results as stated in this section

are directly applicable to the original system (28) and (29). The properties of this transformation are analysed in section VIII.4. The pseudo equilibrium surface is a true equilibrium surface for the rescaled vector field Z^T . It is useful to distinguish the following subsets of Ψ :

$$\begin{aligned} N_\psi &= \{(x, y) \in \Psi \cap M : \text{rank}(D_y g) = m - 1 \text{ and} \\ &\quad \text{the Jacobian of } Z^T \text{ has} \\ &\quad \text{exactly 2 nonzero eigenvalues}\} \end{aligned} \quad (30)$$

$$B_\psi = \Psi \setminus N_\psi \quad (31)$$

Assuming that M is dense in L , which is the typical situation, the Jacobian of Z^T (restricted to L) has at least $(n - 2)$ zero eigenvalues at a point in Ψ (see section VIII.5). Hence N_ψ corresponds to the subset of Ψ where the most regular type behavior takes place. It will be shown that near every point in N_ψ the integral curves of the system locally behave like for a linear system. N_ψ is called the 'nice' set. Its complement B_ψ in Ψ is called the bad set. Eventually generic assumptions will be imposed to make sure that this is a truly lower dimensional subset, so that it suffices to consider the nice set only. Finally, Ξ corresponds to points where Z^T is tangent to the singularity (see section VIII.4). Henceforth points in Ξ will be called *semi-singular*. Again it will be assumed that the most regular type behavior occurs except for truly lower dimensional subsets. Let

$$\begin{aligned} N_\xi &= \{(x, y) \in \Xi : \text{rank} \begin{pmatrix} D_x g & D_y g \\ D_x \Delta & D_y \Delta \end{pmatrix} = m + 1, \\ &\quad \text{and } D_y \{(D_y \Delta) \kappa\} \kappa \neq 0\} \end{aligned} \quad (32)$$

$$B_\xi = \Xi \setminus N_\xi. \quad (33)$$

The condition on the second derivative in the definition of N_ξ ensures that trajectories do not cross over as they touch the singularity. The properties of these various subdivisions of the state space are summarized in the next theorem.

Theorem 5 *If the subsets are not empty, then*

1. $L \setminus S$ is an n -manifold. The solutions of the system (28) and (29) are the integral curves of the vector field Z on $L \setminus S$.
2. R is an $(n - 1)$ -dimensional manifold. At each point in R two trajectories either approach or leave the singularity transversally.
3. N_ψ is an $(n - 2)$ -dimensional manifold. Ignoring orientations, the solution curves are locally equivalent to those of a linear system near the origin.
4. N_ξ is an $(n - 2)$ -dimensional manifold. Trajectories touch the singularity tangentially, but do not cross over.

□

It will also be assumed that the bad sets are truly lower dimensional. More precisely, the following assumption will be made:

(A0) The nice sets N_ψ and N_ξ are dense in Ψ and Ξ , respectively. Furthermore, at every point $\psi \in \Psi$ or $\xi \in \Xi$, the sets B_ψ and B_ξ have dimension at most $(n - 3)$.

If f and g are analytic functions (like it is the case for the power systems model), then L , S , Ψ and Ξ are analytic subsets of $X \times Y$. The nice sets N_ψ and N_ξ are described by extra inequality conditions involving analytic functions and are therefore

semi-analytic. Hence it is possible to stratify L compatible with these sets [25]. In particular, since the bad sets (points where the inequalities do not hold) lie in the boundary of the nice sets, they are locally finite unions of lower dimensional strata and so the extra condition on the dimensions comes for free. In general, Assumption (A0) describes the least degenerate situation which will be true for a typical system.

Equilibria and periodic orbits of Z in $L \setminus S$ are defined as usual, but with the proviso that they do not lie on or intersect the singular set S . Denote the coordinates in the state space by $z = (x, y)$ and for a stable equilibrium z_s , let C denote the connected component of $L \setminus S$ which contains z_s . Without loss of generality, it will be assumed that the determinant $\Delta(x, y)$ is positive in C (otherwise redefine Δ as $-\Delta$). If a trajectory hits the singularity, then, as discussed in section IV.1, its behavior is unpredictable and therefore, if $\Phi_{t_0}(z) \in S$ for some $t_0 > 0$, then z will not be considered as a point in a stable manifold of an equilibrium or a periodic orbit. The region of attraction of a stable equilibrium z_s is defined as

$$A = \{z \in C : \Phi_t(z) \in C \forall t \geq 0, \Phi_t(z) \rightarrow z_s \text{ as } t \rightarrow \infty\}.$$

The results in this section characterize the boundary of the region of attraction. For any equilibrium $z_0 \in C$ define the stable and unstable manifolds as

$$\begin{aligned} W^s(z_0) &= \{z \in C : \Phi_t(z) \in C \text{ for all } t \geq 0, \\ &\quad \Phi_t(z) \rightarrow z_0 \text{ as } t \rightarrow \infty\} \\ W^u(z_0) &= \{z \in C : \Phi_t(z) \in C \text{ for all } t \leq 0, \\ &\quad \Phi_t(z) \rightarrow z_0 \text{ as } t \rightarrow -\infty\}. \end{aligned}$$

Similarly, for a periodic orbit $\gamma_0 \in C$ define the stable and unstable manifolds as

$$\begin{aligned} W^s(\gamma_0) &= \{z \in C : \Phi_t(z) \in C \text{ for all } t \geq 0, \\ &\quad \Phi_t(z) \rightarrow \gamma_0 \text{ as } t \rightarrow \infty\} \\ W^u(\gamma_0) &= \{z \in C : \Phi_t(z) \in C \text{ for all } t \leq 0, \\ &\quad \Phi_t(z) \rightarrow \gamma_0 \text{ as } t \rightarrow -\infty\}. \end{aligned}$$

Finally, for a pseudo equilibrium $\psi \in \Psi$ or a semi-singular point $\xi \in \Xi$, these definitions are as follows

$$\begin{aligned} W^s(\psi) &= \{z \in C : \exists t_0 > 0 \text{ such that } \Phi_t(z) \in C \\ &\quad \text{for } 0 \leq t < t_0 \text{ and } \Phi_t(z) \rightarrow \psi \text{ as } t \rightarrow t_0\} \\ W^u(\psi) &= \{z \in C : \exists t_0 < 0 \text{ such that } \Phi_t(z) \in C \\ &\quad \text{for } 0 \geq t > t_0 \text{ and } \Phi_t(z) \rightarrow \psi \text{ as } t \rightarrow t_0\}. \end{aligned}$$

and verbatim the same for ξ .

Lemma 1 Let $\psi \in N_\psi$ and suppose the Jacobian of Z^T has n_s stable eigenvalues. Then $W^s(\psi)$ is an n_s -dimensional and $W^u(\psi)$ is a $(2 - n_s)$ -dimensional manifold. If $\delta(\psi)$ denotes a δ -sized neighborhood of ψ in N_ψ , then the stable and unstable manifolds of $\delta(\psi)$, $W^s(\delta(\psi))$ and $W^u(\delta(\psi))$, are $(n - 2 + n_s)$ - and $(n - n_s)$ -dimensional manifolds, respectively. $\square$

For $n_s = 0, 1$ or 2 , ψ is said to be a pseudo-source, pseudo-saddle or pseudo-sink, respectively. If N_C denotes a connected component of the nice set N_ψ , then the Lemma implies that $W^s(N_C)$ and $W^u(N_C)$ are manifolds. Moreover, all pseudo equilibria in N_C are of the same type. A pseudo-saddle is called transverse if its stable and unstable manifolds are not tangent to the singularity. In this case, the singularity divides a neighborhood of ψ into two parts, each containing one stable and one unstable trajectory. Again this is the most regular structure possible and it will be assumed that it persists on the compo-

nent except for a truly lower dimensional subset. The analogous Lemma for semi-singular points is trivial since convergence occurs in finite time and we only state it for reference for the stable manifold only.

Lemma 2 Let $\xi \in N_\xi$. Then $W^s(\xi)$ is a semi-trajectory of Z^T converging to ξ in finite time. If $\delta(\xi)$ denotes a δ -sized neighborhood of ξ in N_ξ , then $W^s(\delta(\xi))$ is an $(n - 1)$ -dimensional manifold, if it is nonempty. $\square$

The local behavior of the system in a half-neighborhood near a semi-singular point is very much like for a transverse pseudo-saddle, except that the trajectory hits tangentially and in finite time.

In the statements of the results in this section, in addition to assumption (A0), (which, however, can be relaxed to requiring the conditions only at points in the closure of the region of attraction), the following assumptions are made:

- (A1) (a) Equilibria z_0 and periodic orbits γ_0 in the boundary of the region of attraction, ∂A , are hyperbolic.
(b) Except for sets of dimension at most $(n - 3)$, all pseudo-saddles in ∂A are transverse.
- (A2) Stable manifolds from the set $\{W^s(z_0), W^s(\gamma_0), \overline{W^s(N_C)}\}$ intersect transversally with unstable manifolds from the set $\{W^u(z_0), W^u(\gamma_0), W^u(N_C)\}$. Here N_C stands for a component of either N_ψ or N_ξ .
- (A3) All trajectories on the boundary of the region of attraction converge to an equilibrium point, a periodic orbit, a pseudo equilibrium point or a semi-singular point.

It is sufficient, of course, if all these conditions are fulfilled for the objects which lie in the closure of the region of attraction. Assumptions (A1) and (A2) are satisfied generically. In condition (A2) note that the closure of the stable manifold $W^s(N_C)$ for the entire component N_C is considered. It is allowed that $\overline{W^s(N_C)}$ intersects nontransversally with a stable manifold $W^s(z_0)$ of an equilibrium, for instance. But intersections with unstable manifolds must be transversal. Assumption (A3) will be satisfied if there exists an energy function as described in the next Lemma.

Lemma 3 If there exists a Lyapunov-function $V = V(x, y)$ such that

1. $D_x V f - D_y V (D_y g)^{-1} D_x g f \leq 0$ in A
2. V is proper in the closure of A
3. the ω -limit set in $\overline{A}$ consists only of equilibria and periodic orbits

then assumption (A3) is satisfied. All trajectories in ∂A converge to equilibria, periodic orbits, pseudo equilibria or semi-singular points. $\square$

The main result of this section follows. Recall that the order of an equilibrium point z_0 or a periodic orbit γ_0 is the dimension of its unstable subspace and it is denoted by a superscript.

Theorem 6 (Quasi-Stability Boundary Theorem)

Under assumptions (A) the boundary of the closure of the region of attraction $\partial \overline{A}$ (= quasi-stability boundary) is composed of the following four principal elements;

1. stable manifolds of order 1 equilibria
2. stable manifolds of order 2 periodic orbits
3. stable manifolds of pseudo-saddles or semi-singular points
4. singular boundary pieces.

More precisely

$$\partial\bar{A} = \bigcup_{z_0^1 \in \partial\bar{A}} W^s(z_0^1) \cup \bigcup_{\gamma_0^2 \in \partial\bar{A}} W^s(\gamma_0^2) \cup W^s(N_{n-1}) \cup (S \cap \partial\bar{A}) \quad (34)$$

where $N_{n-1} = N_{n-1}^\psi \cup N_{n-1}^\xi$ and

$$N_{n-1}^\psi = \{\psi \in N_\psi \cap \partial\bar{A} : \text{there exists a } \delta > 0 \text{ such that the } \delta\text{-sized neighborhood of } \psi \text{ in } N_\psi, \delta(\psi) \subset \partial\bar{A} \text{ and } n_s = 1\} \quad (35)$$

and

$$N_{n-1}^\xi = \{\xi \in N_\xi \cap \partial\bar{A} : \text{there exists a } \delta > 0 \text{ such that the } \delta\text{-sized neighborhood of } \xi \text{ in } N_\xi, \delta(\xi) \subset \partial\bar{A}\} \quad (36)$$

□

Equilibrium points, periodic orbits, pseudo equilibria and semi-singular points in the boundary of the region of attraction will be called *anchor points*. The following theorem gives necessary and sufficient conditions for the presence of the various types of anchor points.

Theorem 7

- (a) $z_0^1 \in \partial\bar{A} \iff W^u(z_0^1) \cap A \neq \emptyset \text{ and } W^u(z_0^1) \cap \bar{A}^c \neq \emptyset$
 $z_0^1 \in \partial\bar{A} \iff W^s(z_0^1) \subset \partial\bar{A}$
- (b) $\gamma_0^2 \in \partial\bar{A} \iff W^u(\gamma_0^2) \cap A \neq \emptyset \text{ and } W^u(\gamma_0^2) \cap \bar{A}^c \neq \emptyset$
 $z_0^1 \in \partial\bar{A} \iff W^s(\gamma_0^2) \subset \partial\bar{A}$
- (c) $\nu \in N_{n-1}^\nu \iff \text{there exists a } \delta_0 > 0 \text{ such that for every } \delta < \delta_0, W^u(\delta(\nu)) \cap A \text{ is dense in } W^u(\delta(\nu)), \text{ for any } \delta\text{-sized neighborhood, } \delta(\nu) \text{ in } N_\nu.$
 $\nu \in N_{n-1}^\nu \iff W^s(\nu) \subset \partial\bar{A}$

Here ν labels either ψ or ξ . □

The principal structure of the full boundary is characterized next.

Theorem 8 Under assumptions (A) the set

$$\partial A / \bigcup_{z_0^1 \in \partial\bar{A}} W^s(z_0^1) \cup \bigcup_{\gamma_0^2 \in \partial\bar{A}} W^s(\gamma_0^2) \cup W^s(N_{n-1}) \cup (S \cap \partial\bar{A})$$

has maximal dimension $n - 2$ at every point. □

Thus, modulo a set of dimension strictly less than $n - 1$, the boundary ∂A consists of the $(n - 1)$ -dimensional stable manifolds of equilibria, periodic orbits, pseudo-saddles and semi-singular points, and of a singular segment. A complete description of the full boundary is much more difficult because there may exist components of ∂A of dimension strictly less than $n - 1$ which lie in the interior of $\partial\bar{A}$. Moreover, these components may consist only of part of a stable manifold.

The internal structure of the anchor points is summarized next without making all the assumptions (A). To refine these results, much stronger assumptions are necessary, which, how-

ever, would not be practical.

Theorem 9 Suppose all equilibria z, z_0 and periodic orbits γ, γ_0 are hyperbolic and all pseudo equilibria ψ, ψ_0 and semi-singular points ξ, ξ_0 lie in the nice sets. For the equivalences marked with an (*), also assume that assumption (A2) on the transversal intersections of stable and unstable manifolds holds. Furthermore, ν is used to label points in either N_ψ or N_ξ . Then

1. For an equilibrium point z_0

$$\begin{aligned} z \in \partial W^s(z_0) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(z_0)} \neq \emptyset \\ (*) z \in \partial W^s(z_0) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(z_0) \neq \emptyset \\ \gamma \in \partial W^s(z_0) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(z_0)} \neq \emptyset \\ (*) \gamma \in \partial W^s(z_0) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(z_0) \neq \emptyset \\ \nu \in \partial W^s(z_0) &\iff W^u(\nu) \cap \overline{W^s(z_0)} \neq \emptyset. \end{aligned}$$

[If z_0 is stable, no transversality assumptions are required.]

2. For a periodic orbit γ_0

$$\begin{aligned} z \in \partial W^s(\gamma_0) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\ (*) z \in \partial W^s(\gamma_0) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(\gamma_0) \neq \emptyset \\ \gamma \in \partial W^s(\gamma_0) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(\gamma_0)} \neq \emptyset \\ (*) \gamma \in \partial W^s(\gamma_0) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(\gamma_0) \neq \emptyset \\ \nu \in \partial W^s(\gamma_0) &\iff W^u(\nu) \cap \overline{W^s(\gamma_0)} \neq \emptyset. \end{aligned}$$

3. For a component N_C of the nice set, with $\nu \notin \partial N_C$,

$$\begin{aligned} z \in \partial W^s(N_C) &\iff (W^u(z) \setminus \{z\}) \cap \overline{W^s(N_C)} \neq \emptyset \\ (*) z \in \partial W^s(N_C) &\iff (W^s(z) \setminus \{z\}) \cap \partial W^s(N_C) \neq \emptyset \\ \gamma \in \partial W^s(N_C) &\iff (W^u(\gamma) \setminus \{\gamma\}) \cap \overline{W^s(N_C)} \neq \emptyset \\ (*) \gamma \in \partial W^s(N_C) &\iff (W^s(\gamma) \setminus \{\gamma\}) \cap \partial W^s(N_C) \neq \emptyset \\ \nu \in \partial W^s(N_C) &\iff W^u(\nu) \cap \overline{W^s(N_C)} \neq \emptyset. \end{aligned}$$

□

For the power system model, only polynomials and polynomial functions of sinusoidal functions are present. If the angles are restricted to a variation of 2π , it follows from algebraic stratification theory (essentially, from the Tarski-Seidenberg Theorem [20]), that the numbers of equilibria and the numbers of connected components of N and S are finite. Therefore

Corollary 1 Suppose assumption (A) holds and there exist only finitely many periodic orbits. Then for a power system model there are only finitely many terms in the union

$$\partial\bar{A} = \bigcup_{z_0^1 \in \partial\bar{A}} \overline{W^s(z_0^1)} \cup \bigcup_{\gamma_0^2 \in \partial\bar{A}} \overline{W^s(\gamma_0^2)} \cup \overline{W^s(N_{n-1})} \cup (S \cap \partial\bar{A})$$

□

Suppose the system admits an energy function, V with the properties described in Lemma 3 [16, 17, 37].

Definition 2 The subset of the anchor points whose stable manifolds are $(n - 1)$ -dimensional in $\partial\bar{A}$, are the principal anchor points, i.e.

$$\mathcal{P} := \bigcup_{z_0^1 \in \partial\bar{A}} \{z_0^1\} \cup \bigcup_{\gamma_0^2 \in \partial\bar{A}} \gamma_0^2 \cup N_{n-1}$$

□

Then, it follows by Theorem 6 that the composition of the quasistability boundary is

$$\partial\bar{A} = \overline{W^s(\mathcal{P})} \cup (S \cap \partial\bar{A}).$$

The energy V is decreasing along the trajectories. Suppose α and β are anchor points. Then, by continuity, if $\alpha \in \partial W^s(\beta)$, then $V(\alpha) \geq V(\beta)$. By Theorem 6, every anchor point that is not principal, belongs to the boundary of the principal anchor

points or the boundary of their stable manifolds. Hence, the energy function attains its lowest values at the closure of the set of principal anchor points in the quasi-stability boundary.

Definition 3 Given an initial condition z_0 , the critical anchor points are the principal anchor points with energy lower than $V(z_0)$. $\square$

So, $\mathcal{P}$ can be divided into two sets $\mathcal{P}_c$ and $\mathcal{P}_{nc}$ defined as

$$\begin{aligned}\mathcal{P}_c(z_0) &:= \{\alpha \in \mathcal{P} : V(\alpha) \leq V(z_0)\} \\ \mathcal{P}_{nc}(z_0) &:= \{\alpha \in \mathcal{P} : V(\alpha) > V(z_0)\}\end{aligned}$$

The next lemma enables us to consider only the critical anchor points $\mathcal{P}_c$ for stability tests.

Lemma 4 (Critical Anchor Lemma):

Suppose $z_0 \in C$ is an initial condition. Then,

$$z_0 \text{ is inside (outside) } \bar{A}$$

$$\iff$$

z_0 is inside (outside) the portion of the stability boundary defined by the closure of stable manifolds of the critical anchor points. $\square$

Thus, energy function based methods can be constructed for stability monitoring. This is briefly outlined in section VI.2.2.

VI. COMPUTATIONAL ASPECTS

In the previous sections, the feasibility boundary and the stability boundary for a general system have been characterised. These results provide the basis for developing computational algorithms and software. A brief survey of the computational issues will be given in this section.

VI.1. COMPUTATION OF THE FEASIBILITY BOUNDARY

For a system with or without tap-changer control, the feasibility boundary ∂F consists of zero sets (See the Feasibility Boundary Theorem (Theorem 1) and the Feasibility Boundary theorem with Tap-Changer (Theorem 4) in section V.1). Let us denote the zero sets, $(C_S, C_Z, C_H, C_{Z,TC}$ and $C_{H,TC}$) as $\bar{F}_i(z) = 0$, where $\bar{F}$ denotes a set of functions. Each of the zero sets is associated with a specific bifurcation, as shown in the Quasi-feasibility Boundary Theorem (Theorem 3). For instance, the saddle-node bifurcation occurs at the points in the set C_Z . There are some inequality constraints on the zero sets, but these inequality constraints are violated only at the points of intersection, i.e. when a point belongs to two or more different zero sets. A simple method to construct a component of the boundary is sketched next:

1. Find an initial point on any segment of the boundary.
2. Construct the segment by solving the applicable zero set.
3. Switch to a different zero set on the boundary, at the intersection points.

More efficient algorithms can be developed for fast computation of the boundary. The problem seems to be inherently suited

for parallel computation because the zero sets can be computed independently.

Three of the zero sets C_Z, C_S and $C_{Z,TC}$ involve computation of the determinant of a Jacobian. Excellent algorithms have been developed in the literature for finding such points, but only in the context of the saddle-node bifurcation (C_Z). Modification of such approaches should simplify the computation of all three zero sets C_Z, C_S and $C_{Z,TC}$. The remaining two zero sets C_H and $C_{H,TC}$ include the computation of the characteristic polynomial of the Jacobian (for the determinant of the Hurwitz matrix). Fast parallel algorithms have been designed for the computation of the characteristic polynomial. In fact, when a very large number of processors are available, the fastest method to compute the determinant of a matrix, is by finding its characteristic polynomial [23]. More work is needed in this area to develop specific methods which exploit the special structure of power systems.

VI.2 COMPUTATION OF THE STABILITY BOUNDARY

In computing the stability boundary, the objective may be to compute the entire stability boundary or to test the stability status of a specific initial state. Instead of restricting the analysis to the region of attraction A , practically it is more meaningful to consider the bigger region, the interior of the closure, $\text{int}(\bar{A})$. With the small perturbations always present in the system, any trajectory in this region $\text{int}(\bar{A})$, will converge to the stable equilibrium point. The boundary of this region is the quasi stability boundary, $\partial \bar{A}$.

1. For computing the entire quasi-stability boundary $\partial \bar{A}$, all the structural elements of the boundary have been characterised in section V.3. The number of different elements which constitute the boundary is very large for a realistic large system. So, the computational burden may be quite overwhelming.
2. For testing the stability status of a specific initial state z_0 , it is not necessary to compute the full boundary. Using an energy function, it is sufficient to consider only those pieces of the boundary, which are near z_0 . This approach has been demonstrated for systems where the algebraic constraints are absent [42]. It can be generalised to include the new elements in the boundary such as the singular anchor points. This will be briefly outlined in section VI.2.2.

VI.2.1 Computation of the full boundary

The stability boundary consists of regular sections defined by stable manifolds, and singular sections defined as zero sets (See the Quasi-Stability Boundary Theorem, Theorem 6).

(1) *Regular Sections:* The regular section is composed of stable trajectories converging to one of the four types of anchor points identified below. This section is only defined in terms of trajectories, hence, it is not readily accessible to computation. This is a well-recognised difficulty in computing the stability boundaries. Theorem 7 characterizes the principal anchor points among the various unstable equilibrium points, periodic orbits, pseudoequilibrium points and semi-singular points. Hence, a principal anchor point must satisfy the respective conditions listed below:

Equilibrium points and Periodic orbits:

- (a) An equilibrium point must be of order 1 and a periodic orbit must be of order 2.
- (b) The equilibrium point or the periodic orbit must belong to the state space component C .
- (c) An unstable trajectory must converge to the stable equilibrium point z_s .
- (d) An unstable trajectory leaves $\bar{A}$, i.e. ends up in a voltage collapse.

Pseudoequilibrium points and semi-singular points:

- (a) Pseudoequilibrium points must be pseudo saddles, and semi-singular points must be semi-saddles (See section VIII.4).
- (b) Points must be in the boundary, ∂C .
- (c) Most of the unstable trajectories in a neighborhood converge to the stable equilibrium point z_s . So, a typical unstable trajectory in a neighborhood converges to the point z_s .

Once the principal anchor points are identified, the stable manifolds of these points constitute the regular section. For testing the stability status of an initial state, z_0 , we can ignore a large number of such anchor points which are not critical. This is discussed in section VI.2.2.

(2) *Singular Sections:* The singular boundary is a subset of the singular set. The singular set defines the state space component C , that contains the stable equilibrium point z_s . The region of attraction A , and the interior of its closure $\text{int}(\bar{A})$, are subsets of the component C . Hence, the singular section of the boundary is a subset of the boundary ∂C . The set ∂C by definition is a zero set (See section V.2). The computation of the boundary ∂C , is similar to the computation of the feasibility boundary. Moreover, only points where the trajectories emerge from both sides of the singular surface, are relevant for the stability boundary. If the trajectory that leaves a singular point z_s reaches the stable equilibrium point, then z_s is in the singular section of the boundary. But, for a stability test, it is sufficient to consider only the boundary, ∂C . This is shown in the next section.

VI.2.2 Testing the Stability status of a specific initial state:

For testing whether a given initial state z_0 , converges to the stable equilibrium point z_s , we need to verify whether the state z_0 lies inside the quasi-stability boundary $\partial \bar{A}$. Then, the state z_0 must belong to the component C . This can be tested by considering the boundary ∂C . The boundary ∂C consists of a zero set, namely the singularity S . Hence this test is relatively easy. Moreover, an initial state must also be inside the regular section of the boundary $\partial \bar{A}$.

Suppose the system has an energy function V (See Lemma 3). For the regular section of the boundary, Critical Anchor Lemma (Lemma 4) allows us to ignore the principal anchor points which have energy higher than $V(z_0)$. So, for a stability test, we need to consider the stability boundary only for the critical anchor points in the set $\mathcal{P}_c$. In general, This set will be small. A simple test is outlined next for a stability test based on this analysis. It is emphasized that more efficient methods can be developed for a rigorous stability test.

Suppose an initial point, z_0 is given. The following conditions progressively eliminate points which do not lie in $\text{int}(\bar{A})$.

1. the determinant, $\Delta(z_0)$ (defined in section V.2), has the same sign as $\Delta(z_s)$.
2. z_0 lies inside the boundary, ∂C i.e. z_0 and z_s belong to the same component, C .
3. (Find the critical anchor points) z_0 lies inside the stability boundary as defined by the stable manifolds of the critical anchor points.

If a point z_0 satisfies all three conditions, then $z_0 \in \text{int}(\bar{A})$. So these conditions are necessary and sufficient. Efficient methods are needed for steps 2 and 3 to establish whether the state z_0 is inside or outside. For systems with no singularity, results in this direction are presented in [41].

A quick estimate for the size of the region of attraction can be computed from the singularity. Later on, it is shown (Corollaries 5 and 6) that the singular points do not lie in the set $\text{int}(\bar{A})$. Hence, the shortest distance to the singularity provides an upper bound for the shortest distance to the quasi-stability boundary. This estimate is easy to compute and for distances more than this estimate, there is at least one direction (beyond the singularity), where the region of attraction (or even the bigger set $\text{int}(\bar{A})$) ceases to exist.

VII. ILLUSTRATIVE EXAMPLES

The results of this paper are applicable to any size system. They are also complete in the sense of providing a suitable basis for software development. In a complex situation like the present one, however, a large scale or even moderate scale illustrative example would turn out to be a big pile of data, very hard to gain insight from. The aim of these illustrations is to provide insight for interpreting the results. For this end, the examples are simplified to the maximal level which allows graphical presentations (three dimensions) while still possessing a rudimentary physical identity with the power system. It is hoped that the paper will inspire software developments and many practical examples.

It should be mentioned here that a recent paper [36] contains what amounts to such an illustrative example on a rudimentary system. This paper amounts to a pilot study for the current one on a specific rudimentary system. Much illustrative material with expansive intuitive exploration is included so it is recommended here as an extensive exposition for a small scale illustration of these results, rich in insight. These earlier results are expanded here by the addition of a tap changer ahead of the load. An abbreviated version of the earlier example is provided here for illustration.

VII.1 THE RUDIMENTARY SYSTEM

The system (rudimentary in the sense that it contains the four rudiments of the power system i.e. generation, load, transmission and control both for excitation and tap changer) is shown in Figure 6. Since there is only one inertia in Figure 6, by making the assumption $P_L = P_G = P$, the voltage dynamics is isolated from the angle dynamics (i.e. $\dot{\theta} = 0$). So this assumption is the counterpart of the so-called "classical" model

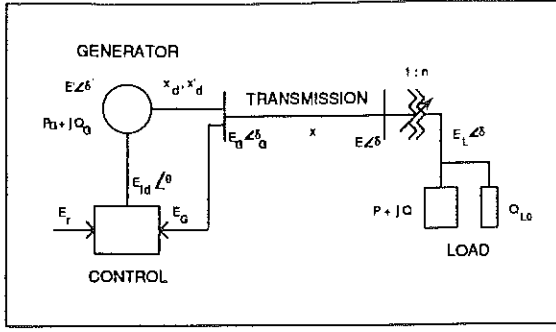


Figure 6. Rudimentary power system model for the example.

which isolates the rotor angle dynamics by assuming constant voltage behind the transient reactance (i.e. $\dot{E}' = 0$). Using a simplified one axis generator model and a simplified first degree control dynamics, this rudimentary case can then be stated as follows:

$$\begin{aligned} \dot{E}' &= \frac{1}{T'_{d0}} \left(-\frac{x_d}{x'_d} E' + \frac{x_d - x'_d}{x'_d} E_G \cos(\delta_G - \delta') + E_{fd} \right) \\ E_{fd} &= \frac{1}{T} \left(-(E_{fd} - E_{fd}^0) - K(E_G - E_r) \right) \\ 0 &= \frac{E' E_G}{x'_d} \sin(\delta_G - \delta') + \frac{E_G E}{x} \sin(\delta_G - \delta) \\ 0 &= \frac{1}{x'_d} (E_G^2 - E_G E' \cos(\delta_G - \delta')) + \\ &\quad \frac{1}{x} (E_G^2 - E_G E \cos(\delta_G - \delta)) \\ 0 &= \frac{E E_G}{x} \sin(\delta - \delta_G) + P \\ 0 &= \frac{1}{x} (E^2 - E_G E \cos(\delta - \delta_G) + Q) \end{aligned}$$

and the tap changer dynamics are defined in the equations (11). Here E_r is the setpoint voltage, E_{fd}^0 is the nominal field excitation, and T, K are control coefficients. The other notations are conventional. Since the angle variables always appear in the form $\delta_G - \delta$ or $\delta_G - \delta'$ in the equations stated above, $E, E_G, \delta_G - \delta$ and $\delta_G - \delta'$ can be eliminated and the equations can be simplified as

$$E_G^2 = \left(\frac{xP}{E} \right)^2 + \left(\frac{xQ + E^2}{E} \right)^2 \quad (37)$$

Further simplification shows that the model can be written as follows:

Dynamic State Equations:

$$\begin{aligned} \dot{E}' &= \frac{1}{T'_{d0}} \left(-\frac{x + x_d}{x'} E'^2 + \frac{x_d - x'_d}{x'} (E^2 + x'Q) + E_{fd} E' \right) \\ \dot{E}_{fd} &= \frac{1}{T} \left(-(E_{fd} - E_{fd}^0) - K(E_G - E_r) \right) \end{aligned} \quad (38)$$

where E_G is defined by equation (37) and $x' = x + x'_d$.

Algebraic State Equation:

$$0 = E'^2 E^2 - (x'P)^2 - (x'Q + E^2)^2 \quad (39)$$

The state space is now three dimensional with variables E, E' and E_{fd} . Accordingly, all manifolds can be directly visualised.

The loadflow manifold, which corresponds to the solution of the algebraic state equation, is two dimensional and is diffeomorphic to the plane. This can be seen by solving equation (39) for E' which gives rise to a global chart. In other words, since E_{fd} is absent in equation (39), the loadflow manifold is a curved wall. All the crosssections, $E_{fd} = \text{const.}$, are identical. Since the state is constrained within the loadflow manifold, all trajectories are displayed in a planar view (a foldout) of the loadflow manifold as shown in Figure 1. All regular points of the loadflow manifold can be calculated by solving equation (39). The singular points where $\frac{\partial E'}{\partial E} = 0$ are defined by

$$E'^2 = 2(x'Q + E^2) \quad (40)$$

It is easily seen that equations (39) and (40) have a unique solution given by the positive square roots of

$$E_{sing}^2 = x' \sqrt{P^2 + Q^2} \text{ and } E_{sing}'^2 = 2x'(Q + \sqrt{P^2 + Q^2})$$

Hence the set of singular points is precisely the line

$$LF_{sing} = \{(E'_{sing}, E_{fd}, E_{sing}) : E_{fd} \in \mathbb{R}_+\}$$

denoted by line a-a in Figure 1. This singular line divides the loadflow manifold into two components LF_{high} and LF_{low} .

$$LF_{high} = \{(E'_{sing}, E_{fd}, E_{sing}) \in LF : E > E_{sing}\}$$

$$LF_{low} = \{(E'_{sing}, E_{fd}, E_{sing}) \in LF : E < E_{sing}\}$$

The singular line a-a splits the dynamics into two planar flows each cut off at line a-a in Figure 1. The trajectories enter or emerge from the singularity at infinite speed in E and at the singular point, the analysis based on the model (38) and (39) simply stops.

The structural features of the phase portrait depend on several parameters which define the parameter space. A distinction has been made between system parameters $P_S = \{x, x_d, x'_d, T'_{d0}\}$ which remain unchanged unless the system itself is changed and the operating parameters $P_O = \{P, Q, E_r, K, T\}$, which may change during system operation. So even in this simple example, the operating parameter space is 6-dimensional. For simplicity, let $E_r = 1$ p.u. at the stable operating point or the loadflow solution x_h . T will be taken to be fixed for this study. The load power factor is assumed to be a constant. This leaves a two dimensional parameter subspace consisting of power P and control gain K representing the most active and hence the most informative subset of the parameter space.

At this point the problem has been reduced to a dynamic state space (E', E_{fd}) and an active parameter space (P, K) which are both two dimensional and can thus be represented in a plane.

VII.2 A BRIEF DESCRIPTION OF THE DYNAMICS

Figure 2 gives the qualitative structure of the bifurcation diagram for parameters P and K , without tap changer control i.e. with the tap changer position $n = 1$. In Figure 7, an actual simulation is given for a specific set of values. At light loads the operating mode is motly represented by parameters in regions 1' and 1'' with two stable equilibria, one to each side of the singular line. The high equilibrium x_h^* changes from a node (in regions 1' and 2') to a focus (in regions 1'' and 2'') along the curve E_{CD} . Since this does not constitute a bifurcation, this

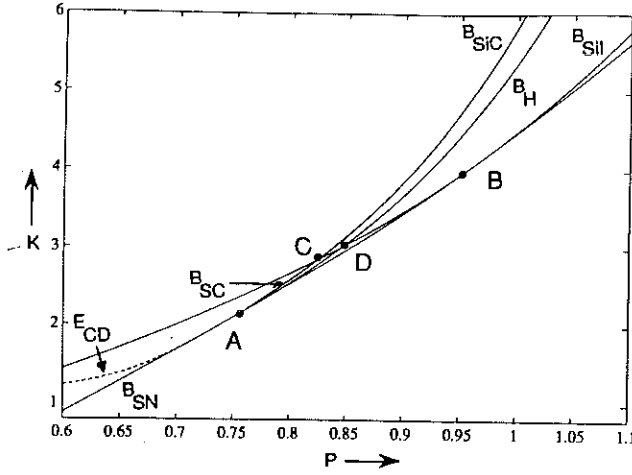


Figure 7. Bifurcation diagram for the parameters: $T'_{d0} = 10$, $T = 1.75$, $x_d = 1.0$, $x'_d = 0.2$, $x = 0.1$, $E_r = 11.0$, $E'_{fd} = 1.6$, $pf = 0.85$ and $Q_{L0} = 0.2$.

line is only dotted in Figure 2. So, the regions 1', 1'' and 2', 2'' constitute the typical regions 1 and 2 respectively.

The feasibility boundary for the high equilibrium x_h is shown in Figure 2 by a heavy line. It consists of a curve $B_{SN} - A - B_H$, connecting the lower half of the saddle node bifurcation curve with the Hopf bifurcation curve at the codimension 2 bifurcation point A. Above and to the left of the feasibility boundary (regions 1, 2, 4 and 6) operation is feasible at the high equilibrium point x_h which is stable here. For the low equilibrium point x_l , the feasibility boundary is actually wider at high control gains than for x_h . It consists of regions 1, 6, 7 and 8 bounded by the singularity induced bifurcation up to point B and beyond point B by the upper half of the saddle node bifurcation. The lower equilibrium is stable throughout this region, but typically is not viable.

The effect of the tap changer control on the feasibility region for the high equilibrium is shown in Figure 4. At low control gains, feasible operation ends at the conventional saddle node bifurcation (B_{SN}). At medium control gains, a tap changer related saddle node bifurcation ($B_{SN,TC}$) acts as the feasibility limit [37]. For high control gains, the Hopf bifurcation (B_H) provides the feasibility boundary.

VIII. PROOFS

In this section the proofs for the results stated in section V will be given. First a singular transformation is introduced that transforms the system into a smooth system and the properties of the transformation are studied. In particular, the singular transformation generates a high dimensional equilibrium surface which will be analysed. After establishing the preliminary lemmas, the main results on the stability boundary will be sketched in this paper, but technical details will only be given in [37].

VIII.1 A SINGULAR TRANSFORMATION

Consider the system defined by equations (28) and (29). The load flow surface L is the constraint surface and S defines the

singularity. The precise definitions of the various surfaces and of the vector fields Z and Z^T have been given in section V.

Lemma 5 *Restricted to $L \setminus S$, the dynamics defined by equations (16) and (17) is locally induced by the vector field Z (i.e. described by an ordinary differential equation).*

PROOF: This follows from the implicit function theorem ($D_y g$ is nonsingular in $L \setminus S$) and the fact that $(D_x g, D_y g)$ defines a normal vector field for the points in $L \setminus S$. $\square$

The vector field Z^T is obtained when the vector field Z is pointwise scaled by the determinant $\Delta(x, y)$. Define

$$(L \setminus S)_+ = \{(x, y) \in L \setminus S : \Delta(x, y) > 0\}$$

$$(L \setminus S)_- = \{(x, y) \in L \setminus S : \Delta(x, y) < 0\}$$

Clearly, $(L \setminus S) = (L \setminus S)_+ \cup (L \setminus S)_-$ as $\Delta(x, y) \neq 0$ in $L \setminus S$.

Lemma 6 *The vector fields Z and Z^T , restricted to $(L \setminus S)_+$, are equivalent. When restricted to $(L \setminus S)_-$, Z and $-Z^T$ are equivalent.*

PROOF: Let t denote the time along integral curves of the vector field Z . Define a new time scale by

$$\frac{d\tau}{dt} = \Delta(x(t), y(t))$$

The rescaled vector field is then Z^T . If $\Delta(x, y) > 0$, the time-orientation is preserved and so Z and Z^T are equivalent on $(L \setminus S)_+$. When $\Delta(x, y) < 0$, as the time orientations are reversed, Z and $-Z^T$ are equivalent. $\square$

In particular, if (x_0, y_0) is an equilibrium point for the vector field Z , then it is also an equilibrium for the new vector field Z^T , and the eigenvalues in the transformed system are simply scaled by $\Delta(x_0, y_0)$ [37]. As curves, the trajectories of Z and Z^T are identical in $L \setminus S$. Whereas Z is not defined on S , Z^T defines a smooth vector field on all of the load flow surface L . It will be shown next that any such trajectory, which approaches the singular surface, has a unique limit and this limit agrees with the limit of the integral curve of the vector field Z^T . Hence Z^T provides a canonical extension for the trajectories of Z . The transformed vector field Z^T has an additional set of equilibrium points in the pseudo equilibrium surface Ψ . The subset M of L is an n -dimensional manifold by implicit function theorem. By definition $L \setminus M \subset S$ and moreover all such points on the load flow surface are even pseudo equilibrium points.

Lemma 7 $L \setminus M \subset \Psi$

PROOF: Let $(x_0, y_0) \in L \setminus M$. Then, at (x_0, y_0) , $g(x_0, y_0) = 0$ and $(D_x g, D_y g)$ is not of full rank. In particular, $D_y g$ has rank less than m . There are two possibilities: (a) If $\text{rank}(D_y g) < m-1$, then $\Delta(x_0, y_0) = 0$ and $\text{adj}(D_y g) = 0$. Therefore $(x_0, y_0) \in \Psi$. (b) If $\text{rank}(D_y g) = m-1$, then necessarily also $\text{rank}(D_x g, D_y g) = m-1$ and so

$$D_x g \in \text{Image}(D_y g). \quad (41)$$

Furthermore, by the definition of the classical adjoint for matrices,

$$\text{adj}(D_y g) D_y g = D_y g \text{adj}(D_y g) = 0$$

i.e.

$$\text{adj}(D_y g) \in \text{kernel}(D_y g) \quad (42)$$

Equations (41) and (42) imply that, at (x_0, y_0) , $\text{adj}(D_y g) D_x g = 0$, i.e. $(x_0, y_0) \in \Psi$. $\square$

Lemma 8 $R = S \setminus (\Psi \cup \Phi)$ is an $(n-1)$ -dimensional embedded submanifold of M and Z^T is transversal to R .

PROOF: Since $D_v g \cdot \kappa = \Delta(D_x g)f$, we have on S that

$$\begin{pmatrix} D_x g & D_v g \\ D_x \Delta & D_v \Delta \end{pmatrix} Z^T = \begin{pmatrix} 0 \\ -D_v \Delta \cdot \kappa \end{pmatrix}. \quad (43)$$

Let $(x_0, y_0) \in R$. As $(x_0, y_0) \notin \Psi$, $\text{rank}(D_v g) = m-1$ and $\text{rank}(D_x g, D_v g) = m$. As also $(x_0, y_0) \notin \Xi$, $D_v \Delta \cdot \kappa \neq 0$ at (x_0, y_0) and so $D_v \Delta$ does not lie in the row span of $D_v g$. But then the Jacobian

$$\begin{pmatrix} D_x g & D_v g \\ D_x \Delta & D_v \Delta \end{pmatrix}$$

has maximal rank $(=m+1)$ everywhere on R . By the implicit function theorem, R is therefore an $(n-1)$ -dimensional manifold. Equation (43) also shows that Z^T is transversal to R . $\square$

For the vector field Z any trajectory that approaches the singular surface is only defined up to the singularity. As has been explained in section IV.1, its behavior after reaching the singularity is unpredictable. But any such trajectory has a well-defined limit on the singularity.

Lemma 9 Any integral curve of the vector field Z which approaches the singular surface has a unique limit on the singularity and this limit agrees with the limit of the solution curve in the transformed system.

PROOF: This is an immediate consequence of the fact that Z^T defines a smooth system on the load flow surface. $\square$

Lemma 9 says that Z^T provides a canonical extension of the vector field Z to the singularity. It should be noted that the orientation of the trajectories is reversed for the points in $(L \setminus S)_-$, as $\Delta(x, y) < 0$. Since the solution curves are identical, from here onwards all the proofs will be done for the transformed system and will be interpreted for the original system. The first application is the proof of the singularity induced bifurcation theorem.

VIII.2 THE PROOF OF THE SINGULARITY INDUCED BIFURCATION THEOREM

The proof uses center manifold theory [11]. First, the existence of the equilibrium locus will be shown. Then, the change in the eigenstructure will be proved in the transformed system.

Applying the implicit function theorem, by Assumption (SI2) there exists a unique equilibrium locus $(x(\mu), y(\mu), \mu)$ near $\mu = \mu_0$. As the functions f and g are smooth, this solution is also smooth. Assumption (SI3') implies that the value of the determinant $\Delta(x(\mu), y(\mu), \mu)$ vanishes only at $\mu = \mu_0$ along the equilibrium locus near $\mu = \mu_0$. The solution locus crosses the singular surface transversally because the determinant $\Delta(x(\mu), y(\mu), \mu)$ is dominated by the linear term. As the equilibrium locus stays away from the singular surface, for $\mu \neq \mu_0$, the eigenvalues of Z are simply scaled by $\Delta(x, y, \mu)$ to yield the eigenvalues of Z^T . But $\Delta(x(\mu), y(\mu), \mu)$ changes sign at $\mu = \mu_0$. It will be shown that $n-1$ eigenvalues of the Jacobian of the vector field Z^T change sign along $(x(\mu), y(\mu), \mu)$ at $\mu = \mu_0$. Scaling back by $\Delta(x, y, \mu)$ to the original system, this proves the result.

By (SI1), $D_v g$ has rank $m-1$ at $(0, 0, \mu_0)$, and by (SI2), $(D_x g, D_v g)$ has full rank m . Hence we can split the coordinates

x and y into $\{x_1, x_2\}$ and $\{y_1, y_2\}$, so that $(D_{x_2} g, D_{y_2} g)$ is nonsingular at $(0, 0, \mu_0)$. Without loss of generality, assume that

$$x_2 = \{x_1\}, \quad x_1 = \{x_2, \dots, x_n\}, \quad y_1 = \{y_1\}, \quad y_2 = \{y_2, \dots, y_m\}.$$

Then, by the implicit function theorem, the equations $g(x, y, \mu) = 0$ can be locally solved as

$$x_2 = x_2(x_1, y_1, \mu), \quad y_2 = y_2(x_1, y_1, \mu).$$

Using this representation, the transformed system can be reduced to the chart coordinates (x_1, y_1) . This gives the following ordinary differential equation:

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, y_1, \mu) \Delta_1(x_1, y_1, \mu) \\ \dot{y}_1 &= g_1(x_1, y_1, \mu) \\ \dot{\mu} &= 0 \end{aligned}$$

where

$$\begin{aligned} f_1(x_1, y_1, \mu) &= (0, I_{n-1}) f(x_1, x_2(x_1, y_1, \mu), y_1, y_2(x_1, y_1, \mu), \mu) \\ g_1(x_1, y_1, \mu) &= (1, 0_{m-1}) \\ &\quad (\text{adj}(D_v g) D_x g f)(x_1, x_2(x_1, y_1, \mu), y_1, y_2(x_1, y_1, \mu), \mu), \\ \Delta_1(x_1, y_1, \mu) &= \Delta(x_1, x_2(x_1, y_1, \mu), y_1, y_2(x_1, y_1, \mu), \mu) \end{aligned}$$

The equation $\dot{\mu} = 0$ is carried along to apply the center manifold theorem. Linearising at $(0, 0, \mu_0)$, the Jacobian is of the form

$$\begin{pmatrix} 0 & 0 & 0 \\ D_{x_1} g_1 & D_{y_1} g_1 & D_{\mu} g_1 \\ 0 & 0 & 0 \end{pmatrix} \quad (44)$$

Define $b_1 = D_{x_1} g_1$, $b = D_{y_1} g_1$ and $b_2 = D_{\mu} g_1$, all evaluated at $(0, 0, \mu_0)$.

Lemma 10 Assumptions (SI1), (SI2), and (SI3') guarantee that b is nonzero.

The proof of this lemma is given below after the conclusion of the main argument.

Let us remove the off-diagonal entries in the Jacobian, defined above, with the linear transformation,

$$\begin{aligned} \tilde{x} &= x_1 \\ \tilde{y} &= y_1 + (b_1/b)x_1 + (b_2/b)\mu \end{aligned}$$

and let the new system be represented by

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{f}(\tilde{x}, \tilde{y}, \mu) \tilde{\Delta}(\tilde{x}, \tilde{y}, \mu) \\ \dot{\tilde{y}} &= \tilde{g}(\tilde{x}, \tilde{y}, \mu) \\ \dot{\mu} &= 0 \end{aligned}$$

Applying the center manifold theorem, there exists a center manifold which is a graph $\tilde{y} = \pi(\tilde{x}, \mu)$. To simplify the eigenvalue computations, let us switch to center manifold coordinates by introducing $\bar{y} = \tilde{y} - \pi(\tilde{x}, \mu)$ and $\bar{x} = \tilde{x}$. In these coordinates the system is given by

$$\begin{aligned} \dot{\bar{x}} &= \bar{f}(\bar{x}, \bar{y}, \mu) \bar{\Delta}(\bar{x}, \bar{y}, \mu) \\ \dot{\bar{y}} &= \bar{g}(\bar{x}, \bar{y}, \mu) \\ \dot{\mu} &= 0 \end{aligned}$$

with $\bar{g}(\bar{x}, 0, \mu) = 0$, due to the tangentiality of the center manifold. Since there is a unique equilibrium locus near $\mu = \mu_0$, $(\bar{x}(\mu), \bar{y}(\mu), \mu)$, for μ near μ_0 this equilibrium locus must lie on the center manifold, i.e. $\bar{y}(\mu) = 0$. It is clear from the Jacobian, defined in (44), that the transformed system has one nonzero

eigenvalue, b , at $(0, 0, \mu_0)$. To study the behavior of the other eigenvalues, let us restrict the analysis to the center manifold, $\bar{y} = 0$. Then the equations reduce to

$$\dot{\bar{x}} = \bar{f}(\bar{x}, 0, \mu) \bar{\Delta}(\bar{x}, 0, \mu) := \hat{f}(\hat{x}, \mu) \hat{\Delta}(\hat{x}, \mu)$$

Expanding $\hat{x}$ and $\hat{\Delta}$ around $(0, \mu_0)$, we get

$$\begin{aligned} \hat{f}(\bar{x}, \mu) &= C_1 \bar{x} + C_2 (\mu - \mu_0) + r_1 \\ \hat{\Delta}(\bar{x}, \mu) &= C_3 \bar{x} + C_4 (\mu - \mu_0) + r_2 \end{aligned}$$

where r_1 and r_2 stand for higher order terms.

Lemma 11 *Assumptions (SI1), (SI2) and (SI3'), imply that C_1 is nonsingular and that $C_4 - C_3(C_1)^{-1}C_2 = c$. (The constant c has been defined in equation (21)).*

The proof is postponed briefly till the proof of the theorem has been given. Lemma 11 allows us to conclude that

$$\bar{x}(\mu) = -(C_1)^{-1}C_2(\mu - \mu_0) + \mathcal{O}(\mu - \mu_0)^2.$$

The Jacobian along the equilibrium locus, $(\bar{x}, 0, \mu)$, has the form,

$$\begin{pmatrix} cC_1(\mu - \mu_0) + \mathcal{O}(\mu - \mu_0)^2 & * \\ 0 & b + \mathcal{O}(\mu - \mu_0) \end{pmatrix}$$

As b is nonzero and cC_1 is nonsingular, the eigenvalues of the transformed system, along the equilibrium locus, stay close to b and the eigenvalues of $cC_1(\mu - \mu_0)$ for small $\mu - \mu_0$. Hence, for the transformed system, $(n-1)$ eigenvalues cross over from $\mathbb{C}^+$ to $\mathbb{C}^-$ or from $\mathbb{C}^-$ to $\mathbb{C}^+$. One eigenvalue stays in the same region (close to b). When transformed back to the original system, the eigenvalues have to be divided by $\bar{\Delta}(\bar{x}(\mu), 0, \mu)$, so that the eigenvalues of the system are close to those of the matrix,

$$\begin{pmatrix} C_1 + \mathcal{O}(\mu - \mu_0) & * \\ 0 & \frac{b}{c(\mu - \mu_0)} + \mathcal{O}(\mu - \mu_0) \end{pmatrix}$$

Thus, one eigenvalue changes sign by diverging to infinity, while the other eigenvalues remain bounded and stay close to those of the matrix C_1 . For a given system, the constants b , c and C_1 can be computed from the definitions. After simplifications, the formulae for these constants are as follows:

$$\begin{aligned} b &= (1, 0_{n-1})(D_{y_1}f - D_1 D_{y_1}g) \\ b_1 &= (1, 0_{n-1})(D_{x_1}f - D_1 D_{x_1}g) \\ b_2 &= (1, 0_{n-1})(D_{\mu}f - D_1 D_{\mu}g) \\ C_1 &= (0, I_{n-1})[(D_{x_1}f - D_1 D_{y_1}g) - (D_{y_1}f - D_1 D_{y_1}g)b_1/b] \\ C_2 &= (0, I_{n-1})[D_{\mu}f - (D_{y_1}f - D_1 D_{y_1}g)b_1/b] \\ C_3 &= (D_{x_1}\Delta - D_2 D_{x_1}g) - (D_{y_1}\Delta - D_2 D_{y_1}g)b_1/b \\ C_4 &= (D_{\mu}\Delta - D_2 D_{\mu}g) - (D_{y_1}\Delta - D_2 D_{y_1}g)b_2/b \\ c &= C_4 - C_2(C_1)^{-1}C_3 \end{aligned}$$

where

$$\begin{aligned} D_1 &= (D_{x_2}f, D_{y_2}f)(D_{x_2}g, D_{y_2}g)^{-1} \\ D_2 &= (D_{x_2}\Delta, D_{y_2}\Delta)(D_{x_2}g, D_{y_2}g)^{-1} \end{aligned}$$

(All the Jacobians are evaluated at $(0, 0, \mu_0)$). $\square$

Now Lemmas 10 and 11 will be proven. In the center manifold coordinates, the equilibrium locus near $\mu = \mu_0$ is described by $(\bar{x}(\mu), 0, \mu)$. It can be seen that

$$\bar{\Delta}(\bar{x}(\mu), 0, \mu) = (C_4 - C_2(C_1)^{-1}C_3)(\mu - \mu_0) + \mathcal{O}(\mu - \mu_0)^2.$$

As $\bar{\Delta}$ defines the determinant Δ in the new coordinates, by definition of c it follows that $c = (C_4 - C_2(C_1)^{-1}C_3)$. All the following Jacobians are evaluated at $(0, 0, \mu_0)$. (SI1) and (SI2) imply that the Jacobian,

$$J_1 := (D_{x_1}f, D_{y_1}f) - (D_{x_2}f, D_{y_2}f)(D_{x_2}g, D_{y_2}g)^{-1}(D_{x_1}g, D_{y_1}g),$$

is nonsingular. Construct a matrix T as follows: The first row of T is the first row of the matrix $\text{adj}(D_{y_1}g)D_{x_1}g$, and the remaining rows are taken as the last $n-1$ rows of the identity matrix I_n . So

$$T := \begin{pmatrix} T_{11} & * \\ 0 & I_{n-1} \end{pmatrix},$$

where

$$T_{11} := (1, 0_{n-1})\text{adj}(D_{y_1}g)D_{x_1}g(1, 0_{n-1})^T \neq 0$$

because

$$\begin{aligned} (1, 0_{n-1})\text{adj}(D_{y_1}g) &\neq 0, & (\text{by the choice of } y_1), \\ (1, 0_{n-1})\text{adj}(D_{y_1}g) &\in \ker(D_{y_1}g), & (\text{by (SI1)}), \\ \text{and } D_{x_1}g &\notin \text{Im}(D_{y_1}g), & (\text{by the choice of } x_1). \end{aligned}$$

It is easy to show that,

$$J_2 := \begin{pmatrix} D_{x_1}f_1 & D_{y_1}f_1 \\ D_{x_1}g_1 & D_{y_1}g_1 \end{pmatrix} = T J_1.$$

Hence J_2 is nonsingular. Carrying out the transformations from the chart coordinates (x_1, y_1, μ) to the center manifold coordinates $(\bar{x}, \bar{y}, \mu)$, it can be shown that

$$J_3 := \begin{pmatrix} D_{\bar{x}}\bar{f} & D_{\bar{y}}\bar{f} \\ D_{\bar{x}}\bar{g} & D_{\bar{y}}\bar{g} \end{pmatrix} = \begin{pmatrix} C_1 & * \\ 0 & b \end{pmatrix},$$

and that J_3 is similar to J_2 . Hence J_3 is nonsingular, which proves that b is nonzero and C_1 is nonsingular. $\square$

VIII.3 PROOF OF THE FEASIBILITY BOUNDARY THEOREM

Points in OP satisfy the equality conditions (a) $f(x, y, p) = 0$ and (b) $g(x, y, p) = 0$, and the inequality constraints (c) $D_{y_1}g$ is nonsingular, and (d) $J = D_x f - D_y f(D_{y_1}g)^{-1}D_{x_1}g$ has all eigenvalues in the open left half-plane. The points in EQ , the set of all equilibrium points, are given by the solutions of the two equations (a) and (b). Hence the boundary of OP in EQ consists of the points where the inequality constraints (c) and (d) are violated. This is also true for a specific component F of OP . Points where $D_{y_1}g$ becomes singular belong to the set C_5 . Condition (d) can be violated either by zero eigenvalues or purely imaginary, but nonzero eigenvalues. Zero eigenvalues occur in the set C_2 . It follows from classical control theory, that the Hurwitz determinant, $H_{n-1}(J)$, vanishes at boundary points of F where the Jacobians $D_{y_1}g$ and J are nonsingular, and J has purely imaginary eigenvalues [47]. Also, if $H_{n-1}(J) = 0$ at some point in ∂F , then, as there can be no eigenvalues of J in $\mathbb{C}^+$, J must have eigenvalues on the imaginary axis. (The presence of eigenvalues which are symmetric about the imaginary axis, not on the imaginary axis, is ruled out since J was stable). By construction, for the points in C_H , J has no zero eigenvalues. Hence the points in $\partial F \cap C_H$ correspond to purely imaginary eigenvalues of J .

We now start to derive the results on the (quasi-) stability boundary. Typically it contains singular segments. A special role is played by the pseudo equilibria and semi-singular points in the closure of the region of attraction. Stable manifolds in the quasi-stability boundary which connect with the singular set are anchored there. In this section a preliminary discussion of semi-singular points is given. The next section contains the analogous results for pseudo equilibria. The main theorems stated in section V.3 are then proved in section VIII.6.

The semi-singular surface Ξ can be divided into a nice set N_ξ and a bad set B_ξ as defined in section V.3.

Lemma 12 *If nonempty, then N_ξ is an $(n-2)$ -dimensional embedded submanifold of M . The vector field Z^T has a nonzero normal component at every point in N_ξ .*

PROOF: For every point in N_ξ there exists a neighborhood so that N_ξ is given as the zeroset of the functions

$$g(x, y) = 0, \Delta(x, y) = 0, D_v \Delta(x, y) \kappa(x, y) = 0$$

with extra inequality conditions imposed. Since $D_v \{D_v \Delta \cdot \kappa\} \neq 0$, it follows that the gradients of these functions are linearly independent on N_ξ and that the normal component of Z^T does not vanish. The result therefore follows from the implicit function theorem. $\square$

The points in the nice set anchor the stability boundary much like equilibrium points and periodic orbits for a Morse-Smale system. Therefore a detailed analysis of the local picture near these points becomes necessary. Let $\xi \in N_\xi$. Since $\xi \notin \Psi$, $\text{rank}(D_v g) = m-1$ and $\text{rank}(D_x g, D_v g) = m$. It follows from the proof of the singularity induced bifurcation theorem, that at $\xi \in M$ local coordinates can be chosen so that the system is described by the ordinary differential equation

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, y_1, \mu) \Delta_1(x_1, y_1, \mu) \\ \dot{y}_1 &= g_1(x_1, y_1, \mu) \\ \dot{\mu} &= 0 \end{aligned}$$

where x_1 is $(n-1)$ -dimensional and y_1 is scalar. Furthermore, Δ_1 denotes the determinant Δ in the new coordinates and g_1 is a component of κ . Choose new coordinates $(\eta_1, \dots, \eta_n)$ which are centered at ξ (i.e. $\eta_i(\xi) = 0$ for all i), as follows:

$$\eta_1 = \Delta_1(x, y), \eta_2 = D_v \Delta(x, y) \kappa(x, y)$$

and complete the coordinate chart with an $(n-2)$ vector η_r from among the components of x_1 to get a diffeomorphism. This can be done since the Jacobian

$$\begin{pmatrix} D_x g & D_v g \\ D_x \Delta & D_v \Delta \\ D_x(D_v \Delta \cdot \kappa) & D_v(D_v \Delta \cdot \kappa) \end{pmatrix}$$

has full rank at ξ . In these coordinates the system can be described as

$$\begin{aligned} \dot{\eta}_1 &= \beta_1(\eta_1, \eta_2, \eta_r), \\ \dot{\eta}_2 &= \beta_2(\eta_1, \eta_2, \eta_r), \\ \dot{\eta}_r &= \beta_r(\eta_1, \eta_2, \eta_r). \end{aligned}$$

The singular surface S is locally given as $\{\eta_1 = 0\}$ and N_ξ is the coordinate plane $\{\eta_1 = 0, \eta_2 = 0\}$. Furthermore, by the

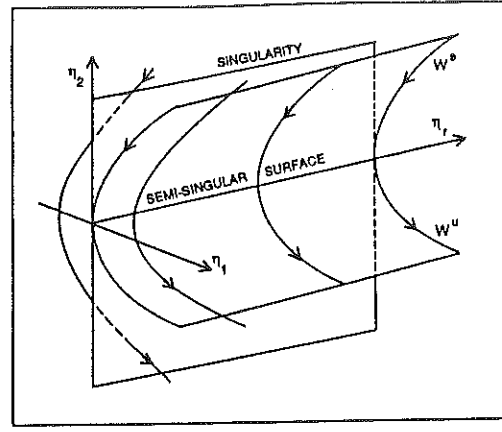


Figure 8. Three dimensional rendering of the local picture near the nice semi-singular surface.

definition of N_ξ ,

$$\begin{aligned} \beta_1(0, 0, \eta_r) &= (D_{v_1} \Delta) g_1|_{N_\xi} = 0, \\ \beta_2(0, 0, \eta_r) &= D_{v_1} (D_v \Delta \cdot \kappa) g_1|_{N_\xi} \neq 0. \end{aligned}$$

There is no equilibrium point or periodic orbit near ξ . Hence the local picture is as shown in Figure 8. It is clear that the flow is tangent to the singularity at any point $\xi \in N_\xi$. But due to our assumption on the second derivative,

$$\begin{aligned} \ddot{\eta}_1 &= \frac{d}{dt} \Big|_{(0,0,\eta_r)} \beta_1(\eta_1, \eta_2, \eta_r) \\ &= (D_{v_1} (D_v \Delta \cdot \kappa) g_1)|_{N_\xi} \neq 0. \end{aligned}$$

Depending on the sign of this term, the nice semi-singular points can be split into

$$N_{se,sa} = \{(x, y) \in N_\xi : D_v (D_v \Delta \cdot \kappa) \kappa > 0\}$$

and

$$N_{se,fo} = \{(x, y) \in N_\xi : D_v (D_v \Delta \cdot \kappa) \kappa < 0\}.$$

It will be shown in the next section that no semi-singular point can lie in the interior of the closure $\bar{A}$ of the region A of attraction. Hence, if U is a local neighborhood of $\xi \in N_\xi$ in M , then only one half of it, namely the intersection U_C of U with the connected component C which contains A , can possibly lie in A . Our analysis stops at the singularity. Note that $\Delta(x, y) > 0$ in C . If $\xi \in N_{se,sa}$, then

$$\frac{d}{dt} \Big|_{\xi} \Delta(x, y) = 0 \text{ and } \frac{d^2}{dt^2} \Big|_{\xi} \Delta(x, y) > 0.$$

In C there exists a unique trajectory which reaches ξ in finite time and immediately leaves the singularity again to the same side. The local picture is as summarized in Figure 9. The phase-portrait resembles the phaseportrait near a continuum of saddlepoints along the η_r axis, restricted to a half-neighborhood. Hence such a point will be called a semi-saddle.

Lemma 13 $W^s(N_{se,sa})$ and $W^u(N_{se,sa})$ are $(n-1)$ -dimensional manifolds.

PROOF: This simply follows from the fact that the flow of Z^T is well-defined and a diffeomorphism near N_ξ . $\square$

For points in $N_{sc,fo}$ we have

$$\frac{d}{dt}|_{\xi} \Delta(x, y) = 0, \text{ but } \frac{d^2}{dt^2}|_{\xi} \Delta(x, y) < 0.$$

Hence there is no trajectory in C which passes through ξ . In particular, $W^s(N_{sc,fo})$ and $W^u(N_{sc,fo})$ are empty. The local phase portrait in C near such a point is like for a continuum of foci along the η_r axis, cut off by the singularity. Hence we call these points semi-foci. Because the intersections of the stable and unstable manifolds with C are empty, semi-foci have no significance in stability boundary calculations. Moreover, as all trajectories in the neighborhood U_C converge to the singularity, $N_{sc,fo} \cap \partial A = \emptyset$.

Eventually we will need necessary and sufficient conditions for a semi-singular point to lie in the boundary of the region of attraction. The following lemmas are instrumental in deriving these results later. It is necessary to restrict the results to the half-neighborhood U_C , which is further divided into three open regions by the local stable and unstable manifolds of N_ξ . This is illustrated in Figure 9. Let U_1 be the region between S and $W^s(N_\xi)$, U_2 the region between $W^s(N_\xi)$ and $W^u(N_\xi)$, and let U_3 be the remaining part. Define a fundamental domain of a stable manifold of $W^s(\xi)$ as a domain G_s such that there exists t_0 with the property that

$$\bigcup_{t < t_0} \Phi_t^T(G_s) = W^s(\xi).$$

Define a fundamental neighborhood as a neighborhood of the fundamental domain that stays away from the unstable manifold. Since the flow of a smooth vector field depends continuously on initial conditions, the following lemmas are easily seen to be correct. Similar results hold for the unstable manifold.

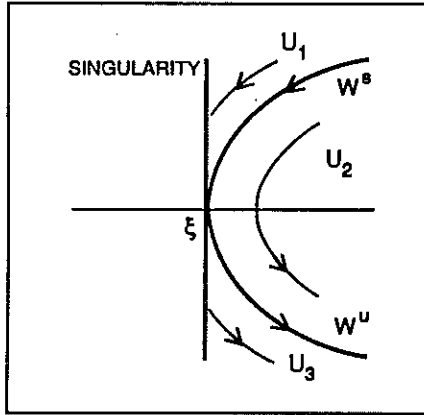


Figure 9. Two dimensional rendering of the half neighborhood around the semi-saddle surface. $(n-2)$ dimensional semi-saddle surface is represented by the point ξ .

Lemma 14 Let $\xi \in N_{sc,sa}$ and let $\delta(\xi)$ be a δ -sized neighborhood of ξ in $N_{sc,sa}$. Consider an $(n-1)$ -dimensional disk D transversal to $W^s(\xi)$ at $q \in W^s(\xi)$. Then there exists a $\delta > 0$ such that for sufficiently large t_0 the image of the disk under the flow of Z^T , $\Phi_{t_0}^T(D)$, comes arbitrarily close to any point in $W^s(\delta(\xi))$.

Define the stable manifold $W^{sc}(\xi)$ of a neighborhood of $\xi \in N_{sc,sa}$, as $W^{sc}(\xi) = W^s(\delta(\xi))$ for some $\delta > 0$. Similarly, define $W^{uc}(\xi)$ as $W^u(\delta(\xi))$ for some $\delta > 0$.

Corollary 2 Suppose $\xi \in N_{sc,sa}$. Let D_u be a 1-dimensional disk in $W^u(\xi)$ and let D be another 1-dimensional disk transversal to $W^{sc}(\xi)$ at $q \in W^s(\xi)$. Then the image of the disk, $\Phi_t^T(D)$, comes arbitrarily close to D_u . Moreover

$$D_u \subset \bigcup_{t > 0} \Phi_t^T(D).$$

Corollary 3 Let $\xi \in N_{sc,sa}$ and let $FN^s(\xi)$ be a fundamental neighborhood of the stable manifold of ξ in C . Then there exists a neighborhood U and a $\delta > 0$ such that

$$\bigcup_{t > 0} \Phi_t^T(FN^s(\xi)) \supset U_C \setminus (W^u(\delta(\xi)) \cup U_3).$$

Here U_C and U_3 are as shown in Figure 9.

VIII.5 ANALYSIS OF THE PSEUDO EQUILIBRIUM SURFACE

The pseudo equilibrium surface Ψ can be split into a nice set N_ψ and a bad set B_ψ as defined in equations (30) and (31). The properties of the set N_ψ will be explored in this section.

Lemma 15 If nonempty, then N_ψ is an $(n-2)$ -dimensional embedded submanifold of M .

PROOF: The proof is similar to the proof of Lemma 12. For every point in N_ψ , we can find a suitably small neighborhood so that N_ψ can be locally described as the solution of $m+2$ equations,

$$g(x, y) = 0, \Delta(x, y) = 0, \text{ and } \kappa_1(x, y) = 0,$$

for some component $\kappa_1(x, y)$ of κ , and with extra inequality conditions imposed in terms of the eigenvalues. These conditions imply that the Jacobian of these functions is of full rank. So N_ψ is an $(n-2)$ -dimensional manifold by the implicit function theorem. $\square$

Next the dynamic behavior for the vector field Z^T is analysed near the nice pseudo equilibrium points.

Lemma 16 For every point $\psi \in N_\psi$, there exists a neighborhood U of ψ in M , and a coordinate chart $(\eta_1, \dots, \eta_n)$ defined on U , such that

(o) the chart is centered at ψ , i.e. $\eta_i(\psi) = 0$ for all i

(i) the dynamics take the form

$$\dot{\eta}_i = f_i^1 \eta_1 + f_i^2 \eta_2$$

for some functions $f_i^k : \eta(U) \subset \mathbb{R}^n \rightarrow \mathbb{R}$, $k = 1, 2$ and $i = 1, \dots, n$

(ii) $S \cap U = \{\eta_1 = 0\}$.

PROOF: By definition of N_ψ , $\text{rank } D_\psi g = m-1$ and $\text{rank } (D_\psi g, D_\psi \Delta) = m$. Hence, as in the proof of the singularity induced bifurcation theorem, the flow of Z^T is diffeomorphic to the solutions of ordinary differential equations of the form

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, y_1) \Delta_1(x_1, y_1), \\ \dot{y}_1 &= g_1(x_1, y_1) \end{aligned}$$

where $\dim x_1 = n-1$ and y_1 is scalar. Furthermore, Δ_1 describes Δ in the new coordinates and g_1 is an appropriate component of κ . Because of the condition on the eigenvalues, the Jacobian

$$J_N := \begin{pmatrix} D_{x_1} \Delta_1 & D_{y_1} \Delta_1 \\ D_{x_1} g_1 & D_{y_1} g_1 \end{pmatrix}$$

has rank 2 at $(0, 0)$. The lemma follows by choosing as coordinates

$$\eta_1 = \Delta_1(x_1, y_1), \text{ and } \eta_2 = g_1(x_1, y_1)$$

and then completing the basis from the components of x_1 . The special form of the dynamics is a consequence of the form of Z^T . $\square$

In these coordinates, the origin is the pseudo equilibrium and the Jacobian of the vector field Z^T has $n - 2$ zero eigenvalues and 2 nonzero eigenvalues. Hence, the equilibrium points in the set N_ψ are nonhyperbolic. But they are special in the following sense:

Lemma 17 *The local center manifold for any point in the nice set is N_ψ itself.*

PROOF: Consider any point $\psi \in N_\psi$. Clearly N_ψ is a center manifold because N_ψ is invariant and satisfies the tangentiality conditions. Suppose CM is some other center manifold. Then it must contain equilibrium points that lie arbitrarily close to ψ . Hence N_ψ belongs to CM , locally. But N_ψ itself is a $(n - 2)$ -dimensional surface. Thus, locally, it is the center manifold. $\square$

The set N_ψ can be split into three different subsets, depending on the stability status. Define

$$\begin{aligned} N_{so} &:= \{\psi \in N_\psi : \text{the Jacobian of } Z^T \\ &\quad \text{has two eigenvalues in } \mathbb{C}^+\} \\ N_{sa} &:= \{\psi \in N : \text{the Jacobian of } Z^T \text{ has one} \\ &\quad \text{eigenvalue in } \mathbb{C}^- \text{ and one in } \mathbb{C}^+\} \\ N_{si} &:= \{\psi \in N : \text{the Jacobian of } Z^T \\ &\quad \text{has two eigenvalues in } \mathbb{C}^-\} \end{aligned}$$

So $N_\psi = N_{so} \cup N_{sa} \cup N_{si}$. Note that any component of N_ψ entirely lies in one of the three sets. Let us call the points in N_{so} , N_{sa} and N_{si} pseudo-sources, pseudo-saddles and pseudo-sinks, respectively. This definition is motivated by the next lemma.

Lemma 18 *For $\psi \in N_\psi$, the flow Φ_t^T is locally equivalent to a linear system defined as:*

$$\dot{x}_u = x_u, \quad \dot{x}_s = -x_s, \quad \dot{x}_c = 0, \quad x_c \in \mathbb{R}^{n-2}.$$

The dimensions of x_u and x_s depend on the particular set in N , i.e.

$$\begin{aligned} \dim(x_u) &= 2 \text{ and } \dim(x_s) = 0, \text{ when } \psi \in N_{so} \\ \dim(x_u) &= 1 \text{ and } \dim(x_s) = 1, \text{ when } \psi \in N_{sa} \\ \dim(x_u) &= 0 \text{ and } \dim(x_s) = 2, \text{ when } \psi \in N_{si} \end{aligned}$$

PROOF: It is known that for nonhyperbolic equilibrium points, the flow can be classified depending on the flow on a center manifold [3]. For $\psi \in N_\psi$, the flow on the center manifold ($= N_\psi$) is trivially the identity map (since $\dot{x}_c = 0$). Hence the result follows. $\square$

The last lemma states that the flow near a nice pseudo equilibrium point looks like that of a linear system. Together with Lemma 15 this proves part 3 of Theorem 5. The remaining statements in Theorem 5 have already been verified (part 1 is Lemma 5, part 2 is Lemma 8 and part 4 is Lemma 12). For hyperbolic equilibrium points, stronger properties are true, for instance, the flow is locally *conjugate* to the flow of a linear system (Hartman-Grobman theorem) [14]. But equivalence is sufficient for our proofs because we are mainly interested in the features of the phase portraits, such as the boundary of the region of attraction. Hence for points in N_ψ there exists a well-defined

local "picture" from a dynamical system point-of-view.

Corollary 4 *$W^s(N_\psi)$ and $W^u(N_\psi)$ are manifolds,*

$$\begin{aligned} \dim(W^s(N_{so})) &= n - 2, & \dim(W^u(N_{so})) &= n, \\ \dim(W^s(N_{sa})) &= n - 1, & \dim(W^u(N_{sa})) &= n - 1, \\ \dim(W^s(N_{si})) &= n, & \dim(W^u(N_{si})) &= n - 2. \end{aligned}$$

PROOF: Locally, near any point $\psi \in N_\psi$, the result follows from Lemma 18. For a point $z \in W^s(\psi)$ (or $W^u(\psi)$), a neighborhood of z in $W^s(\psi)$ can be described as the flow in finite time of a local neighborhood near ψ . As the flow of the smooth vector field Z^T defines a diffeomorphism, the proof is complete. $\square$

It is clear from the definition that the region of attraction of a stable equilibrium point, z_s , is a subset of the connected component C of $L \setminus S$ which contains z_s . Hence we restrict the analysis to this component C for the stability boundary proofs. As will be shown now, it follows from the form of the local coordinates near nice singular points that *any singular point in the stability boundary ∂A will belong to the quasi-stability boundary $\partial \bar{A}$* . Hence, for any pseudo equilibrium point, only a part of the local neighborhood will be in $\bar{A}$.

Lemma 19 *$R \cap \text{int}(\bar{A}) = \emptyset$*

PROOF: Suppose $z \in R$ and take a neighborhood of z . By Lemma 8, the trajectories of Z^T cross R transversally near z and the determinant $\Delta(x, y)$ changes sign across R . Then the part of the neighborhood where Δ has different sign cannot be in C , hence also not in A . So $z \notin \text{int}(\bar{A})$. $\square$

Corollary 5 *If the nice sets N_ψ and N_ξ are dense in Ψ and Ξ , respectively, then R is dense in S and thus $S \cap \text{int}(\bar{A}) = \emptyset$.*

PROOF: The structure of the local coordinates given earlier implies for both nice pseudo equilibrium points and nice semi-singular points that N_ψ and N_ξ have points in R arbitrarily close by, i.e. $\bar{R} = S$. $\square$

Clearly it is only necessary to assume the density conditions for those components of the nice set which intersect the stability boundary. This gives

Corollary 6 *If $\overline{N_\psi \cap \partial A} = \Psi \cap \partial A$ and $\overline{N_\xi \cap \partial A} = \Xi \cap \partial A$, then*

$$S \cap \text{int}(\bar{A}) = \emptyset.$$

$\square$

Pseudo-saddles play an important role in the computations of the stability boundary. Hence this set will now be analysed in more detail. The following lemmas enable us to give necessary and sufficient conditions for a pseudo-saddle to lie in the boundary of the region of attraction. The local dynamic behavior near a hyperbolic equilibrium point or a hyperbolic periodic orbit is well-understood. An important result, about properties of the stable and unstable manifolds, which is a key technical tool in the detection of points in the stability boundary, is the λ -lemma by Palis [29]. A version of the λ -lemma is applicable for nonhyperbolic diffeomorphisms [29]. At a hyperbolic periodic orbit, the flow has a 1-dimensional center manifold. For a pseudo-saddle, the flow has an $(n - 2)$ -dimensional center manifold in N . Define the stable manifold $W^{sc}(\psi)$ of a neighborhood of $\psi \in N_{sa}$, as $W^{sc}(\psi) = W^s(\delta(\psi))$ for some $\delta > 0$. Similarly, define $W^{uc}(\psi)$ as $W^u(\delta(\psi))$ for some $\delta > 0$.

Lemma 20 Suppose $\psi \in N_{sa}$ and consider a 1-dimensional disk D transversal to $W^{sc}(\psi)$ at $q \in W^s(\psi)$. Then, for any $\nu > 0$, there exists a $t_0(\nu)$ such that Φ_t^T is ν C^1 -close to $W^u(\psi)$ for all $t > t_0(\nu)$. $\square$

This lemma is a special case of the λ -lemma for nonhyperbolic diffeomorphisms [29]. Lemmas 16, 17, and 18 allow us to use this result. A proof and the application are given in [37]. The lemma states that the image of the disk D under the flow, $\Phi_t^T(D)$, locally comes arbitrarily close to any point in $W^u(\psi)$. A more useful result is obtained by considering a disk of dimension $n - 1$.

Corollary 7 Let $\psi \in N_{sa}$ and consider an $(n - 1)$ -dimensional disk D transversal to $W^s(\psi)$ at $q \in W^s(\psi)$. Then there exists a $\delta > 0$ such that for any $\nu > 0$ there exists a $t_0(\nu)$ such that Φ_t^T is ν C^1 -close to $W^u(\delta(\psi))$ for all $t > t_0(\nu)$.

PROOF: Specifically, the disk D contains a 1-dimensional disk D_q that transversally intersects with $W^{sc}(\psi)$ at $q \in W^s(\psi)$. As the transversal intersections persist for points $\tilde{q}$ near q we can similarly find disks $D_{\tilde{q}}$ with $D_{\tilde{q}}$ transversal to $W^{sc}(\psi)$ at $\tilde{q}$. Choosing δ as the size of this neighborhood, the proof follows from the λ -lemma. $\square$

Corollary 8 Suppose $\psi \in N_{sa}$. Let D_u be a disk in $W^u(\psi)$ and let D be a 1-dimensional disk transversal to $W^{sc}(\psi)$ at $q \in W^s(\psi)$. Then $\Phi_t^T(D)$ comes arbitrarily close to D_u . Moreover

$$D_u \subset \bigcup_{t>0} \overline{\Phi_t^T(D)}.$$

PROOF: The proof follows from the λ -lemma. $\square$

For hyperbolic equilibria and periodic orbits, roughly, fundamental domains for the stable manifold (unstable manifold) are defined, so that the flow of such a domain will cover the whole stable (unstable) manifold in a special way. For a precise definition, see [29]. We can define a fundamental domain for the pseudosaddles in a similar way. Roughly, for a pseudosaddle ψ , the fundamental domain of the stable manifold, $G^s(\psi)$, is a domain with the property that

$$\bigcup_{t \in \mathbb{R}} \Phi_t^T(G^s(\psi)) = W^s(\psi).$$

The fundamental neighborhood of the stable manifold of $\psi \in N_{sa}$, denoted by $FN^s(\psi)$, is defined as any neighborhood of $G^s(\psi)$ which is disjoint from the unstable manifold $W^{uc}(\psi)$. More details on these definitions are provided in [37].

Corollary 9 Let $\psi \in N_{sa}$ and let $FN^s(\psi)$ be a fundamental neighborhood of the stable manifold of ψ . Then there exists a neighborhood U and a $\delta > 0$ such that

$$\bigcup_{t>0} \Phi_t^T(FN^s(\psi)) \supset U \setminus W^u(\delta(\psi)).$$

PROOF: $FN^s(\psi)$ contains a $(n - 1)$ -dimensional disk, transversal to $W^s(\psi)$ at $q \in G^s(\psi)$. The result follows from the λ -lemma and the definition of the fundamental domain. $\square$

By reversing the time-scale in the proofs, the results on the λ -lemma, can be restated for the unstable manifold of a pseudo-saddle [37]. This is straightforward. These local results can be extended to global stable and unstable manifolds provided the disks considered are invariant [37]. For instance, when an un-

stable manifold intersects with a stable manifold transversally, the lemmas developed here are directly applicable.

The λ -lemma and its corollaries as stated above only hold if full neighborhoods of the pseudo-saddles are allowed. However, for our purposes, this is not quite good enough since the singularity will always cut off half of the neighborhood (see Corollary 6). we can only work with truncated trajectories and therefore an even closer look at a local neighborhood of a pseudo-saddle is required.

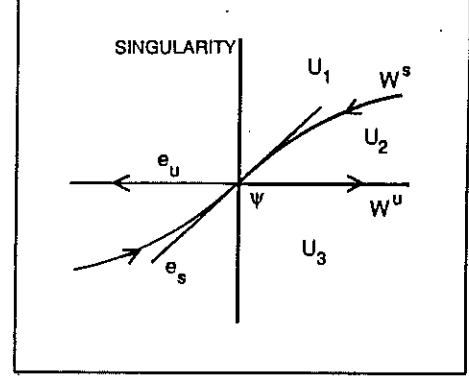


Figure 10. Two dimensional representation of the local picture around the pseudo saddle surface. $(n - 2)$ dimensional pseudo saddle surface is depicted by the point Ψ .

Consider any $\psi \in N_{sa} \cap \partial \bar{A}$. Choosing the coordinates as in Lemma 16, the Jacobian of the dynamics at ψ has $n - 2$ zero eigenvalues, one stable and one unstable eigenvalue. The eigenvectors for the zero eigenvalues are easily found to be $e_3, e_4, e_5, \dots, e_n$, where e_i is the unit vector along the η_i axis. Let the eigenvalues of the stable and unstable eigenvalues be denoted by e_s and e_u respectively. The local "picture" depends on how these are oriented with respect to the singular surface $\{\eta_1 = 0\}$. Generically we can assume that both e_s and e_u do not belong to the singular set, i.e. that ψ is transverse (Assumption (A1)(b)). Then the singular surface divides any local neighborhood into two parts, such that each contains one stable and one unstable trajectory of ψ [37]. This is summarized in Figure 10. Moreover, suppose U is a small neighborhood of ψ and let U_C denote the part of this neighborhood in C . Then the local stable and unstable manifolds of the pseudo-saddle surface further divide this neighborhood U_C into three parts as follows: a part that lies between the singularity and the stable manifold denoted by U_1 , a part U_2 that lies between the stable and unstable manifolds, and the remaining part U_3 . Henceforth, the stable and unstable manifolds are taken to be only those which lie in C . These would agree with the definitions given in section V.3 for the stable and unstable manifold of the vector field Z . By analysing each of the neighborhoods U_i the following generalizations of the λ -Lemmas and its Corollaries are derived. The proofs are similar.

Lemma 21 Suppose $\psi \in N_{sa}$ is transverse. Consider an $(n - 1)$ -dimensional disk D transversal to $W^s(\psi)$ at $q \in W^s(\psi)$ (in C). Then there exists a $\delta > 0$ such that for any $\nu > 0$ there exists a $t_0(\nu)$ such that the flow of the disk, $\Phi_t^T(D)$, is ν C^1 -close to $W^u(\delta(\psi))$ (in C), for all $t > t_0(\nu)$. $\square$

Corollary 10 Suppose $\psi \in N_{sa}$ is transverse and consider a 1-dimensional disk D_u in $W^u(\psi)$ in C . Let D be another 1-dimensional disk, transversal to $W^{sc}(\psi)$ at $q \in W^s(\psi)$ in C . Then the image of the disk D under the flow, $\Phi_t^T(D)$, comes arbitrarily close to D_u . Moreover,

$$D_u \subset \bigcup_{t>0} \overline{\Phi_t^T(D)}$$

□

Corollary 11 Let $\psi \in N_{sa}$ be transverse and consider a fundamental neighborhood $FN^s(\psi)$ of the stable manifold of ψ in C . Then there exists a neighborhood U and a $\delta > 0$ such that

$$\bigcup_{t>0} \Phi_t^T(FN^s(\psi)) \supset U_C \setminus (W^u(\delta(\psi)) \cup U_3).$$

Here U_C and U_3 are as defined in Figure 10.

□

As a first major step in analysing the stability boundaries, the following lemmas show that the stable manifolds of the nice pseudo equilibrium points lie on the boundary $\partial \bar{A}$ whenever they lie in ∂A . This is intuitively clear because there are trajectories arbitrarily close to the stable manifolds which converge to the singularity (hence, not in A). So, the stable manifolds of the pseudo-saddles constitute part of the quasi-stability boundary $\partial \bar{A}$. The proofs follow from the local equivalence lemma (Lemma 18) and the analysis of the local neighborhoods.

Lemma 22 $N_{si} \cap \bar{A} = \emptyset$

PROOF: It can be seen that all trajectories in a local neighborhood of a pseudo-sink contained in C either converge to the pseudo-sink or to the singularity. So a pseudo-sink cannot be on the boundary ∂A . □

Lemma 23 $W^s(N_{sa}) \cap \text{int}(\bar{A}) = \emptyset$

PROOF: The proof follows from our analysis based on the illustration, Figure 10. [37] □

Corollary 12 $W^s(N) \cap \text{int}(\bar{A}) = \emptyset$

PROOF: Note that $N_{sa} \cap \text{int}(\bar{A}) = \emptyset$. Pseudo-sinks and pseudo-saddles have been considered in the previous two lemmas. □

VIII.6 STABILITY BOUNDARY RESULTS

In this section the results of section V.3 are proven using the transformed vector field Z^T . (Theorem 5 has already been verified in the previous section.) To begin with, invariance properties of the stable and unstable manifolds will be analysed. Next necessary and sufficient conditions to check the presence of anchor points on the stability boundary will be developed. This then allows to characterise the stability boundary. We start with a rather intuitive statement.

Lemma 24 The stability boundary ∂A has maximal dimension $(n-1)$ at every point.

PROOF: $\partial A \cap M$ has maximal dimension $n-1$ because M is an n -dimensional manifold and A is open in M . Furthermore, by Lemma 7 $L \setminus M \subset \Psi$ and Ψ has maximal dimension $n-2$ by Assumption (A0) and Lemma 15. Hence ∂A has maximal

dimension $n-1$. □

As noted in the previous sections, because of the presence of the singularity some trajectories of the system are not defined for all time since trajectories may start from or end up at the singular surface. Thus, for a system with singularities, the vector field Z is not complete (in fact, it is not even smooth). The transformed vector field Z^T is smooth by construction. But we want to study the boundary of the region of attraction for the vector field Z , not for Z^T . Hence the trajectories of Z^T are truncated at the singularity. The correct region of attraction, A , is therefore described in terms of the flow induced by Z^T as

$$A = \{z \in C : \Phi_t^T(z) \in C \ \forall t > 0, \Phi_t^T(z) \rightarrow z_s \text{ as } t \rightarrow \infty\}$$

Similarly, the stable and unstable manifolds are defined as in section V.3. We do not allow trajectories to exit C and return at a later time. From the definitions, it is clear that any trajectory in a stable manifold $W^s(\alpha)$ stays there forever in positive time when α is an equilibrium point, a periodic orbit or a pseudo equilibrium point. For a semi-saddle α this is not true. In general $W^s(\alpha)$ is not invariant in negative time because some trajectories might have entered from the singularity. A set of lemmas are presented next which analyse the invariance properties of the region of attraction and the other stable manifolds. A set $\bar{A}$ is said to be positive time-invariant if

$$\bigcup_{t>0} \Phi_t^T(\bar{A}) = \bar{A}.$$

The proofs of these lemmas are technical and are therefore omitted here [37].

Lemma 25 A is locally time-invariant. □

Lemma 26 For any $z \in \bar{A} \setminus \Psi$,

$$\left(\bigcup_{t>0} \Phi_t^T(z)\right) \cap S \subset \Xi.$$

□

Lemma 27 A , $\partial A \setminus (S \cup W^s(\Xi))$ and $\bar{A} \setminus W^s(\Xi)$ are positive time-invariant. □

Lemma 28 The largest invariant set in $\bar{A}$ is

$$I := \bar{A} \setminus \bigcup_{t \in \mathbb{R}} \Phi_t^T(S)$$

□

Similar invariance properties hold for any other stable manifold. Furthermore,

Lemma 29 Suppose α is a hyperbolic equilibrium point, a hyperbolic periodic orbit or a nice pseudo equilibrium point. Then $W^s(\alpha)$, $\partial W^s(\alpha) \setminus (S \cup W^s(\Xi))$ and $\bar{W}^s(\alpha) \setminus W^s(\Xi)$ are positive time-invariant. □

Using these invariance properties and the λ -lemmas, the results summarized as Theorem 9 can be proved. The full proof is long and therefore only some special cases are verified in this paper. Also, unless mentioned explicitly, sufficiency is usually a straightforward argument using invariance properties of the corresponding objects. Therefore only necessity will be addressed in the proofs. For more details, see [37].

Lemma 30 Suppose z_0 and z_1 are hyperbolic equilibrium points and assume that the intersection of any stable manifold with an unstable manifold is transversal (Assumption (A.2)). Then

$$z_1 \in \partial W^s(z_0) \iff (W^s(z_1) \setminus \{z_1\}) \cap \partial W^s(z_0) \neq \emptyset$$

PROOF: Let $z_1 \in \partial W^s(z_0)$. Choose a suitably small ϵ -sized neighborhood $B_\epsilon(z_1)$ around z_1 which does not contain any other anchor point or singular points. Consider a sequence $\{\hat{z}_n\} \in W^s(z_0) \cap B_\epsilon(z_1)$, such that $\hat{z}_n \rightarrow z_1$ as $n \rightarrow \infty$. If one of these points $\hat{z}_n$ lies in $W^u(z_1)$, then the result follows from the λ -lemma as follows: Suppose $\hat{z}_{n_0} \in W^u(z_1)$. Then

$$(W^u(z_1) \setminus \{z_1\}) \cap W^s(z_0) \neq \emptyset$$

By assumption $W^u(z_1)$ transversally intersects with $W^s(z_0)$. Let D_ϵ be a disc of dimension k , where k is the order of z_1 , transversal to $W^s(z_1)$ at some $\tilde{z} \in G^s(z_1)$, the fundamental domain. By the λ -lemma, $\bigcup_{t>0} \Phi_t^T(D_\epsilon)$, comes arbitrarily close to $\hat{z}_{n_0}$. By the persistence of transversal intersections, it follows that

$$\bigcup_{t>0} \Phi_t^T(D_\epsilon) \cap W^s(z_0) \neq \emptyset$$

So $\Phi_t^T(D_\epsilon) \cap W^s(z_0) \neq \emptyset$ for some $t > 0$. Then also $D_\epsilon \cap W^s(z_0) \neq \emptyset$ by invariance. As $\epsilon \rightarrow 0$, it follows that $\tilde{z} \in \overline{W^s(z_0)}$, i.e. $\tilde{z} \in \partial W^s(z_0)$. As $\tilde{z}$ is any point in $G^s(z_1)$, $G^s(z_1) \subset \partial W^s(z_0)$. Hence $W^s(z_1) \subset \partial W^s(z_0)$. In this case therefore the result is much stronger than the claim.

Suppose now that $\hat{z}_n \notin W^u(z_1)$ for all n . Consider the trajectories which reach $\hat{z}_n$. Let $\tilde{z}_n$ denote the entry point into the neighborhood $B_\epsilon(z_1)$ for the trajectory that reaches $\hat{z}_n$, i.e.

$$\tilde{z}_n := \left(\bigcup_{t<0} \Phi_t^T(\hat{z}_n) \right) \cap \partial B_\epsilon(z_1).$$

Then the sequence $\{\tilde{z}_n\}$ remains bounded and at least a subsequence has a limit. Suppose $\lim_{n \rightarrow \infty} \tilde{z}_n = \bar{z}$. Then $\bar{z} \in \partial B_\epsilon(z) \cap \overline{W^s(z_0)}$, and, by the continuity of the flow, as $\lim_{n \rightarrow \infty} \hat{z}_n = z_1$, also $\bar{z} \in W^s(z_1)$. This completes the proof. $\square$

Other results in Theorem 9 which involve only equilibrium points and periodic orbits can be proved in a similar fashion. Let us also look at a proof which considers pseudo equilibrium points.

Lemma 31 Suppose z_0 is a hyperbolic equilibrium point and let $\psi \in N_\psi$. Then

$$\psi \in \partial W^s(z_0) \iff W^u(\psi) \cap \overline{W^s(z_0)} \neq \emptyset$$

PROOF: Suppose $\psi \in \partial W^s(z_0)$ and assume first that ψ is a pseudo-source. Consider a fundamental domain $G^u(\psi)$ in C and take an ϵ -sized fundamental neighborhood $FN_\epsilon^u(\psi)$ in C [37]. Then, using the λ -lemma, it is easy to see that

$$\bigcup_{t<0} \Phi_t^T(FN_\epsilon^u(\psi)) \supset U_C \setminus (\delta(\psi) \cup U_1)$$

where U_C denotes a neighborhood in C , and U_1 is such that $U_1 \cap W^s(z_0) = \emptyset$. Clearly, $\delta(\psi) \cap W^s(z_0) = \emptyset$. But $U_C \cap W^s(z_0) \neq \emptyset$. Therefore

$$\bigcup_{t<0} \Phi_t^T(FN_\epsilon^u(\psi)) \cap W^s(z_0) \neq \emptyset.$$

By the invariance properties, $FN_\epsilon^u(\psi) \cap W^s(z_0) \neq \emptyset$. Letting $\epsilon \rightarrow 0$, this implies that $G^u(\psi) \cap \overline{W^s(z_0)} \neq \emptyset$. Therefore $W^u(\psi) \cap$

$$\overline{W^s(z_0)} \neq \emptyset.$$

If ψ is a pseudo-saddle, then the proof is similar, except that in this case

$$\bigcup_{t<0} \Phi_t^T(FN_\epsilon^u(\psi)) \supset U_C \setminus (W^s(\delta(\psi)) \cup U_1).$$

The lemma follows because $W^s(\delta(\psi)) \cap W^s(z_0) = \emptyset$. $\square$

This concludes our general discussion of some of the equivalences in Theorem 9. We now specialize some of these conditions for anchor points. From here onwards, assumptions (A) will be in effect. First equilibrium points and periodic orbits are considered. Because the stability boundary is at most $(n-1)$ -dimensional, among these the most interesting ones are the ones with a $(n-1)$ -dimensional stable manifold.

Lemma 32 Suppose z_0 is an order 1 equilibrium point. Then

- (i) $z_0 \in \partial A \iff (W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset$
- (ii) $z_0 \in \partial A \iff W^s(z_0) \subset \partial A$

PROOF: Sufficiency of both conditions follows as usual from invariance properties. For necessity, suppose $z_0 \in \partial A$. By Lemma 30, $(W^u(z_0) \setminus \{z_0\}) \cap \overline{A} \neq \emptyset$. Let $z \in (W^u(z_0) \setminus \{z_0\}) \cap \overline{A}$. If moreover $z \in A$, then claim (i) follows. Otherwise $z \in \partial A$. By Assumption (A3), $z \in W^s(\alpha)$ for some anchor point α . So $z \in W^u(z_0) \cap W^s(\alpha)$. By Assumption (A2), $W^s(\alpha)$ transversally intersects with $W^u(z_0)$ at z . But $W^u(z_0)$ is of dimension 1, and the intersection is along $W^u(z_0)$. Hence $W^s(\alpha)$ must be n -dimensional and so α is a stable equilibrium. Contradiction. So $(W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset$.

To prove the necessity of (ii), suppose $z_0 \in \partial A$. By (i) $(W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset$. Using an argument based on the λ -lemma, i.e. by considering the flow of a 1-dimensional disk transversal to any local point in the stable manifold, it can be shown that $z \in \partial A$ for any $z \in G^s(z_0)$. Therefore $G^s(z_0) \subset \partial A$ and so $W^s(z_0) \subset \partial A$. $\square$

Lemma 33 Suppose γ_0 is an order 2 periodic orbit. Then

- (i) $\gamma_0 \in \partial A \iff (W^u(\gamma_0) \setminus \gamma_0) \cap A \neq \emptyset$
- (ii) $\gamma_0 \in \partial A \iff W^s(\gamma_0) \subset \partial A$.

This proof is slightly lengthy and will be split into two separate lemmas. Let t_γ denote the period of the periodic orbit γ_0 . Define $h(z) = \Phi_{t_\gamma}^T(z)$ and note that h is a diffeomorphism. Using h , stable and unstable manifolds for each $z_\gamma \in \gamma_0$ are defined as follows (h^n denotes the n^{th} iterate):

$$W^s(z_\gamma) := \{z \in C : h^n(z) \rightarrow z_\gamma \text{ as } n \rightarrow \infty \text{ and } h^n(z) \in C \forall n\}$$

$$W^u(z_\gamma) := \{z \in C : h^n(z) \rightarrow z_\gamma \text{ as } n \rightarrow -\infty \text{ and } h^n(z) \in C \forall n\}$$

Then $W^s(z_\gamma)$ and $W^u(z_\gamma)$ are manifolds and λ -lemmas hold for these as well. Using these, the next lemma can be proven [37]:

Lemma 34 Suppose γ_0 is an order 2 periodic orbit. Then

$$\begin{aligned} \gamma_0 \in \partial A &\iff (W^u(z_\gamma) \setminus \{z_\gamma\}) \cap \overline{A} \neq \emptyset \forall z_\gamma \in \gamma_0 \\ \gamma_0 \in \partial A &\iff (W^s(z_\gamma) \setminus \{z_\gamma\}) \cap \partial A \neq \emptyset \forall z_\gamma \in \gamma_0. \end{aligned}$$

$\square$

For an order 2 periodic orbit each $W^u(z_\gamma)$ is one-dimensional and the set $W^u(\gamma_0) \setminus \gamma_0$ has two connected components. Using Lemma 34, it can be shown that at least one such component of $W^u(\gamma_0) \setminus \gamma_0$, denoted by W_c^u , lies in $\bar{A}$. If $W_c^u \cap A \neq \emptyset$, then (i) of Lemma 33 follows. Otherwise $W_c^u \subset \partial A$. But this is not possible as shown in the next lemma.

Lemma 35 *Suppose α is an equilibrium point, a periodic orbit or a component of the nice set (N_ψ or N_ℓ), and let $W^u(\alpha)$ be k -dimensional. Then $W^u(\alpha) \cap \partial A$ is not k -dimensional at any point.*

PROOF: Suppose $W^u(\alpha) \cap \partial A$ is k -dimensional at some $z_0 \in W^u(\alpha) \cap \partial A$. Then there exists a k -dimensional neighborhood N_0 of z_0 in $W^u(\alpha) \cap \partial A$ that belongs to ∂A . Any trajectory in ∂A must converge to some anchor point by Assumption (A3). So, for any $z \in N_0$, also $z \in W^s(\beta)$ for some other anchor point. Hence $W^u(\alpha)$ and $W^s(\beta)$ intersect at z . As z is any point in N_0 , this set must be exhausted by intersections with the stable manifolds. But, by Assumption (A2) such intersections are transversal. The stable manifolds of an equilibrium point, a periodic orbit, or a component of a nice set in ∂A have maximal dimension $n-1$. So any transversal intersection of such a set with N_0 results in a subset of N_0 with positive codimension. Also any bad pseudo equilibrium point must be on the boundary of a component of the nice set by Assumption (A0). Since ∂A has maximal dimension $n-1$, the closure of the stable manifold of N_0 , a component of N , has maximal dimension $n-1$ in ∂A . Furthermore, by Assumption (A2), the intersection with the closure is also transversal. Once again, this results in a subset of N_0 with the maximal dimension $k-1$. But a countable union of $(k-1)$ -dimensional sets cannot be k -dimensional. Contradiction. Hence $W^u(\alpha) \cap \partial A$ is not k -dimensional anywhere. $\square$

This completes the proof of Lemma 35. This lemma is instrumental in proving Lemma 33. So far, we have shown that $\gamma_0 \in \partial A$ implies that at least one component of $(W^u(\gamma_0) \setminus \gamma_0)$, denoted by W_c^u , entirely lies in $\bar{A}$. Lemma 35 implies that W_c^u cannot belong to ∂A . Hence $W_c^u \cap A \neq \emptyset$. This proves the first statement. The second statement follows using (i) and the λ -lemma for periodic orbits. [37]. $\square$

As pointed out in section V.3, the boundary of the closure of the region of attraction, $\partial \bar{A}$, is practically more important than ∂A . It also turns out that $\partial \bar{A}$ has a simpler structure than ∂A . The main property of $\partial \bar{A}$, which simplifies the analysis, is that it is $(n-1)$ -dimensional on a dense subset.

Lemma 36 $\partial \bar{A} \cap M$ is $(n-1)$ -dimensional.

PROOF: Suppose $z \in \partial \bar{A} \cap M$. As $z \in M$, there exists a homeomorphism h and a small neighborhood U of z so that $h(U)$ is open in $\mathbb{R}^n$. Define $U_1 = U \cap \text{int}(\bar{A})$. Then U_1 is open and $z \in \partial U_1$, as $z \in \partial \bar{A}$. As $z \in \text{int}(U)$, locally near z , $\partial \bar{A}$ coincides with ∂U_1 . As h is a homeomorphism, $h(U_1)$ is open in $\mathbb{R}^n$ and $h(z) \in \partial h(U_1)$ (h takes boundary points to boundary points). It is easy to see that $z \in \partial \bar{U}_1$, as $z \in \text{int}(U)$ and $z \in \partial \bar{A}$, and, locally near z , $\partial \bar{U}_1$ agrees with $\partial \bar{A}$. Therefore $h(z) \in \partial h(\bar{U}_1)$. As h is a homeomorphism, locally near $h(z)$, $\partial h(\bar{U}_1) = \partial h(U_1)$. Furthermore, $h(U_1)$ is open in $\mathbb{R}^n$. Hence $\partial h(U_1)$ has dimension $n-1$ [10]. Therefore, locally near $h(z)$, $\partial h(\bar{U}_1)$ has dimension $n-1$. As homeomorphisms preserve dimensions, $\partial \bar{U}_1$ has di-

mension $n-1$ locally and so $\partial \bar{A}$ has dimension $n-1$ near z . As z is any point in $\partial \bar{A} \cap M$, this completes the proof. $\square$

Lemma 37

$$\overline{(\partial \bar{A} \cap M)} = \partial \bar{A}$$

PROOF: $\partial \bar{A} = (\partial \bar{A} \cap M) \cup (\partial \bar{A} \setminus M)$. Now, $\partial \bar{A} \setminus M \subset \partial \bar{A} \cap \Psi$ by Lemma 7. Moreover, by the definition of N_ψ and B_ψ ,

$$\partial \bar{A} \setminus M \subset \partial \bar{A} \cap B_\psi \subset \partial A \cap B_\psi$$

By Assumption (A0), it follows that

$$\partial \bar{A} \setminus M \subset \partial A \cap B_\psi \subset \partial(\partial A \cap N_\psi)$$

But, $\partial A \cap N_\psi = \partial \bar{A} \cap N_\psi$ by Corollary 6. So,

$$\partial \bar{A} \setminus M \subset \partial(\partial A \cap N_\psi) \subset \partial(\partial \bar{A} \cap M)$$

Thus, $\overline{(\partial \bar{A} \cap M)} = \partial \bar{A}$. $\square$

Moreover, the same result holds for the full boundary also, as shown in the next lemma.

Lemma 38

$$\overline{(\partial A \cap M)} = \partial A$$

PROOF: By Assumption (A0) both $\overline{N_\psi \cap \partial \bar{A}} = \Psi \cap \partial A$ and $\overline{N_\ell \cap \partial \bar{A}} = \Xi \cap \partial A$. By Lemma 7, $L \setminus M \subset \Psi$ and therefore

$$\begin{aligned} \overline{M \cap \partial \bar{A}} &= \overline{(M \setminus N) \cap \partial \bar{A} \cup N \cap \partial \bar{A}} \\ &= \overline{(M \setminus N) \cap \partial \bar{A} \cup (\Psi \cap \partial A) \cup (\Xi \cap \partial A)} \\ &= \overline{((M \setminus N) \cup \Psi \cup \Xi) \cap \partial \bar{A}} = \overline{L \cap \partial \bar{A}} = \partial A. \end{aligned}$$

$\square$

The quasi-feasibility boundary $\partial \bar{A}$ is $(n-1)$ -dimensional in M . By Assumption (A3), the boundary $\partial \bar{A}$ is a subset of the stable manifolds of the anchor points, with possibly a singular boundary, i.e.

$$\begin{aligned} \partial \bar{A} \subset \partial A \subset \bigcup_{z_0 \in \partial A} W^s(z_0) \cup \bigcup_{\gamma_0 \in \partial A} W^s(\gamma_0) \\ \cup W^s(\Psi \cap \partial A) \cup W^s(\Xi \cap \partial A) \cup (S \cap \partial A). \end{aligned} \quad (45)$$

As $\partial \bar{A}$ is $(n-1)$ -dimensional on a dense subset, we will only analyse the stable manifolds, which are $(n-1)$ -dimensional. We will later prove that this is indeed sufficient. The next two Lemmas prove Theorem 7.

Lemma 39 *Suppose z_0 is an order 1 equilibrium point.*

- (i) $z_0 \in \partial \bar{A} \iff (W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset$ and $(W^u(z_0) \setminus \{z_0\}) \cap \bar{A}^c \neq \emptyset$
- (ii) $z_0 \in \partial \bar{A} \iff W^s(z_0) \subset \partial \bar{A}$

PROOF: The sufficiency follows from invariance properties. For the necessity of the first statement, suppose $z_0 \in \partial \bar{A}$. Then $z_0 \in \partial A$. By Lemma 32, $(W^u(z_0) \setminus \{z_0\}) \cap A \neq \emptyset$. The remaining part of the proof is by contradiction. Suppose $(W^u(z_0) \setminus \{z_0\}) \cap \bar{A}^c = \emptyset$. Then, $W^u(z_0) \subset \bar{A}$. Again by Lemma 32, $W^s(z_0) \subset \partial A$. So both the stable and unstable manifolds of z_0 lie in $\bar{A}$. As z_0 is an equilibrium of order 1, $W^u(z_0)/\{z_0\}$ consists of exactly two trajectories. By Lemma 32, at least one of these belongs to A . The other unstable trajectory of z_0 must also be in A because the presence of this trajectory in ∂A is ruled out by Lemma 35. Therefore $(W^u(z_0) \setminus \{z_0\}) \subset A$. So we can choose a fundamental neighborhood of the unstable manifold, $FN^u(z_0)$, so that $FN^u(z_0) \subset A$. By the corollary to the λ -lemma [29], there exists a local neighborhood LN such that

$$\bigcup_{\epsilon > 0} \Phi_t^T(FN^u(z_0)) \supset LN \setminus W^s(z_0).$$

By local invariance we conclude that $LN \subset \bar{A}$, as $W^s(z_0) \subset \bar{A}$. Therefore the neighborhood LN contains z_0 and is a subset of $\bar{A}$. This implies that $z_0 \in \text{int}(\bar{A})$. Contradiction.

To prove the necessity of (ii), let $z_0 \in \partial \bar{A}$. In particular also $z_0 \in \partial A$, and by Lemma 32 $W^s(z_0) \subset \partial A$. By (i), we also know that $W^u(z_0) \setminus \{z_0\} \cap \bar{A}^c \neq \emptyset$. Note that $\bar{A}^c$ is open and satisfies invariance properties similar to A , but has negative time-invariance, instead of positive time-invariance; $\bar{A}^c$ also has the local invariance properties. Hence, carrying out the proof of Lemma 32 with $\bar{A}^c$ replaced by A , we can conclude by the λ -lemma that $W^s(z_0) \subset \partial(\bar{A}^c)$. Therefore $W^s(z_0) \subset \partial \bar{A}$. $\square$

The proof of Theorem 7, part (b) for order 2 periodic orbits is almost identical and can be found in [37]. The other main contribution to the quasi-stability boundary is from the nice subsets defined as N_{n-1} in equation (35).

Lemma 40 Suppose $\psi \in N_{n-1}^\psi$. Then for every sufficiently small $\delta > 0$,

$$W^u(\delta(\psi)) \setminus \delta(\psi) \cap A \text{ is dense in } W^u(\delta(\psi)) \setminus \delta(\psi)$$

and $W^s(\delta(\psi)) \subset \partial \bar{A}$.

PROOF: The points in N_{n-1}^ψ are transverse pseudo-saddles. So each point $\psi \in N_{n-1}^\psi$ has one stable trajectory and one unstable trajectory. If $\psi_0 \in N_{n-1}^\psi$, then, by definition of N_{n-1}^ψ , there exists a $\delta_0 > 0$ such that $\delta_0(\psi_0) \subset \partial A$. Let $\delta < \delta_0$ be small and consider any point $\psi \in \delta(\psi_0)$. Applying Lemma 31, it follows that $W^u(\psi) \cap \bar{A} \neq \emptyset$ for each $\psi \in \delta(\psi_0)$. So $W^u(\delta(\psi_0)) \subset \bar{A}$. But $W^u(\delta(\psi_0))$ is $(n-1)$ -dimensional. From Lemma 35, we see that $W^u(\delta(\psi_0)) \cap \partial A$ cannot be $(n-1)$ -dimensional at any point. Hence $W^u(\delta(\psi_0)) \cap A$ is dense in $W^u(\delta(\psi_0))$.

To prove the second statement, note that, by (i) for sufficiently small $\delta > 0$ we have

$$W^u(\delta(\psi)) \cap A \neq \emptyset.$$

The proof now follows again from the λ -lemma. Suppose first that $W^u(\delta(\psi)) \cap A \neq \emptyset$ for all $\delta > 0$. Take any point $z \in G^s(\psi)$, the fundamental domain. Consider an ϵ -sized $(n-1)$ -dimensional disk D_ϵ , transversal to $W^s(\psi)$ at z . Applying Corollary 7, the flow of D_ϵ comes arbitrarily close to the unstable manifold of an δ^* sized neighborhood of ψ in N_ψ . But there is at least one point of transversal intersection of $W^u(\delta^*(\psi))$ with A . Hence, as transversal intersections persist,

$$\left(\bigcup_{\epsilon > 0} \Phi_t^T(D_\epsilon)\right) \cap A \neq \emptyset.$$

Then, repeating the same argument as in Lemma 30 and 31, it follows that $D_\epsilon \cap A \neq \emptyset$, and $z \in \bar{A}$ by letting $\epsilon \rightarrow 0$. As z is any point in $G^s(\psi)$, we have proved that $W^s(\psi) \subset \bar{A}$, i.e. $W^s(\psi) \subset \partial A$. Furthermore, by Lemma 23, $W^s(N_{n-1}) \cap \text{int}(\bar{A}) = \emptyset$ and thus $W^s(\delta(\psi)) \subset \partial \bar{A}$. $\square$

A similar result holds for semi-saddles, as stated in Theorem 7. We have now established the required necessary and sufficient conditions to check the presence of the principal anchor points on the quasi-stability $\partial \bar{A}$. Now we will prove that the stable manifolds of these points, possibly along with the singularity, constitute the quasi-stability boundary.

Proof of Theorem 6 on the quasi-stability boundary:

Assumption (A3) implies equation (45). By Lemmas 39 and 40, we have proved that

$$QS := \bigcup_{z_0 \in \partial \bar{A}} W^s(z_0) \bigcup_{\gamma_0 \in \partial \bar{A}} W^s(\gamma_0) \cup W^s(N_{n-1}) \cup (S \cap \partial \bar{A}) \subset \partial \bar{A}$$

We therefore need to show that the difference between $\partial \bar{A}$ and QS can be compensated for by taking the closure. First, let us show that N_{n-1} provides the principal boundary for the points in $\Psi \cap \partial A$ and $\Xi \cap \partial A$.

Lemma 41 $W^s(B_\psi \cap \partial A) \cap \partial A$ has maximal dimension $n-2$ at every point in the set.

PROOF: The proof uses Assumption (A0) that $N_\psi \cap \partial A$ is dense in $\Psi \cap \partial A$. Since the components of N_ψ are $(n-2)$ -dimensional manifolds, the intersections of the closures of the components N_C constitute the bad set $B_\psi \cap \partial A$. Furthermore, it can be shown that, if $W^s(B_\psi \cap \partial A)$ has maximal dimension $n-1$ at any point, then some of the intersecting surfaces are pseudo-sinks. This will rule out such a point being on the boundary of ∂A [37]. $\square$

Lemma 42 $\partial A \cap W^s((\Psi \cap \partial A) \setminus N_{n-1}^\psi)$ has maximal dimension $n-2$ at every point.

PROOF: Note that

$$(\Psi \cap \partial A) \setminus N_{n-1}^\psi = (B_\psi \cap \partial A) \cup (N_{s_0} \cap \partial A) \cup ((N_{s_a} \cap \partial A) \setminus N_{n-1}^\psi)$$

The result has already been proven in Lemma 41 for the bad pseudo equilibrium points $B_\psi \cap \partial A$. Pseudo-sources have no stable manifold. Among the pseudo-saddles, by construction only the points in N_{n-1}^ψ have stable manifolds of dimension $n-1$ in ∂A . Hence, the lemma follows. $\square$

For semi-saddles, by Assumption (A0) B_ξ has maximal dimension $(n-3)$. By construction $W^s(N_\xi \cap \partial A \setminus N_{n-1})$ has therefore dimension $(n-2)$ at most. Hence the same statement as in Lemma 42 also holds for Ξ .

Only the equilibrium points of order 1 have stable manifolds of dimension $n-1$. All higher order equilibrium points have stable manifolds of dimension strictly less than $n-1$. Similarly, the periodic orbits of order higher than 2 have stable manifolds of maximal dimension $n-2$. All the singular points in the boundary $\partial \bar{A}$ have been grouped as $S \cap \partial \bar{A}$. In fact, by Corollary 6, $S \cap \partial A = S \cap \partial \bar{A}$, as $S \cap \text{int}(\bar{A}) = \emptyset$.

By Lemmas 36 and 37, $\partial \bar{A}$ is $(n-1)$ -dimensional on a dense subset, $\partial \bar{A} \cap M$. Rewriting equation (45), we see that

$$\partial \bar{A} \subset QS \cup D,$$

where D has maximal dimension $n-2$ at any point. Also, $QS \subset \partial \bar{A}$ and thus

$$\partial \bar{A} = QS \cup D_0,$$

where D_0 has maximal dimension $n-2$ at any point. But $\partial \bar{A} = \overline{\partial \bar{A} \cap M}$. Because $\partial \bar{A} \cap M$ is $(n-1)$ -dimensional, and D_0 has maximal dimension $(n-2)$ at any point, we get

$$\overline{(\partial \bar{A} \cap M) \setminus D_0} = \overline{\partial \bar{A} \cap M} = \overline{QS} = \partial \bar{A}$$

This completes the proof. $\square$

Proof of Theorem 8 on the stability boundary:

Assumption (A3) implies equation (45). From Lemmas 32 and 33, we get

$$SB := \bigcup_{z_0 \in \partial A} W^s(z_0) \bigcup_{\gamma_0^2 \in \partial A} W^s(\gamma_0^2) \cup W^s(N_{n-1}) \cup (S \cap \partial A) \subset \partial A$$

By Lemma 42, $\partial A \cap W^s((\Psi \cap \partial A) \setminus N_{n-1}^\Psi)$ has maximal dimension $n-2$ everywhere and the same holds for $\partial A \cap W^s((\Xi \cap \partial A) \setminus N_{n-1}^\Xi)$. Any higher order (> 1) equilibrium point or a higher order (> 2) periodic orbit has a stable manifold of dimension strictly less than $n-1$. Hence we can write $\partial A = SB \cup D_1$, where D_1 has maximal dimension $n-2$ at any point. This proves Theorem 8. $\square$

It should be noted that $\dot{S}B \supset QS$. In general SB is a bigger set, as there may be $(n-1)$ -dimensional pieces of the stability boundary in the interior of $\bar{A}$. Yet, SB may not be a dense subset of ∂A because some components of the stability boundary in $\text{int}(\bar{A})$ may have maximal dimension less than $n-1$. A trivial example would be a source in the interior. But such points are not practically interesting. We can characterise the full boundary, ∂A , after making much stronger assumptions [37]. The validity of such assumptions for a general power system has to be considered. This question is mainly theoretical. From a practical point-of-view, the computation of the quasi-stability boundary is sufficient.

Proof of Lemma 3 on the energy function:

We will show that the function V is an energy function for the transformed vector field Z^T . Since Z^T is a scaled version of Z , property 1) in Lemma 3 simply translates to Z^T by multiplying by $\Delta(x, y)$ to get

$$(D_x V) f \Delta - D_y V \text{adj}(D_y g) D_x g f \leq 0 \text{ in } A.$$

Therefore $(D_x V, D_y V) \cdot Z^T \leq 0$ in A . The function V is decreasing along the trajectories of the vector field Z^T in A . By continuity this is also true in $\bar{A}$. The property 2) in Lemma 3 implies that V is an energy function for Z^T in $\bar{A}$. By property 3), the ω -limit set for the vector field Z consists of equilibrium points and periodic orbits in $\bar{A}$.

Lemma 43 *For the vector field Z^T any trajectory in ∂A , that approaches a singularity, has a pseudo equilibrium or a semi-singular point as its limit on the singularity.*

PROOF: By Lemma 9, such a trajectory has a unique limit on the singularity. Nothing needs to be shown if this point lies in Ψ or Ξ . Otherwise there exists a trajectory that reaches z in positive time (namely the trajectory under study), and $z \in \partial A$ as ∂A is closed. By Lemma 8, the flow Φ_t^T is transversal to R at z . Therefore all the trajectories in a neighborhood approach the singularity in positive time, hence do not belong to A . Thus $z \notin \partial A$. Contradiction. So $z \in \Psi$ or $z \in \Xi$. $\square$

The trajectories of Z and Z^T are identical in C . Hence, for the vector field Z , any trajectory in ∂A that approaches the singularity, has a pseudo equilibrium or a semi-singular point as its limit. If the trajectory does not approach the singular surface, then the energy V decreases continuously, and V is proper. So the trajectory has a ω -limit point. By property 3) in Lemma 3,

such a point is either an equilibrium point or a periodic orbit. This completes the proof of Lemma 3.

It is easy to see from the proof that the property 2) of the function V is equivalent to
(2') If $\bigcup_{t>0} V(\Phi_t^T(z))$ is bounded, then $\bigcup_{t>0} \Phi_t^T(z)$ is bounded for all $z \in \bar{A}$.

Suppose such an energy function V is available. Then, define the set $\mathcal{R}(e)$ as the connected component of the set

$$\{z \in C : V(z) < e\}$$

that contains the stable equilibrium point z_s .

Lemma 44 *Suppose $e_{\max}$ is the largest number such that $\overline{\mathcal{R}(e_{\max})}$ does not intersect with unstable equilibrium points, unstable periodic orbits, pseudo equilibrium points and semi-singular points. Then,*

$$\mathcal{R}(e_{\max}) \subset A.$$

PROOF:

By definition $\mathcal{R}(e) \cap S = \emptyset$. Thus, $\mathcal{R}$ does not contain singular boundary points for any number, e . Moreover, $\overline{\mathcal{R}(e_{\max})}$ does not contain equilibrium points, periodic orbits, pseudo equilibrium points and semi-singular points. Hence, $\mathcal{R}(e_{\max})$ does not contain any other boundary point. So, $\mathcal{R}(e_{\max}) \subset A$. $\square$

Energy V is decreasing along trajectories. This property along with continuity allows us to prove the next result:

Lemma 45 *Suppose α is an anchor point. $\mathcal{R}(V(\alpha)) \subset \bar{A} \iff \alpha$ is the point in the set, $\bar{\mathcal{P}}$ with the lowest energy.*

PROOF:

As noted in section 6, the function V attains its lowest values on the closure of the principal anchor points in the quasi-stability boundary. If α_l is the point in the set $\bar{\mathcal{P}}$, with the lowest energy, then, $\mathcal{R}(V(\alpha_l))$ does not contain any anchor point. The energy at the stable manifolds will be even higher. So, they do not intersect with $\mathcal{R}(V(\alpha_l))$. So, $\mathcal{R}(V(\alpha_l)) \subset \bar{A}$. The necessity can also be seen similarly. $\square$

Using this lemma, we can prove the method to locate the critical anchor points.

Proof of Lemma 4 (Critical Anchor Lemma):

z_0 is inside $\text{int}(\bar{A}) \iff$ There exists a path from z_0 to z_s with energy increasing along the path, in $\text{int}(\bar{A})$.

Note that $\text{int}(\bar{A})$ and C are path-connected. Suppose z_0 lies inside the stability boundary defined by the critical anchor points. Then, the point with the lowest energy α_l either belongs to the set $\mathcal{P}_c$, or is in the boundary, $\partial \mathcal{P}_c$. So, the set $\mathcal{R}(V(\alpha_l))$ is a subset of $\text{int}(\bar{A})$ and so contains z_s . As z_0 lies inside the boundary defined by the closure of the stable manifolds in $\mathcal{P}_c$, then, $\mathcal{R}(V(\alpha_l))$ and z_0 are on the same side of the stable manifolds of the critical anchor points. Hence, it is possible to find a path with energy increasing from z_s to z_0 without crossing ∂C or the stable manifolds of the critical anchor points. Hence, $z_0 \in \text{int}(\bar{A})$. For the sufficiency, if z_0 lies outside the boundary defined by $\mathcal{P}_c$, then any path that connects z_s to z_0 must cross one of the stable manifolds of the principal anchor points, i.e. the path crosses $\partial \bar{A}$. Hence, $z_0 \notin \text{int}(\bar{A})$. $\square$

IX. PRACTICAL IMPLICATIONS OF THE RESULTS PRESENTED IN THIS PAPER

A principal objective of system operation is to maintain security. As explained in section I, system security depends on adequate safety margins of feasibility, viability, and transient stability for the operating point to survive normal operating activities and changes and also a specified set of potential disturbances such as the "first contingencies." What then is needed to achieve this goal?

1. One must "*know the territory*." Certainly in the sense of a largely subconscious model of system behavior built up in the operators mind through experience. The capability of a good operator in this area should not be underestimated. However, the complexity and size of today's systems necessitates computer based techniques to evaluate the existence and the margin of feasibility, viability and transient stability which are the main indicators of system operating conditions.
2. This means basically to understand the *taxonomy* of the system in a specific quantitative manner. Such a taxonomy in a physically and mathematically precise sense is offered here, as a much reinforced basis for future developments of system operating and planning technologies as well as more abstract studies. Some such developments are already implied by the results of this paper.
3. In the parameter space the *feasibility boundary* is identified as being composed of five bifurcation surfaces (saddle node, Hopf, singularity induced and tap changer induced ones. Thus it is defined as zero sets by algebraic equations and hence relatively easily computable. This then provides information not just on whether the (existing or proposed) operating point is feasible (this can be checked by direct computation) but also on its distance from the boundary that is its margin of security. Information is further gained on the security of a proposed operating change in the parameter space, specifically its security against any collapse event during the proposed change. (The troublesome automatic tapchanger problem is directly computed as one (or two) additional (restrictive) segments to the feasibility boundary.)
4. Also in the parameter space, the *viability boundary* is quite directly identified by static system model and the limits of viability (that is acceptable $\pm$ deviations from the normal) in voltage, current, frequency, etc. No details are discussed in this paper, but once the boundary is defined, the viability of an operating point and its security in face of possible changes can be evaluated readily along the lines discussed for feasibility in the preceding subsection 4.
5. In the state space, the practically very *important role of the singularity* is clarified. It is noted that the quasi-stationary assumption, which is the basis of the $\dot{x} = f, 0 = g$ model, loses validity its variables simply cease to exist not just at the singularity but in a local band around it. Once the system gets into this band, it dissolves into random processes with unpredictable destinations. Thus the singularity represents the physical limit of predictable system behavior in the state space.

6. The stability boundary thus consists of regular and singular sections. For unconstrained systems, it has been shown that the boundary is regular and it consists of stable manifolds of unstable equilibria and unstable periodic orbits. For our system, which includes singularities, the boundary has regular sections as well as singular sections. The regular sections are anchored at the unstable equilibrium points and unstable periodic orbits, along with new anchor points which are high-dimensional surfaces in the singular set, S . The regular sections are composed of the stable manifolds of the anchor points. Hence, these are only defined as a composite of trajectories, which makes computation difficult. The singular sections consist of zero sets. These can be computed relatively easily and they may even simplify the computation of the regular boundary. The complete structure is worked out here. This is the basis of computational approaches for finding stability margins or maximum clearing times.
7. The taxonomy directly leads to a *classification* of the various voltage collapse phenomena. The three main divisions are parameter space, state space phenomena and special operating or control practices such as locally controlled tap changes, locally controlled load dropping or protective relaying.

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Closure on “A Taxonomy of the Dynamics of the Large Power System with Emphasis on its Voltage Stability”

A sizeable segment of the discussions at the workshop on this paper and the other papers seemed to center around the interface between the mathematical studies and the representation of the physical reality, assumed in the papers. This area never seems to have received a serious, detailed and in-depth study. Since the answers to so many of the questions depend on a thorough understanding of this area, we have decided to provide in this closure, at least the beginnings of the such a study. In the absence of much prior work, it is necessary to proceed step by step and in an often tedious manner, in this first attempt. For this, we ask the reader's indulgence. A few matters concerning the nonmodulated parts of the system such as generators, controls and some aspects of the loads will be discussed first. Following this, the bulk of the text is devoted to the modulated system region i.e. mostly the transmission system. The role of the loadflow manifold (loadflow surface, to be more precise, defined as LF in the paper) as imbedded in a larger state space including the transmission system dynamics $\dot{y} = g(x, y)$ is discussed with an eye on what is doable and what is knowable currently. Limitations of the quasi stationarity and system behavior outside the loadflow manifold are studied with emphasis on how far one should carry arbitrary representations such as lumped RLC models of the transmission system and the importance of keeping a physically valid formulation of problems under study.

An Engineering Analysis of the Dynamics of the Power System and the Quasi Stationary Assumption

I. GENERATOR ELECTROMAGNETICS

Physical magnetic fields cross the rotor along its two symmetry axes (d, q). Various components of these fields couple with various coils on the stator and the rotor and result in corresponding dynamic relationships. For instance a simplified dynamic equation along the direct axis—single axis model—would be

$$\dot{E}' = \frac{1}{T_{d0}} \left(-\frac{x_d}{x_d'} E' + \frac{x_d - x_d'}{x_d'} E \cos(\delta - \delta') + E_{fd} \right) \quad (1)$$

not considering saturation at the moment. Note that none of the material in this paper is restricted to the use of this simple model. It is only used for simplicity to represent generator dynamics. The results apply to models of any detail which would include both axes ($E_d, E_q, E_q', E_q'', E_d'', E_{fd}$). Note that by normalisation (p.u. system) the various voltages here also represent the corresponding fluxes. None of these are AC in the modulated or phasor sense but they are real physical quantities (not AC) stationary or slowly varying (transient T' or subtransient T'' time constants, with T'' marginally slow for quasi stationarity). The latter is the generator terminal voltage at the stator terminals transformed via Blondel-Park-Concordia transformation to an equivalent component which is synchronous with the rotor and so fixed in position relative to the d, q axes.

All variables in the model of (1), and in more detailed dynamic models, represent physical entities (like fields, fluxes and voltages) which are physically stationary with respect to the

rotor and stationary or slowly varying (T' , T'') in time. Fast events at the machine terminals will have little response within this slow dynamics.

Now the rotor is rotating at 3600 rpm and by so doing induces a 60-cy AC voltage in the stator coils proportional to e_G . The voltage e_G is an internal voltage which is varying with the generator dynamics (T' , T'') and i_G is the generated current. The voltage $e_G(t)$ is an actual induced voltage but not typically modulated, however a fictitious modulated form equivalent by Blondel transformation, would be

$$e_G(t) = \sqrt{2}E_G(t)\sin(\omega t + \delta), \quad \omega = 377 \text{ rad/sec} \quad (2)$$

$$i_G(t) = \sqrt{2}I_G(t)\sin(\omega t + \delta + \varphi) \quad (3)$$

Only the terminal or bus voltage $e(t)$ is measurable and recordable as a modulated sine wave. The voltage $e_G(t)$ is then connected to the terminal or bus voltage $e(t)$ by the leakage reactance L_G .

$$e_G(t) - L_G \dot{i}_G(t) = e(t) \quad (4)$$

For instance, if there is no subtransient, then $e_G(t) = e'(t)$ and $L_G = L'$.

$$e'(t) = \sqrt{2}E'(t)\sin(\omega t + \delta') \quad (5)$$

$$i'(t) = \sqrt{2}I'(t)\sin(\omega t + \delta' + \varphi') \quad (6)$$

$$e'(t) - L' \dot{i}'(t) = e(t) \quad (6a)$$

So $e_G(t)$ or $e'(t)$ in equations (2) through (4) is then the *input* into the system, maintained at the generator terminal or at the internal point in the generator respectively. Note that the rotation of the machine acts as the modulating agent providing the link between the (physically) non AC, nonmodulated dynamics inside the machine and the modulated AC world on the outside.

II. GENERATOR ELECTROMECHANICS

Putting everything together for the transmission system an extremely complex but linear differential equation model results consisting of ordinary and partial differential equations in voltage and current. Linearity is lost when coupling to the generator is considered, which on the mechanical side must be in power. Furthermore the instantaneous power values $p_G(t) = e_G(t)i_G(t)$ on the network side vary at 120 cycles because of the modulation on the network side and because the turbine generator inertia does not respond significantly at that speed. So the coupling to the mechanical equation of which

$$J\ddot{\delta} + d\dot{\delta} = P_T - P_G \quad (7)$$

is a rudimentary example where P_T of course is the turbine power and P_G , the generator instantaneous output power, has the form

$$p_G(t) = e_d(t)i_d(t) + e_q(t)i_q(t) \quad (8)$$

on the nonmodulated internal sides of the generator terminals (physical but not readily measured) and, using (2) and trigonometric identities

$$p_G(t) = e(t)i(t) = EI \cos \varphi (1 - \cos 2(\omega t + \delta)) + EI \sin \varphi \sin 2(\omega t + \delta) \quad (9)$$

on the modulated, network side of the generator. $P_G(t)$ is identical in (8) and (9) except for the averaging effect of (7) on (9) when entering the generator.

The physical generator output p_G and $\bar{p}_G(t)$, the average electrical power may both be varying at the rate of E and I in a frequency range of less than 5Hz (which covers most electrical and mechanical swings of the system.) However at 60Hz or 120Hz, the generator inertia acts as a perfect demodulator permitting not even a ripple. Thus the rotation of the generator and its inertia take the role of respectively modulator and demodulator between respectively the nonmodulated e_d, e_q, i_d, i_q components within the generator and the modulated three phase components E, I, δ etc. on the network side. All of these can be time varying at a modest speed corresponding to operating changes and network swings as occur in stability crises.

III. THE LOADS

Very little is known about the actual behavior of composite loads. Any practical rendition of the composite load in terms of its circuit type elements is of course out of the question but it is reasonable for analytical purposes to assume the *existence* of this kind of model. A capacitor bank or a reactor (or SVC) at a bus is a special type of load directly modeled by single circuit components L or C .

Load models are discussed in more detail by the authors' comment on the paper by Liu et. al. [1] in this volume and in Section IV.2 of the Taxonomy paper. As observed there, bus voltage (E, δ) serves as inputs to which the load responds according to its own dynamics. The latter would also possess a general frequency range of less than 5Hz from observational evidence ([13], [21], [28], Taxonomy).

So from the viewpoint of the following study of the transmission system dynamics, load buses will appear much like generator buses with instantaneous load power of the form

$$p_L(t) = e_L(t)i_L(t) = E_L I_L \cos \varphi_L (1 - \cos 2(\omega t + \delta_L)) + E_L I_L \sin \varphi_L \sin 2(\omega t + \delta_L) \quad (10)$$

Here it is sufficient to note that the internal electromagnetics generating the load input $p_L(t)$ or $i_L(t)$ is slow at least in the sense that signals are in a frequency range of less than 10Hz.

IV. THE TRANSMISSION SYSTEM—ISSUES OF MODULATION

An extensive network connects the buses consisting of transmission lines (3 phase but it is enough to look at one under symmetrical conditions) connecting nodes, i.e. buses which also supply generator loads and other elements. The bus voltages and branch currents are modulated AC quantities, typically

$$e(t) = \sqrt{2}E(t)\sin(\omega t + \delta(t)) \quad (11)$$

$$i(t) = \sqrt{2}I(t)\sin(\omega t + \delta(t) + \varphi(t)) \quad (12)$$

The network itself consists of transmission lines with lengths varying roughly between 10-500 miles with notable exceptions in both directions. Commonly the lines are represented by nominal π circuits putting the total line impedance on top and the lumping together capacitances of each half line into the legs. This of course is convenient because then the complicated transmission network is just an electrical network composed of resistors, capacitances and reactances and subject to the basic circuit equations, i.e. Kirchhoff equations, Ohms law etc. and the basic circuit dynamic equations like $i_C(t) = C\dot{e}_C(t)$, $e_L(t) = L\dot{i}_L(t)$, $e_r(t) = ri_r(t)$, $p(t) = e(t)i(t)$ etc. (Note: here $\dot{i}$ denotes the derivative of i). However informed caution is needed in using this model because in reality the transmission line is not a π circuit or T circuit but a distributed parameter device described by partial differential (wave) equations in time and distance along the line.

If the line is carrying a single frequency current with constant amplitude, or at most very slowly varying amplitude, then a π circuit representation may be possible but even then the simple minded "nominal" π circuit described above may need to be replaced by a more refined "equivalent" π circuit to compensate for the distributed constant effect ([43], Taxonomy). If this is done the results are accurate as long as the carrier is of constant frequency and the modulation varies slowly. At 60cy the difference between nominal and equivalent π is not excessive except at very long lines, so either can be used as long as the modulation is slow enough. Operating changes and conventional real and reactive power swings (i.e. angle and voltage swings) tend to be slower than 0.1 sec with most being much slower. In this range, the "quasi stationary" range π circuit representation is usually acceptable, provided that the fast modes associated with the resulting RLC "transmission network" do not get excited during the events. In fact one way of defining quasi stationarity would be the validity of π circuit representation.

One must remain fully conscious however of the condition that the modes of the fictitious RLC network which would be fast do not get excited or other fast moving modulating signals do not enter. When there are fast events, the RLC transmission system and in fact even the phasor type AC dynamics (reactive power impedance, phasor etc., etc.) lose validity. This fact is often overlooked in the literature. It will be discussed in more detail later. The next section however will deal with the slower phenomena covered by quasi stationarity and its region of validity.

V. THE QUASI STATIONARY DOMAIN

This is (fortunately) the domain where operating changes and most of conventional voltage or angle stability swings (except those connected with equipment loss) occur. In this chapter the range and character of those phenomena will be established. To avoid pitfalls and unwarranted conclusions it will be necessary to proceed step by step from the fundamentals. Four assumptions will be needed as will develop in what follows.

ASSUMPTION #1 A π circuit based RLC network representation for the transmission system is valid with small error within the quasi stationary range.

ASSUMPTION #2 The transient modes associated with the RLC transmission system model are not excited within the quasi stationary range.

ASSUMPTION #3 The quasi stationary AC system dynamics remains valid with small error within the quasi stationary range.

ASSUMPTION #4 There exists an open component region C , where the operating point is located on the load flow manifold and the trajectories stay within this manifold.

The normal form of the constrained differential equation model of the power system

$$\dot{x} = f(x, y) \quad (13)$$

$$0 = g(x, y) \quad (14)$$

contains in its ordinary differential equation part $\dot{x} = f(x, y)$, the dynamics (with response times typically slower than 0.1 sec) of generators, various controls, some load components etc. Because of the structure of the power system, f consist of separate segments for each individual generator, SVC, etc. which are not coupled to each other even when connected to the same bus except through the connection with the local bus and the transmission network which connects the buses. For instance for the generators this connection point will be identified at an internal generator voltage such as $x_k = E'$, $x_{k+1} = \delta'$ or the voltage induced in the armature winding such as $x_k = E_G$, $x_{k+1} = \delta_G$ and the generator current $x_{k+2} = I_G$, $x_{k+3} = \varphi_G$ or preferably the instantaneous power flow on the generator side and on the network side.

So no further discussion is needed of the $\dot{x} = f(x, y)$ component. The coupling itself is assumed here to be through an output impedance L_{Gi} such as transient reactance or armature leakage between internal generator voltage e_{Gi} and the bus voltage e_i such that for generators

$$e_{Gi}(t) - e_i(t) = L_{Gi}\dot{i}_{Gi}(t) = x_i - y_i \quad (15)$$

The latter is equivalent to a lossless transmission line between internal voltage E'_i and bus i voltage E_i . Armature resistance can be easily accommodated. The constraint equation $g(x, y) = 0$ will now be derived step by step.

V.1 Step by step to the algebraic constraint $g(x, y) = 0$

The transmission network as now defined consists of buses connected to generators, reactors, capacitors (composed of the applicable legs of the π circuits representing lines and any actual capacitor banks attached), other devices, and loads (any impedance components of loads can be attached as parallel capacitors and inductors to the bus) coupling to the loads will be through (15) in the manner of the generators. Transmission lines then are represented by lumped resistances R_{ij} and inductances L_{ij} in series. This is still a rather limited model of arbitrary size, but it exhibits all the features pertinent to this discussion. The lumped RLC representation of the network however will require the rigorous scrutiny given later.

Clearly at the coupling points on the network side the coupling quantities such as voltage e_{Gi} (2), current i_{Gi} (3) and power $p_{Gi}(t)$ (9) are modulated signals. Consequently it is easy to see that within the network and because of its linearity the modulated sinusoidal character of the voltages and currents will persist throughout within the quasi stationary region on the load flow manifold. The system equations for ordinary operation can be written in the form of Kirchhoff's equations. These equations will be written in the form of power balance to maintain contact with the outside world which is in turbine or load power. Writ-

ing these equations requires only the basic circuit relations for R , L and C that is $e_R = iR$, $i_C = C\dot{e}$, $e_L = L\dot{i}$. Two of these however involve differentiation of modulated signals of the type of (2),(3). These require some special attention.

For example, when a voltage $e(t)$ as defined in (2) is attached to a capacitor, the current

$$\begin{aligned} i_C(t) &= \sqrt{2}I \sin(\omega t + \delta + \varphi) = C\dot{e}(t) \\ &= \sqrt{2}CE \sin(\omega t + \delta) + \sqrt{2}CE(\omega + \dot{\delta}) \cos(\omega t + \delta) \end{aligned} \quad (16)$$

is also a modulated sinusoid but with a changed phase. The instantaneous power (which can only be "real") is

$$\begin{aligned} p_C(t) &= e(t)i_C(t) \\ &= CE\dot{E}(1 - \cos 2(\omega t + \delta)) + CE^2(\omega + \dot{\delta}) \sin 2(\omega t + \delta) \end{aligned} \quad (17)$$

which still contains a modulated sinusoid although at a double frequency. Conversely the voltage on an inductor carrying a current $i(t)$ as shown in (3) is

$$\begin{aligned} e_L(t) &= L\dot{i}(t) \\ &= \sqrt{2}L\dot{I} \sin(\omega t + \delta) + \sqrt{2}LI(\omega + \dot{\delta}) \cos(\omega t + \delta) \end{aligned} \quad (18)$$

$$\begin{aligned} p_L(t) &= e_L(t)i(t) \\ &= LI\dot{I}(1 - \cos 2(\omega t + \delta)) + I^2L(\omega + \dot{\delta}) \sin 2(\omega t + \delta) \end{aligned} \quad (19)$$

and for a transmission line with inductance L_{ij} and resistance R_{ij} ,

$$\begin{aligned} e_i(t) - e_j(t) &= e_{ij}(t) = \dot{i}_{ij}(t)L_{ij} + i_{ij}R_{ij} \\ &= \sqrt{2}L_{ij}\dot{I}_{ij} \sin(\omega t + \delta_i + \varphi_{ij}) \\ &\quad + \sqrt{2}L_{ij}I_{ij}(\omega + \dot{\delta}_i + \dot{\varphi}_{ij}) \cos(\omega t + \delta_i + \varphi_{ij}) \\ &\quad + \sqrt{2}R_{ij}I_{ij} \sin(\omega t + \delta_i + \varphi_{ij}) \end{aligned} \quad (20)$$

and the corresponding equation for power flow into the line at bus i toward bus j is given as

$$\begin{aligned} p_{ij}(t) &= e_i(t)i_{ij}(t) \\ &= E_i I_{ij} \cos \varphi_{ij} (1 - \cos 2(\omega t + \delta_i)) \\ &\quad + E_i I_{ij} \sin \varphi_{ij} \sin 2(\omega t + \delta_i) \end{aligned} \quad (21)$$

Note that E , I amplitudes here are expressed in RMS anticipating the phasor form which will develop later.

Now all the component expressions are in place for the instantaneous network constraints for the network, one node of which is sketched in Figure 1. Three sets of equations are needed.

1. Node equations for instantaneous power balance at node i

$$p_{G_i}(t) - p_{L_i}(t) = p_{C_i}(t) + \sum_j C_{ij} p_{ij}(t) \quad (22)$$

$$e_i(t)i_{G_i}(t) - e_i(t)i_{L_i}(t) = e_i(t)i_{C_i}(t) + \sum_j C_{ij} e_i(t)i_{ij}(t) \quad (23)$$

where C_{ij} is one if buses i and j are connected, and zero otherwise. Instantaneous power of course can only be real. The power form (instead of current) is necessitated by the presence of non-electrical real power inputs at the generators and the loads. All components for those are already defined in equations (9)-(21). Note that loads L_i are treated like generators in these equations. Both are of course sources or sinks of real power. Internal dynamics of the loads are not well defined but should not affect the results unless, P_{L_i} is empty except for an impedance component, which would be unrealistic.

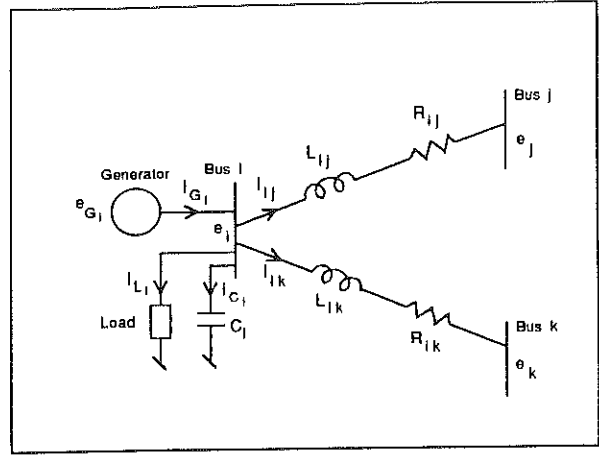


Figure 1. Sketch for the instantaneous state equations

2. Transmission line flow equations (in lieu of loop equations a la Kirchhoff). These are already defined for the transmission line in (20)-(21). For uniformity, let us redefine equation (20) in terms of the power flow by multiplying with i_{ij} .

$$e_i(t)i_{ij}(t) - e_j(t)i_{ij}(t) = L_{ij}i_{ij}\dot{i}_{ij} + R_{ij}i_{ij}^2 \quad (24)$$

3. Generator coupling equations. Let us multiply equation (15) by the current i_{G_i} to rewrite the equation in terms of power flow.

$$e_{G_i}(t)i_{C_i}(t) - e_i(t)i_{G_i}(t) = L_{G_i}i_{G_i}\dot{i}_{G_i}(t) \quad (25)$$

The combined forms for each of the three network equations will now be displayed. These couple to equations (7)-(9) and (10) through p_{G_i} and p_{L_i} .

1. Bus power balance (from (23))

$$\begin{aligned} p_{G_i} - p_{L_i} &= E_i I_{G_i} \cos \varphi_{G_i} (1 - \cos 2(\omega t + \delta_i)) \\ &\quad + E_i I_{G_i} \sin \varphi_{G_i} \sin 2(\omega t + \delta_i) \\ &\quad - E_i I_{L_i} \cos \varphi_{L_i} (1 - \cos 2(\omega t + \delta_i)) \\ &\quad - E_i I_{L_i} \sin \varphi_{L_i} \sin 2(\omega t + \delta_i) \\ &= \sum_j C_{ij} [E_i I_{ij} \cos \varphi_{ij} (1 - \cos 2(\omega t + \delta_i)) \\ &\quad + E_i I_{ij} \sin \varphi_{ij} \sin 2(\omega t + \delta_i)] \\ &\quad + \omega C_i E_i^2 (1 - \cos 2(\omega t + \delta_i)) \frac{\dot{E}_i}{E_i \omega} \\ &\quad + \omega C_i E_i^2 \sin 2(\omega t + \delta_i) \left(1 + \frac{\dot{\delta}_i}{\omega}\right) \end{aligned} \quad (26)$$

2. Transmission line power balance (from (24))

$$\begin{aligned} 0 &= E_i I_{ij} \cos \varphi_{ij} (1 - \cos 2(\omega t + \delta_i)) \\ &\quad + E_i I_{ij} \sin \varphi_{ij} \sin 2(\omega t + \delta_i) \\ &\quad - E_j I_{ij} \cos(\delta_i + \varphi_{ij} - \delta_j) (1 - \cos 2(\omega t + \delta_j)) \\ &\quad - E_j I_{ij} \sin(\delta_i + \varphi_{ij} - \delta_j) \sin 2(\omega t + \delta_j) \\ &\quad - \omega L_{ij} I_{ij}^2 (1 - \cos 2(\omega t + \delta_i + \varphi_{ij})) \frac{\dot{I}_{ij}}{I_{ij} \omega} \\ &\quad - \omega L_{ij} I_{ij}^2 \sin 2(\omega t + \delta_i + \varphi_{ij}) \left(1 + \frac{\dot{\delta}_i + \dot{\varphi}_{ij}}{\omega}\right) \\ &\quad - R_{ij} I_{ij}^2 (1 - \cos 2(\omega t + \delta_i + \varphi_{ij})) \end{aligned} \quad (27)$$

3. Generator Coupling Equations (from (25))

$$\begin{aligned}
0 = & E_i I_{G_i} \cos(\delta_i + \varphi_{G_i} - \delta_{G_i})(1 - \cos 2(\omega t + \delta_{G_i})) \\
& + E_i I_{G_i} \sin(\delta_i + \varphi_{G_i} - \delta_{G_i}) \sin 2(\omega t + \delta_{G_i}) \\
& - E_i I_{G_i} \cos \varphi_{G_i} (1 - \cos 2(\omega t + \delta_i)) \\
& - E_i I_{G_i} \sin \varphi_{G_i} \sin 2(\omega t + \delta_i) \\
& - \omega L_{G_i} I_{G_i}^2 (1 - \cos 2(\omega t + \delta_i + \varphi_{G_i})) \frac{\dot{I}_{G_i}}{I_{G_i} \omega} \\
& - \omega L_{G_i} I_{G_i}^2 \sin 2(\omega t + \delta_i + \varphi_{G_i}) \left(1 + \frac{\dot{\delta}_i + \dot{\varphi}_{G_i}}{\omega}\right) \quad (28)
\end{aligned}$$

Note that the I variables can be eliminated between the three instantaneous sets of equations (26), (27) and (28) although it would be difficult in this form to actually do it. Simplifications of form are clearly needed.

First let us introduce a short hand notation for (26)-(28)

$$G(x(t), y(t), \dot{y}(t)) = 0 \quad (29)$$

Note that y consists of voltages E_i in the network, mostly bus voltages, phase angles δ_i of the same and also currents I_{ij} and their phase angles φ_{ij} , G and y are redundant sets of equations and variables respectively. Currents I will eventually be eliminated between (26) and (27)-(28). To reach that goal, first $y(t)$ which are slow varying (due to slow variation in x) time functions will be split like

$$y(t) = y(t_0) + \Delta y(t) = y(t_0)[1 + \varepsilon(t)] \quad (30)$$

where t_0 is a specific time, $t \geq t_0$ is the running time and so $\varepsilon(t)$ is the relative increment in $y(t)$ which builds up beyond t_0 . This ε increment has several components not all of which will be attached to any particular $y_k(t_0)$ component.

1. In equations (26)-(28) there are observable terms of the type $\dot{E}$, $\dot{I}$, $\dot{\delta}$, $\dot{\varphi}$ which of course are time varying. For illustration, let us look at one component of (26)-(28) say $p_C(t)$ defined in (17). This can readily be expressed in a simpler form as

$$\begin{aligned}
p_C(t) &= e(t) i_C(t) \\
&= \omega C E^2 \left[\left(1 + \frac{\dot{\delta}}{\omega}\right) \sin 2(\omega t + \delta) \right. \\
&\quad \left. + \frac{\dot{E}}{E \omega} (1 - \cos 2(\omega t + \delta)) \right] \quad (31)
\end{aligned}$$

This form allows us to separate error terms in a normalized form e.g. $\dot{\delta}/\omega$ for variation of angular velocity, i.e. rate of angular variation relative to steady state angular velocity and $\dot{E}/(\omega E)$ for normalized voltage variation rate $\dot{E}/E$ relative to static angular velocity ω . Note that ω defines the frequency of the carrier, the reference point in quasi stationarity. These terms then represents one typical component of ε which will be denoted $\varepsilon'(t)$.

2. Another component is implied directly by the form of (30).

$$\varepsilon''(t) = \frac{1}{y(t_0)} \int_{t_0}^t \dot{y}(t) dt \quad (32)$$

So, the total error is due to both $\varepsilon'(t)$ and $\varepsilon''(t)$. Here we note that both errors are normalised to a unit value of y and both are linear in $\dot{y}(t)$ that is

$$\mathcal{O}(\varepsilon) = \mathcal{O}(\varepsilon') = \mathcal{O}(\varepsilon'') = \dot{y} \quad (33)$$

Now separate G as follows

$$G(x(t), y(t)) = G(x(t), y(t))|_{t_0} + G_\varepsilon(x, y, \varepsilon(t)) \quad (34)$$

This can be done term by term. Though there are only finitely many such terms, we may neglect some higher order error terms in the simplification. Also, it should be noted that this term by term separation will remain reversible after linear operations like averaging in time. Furthermore, in $G(x(t), y(t))|_{t_0} = G_0(x, y)$, everything except the carrier $\sin(\omega t + \dots)$ is independent of the running time t . Since the preceding manipulation affected only the modulating signals E , I , δ etc., the $\sin(\omega t + \dots)$ carriers are still intact in both G_0 and G_ε .

A. Let us examine G_0 first. Since $x(t_0)$ and $y(t_0)$ are now not time varying, the only time variation remaining in G_0 is due to the trigonometric functions of the type $(1 - \cos 2(\omega t + \delta_0))$ or $\sin 2(\omega t + \delta_0)$, with the phase δ_0 a constant. As was pointed out in Section I, the generator inertias are perfect smoothing filters at 60Hz or 120Hz power variation. Hence, the only physically visible aspects of these terms will be their averages over $\pi/\omega = 1/120$ seconds or integer multiples thereof. Let us look at a typical term in (26)-(28) such as

$$E_i I_{G_i} \cos \varphi_{G_i} (1 - \cos 2(\omega t + \delta_i)) + E_i I_{G_i} \sin \varphi_{G_i} \sin 2(\omega t + \delta_i) \quad (35)$$

Clearly the average is

$$P_{G_i} = E_i I_{G_i} \cos \varphi_{G_i} \quad (36)$$

However the term

$$Q_{G_i} = E_i I_{G_i} \sin \varphi_{G_i} \quad (37)$$

represents a quantity of energy which is swinging at peak rate Q_G and frequency 2ω across the generator bus interface. This is valuable information and indeed Q_G is carried under the name of "reactive power." One should not forget that reactive power *does not flow*, it only swings with zero average energy transfer, although its defining equations look very much like those of the real power. Now introducing phasor notations

$$\hat{E}_i = E_i / \underline{\delta}_i = E_i e^{j\delta_i} \text{ etc.} \quad (38)$$

one can write

$$S_{G_i} = P_{G_i} - jQ_{G_i} = \hat{E}_i \hat{I}_i^* \quad (39)$$

B. Time averaging over $t - t_0 = \pi/\omega$ can now be applied to $G_\varepsilon(x, \varepsilon(t))$. The terms in here are time functions, actually the derivatives $\dot{E}$, $\dot{I}$, $\dot{\delta}$, $\dot{\varphi}$ multiplied by sinusoids at the rate of 2ω . Term by term, they are not complicated, so the averaging can actually be performed at a given t_0 for a given system. More specifically, the time varying terms also possess a structure

$$k_c(t_0) \varepsilon'(t) (1 - \cos 2(\omega t + \delta)) + k_s(t_0) \varepsilon'(t) \sin 2(\omega t + \delta) \quad (40)$$

or

$$k_c(t_0) \varepsilon''(t) (1 - \cos 2(\omega t + \delta)) + k_s(t_0) \varepsilon''(t) \sin 2(\omega t + \delta) \quad (41)$$

These need now be time averaged on t over $t_0 - \frac{1}{120}$ to t_0 . The terms

$$\bar{\varepsilon}_1 = 120 \int_{t_0 - \frac{1}{120}}^{t_0} \varepsilon'(t) (1 - \cos 2(\omega t + \delta)) dt \quad (42)$$

$$\bar{\varepsilon}_2 = 120 \int_{t_0 - \frac{1}{120}}^{t_0} \varepsilon''(t) (1 - \cos 2(\omega t + \delta)) dt \quad (43)$$

can be integrated out without much trouble or it can simply be observed that they are still of the order of magnitude of the relative error rate terms $\dot{E}/(E\omega)$ or $\dot{\delta}/\omega$. These $\bar{\epsilon}$ terms provide the error terms in the real power equations. For reactive power, if integration by parts is applied to the terms

$$\bar{\epsilon}_3 = 120 \int_{t_0 - \frac{1}{120}}^{t_0} \epsilon'(t) \sin 2(\omega t + \delta) dt \quad (44)$$

$$\bar{\epsilon}_4 = 120 \int_{t_0 - \frac{1}{120}}^{t_0} \epsilon''(t) \sin 2(\omega t + \delta) dt \quad (45)$$

before time averaging $G_0(x, y)$, then we can observe that $\bar{\epsilon}$ will readily rejoin the reactive power equations. Then, these error terms $\bar{\epsilon}$ can be combined (possibly with a second order small discrepancy) to $g(x, y)$ as

$$\left[\begin{array}{c} P(x, y(1 + \bar{\epsilon})) \\ Q(x, y(1 + \bar{\epsilon})) \end{array} \right] = g(x, y(1 + \bar{\epsilon})) = 0 \quad (46)$$

Or letting $y' = y(1 + \bar{\epsilon})$, for the complete system,

$$\dot{x} = f(x, y') \quad (47)$$

$$0 = g(x, y') \quad (48)$$

This applies to any fixed time $t = t_0$ where the error is to be evaluated. Here y is the true value, but the solution obtained in solving the $\dot{x} = f(x, y'), 0 = g(x, y')$ problem is y' so that the use of $g(x, y') = 0$ results in y values which may be in error up to a relative or percentage level of $\bar{\epsilon}$ term by term. This then provides a basis for judging the accuracy at various points of the quasi stationary region and particularly at its boundary. Note that $\bar{\epsilon}$ and hence $\bar{\epsilon}$ depend only on the relative rates of change $\dot{E}, \dot{\delta}$ so indeed the error should be small at slow variations of voltage and current. It is important that the errors can be actually computed by simple if tedious operations as described, or overall relative order of magnitude can be judged. Furthermore, the relative error is identified individually for each y state variable. Also, it is of a uniform relative magnitude in the order of relative magnitudes of the velocities such as $\dot{E}/(E\omega)$ or $\dot{\delta}/\omega$.

V.2 The region of quasi stationarity and its boundary

As was established in the paper a unique solution of $g(x, y) = 0$ exists in the interior of open components C of the state space. These components make up the load flow manifold. They are separated by a singular surface (and its vicinity). Equilibrium or operating points and their regions of attractions are situated on individual components. All predictable transient behavior takes place in the interior of C . And finally C is the Lebensraum of quasi stationarity. The following claim can be made.

The operating point and all predictable swings and transient behavior connected with angle and voltage stability of the power system are within the quasi stationary range and can be studied using the algebraically constrained ordinary differential equation model of the form (13)-(14). To prove this, one has to estimate the errors caused by time variation of states. The normal frequency and time constant ranges of typical phenomena are known and the equations of Section V.1 define the relative errors associated with various circuit elements e.g. $\epsilon' = \dot{E}/(E\omega)$. Note that $\dot{E}/E$ normalizes the voltage magnitude out of the picture so that $\dot{E}/E$ is a dimensionless time rate representation of the voltage variation. This compared with the rate of change ω of the phase (rotor) angle at 60Hz gives the relative magnitude of the error vis a vis the dominant stationary contribution from the

same source. $\dot{\delta}/\omega$ and $\dot{\delta}/\omega$ are the other two type of errors connected with the derivative terms, ϵ' in (26)-(28). The errors connected with the local time variation of y (denoted ϵ'' above (32) is in the same order of magnitude. The order of ϵ which consists of both ϵ' and ϵ'' , is of order $\dot{E}/(E\omega)$ or $\dot{\delta}/\omega$. Same is true for the average $\bar{\epsilon}$.

Swings involved in angle stability typically have a frequency (or comparable time constant of less than 2 Hz). So the error for angle stability swings is in the order of magnitude of

$$\epsilon \approx \frac{\dot{E}}{E\omega} \approx \frac{\dot{\delta}}{\omega} < \frac{2\text{Hz}}{60\text{Hz}} \sim 3\% \quad (49)$$

So the relative error from this source is $\bar{\epsilon} < 3\%$. To neglect these errors would be a good and safe approximation.

Voltage swings tend to be somewhat faster up to may be 6 Hz but then again midrange swings, which are much slower, tend to be more critical here. Thus, for voltage stability the error of quasi stationary computation will be less than 6/60~10%. This maximum is stretching the limit but the bulk of the cases will be safely in the quasi stationary region.

Under typical stability crises circumstances, one need not be specially concerned with quasi stationarity. Furthermore the state will stay on the component C of the load flow manifold which also contains the operating point.

V.3 The role of the RLC network model during quasi stationarity

As defined in equations (26)-(28), the system equation written as $G(x, y, \dot{y}) = 0$ looks like a differential equation in y . It is a differential equation of course and taken together with $\dot{x} = f(x, y)$, it is a well posed problem mathematically. In this problem the Jacobian will have two well defined and well separated sets of eigenvalues. Those mostly associated with x are relatively slow, generally slower than 0.1 seconds, often much slower. These basically define the dynamics of generators (voltage and rotor angle) some load components and other devices. The other group is connected with the dynamics of the transmission system, which is modeled here as an RLC network based on π circuit representation of the transmission lines. The eigenvalues here are faster or much faster than 60cy. In the quasi stationary condition trajectories stay within a component of the state space located on the load flow manifold (Assumption #4). If so, then the y dynamics does not get excited in fact this is an equilibria surface of this y dynamics (actually this is so by definition of the load flow surface.) Of course the actual operating point will still track the $\dot{y}$ changes within the $g(x, y) = 0$ load flow but this is in the nature of tracking a variation imposed from the outside from $\dot{x} = f(x, y)$, a sort of forced slow variation not a transient response in the sense of y .

Consequently the trajectories defined by the $\dot{x} = f, 0 = g$ model indeed define the correct and precise dynamics of the quasi stationary system and the error defined in equations (31) and (32) are not errors of the trajectories themselves but errors in the phasor-based computations used. Fortunately as pointed out in Section V.1 (33), these are small and in fact very small over most of the quasi stationary range. It is interesting and important that the errors show up term by term in the computation. So each y has a potential relative error $\bar{\epsilon}$ of the same order and this error depends on the relative velocity of the various coordinates so that it is small if the relative velocity is small.

This is the essence of quasi stationarity. Furthermore equations (42)–(45) for the fast time offer a way of actually computing the errors for each state y at least in principle. Normally one needs to consider the overall order of these errors as was done in Section V.2.

It should be emphasized that there is no problem with the validity of the π circuit based $\dot{y} = g(x, y)$ model as used in this section i.e. within the load flow manifold in quasi stationary conditions where motions are slow. At those speeds π circuits give good approximation for transmission lines.

The RLC π equivalent model implies the presence of another signal component, a steady state oscillation in the network when the system is carrying a steady state AC signal. Natural frequencies of the transmission system in the RLC model seem to be higher than 60Hz but in the same order of magnitude. This closeness indicates that a steady 60Hz signal would excite some resonant oscillation at the natural frequencies of the network. These would not change the observations made here. They would only be superimposed on the 60Hz quasi stationary signals with no effect. Some amount of energy would be swinging in a situation somewhat akin to reactive power. This phenomenon does not seem to have been observed, another doubt thrown on the validity of the RLC network model outside the load flow manifold.

VI. OUTSIDE THE QUASI STATIONARY DOMAIN

Assumptions #1–#4 are satisfied within the quasi stationary domain but generally not outside of it. Let us investigate what happens to these assumptions outside the domain.

Load flow manifolds and open components may or may not exist depending on the parameter values, but in any case the state and the trajectories may or may not be imbedded in them. So the algebraic constraint $0 = g(x, y)$ does not apply (Assumption #4 is not valid). Outside the load flow manifold, i.e. in an enlarged statespace it cannot be assumed that voltages and currents are modulated AC quantities. Thus the quasi stationary assumption does not apply (Assumption #3 is out). This means that the $0 = g(x, y)$ equation or $\dot{y} = g(x, y)$ equation ceases to exist. One has to revert to the $G(x, y, \dot{y})$ formulation of instantaneous dynamics of (26), (27) and (28) with substantial modifications. The internal generator voltages retain their form of $e_{G_i}(t) = E_{G_i} \sin(\omega t + \delta_{G_i})$. Also the $\dot{x}$ of (x, y) dynamics (which is not modulated) will stay unaffected in form but the various generators are not necessarily synchronized any more. There is no basis to assume that bus voltages or line currents will be modulated signals like $e_i(t) = E_i \sin(\omega t + \delta_i)$ or $i_{ij}(t) = I_{ij} \sin(\omega t + \delta_i + \varphi_{ij})$ unless it turns out after solving the differential equations that they are. Within the $G(x, y, \dot{y})$ equations (26)–(28), these variables must be represented as instantaneous variables simply as $e(t)$ and $i_{ij}(t)$. This leaves the system equations

$$\dot{x} = f(x, y) \quad (50)$$

$$0 = G(x, y, \dot{y}) \quad (51)$$

where the $G(x, y, \dot{y})$ equation after modifying (26)–(28) as postulated becomes (These still couple to equations (7)–(8) and (10) through p_{G_i} and p_{L_i})

1. Bus power balance equations

$$\begin{aligned} p_{G_i}(t) - p_{L_i}(t) &= e_i(t)i_{G_i}(t) - e_i(t)i_{L_i}(t) \\ &= C_i e_i(t) \dot{i}_i(t) + \sum_j C_{ij} e_i(t) i_{ij}(t) \end{aligned} \quad (52)$$

2. Transmission line equation

$$e_i(t) - e_j(t) = L_{ij} \dot{i}_{ij} + R_{ij} i_{ij}(t) \quad (53)$$

3. Generator coupling equation

$$E_{G_i} \sin(\omega t + \delta_i) - e_i(t) = L_{G_i} \dot{i}_{G_i}(t) \quad (54)$$

where p_{G_i} and p_{L_i} are still the inputs. Note that y variables in (51) are different from those in (29). Equation (29) is written in terms of RMS voltages and phases (slowly time varying), whereas in (51), the y variables are instantaneous voltages and currents, which are not necessarily sinusoidal. Also, though the transmission network has been represented as a linear RLC circuit, the equations would be still nonlinear as power balance equations.

VI.1 Feasibility of studies connected with transmission system dynamics

What happened so far is that the phasor calculus based differential equations (13) and (14), were replaced by an instantaneous equation of not generally modulated signals of the form

$$\dot{x} = f(x, y) \quad (55)$$

$$0 = G(x, y, \dot{y}) \quad (56)$$

in an enlarged dynamic state space of $X \times Y$, $x \in X \subset \mathbb{R}^n$, $y \in Y \subset \mathbb{R}^m$. To make studies in the enlarged dynamic state space of $X \times Y$, it will be necessary to have them on the complex (but not constrained) differential equations of (52)–(54) combined with $\dot{x} = f(x, y)$. Since there are inputs present in a modulated form and the network is linear in $e(t)$ and $i(t)$ modulated components may occur on parts of expressions. The point is that one should not guess but work out the solution from a physically based system representation like (52)–(54). The problem formulation should manifest mathematical and physical validity. The validity of the RLC model for the network is quite limited as will be shown.

In the enlarged statespace the eigenvalues of the Jacobian increase in number from n to $n + m$. More importantly these form two distinct groups. The eigenvalues associated with the x states represent relatively slow dynamics, slower than 0.1Hz. The eigenvalues associated with the y states are generally faster than 6Hz. This separation raises some very interesting possibilities.

1. Utilizing the joint (x, y) dynamics formally as presented in (55) and (56). One could ask the stability of trajectories within components in this larger space as against the constrained space of (13), (14) this would establish the stability of state space components within the load flow manifold and the possibility of jumps between components. This in fact can be rather readily computed using the model of (55) and (56) formally, assuming that it is valid.
2. A dynamic study of the system behavior within the full $X \times Y$ space (like the one carried out in this paper for inside the loadflow manifolds) could be carried out. Equilibria, stability boundaries, singularities, extension of pseudo equilibria could be pursued for the enlarged space. The

results of the constrained case as presented in this paper could be imbedded in the full state space. The gradual dissolution of the phasor type quasi stationary dynamics around the singularity would still present problem and introduce random effects. Clearly Assumption #2 is out in all this. The y dynamics would become very active actually dominant. Also the dynamics of (55) and (56) in the absence of the phasor simplification is much more complex than the dynamics of (13) and (14), so results would come with even more difficulty. But the effort would be worth while provided the lumped constant RLC model used in (55) and (56) is valid. Two circumstances, when the system is off the load flow manifold will be treated separately in Sections VI.3 and VI.4.

3. The separation of fast and slow eigenvalues for the x and y subspaces present an ideal hunting ground for singular perturbation type studies, provided the RLC dynamics is valid. But a customary dash in this direction is definitely premature.

VI.2 Validity of the lumped RLC transmission system model

The hitch is the validity of the transmission system dynamics which is the basis of the dynamics of $G(x, y, \dot{y})$ as given in equations (52)–(54). This is a lumped constant dynamics based on π circuit representation. This approach reduces the transmission system into a basic complex RLC circuit.

As discussed in detail in the book ([43], Taxonomy) and elsewhere, simple “nominal” π representation (lumping capacitances of each half line at the line ends) or an “equivalent” π which takes into consideration the realistic wave motion effects can give respectively good or precise results for steady state 60cy flow across transmission lines or transmission networks. Slow modulation which does not excite the transient modes of the network i.e. quasi stationarity (Assumption #2) will work well. So Assumption #1 will be indeed valid in the quasi stationary range as postulated in Section V.1. Validity can be extended to different carrier frequencies as long as the condition is stationary but the equivalent π circuits will be frequency dependent and increasingly different from the nominal π with increasing carrier frequency. This leads to the fact that π equivalents lose all validity in transient conditions especially at fast transients which occur on the transmission system where the frequency components range at least in the hundreds of Hz and of course any idea of modulation disappears. So Assumption #1 is definitely not applicable outside the quasi stationary range.

This leaves the proposed endeavors 1, 2, and 3 outlined in Section VI.1, without any valid model to use. It will be an arduous and long research effort to bridge this gap.

It is easy to see however that there is great physical difference between the reality and the lumped RLC model. Unlike the latter, on the real system there are no lumped capacitances at the buses except where capacitor banks are connected. In the π circuit model, charge or energy dumps at arbitrary rates into the capacitive bucket at the bus, whereas on the real system it is forced to flow through a narrow pipe of a transmission line in the form of a wave which flows at only one speed regardless of the imposed voltage or energy. Transients ranging over the transmission system are often set in motion by lightning. These consist of traveling waves bouncing back and branching at the buses. Lightning arrestors interface with them and limit their

heights. The same mechanisms can be set in motion by events on the system itself. The lumped parameter model would swing away smoothly like a laboratory circuit model.

VI.3 Events following loss of major equipment

When a large generator or transmission line is lost (say without a fault) suddenly, the system state will stay momentarily constant but the system itself and so the load flow manifold would have changed. So the state will find itself momentarily off the manifold. On the other hand because of the network nature of the system events like this typically have a geographically localized effect, there will be little change in the surface at distant buses. So the system state would be likely to return to the new load flow manifold. This indeed is the experience in power system operation. In fact, if the RLC model were valid outside the load flow manifold it would be a rather easy matter to find the post change trajectory in the joint $X \times Y$ space, possibly by the use of singular perturbation techniques. Unfortunately as explained above there is no reason to expect that the lumped RLC model of the transmission network is valid outside the load flow manifold ($0 = g(x, y)$) so such studies would be fictitious and futile. There are papers in the literature which use singular perturbation by eclectically singling out the “dynamics” of a single element such as one component of a load. Even if correct, this certainly would be only a part of the dynamics and any analysis based on them should be considered fictitious. As mentioned above, with moderate breakup the system usually returns to the same component of the load flow, and hopefully to the same region of attraction. In any case it will eventually land on some component which is stable. Once there, it resumes its quasi stationary character subject to the behavior studied in this paper. Note that in case of faults, some very fast changes occur within a time interval of as short as three cycles, if all goes well with the protection. During this period, quasi stationarity will also be lost, at least part of the time, although computing the trajectories during the fault would be typically justified since this is a system swing of limited speed. Whatever be the accuracy of these under fault calculations, the point is that at the end the system is left in a state on the loadflow manifold which is quasi stationary.

VI.4 Events at the Singular Boundary

When the Jacobian $D_y g$ is singular then the existence conditions of a solution (x, y) on the load flow surface are not satisfied. So when this point is reached the state has to leave the surface. This precise situation is a mathematical abstraction, but very physical events take place near the singularity. The quasi stationary model is valid up to some neighborhood of the singularity. Some of the y variables will move at a high speed near the singularity. This can be observed in Figure 5 of the Taxonomy paper where a trajectory approaches the singularity. Quasi stationarity clearly disappears in a neighborhood close to the singularity. This means that equations $g(x, y) = 0$ simply disappear, or cease to exist. Note that the terms such as $E/(\omega E)$ also disappear along with the rest of the equation since they also depend on quasi stationarity. In fact, the entire phasor based AC dynamics operations (reactive power, reactances, impedances, power flow equations, equivalent π and T circuits etc. etc.) lose meaning here. The RLC model which was used above in Sections V and VI to study transmission system dynamics is valid only on the load flow manifold while quasi stationarity prevails. It is not valid outside the quasi stationary

region and of course the singularity and its vicinity is quite outside of this region. In this region a valid model would need to accommodate the wave character of transmission line behavior. Then the transients consist of traveling waves bouncing back and forth between buses upto a few hundred miles away. Since these waves run 3,000 miles in $1/60$ of a second (the 60 Hz wavelength) these transients will be very fast even compared to the 60 cycle carrier. No quasi stationarity applies here. It would be very difficult to develop a model for these conditions. Currently none exists.

The $\dot{x} = f(x, y)$ functions survive (these are not modulated) but become decoupled since the y 's, the only coupling features between generator buses, lose their meaning. So this is a true voltage collapse. No trajectory can be predicted even theoretically. Trajectories dissolve into a random process. This happens because the signals $e(t)$ or $i(t)$ are inherently not synchronized with the 60 Hz carrier. This is the basic practice on the power system and it would be both unpractical and irrelevant to attempt such synchronization. The absence of synchronization implies the existence of a random phase component Δ (like $\sin(\omega t + \delta + \Delta)$) in the sinusoids where Δ would be random with a uniform distribution over 0 to 2π . This Δ then determines the initial state and hence the ensuing trajectory, when quasi stationarity changes into instantaneous dynamics (actually at a mushy boundary). This is a typical start of a random process.

In fact nothing can be predicted about the state except that it will take off on a random trajectory which will take it on an unpredictable trip ending at some state space component C . Once there, then the quasi stationarity applies again but it is far from guaranteed that the state will then settle down to a stable and viable equilibrium. For all practical purposes a voltage collapse occurs when approaching a singularity. The system is broken up by the selective protection and will need to be restored.

The reasonable thing is to study the $\dot{x} = f, 0 = g$ system equations and consider the singular surface as a barrier to predictable characterisation (which it certainly is).

VII. REFERENCES

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Abstract - This investigation of voltage collapse is based upon a power system model which includes generator dynamics, and maintains network topology. The model is described by a set of differential-algebraic equations. The paper explores the connection between voltage collapse and singularity of the algebraic equations of the model. This connection is investigated in terms of both large and small disturbance behaviour. The major factors involved in long term voltage collapse are explored, with a view to clarifying their influence on the stability of weak systems.

Keywords - voltage collapse, differential-algebraic systems, small disturbance stability, large disturbance dynamics, load modelling

I. INTRODUCTION

Numerous power systems throughout the world have suffered failures which have been associated with abnormal voltage behaviour. A mechanism for such failures has been described by Lachs [1] and Brownell and Clark [2]. Typically a sequence of events occurs which weakens the power system, and ultimately results in its failure. These events may include line and generator tripping, load increases, and limiting or loss of reactive power support. The frequency and severity of these collapses has resulted in focusing a substantial research effort on the area of voltage instability/collapse. Even so, this subject is still not well understood.

As voltage collapse is approached, possibly due to some parameter variation, the normal operating point and another solution point of the network equations move toward each other. Continued parameter variation results in a bifurcation as these two points coalesce then disappear. In the vicinity of a bifurcation point the power system becomes very difficult to control. After the disappearance of the operating point, the system breaks down.

Even though voltage collapse is, in many respects, a long term phenomenon, the ultimate failure of a power system occurs quickly, and is therefore a short term problem. Fundamental to the understanding of the voltage collapse mechanism is an understanding of this ultimate mode of failure.

This paper initially focuses upon system failure mechanisms at the point of final collapse. Generally in voltage collapse studies, generator angle dynamics are neglected. This simplification is based upon observations of actual system failures which indicate that angles remain bounded whilst voltages fall abnormally. However in this investigation angle dynamics have been included so that their role in the failure mechanism can be clarified. Having investigated ultimate failure modes, the paper then explores some long term mechanisms which can drive a system to a point of collapse. Some insights are provided into the weakening process that the system undergoes during that long term period.

A structure preserving model developed in Section 2 incorporates both machine dynamics and network topology. This model consists of a set of differential-algebraic equations. The algebraic equations introduce

some complexities into the behaviour of the model. In particular, at points of algebraic singularity, the model breaks down. This is considered in Section 3. In Section 4, energy function concepts are introduced. These concepts are used to enable collapse mechanisms to be viewed in terms of energy behaviour. It is shown in Section 5 that there exists a strong connection between short term voltage collapse and the algebraic singularity of the DA model. In Section 6, some long term mechanisms which can drive a system to a point of collapse are investigated. Realistic collapse behaviour is simulated. Conclusions are given in Section 7.

II. POWER SYSTEM MODEL

This section initially considers the development of a model which is suitable for the investigation of the (short term) behaviour of a power system at the point of final failure following a slow collapse. This short term model consists of a set of differential-algebraic equations. A locally equivalent ODE representation is explored. (This model is developed in some detail to enable later investigation of its DA structure.) Additions to the model which allow the representation of some aspects of long term collapse behaviour are then considered.

2.1 Short Term Power System Model.

A sufficiently accurate model of short term power system behaviour is obtained when machines are represented using the classical machine model. Other generator/exciter models could be substituted if necessary. However, this machine representation simplifies the details of the model without detracting from the main concepts. In accordance with this model, the synchronous machines are represented by a constant voltage $|E_i|$ in series with transient reactance.

Consider a network consisting of n_0 buses connected by transmission lines and transformers. There are generators at m of these buses. Hence, the network is augmented with m fictitious buses representing the generator internal buses, in accordance with the classical machine model. The total number of buses in the augmented system is therefore $n_0+m:=n$.

Let the complex voltage at the i^{th} bus be the (time varying) phasor $V_i = |V_i| \angle \delta_i$ where δ_i is the bus phase angle with respect to a synchronously rotating reference frame. The bus frequency deviation is given by $\omega_i = \dot{\delta}_i$. The model of interest is based on machine reference angles, with the n^{th} bus taken as the reference. The internodal angles $\alpha_i := \delta_i - \delta_n$ are used. Define

$$\begin{aligned} |V| &= [|V_1|, \dots, |V_{n_0}|]^t \\ \alpha &= [\alpha_1, \dots, \alpha_{n-1}]^t = [\alpha_\ell^t \quad \alpha_g^t]^t \\ \omega_g &= [\omega_{n_0+1}, \dots, \omega_n]^t \end{aligned}$$

where 't' denotes matrix transpose, α_ℓ is the

n_0 -vector of load angles, and $\underline{\alpha}_g$ is the $(m-1)$ -vector of generator angles.

Let P_{bi} and Q_{bi} denote the total real and reactive power leaving the i^{th} bus via transmission lines. Define

$$\tilde{P}_b = [P_{b1}, \dots, P_{b, n-1}]^t = [\tilde{P}_\ell^t \tilde{P}_g^t]^t$$

where subscripts ℓ, g are again used to denote loads and generators respectively.

Now consider the modelling of loads. Denote the real and reactive power demand at the i^{th} bus by P_{di} and Q_{di} respectively. In general these powers are functions of voltage and frequency. In this investigation, the loads shall be assumed to satisfy

$$P_{di} = P_{di}^0 |V_i|^{C_i} \quad i=1, \dots, n_0 \quad (1a)$$

$$Q_{di} = Q_{di}^0 |V_i|^{n_i} \quad (1b)$$

In [3,4], it is shown that combining the power balance equations with the usual generator swing equations gives the model

$$\dot{\underline{\omega}}_g = -\underline{M}_g^{-1} \underline{D}_g \underline{\omega}_g - \underline{M}_g^{-1} \underline{T}_g^t (\tilde{P}_g(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) - \tilde{P}_M^0) \quad (2a)$$

$$\dot{\underline{\alpha}}_g = \underline{T}_g \underline{\omega}_g \quad (2b)$$

$$\underline{0} = \tilde{P}_\ell(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) + P_d(|\underline{V}|) := \underline{f}_\ell(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \quad (3a)$$

$$\underline{0} = [\underline{V}]^{-1} (Q_b(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) + Q_d(|\underline{V}|)) := \underline{g}(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \quad (3b)$$

$\underline{M}_g, \underline{D}_g$ are diagonal matrices of inertia and damping constants. $\tilde{P}_M^0$ is the $(m-1)$ -vector of mechanical powers excluding the reference machine. (Some transformation of powers is often required to achieve the precise form (2) [5].) $\underline{T}_g$ is a specially structured matrix of ± 1 entries and $[\underline{a}]$ denotes $[\text{diag}\{a_i\}]$ for vector $\underline{a}$. For later analysis, we define

$$\tilde{f}_g(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) := \tilde{P}_g(\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) - \tilde{P}_M^0 \quad (4)$$

Note that this structure preserving model (SPM) consists of a set of differential-algebraic equations. The system variables are clearly $\underline{\omega}_g \in \mathbb{R}^m, \underline{\alpha}_g \in \mathbb{R}^{m-1}, \underline{\alpha}_\ell \in \mathbb{R}^{n_0}$, and $|\underline{V}| \in \mathbb{R}_+^{n_0}$.

The structure of the solution set for (3) is of particular importance in the analysis of the differential-algebraic model (2),(3). This set is therefore defined as

$$G := \{ \underline{z} : \underline{f}_\ell(\underline{z}) = \underline{0}, \underline{g}(\underline{z}) = \underline{0} \} \quad (5)$$

where $\underline{z} = (\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|)$. Typically G defines a differentiable $(m-1)$ -manifold which shall be referred to as the constraint manifold.

2.2 Local ODE Representation of the SPM.

Differential-algebraic equations of the form describing the SPM (2),(3) are theoretically problematic [6]. Fortunately, it can be shown that the model is locally equivalent to a set of ordinary differential equations for almost all operating states.

The load bus variables $\underline{\alpha}_\ell, |\underline{V}|$ are related to the generator angles $\underline{\alpha}_g$ by the $2n_0$ algebraic equations (3). (The constraint manifold defines the space of solutions of (3).) Define the Jacobian

$$\underline{J}_{\ell\ell} = \begin{bmatrix} \frac{\partial \underline{f}_\ell}{\partial \underline{\alpha}_\ell} & \frac{\partial \underline{f}_\ell}{\partial |\underline{V}|} \\ \frac{\partial \underline{g}}{\partial \underline{\alpha}_\ell} & \frac{\partial \underline{g}}{\partial |\underline{V}|} \end{bmatrix} \in \mathbb{R}^{2n_0 \times 2n_0} \quad (6)$$

Then, by the implicit function theorem [7], if $\det \underline{J}_{\ell\ell} \neq 0$, locally the load bus variables can be written explicitly in terms of the generator angles as

$$\underline{\alpha}_\ell = \underline{\phi}(\underline{\alpha}_g), \quad |\underline{V}| = \underline{\psi}(\underline{\alpha}_g) \quad (7)$$

An equivalent differential equation form of the SPM can therefore be obtained locally by substituting (7) into (2). Setting

$$\tilde{P}_g^*(\underline{\alpha}_g) := \tilde{P}_g(\underline{\alpha}_g, \underline{\phi}(\underline{\alpha}_g), \underline{\psi}(\underline{\alpha}_g)) \quad (8)$$

gives the model

$$\dot{\underline{\omega}}_g = -\underline{M}_g^{-1} \underline{D}_g \underline{\omega}_g - \underline{M}_g^{-1} \underline{T}_g^t (\tilde{P}_g^*(\underline{\alpha}_g) - \tilde{P}_M^0) \quad (9a)$$

$$\dot{\underline{\alpha}}_g = \underline{T}_g \underline{\omega}_g \quad (9b)$$

Equations (9) define ordinary differential equations which are locally equivalent to the SPM.

Points at which $\det \underline{J}_{\ell\ell} \neq 0$ shall be referred to as voltage causal points because load bus voltages can be predicted from generator angles. Most points on the constraint manifold are causal, so it is natural to seek an extension of local solvability to solvability over regions of the constraint manifold. This extension is considered in Section 3.

2.3 Long Term Power System Model.

It is clear from [1] that many factors, such as tap changing transformers, load dynamics and generator reactive power limiting, contribute to the mechanisms of long term voltage collapse. However it has been found that a good appreciation of these long term mechanisms can be gained through the use of a model consisting of the short term power system representation (2),(3) augmented by a model describing on-load tap changer behaviour. (This gives an adequate representation of load dynamics through the combined effects of tap changing transformers and the voltage dependence of the loads (1).) Generator reactive power limits have not been explicitly modelled. However the consequences of reactive power limiting are considered in Section 6.2.

The tap changing transformer model consists of an AVR which forces the tap to change whenever the error between the regulated bus voltage and the setpoint voltage exceeds a preset deadband. Taps vary in discrete steps. In this model, tapping continues at a

constant rate until the voltage error falls below the deadband, or a tap limit is encountered.

III VOLTAGE CAUSAL REGIONS AND THE IMPASSE SURFACE

Section 2.2 introduced the idea of mathematically reducing the differential-algebraic equations describing the SPM to a locally equivalent set of differential equations. This reduction is valid at points where $\det J_{\ell\ell} \neq 0$. Typically a major part of the constraint manifold satisfies that condition. However the constraint manifold will in general also contain non-causal points, i.e., points at which equivalence is not possible. This section initially investigates a decomposition of the constraint manifold into regions of equivalence separated by surfaces of non-equivalence. The influence of load indices on this decomposition is then considered.

3.1 Constraint Manifold Decomposition.

It is convenient in this decomposition to index the regions of equivalence by the number of negative real eigenvalues of $J_{\ell\ell}$. Regions of solvability must exclude non-causal points. Therefore, the constraint manifold shall be decomposed into open sets C_ℓ where

$$C_\ell = \left\{ \underline{z} : \det J_{\ell\ell} \big|_{\underline{z}} \neq 0, \underline{z} \in G, J_{\ell\ell} \big|_{\underline{z}} \text{ has } \ell \text{ negative real eigenvalues} \right\} \quad (10)$$

It is possible that these sets are not connected. Therefore each C_ℓ is partitioned into its connected components $C_{\ell 1}, \dots, C_{\ell k}$, i.e.,

$$C_\ell = \bigcup_{i=1}^k C_{\ell i} \quad (11a)$$

$$C_{\ell i} \cap C_{\ell j} = \emptyset \quad i \neq j, i, j = 1, \dots, k \quad (11b)$$

The size of sets $C_{\ell i}$ depend greatly on the load model parameters. In fact [8] investigates conditions which ensure $C_0 = G$, i.e., $C_\ell = \emptyset$, $\ell \neq 0$. This is considered further in Section 3.2.

From (7), it can be seen that the functions ϕ, ψ , which enable the establishment of the equivalent ODE representations, are functions of generator angles only. Therefore it is convenient to project the sets $C_{\ell i}$ onto their generator angle components, as

$$A_{\ell i} = \left\{ \underline{\alpha}_g : (\underline{\alpha}_g, \underline{\alpha}_\ell, |\underline{V}|) \in C_{\ell i} \right\} \quad (12)$$

The following theorem establishes the desired extension from local solvability to solvability over the disjoint regions $C_{\ell i}$.

Theorem 3.1 On each $C_{\ell i}$, the set G is represented

by unique continuous functions $\phi_{\ell i} : A_{\ell i} \rightarrow \mathbb{R}^{n_0}$, $\psi_{\ell i} : A_{\ell i} \rightarrow \mathbb{R}^{n_0}$ such that $\underline{\alpha}_\ell = \phi_{\ell i}(\underline{\alpha}_g)$, $|\underline{V}| = \psi_{\ell i}(\underline{\alpha}_g)$.

The proof can be found in [9].

The regions $C_{\ell i}$ are referred to as voltage causal regions, and ℓ as the causal region index. Within each $C_{\ell i}$, the load bus voltages and angles are continuously dependent on the generator angles. Hence,

over each of these regions, the DA system (2),(3) is equivalent to a local ODE representation (9). The DA system can therefore be decomposed into multiple ODE systems, one on each voltage causal region.

The boundaries of the sets $C_{\ell i}$ are known as the *impassé surface*, a term borrowed from circuit theory [10]. The impassé surface $\mathcal{I}$ is defined as

$$\mathcal{I} := \left\{ \underline{z} \in G : \det J_{\ell\ell} \big|_{\underline{z}} = 0 \right\} \quad (13)$$

It is shown in [9] that in general $\mathcal{I}$ is composed of intersecting manifolds. Note that

$$G = \left[\bigcup_{i=0}^{2n_0} C_i \right] \cup \mathcal{I} \quad (14)$$

i.e., the constraint manifold G consists of disjoint open sets $C_{\ell i}$ which are separated by the impassé surface.

Functions ϕ, ψ are not defined at points on the impassé surface, as the implicit function theorem requires that $\det J_{\ell\ell} \neq 0$. The DA model breaks down.

If a trajectory encounters the impassé surface, then at the point of intersection the SPM cannot be used to predict voltage behaviour. System behaviour at (and near) the impassé surface shall be investigated further in the following sections. Load bus voltage response is of particular interest.

3.2 Influence of Load Indices.

It has been shown in [8] that variation of the load indices used in (1) to model loads may cause significant structural changes to the causal region/impassé surface decomposition of the constraint manifold. In fact, conditions which ensure global voltage causality, i.e., $C_0 = G$, are considered. As part of this investigation, the following conjecture was proposed.

Conjecture 3.2 Suppose

(i) The admittance matrix of the load bus network is negative definite.

(ii) $P_i, Q_i \geq 0 \quad i=1, \dots, n_0$

and for all non-zero real and reactive loads, either

(iia) $\zeta_i \geq 1 \quad i=1, \dots, n_0$

(iib) $\eta_i = 2 \quad i=1, \dots, n_0$

or

(iva) $\zeta_i = 2 \quad i=1, \dots, n_0$

(ivb) $\eta_i \geq 1 \quad i=1, \dots, n_0$

Then $\det J_{\ell\ell} \neq 0$ at all points on the constraint manifold, i.e., $C_0 = G$.

It is shown in [8] that this conjecture can be proven when $n_0=1$ or 2.

Example 1.

Figure 1 illustrates the effect of the variation of load indices on the constraint manifold, and hence on system dynamics. Power-angle curves for a simple power system like that of Figure 1a, obtained using different values of real power load index ζ , are shown in Figure 1b. A reactive power load index of $\eta=2$ was used in all cases. These curves show the relationship between electrical power and angle given by (8). Because (8) is single valued on each causal region, the curves reflect the variation, with load indices, of the constraint manifold structure. (Power-angle curves

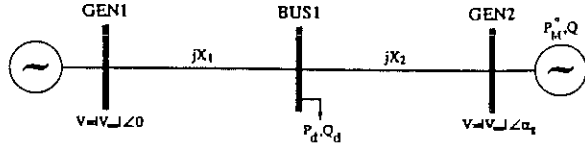


Figure 1a: Two Generator System.

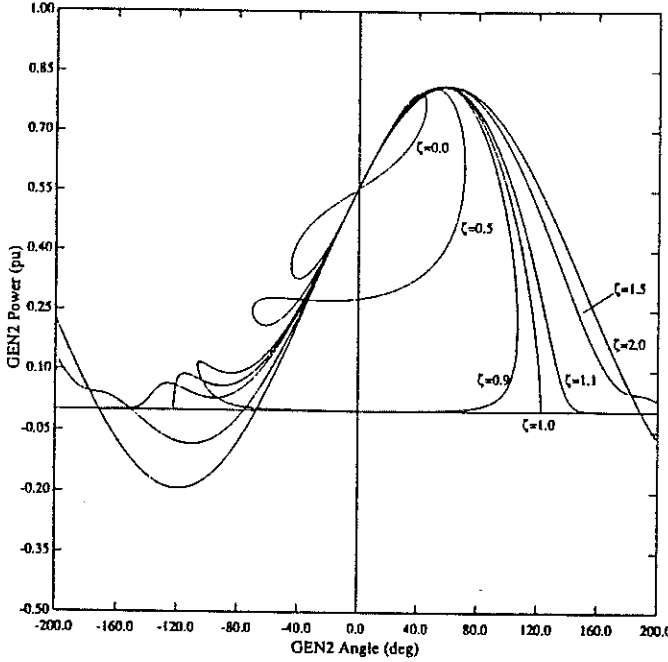


Figure 1b: Power-Angle Curves, Varying Load Indices.

were chosen as they also provide insights into changes in system dynamic behaviour.)

When $\zeta > 1$, the curves are single valued (for each value of angle), indicating global voltage causality. However for $\zeta < 1$, the curves are multivalued, indicating that the constraint manifold is decomposed into two causal regions. The bifurcation points (in terms of angle) of these curves correspond to the impasse surface for each case. $\square$

IV ENERGY FUNCTIONS

The development of an energy function for the SPM has been studied previously [3]. A useful energy function is given by

$$V(\omega_g, z) = \frac{1}{2} \omega_g^T P_1(\mu) \omega_g + \int_z^z \langle h(\lambda), d\lambda \rangle \quad (15)$$

where $z = (\alpha_g, \alpha_\theta, |V|)$. $h(z) = [\tilde{f}_g(z), f_\theta(z), g(z)]$

and z^s denotes a stable EP. Note that $P_1(\mu)$ must satisfy $P_1(\mu) > 0$. Conditions governing this matrix are discussed in [3]. The first term of (15) is often referred to as the kinetic energy function, whilst the other term is known as the potential energy function.

It can be seen from (15) that the potential energy function is dependent on the algebraic variables α_θ , $|V|$. But in Section 3.1, it was indicated that within each causal region, there exist unique continuous functions $\alpha_\theta = \phi(\alpha_g)$ and $|V| = \psi(\alpha_g)$. Therefore, referring to (8), the potential energy function on any causal region may be written,

$$\begin{aligned} PE^*(\alpha_g) &= \int_{\alpha_g}^{\alpha_g} \langle \tilde{f}_g(\alpha_g, \phi(\alpha_g), \psi(\alpha_g)), d\alpha_g \rangle \\ &= \int_{\alpha_g}^{\alpha_g} \langle \tilde{f}_g^*(\alpha_g), d\alpha_g \rangle \end{aligned} \quad (16)$$

These local potential energy functions are dependent on α_g only, so may be conceptualized as $(m-1)$ -hypersurfaces (or sheets) in α_g -space. Note that each $PE^*(\alpha_g)$ is unique over its causal region. Further information on these potential energy sheets may be found in [4,8,9].

Example 2.

The potential energy function for a two generator system of the form shown in Figure 1a is shown in Figure 2. A real power load index of $\zeta=0$ was used in this case, resulting in the constraint manifold being composed of two causal regions and the impasse surface. The PE function consists of two PE sheets which join at the impasse surface, points A and B. The sheet corresponding to C_1 lies above that of C_0 . $\square$

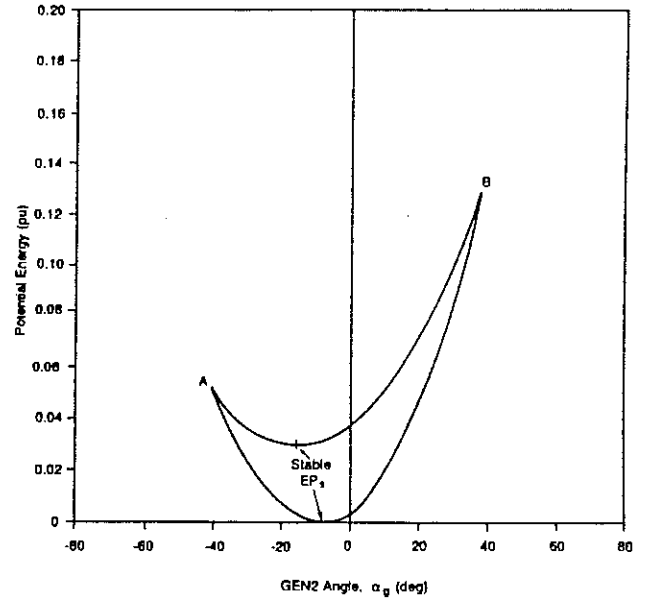


Figure 2: Potential Energy Sheets.

V VOLTAGE CAUSALITY AND VOLTAGE COLLAPSE

The generally accepted view of voltage collapse is one of voltages falling uncontrollably, but generator angles (initially at least) behaving normally. A breakdown occurs in the normal relationship between voltages and angles. In the power system model, such a breakdown occurs at non-causal points. Therefore, a strong connection appears to exist between the concepts of voltage causality and voltage collapse. This connection shall now be investigated further.

5.1 SPM Linearization and Causality.

The response of the power system to small perturbations often provides important qualitative information about large disturbance behaviour. Therefore, the initial investigation of voltage causality effects shall be directed toward the linearized SPM. Of particular interest is the relationship between causality and stability of EPs.

Linearizing (2),(3) gives

$$\dot{\Delta\omega}_g = -M_g^{-1} D_g \Delta\omega_g - M_g^{-1} T_g^t \tilde{\Delta f}_g \quad (17a)$$

$$\dot{\Delta\alpha}_g = T_g \Delta\omega_g \quad (17b)$$

and

$$\begin{bmatrix} \Delta f_\ell = 0 \\ \Delta g_\ell = 0 \\ \tilde{\Delta f}_g \end{bmatrix} = \begin{bmatrix} J_{\ell\ell} & \vdots & \frac{\partial f_\ell}{\partial \alpha_g} \\ & \ddots & \frac{\partial g_\ell}{\partial \alpha_g} \\ \frac{\partial f_g}{\partial \alpha_\ell} & \frac{\partial f_g}{\partial |V|} & \frac{\partial f_g}{\partial \alpha_g} \end{bmatrix} \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \end{bmatrix}$$

$$= \begin{bmatrix} J_{\ell\ell} & J_{\ell g} \\ J_{g\ell} & J_{gg} \end{bmatrix} \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \end{bmatrix} = F \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \end{bmatrix} \quad (18)$$

Solving for $\tilde{\Delta f}_g$ in (18), and substituting into (17) yields the linearized version of the SPM as

$$\begin{bmatrix} \Delta \alpha_g \\ \Delta \omega_g \end{bmatrix} = \begin{bmatrix} 0 & T_g \\ \dots & \dots \\ -M_g^{-1} T_g^t F & -M_g^{-1} D_g \end{bmatrix} \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \end{bmatrix} = A \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \\ \Delta \omega_g \end{bmatrix} \quad (19)$$

where

$$F = J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g} \quad (20)$$

The operating point is small disturbance stable if and only if all eigenvalues of A lie in the open left half of the complex plane [11], i.e., they all have negative real parts. Further, EP type is given by the number of eigenvalues of A which lie in the open right half of the complex plane. At a type-k EP, A will have k eigenvalues with positive real parts. It can be shown [12] that due to the relationship between A and F , generally EP type is given by the number of negative eigenvalues of F .

The relationship between load bus variables and generator angles, in this linearized SPM, is found by rearranging (18), so that,

$$\begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \end{bmatrix} = -J_{\ell\ell}^{-1} J_{\ell g} \Delta \alpha_g \quad (21)$$

If $J_{\ell\ell}$ is nonsingular, i.e., at causal points on the constraint manifold, small disturbances in the load bus variables $\Delta \alpha_g$, $\Delta |V|$ can be predicted from small disturbances of the generator angles $\Delta \alpha_g$. At such points, voltages cannot behave abnormally. However, if the system is linearized around a point where $J_{\ell\ell}$ is singular, voltages become infinitely sensitive to generator angles. Hence the potential for voltage collapse exists at non-causal points.

5.2 Parameter Variations and Voltage Collapse.

The usual bifurcation analysis approach to investigating voltage collapse, by studying the singularity of the full power flow Jacobian, does not discriminate between generator angles and the dependent load bus voltages and angles. This makes it impossible to

differentiate between voltage collapse and angle separation at bifurcation points. Kwatny et al [13] have studied the roles played by the different groups of variables in voltage collapse/angle separation for essentially the model (2),(3). However their choice of grouping of variables disguised the role of voltage causality in distinguishing between those phenomena. This role is clarified in the following analysis.

At an equilibrium point, if parameters are represented by p , the power flow equations can be written

$$\begin{bmatrix} f_\ell(\alpha_g, \alpha_\ell, |V|, p) \\ g_\ell(\alpha_g, \alpha_\ell, |V|, p) \\ \tilde{f}_g(\alpha_g, \alpha_\ell, |V|, p) \end{bmatrix} = 0 \quad (22)$$

With reference to (18), linearizing (22) about the equilibrium point gives

$$\begin{bmatrix} \Delta f_\ell = 0 \\ \Delta g_\ell = 0 \\ \dots \\ \tilde{\Delta f}_g = 0 \end{bmatrix} = F \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \\ \Delta \alpha_g \end{bmatrix} + \begin{bmatrix} P_\ell \\ P_g \end{bmatrix} \Delta p \quad (23)$$

where

$$P_\ell = \begin{bmatrix} \frac{\partial f_\ell}{\partial p} \\ \frac{\partial g_\ell}{\partial p} \end{bmatrix}, \quad P_g = \begin{bmatrix} \frac{\partial \tilde{f}_g}{\partial p} \end{bmatrix}$$

Application of Schur's formula to F , using the partitioning of (18), gives

$$\det F = \det J_{\ell\ell} \det(J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g}) \quad \text{if } J_{\ell\ell} \text{ nonsingular} \quad (24a)$$

$$\det F = \det J_{gg} \det(J_{\ell\ell} - J_{\ell g} J_{gg}^{-1} J_{g\ell}) \quad \text{if } J_{gg} \text{ nonsingular} \quad (24b)$$

Further, rearranging (23) leads to

$$(J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g}) \Delta \alpha_g = -(P_g - J_{g\ell} J_{\ell\ell}^{-1} P_\ell) \Delta p \quad (25a)$$

$$(J_{\ell\ell} - J_{\ell g} J_{gg}^{-1} J_{g\ell}) \begin{bmatrix} \Delta \alpha_\ell \\ \Delta |V| \end{bmatrix} = -(P_\ell - J_{\ell g} J_{gg}^{-1} P_g) \Delta p \quad (25b)$$

When referring to (24a) and (25a), recall from (20) that

$$F = J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g}$$

At a bifurcation point, F is singular. So, if $J_{\ell\ell}$ is nonsingular, i.e., if the point is causal,

(24a) gives that $\det(J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g}) = \det F = 0$.

Equation (25a) shows that this condition implies infinite sensitivity of generator angles to parameter variations. (This is typical of loss of small disturbance stability.) But, for all realistic power systems, it is easy to show that J_{gg} is always positive definite. Therefore (24b) implies that when

F is singular, $\det(J_{\ell\ell} - J_{\ell g} J_{gg}^{-1} J_{g\ell}) = 0$. By (25b), load bus variables are also infinitely sensitive to

parameter variations. This result does not indicate voltage collapse however. Because $J_{\ell\ell}$ is nonsingular, (21) ensures that load bus variables follow generator angles, the independent variables. Therefore, because generator angles are infinitely sensitive to parameters, load bus variables must be also.

However, the situation changes when both $\mathbb{E}$ and $J_{\ell\ell}$ are singular. Equation (24b) gives that $\det(J_{\ell\ell} - J_{\ell g} J_{gg}^{-1} J_{g\ell}) = 0$, so by (25b), load bus variables are infinitely sensitive to parameter variations. But nothing can be said about $\det(J_{gg} - J_{g\ell} J_{\ell\ell}^{-1} J_{\ell g})$ because Schur's formula is not applicable. Therefore, these conditions do not give rise to normal angle instability behaviour. This situation is typical of voltage collapse.

5.3. Short Term Voltage Collapse from a Dynamic Perspective.

Voltage collapse as described in Section 5.2 assumes that parameters vary slowly, so that generator dynamics can follow the changes closely. This assumption enables a good first approximation for voltage collapse conditions. However, in reality, parameter changes can occur quickly, causing a transient response from generators. For example, when shunt devices are switched, the network undergoes a step change. Even loads cannot be considered as slowly changing. Whilst it is certainly true that the average value of load varies slowly, it is well accepted that random (step) variations about that average occur with a broad band frequency spectrum [14].

Step variations in load are usually small. Therefore, they have negligible effect on stability at a robust operating point. However, at a marginally stable operating point, such variations may be of sufficient magnitude that they push the system out of the region of stability, and some form of system failure occurs. (There are many reasons why a system may be operating close to its limit. This is considered in Section 6.)

It is possible that a small disturbance, such as a load step, could lead to angle instability, i.e., generator separation. Alternatively, in response to the disturbance, the system trajectory could encounter the impasse surface, i.e., a non-causal point. At such a point, the relationship between generator angles and bus voltages breaks down. The rate of decline of voltage becomes infinite due to infinite sensitivity to generator angle movement. (This is indicative of voltage collapse.) Angle behaviour becomes undefined. In the real system, unmodelled parasitic effects dominate behaviour to the extent that the trajectory must move off the constraint manifold, i.e., the algebraic equations (3) describing the network are no longer satisfied. The SPM cannot predict such behaviour.

However, an indication of this behaviour may be gained by considering a disturbance which is just severe enough to ensure the system trajectory encounters the impasse surface tangentially, i.e., the disturbance is critically cleared. At each point along the trajectory up to the impasse surface, bus voltages are given by a unique (vector) function of generator angles, i.e., the dynamic variables. However, this relationship breaks down at the impasse surface. Several angle-voltage functions come together. Therefore, as dynamics move the angles away from the impasse surface, voltages may respond by following any

of these different functions. As a consequence, the system may ultimately settle to different stable operating points.

Example 3.

An example of this type of behaviour is easily obtained using a two generator network of the form shown in Figure 1a. The load is modelled with a real power index of $\xi=0$. The disturbance, for this illustration, involved a step increase in load at GEN2, then its return to normal a short time later. The angle and voltage responses are shown in Figure 3. Notice that at the impasse surface, system behaviour bifurcates. Two possible paths can be followed to two totally different stable operating points, one in C_0 and the other in C_1 . The bus voltage at the stable point in C_1 is significantly lower than at the normal stable point. Further, nowhere does the generator angle deviate from its predisturbance value by more than 32 degrees. Angle instability is not an obvious consideration.

In terms of the PE function of Figure 2, the critically cleared disturbance provides the system with

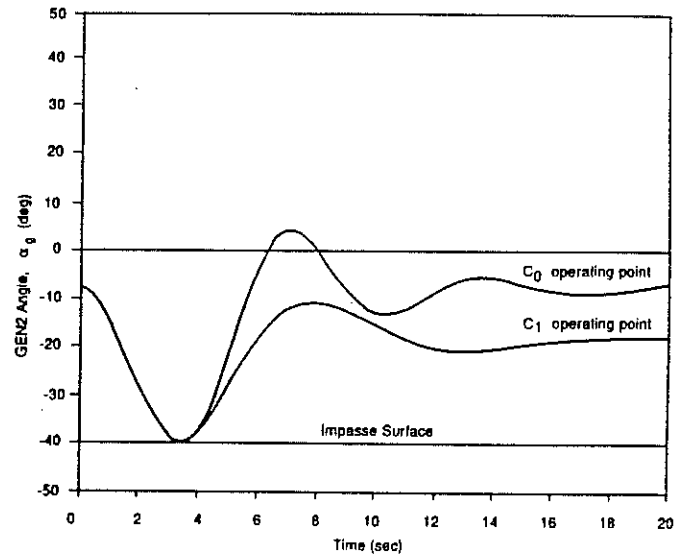


Figure 3a: Trajectory Bifurcation, Gen. Angle.

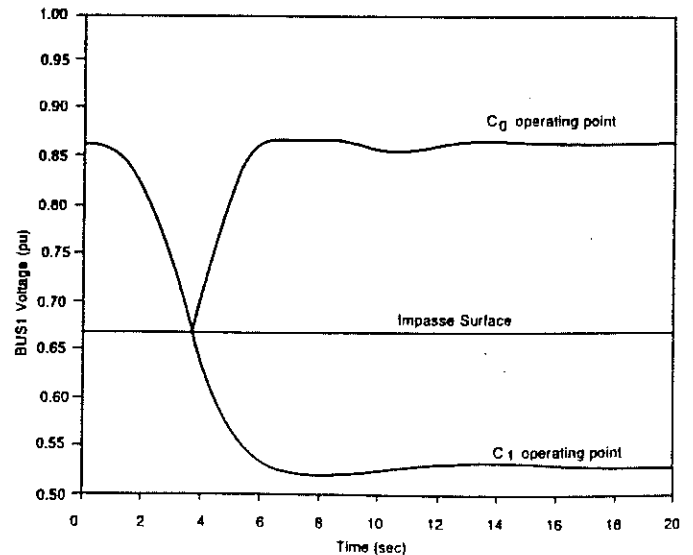


Figure 3b: Trajectory Bifurcation, Bus Voltage.

just enough energy to climb the potential energy well to point A, i.e., it has zero kinetic energy at point A. From there, the system may proceed in one of two directions, onto either the higher or lower energy sheet. As both sheets contain a stable EP, at their lowest points, the system can easily settle on either. □

In the SPM, bifurcation of system behaviour can only occur if the trajectory is tangential to the impasse surface. However, in the real system, because of the rapid decline of voltage near the impasse surface, parasitic effects may have an influence if the trajectory passes 'close to' the impasse surface. These parasitic effects may enable the system to jump from one causal region to another. Likewise, when the system undergoes a step change, e.g., switching or load steps, the parasitic effects may cause a jump between causal regions. Certainly real voltages and currents cannot undergo step changes, as is implicitly assumed through the algebraic equations of the SPM.

Once in that other causal region, it is possible that the system may try to settle to a non-normal operating point, as in the example. Invariably at least one bus will have low voltage at that point, so overloads are likely. Protection action, such as line tripping or load shedding, will usually occur, disturbing the system further. System disintegration may well follow.

VI LONG TERM MECHANISMS OF VOLTAGE COLLAPSE

It has been shown in the preceeding sections that when a power system fails, it may do so in one of two ways, viz., angle separation or loss of voltage causality. In a typical voltage collapse scenario, some long term dynamic behaviour drives the system to such a point of failure. This section focuses upon those long term dynamics, with a view to providing some insights into the weakening process that the system undergoes during this period.

The three major factors contributing to long term collapse are load variations, reactive power limits and tap changing. Each will be considered. They will be investigated for both globally causal and non-causal systems where the differences in system behaviour are significant. After exploring these factors, examples will be given which illustrate dynamic voltage collapse via the two different system failure modes.

6.1 Load Variation.

The effect of load variation on system stability shall be explored through examples based on a simple power system of the form shown in Figure 1a. Initially the load will be modelled with indices of $\zeta, \eta=2$. This ensures global causality. Load indices of $\zeta=0, \eta=2$ will then be used to demonstrate the consequences of loss of causality.

6.1.1 Globally Causal System ($\zeta, \eta=2$).

Figure 4a shows power-angle curves for a network similar to Figure 1a, for various levels of real power base load P_d^0 at the load bus BUS1. Corresponding PE-angle curves are shown in Figure 4b. The power generated at GEN2 for this case is 0.5pu. Each power-angle curve, other than that corresponding to a base load of 4.0pu, intersects the $P_{GEN2}=0.5pu$ line at two points, the equilibrium points of the system. The lower angle EP is stable (SEP), while the higher one is unstable (UEP). A measure of the stability of the system is given by the value of potential energy at

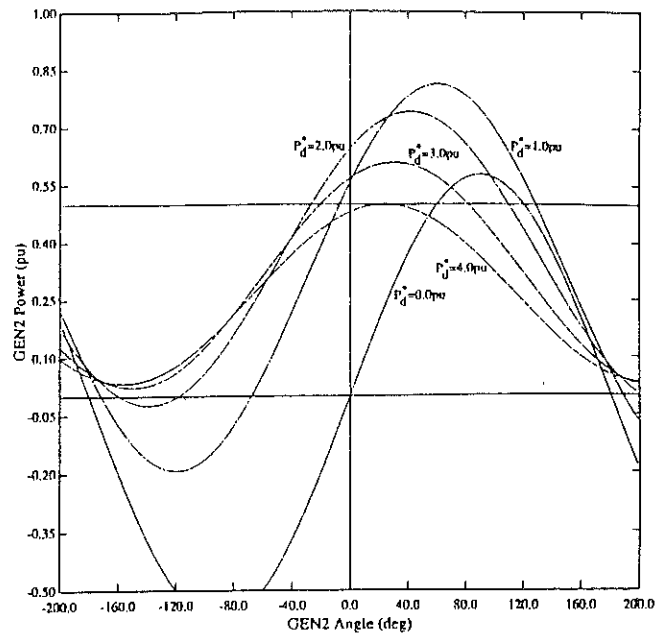


Figure 4a: Power-Angle Curves, Load Variation.

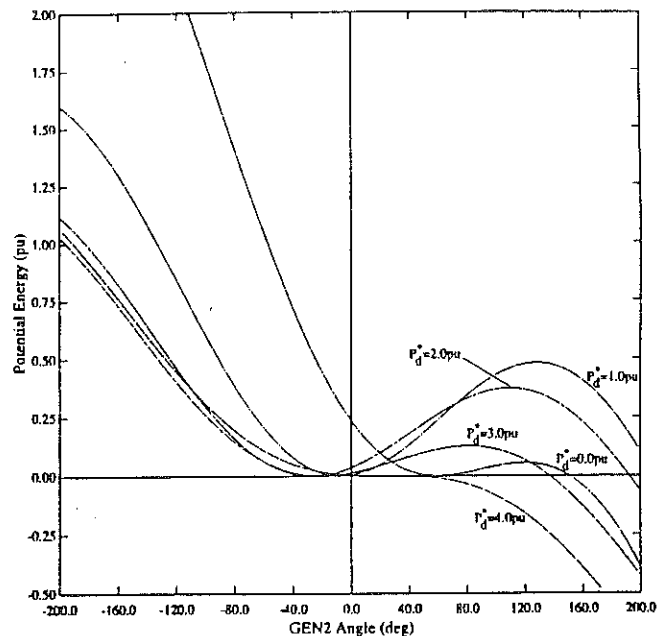


Figure 4b: Potential Energy Curves, Load Variation.

the UEP, i.e., the lowest peak on each PE curve. Therefore, it can be seen that as the real power base load increases from 0.0pu to about 1.0pu, system stability improves. As base load increases further, system stability decreases. Figure 4a shows that as base load is increased to 4.0pu, the SEP and UEP coalesce. Such a load could only just be supplied. Stability could not however be maintained as the stability margin has fallen to zero, as shown in Figure 4b. The region of stability of the SEP has evanesced.

The fact that stability can actually reduce with a drop in load (as indicated in this case when base load drops from 1.0pu to 0.0pu) means that schemes which shed load to protect systems from voltage collapse must be carefully thought through. The location and amount of load that is shed could be quite critical.

6.1.2 Non-Causal System ($\zeta=0, \eta=2$).

The power-angle curves for this case are shown in

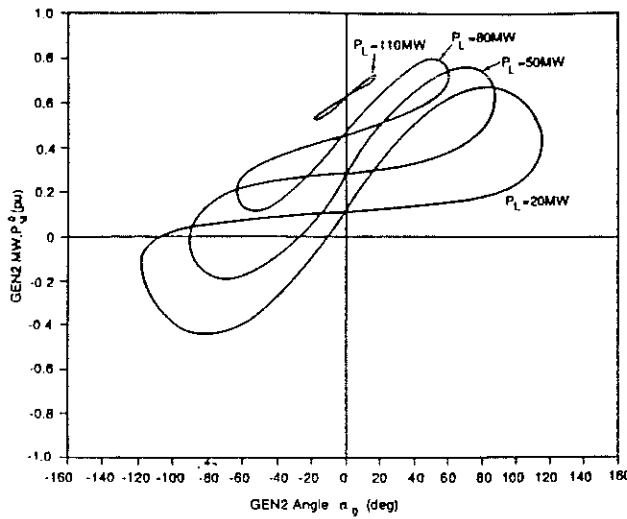


Figure 5: Power-Angle Curves, Load Variation.

Figure 5. As load increases, the capacity for transferring power across the network, between machines, is reduced. Also note though that the load variation directly affects the size and topology of the constraint manifold. Increasing load causes a contraction of the constraint manifold, i.e., loss of voltage causality (short term voltage collapse) would occur at smaller and smaller angular deviations. System dynamic behaviour is therefore influenced significantly by load variation.

Note that as load increases, the distance between the points where $\underline{F}$ is singular (extreme power points) and those at which $\underline{J}_{\theta\theta}$ is singular (extreme angle points) reduces. In the limit, the constraint manifold shrinks to a single point where both $\underline{F}$ and $\underline{J}_{\theta\theta}$ are singular. This corresponds to a static voltage collapse point as established in Section 5.2.

6.2 Reactive Power Limits.

Generators can incur transient reactive power overloads. However over a longer period, over-excitation limiters (whether automatic or manual) act to reduce the machines reactive power output. This is achieved by lowering the terminal voltage. The system of Figure 1a is again used to demonstrate the effect on stability of this downward trend in voltage. Figure 6 shows the power-angle curves for various values of terminal voltage at GEN2. As the terminal voltage falls, the power transfer capability of the network, and hence the stability margin, also falls.

In contrast to generators, static var compensators (SVCs) have limits (on susceptance) which must be enforced in the transient period as well as over the long term. The influence of SVC limits on system stability and the potential energy calculation has been investigated in [15]. To illustrate these effects, the system of Figure 1a is augmented by an SVC at BUS1. Corresponding power-angle and PE-angle curves are shown in Figure 7. Notice that the stability margin increases with SVC limit. However the stability margin cannot be increased arbitrarily. A maximum value is approached asymptotically as the limit is increased.

6.3 On-Load Tap Changers.

In a normal healthy power system, transformer tapping has a negligible effect on system stability. However in a system that has previously been weakened, for example through line outages and/or load increases,

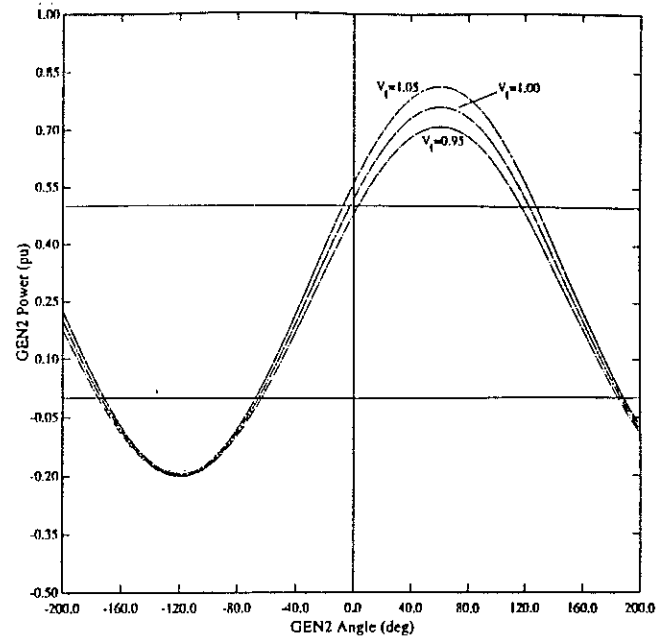


Figure 6: Power-Angle Curves, Varying Terminal Volts.

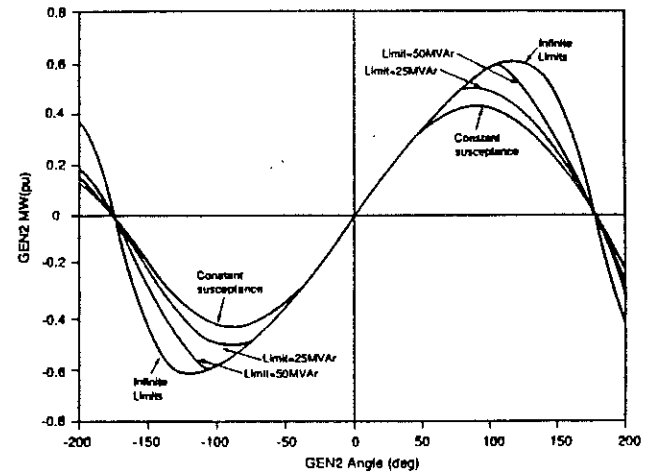


Figure 7a: Power-Angle Curves, Various SVC Limits.

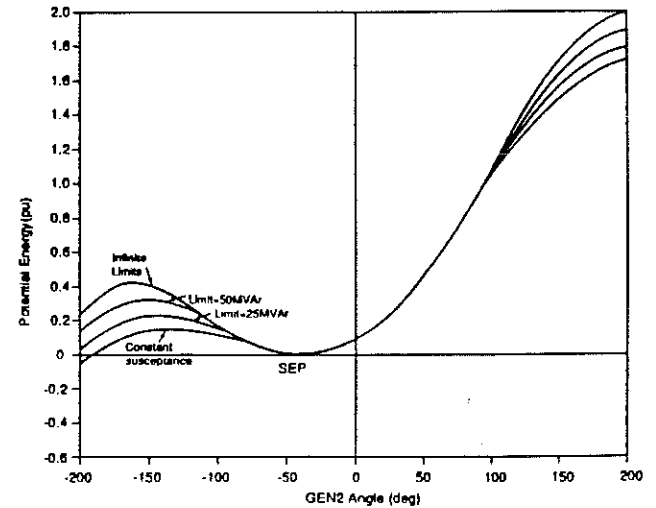


Figure 7b: Potential Energy, Various SVC Limits.

tapping effects may become significant. These effects are complex, and depend largely on the location within the network of the transformers.

Tapping of a generator transformer to raise the voltage profile of the transmission system will result in an improvement in system stability. However if that generator is separated from the major load centre and other generators by a large impedance, then the improvement in stability may be negligible.

Alternatively, tapping of load transformers to restore load bus voltages usually has a detrimental effect on stability. When a transformer taps to raise the load side voltage, invariably the transmission side voltage falls. This lower transmission voltage results in a reduction in the power transfer capability of the network and hence in the stability margin. This can be illustrated using the network of Figure 8. Initially the load will be modelled with indices of $\zeta, \eta=2$ to ensure global voltage causality. Then load indices of $\zeta=0.9, \eta=2$ will be used to show the effects of loss of causality.

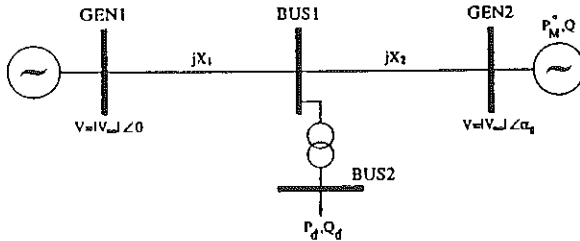


Figure 8: Two Generator System, with Transformer.

6.3.1 Globally Causal System ($\zeta, \eta=2$).

Figure 9a shows power-angle curves for this network for increasing tap, i.e., increasing boost to the load bus voltage. Corresponding PE-angle curves, based on $P_{GEN2}=0.5\text{pu}$, are shown in Figure 9b. Notice that as taps increase, the stability margin reduces significantly, i.e., the region of attraction of the SEP evanesces. Continued tapping results in the loss of all EPs, and therefore in system instability. The dynamics of such a system failure are discussed in Section 6.4.1.

6.3.2 Non-Causal System ($\zeta=0.9, \eta=2$).

Power-angle curves for this case are shown in Figure 10. Notice that an increase in tap not only

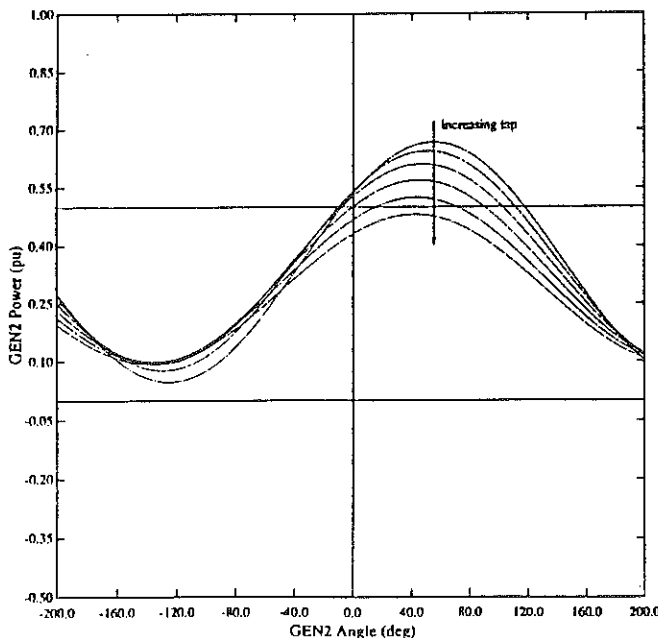


Figure 9a: Power-Angle Curves, Increasing Tap.

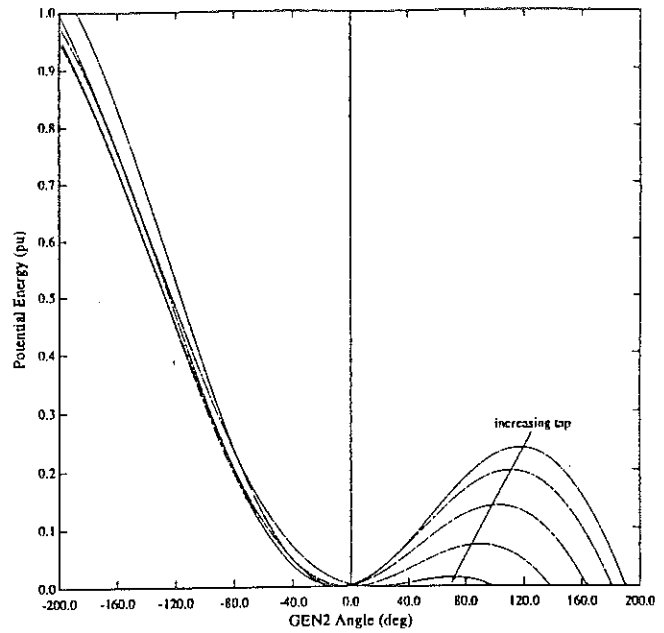


Figure 9b: Potential Energy. Increasing Taps.

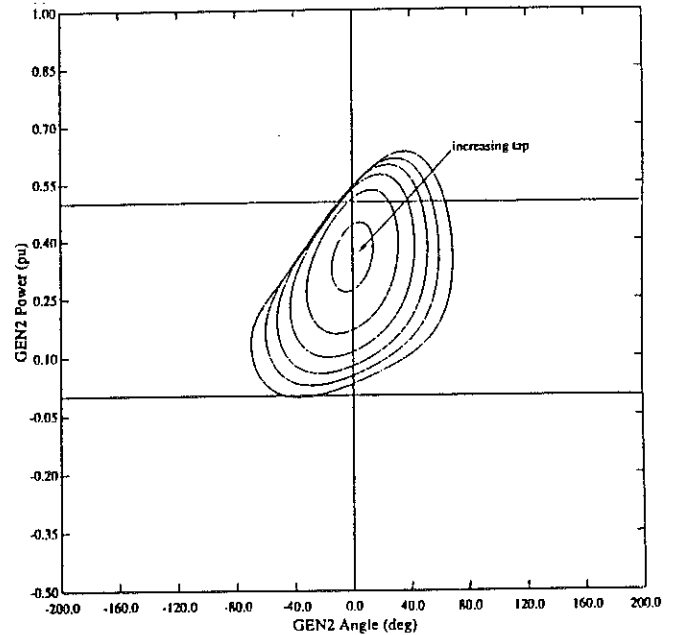


Figure 10: Power-Angle Curves, Increasing Tap.

reduces the power transfer capability of the system, but also causes a contraction of the constraint manifold. As with the globally causal system, an increase in tap can result in the loss of all EPs. The dynamics of the inevitable system failure are discussed in Section 6.4.2.

6.4 Dynamic System Failure Modes.

The network of Figure 8 can be used to demonstrate two modes of power system failure. These failure modes correspond to causal and non-causal systems. For this demonstration, assume that over time system conditions have changed such that the load has moved to within 1MW of its critical value. (The system is in steady state though.) Therefore, an increase in load of 1MW will result in some form of system collapse. The two load modelling cases of Section 6.3 will be considered separately.

6.4.1 Globally Causal System ($\zeta, \eta=2$).

The voltage at the load bus BUS2, and the angle at GEN2 are plotted in Figure 11 for a load step of 1MW, i.e., past the critical value. The increase in load caused a drop in bus voltage which the transformer AVR tried to correct. The transformer tapped every 10sec attempting (unsuccessfully) to raise the load bus voltage. Each time it tapped, the system moved to a lower power-angle curve of Figure 9a, and the stability margin reduced. The fifth tap change, at 50sec, moved the system onto the lowest curve of Figure 9a, so there was no longer an SEP. Angle separation quickly followed. Notice that voltages dropped rapidly, but only in response to angle movement.

The movement in the power-angle plane shown in Figure 12 clearly illustrates the jumps (at each tap step) between power-angle curves. Note that if taps had been frozen at any of the first four tap steps, the system would have settled to a new SEP. (The load bus voltage would have been low though.)

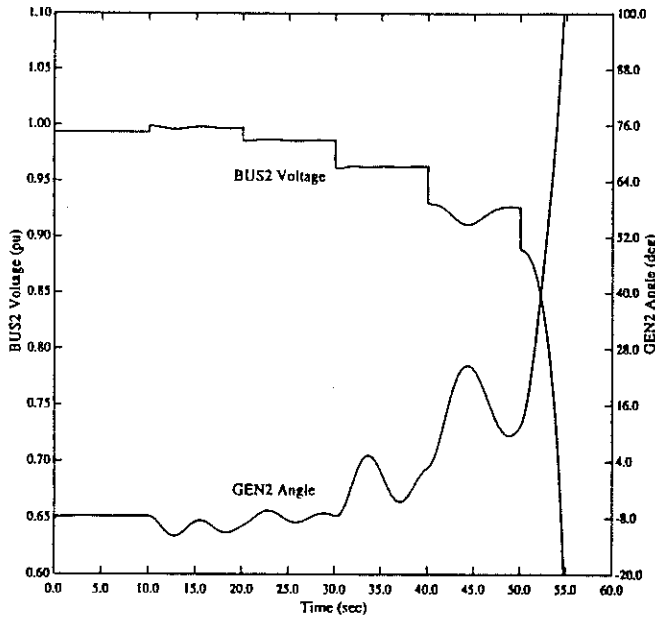


Figure 11: System Collapse, Causal System.

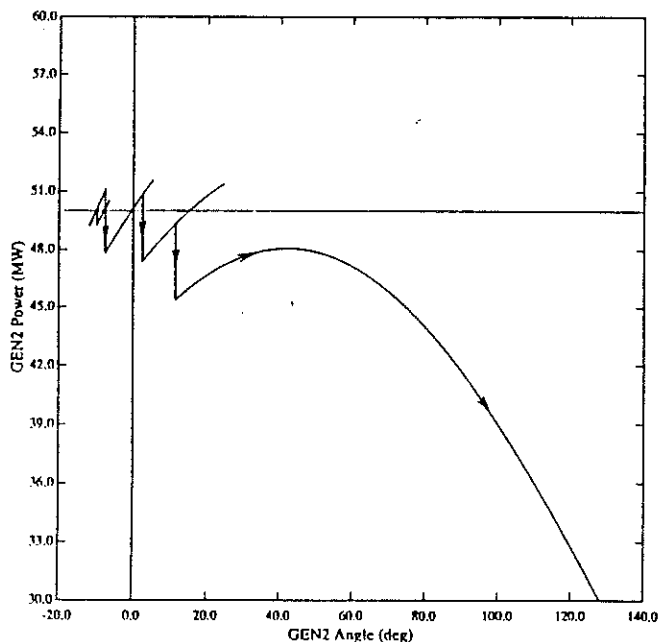


Figure 12: Trajectory in the Power/Angle Plane.

6.4.2 Non-Causal System ($\zeta=0.9, \eta=2$).

As in the previous case, the final system collapse was initiated by a step increase in load of 1MW. BUS2 voltage and GEN2 angle are plotted in Figure 13. Again the transformer tapped every 10sec, moving the system to ever smaller power-angle curves of Figure 10. This movement is shown in Figure 14. At 50sec the tap change moved the system to the inner-most power-angle curve of Figure 10, so the SEP was lost. GEN2 angle started to increase in a manner similar to angle separation. However the impasse surface was quickly encountered, at point A on Figure 14. Voltage became infinitely sensitive to angle movement, so fell rapidly. The power system model breaks down at the impasse surface, so subsequent system behaviour cannot be predicted.

Notice in this case that the generator angle remains bounded (in fact less than 15°) throughout the system collapse. This case has all the characteristics of "traditional" voltage collapse.

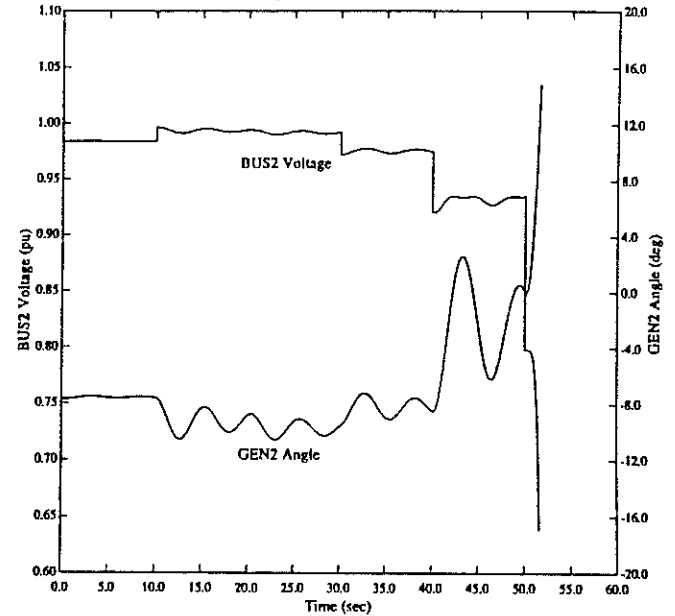


Figure 13: System Collapse, Non-Causal System.

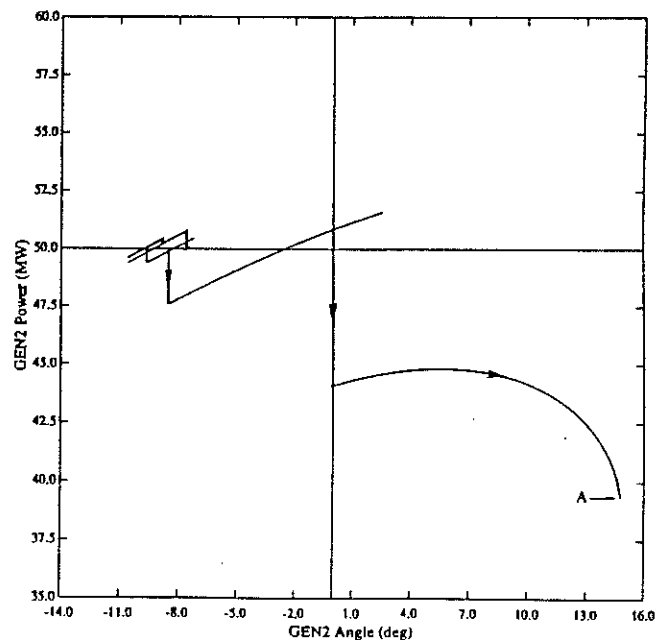


Figure 14: Trajectory in the Power/Angle Plane.

VII CONCLUSIONS

A power system model which maintains network topology, whilst including generator dynamics, has been developed. This model is described by a set of differential-algebraic equations. Conditions are given under which the model can be reduced to a set of ordinary differential equations. Points at which these conditions are satisfied are referred to as causal points.

The paper has explored the connection between voltage collapse and the loss of voltage causality. It is shown that in terms of the small disturbance behaviour of power systems, at a non-causal point load bus voltages become infinitely sensitive to generator angle movements. Further, it has been found that lack of causality plays an important role in voltage collapse resulting from parameter variations.

In terms of the short term large disturbance behaviour of power systems, it has been found that a trajectory which passes near a non-causal point may bifurcate, with the system ultimately settling to one of a number of possible stable equilibrium points. Again a close relationship is shown to exist between short term voltage collapse and loss of voltage causality.

The paper has investigated some long term mechanisms of voltage collapse. It is shown that load variation, reactive power limiting and tap changing can significantly influence the stability of weak systems. It is further shown that this influence varies depending on whether or not the system contains non-causal points. Two mechanisms of system failure are demonstrated. The second, which involves non-causal behaviour, exhibits all the characteristics observed in "traditional" voltage collapse.

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IMMEDIATE CHANGE IN STABILITY AND VOLTAGE COLLAPSE WHEN GENERATOR REACTIVE POWER LIMITS ARE ENCOUNTERED

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Abstract: When a generator of a heavily loaded electric power system reaches a reactive power limit, the system can become immediately unstable and a dynamic voltage collapse leading to blackout may follow. We study the statics and dynamics of this mechanism for voltage collapse by example and by the generic theory of saddle node and transcritical bifurcations. Load power margin calculations can be misleading if the immediate instability phenomenon is neglected.

1. INTRODUCTION

Voltage collapse is an instability of heavily loaded electric power systems which leads to declining voltages and blackout. It is associated with bifurcation and reactive power limitations of the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackouts will occur if suitable monitoring and control measures are not taken. It is important to understand mechanisms of voltage collapse so that voltage collapse blackouts may be effectively prevented. Most of the current approaches to analyzing voltage collapse are represented in [1].

One aspect of voltage collapse is that power systems become more vulnerable to voltage collapse when generator reactive power limits are encountered [2,3,4,5,6,7,8]. The effect of a generator reactive power limit is to immediately change the system equations. For example, the effect of a generator excitation current limit may be simply modelled by replacing the equation describing a constant output voltage magnitude by an equation describing a constant excitation current. Although the system state is unchanged, the immediate change in the system equations causes a discontinuous change in the stability margin of the system. The case in which the stability margin decreases when the reactive power limit is encountered but the system remains stable is familiar [2,5,9,7,6]. We study the case in which the system becomes immediately unstable when the reactive power limit is encountered. This possibility was mentioned by Borremans et al [4] but otherwise appears to have been overlooked. The immediate instability of the system can lead to voltage collapse and the main purpose of this paper is to study the statics and dynamics of this mechanism for voltage collapse.

We analyze a power system example and demonstrate that at lesser loadings, encountering the reactive power

limit is expected to decrease, but not destroy stability. At sufficiently high loadings, encountering the reactive power limit will immediately destabilize the system and can precipitate a voltage collapse along a trajectory which is one part of the unstable manifold of an unstable equilibrium. The movement along this trajectory is a new model for the dynamics of voltage collapse. Moreover we can argue using the theory of saddle node and transcritical bifurcations that the immediate instability and subsequent dynamic voltage collapse are likely to be typical for a general, heavily loaded power system. The results have important implications for correctly measuring the proximity to voltage collapse using load power margins; it seems that the load power margin to bifurcation can be a misleading indication of system stability unless the possibility of immediate voltage collapse is taken into account.

2. POWER SYSTEM MODEL

This section summarizes the 3 bus power system example of [10,11] (see figure 1) which consists of two generators (one is slack bus) and a dynamic load with capacitive support. The parameter values used in this paper are identical to those of [10,11] except that the generator damping D has been increased. This eliminates the Hopf bifurcations and other oscillatory phenomena discovered in [12,13] at high loadings. A well designed power system stabilizer would suppress these oscillations. Note that the value of D has no effect on the loading at which the saddle node bifurcation occurs.

The load model includes a dynamic induction motor model with a constant PQ load in parallel. The combined model for the motor and the PQ load is

$$P_l = P_0 + P_1 + K_{pw}\dot{\delta} + K_{pv}(V + T\dot{V}) \quad (2.1)$$

$$Q_l = Q_0 + Q + K_{qw}\dot{\delta} + K_{qv}V + K_{qv2}V^2 \quad (2.2)$$

Q is a parameter which varies with the load reactive power demand. The capacitor is accounted for by adjusting E_0 , Y_0 and θ_0 to E'_0 , Y'_0 and θ'_0 to give the Thevenin equivalent of the circuit with the capacitor.

The system can be described by following differential equations [10]

$$\dot{\delta}_m = \omega \quad (2.3)$$

$$M\dot{\omega} = -D\omega + P_m + E_m Y_m V \sin(\delta - \delta_m - \theta_m) + E_m^2 Y_m \sin \theta_m \quad (2.4)$$

$$K_{qw}\delta = -K_{qv2}V^2 - K_{qv}V + Q_l - Q_0 - Q \quad (2.5)$$

$$TK_{qw}K_{pv}\dot{V} = K_{pw}P_{qv2}V^2 + (K_{pw}K_{qv} - K_{qw}K_{pv})V + K_{qw}(P_l - P_0 - P_1) - K_{pw}(Q_l - Q_0 - Q) \quad (2.6)$$

where the real and reactive powers supplied to the load by the network are

$$P_l = -E'_0Y'_0V \sin(\delta + \theta'_0) - E_mY_mV \sin(\delta - \delta_m + \theta_m) + (Y'_0 \sin \theta'_0 + Y_m \sin \theta_m)V^2 \quad (2.7)$$

$$Q_l = E'_0Y'_0V \cos(\delta + \theta'_0) + E_mY_mV \cos(\delta - \delta_m + \theta_m) - (Y'_0 \cos \theta'_0 + Y_m \cos \theta_m)V^2 \quad (2.8)$$

E_m is the terminal voltage of the generator and E is the internal voltage of the generator.

The state vector is $x = (\delta_m, \omega, \delta, V)$ and M , D and P_m are the generator inertia, damping and mechanical power respectively. The load parameter values are $K_{pw} = 0.4$, $K_{pv} = 0.3$, $K_{qw} = -0.03$, $K_{qv} = -2.8$, $K_{qv2} = 2.1$, $T' = 8.5$, $P_0 = 0.6$, $Q_0 = 1.3$, $P_1 = 0.0$ and the network and generator parameter values are $Y_0 = 20.0$, $\theta_0 = -5.0$, $E_0 = 1.0$, $C = 12.0$, $Y'_0 = 8.0$, $\theta'_0 = -12.0$, $E'_0 = 2.5$, $Y_m = 5.0$, $\theta_m = -5.0$, $E_m = 1.0$, $P_m = 1.0$, $D = 0.12$, $M = 0.3$, $X_s = 0.15$.

3. MODEL OF A GENERATOR REACTIVE POWER LIMIT

The two main causes of the reactive power output Q_g reaching limit in a generator are the excitation current limit and the stator thermal limit [14, 15]. The two limits have similar overall effects on the system and we only consider the excitation current limit.

Because the generator internal voltage E is proportional to the excitation current, the excitation current limit may be modelled by E encountering a limit E^{lim} . Before the limit is encountered, $E < E^{lim}$ and E_m is controlled so that $E_m = E^{imp}$, the terminal voltage imposed by the regulator [5]. When the limit is encountered, $E = E^{lim}$ and E_m varies with $E_m < E^{imp}$. Encountering the limit may be thought of as changing from the constraint of constant E_m to the constraint of constant E . The effect of the limit on the system differential equations (2.3)-(2.6) is to replace the constant E_m by an expression involving the system state and E^{lim} . We now derive the necessary equations for this replacement.

Write the powers delivered to the network by the generator as:

$$P_g = EE_mY_s \sin \delta_s \quad (3.1)$$

$$Q_g = EE_mY_s \cos \delta_s - E_m^2Y_s \quad (3.2)$$

where δ_s is the difference of the angle between E and E_m and Y_s is $1/X_s$. Squaring and adding (3.1) and (3.2) yields

$$(Q_g + E_m^2Y_s)^2 + P_g^2 = (EE_mY_s)^2 \quad (3.3)$$

Real and reactive power balance at the generator gives

$$P_g = -E_m^2Y_m \sin \theta_m + E_mY_mV \sin(\delta_m - \delta + \theta_m) \quad (3.4)$$

$$Q_g = E_m^2Y_m \cos \theta_m - E_mY_mV \cos(\delta_m - \delta + \theta_m) \quad (3.5)$$

A quadratic equation in E_m can be obtained from (3.3) to (3.5):

$$E_m^2(Y_m \cos \theta_m + Y_s)^2 - 2E_mY_mV(Y_m \cos \theta_m + Y_s) \cos(\delta_m - \delta + \theta_m) + Y_m^2V^2 \cos^2(\delta_m - \delta + \theta_m) + E_m^2Y_m^2 \sin^2 \theta_m - 2E_mY_m^2V \sin \theta_m \sin(\delta_m - \delta + \theta_m) + Y_m^2V^2 \sin^2(\delta_m - \delta + \theta_m) - E^2Y_s^2 = 0 \quad (3.6)$$

The limited system equations are obtained by solving (3.6) with E replaced by E^{lim} for the positive solution of E_m and substituting it into (2.3)-(2.6). It remains to compute the value of E^{lim} . Suppose Q_g reaches its limit at $Q = Q^{lim}$. Then E^{lim} is computed from equations (3.3)-(3.5) where δ_m , δ , V are equilibrium solutions of (2.3)-(2.6) when $Q = Q^{lim}$.

4. IMMEDIATE VOLTAGE COLLAPSE WHEN A LIMIT IS ENCOUNTERED

We present two cases of the example power system encountering an excitation current limit. It is convenient to write J for the Jacobian of the unlimited system evaluated at the operating equilibrium x_0 and J^{lim} for the Jacobian of the limited system evaluated at x_0 . The first case is well known and the bifurcation diagrams of the unlimited and limited systems are shown in figure 2(a). The limit is encountered at $Q = Q^{lim} = 11.0$. When the limit is encountered the system changes structure and J changes to J^{lim} . The eigenvalues of J^{lim} differ from the eigenvalues of J but their real parts remain negative (see table 1, case (a)). The equilibrium at x_0 of the limited system has reduced stability but remains stable and the system reactive power margin $Q^{lim} - Q^*$ is reduced. Note that the upper portion of each of the bifurcation diagrams is stable and the lower portion is unstable; in this case x_0 is on the upper and stable portion of both the unlimited and limited system bifurcation curves.

In the second case, the limit is encountered at the higher system loading $Q = Q^{lim} = 11.4$. The system changes so that J^{lim} has an eigenvalue with positive

real part (see table 1, case(b)). In this case x_0 is on the upper and stable portion of unlimited system bifurcation curve and the lower and unstable portion of limited system bifurcation curve (see figure 2(b)). The operating equilibrium x_0 becomes immediately unstable when the limit is encountered and the system dynamics will move the system state away from x_0 . Now we describe the dynamic consequences of the instability; see [16,17] for background in dynamical systems and bifurcations.

The unstable equilibrium x_0 has a one dimensional unstable manifold W^u which consists of two trajectories W_-^u and W_+^u leaving x_0 and x_0 itself (see the idealized sketch of figure 3). W^u is a smooth curve passing through x_0 which is tangent at x_0 to the eigenvector v of J^{lim} associated with the positive eigenvalue. x_0 also has a three dimensional stable manifold W^s which divides the state space near x_0 into two parts. Note that the stable manifolds sketched in figure 3 have an additional dimension which is not shown.

The system state is initially at x_0 but cannot remain there because of the inevitable small perturbations on the state. If the state is perturbed from x_0 to one side of W^s , it will be attracted towards W_-^u and we can simply approximate the dynamics by motion along W_-^u . Similarly, if the state is perturbed from x_0 to the other side of W^s , it will be attracted towards W_+^u and we can simply approximate the dynamics by motion along W_+^u . In short, the dynamical consequences of the immediate instability are motion along the trajectory W_-^u or the trajectory W_+^u . Either outcome is possible and there seems no reason to regard W_-^u or W_+^u as more likely.

We integrated the differential equations of the limited system starting near x_0 to find the outcome of motion along W_-^u or W_+^u . (The initial conditions were chosen to be $x_0 \pm 0.001v$ and a semi-implicit trapezoidal integration method [18] was used since differential equations are stiff near a saddle node bifurcation.) The corresponding time histories of V are shown in figure 4. The trajectory W_-^u tends to the nearby stable equilibrium x_1^{lim} and the initial portion of the slow, oscillatory convergence of V to x_1^{lim} is shown in the upper graph of figure 4. (This may not occur in practice because the voltage control system could prevent the voltage from rising [9].) The trajectory W_+^u diverges so that V decreases and the motion along W_+^u shown in the lower graph of figure 4 is a voltage collapse. Thus we describe a model of a new mechanism for voltage collapse:

The operating equilibrium x_0 becomes immediately unstable when a reactive power limit is encountered and one of the possible dynamical consequences is voltage collapse along part of the unstable manifold of x_0 .

This model of immediate voltage collapse has some similarities with the center manifold collapse model [10]; the voltage collapse dynamics can be modelled by movement along a particular trajectory. However, the trajectory is part of an unstable manifold instead of the unstable part of a center manifold and there is also the possibility of convergence to a nearby stable equilibrium along the other part of the unstable manifold.

5. APPROXIMATE ANALYSIS SHOWING THAT IMMEDIATE VOLTAGE COLLAPSE CAN OCCUR FOR HIGH LOADING

We expect the immediate instability of our example to be typical for sufficiently high loadings. This can be demonstrated analytically by approximating the equations of the example system as follows. First consider the relationship between V , E and E_m . Write $\alpha = \delta_m - \delta$, and approximate the line impedance to be purely reactive so that $\theta_m = 0$. Equation (3.6) becomes

$$E_m^2(Y_m + Y_s)^2 - 2E_m Y_m V(Y_m + Y_s) \cos \alpha + Y_m^2 V^2 - E^2 Y_s^2 = 0 \quad (5.1)$$

Assuming that α is small ($\leq 20^\circ$), we approximate $\cos \alpha \approx 1$, and write $Y_m' = Y_m + Y_s$ to obtain the reduced equation:

$$E_m = \frac{E Y_s}{Y_m'} + \frac{Y_m V}{Y_m'} \quad (5.2)$$

We substitute (5.2) into system differential equations (2.5) and get a decoupled dynamical equation for the limited system with the left hand side set to zero:

$$Q_0 + Q - \left(-K_{qv} + E_0' Y_0' \cos \delta + \frac{E Y_m Y_s}{Y_m'} \cos \alpha \right) V + \left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'} \cos \alpha \right) V^2 = 0 \quad (5.3)$$

Using the approximations of [11], we suppose all angles are small:

$$Q_0 + Q - \left(-K_{qv} + E_0' Y_0' + \frac{E Y_m Y_s}{Y_m'} \right) V + \left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'} \right) V^2 = 0 \quad (5.4)$$

Equation (5.4) describes the bifurcation curve (the relation between V and Q) of the limited system as shown in figure 2(b). The gradient of (5.4) is

$$\frac{dQ}{dV} = -K_{qv} + E_0' Y_0' + \frac{E Y_m Y_s}{Y_m'} - 2V \left(K_{qv2} + Y_0' + Y_m - \frac{Y_m^2}{Y_m'} \right) \quad (5.5)$$

For comparison, we write the equation of the bifurcation curve for the unlimited system as [11]:

$$Q_0 + Q - (-K_{qv} + E_0' Y_0' + E_m Y_m) V + (K_{qv2} + Y_0' + Y_m) V^2 = 0 \quad (5.6)$$

The gradient of (5.6) is

$$\frac{dQ}{dV} = -K_{qv} + E'_0 Y'_0 + E_m Y_m - 2V(K_{qv2} + Y'_0 + Y_m) \quad (5.7)$$

At the intersection of two curves, we can substitute (5.2) into (5.5). Then (5.5) becomes:

$$\frac{dQ}{dV} = -K_{qv} + E'_0 Y'_0 + E_m Y_m - 2V(K_{qv2} + Y'_0 + Y_m) + V \frac{Y_m^2}{Y'_m} \quad (5.8)$$

Comparing (5.7) and (5.8), it is clear that the gradient of the bifurcation curve of the unlimited system is always larger than the gradient of the bifurcation curve of the limited system at their intersection since VY_m^2/Y'_m is always positive.

Now suppose the limit is encountered at the saddle node bifurcation of the unlimited system so that $Q^{lim} = Q^*$, the value of Q at the saddle node bifurcation. For the unlimited system we have the gradient $\frac{dQ}{dV}|_{V_*} = 0$ at the saddle node bifurcation point where V_* is the voltage at the saddle node bifurcation. For the limited system, it follows that $\frac{dQ}{dV}|_{V_*} > 0$ and hence $\frac{dV}{dQ}|_{Q^*} > 0$. Therefore the two curves cross each other at the bifurcation point as shown in figure 5. Because the lower portion of the curve is unstable, the equilibrium x_0 becomes unstable and J^{lim} has a positive eigenvalue when the limit $Q^{lim} = Q^*$ is encountered.

Now we suppose that the limit is encountered at a loading Q^{lim} less than and close to Q^* . Since the eigenvalue of J^{lim} is a continuous function of Q^{lim} , we conclude that the equilibrium x_0 will become immediately unstable when the limit is encountered for Q^{lim} sufficiently close to Q^* .

6. IMMEDIATE INSTABILITY AND VOLTAGE COLLAPSE IN A GENERAL POWER SYSTEM MODEL

The particular example presented above shows that a sufficiently heavily loaded but stable system can become immediately unstable when a reactive power limit is encountered. The dynamical consequences of this instability are either collapse along the unstable manifold trajectory W_+^u or convergence to a nearby stable equilibrium along the unstable manifold trajectory W_-^u . Now we argue that this description is expected to apply in the general case. Our main assumptions are that applying a reactive power limit does not increase the margin of system stability, the phenomena occurring are generic and a simplification that only one bifurcation occurs.

Consider a general power system modelled by smooth parameterized differential equations $\dot{x} = f(x, \lambda)$, where $x \in \mathbb{R}^n$ is the system state and $\lambda \in \mathbb{R}^m$ are slowly changing system parameters such as real and reactive

load powers [10]. We suppose the system is operated at a stable equilibrium x_0 when the parameters are λ_0 . When a reactive power limit is encountered the system equations immediately change to $\dot{x} = f^{lim}(x, \lambda)$ but the position of the equilibrium x_0 is unchanged. That is, $0 = f(x_0, \lambda_0) = f^{lim}(x_0, \lambda_0)$.

Now suppose that the equations $\dot{x} = f(x, \lambda)$ are gradually changed into the equations $\dot{x} = f^{lim}(x, \lambda)$. This can easily be done by combining f and f^{lim} into new equations

$$\dot{x} = g(x, \lambda_0, k) \quad (6.1)$$

with a parameter k so that $g(x, \lambda_0, 0) = f(x, \lambda_0)$ and $g(x, \lambda_0, 1) = f^{lim}(x, \lambda_0)$. We also require that

$$g(x_0, \lambda_0, k) = 0 \quad \text{for} \quad k \in [0, 1] \quad (6.2)$$

In short, we construct a homotopy joining f and f^{lim} which preserves the equilibrium at x_0 . Note that λ is fixed at λ_0 as the parameter k is varied. k gradually increasing from 0 to 1 has the effect of gradually applying the reactive power limit to the system. This allows the change in structure between f and f^{lim} to be studied using bifurcation theory. However, we do not seek to represent the manner in which the reactive power limit is applied in practice by the gradual increase in k .

(6.1) is a one parameter system of differential equations with the restriction (6.2) of an equilibrium at x_0 . If we assume that this is a generic one parameter system of differential equations, then the only bifurcations which can occur are the transcritical bifurcation and the Hopf bifurcation [16]. (In the set of all smooth one parameter differential equations without restrictions or symmetries, the generic bifurcations are the saddle node bifurcation and the Hopf bifurcation. An equilibrium disappears in a saddle node bifurcation and therefore the restriction (6.2) precludes saddle node bifurcations.) In a generic transcritical bifurcation two equilibria approach each other, coalesce and then separate with an exchange of stability. The Jacobian has a single, simple zero eigenvalue at the bifurcation. If one of the equilibria is stable before the bifurcation, then the other is type one unstable. After the bifurcation the equilibria are also stable and type one unstable but each equilibrium has changed its stability. Figure 6 shows a typical bifurcation diagram for the transcritical bifurcation in which the solid line indicates stable equilibria and the dashed line denotes unstable equilibria. In the generic Hopf bifurcation, the equilibrium changes stability by interacting with a limit cycle. The Jacobian has a single, simple pair of imaginary eigenvalues at the bifurcation. Of course it is also generic for there to be no bifurcation for $k \in [0, 1]$.

We first consider the special case of encountering a reactive power limit at the point of voltage collapse; that is, λ_0 is chosen so that $\dot{x} = f(x, \lambda)$ with parameter λ has a generic saddle node bifurcation at (x_0, λ_0) . Then x_0 is a degenerate equilibrium formed by the coalescence of a stable equilibrium and a type one unstable equilibrium and the Jacobian of f evaluated at (x_0, λ) is singular [10]. Since the Jacobian of f evaluated at (x, λ_0) and the Jacobian of g evaluated at $(x, \lambda_0, 0)$ are

identical, the Jacobian of g evaluated at $(x, \lambda_0, 0)$ is also singular. Therefore $(x_0, \lambda_0, 0)$ is also a bifurcation point of the system (6.1) with k as parameter. If we consider only generic phenomena, then the bifurcation of (6.1) at $(x_0, \lambda_0, 0)$ is a transcritical bifurcation. The Hopf bifurcation is precluded by the single zero eigenvalue and absence of nonzero imaginary eigenvalues of the Jacobian at x_0 at the generic saddle node bifurcation of $\dot{x} = f(x, \lambda)$ with λ as parameter.

In a generically occurring transcritical bifurcation the bifurcation diagram (suitably reduced to the center manifold of the suspended system [17]) is as shown in figure 6. There are two possible "directions" through the bifurcation as k is increased from zero so that the equilibrium x_0 is either stable or unstable for small positive k . Since reactive power limits are generally observed to limit system performance, it seems likely that partially applying a reactive power limit (imposing small positive k) should destabilize rather than stabilize the system. Thus the more usual case to consider should be x_0 unstable for small positive k . For small positive k there is a type one unstable equilibrium x_1 close to x_0 and part of the unstable manifold W^u of x_0 is a trajectory tending to x_1 . In this case, if we further assume for simplicity that there are no further bifurcations as k increases from a small positive value to one, then we can conclude that at $k = 1$, the system $\dot{x} = g(x, \lambda_0, 1) = f^{lim}(x, \lambda_0)$ has x_0 unstable and type one. Moreover, there is a stable equilibrium x_1 in the vicinity and the part W^u of the unstable manifold of x_0 is a trajectory tending to x_1 .

The assumption of genericity of the one parameter differential equations (6.1) implies that the occurrence of the transcritical bifurcation is robust to small changes in (6.1). In particular, if we consider a sufficiently heavily loaded but still stable system $f(x, \lambda'_0)$ with λ'_0 chosen close to λ_0 so that $f(x, \lambda'_0)$ has a stable equilibrium x'_0 , then the corresponding homotopy $g(x, \lambda'_0, k)$ will have a transcritical bifurcation as k increases from zero to one and we can extend the conclusions for the case of a reactive power limit encountered at the saddle node bifurcation to the case of a reactive power limit encountered just before the saddle node bifurcation. The only difference is that x_0 is stable for $k = 0$ and the transcritical bifurcation will occur at a positive value of k . Figure 6 shows the bifurcation diagram for the change in stability of the example system in case (b) (the black circles are the data we computed).

7. DISCUSSION AND CONCLUSIONS

The instantaneous change in the system equations when a generator reactive power limit is encountered causes the system dynamics and structure to instantaneously change, although the system state is unchanged. In particular, the Jacobian at the operating equilibrium and the closest unstable equilibrium change discontinuously. It follows that most of the voltage collapse indices proposed in the literature which are functions only of the system *before* a reactive power limit is encountered are *discontinuous* when the reactive power limit is encountered. We emphasize this point because the literature often neglects this somewhat unpalatable discontinuity or misleadingly describes it as a discon-

tinuity in the derivative of the index. Exceptions are indices such as the total generated reactive power index of Begovic and Phadke [6] which are a functions of the system operating point only and hence continuous when a reactive power limit is encountered. Two other exceptions are the energy function index of Overbye and DeMarco [19] and the load power margin index when proper account is taken of the reactive power limits as, for example, in Van Cutsem [9].

One important consequence of a generator reactive power limit, causing an immediate instability is that the load power margin can be misinterpreted to give an incorrect measure of system stability. A load power margin measures the load increase that the system can sustain before bifurcation but is only valid when the system is operated at a stable equilibrium. If the operating equilibrium becomes immediately unstable, the load power margin is incorrect because the system is (at least momentarily) at an *unstable* equilibrium. (If the system were operated at the nearby stable equilibrium, then the load power margin would be a valid measure of system stability. We note that the system state might converge to the nearby stable equilibrium as a result of the immediate instability but the possibility of voltage collapse as a result of the immediate instability is at least as likely.)

In case (a) the example power system retains stability when the reactive power limit is encountered and the reactive power margin $Q^* - Q$ does measure the system stability (see figure 2(a)). In the more heavily loaded case (b), the system becomes immediately unstable when the reactive power limit is encountered and the system may immediately collapse whereas the reactive power margin $Q^* - Q$ can be misinterpreted as a positive margin of stability (see figure 2(b)). We conclude that the stability of the limited system equilibrium should be checked when load power margins are computed.

Borremans et al [4] recognized that a generator reactive power limit could precipitate a voltage collapse in the manner of figure 2(b), but regarded this possibility as more theoretical than the case of figure 2(a). We show by an example and general arguments that immediate voltage collapse is likely to be typical for a sufficiently highly loaded system encountering a generator reactive power limit. However, our results have not established that immediate voltage collapse occurs over a significant range of loadings up to the bifurcation. That is, we have not excluded the possibility that the immediate voltage collapse might only exist for a small interval of loadings before bifurcation. (Our power system example is not conclusive in this regard because of its small size and the lack of validation of the load model.) We conclude that immediate instability when a generator reactive power limit is encountered is a plausible cause of voltage collapse whose relative importance is not yet established.

We also study the simplest and most likely dynamical consequences of the immediate instability in a general power system using the theory of generic saddle node and transcritical bifurcations. The dynamical consequences are either convergence to a nearby stable equi-

librium or a voltage collapse. The voltage collapse dynamics may be modelled by movement along a specific trajectory which is part of the unstable manifold of the unstable equilibrium. This is a new model for voltage collapse dynamics with some similarities with the center manifold theory of voltage collapse [10]. The dynamics of voltage collapse are easy to simulate by numerical integration along the unstable manifold and were illustrated using the example power system.

We hope our example and general analysis will encourage more study of the interaction of system limits and voltage collapse. In particular, we note the usefulness of the transcritical bifurcation in understanding how encountering a system limit can affect the system behaviour.

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Case	Q^{lim}	Eigenvalues of J			Eigenvalues of J^{lim}		
(a)	11.000	-14.686	-0.110±3.721j	-126.001	-7.602	-0.017±3.379j	-142.961
(b)	11.400	-3.734	-0.040±3.040j	-94.627	3.977	-0.904±3.388j	-106.645

Table 1

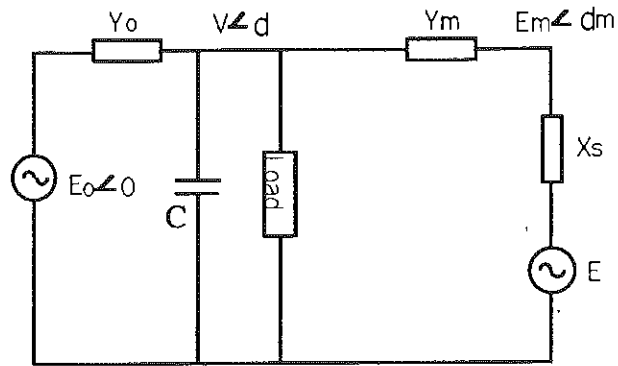


Figure 1. 3 – bus example power system

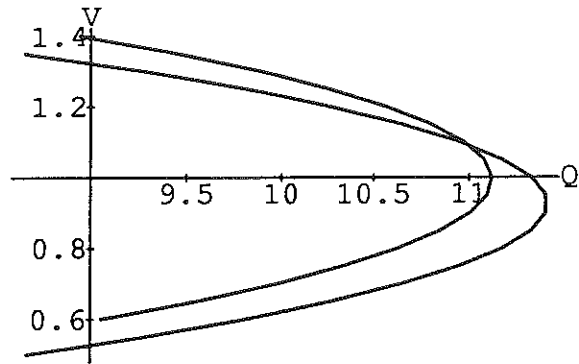


Figure 2(a). Limit encountered at a lesser loading

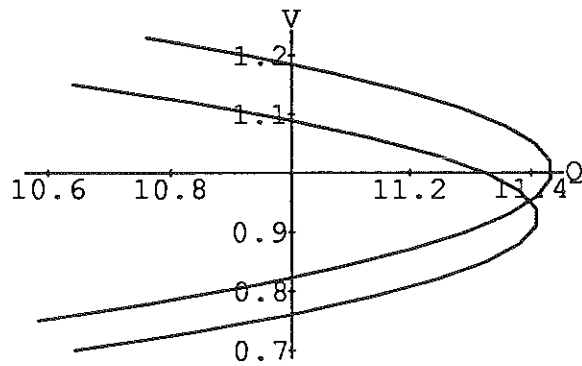


Figure 2(b). Immediate instability when limit is encountered

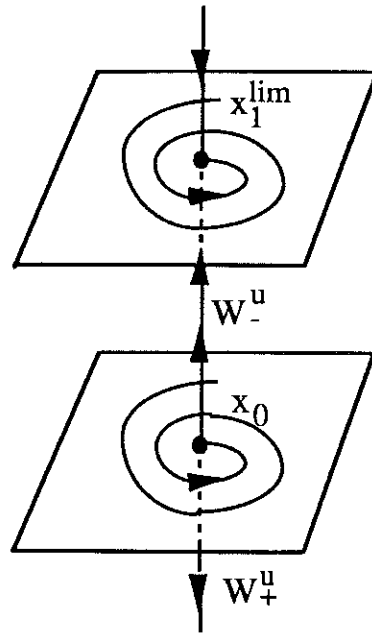


Figure 3. Idealized sketch of limited system dynamics

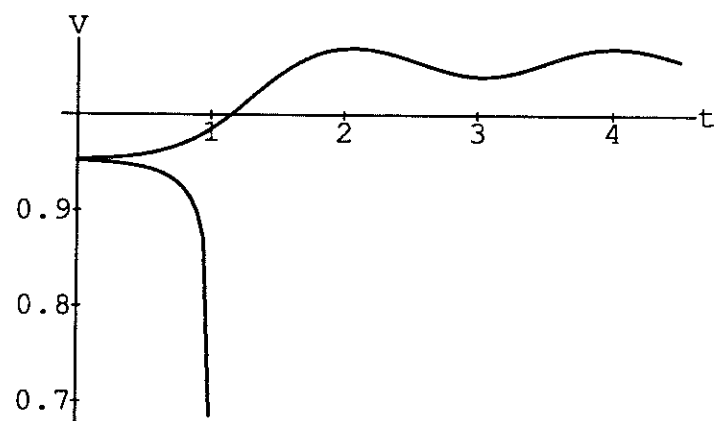


Figure 4. Possible dynamics caused by immediate instability

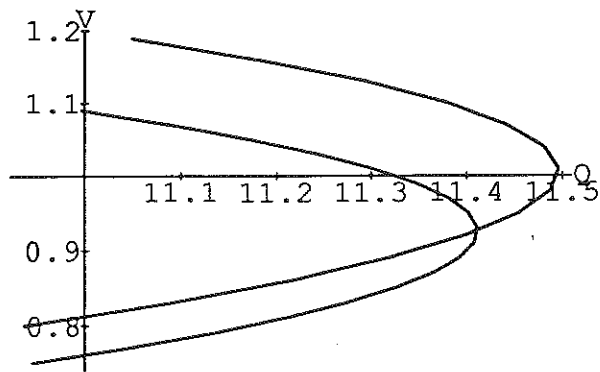


Figure 5. Immediate instability at a saddle node bifurcation

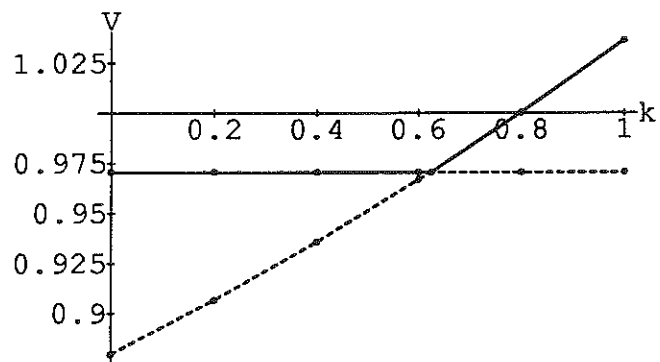


Figure 6. Transcritical bifurcation diagram when limit is gradually applied

VOLTAGE COLLAPSE OR STEADY STATE STABILITY LIMIT

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ABSTRACT

Following some confusion found in the literature regarding the relation between Voltage Collapse (VC) and the classical Steady State Stability Limit (SSSL), the main similarities and differences between these two concepts are identified. It is shown that they are in fact two different phenomena.

Analytical expressions to characterize VC and SSSL are extensively supported by graphical outputs. Three dimensional surfaces are used to show the locus of the operating points where these phenomena occur. It is found that there are two common points where both phenomena occur at the same time, when the total differential of P or Q equal zero ($dP=0$ or $dQ=0$) and this coincides with the power factor angle being equal to the negative of the transmission line angle or the negative of the transmission line angle $\pm 180^\circ$ respectively. From these two conditions, only $dP=0$ occurs with $P>0$; therefore it is actually one common point to be considered under practical situations.

The relation of Voltage Collapse and Steady State Stability Limit with the singularity of the power flow Jacobian and the System state matrix is also presented. It is shown that $\partial P/\partial V$ or $\partial Q/\partial V$ in the Jacobian matrix do not necessarily have to be zero for Voltage Collapse to occur.

INTRODUCTION

Historically, steady-state stability was studied before transient stability [1], but as it was recognised that short circuits were the most severe hazard to stable operation, attention has been focused mainly on transient stability and the different ways to improve it.

As the Steady-State Stability Limit (SSSL) of a system is greater than its Transient Stability Limit (TSL) under the same operating conditions, it seemed fully justifiable to concentrate the attention on TSL.

With the relatively recent appearance of the term "Voltage Collapse" (VC), power system engineers have turned again to SSSL studies. As the VC term was associated with the instability phenomenon generally involving the transference of important amounts of power over long distances (saturating the transmission lines capacity) and could be triggered from the steady-state by a small change in the load, similarities between these two issues were brought to the point that some authors see the VC issue as the SSSL revisited.

A definition of SSSL, taken from a Classical Reference Book on Power System Stability [1], is:

"[SSSL] is the maximum power at the receiving end of the circuit that can be transmitted without loss of synchronism if the load is increased in very small steps and if the field currents are changed after each increment of load so as to restore the normal operating conditions (usually constant terminal voltages)."

It is implied in this definition that reactive support is available at the PV nodes to keep the terminal voltage constant; perhaps supplied by synchronous machines or static compensators.

Note that VC can also be caused by the increase of load in very small steps, although loss of synchronism (loss of SSS) occurs, as opposed to VC, with the loss of synchronising power as the electrical angle between machines grows wider. This type of instability is sometimes referred to as "angle instability".

Following a more recent, but also more general definition of SSSL presented in reference [2], which for the sake of completeness is also quoted here,

"...The steady-state stability limit is a steady-state operating condition for which the power system is steady-state stable but for which an arbitrarily small change in any of the operating quantities in an unfavourable direction causes the power system to lose stability..."

the VC type of instability could also be classified under this heading because once at the limit, just before VC occurs, the power system could be in a steady-state operating condition and the instability triggered by a small change of an operating quantity.

In the last paragraph of reference [2], discussion of the classification of instability according to the nature of the cause, involves the following comment:

"...Instability can also be caused by insufficient reactive support, resulting in a voltage collapse even when sufficient synchronising and damping torques are available. The term "voltage instability" has been used to describe this [phenomenon]..."

The authors close with:

"...the concept of 90° limit does not appear as there is no voltage support. Many practical cases of instability are blends of both "angle" and "voltage" instability. However, these terms are not yet widely used and are only mentioned in this section because recent voltage problems on many systems may make them important in the near future."

These extracts from references [1,2], show the feeling on this subject of a great number of power system engineers: on one hand, VC seems to be what is known as the steady-state stability limit, but on the other hand, it needs special clarification and some precautions should be taken if comparisons are taken further.

The confusion is evident from the results of a CIGRE questionnaire [3] which reports on the Voltage Stability section:

"...In many answers, there is no proper boundary between the voltage and steady-state stability problems..."

When deciding whether these two terms are the same or not some authors on VC do not mention the relation at all, others choose sides implicitly through their mathematical formulation, and still others accept that these two subjects differ from each other in essence; this is the case of reference [4], in which it is stated that:

"...This result theoretically establishes a link between voltage and phase coherency. It also suggests a link between voltage collapse and steady-state angle stability although no such link has been theoretically established to date."

The confusion on this matter was the motivation for the study reported in this paper, which fills one of the important conceptual gaps found in the VC literature.

CLASSICAL STEADY-STATE STABILITY LIMIT

Following the SSSL definitions given in the previous section, consider as a test case, the two node system composed of a synchronous generator, a transmission line and a synchronous motor (G-T-M) as shown in Fig. 1A.

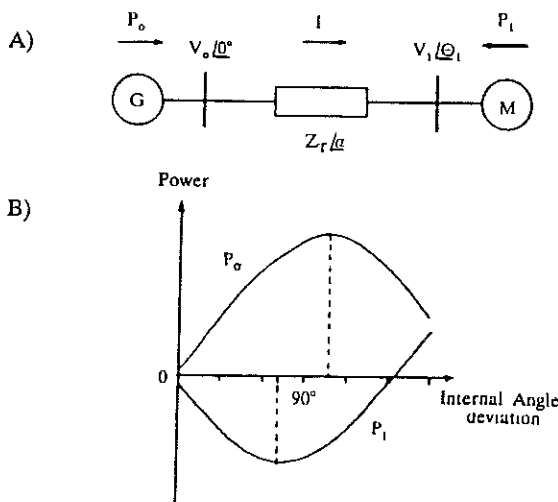


Figure 1 A) Two node test system and its B) Power Angle curves for constant excitation.

When specifying the maximum power that can be transmitted through the line, we should keep in mind that for a line with losses modelled ($R \neq 0$), the maximum power at the sending end of the line is greater than that at the receiving end and occurs at a different voltage angle separation (Fig. 1B). From a practical point of view, we are more interested in the maximum power at the receiving end of the line;

once there is no point in sending more power into the line if by doing so, only the losses are increased instead of the received power. So, we will concentrate our attention on the receiving end of the line.

The power at the receiving end node of Fig. 1A is given by:

$$\begin{aligned}\bar{S}_1 &= \bar{V}_1 \cdot \bar{I}^* = \bar{V}_1 \cdot \left(\frac{\bar{V}_1 - \bar{V}_0}{\bar{Z}_T} \right)^* = V_1 e^{j\theta_1} \cdot \left(\frac{V_1 e^{-j\theta_1} - V_0}{Z_T e^{-j\alpha}} \right) = \\ &= \frac{V_1^2}{Z_T} e^{j\alpha} - \left(\frac{V_1 \cdot V_0}{Z_T} \right) e^{j(\theta_1 + \alpha)}\end{aligned}$$

or

$$P_1 = \frac{V_1^2}{Z_T} \cdot \cos \alpha - \frac{V_1 \cdot V_0}{Z_T} \cos (\theta_1 + \alpha) \quad (1)$$

$$Q_1 = \frac{V_1^2}{Z_T} \cdot \sin \alpha - \frac{V_1 \cdot V_0}{Z_T} \sin (\theta_1 + \alpha) \quad (2)$$

If the voltages are considered constant, it is straightforward from Eqn.1 that the maximum power $P_{1\max}$ occurs when $\theta_1 = -\alpha$. For long EHV and UHV transmission lines the reactance of the line is much greater than its resistance ($X \gg R$) and consequently it is a common practise to neglect the later; thus making $\alpha = 90^\circ$. Usually the SSSL has been based on this angle characterising the critical angular difference θ_1 as either -90° or the (-)ve value of the angle α . Substitution of $\alpha = 90^\circ$ and $\theta_1 = -\alpha$ in Eqns.1 and 2 gives:

$$P_{1\max} = - \frac{V_1 \cdot V_0}{Z_T} \quad (3)$$

and the corresponding reactive power required to sustain V_1 constant under this condition.

$$Q_1(P_{1\max}) = - \frac{V_1^2}{Z_T} \quad (4)$$

VOLTAGE COLLAPSE

Whenever we hear "Voltage Collapse", the first thing that comes to mind is the V-P curve shown in Fig. 2B, which corresponds to a particular power factor of the load. The test system used to obtain this curve, is shown in Fig. 2A, which represents a generator, a transmission line and a non-synchronous load (G-T-L).

The corresponding V-P curve, can be obtained if the load impedance magnitude varies over a wide range ($\infty, 0$) representing all the values of power taken from the open circuit to the short circuit at the load end.

Starting with an infinite load impedance (Z_L), which is equivalent to having an open circuit at node 1 or zero load, impedance Z_L is

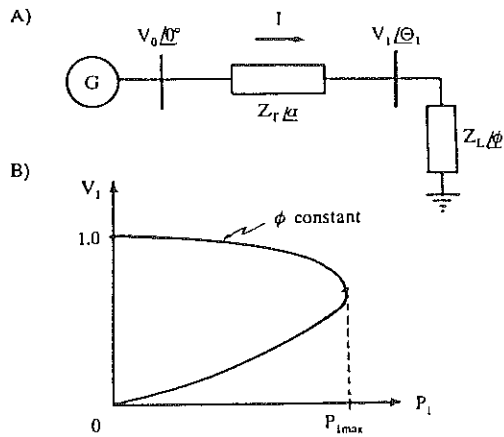


Figure 2 A) Two node test system and its B) V-P Characteristic.

continuously decreased to simulate the loading; the power at that node increases and so does the current I . This increase in the current, produces a voltage drop across the transmission line reducing V_1 .

The power P_1 calculated as the product $V_1 I$ increases for the rapid current increase is higher than the initially slow reduction in V_1 . This increase in P_1 passes through a maximum value and then diminishes as the load impedance Z_L keeps on decreasing, once the increase of the current I tends to saturate and is not enough to compensate the sharp reduction of the voltage V_1 in the product $V_1 I$. Further reduction of Z_L (i.e. increase of the load) will cause further reduction in the power P_1 proportional to the reduction of V_1 .

Note that as in the previous section, there is a maximum power P_1 that can be reached through this process. This maximum power transfer occurs when the load impedance magnitude Z_L is equal to the magnitude of the transmission line impedance Z_T .

The development of the expression to calculate P_{1max} (Eqn. 5) for this case is not as straightforward as in the previous section and therefore not presented here. The reader can find the details in reference [5].

$$P_{1max} = \frac{V_0^2 \cos \phi}{2Z_T [1 + \cos(\alpha - \phi)]} \quad (5)$$

and the associated reactive power, which is calculated according to the power factor,

$$Q_{1 P1max} = P_{1max} (\tan \phi) \quad (6)$$

If we assume as we did in Eqn. 3, that $X \gg R$ and substitute $\alpha = 90^\circ$ in Eqn. 5, we find that as opposed to what happened in the previous case, P_{1max} increases up to infinity, when $\phi = -90^\circ$ and so does $Q_{1 P1max}$. The differences have started to appear. Note that Eqn. 5 does not contain the variable V_1 and Eqn. 3 does. In fact, if we develop the expression to calculate V_{1crit} for the points P_{1max} and $Q_{1 P1max}$ we have:

$$V_{1crit} = \frac{V_0}{2 \cos \frac{\alpha - \phi}{2}} \quad (7)$$

which will also tend to infinity for $\alpha = 90^\circ$ and $\phi = -90^\circ$ as opposed to the constant value V_1 in Eqn. 3.

An explanation of the voltage collapse phenomenon from the Power System Operation point of view can be found in a number of references (e.g. [6]); this paper limits itself to addressing the relevant points for comparison purposes with the SSSL.

CONDITIONS DEFINING THE STEADY-STATE STABILITY LIMIT

Recalling the very old fashioned, but also very illustrative Power Circle Diagram technique [7] and applying it to the test system of Fig. 1A, we can obtain the locus of the active and reactive power at the receiving end node shown in Fig. 3 for constant values of V_0 and V_1 .

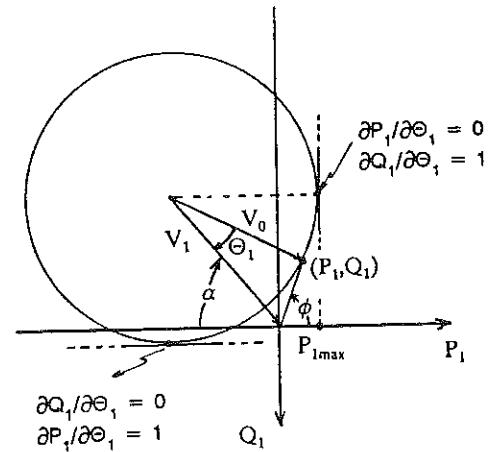


Figure 3 Receiving end Power Circle Diagram.

Phasor notation was oversimplified to give a clearer picture of the interrelation between variables and facilitate a qualitative analysis.

It is obvious from this diagram that the maximum active power P_{1max} occurs at the point where the tangent to the power circle becomes vertical, i.e. $\partial P_1 / \partial \theta_1 = 0$ and $\partial Q_1 / \partial \theta_1 = 1$.

This point also corresponds to the point where the displacement angle between sending and receiving-end voltages θ_1 is equal to the negative of the line impedance angle α (V_0 was taken as the reference). This is in accordance with Eqn. 1, from where we observed that the maximum P_1 occurred when $\theta_1 = -\alpha$. Analogously, the maximum reactive power Q_{1max} will occur at the point where the tangent to the power circle becomes horizontal, i.e. $\partial P_1 / \partial \theta_1 = 1$ and $\partial Q_1 / \partial \theta_1 = 0$.

A family of curves like the one in Fig. 3 may be drawn for the same value V_0 (1.0 p.u.) and different values of V_1 . The circles are eccentric with the centres lying on the same straight line at various distances

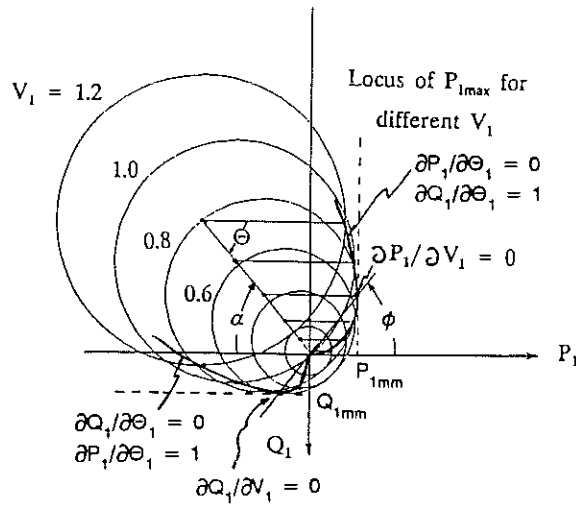


Figure 4 Locus of P_1 , Q_1 for $V_0 = 1.0$ pu and various V_1 showing P_{1max} and Q_{1max} for each voltage level V_1 . Line resistance $R \neq 0$.

from the origin; as shown in Fig. 4. The radii are proportional to the corresponding V_1 .

The curve shows the locus of P_{1max} and Q_{1max} for different V_1 ; note that there is a point P_{1mm} where P_{1max} is maximum and a point Q_{1mm} where Q_{1max} reaches a maximum value as well. At these points, apart from the previous conditions $\partial P_1 / \partial \Theta_1 = 0$, $\partial Q_1 / \partial \Theta_1 = 1$ for P_{1mm} and $\partial P_1 / \partial \Theta_1 = 1$, $\partial Q_1 / \partial \Theta_1 = 0$ for Q_{1mm} , other conditions are satisfied, $\partial P_1 / \partial V_1 = 0$ and $\partial Q_1 / \partial V_1 = 0$ respectively. Note that Q_{1mm} does not occur under practical situations, once $P < 0$ meaning that the active power would be flowing from load to generation.

As P_1 and Q_1 are functions $f_{P,Q}(V_1, \Theta_1)$, the total differentials will then be:

$$\begin{aligned} dP_1(V_1, \Theta_1) &= \frac{\partial P_1}{\partial V_1} dV_1 + \frac{\partial P_1}{\partial \Theta_1} d\Theta_1 = 0 \\ dQ_1(V_1, \Theta_1) &= \frac{\partial Q_1}{\partial V_1} dV_1 + \frac{\partial Q_1}{\partial \Theta_1} d\Theta_1 = 0 \end{aligned} \quad (8)$$

Another important fact at these points is that the power factor angle ϕ coincides with the line impedance angle α and $\alpha \pm 180^\circ$ respectively.

If we produce the same family of curves for the case when the line impedance angle $\alpha = 90^\circ$, i.e. when the line resistance is neglected; the corresponding circles will look like Fig. 5.

As resistance is neglected, there are no I^2R losses and all active power sent at one end of the line is received at the other end; this is the reason for which the locus of P_{1max} appears as a parabola whose maximum P_{mm} occurs at infinity.

Note that as P_{1max} increases, the circles touching its locus correspond to ever higher values of V_1 , implying the requirement of large amounts of Q_1 injected at that end in order to raise the voltage to the required level. Of course, in this case the condition $\partial P_1 / \partial V_1 = 0$ will not be satisfied for any point on the locus of P_{1max} .

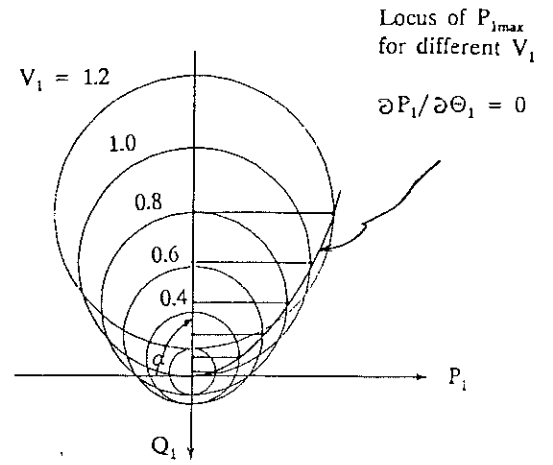


Figure 5 Locus of P_1 , Q_1 for $V_0 = 1.0$ pu and various V_1 showing P_{1max} for each voltage level V_1 . Line resistance $R=0$.

CONDITIONS DEFINING VOLTAGE COLLAPSE

According to Fig. 2, voltage collapse would occur at the point where $P_1 = P_{1max}$. By looking only at that curve, one could easily make the mistake of defining (without precaution) one and only one condition for voltage collapse, namely $\partial P_1 / \partial V_1 = 0$.

This would be the case only for the particular constant power factor for which that curve was drawn. In fact, several authors have used this condition without clearly stating that the expression for P_1 can not contain Θ_1 explicitly, but only as a function of V_1 ; otherwise the derivative of the term containing Θ_1 in $\partial P_1 / \partial V_1 = 0$ will be considered to be constant when in fact Θ_1 varies with V_1 [8]. If we built a family of curves for different power factors, it will look like the one in Fig. 6.

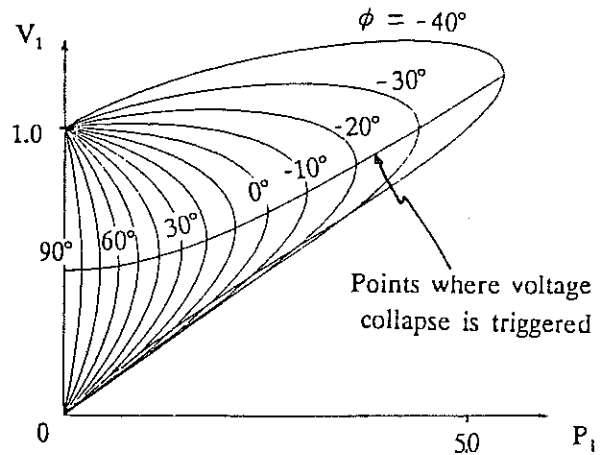


Figure 6 V-P curves for different power factors. Line resistance $R=0$.

If the power factor is allowed to vary, i.e. if P_1 and Q_1 are independent of each other, the operating point will be allowed to move from one curve to the other and it could reach voltage instability

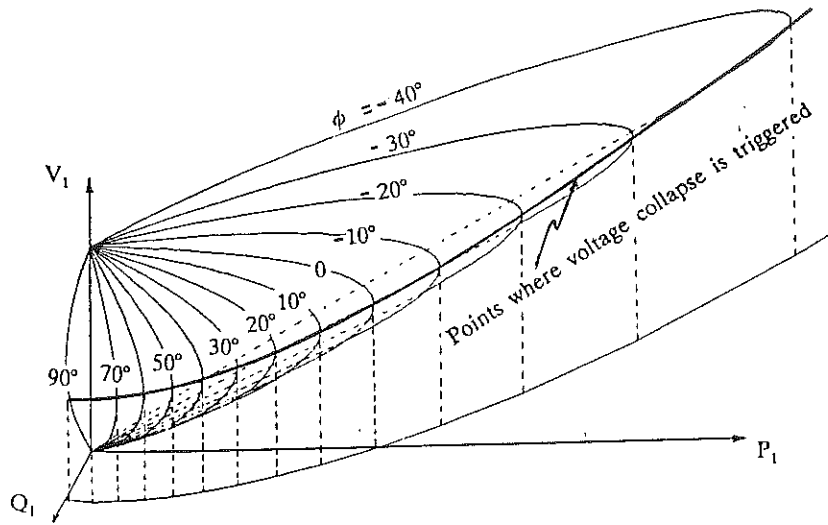


Figure 7 3-D Surface V_1 - P_1 - Q_1 of the feasibility region for V_1 and $P_1 \geq 0$. Line resistance $R=0$.

at any point in the locus of the voltage collapse points shown in Fig. 6. These curves correspond to a line with no resistance, i.e. $\alpha = 90^\circ$.

To present a clearer picture of how the operating point moves when P_1 and Q_1 vary, Fig. 7 shows the 3-D surface V - P - Q of the feasibility region for V_1 and P_1 positive. The curve shows the points where voltage collapse is triggered (critical points) and their projection on the P - Q plane. Note that the projection on the plane V - P of each one of the curves for constant power factor shown in Fig. 7 corresponds to one of the curves shown in Fig. 6.

The stable operating points are those lying on the part of the surface above the critical points [6]. The critical points will be characterised by an infinite vertical plane approaching the surface and touching it at only one point.

In the case of a lossless line ($R=0$), the projection of the critical points on the P - Q plane will be a parabola on the Q_1 axis extending its arms in the $-Q_1$ direction. The maximum value of P_1 will be, as in the cases of the SSSL shown in Fig. 5, at infinity.

Note that there will be no infinite vertical plane parallel to the V_1 - Q_1 plane touching the surface at one point; i.e. there is no finite maximum of the surface on the plane V_1 - Q_1 with respect of P_1 . In other words, the partial derivative $\partial P_1 / \partial V_1$ will not be equal to zero at any point. Care should be taken when working with the projections on the V_1 - P_1 plane (Fig. 6), once it seems that all critical points fulfill the requirement of $\partial P_1 / \partial V_1 = 0$, analogously when working with the projections on the V_1 - Q_1 plane, it may seem that for all the critical points $\partial Q_1 / \partial V_1 = 0$.

One condition when VC occurs at $\partial P_1 / \partial V_1 = 0$ is the case of the transmission line with resistance ($R \neq 0$). Figure 8 shows the same 3-D surface V_1 - P_1 - Q_1 of the feasibility region for the case of the line with losses, i.e. $\alpha < 90^\circ$ and the projection of the critical points, which again is a parabola. In this case the parabolic axis has been shifted counterclockwise by an angle of $90^\circ - \alpha$, and its right arm crosses now the Q_1 axis.

For this condition, there is a vertical plane parallel to the plane V_1 - Q_1

touching the surface in only one point. At this point, $\partial P_1 / \partial V_1 = 0$, but only at this point; not at all the critical points where VC occurs.

This point will again be in the curve corresponding to a power factor angle $\phi = -\alpha$.

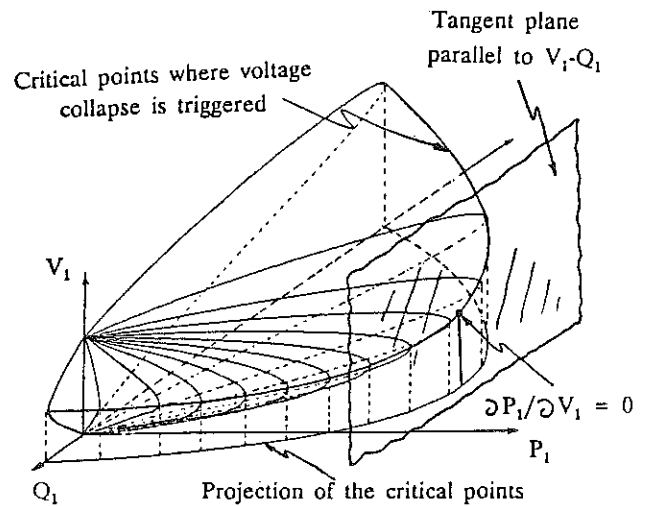


Figure 8 3-D Surface V_1 - P_1 - Q_1 of the feasibility region for V_1 and $P_1 \geq 0$. Line resistance $R \neq 0$.

RELATION BETWEEN VOLTAGE COLLAPSE AND THE STEADY-STATE STABILITY LIMIT

From the 3-D surface shown in Fig. 8, we can obtain the projection on the plane P - Q of the points corresponding to a given voltage magnitude V_1 , i.e. we can obtain a section of the surface by cutting it horizontally at a height V_1 . The resulting section is a circle on the plane P - Q . Thus, the surface of Fig. 8 can be seen as the superposition of a number of eccentric circles with radii proportional to V_1 . Recalling that the power circle diagram shown in Fig. 3 for the receiving end is also a circle on the P - Q plane containing operating

points for a fixed voltage; it is possible to establish the links between VC and SSSL. Do not forget that the VC surface (Fig. 8) was built for the G-T-L system which implies a variable receiving end voltage (Fig. 2A), whereas the circle diagram of Fig. 3 was built for the receiving end voltage constant. For a particular operating point in time (snapshot), it is possible to have the same P , Q and V at the receiving end of both test systems (Fig. 1A and Fig. 2A), but this does not mean that they both will necessarily be operationally stable. It could well be that for a given set P , Q , V , the test system G-T-M is operating stable, whereas the G-T-L test system operating at the same point is captured in a snapshot of its trajectory towards instability.

Figures 9 to 12 put the VC and SSSL in the same context. Figs. 9 and 12 show the point where the critical conditions defining VC and SSSL meet for the case of line resistance $R \neq 0$ and $P > 0$. At this point, in Fig. 9 the vertical plane tangent to the surface (VC) coincides with the horizontal line parallel to the Q_1 axis touching the surface at the maximum value of P_1 this test system can reach P_{1mm} (SSSL). There is one second point where VC and SSSL coincide (Q_{1mm} in Fig. 4), but as mentioned before, it does not represent a practical situation as the active power would flow from load to generation ($P < 0$).

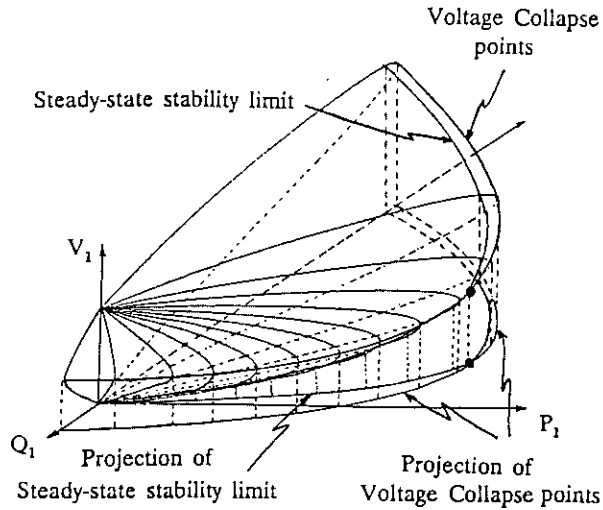


Figure 9 3-D Surface V_1 - P_1 - Q_1 showing the points where VC and SSSL occur. Line resistance $R \neq 0$.

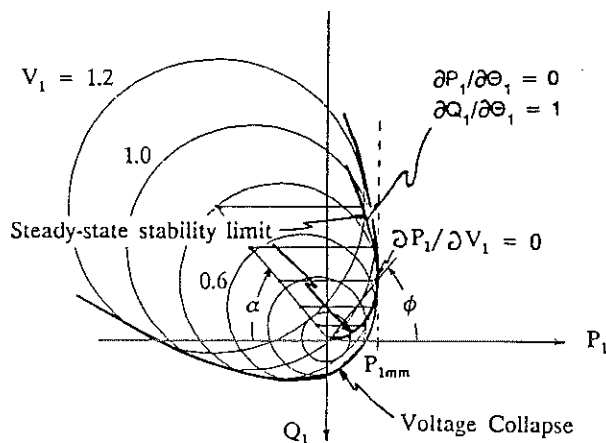


Figure 10 Projection on the P_1 - Q_1 plane of the surface shown in Fig. 9.

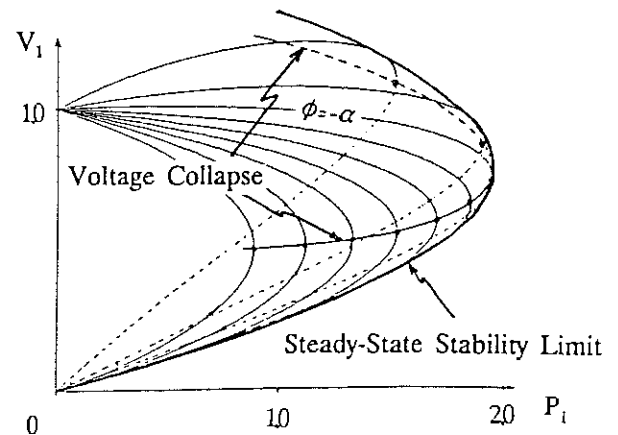


Figure 11 Projection on the V_1 - P_1 plane of the surface shown in Fig. 9.

Compare Fig. 9 with Figs. 4 and 8; and note once again, that this point occurs at a power factor angle $\phi = -\alpha$.

Similarly figure 12 shows the V_1 , Θ_1 voltage solution locus for $P > 0$. In this case the curve of critical points of the SSSL is parallel to the V_1 - P_1 plane, cuts the Θ_1 axis at an angle $\Theta_1 = -\alpha$ and passes through the highest point of the surface, where $P_1 \approx P_{1mm}$. Note that the locus of the critical points for VC passes also through P_{1mm} .

The loci of the critical points for VC and SSSL can also be appreciated in Figs. 10 and 11, which show a projection of the 3-D surface on the P - Q and V - P planes respectively.

Note that in all the cases there is only one point ($P > 0$) P_{1mm} , where both conditions occur at the same time. The locus for the SSSL corresponds to operating conditions for which the voltage displacement angle $\Theta_1 = -\alpha$; on the other hand, as a contrast, VC can occur even for very low values of Θ_1 as seen in figures 4, 10 and 12.

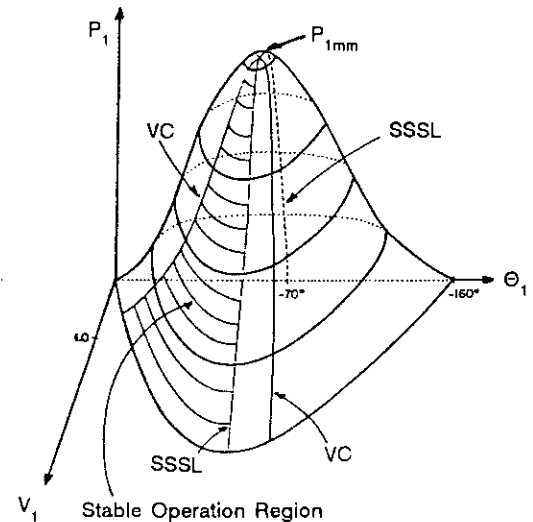


Figure 12 3-D Surface P_1 - Θ_1 - V_1 showing the points where VC and SSSL occur. Line resistance $R \neq 0$.

Figure 12 shows also the area of the surface where operation is stable from the point of view of both angle and voltage magnitude.

Focusing the analysis on the case of active power $P_1 > 0$ which represents practical situations (power flowing from source to load), forces $\partial Q_1 / \partial \theta_1$ to be $\neq 0$ as can be seen from Fig. 10. The observation that in practice this derivative is always different from zero is very important to draw conclusions when analyzing the relation with the power flow Jacobian matrix in the next section, once $\partial Q_1 / \partial \theta_1$ is a submatrix of the Jacobian.

RELATION WITH THE SINGULARITY OF THE POWER FLOW JACOBIAN AND THE SYSTEM STATE MATRIX

Since Venikov [9] recognised that voltage instability was closely related to the singularity of the Jacobian matrix of the load flow equations, several authors have used this relation in one way or another to approach the voltage collapse problem (e.g. [10,11]).

To justify this statement, the determinant of the Jacobian matrix (polar form) is calculated for each one of the operating points defining the 3D surface of Fig. 9. The results are shown graphically in Fig. 13, from where it can be seen that the value of the normalised determinant is a continuous surface crossing the plane P-Q.

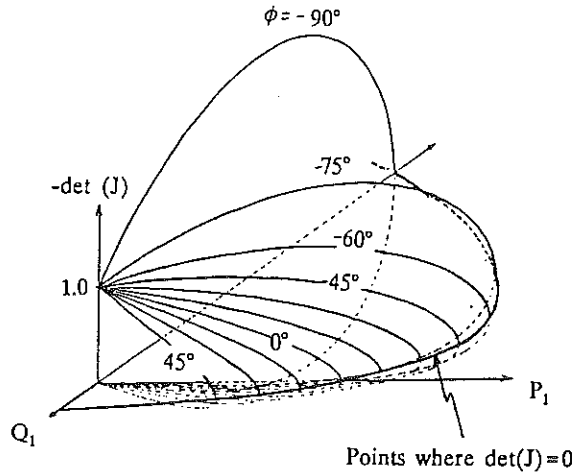


Figure 13 3-D surface of the normalized determinant of the Jacobian matrix.

The intersection of this new surface with the plane P-Q, corresponds to the projection on the plane P-Q of the critical points shown in Fig. 9. This demonstrates that when voltage collapse occurs, the determinant of the Jacobian matrix is actually equal to zero. That is why it is a common practice to imply the proximity to voltage collapse from the non-convergence of the load flow solution, although the non-convergence can also be caused by some other reason (e.g. numerical convergence problems).

As a very illustrative example, Table I presents the values of the four submatrices $\partial P / \partial \theta$, $\partial P / \partial V$, $\partial Q / \partial \theta$ and $\partial Q / \partial V$ of the Jacobian matrix together with its determinant for some of the Voltage Collapse points shown in Fig. 9 (calculated for a line angle $\alpha = 60^\circ$).

Table I

Conditions at Voltage Collapse for different Power Factors

p.f	J	V ₁	P ₁	Q ₁	$\frac{\partial P}{\partial \theta}$	$\frac{\partial P}{\partial V}$	$\frac{\partial Q}{\partial \theta}$	$\frac{\partial Q}{\partial V}$
-90°	0	1.932	0	-3.73	-0.50	-0.97	-1.87	-3.61
-70°	0	1.183	0.479	-1.32	-0.10	-0.19	-1.18	-2.14
-60°	0	1.000	0.500	-0.87	0.0	0.0	-1.00	-1.73
-50°	0	0.872	0.489	-0.58	0.08	0.12	-0.87	-1.42
-30°	0	0.707	0.433	-0.25	0.18	0.26	-0.68	-0.97
-10°	0	0.610	0.363	-0.07	0.26	0.30	-0.55	-0.64
0°	0	0.577	0.333	0.00	0.29	0.29	-0.50	-0.50
10°	0	0.552	0.300	0.05	0.32	0.27	-0.45	-0.38
30°	0	0.518	0.232	0.13	0.37	0.19	-0.37	-0.19
50°	0	0.502	0.162	0.19	0.41	0.07	-0.29	-0.05

Note from Table I that although at all those points $\det |J| = 0$, the submatrices are not zero except for the case of power factor $\phi = -\alpha$, where $\partial P / \partial \theta = \partial P / \partial V = 0$ as seen on Fig. 10. Therefore, the condition defining voltage collapse is $\det |J| = 0$. From the load flow standard form equation:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix} \quad (9)$$

we have that:

$$\det |J| = \left(\frac{\partial P}{\partial \theta} \cdot \frac{\partial Q}{\partial V} \right) - \left(\frac{\partial Q}{\partial \theta} \cdot \frac{\partial P}{\partial V} \right) = 0, \quad (10)$$

as opposed to the conditions defining the SSSL, where $\partial P / \partial \theta = 0$.

From Eqn. 10 it is seen that, for the condition $\partial P / \partial V = 0$ to be valid as a characterisation of VC, one of the diagonal submatrices in Eqn. 9 should be equal to zero. That is why in a previous section it was said that for this condition to be used to identify the points where VC occur, it is necessary that θ_1 does not appear explicitly in the equation of P_1 but as a function of V ; thus forcing the term $\partial P / \partial \theta = 0$. The analogous reasoning is also valid for $\partial Q / \partial V = 0$.

In order to take θ_1 out of the expression for P_1 , the following procedure can be followed starting from the basic active and reactive power equations 1 and 2.

- Eliminate P_1 and Q_1 replacing them by $\tan \phi$.

$$\tan \phi = Q_1 / P_1 \quad (11)$$
- After several steps of algebraic manipulations, it is possible to obtain from Eqn. 11 the expression for $\cos (\theta_1 + \alpha)$.

$$\cos(\Theta_1 + \alpha) = \frac{V_1}{V_0} \cdot \sin \phi \cdot \sin(\phi - \alpha) \pm \cos \phi \sqrt{1 - \left[\frac{V_1}{V_0} \sin(\phi - \alpha) \right]^2} \quad (12)$$

• Substitution of Eqn. 12 back into Eqn. 1 will give us the expression of P_1 which is not a function of Θ_1 .

$$P_1 = -\frac{V_1^2}{Z_T} \left[\sin \phi \cdot \sin(\phi - \alpha) + \cos \phi \sqrt{\frac{V_0^2}{V_1^2} - \sin^2(\phi - \alpha)} - \cos \alpha \right] \quad (13)$$

To check the validity of this expression, the partial derivative of P_1 with respect to V_1 is made equal to zero, which leads us to,

$$V_1 = \frac{V_0}{\sqrt{2} \sqrt{1 \pm \cos(\phi - \alpha)}} = \frac{V_0}{2 \cos\left(\frac{\phi - \alpha}{2}\right)} \quad (14)$$

This expression for V_1 at the point where voltage collapse occurs is exactly the same as the one arrived at by Barbier and Barrett [5] and Borremans et.al. [12].

Applying a similar treatment to Eqn. 2, we get both P_1 and Q_1 as functions of V_1 only. Equation 8 would look under this condition as:

$$\begin{aligned} dP_1(V_1) &= \frac{\partial P_1}{\partial V_1} dV_1 = 0 \\ dQ_1(V_1) &= \frac{\partial Q_1}{\partial V_1} dV_1 = 0 \end{aligned} \quad (15)$$

or in a matrix form similar to Eqn. 9,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} 0 & \frac{\partial P}{\partial V} \\ 0 & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta \Theta \\ \Delta V \end{bmatrix} \quad (16)$$

Note that only in this case the condition for voltage collapse (Eqn. 10) can then be equally substituted by:

$$\frac{\partial P}{\partial V} = 0 \quad \text{or} \quad \frac{\partial Q}{\partial V} = 0, \quad (17)$$

but this is valid only when P and Q are not functions of Θ . Several authors have used this condition (Eqn. 17) to characterize VC without pointing out the restrictions of its validity, perhaps due to the wrong appreciation of the V-P, V-Q curves like the ones shown in figure 6, which are only the projection of a more complicated surface.

Going back to the relation between VC, SSSL and the singularity of the Jacobian, it should be noted that the Jacobian matrix in Eqn. 9 contains terms directly related to the connectivity of the network; that

is the reason for referring to the strength of a system from a VC point of view with the term "Structural Stability" [4].

On the other hand, no direct relation exists between the SSSL and the singularity of the Jacobian. From Fig. 3 it is seen that for angle instability to occur it is a condition that $\partial P/\partial \Theta = 0$ or $\partial Q/\partial \Theta = 0$. At the same time it is evident from the same figure that $\partial Q/\partial \Theta \neq 0$ for values of $P > 0$; what is even more, at the point where $\partial P/\partial \Theta = 0$ the variation $\partial Q/\partial \Theta = 1$ as shown on Table I. Besides that, it is seen in Fig. 4 that except from the point where VC coincides with the SSSL, $\partial P/\partial V \neq 0$. Thus, if the off-diagonal terms of the Jacobian matrix of Eqn. 9 are different from zero and $\partial P/\partial \Theta = 0$, it is obvious that in general the angle instability will not create eigenvalues $\lambda = 0$ in the power flow Jacobian matrix. This is graphically shown in Fig. 10, where the locus for VC (where the Jacobian happens to be singular) is different from the locus for SSSL.

The generation of eigenvalues $\lambda = 0$ due to angle instability does not take place in the load flow Jacobian matrix but in the more general State Matrix of the system; from which the load flow Jacobian is only a submatrix.

A general dynamic model is represented by

$$\dot{X} = A_{\text{SYS}} X \quad (18)$$

where X is the vector of state variables and A_{SYS} is the state matrix of the system.

For a multimachine dynamic system, the state vector can contain variables related to the generator, the automatic voltage regulator, the turbine, boiler, their controls and on the network side, related to the loads and dynamically modelled components.

As the time constant spectrum is very large, some of these state variables can be eliminated from the vector according to the particular needs of the study, i.e. if the study is focused for example on the subtransient or transient period, all the slow varying state variables can be considered constant without any significant loss of accuracy.

For our purposes the following reduced order model is taken from reference [13],

$$d\delta_i/dt = \omega_i - \omega_{ss} \quad i=2, \dots, m \quad (19)$$

$$M_i d\omega_i/dt = T m_i + P_{li} - \sum_{k=1}^n V_i V_k Y_{ik} \cos(\Theta_i - \Theta_k - \alpha_{ik}) \quad i=2, \dots, m \quad (20)$$

$$\begin{aligned} 0 &= V_i^2 \sin(\Theta_i - \delta_i) - X q_i Q_{li} \sin(\Theta_i - \delta_i) - \\ &X q_i P_{li} \cos(\Theta_i - \delta_i) + X q_i \sum_{k=1}^n V_i V_k Y_{ik} \cos(\delta_i - \Theta_k - \alpha_{ik}) \quad i=2, \dots, m \end{aligned} \quad (21)$$

$$0 = P_{li} - \sum_{k=1}^n V_i V_k Y_{ik} \cos(\Theta_i - \Theta_k - \alpha_{ik}) \quad i=m+1, \dots, n \quad (22)$$

$$0 = Q_{li} - \sum_{k=1}^n V_i V_k Y_{ik} \sin(\Theta_i - \Theta_k - \alpha_{ik}) \quad i=m+1, \dots, n \quad (23)$$

$$V_i = V_{ref_i} \quad i=1, \dots, m \quad (24)$$

where m is the number of machines and n is the number of nodes

The corresponding linearized model is then

$$\begin{bmatrix} \Delta\delta \\ M \Delta\omega \\ \dots \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & I & : & 0 & 0 \\ 0 & 0 & : & A & B \\ \dots & \dots & \dots & \dots & \dots \\ E & 0 & : & F & G \\ 0 & 0 & : & C & D \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \\ \dots \\ \Delta\theta_g \\ \Delta Q_L V_L \end{bmatrix} \quad (25)$$

where the Jacobian is

$$J = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (26)$$

Rearrangement of rows and columns of Eqn. 25 gives Eqn. 27.

$$\begin{bmatrix} \Delta\delta \\ 0 \\ \dots \\ M \Delta\omega \\ 0 \end{bmatrix} = \begin{bmatrix} I & 0 & : & 0 & 0 \\ 0 & E & : & F & G \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & : & J \\ 0 & 0 & : & \dots \end{bmatrix} \begin{bmatrix} \Delta\omega \\ \Delta\delta \\ \dots \\ \Delta\theta_g \\ \Delta Q_L V_L \end{bmatrix} \quad (27)$$

It was shown before that at the SSSL matrix J is non-singular; therefore, the State Matrix will be singular if and only if submatrix E; which relates to the change in internal generator angles, is singular. Submatrix E is not a connectivity type matrix, and is more related to the operating point of the system. For this reason, the steady-state angle stability is sometimes referred to as "Operational Stability", as opposed to the term "Structural Stability" mentioned before, often associated with the voltage stability of a system.

One conclusion from Eqn. 27 is that if a steady-state instability occurs, it has to be either due to the singularity of the Jacobian J (VC) or due to the singularity of the submatrix E (SSSL).

Preliminary results show that the cause of any steady-state instability is either reach of VC or reach of SSSL. However further studies are needed using the full (non-reduced) order model.

CONCLUSIONS

Both Voltage Collapse and Steady State Stability Limit are put in the same framework, from where the main similarities and differences have been clearly identified.

The approach taken in this paper to study VC and SSSL clarify some of the doubtful points related to VC pointed out by several authors. It was shown that VC and SSSL are in fact two different concepts; the locus of operating points defining VC and SSSL are different and under practical conditions coincide at only one point, where the power factor angle equals the negative of the line impedance angle.

The relation of VC and SSSL with the singularity of the power flow Jacobian and the System State Matrix was presented. VC will cause

singularity of the Jacobian and the System State Matrix. SSSL will cause singularity of the System State Matrix only, except for the case when both, SSSL and VC occur at the same time.

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Voltage Collapse Mechanisms : A Case Study

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Abstract. There is a need for a better understanding of voltage collapse dynamics and for a unified view of existing ways of analysis. This paper offers some views on this problem, on the basis of a detailed case study carried out on a simple system. The analysis relies on the decomposition into a slow discrete control and a fast continuous-time sub-system, both contributing to (in)stability. The test system has been made as simple as possible while maintaining essential aspects of voltage dynamics : load behaviour with voltage, on-load tap changer action and protecting devices acting on machine excitation levels. It exhibits a wide variety of behaviours within the time frame of 2-3 minutes following a disturbance.

Keywords. Voltage dynamics, nonlinear system stability, voltage stability.

1 Introduction

During the last decade, voltage stability has become a major concern for the security of many power systems [1].

Although the factors taking part are known since the early manifestations of this problem [2] and their effects are understood qualitatively and can be reproduced in digital simulations, it is however widely agreed that there is a *lack of a unified, coherent, system theory-based view* of the dynamics of voltage collapse.

Voltage stability is primarily related to the existence of a "loadability limit", i.e. an upper limit of the power which can be consumed, in (a region of) a power system. Among others, this limit is dictated by electrical distances and by reactive power production capabilities. Hence, major causes of voltage instability are schematically : (i) load increases up to the loadability limit, and (ii) generation or transmission incidents bringing this limit down below the pre-disturbance load level.

Preventive voltage security assessment aims at keeping the system away from the above limit, even under contingencies. Research efforts have been paid to improve voltage stability indicators [1, 3], in order for instance to take into account reactive power limitations [4], to identify weak system areas [5], etc. *To this purpose, tools based on the standard load flow equations are generally deemed adequate.*

This is no longer true as long as dynamics is concerned. Indeed, while power flow equations can adequately yield the loadability limit (or only give some indication of its prox-

imity), they do not explain what happens once the system meets this limit. There are however several motivations for a deeper understanding of what happens then; among them, the coming development of an *adequate emergency control* [6] requires to address fundamental questions such as "how does the system behave beyond the critical point, how is it then controllable, how fast the controls need to be, etc ?".

In the author's opinion, some confusion has been introduced by early works attempting to interpret the load flow equations as the equilibrium equations of a dynamic model capable to render the unstable system behaviour. The heritage of this is a dynamic analysis in terms of constant power loads, a PV/PQ bus representation of generators, or the interpretation of the load flow Jacobian as the state matrix of the system linearized dynamics, etc. As was quoted in Ref. [7], the load flow equations merely yield the operating point *of the transmission part* of the system.

Beside the development of preventive, static indicators (not considered here; see e.g. [1, 8]), several ways of investigations have been opened in the dynamic analysis. This includes non exhaustively :

- steady-state (i.e. local) stability analysis of the transformer On-Load Tap changer (OLTC) dynamics [9]
- analysis of Jacobian singularity [10, 11, 7]
- large perturbation analysis of the tap changer dynamics [12]
- nonlinear discrete system analysis of the latter [13]
- bifurcation theory [14, 15]
- closer investigations on load dynamics [16].

All these approaches are complementary views on the problem. *There is however a need to think about the power system in terms of a whole, unique system.* The recent contributions of Refs. [17, 18] go along this direction.

This paper reports on the author's attempts to clarify his own view on the above dynamic issue. It relies on a rather general system-oriented framework, applied to the detailed analysis of a simple system. The adopted viewpoint is voltage instability following a major incident; this slightly departs from the traditional analysis of a slow increase in load, which it encompasses however.

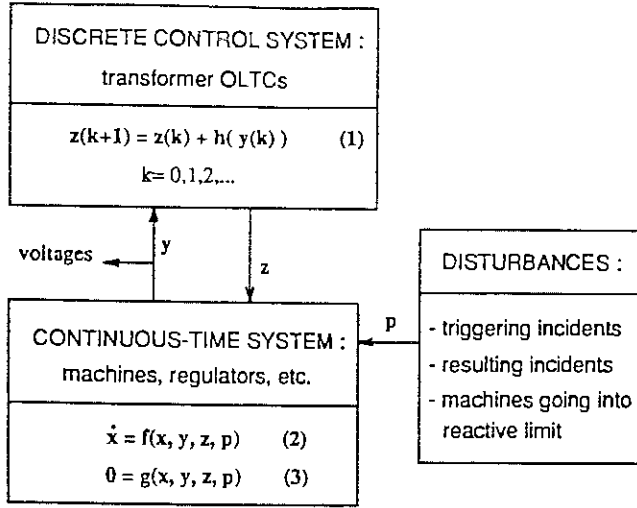


Figure 1: Basic framework of the analysis.

2 Framework of the analysis

2.1 The system picture

We propose to study voltage instability, from a dynamic viewpoint, within the framework of a mixed discrete/continuous-time system, as represented in Figure 1.

The continuous-time sub-system is made up of machines, Automatic Voltage Regulators (AVRs), governors, loads (represented statically or dynamically), static or synchronous compensators, etc., all connected through the network. This is the system commonly modeled in "short-term" dynamic simulations. It is described by mixed differential/ algebraic equations of the type (2) and (3) (Fig. 1) involving state (x) and algebraic (y) variables. The y vector usually includes the network bus voltages.

The discrete control system is made up of all the transformer OLTCs, whose important role in voltage stability is well-known. Each OLTC is supposed to control its transformer ratio r according to (see Figure 2) :

$$r(k+1) = \begin{cases} r(k) + \Delta r & \text{if } V_2 > V_2^0 + \epsilon \\ r(k) & \text{if } |V_2 - V_2^0| < \epsilon \\ r(k) - \Delta r & \text{if } V_2 < V_2^0 - \epsilon \end{cases} \quad (4)$$

$$\text{with } r_{\min} \leq r(k+1) \leq r_{\max} \quad (5)$$

where V_2 is the controlled secondary voltage, V_2^0 the corresponding set-point, ϵ a dead band (or sensitivity threshold or allowable deviation), Δr the ratio step size. The changes in ratio occur at time instants $k \Delta t$ where Δt is a variable time interval. All relations of the type (4) are grouped into Eq. (1) of Fig. 1.

Remarks. 1. Note that for the sake of simplicity, a single time counter k has been adopted, while the various OLTCs are not synchronized. This formal simplification does not affect the

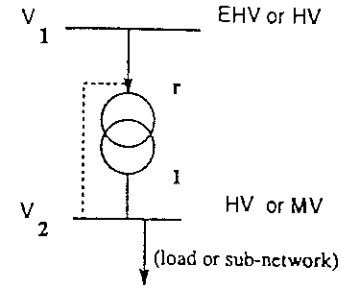


Figure 2: Transformer OLTC notation.

analysis.

2. Note that we adopt the viewpoint proposed in Ref. [13] and describe the OLTCs action through a difference rather than a differential equation. The basic motivation is to keep intact the true nature of this control and to avoid introducing artificial state variables which need to be distinguished from those of the continuous-time system in the subsequent analysis phase.

Both sub-systems interact : (i) through the transformer ratios (z), which are involved in the network and the load modellings. z is a fixed parameter with respect to the time-varying x and y ; (ii) through the y vector of voltages, whose some components "drive" the discrete control system.

Disturbances add to the picture. These include : (i) triggering incidents affecting the transmission and/or generation system; (ii) resulting incidents (such as generator trippings caused by too low voltages), and (iii) the lowering of machine excitation levels - and reactive support - after temporary overloads. The effect of these disturbances is to suddenly change the value of parameters p involved in the f and g functions of Eqs. (2,3).

2.2 Time decoupling and stability

A well-known property of this composite system is the time decoupling which results from the initial delay and the relative slowness of the discrete control system with respect to its continuous-time counterpart [18].

This time decoupling suggest that two types of instability can take place (and coexist) :

- (i) following some change in z or in p , the continuous-time system loses its stability;
- (ii) after each change in z and p , the continuous-time system is able to reach a new equilibrium but the discrete control system is unstable and drives the whole system in unacceptable regions of operation.

Assuming a good time decoupling holds, the following analyses can be performed.

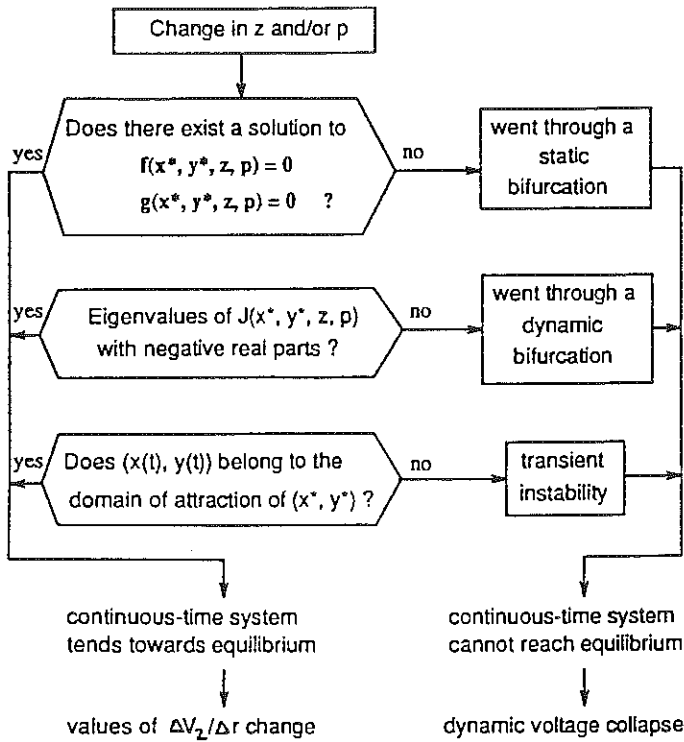


Figure 3: Stability of the continuous-time system.

2.3 Stability of the continuous-time system

Let us assume that the continuous-time system is at (or close to) one of its stable equilibrium points. At some time t , a change in z by the discrete control system or a change in p by the disturbances makes this system enter a new "configuration" (in a wide sense). Figure 3 sketches various scenarios which can result therefrom :

1. the new configuration has no equilibrium point any longer : the variations in p or z made the system go through a static bifurcation, i.e. a structural change in the number of equilibrium points;
2. the controlling stable equilibrium point has turned into an unstable one, hence unable to attract the system trajectory any further : a dynamic bifurcation has occurred, i.e. a structural change in the nature of the equilibrium point(s);
3. a new stable equilibrium exists but the system is unable to reach it : in the state space, the system representative point at time t (when entering the new configuration) does not belong to the domain of attraction of the above stable equilibrium;
4. none of the above catastrophes occurs : the system tends towards a new equilibrium, until new changes in z or p occur (if any).

The first three scenarios all lead to a dynamic instability (usually characterized by monotonic decreases in the voltages) but

a deeper analysis allows to distinguish between them.

Scenario 1 is revealed by the impossibility to solve the equilibrium equations (see Fig.3) with the new z and p .

Scenario 2 can be detected through an eigenvalue analysis of the linearized system :

$$\dot{\Delta x} = J \Delta x \quad (6)$$

$$\text{with } J = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \left(\frac{\partial g}{\partial y} \right)^{-1} \frac{\partial g}{\partial x} \quad (7)$$

obtained by eliminating Δy from :

$$\begin{bmatrix} \dot{\Delta x} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = J_u \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (8)$$

where $\frac{\partial g}{\partial y}$ is assumed to be regular.

The linearization is made around the new equilibrium point (x^*, y^*) at which both the reduced (J) and the unreduced (J_u) Jacobians have to be evaluated.

A dynamic bifurcation leading to aperiodic instability corresponds to a real eigenvalue *crossing* the imaginary axis. At the bifurcation point, J has a zero eigenvalue and hence is singular. Moreover, since :

$$\det J_u = \det \frac{\partial g}{\partial y} \cdot \det J \quad (9)$$

J_u is also singular at this point.

The remaining unstable cases are the concern of Scenario 3. The latter is also the basic mechanism of transient angle instability, where the lack of attraction towards a stable equilibrium point results from the importance and the duration of the disturbance (usually a short-circuit). Here, the changes in z and p are not likely to have such a strong destabilizing effect on the continuous-time system but Scenario 3 remains possible from a theoretical viewpoint.

2.4 Stability of the discrete control system

Let us assume now that the continuous system lets itself be driven from equilibrium to equilibrium by the discrete one and let us focus on the latter.

Equations (4,5) do not follow the standard modelling of a discrete-time system. Nevertheless, the usual notions of equilibrium and stability fit to the present model as well [13].

The discrete control system is at the equilibrium if :

- (i) $|V_2 - V_2^0| < \epsilon$ for all OLTCs within their bounds (5), and
- (ii) for those blocked at their lower (resp. upper) bound, decreasing (resp. increasing) this bound would result in reducing the $|V_2 - V_2^0|$ deviation.

Such an equilibrium point is asymptotically stable if, following a small perturbation, the system comes back (close) to this point in a finite number of steps. By extension, if this holds, the discrete control system is said to be stable.

Generally speaking, after a major disturbance, the above system can loose stability either by disappearance of any equilibrium point or by lack of attraction towards an equilibrium [12, 19].

The stability can also be judged in terms of the V_2 controlled voltages. Because of the logic in (4) and since

$$V_2(k+1) - V_2(k) \simeq \frac{\Delta V_2}{\Delta r}(k) \Delta r \quad (10)$$

a stable discrete control system necessarily satisfies :

$$\frac{\Delta V_2}{\Delta r}(k) < 0 \quad (11)$$

at all steps following some step k . Note that $\Delta V_2 / \Delta r$ is the sensitivity of V_2 wrt r in the steady-state, i.e. assuming the continuous-time system has returned to equilibrium after being disturbed by Δr .

Clearly, a stable (resp. unstable) control system can momentarily exhibit positive (resp. negative) values of the above sensitivity but a change in sign must take place later on. This change in sign is dictated by a change in the equilibrium point of the continuous-time sub-system. As will be discussed later on, it is closely related to the reaching of a loadability limit, i.e. to the traditional view of voltage stability, which is thus encompassed in the present approach.

2.5 Voltage instability time frames

Different time frames have been observed in experienced voltage incidents [20]. Within the context of the above simplified modelling :

- "short-term" voltage instability deals with cases where following a major disturbance, the system loses its stability well before the discrete control system has time to act; hence the continuous system is responsible for the instability. This can result for instance from stringent load dynamics, capable to recover the pre-disturbance demand level very fast (high proportion of induction motors, etc.). Incidentally, within this time frame, angle and voltage dynamics can be hardly decoupled;
- "mid-term" voltage instability corresponds to the many cases where instability "takes some time to develop". After an initiating disturbance :
 - the system installs itself at an intermediate equilibrium in which the discrete control system has not reacted yet while the transients associated with the continuous system have (almost) already died out [6];

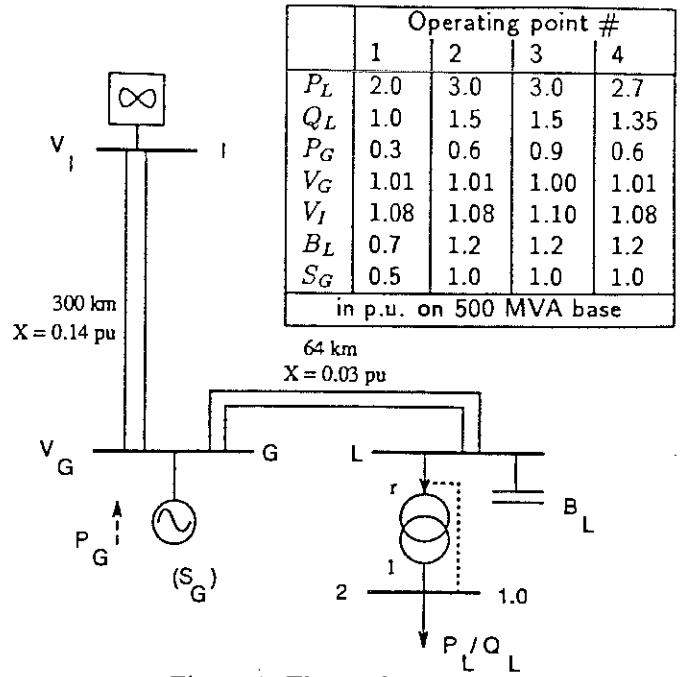


Figure 4: The study system.

- the subsequent system evolution is characterized by the response of the continuous system to the repeated control actions $z(k+1) - z(k)$.
- "long-term" instability deals notably with controlled (e.g. thermostatic) loads which tend to restore their power level after a sustained voltage depression.

The time frame considered in this paper is basically the second one but the approach could deal with the first as well, if adequate load modelling is introduced. The third time frame could be also added easily.

3 The study system

3.1 Overview

The one-line diagram of the study system is given in Figure 4. The power required at bus L comes to the great extent from the infinite bus I through a relatively long double-circuit 400 kV line. Some support is however provided by the generator of bus G, which is much closer to the load. The load is fed through Bus 2, whose voltage is controlled by the transformer automatic OLTC. Figure 4 also gives the load flow data of various operating points used in the sequel, along with the amount of shunt compensation B_L and the generator rated apparent power.

The system has been devised as simple as possible while retaining all realistic ingredients needed to reproduce voltage phenomena : voltage dependence (and - where needed - dynamics) of the load, effect of the OLTC discrete control and effect of forcing / reducing the generator excitation level.

The machine at bus G has been modelled dynamically, as explained in the sequel. The motivation for including this generator in addition to the infinite bus was, first, to simulate the impact of (local) reactive power restrictions and, second, to allow angle dynamics to play its role along with voltages, which is not possible with the classical single-generator examples.

3.2 Load modelling

As is well-known, load modelling is probably the main issue in voltage stability analysis. Our purpose is not to discuss the validity of modellings but rather to see the effect on the system behaviour of various load models of the type commonly used in the simulation of large real-life systems.

Studies such as those reported in [22, 23] have shown that the typical response of a load to a *step change* in voltage consists of three main phases :

1. an initial transient phase during which the induced electro-magnetic and electro-mechanical disturbances settle down. This period does not usually last for more than one second;
2. a new steady-state phase, characterized by the static voltage sensitivity of the load;
3. a recovery phase under the effect of the OLTCs.

As regards the second phase, an elementary, although widely used modelling is of the type :

$$P = P_0 \left(\frac{V}{V_0} \right)^\alpha \quad Q = Q_0 \left(\frac{V}{V_0} \right)^\beta \quad (12)$$

where (P_0, Q_0, V_0) refer to the corresponding pre-disturbance and (P, Q, V) to the current operating point. The sensitivity wrt frequency is not considered here.

Adopting (12) as the load model merely consists in neglecting the first, transient phase. In order to take the latter into account, some dynamic modelling is required, whose identification is even more delicate than obtaining reliable α and β parameters ! One of the proposed solutions is to use a composite equivalent load model made up of a static (impedance) part in parallel with a dynamic induction motor and possibly other components [16, 22, 23]. In this study, a "black-box" modelling has been used when it was required to appraise the effect of the above load transient. It consists in treating the load as an *equivalent dynamic* admittance $g_{eq} - jb_{eq}$ which varies with time in order to fit asymptotically the static behaviour described by Eqs. (12). This is further discussed in Section 4.6.

Finally, the OLTC is modelled as follows. When a disturbance causes the secondary voltage V_2 to deviate by more than a tolerance ϵ from its set-point value V_2^0 , the OLTC reacts

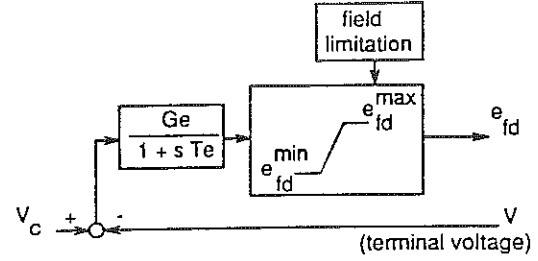


Figure 5: AVR modelling.

but after an initial dead-time τ_1 . It then changes the ratio by $\pm \Delta r$ each τ_2 seconds until V_2 approaches V_2^0 by less than ϵ (which occurs in stable cases only). The simulations have been performed with : $V_2^0 = 1$, $\epsilon = 0.01$, $\Delta r = 0.015$, $r_{min} = 0.80$, $r_{max} = 1.1$ pu and $\tau_1 = 20$, $\tau_2 = 10$ s. The transformer leakage reactance is neglected for the sake of simplicity.

3.3 Generator modelling

The machine at bus G is represented by a classical third-order model describing the rotor motion and the field flux decay [24]. The relevant parameters are (in the machine MVA base) : $X_d = X_q = 2.1$, $X'_d = 0.4$, $D = 3$ pu and $H = 3.5$, $T'_{d0} = 8.0$ s. The mechanical power is considered constant.

It is equipped with an Automatic Voltage Regulator (AVR) represented by a simple first-order lag in cascade with a limiter, as shown in Figure 5. Moreover, a rotor protection is assumed to monitor the excitation voltage e_{fd} . Once e_{fd} has exceeded a threshold value e_{fd}^{per} for more than τ seconds, the protection lowers the ceiling voltage e_{fd}^{max} (and hence e_{fd}) to the e_{fd}^{per} threshold value. The latter corresponds to the maximum permanent admissible excitation level and is obtained from the machine capability curves. The following choices have been made : $G_e = 50$, $e_{fd}^{min} = 0.$, $e_{fd}^{max} = 5.$, $e_{fd}^{per} = 2.69$ pu and $\tau = 40$ s. T_e is equal to 0.1 s or simply set to zero when a fast AVR is assumed.

3.4 The overall modelling

The system modelling is classical and is described hereafter for the sake of completeness (see however the ending remark).

The state vector is :

$$x = [\delta \quad \omega \quad e'_q \quad e_{fd}]^T \quad (13)$$

The first three components are the rotor angle (with respect to the infinite bus), the rotor speed and the e.m.f. behind transient reactance, respectively. e_{fd} is a state variable as long as $T_e \neq 0$ and provided it is not under limit; otherwise it is eliminated. The algebraic vector is :

$$y = [v_{xG} \quad v_{xL} \quad v_{yG} \quad v_{yL}]^T \quad (14)$$

i.e. the projections of the G and L bus voltage phasors on two rectangular axes rotating at the nominal speed (the constant infinite bus voltage is treated as a parameter and the OLTC is absorbed in the load modelling).

The differential equations (2) of Fig. 1 take on the form :

$$\frac{d}{dt}\delta = \omega \quad (15)$$

$$\frac{d}{dt}\omega = -\frac{D}{2H}\omega + \frac{\omega_B}{2H}[P_m - P_e(\delta, e'_q, v_{xG}, v_{yG})] \quad (16)$$

$$\frac{d}{dt}e'_q = \frac{1}{T'_{d0}}[e_{fd} - e_q(\delta, e'_q, v_{xG}, v_{yG})] \quad (17)$$

$$\frac{d}{dt}e_{fd} = -\frac{1}{T_e}e_{fd} + \frac{G_e}{T_e}(V_c - \sqrt{v_{xG}^2 + v_{yG}^2}) \quad (18)$$

There are four algebraic equations of the type (3), originating from the basic Y bus admittance matrix relationship :

$$\begin{bmatrix} i_{xG}(\delta, e'_q, v_{xG}, v_{yG}) \\ -i_{xL}(v_{xL}, v_{yL}, r) \\ i_{yG}(\delta, e'_q, v_{xG}, v_{yG}) \\ -i_{yL}(v_{xL}, v_{yL}, r) \end{bmatrix} = \begin{bmatrix} 0 & -B \\ B & 0 \end{bmatrix} \begin{bmatrix} v_{xG} \\ v_{xL} \\ v_{yG} \\ v_{yL} \end{bmatrix} \quad (19)$$

where the LHS vector includes the projections of the currents injected at buses G and L and where $Y = jB$ (2×2 matrix) for this lossless system.

The system dynamics has been integrated using a variable step size Runge-Kutta algorithm. In the course of computing $\dot{x}$ for a given x , the Newton method has been used to solve Eq. (19) with respect to y .

Remark. Note that the above formulation (more particularly the algebraic equations) does deliberately *not* involve powers as is done in many studies aimed at bridging the gap between the load flow and the dynamic analyses. Rather, it relies on the fundamental voltage-current relationships (19) used in most dynamic simulation programs.

4 Simulation results

4.1 Case 1

Initially the system operating point is the first one on Figure 4. The load behaves as a *constant impedance* ($\alpha = \beta = 2$). The machine AVR is considered infinitely fast ($T_e = 0$) with respect to the studied phenomena.

The studied disturbance is the tripping, at time $t = 1$ s, of one circuit between G and I which makes the line reactance increase from 0.14 to 0.28 pu.

The system behaviour within a 3-minute time interval following the incident is described by Figures 6 to 12, which show respectively the voltage at bus L (V_L), the secondary voltage (V_2), the transformer ratio (r), the machine field voltage

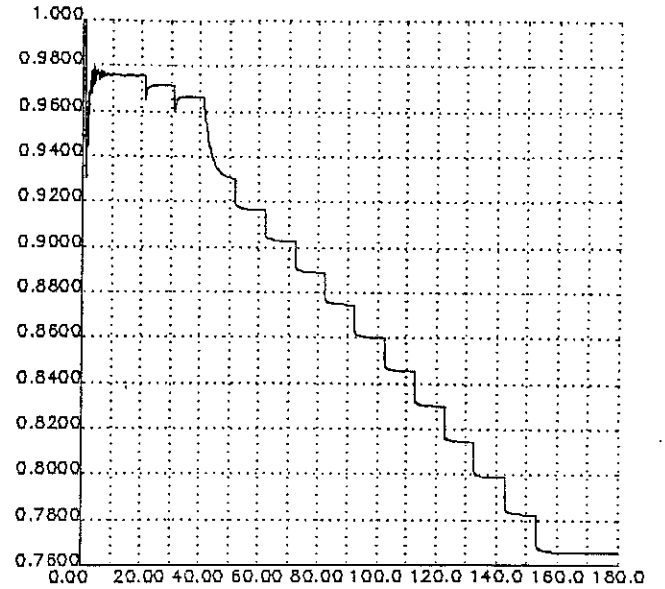


Figure 6: Case 1. V_L voltage.

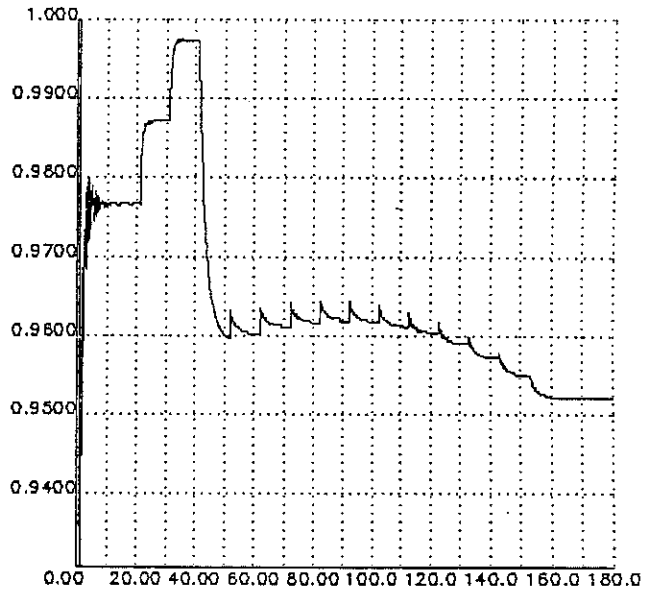


Figure 7: Case 1. V_2 (secondary) voltage.

(e_{fd}), its reactive power production (Q_G), the load active power (P_L) and the machine rotor angle (δ).

Different, successive phases can be distinguished in the system dynamics :

- a phase of electromechanical transients : during the latter, the field voltage goes to ceiling for about 0.4 s, which denotes the severity of the disturbance;
- an intermediate equilibrium lasting from about $t = 11$ to $t = 21$ s : during the latter, the voltage profile is supported by the machine reactive power production. The latter has increased dramatically, in order to compensate for the increase in XI^2 network losses and the decrease in $B_L V_L^2$ shunt reactive production, in spite of a 5% reduction in the

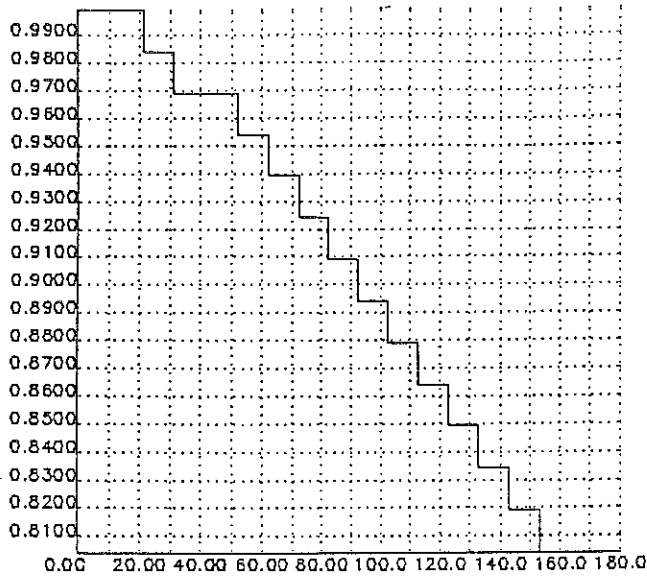


Figure 8: Case 1. Transformer ratio r .

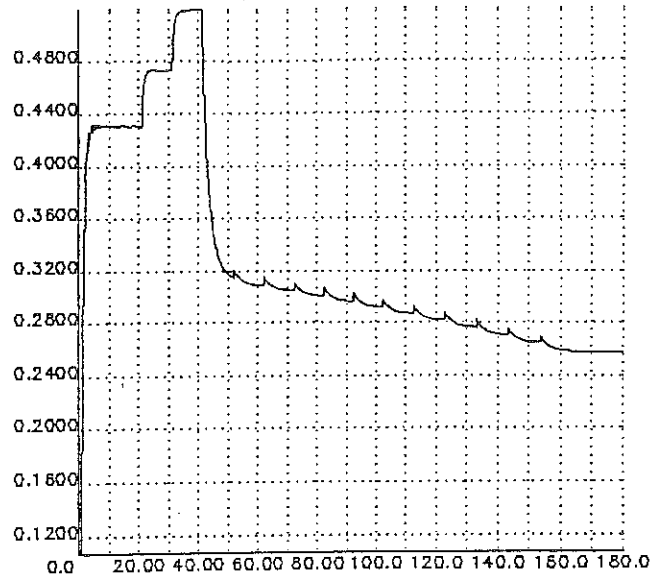


Figure 10: Case 1. Reactive power generation Q_G .

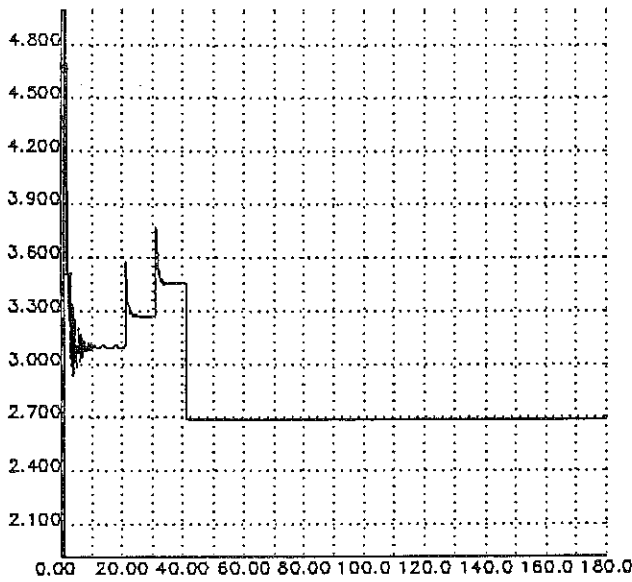


Figure 9: Case 1. Field voltage e_{fd} .

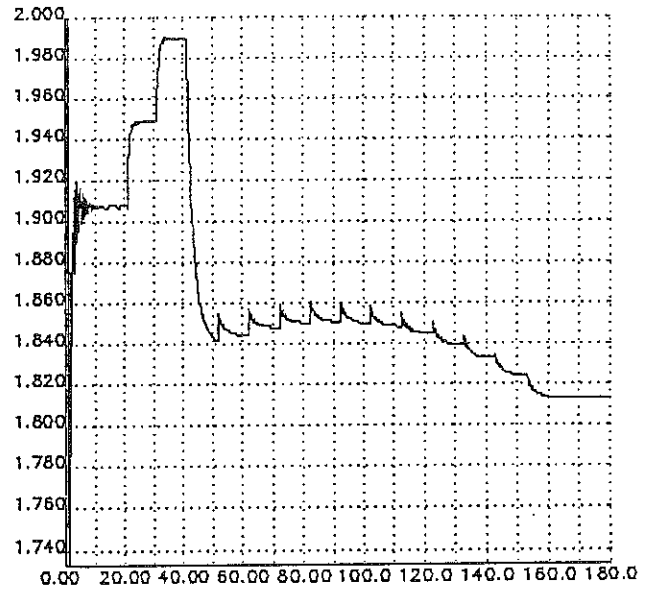


Figure 11: Case 1. Load active power P_L .

active and reactive load. This support is obtained at the expense of the machine overexcitation ($e_{fd} = 3.1 > 2.69$);

- at time $t \simeq 21$ s, the OLTC starts acting. Two steps are sufficient to bring the secondary voltage V_2 (20 s later) within the $[0.99 \ 1.01]$ pu interval. This goes with an increase in the load demand and hence in the line currents, as well as with a decrease in the V_L voltage and hence in the capacitor support. Both contribute to further overload the machine ($e_{fd} = 3.45 \gg 2.69$);
- the reduction in the machine excitation level takes place at $t \simeq 41$ s, after a 40 s overload. From there on, the machine operates with constant excitation ($e_{fd} = 2.69$) and the voltage profile sags dangerously;
- the corresponding severe drop in V_2 causes the OLTC to

get back to work. Its subsequent action on the V_2 voltage and P_L load consists however of two phases : one in which the latter quantities are indeed restored (but rather marginally) and one in which the OLTC logic is reversed. A maximum load power of about 1.85 pu is obtained at time $t \simeq 90$ s;

- eventually, the sustained degradation process stops when the transformer ratio hits its limit. Note that the system settles down at a stable equilibrium. In fact it is perfectly stable with respect to all the components acting in the above described time frame, but not with respect to the long-term behaviour of the load : namely, any further attempt of the consumer to increase the obtained power P_L through an increase in the load admittance will eventually result in a lower load and a further, useless drop in the V_L

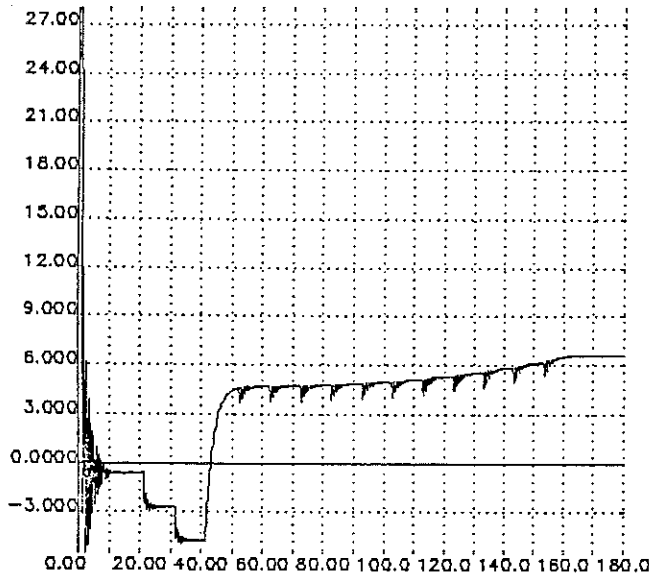


Figure 12: Case 1. Rotor angle δ .

voltage [2];

- clearly, the final abnormal voltage can trigger other incidents, not considered here; among them, the tripping of generators (e.g. owing to low auxiliaries voltage) is a major risk of cascading events.

The above scenario is one of the basic voltage instability mechanisms; as will be shown in the sequel, other mechanisms can result from other load behaviours. Also, it is interesting to quote that some experienced voltage collapses precisely resulted in a dramatically depressed but stabilized voltage profile, see e.g. the one described in Ref. [25].

Let us now analyze the above simulation from the traditional viewpoint of the so-called *PV* curves. Figure 13 shows three *network PV* characteristics and two *load PV* characteristics.

As regards the network characteristics :

- Curve 1 gives V_L as a function of P_L in the healthy configuration;
- Curve 2 relates to the configuration after the line has tripped but while the machine voltage is still controlled;
- Curve 3 relates to that after field voltage limitation.

For a given load behaviour (constant impedance in the present case) there is a family of load characteristics, since :

$$P_L = P_0 \left(\frac{V_2}{V_2^0} \right)^\alpha = P_0 \left(\frac{V_L}{r V_2^0} \right)^\alpha \quad (20)$$

$$Q_L = Q_0 \left(\frac{V_2}{V_2^0} \right)^\beta = Q_0 \left(\frac{V_L}{r V_2^0} \right)^\beta \quad (21)$$

where r takes on several values. The two load curves of Figure 13 correspond to respectively the initial and the final

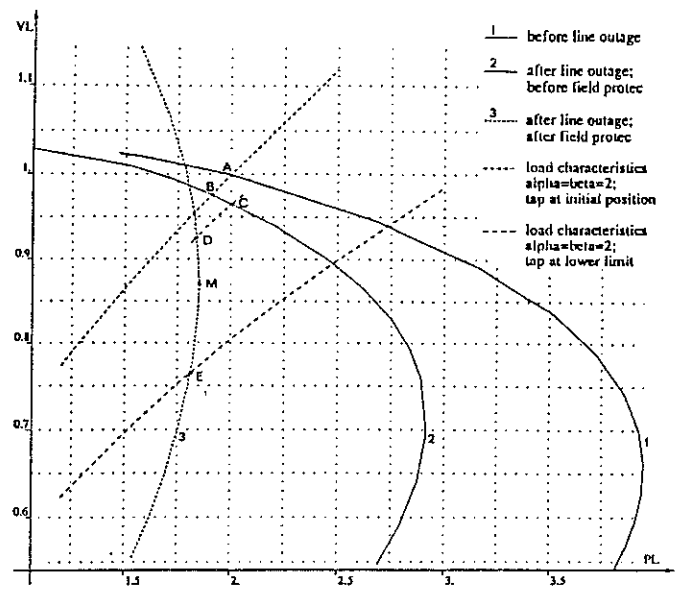


Figure 13: Case 1. Network and load *PV* curves.

(lower limit) value of r . The former passes through ($P_L = 2$, $V_L = 0.995$), the initial operating point.

Remarks. To avoid some confusions quoted in the literature, the following remarks are noteworthy.

1. The network curves are *equilibrium characteristics*, i.e. each point corresponds to one equilibrium point of the continuous system, for a given demand level, in the configuration of concern. From a practical viewpoint, they have been obtained by imposing step changes in the P_0 and Q_0 parameters of the load model (12) and by noting the value reached by V_L after transients have died out. Curves 1 and 2 have been obtained with e_{fd} unlimited and curve 3 with $e_{fd} = e_{fd}^{per}$ ($= 2.69$). Incidentally, this procedure allows the lower parts of the *PV* curves to be obtained as easily as the upper ones, which would not be possible with a standard load flow.
2. These curves do *not* coincide with those obtained from a load flow treating bus G as a *PV*-bus. This is discussed in Section 4.2.
3. The V_L voltage depends on both P_L and Q_L . When plotting V_L as a function of P_L (resp. Q_L) it is required to specify how Q_L (resp. P_L) varies meanwhile. Clearly, the network characteristics is not totally independent of the load characteristics. For *PV* curves, a constant power factor is a common assumption. The latter is, *generally speaking, incorrect within the context of Figure 13*. Indeed, in this figure, each point of a *PV* curve obeys Eqs. (12) and hence the power factor is constant only if $\alpha = \beta$, which should be considered as a particular case. Only the increase in (P_0, Q_0) obeys a constant power factor.
4. A network *PV* curve being an equilibrium characteristics, the point representative of the system trajectory will depart from this curve as long as transients have not died out. This holds for the load characteristics as long as the load transients are represented but if a purely static model is used, the system representative point must stay on the load char-

acteristics at any time and jump instantaneously from one characteristic to the next when r varies.

Coming back to the above simulation results, Figures 6 to 12 clearly reveal a succession of equilibrium points. The latter can be easily identified on Figure 13: *A* corresponds to the prefault operating conditions, *B* to the equilibrium after the electromechanical transients, *C* to that after the OLTC has momentarily restored the load. The subsequent equilibrium points are situated along Curve 3, in between Points *D* and *E*.

Within the framework of Section 2, the above case corresponds to an *unstable discrete control system* imposing to the continuous-time system a sequence of equilibrium points which it succeeds to follow. The instability of the discrete control system results from the absence of an equilibrium point for the latter, in the final configuration. Indeed such an equilibrium would require $V_2 = V_2^0 = 1$ pu and hence $P_L = P_0(V_2/V_2^0)^2 = 2$ pu while the vertical $P_L = 2$ does not cross the network characteristics 3 any longer. The change in sign of $\Delta V_2/\Delta r$ (see § 2.4) occurs at time $t = 90$ s; it also corresponds to a maximum value of the P_L load equal to 1.85, a value which can be easily found back on Figure 13.

Precisely, $P_L = 1.85$ is the contingency-constrained *loadability limit*, i.e. the ultimate load which can be tolerated in order the system to be stable after the line tripping. This value would obviously leave no security margin.

4.2 Further discussion of Case 1

The above simulation clearly shows that *the continuous-time system operates in a quite stable way on the lower part of the PV curve*. Moreover, the crossing of the maximum load point *M* does not cause any degeneracy of the system Jacobian matrices J and J_u .

In fact Point *M* would cause a system singularity if a *constant power load was assumed*; indeed, in such a case, as the load power would approach that of Point *M*, two equilibrium points would converge to each other, coalesce and then disappear. This would cause the corresponding Jacobian to be singular at Point *M* but this Jacobian would be a crude approximation of the true one (J), obtained by replacing the correct load behaviour by a constant power approximation in the LHS vector of Eq.(19).

As quoted above, the *PV* curves of Figure 13 are obtained as a set of equilibrium points rather than the output of a standard load flow. In order to illustrate the discrepancy which can exist between the two approaches, Figure 14 compares the *PV* curves 2 and 3 of Figure 13 with those obtained by means of repeated load flow calculations. Point *D* of the latter corresponds to the load flow divergence. Both curves correspond to a constant power factor (which is correct, according to a previous remark, since $\alpha = \beta$) and to coherent

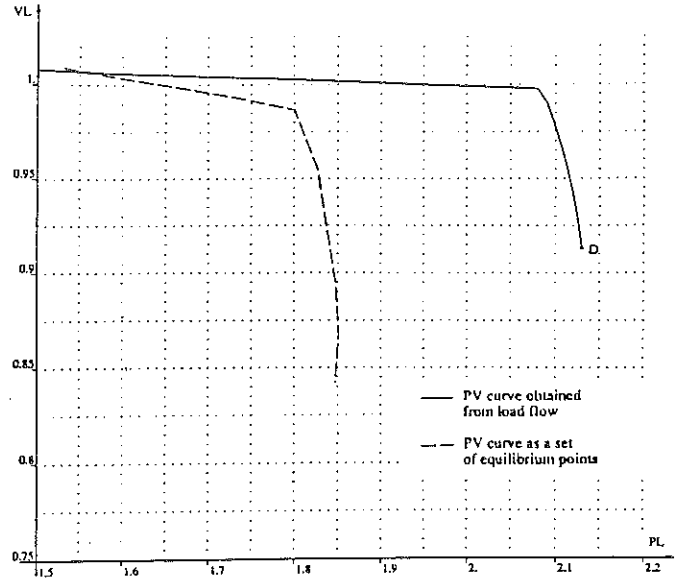


Figure 14: Case 1. Load flow vs equilibrium PV curves.

assumptions about the reactive power capability of Generator *G* ($e_{fd}^{max} = e_{fd}^{per} = 2.69$ and $Q_G^{max} = 0.66$ pu, respectively). As can be seen, there is a discrepancy between both curves. This comes from the fact that the load flow treats Bus *G* as a *PV* bus, with V_G fixed at its *pre-fault* set-point (1.01 pu) while the equilibrium characteristics takes into account the steady-state behaviour of the machine and its AVR. As a result, the V_G voltage decreases following the line tripping and sags as the load increases; this, in turn, influences the machine reactive power production and hence the break-point corresponding to its maximum reactive capability. Both effects yield a lower loadability limit with respect to the load flow. Otherwise stated, it is necessary to use some slightly lower value of V_G and Q_G^{max} in the load flow in order to make it coincide with the true system behaviour¹.

In fact, the result of Figure 14 is a practical illustration of the theoretical derivations of Ref. [7]. In the latter, the authors show that, generally speaking, the singularity of the dynamic system Jacobian J does not coincide with that of the load flow Jacobian J_{LF} , unless unrealistic simplifications are made in the system modelling. Figure 14 shows indeed that *assuming a constant power load*, J is singular for $P_L \simeq 1.85$ and J_{LF} for $P_L \simeq 2.13$, i.e. for a more optimistic load level. Moreover, note that this reasoning as well as the derivation of Ref. [7] relies on a constant power load assumption; as already shown, the load behaviour even more contributes to make the load flow Jacobian inadequate to predict the dynamic behaviour of the continuous-time system.

¹Ref. [26] proposed an easy way to take the AVR effect into account in a standard load flow. The method is also illustrated in Ref. [4]

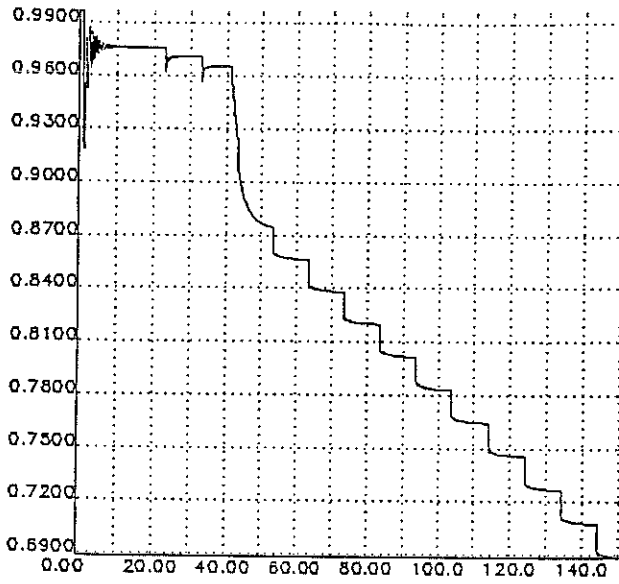


Figure 15: Case 2. V_L voltage.

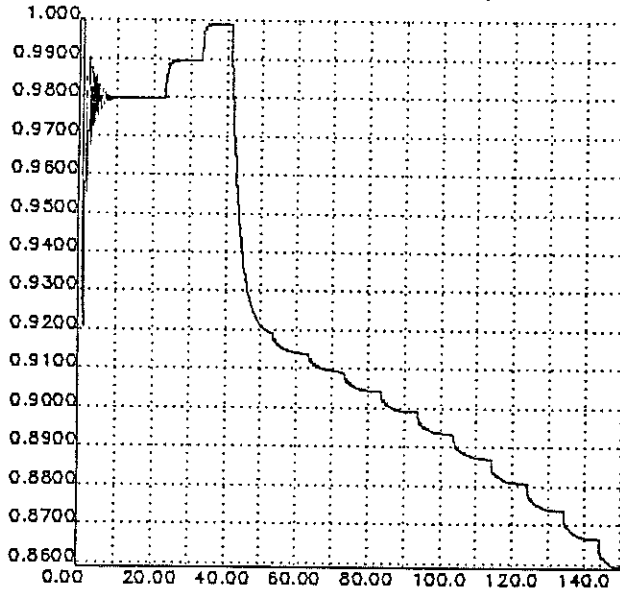


Figure 16: Case 2. V_2 voltage.

4.3 Case 2

This case offers many similarities with the preceding one : constant impedance load, zero AVR time constant, same disturbance. The system initial operating point is however the second one on Figure 4; it corresponds to a higher load level as well as a higher generation capability at bus G.

Figures 15 and 16 give the time evolutions of the V_L and V_2 voltages, respectively. As can be seen, they are quite similar to those of Case 1, i.e. a succession of equilibrium points, unstable from the discrete control system viewpoint, stable from the continuous-time one. The only difference lies in the fact that the transformer OLTC exhibits a reversed logic as soon as the machine field voltage is limited. This behaviour is easily analyzed through the corresponding PV curves of

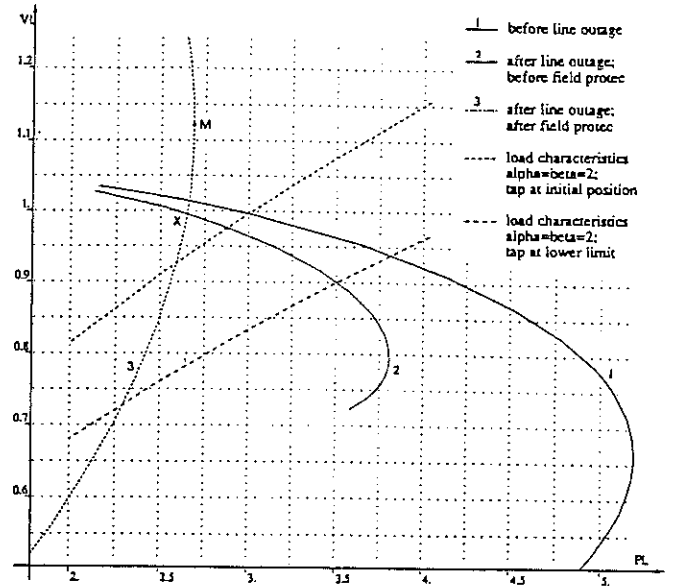


Figure 17: Case 2. Network and load PV curves.

Figure 17. As can be seen, when the system enters its reactive power limited configuration, the resulting equilibrium point falls on the lower part of PV Curve 3. Any further attempt to increase V_2 results in the opposite effect.

Moreover, comparing Figures 17 and 13, it appears that M , the maximum load point of Curve 3, is now *above* Curve 2. As a result, the contingency-constrained loadability limit of the system does no longer correspond to Point M (as in Fig. 13) but rather to Point X , for which $P_L = 2.625$ pu.

This result can be easily ascertained by a standard load flow calculation, previously adapted to match the equilibrium characteristics around Point X (see discussion of preceding section) : this amounts to choose $V_G = 0.9917$ pu (instead of 1.01 : see Fig. 4) and $Q_G^{max} = 316.3$ MVar ≈ 0.6326 pu. Table 1 shows the maximal active and reactive power mismatches at the successive iterations of three load flow runs, corresponding to three load levels around Point X . $P_L = 2.624$ yields perfect convergence in 4 iterations, $P_L = 2.72$ causes divergence and, in between, $P_L = 2.65$ makes the algorithm oscillate between two solutions.

As can be better seen from Figure 18, for $P_L = 2.72$ there is no solution once the reactive limit is enforced. For $P_L = 2.65$, the algorithm could rightfully not choose between : (i) Point A, where the machine voltage is at the set-point value but the reactive production is above its limit, and (ii) Point B, where the reactive limit is enforced but the machine voltage *rises* above its set-point value. (In fact, at this point, the rise in V_G decreases the XI^2 losses between G and L and increases the $B_L V_L^2$ support, both counterbalancing the increased reactive transfer from G to I). Clearly, X is the ultimate point for which both constraints can be satisfied.

Note however that the load flow Jacobian (with constant power loads) is not singular at this point, since the

Table 1

before iter #	max. mismatch P (MW)	max. mismatch Q (MVar)	before iter #	max. mismatch P (MW)	max. mismatch Q (MVar)
$P_L = 2 \quad Q_L = 2.624 \text{ pu}$			$P_L = 2 \quad Q_L = 2.650 \text{ pu}$		
1	1312.0	94.8	1	1325.0	200.8
2	37.1	52.3	2	29.9	55.1
3	0.3	0.2	3	0.3	0.2
4	0.0	0.0	4	0.0	16.8
$P_L = 2 \quad Q_L = 2.720 \text{ pu}$			$Q_G = 333.1 \quad \text{PV} \rightarrow \text{PQ}$		
1	1360.0	218.3	5	87.5	1005.9
2	32.9	58.3	6	9.0	56.6
3	0.4	0.3	7	0.1	0.2
4	0.0	65.0	8	0.0	16.8
5	188.9	33.2	9	87.5	1006.0
6	106.9	18.8	10	9.0	56.6
7	849.7	149.0	11	0.1	0.2
8	252.0	41.5	12	0.0	16.8
9	109.9	18.3	13	87.5	1005.9
10	300.6	62.1	14	9.0	56.6
11	80.4	27.3	15	0.1	0.2
12	348.5	57.5			
13	124.0	20.3			
14	162.3	30.5			
15	98.2	22.3			

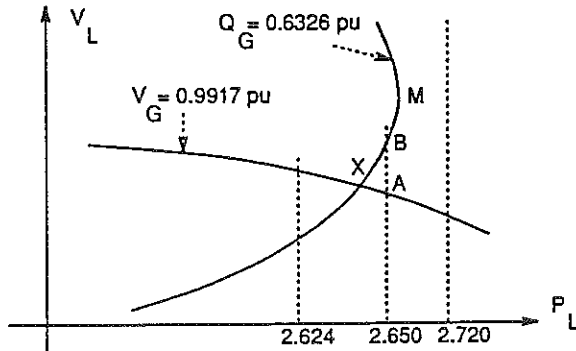


Figure 18: Case 2. Detailed view around Point X.

$\Delta V_L / \Delta P_L$ slope is not infinite.

As a conclusion, the two cases show that a loadability limit can be reached in two ways :

- after some generators have reached their reactive limits, the load can still be increased up to a maximum thanks to the reactive support of farther (non limited) generators; at this maximum, the load flow Jacobian (with constant power loads) is singular;
- just after some generator has hit its reactive limit, no reactive support can be obtained any more from the remaining (non limited) ones, for they are electrically too far away for the current load level. At this limit, the load flow Jacobian is not singular.

In Case 2, the higher reactive reserve available on G allows to satisfy a higher load level, which results in a limit of type B.

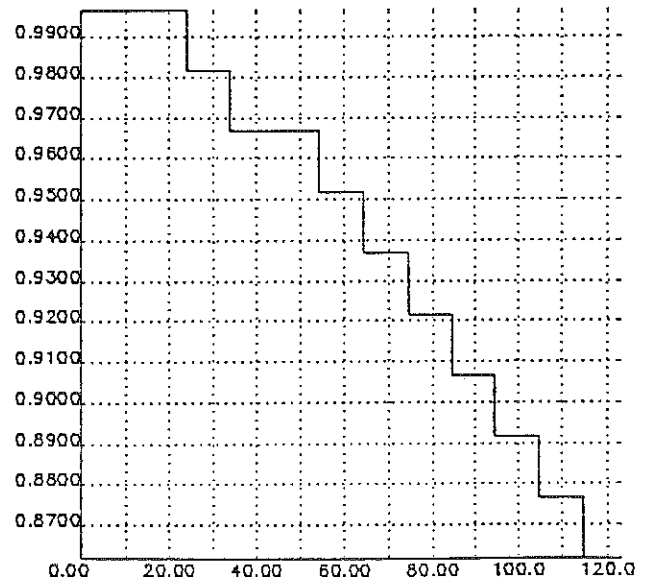
4.4 Case 3

This case involves the same operating point and triggering incident as in Case 2. The load however is assumed to behave as a *constant* active and reactive current ($\alpha = \beta = 1$).

Figures 19 to 22 show the time evolution of r , V_L , V_2 and δ respectively. As can be seen, the system behaviour significantly departs from the previous one, since about 2 minutes after the line tripping, the voltages plunge towards zero (the simulations were stopped once V_L reached 0.1 pu). Before, a phase of reserved action of the OLTC is observed.

From the viewpoint of the proposed system framework, this simulation corresponds to an unstable discrete control system, leading to a loss of stability of the continuous-time one.

This can be easily verified on the PV curves of Figure 23. The network characteristics are unchanged with respect to Case 2 (since $\alpha = \beta$), while the load characteristics are straight lines; two are shown, corresponding to the initial tap position ($r \approx 1$) and to the lower limit ($r = 0.8$). Clearly, below some value of r , the network and load characteristics do not intersect and hence there is no equilibrium any longer. Indeed, the change in r from 0.892 to 0.877 which occurs at time $t \approx 105$ s, is seen by the continuous system as a change in the z parameter causing a static bifurcation (see Figure 3). Since there is no delay in the load model, the point representative of the system trajectory slides along the load characteristics. The absence of an equilibrium for the system as seen from Bus L implies the absence of an equilibrium for the system as a whole. This is particularly well revealed by Figure 21 in

Figure 19: Case 3. Transformer ratio r .

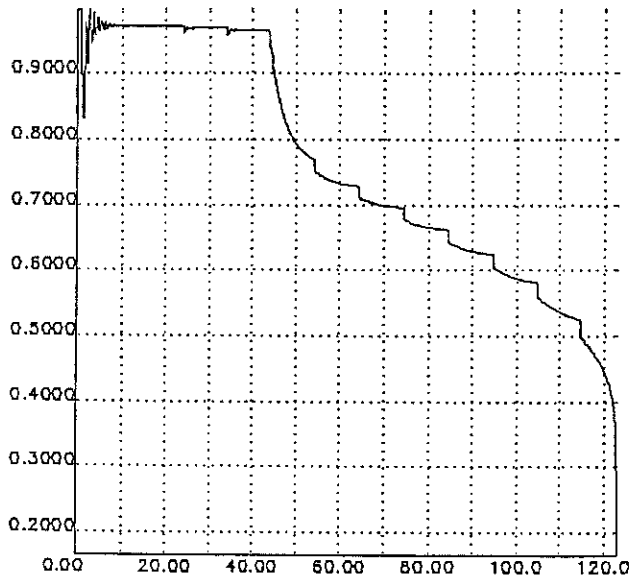


Figure 20: Case 3. V_L voltage.

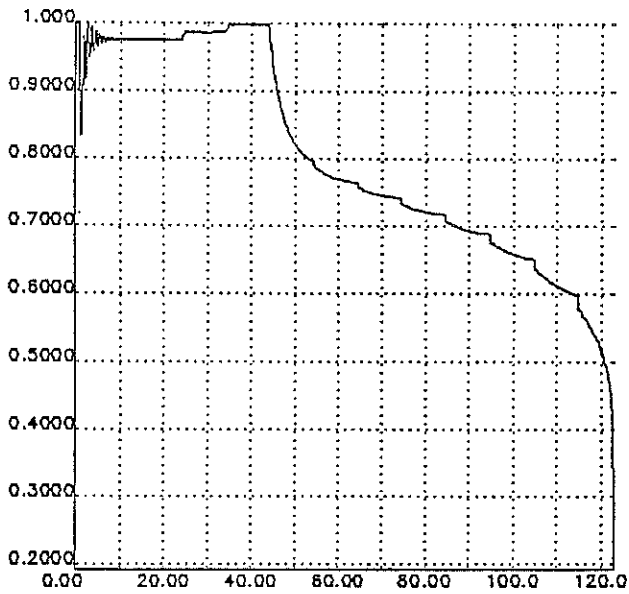


Figure 21: Case 3. V_2 voltage.

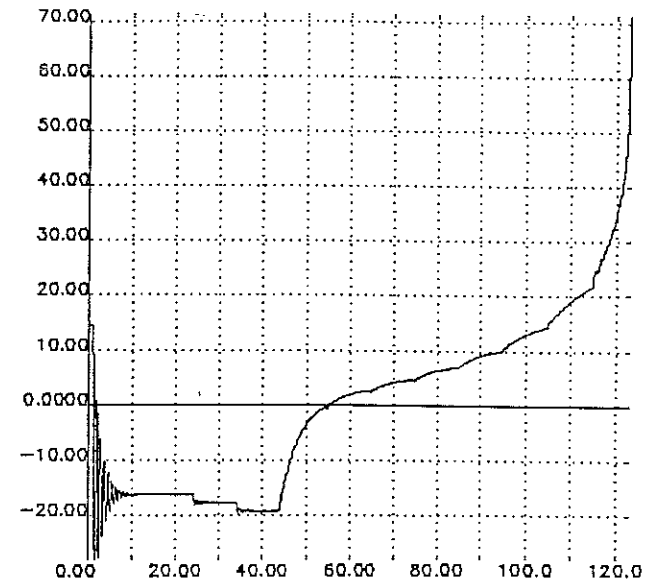


Figure 22: Case 3. Rotor angle δ .

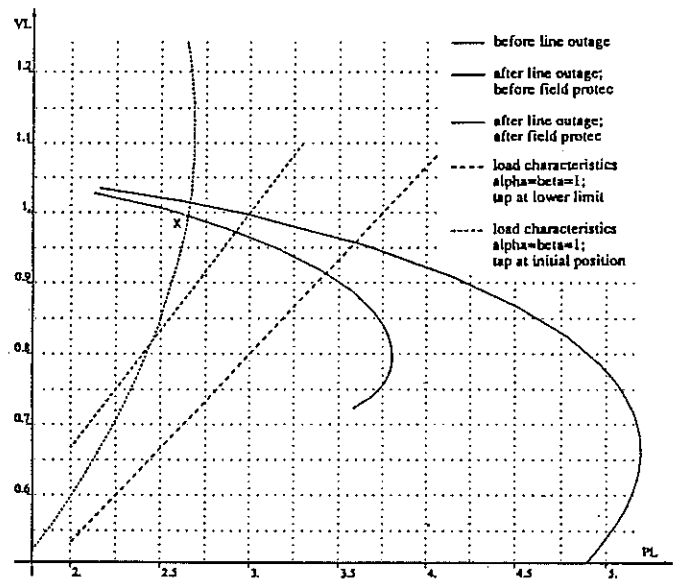


Figure 23: Case 3. Network and load PV curves.

which the rotor angle motion recalls that of a machine going out of step.

The above bifurcation occurs when the load characteristics is tangent to the network one. Note that *owing to the variation of the load with voltage, this occurs far below the loadability limit point X of Figure 23.*

At the point of tangency, the Jacobian J is singular. This is confirmed by an eigenvalue analysis, carried out at the successive equilibrium points of the system. From a practical viewpoint, "snapshots" of the x and y vectors have been taken at time instants where the system was reasonably come back to equilibrium, namely as late as possible before each next disturbance or change in r . A plot of the eigenvalues is shown in Figure 24. Since the AVR is assumed infinitely fast ($T_e = 0$), the system has 3 eigenvalues. Those relative

to the pre-disturbance state are marked with a $\otimes$. After the line tripping and before the field voltage limitation, the system goes through 3 equilibrium points whose eigenvalues are marked with a Δ . The steady-state stability has worsened but the effect of the OLTC is marginal. Finally, once under constant excitation, the system goes through 6 equilibrium points (marked with O's on Fig. 24); in this case, the OLTC is very detrimental, since a real negative eigenvalue tends towards zero.

Figure 24 shows a seventh, real positive eigenvalue. The latter is relative to the system state at time $t \simeq 114$ s (before the last change in r : see Fig. 19). At that time, while the system seems to be (almost) at equilibrium, there is already no equilibrium any longer; in fact, the voltage collapse takes

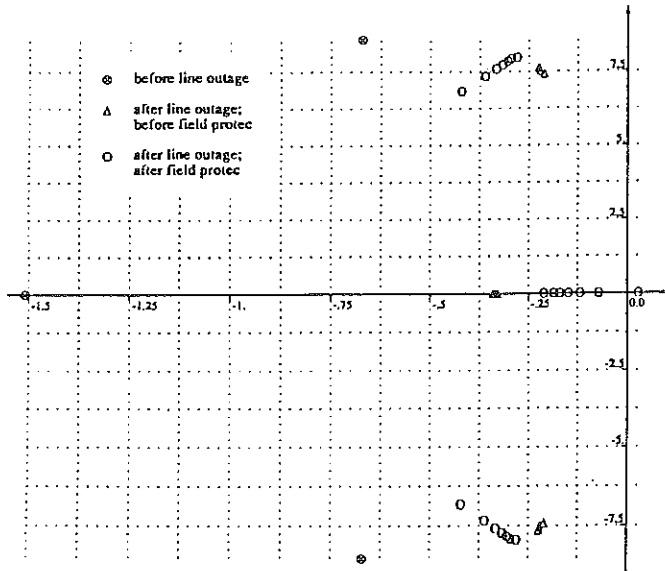


Figure 24: Case 3. Successive system eigenvalues.

some time to develop. As a consequence, Figure 24 should not be interpreted as a real eigenvalue *crossing* the imaginary axis: when zero is reached, there is no equilibrium point and hence no correct eigenvalue analysis any longer.

4.5 Case 4

In this case the load is again a *constant impedance* ($\alpha = \beta = 2$). The operating point is however the third one in Figure 4 and the triggering event is the tripping of one circuit between G and I along with one between G and L.

Figures 25 and 26 show the evolutions of the V_L voltage and of the rotor angle δ . As can be seen: (i) the discrete control system is unstable as in Cases 2 and 3; (ii) the continuous-time system loses its stability following the change in r at time $t \simeq 112$ s. This instability differs however from that of Case 3, since it is linked to the loss of synchronism of the machine. The V_L voltage does not plunge towards zero but rather oscillates owing to the rotor aperiodic motion (the numerical integration has been stopped arbitrarily). This case is a typical example of angular instability following a voltage problem: the loss of synchronism is the result, not the cause.

These results are fully confirmed by an eigenvalue analysis. Figure 27 shows the successive locations of the three system eigenvalues. This figure has been obtained with the same dynamical modelling and using the same procedure as for Figure 24. As can be seen, there is again a real negative eigenvalue tending towards zero. The similarity between both eigenvalue loci is particularly noteworthy. In fact, although the phenomena are different, in both cases the system undergoes a static bifurcation. In Figure 24, the latter results from the "separation" of the load and the network (equilibrium) characteristics at bus L. In Figure 27, it results from the separation of the mechanical input and electrical output characteristics of the

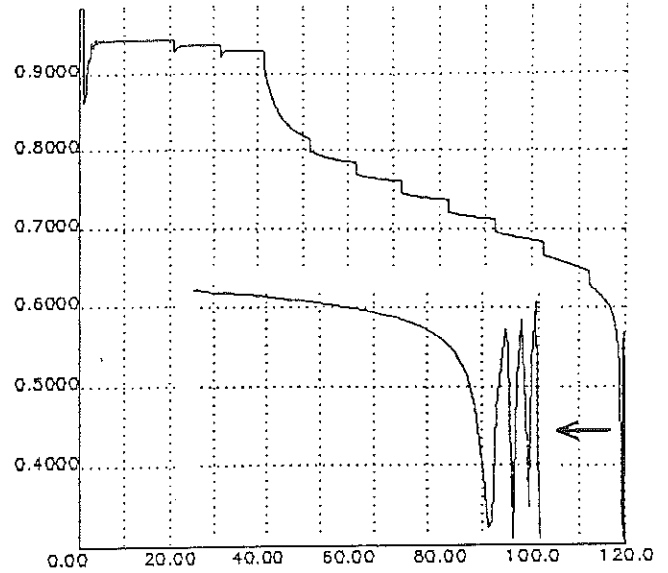


Figure 25: Case 4. V_L voltage.

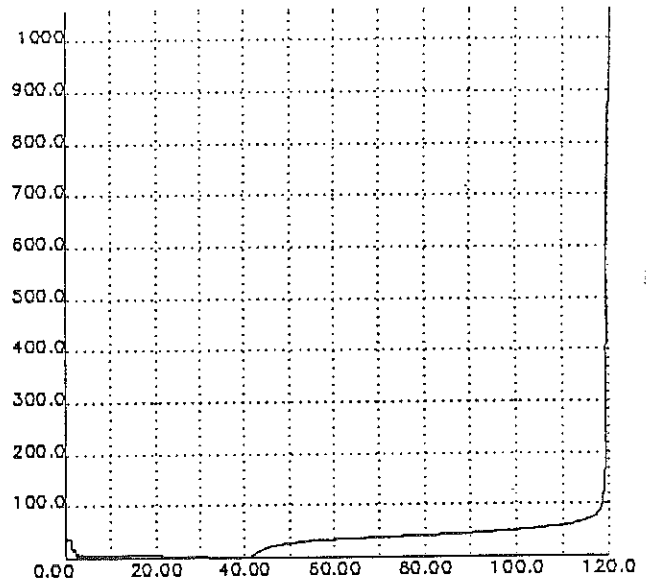


Figure 26: Case 4. Rotor angle δ .

machine. As outlined in Figure 28, the switching to constant excitation and the subsequent disturbances imposed by the OLTC decrease the $P_e(\delta)$ curve until it does not cross the P_m horizontal any longer. In the simulated example, such a separation is furthered by the higher P_m , the stronger incident and the pronounced decrease in load with voltage. Finally, the remark of the last section about the real positive root still holds.

4.6 Case 5

When going from Case 2 to Case 3, a less voltage sensitive load was considered. The simulation hereafter goes further by considering a load such that $\alpha = \beta = 0.5$. The initial operating point is the fourth one in Figure 4 and $T_e = 0.1$ s.

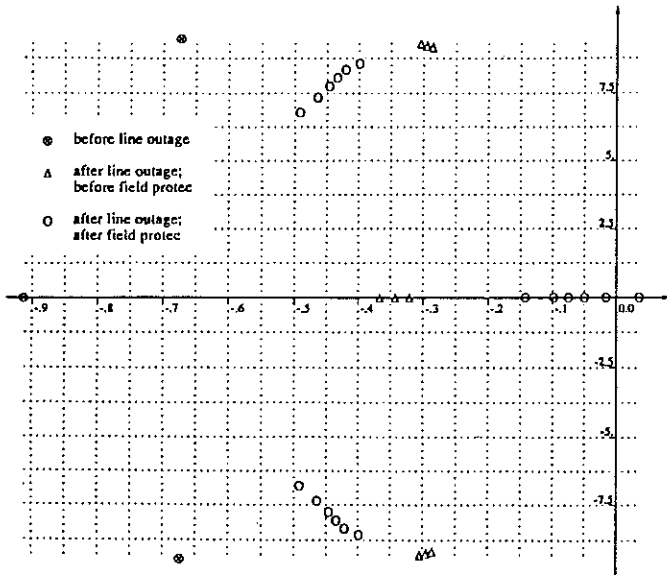


Figure 27: Case 4. Successive system eigenvalues.

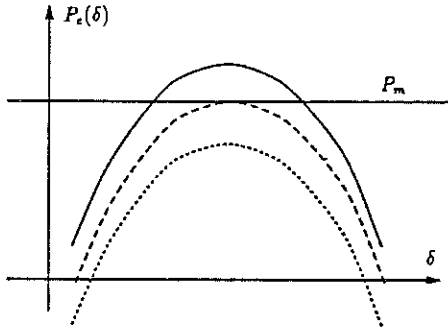


Figure 28: Bifurcation originating from the machine.

The triggering incident is the loss of one circuit between G and I.

With the problem so stated, numerical integration has not been possible owing to the occurrence of *singularities*. The latter are characterized by a divergence of the numerical algorithm. This occurs when the $\partial g/\partial y$ matrix approaches singularity. In such a case, the dynamic system trajectory is no longer defined [14]. In practice, numerical problems followed from the impossibility to solve the algebraic equations (19) for a given x , along the trajectory. As the integration algorithm tried some value of x in the vicinity of the singularity, Eqs. (19) stopped having a solution and the Newton algorithm diverged².

These problems are often quoted in the literature as being caused by an inadequate load modelling. The way singularity and load modelling relate can be intuitively explained as follows. Using a purely static load model amounts to consider that the continuous-time system is infinitely slow with respect to the load. Hence, at any time t , the system op-

²With variable time step algorithms, the step size is reduced when approaching the singularity, until it becomes unacceptably small

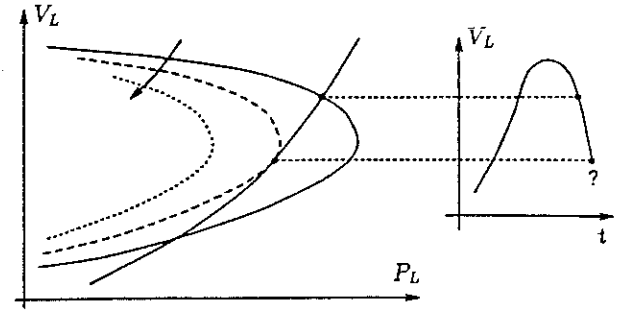


Figure 29: Intuitive view of singularities

erating point can be considered as the intersection between a "dynamic" network PV curve and the load characteristics. Now, stringent conditions (imposed e.g. by the load itself) can make this dynamic characteristics shrink so much that it does no longer intersect the load characteristics, as outlined in Figure 29³. In such a case, neglecting the transient phase of the load dynamics (see §3.2) becomes unacceptable⁴.

To take the load transient phase into account, a "black-box" modelling has been used, in which the load behaves as an equivalent dynamic admittance $g_{eq} - jb_{eq}$ varying with time so as to asymptotically fit the static behaviour of Eq. (12). More precisely, the instantaneous load consumption is given by :

$$P(V, t) = g_{eq}(V, t) V^2 \quad (22)$$

$$Q(V, t) = b_{eq}(V, t) V^2 \quad (23)$$

where g_{eq} and b_{eq} are additional state variables obeying :

$$\frac{d}{dt}g_{eq} = \frac{1}{T_l} \left(\frac{P_0(\frac{V}{V_0})^\alpha}{V^2} - g_{eq} \right) = \frac{1}{T_l} \left(P_0 \frac{V^{\alpha-2}}{V_0^\alpha} - g_{eq} \right) \quad (24)$$

$$\frac{d}{dt}b_{eq} = \frac{1}{T_l} \left(\frac{Q_0(\frac{V}{V_0})^\beta}{V^2} - b_{eq} \right) = \frac{1}{T_l} \left(Q_0 \frac{V^{\beta-2}}{V_0^\beta} - b_{eq} \right) \quad (25)$$

in which T_l is an equivalent time constant. One easily verifies that setting the above time derivatives to zero or making T_l tend towards zero yields Eq. (12).

Remark. Within a short (as compared to T_l) time interval following a discontinuity in V , the above load behaves as a mere impedance. The induction motor exhibits a similar behaviour although its dynamics somewhat differs from Eqs. (24-25); for the latter $\alpha \simeq 0$ and the role of g_{eq} , b_{eq} is played by the motor slip s .

In the above described simulation case, a time constant T_l as small as 0.1 s was found sufficient to remove all singularities ! The resulting behaviour of V_L is shown in Figure 30. The system undergoes a fast voltage collapse within a 25 s

³The latter shows dynamic, not equilibrium network characteristics

⁴Point E_u of Ref. [17] appears to be a singularity resulting from a static, constant power load assumption in the presence of the machine dynamics

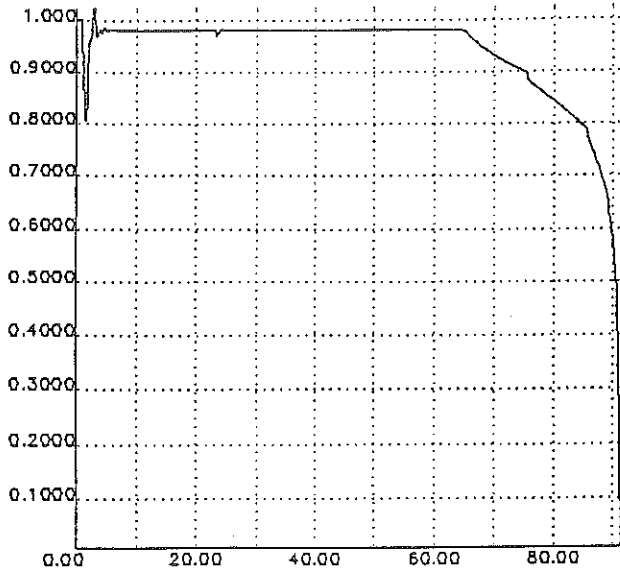


Figure 30: Case 5. V_L voltage.

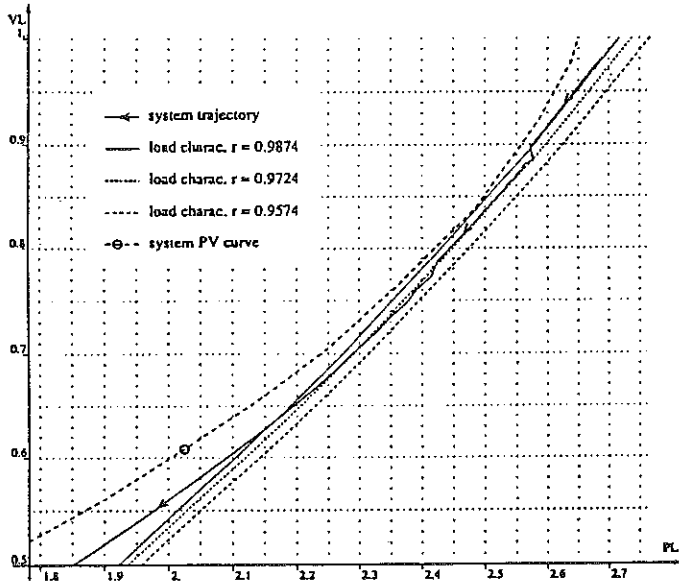


Figure 31: Case 5. PV curves and system trajectory.

interval following the field voltage reduction (at time $t \simeq 64$ s). Indeed, the corresponding PV curves of Figure 31 indicate that as soon as the system enters this final configuration, there is no equilibrium any longer. The system trajectory is also shown : owing to the load delay T_l it departs from the static characteristics, first slightly, then significantly, once the voltage drops fast. The larger T_l , the larger the gap.

It has been also observed that the smaller the separation between the network and the load PV curves, the longer it takes to the system to collapse. Referring to Figure 31 for instance, if the transformer ratio is blocked at the 0.9874 value, it takes as much as one minute to collapse. Blocking the ratio at 0.9724, it takes about 30 seconds. The existence of such a delay was already quoted in Case 3.

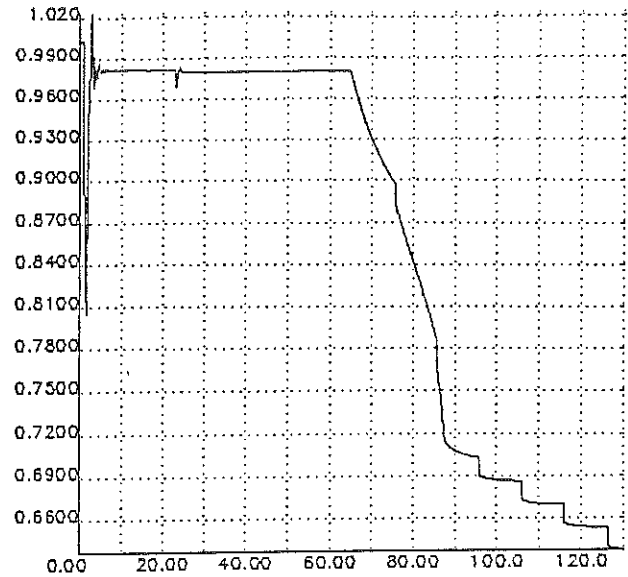


Figure 32: Case 5. V_L voltage; discontinuous load model.

Within the context of load modelling, it may be also essential to consider discontinuous load characteristics in the presence of significant voltage changes. The discontinuity arises e.g. due to motor stalling or to the extinction of discharge lighting. A compromise commonly used is to switch to a constant impedance modelling as long as the low voltage persists [23]. This can be done easily in the above modelling by upper bounding g_{eq} and b_{eq} . For instance, if the change from Model (12) to a constant impedance takes place below a V_s threshold voltage, one simply has :

$$g_{eq} < g^{max} = \frac{P_0}{V_s^2} \left(\frac{V_s}{V_2^0} \right)^\alpha \quad (26)$$

$$b_{eq} < b^{max} = \frac{Q_0}{V_s^2} \left(\frac{V_s}{V_2^0} \right)^\beta \quad (27)$$

Figure 32 shows the behaviour of V_L when switching to constant impedance for $V_s = 0.75$ pu. As can be seen, the voltage first drops as dramatically as in Figure 30 and eventually behaves as in Case 2 (unstable OLTC / stable continuous-time system).

5 Conclusions

In this paper voltage instability mechanisms have been analyzed from a system theoretic viewpoint. One aspect of the analysis consisted in separating the effects of the discrete control performed by the OLTCs from the remaining continuous-time system. The true dynamics of the latter has been considered, e.g. instead of a PV-PQ representation of the generators. The load behaviour was of the type generally assumed in large-scale dynamic studies.

In spite of the example simplicity, several salient features of voltage collapse mechanisms have been brought to the fore :

- the instability of the discrete control system is related to the power system loadability limit. From a practical viewpoint, it shows itself as a reversed logic beyond the maximum load point; this leads to a useless decrease of primary and secondary voltages;
- the stability of the continuous-time system is not linked to this loadability limit;
- rather, the incidents, the reductions in machine excitation levels and the repeated actions of the OLTCs all cause the continuous-time system to go through successive equilibrium points whose stability can deteriorate until a bifurcation occurs;
- however, depending upon the load behaviour, such a bifurcation does not necessarily occur : the instability can result from the discrete control system only, resulting in a dramatically depressed but "stabilized" voltage profile;
- two kinds of bifurcations are : (i) those resulting from the separation of the load and network characteristics at some bus of the system, and (ii) those resulting from the loss of stability of a machine;
- in both cases, there is a real negative eigenvalue tending towards zero; the reaching of zero (and hence the Jacobian singularity) is not linked to the reaching of the loadability limit;
- standard load flow relationships are inadequate to describe the system dynamics close to or beyond this limit. Major discrepancies result from the PV-PQ generator modelling and from the constant power load assumption;
- in all the tested unstable cases, the instability of the discrete control system has been met first; from there on and until a bifurcation occurs (if any), the system operates on the lower part of the PV curves. From the emergency control viewpoint, within this time interval, the short-term system salvaging is still possible through the OLTC control (blocking, reversed operation);
- in all aspects of the study, load sensitivity to voltage played an important role. The static bifurcation of type (i) above is probably the case which requires the most careful checking of the model validity : the switching to a less severe modelling can make the system switch to another form of instability.

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COMPUTATIONAL EXPERIENCE WITH THE POINT OF COLLAPSE METHOD ON VERY LARGE AC/DC SYSTEMS

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Abstract: This paper describes the implementation of the Point of Collapse (PoC) method for the computation of saddle-node bifurcation (voltage collapse) points in large ac/dc systems. A comparison of the performance of this method against continuation methods is presented for real systems of up to 2200 buses. The paper also discusses the computational details of the implementation of the PoC and continuation methods, and the unique challenges encountered due to the presence of high voltage direct current (HVDC) transmission, area interchange control, regulating transformers, and voltage and reactive power limits. Finally, the characteristics and capabilities of a robust point of collapse power flow program, one of the outcomes of this work, are also presented.

Keywords: Voltage collapse, large ac/dc systems, saddle-node bifurcation, point of collapse, continuation methods.

1 Introduction

During the last few years several known methodologies for detecting saddle-node bifurcations¹ in dynamic systems using steady state analysis techniques [1], have been tailored and applied to the determination of loadability limits of power systems. Specifically, the PoC method introduced in 1988 [2], and its variations [3], have been shown to be computationally feasible and well suited for determining proximity to voltage collapse in integrated ac/dc dynamic networks [4, 5]. Continuation methods have also proven to be a good way of calculating bifurcation points in ac power systems, as shown in [7, 6].

This paper presents brief quantitative and qualitative descriptions of these two methods, and describes the additional modifications needed to handle an arbitrary number of ac limits and dc links. A detailed account of the implementation of these methods in C code and the characteristics of the resulting program are also presented. This tool is then used for determining voltage profiles and loadability margins of several ac/dc systems. The results and performances are analyzed and compared for both methods.

2 The Point of Collapse Method

A general one-parameter nonlinear dynamic system can be represented by the vector field

$$\dot{z} = f(z, \lambda) \quad (1)$$

¹In this paper dynamic saddle-node bifurcations, or voltage collapse points, will be considered to be detectable by looking only for singularities of the steady state power flow Jacobian, since, under certain assumptions, saddle-node bifurcations of ac/dc dynamic systems with algebraic constraints can be shown to occur when the corresponding power flow Jacobian becomes singular [4, 5].

where $z \in \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. This system presents a bifurcation at the equilibrium point (z_0, λ_0) when the corresponding linearization (Jacobian) is singular. Saddle-node bifurcations, as opposed to other kind of bifurcations, are typical in practice [1, 8, 9], and are characterized by the steady state Jacobian $D_z f(z_0, \lambda_0)$ having a simple and unique zero eigenvalue, with nonzero right eigenvector v and left eigenvector w . This condition can be summarized by the set of vector equations (2) for the right eigenvector, and (3) for the left eigenvector.

$$\begin{aligned} f(z, \lambda) &= 0 \\ D_z f(z, \lambda) v &= 0 \\ \|v\| &\neq 0 \end{aligned} \quad (2)$$

$$\begin{aligned} f(z, \lambda) &= 0 \\ D_z^T f(z, \lambda) w &= 0 \\ \|w\| &\neq 0 \end{aligned} \quad (3)$$

The solution to these equations yield the saddle-node bifurcation point.

For power system dynamic models, the parameter λ typically stands for load increase throughout the network. One can solve equations (2) or (3) to determine the point of voltage collapse (z_0), and the maximum loading factor (λ_0). Equations (2) are used by Seydel in [1] to determine saddle-node bifurcations of general dynamic systems, and were first applied to voltage stability analysis of ac systems in [2]. A variation of equations (3) was used in [3] to determine voltage collapse points of purely reactive ac systems; here the nonzero condition for the left eigenvector, i.e., $\|w\| \neq 0$, is replaced by the saddle-node bifurcation condition (4). In all these cases the Jacobian can be shown to be nonsingular at the bifurcation point [5], which makes these methods numerically appealing.

$$w^T \frac{\partial f}{\partial \lambda} \Big|_{(z_0, \lambda_0)} \neq 0 \quad (4)$$

When these methods are applied to real sized ac/dc systems, there are several factors that have to be taken into account in order to obtain consistent and reliable results. The first consideration is the need for having good initial guesses for the system variables, particularly the eigenvectors, otherwise a Newton-Rapson approach for obtaining the solution to the PoC equations either yields undesirable results or does not converge. This becomes an important issue for large systems with an arbitrary number of ac/dc state variable limits.

AC limits in this paper are considered to be the classical voltage regulating transformer and phase shifter tap limits,

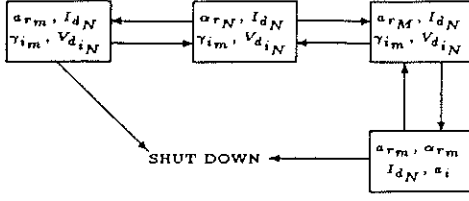


Fig. 1: Control modes of HVDC link for power flow solution, when ac bus voltage at the rectifier side changes. Margin switching at the inverter side is assumed.

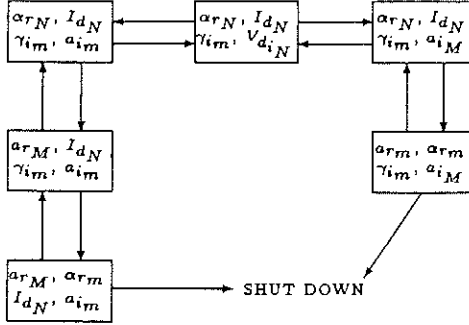


Fig. 2: Control modes of HVDC link for power flow solution, when ac bus voltage at the inverter side changes. Margin switching at the inverter side is assumed.

and voltage and reactive power limits at generator buses. The HVDC system introduces additional state variable limits, namely, commutation transformer tap limits, firing angle and extinction angle limits, and current and power limits. These steady state limits represent also dynamic control limits. In the case of the ac system, enforcing limits does not present major difficulties since they can be modelled by a change of equations on the steady state system model. However, when a variable reaches an operational limit in the HVDC link, the system changes control mode, which can be modelled by a change in the state variables used to simulate the dc steady state behavior. Therefore, a logical scheme has to be devised in order to accurately represent the dynamic control logic at both converter stations. Figures 1 and 2 depict the transition logic used in this paper to simulate the switching between different control modes for the rectifier and inverter, when the ac bus voltage at either side of the dc line changes. In these figures, the boxes represent the four dc variables that must be kept constant in order to obtain a solution to the ac/dc power flow. For power control just substitute active power for dc current in both figures. The notation in these figures is as follows: r stands for rectifier and i for inverter; α is the firing angle, whereas γ is the extinction angle; I_d is the dc link current and V_d is the corresponding voltage; N represents a nominal value, M is a maximum value, and m is a minimum value. The reader is referred to [10, 11] for a detailed explanation on the different modelling and control issues in HVDC systems.

The state variables z are first initialized to the values obtained from a base case power flow solution. The eigenvectors, v and w , are set to the values yielded by 4 or 5

iterations of the inverse power method [12] applied to the initial power flow Jacobian, and the parameter λ is initially set to zero. After testing the method in several ac/dc and ac systems, these initial guesses prove to be unreliable, particularly when ac and dc limits are reached. Hence, to improve the convergence characteristics of the method, new values for the eigenvectors are calculated every time the ac/dc system reaches a state variable limit. This is done by applying a few iterations of the inverse power method to the Jacobian of the new set of ac equations and dc variables evaluated at the switching point.

For systems relatively far from the bifurcation point, the initial guesses described above are not sufficient to obtain consistent results. By initially loading the system one can resolve this difficulty. This initial load can be calculated using the tangent vector to the system branch [8, 1] at the base case (see figure 3), which is a technique used in continuation methods to find system voltage profiles [7, 6]. This tangent vector, $dz/d\lambda$, can be found by a factorization and a repeat solution of the base case power flow Jacobian, $D_z f(z_1, 0) = \frac{\partial f}{\partial z} \Big|_1$, assuming a linear loading pattern, as shown in equation (5).

$$f(z, \lambda) = 0 \Rightarrow \frac{df}{d\lambda}(z_1, 0) = \frac{\partial f}{\partial z} \Big|_1 \frac{dz}{d\lambda} + \frac{\partial f}{\partial \lambda} \Big|_1 = 0$$

$$\frac{dz}{d\lambda} = -D_z^{-1} f(z_1, 0) \frac{\partial f}{\partial \lambda} \quad (5)$$

The amount of added load comes from the normalization of the tangent vector, i.e.,

$$\Delta \lambda = \frac{k}{\|dz/d\lambda\|_1} \quad (6)$$

$$\Delta z = \Delta \lambda \frac{dz}{d\lambda} \quad (7)$$

Equation (7) is used to calculate an initial guess ($z_1 + \Delta z$) to obtaining the solution of the power flow at the new load setting. Several tests have demonstrated that an actual solution to this new power flow is not required in order to obtain a set of good initial values for the PoC variables.

The scaling constant k in equations (6) was used to simulate different system loadings. Furthermore, after experimenting with several values of k in a variety of ac/dc and ac only systems, one can observe that the bifurcation point is closely related to the number of generators in the system, especially when reactive power limits are enforced. This led the authors to conclude that a good value for k is

$$k = \sqrt{\text{Number of Generators}}$$

This value is used unless more than 10 % of the generators reach limits when the system is initially loaded. If the 10 % limit is reached, the value of k is reduced by half until this criterion is met. Although this idea worked in most of the cases tested, one cannot guarantee that this will always yield the desired results. Since this process is used only to obtain an initial guess for the PoC equations, the value used for k can be changed by the user.

Finally, experience with the PoC method has demonstrated that the left eigenvector equations yield better results than their right eigenvector counterpart. Also, using a norm one condition as the third PoC equation in (3), i.e., $\|w\|_1 = \max\{w_i\} = 1$, has proven to be a more reliable way for finding the voltage collapse point.

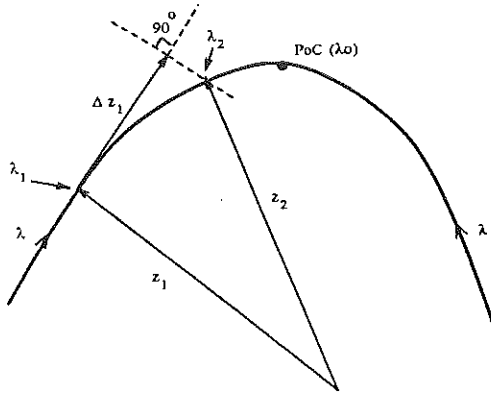


Fig. 3: Continuation method geometry in state space.

3 Parameterized Continuation Methods

Continuation methods present another way of determining proximity to saddle-node bifurcations in dynamic systems. These methods are thoroughly described by Seydel in [1] for general systems, and were applied to the analysis of voltage collapse in ac only systems in [7, 6]. Reference [6] shows a direct application of the parameterized continuation methods to the ac power flow equations. In [7], although the authors do not use the parameterization approach, they introduce the idea of a perpendicular intersection to improve the convergence characteristics of these methods. The software presented on this paper uses both techniques, i.e., parameterization and perpendicular intersection, to trace the branch (voltage profiles) of realistic ac/dc networks.

These methods consist of a three step approach to tracing the equilibrium points as one parameter in the system changes, i.e., find the solutions to the power flow equations $f(z, \lambda) = 0$, for a given set of parameter values. Typically the loading factor λ is the varying parameter; however, as the system gets closer to bifurcation the classical power flow Jacobian becomes ill-conditioned. These could create numerical havoc in the first step of the method, but a change of parameter (parameterization), e.g., switching from λ to a bus voltage $z_i \in z$, makes the system Jacobian nonsingular at the voltage collapse point.

Figure 3 shows graphically how these methods work. The manifold, depicted as a boldface curve, represents the system equilibria as the system parameter changes. Assuming that the system is initially at the state (z_1, λ_1) , one can move to the new equilibria (z_2, λ_2) by using $\Delta\lambda$ and the scaled tangent vector Δz_1 defined in equations (6) and (7), respectively, since for a small step $z_2 \approx z_1 + \Delta z_1$ and $\lambda_2 \approx \lambda_1 + \Delta\lambda$. To obtain the actual values of z_2 and λ_2 , one can use the perpendicular plane to the tangent vector to find the desired point in the branch. Mathematically this can be summarized in the following three steps:

1. Find the step $\Delta\hat{z}$ and Δp in state space and parameter space by solving equation (8).

$$D_z f(\hat{z}_1, p_1) \frac{d\hat{z}}{dp} = - \frac{\partial f}{\partial p} \Big|_1 \quad (8)$$

$$\Rightarrow \Delta p = \frac{k}{\|d\hat{z}/dp\|}$$

$$\Delta\hat{z} = \Delta p \frac{d\hat{z}}{dp}$$

The parameter p is initially set to λ , and the state variables $\hat{z}$ are equivalent to z . However, as the process approaches the bifurcation, p is likely to change to one of the ac bus voltages, according to step 3, with the loading factor λ becoming part of $\hat{z}$. Although the constant k is user defined, $k = 1$ has given good results.

2. Find the intersection between the perpendicular plane to the tangent vector and the branch, i.e., solve equations

$$f(\hat{z}, p) = 0 \quad (9)$$

$$\Delta p (p - p_1 - \Delta p) + \Delta\hat{z}^T (\hat{z} - \hat{z}_1 - \Delta\hat{z}) = 0$$

where p_1 and $\hat{z}_1$ come from the previous iteration. By initially setting $\hat{z}$ to $\hat{z}_1 + \Delta\hat{z}$ and p to $p_1 + \Delta p$, solving this set of equations usually takes one or two iterations. Nevertheless, if the process fails to converge, the steps $\Delta\hat{z}$ and Δp are cut in half until convergence is attained. Enforcing ac and dc limits is done with ease, since the initial guesses are good estimates of the actual solution. If the initial values violate ac/dc limits, the steps are cut so that all limits are met, and then the continuation equations (9) are solved. When $|\Delta p|$ is cut to less than certain tolerance (typically 10^{-4}), the corresponding ac equations, or dc state variables, are changed accordingly to the variable limits that have been reached.

3. Check the relative change in all the system variables, and trade p with the variable that has the largest change. In other words,

$$p \leftarrow \max \left\{ \left| \frac{\Delta\hat{z}_1}{\hat{z}_1} \right|, \left| \frac{\Delta\hat{z}_2}{\hat{z}_2} \right|, \dots, \left| \frac{\Delta\hat{z}_n}{\hat{z}_n} \right|, \left| \frac{\Delta p}{p} \right| \right\}$$

The experience with the method has demonstrated that as the process approaches the bifurcation, p changes from λ to the system bus voltage that is varying the most, and after a few iterations of the method it returns back to λ . However, in all the cases tested, one can obtain good results without switching parameters.

By changing the parameter p from λ to a state variable $z_i \in z$, one guarantees that the Jacobian of equations (8) is nonsingular at the bifurcation point [1]. Furthermore, assuming that the branch is a C^1 curve [8, 9], it can be formally shown that the Jacobian of equations (9) is also nonsingular at the voltage collapse point, even for $p = \lambda$ (singular power flow Jacobian).

The method naturally goes around the collapse point, allowing the user to trace the "unstable" side of the branch. However, this turning point must be detected in order to change the sign of Δp in equations (8). For $p = \lambda$, the bifurcation point can be detected by looking for a sign change in the determinant of the power flow Jacobian, which is a by-product of its factorization. On the other hand, for $p = z_i$, the turning point is detected by looking for a sign change in $\Delta\lambda$.

4 The Point of Collapse Power Flow

This program is a portable C code implementation of the methods described above. It has been successfully tested in several Unix workstations, namely, SUN-SPARC IPC, HP, and DELL, and is also running in PC machines under DOS 3.3 (a Windows 3.0 version is currently under development). The program is capable of handling any size ac/dc or ac systems, limited only by the available memory and swap space. It reads WSCC [13], EPRI [14], and Common Format [15] input files, and produces a variety of ASCII and binary files, depending on the user defined options. It was designed to be a production type software for research and commercial applications, hence, speed and versatility were the main concerns during the development of this program.

The program solves the ac/dc power flow using a Newton-Rapson iterative process with automatic step-size adjustment, which makes it numerically robust. The factorization routines were directly taken from the Sparse Matrix Manipulation System (SMMS) [16]. The PoC and continuation methods were implemented with all the practical details explained in the two previous sections, so that a solution to the voltage collapse problem can be obtained for systems with a diversity of operational limits.

To test the validity and performance of the program, results were compared against three commercial software packages: EPRI's power flow version 3 [13] for SUN and version 5 [14] for DELL, and Electrocon's power flow version 3.10 for PC-DOS machines [17]. The results and performance yielded by this program are shown and discussed in the next section. Figure 4 shows some of the user defined options of the PoC power flow.

5 Results

The program was tested in a variety of ac/dc and ac systems, ranging from 5 to 2158 buses. Figures 5 and 6 depict two of the ac/dc tests systems used for simulations. The 14 bus system was designed by the authors, whereas the 173 bus network is a reduced system used in a joint EPRI research project currently under way at the University of Wisconsin, Madison, and CEPEL, Brazil. A 133 bus system (a modification of the 173 bus system), and a real 2158 ac/dc bus system were also used.

Figures 7 and 8 show two different sets of voltage profiles obtained by applying the continuation method to the 173 and 2158 ac/dc bus systems, respectively. The first profile shows the sharp voltage changes due to the reactive power limits at the generation buses. This is an issue already reported in [3], and is produced by the loss of reactive support throughout the network when a generator or group of generators reach a Q limit, hence, sharply reducing the maximum loadability margin of the system. Figure 8 shows another interesting phenomenon where the system becomes immediately unstable when a generator reaches a reactive limit. This was also observed in the 133 bus system, although, from the practical point of view, it seems to occur rather close to the bifurcation. (For a detailed explanation of this kind of instability the interested reader is referred to the work by Dobson et al in [18].) Also, it was confirmed that active power control in the HVDC link, as opposed to constant current control, reduces the loadability margin of the

System	P.F. time	Continuation		PoC	
		λ_0 [%]	time	λ_0 [%]	time
14 AC buses 2 DC bipoles 21 AC elem. 5 Gen. 2 Areas	0.4s	11.567	7.9s	11.5683	3.6s
173 AC buses 255 AC elem. 29 Gen.	6.5s	2.7591	43.3s	2.76086	25.4s
173 AC buses 2 DC bipoles 255 AC elem. 29 Gen.	5.7s	2.0254	44.1s	2.0275	23.3s
133 AC buses 213 AC elem. 41 Gen.	4.3s	4.9983†	45.5s	4.99949	29.9s
133 AC buses 2 DC bipoles 213 AC elem. 41 Gen.	5.1s	5.0021†	52.1s	5.00339	34.9s
2158 AC buses 2 DC bipoles 3402 AC elem. 384 Gen. 62 V reg. trf. 8 PQ reg. trf. 23 Areas	10.3m	15.275†	5h 11.0m	15.2986	1h 31.2m
2158 AC buses 2 DC bipoles 3402 AC elem. 384 Gen.	2.8m	32.688†	53.4m	32.6799	38.4m
2158 AC buses 3402 AC elem. 384 Gen.	2.8m	32.686†	1h 5.5m	32.6786	29.5m

†Q limit instability

‡Convergence problems

Table 1: Continuation and PoC method results for SUN SPARC-IPC Unix workstation.

ac/dc system [19].

Table 1 shows the maximum loading factors λ_0 , and the time performance for different ac/dc and ac systems obtained in a SUN-SPARC IPC Unix workstation. The times shown include all program operations, including input and output, which in a dedicated Unix workstation correspond to real times. The ac only systems in this table were simulated by treating the HVDC links as constant active and reactive power injections. Notice that for the 2158 bus system, several types of regulating transformers, area interchange control, bus voltage and reactive limits are included, which significantly increases the computation time.

The PoC method consistently performed faster, by at least a factor of 2, than the continuation methods. However, continuation methods have the advantage of producing voltage profiles, and as a by-product yield unstable equilibrium points that can be useful for system stability analysis. Another advantage of the continuation methods is that they are able to detect immediate instabilities due to reactive power limits, although in practice these points and the bifurcations are very close. Finally, the step cutting technique used in the continuation methods to improve convergence has the disadvantage of yielding slightly wrong results, as shown by ‡ in table 1. This consistently occurred in systems with sharp turning points.

6 Conclusions

A detailed description of the implementation of the PoC and continuation methods is presented, including the prac-

tical considerations that allow to obtain consistent results in a diversity of ac and ac/dc systems. A brief description of the computational characteristics and requirements of these methods is also shown. Finally, some of the test cases and results are discussed, in particular those related to a real ac/dc 2158 bus network with a variety of operational limits and controls.

The PoC method yielded time performances that justify its use as a production type tool. This performance can be significantly improved by using specialized sparse matrix techniques, like block factorization as suggested in [3], to reduce computation times. On the other hand, the continuation methods, although slower, produce a series of additional information that can be used in the analysis of other stability issues. For these two reasons the authors believe that this two methods complement each other, particularly for systems where one of the methods might fail in producing the desired results.

Acknowledgements

The support of this work by the Electric Power Research Institute, project RP 2675-4 under the direction of Stig Nilsson, is gratefully acknowledged.

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pflow [-aAbBcCFGhHilJjKkLlMmNnPpQqRrStTvVwW] input_file output_file
Options: -a      Turns off tap and angle limits in regulating transformers.
        -A      Turns off interchange area control.
        -Bnum   PQ bus number 'num' where the voltage is fixed in order to
                find the loading factor (lambda) for voltage collapse studies.
                Must be used with -K and -v options.
        -cfile  Increases the loading factor lambda using a modification of
                Iba's homotopy continuation method for finding voltage profiles.
                The output (optional 'file') is a list of 8 random AC voltages
                plus 3 additional variables for each DC bus. Must be used with
                -K option.
        -Cfile  Point of Collapse studies, i.e., find the maximum loading factor
                lambda for a given generation and load direction. The base case
                loading can be initialized using the -L option; nevertheless,
                the program calculates an initial loading of the system before
                the PoC method is applied. The left eigenvector is written
                in 'file' (optional). Must be used with -K option.
        -ffile  Output EPRI binary information to file (Interface to
                SSSP and ETMSP programs).
        -Hfile  Increases the loading factor lambda using a parameterized
                homotopy continuation method for finding voltage profiles.
                The output (optional 'file') is a list of 8 random AC voltages
                plus 3 additional variables for each DC bus. Must be used with
                -K option.
        -I      Input data in IEEE common format.
        -kval   Factor 'val' used in the homotopy continuation method for
                finding the increments in the loading factor lambda (def. 1).
                Must be used with the -H option.
        -Kfile  Read generation and load distribution factors from 'file'.
                The data is all p.u. and must be separated by spaces, i.e.,
                BusNumber BusName DPG DPl DQl. If the input variables are
                unknown give them a value of zero. The generation factors
                are normalized for each area, i.e.,  $||Pg||=1$  in each area.
                Buses not in the list are assumed to have zero distribution
                factors. If BusName has spaces, wrap it in double or single
                quotes.
        -Jfile  Write the Jacobian of the solved case in I J VALUE format in
                'file.jac'. The equation mismatches and the system variables
                are also written in 'file.mis' and 'file.var', respectively.
                If no 'file' is given the program writes to standard output.
        -Lval   Loading factor 'val' (def. 0). Simulates load changes in
                conjunction with the load distribution factors (-K option).
        -n      Turns off all AC system limits.
        -N      Turns off all AC system controls.
        -p      Turns off P and Q limits in regulating transformers.
        -P      Turns off P and Q control by regulating transformers.
        -q      Turns off Q limits in PV buses.
        -Q      Turns off remote voltage generator control. The generators
                will just control their terminal voltage to its initial value.
        -r      Turns off V limits in regulating transformers and PV buses.
        -R      Turns off V control by regulating transformers.
        -vmag   Voltage magnitude 'mag' at the first PQ bus (unless otherwise
                specified by -B option) to find the corresponding loading factor
                for voltage collapse studies. Must be used with -K option.
        -Vfile  Read initial guesses for AC bus voltages from 'file'.
                The data must be separated by spaces, i.e.,
                BusNumber BusName V.mag V.ang(deg). If the input variables
                are unknown give them a value of zero. Buses not in the list
                are given a flat start. If BusName has spaces, wrap it in
                double or single quotes.
        -Wfile  Similar to -w option, but the solved case is written in IEEE
                TAPE common format.

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Fig. 4: Part of the PoC power flow help features.

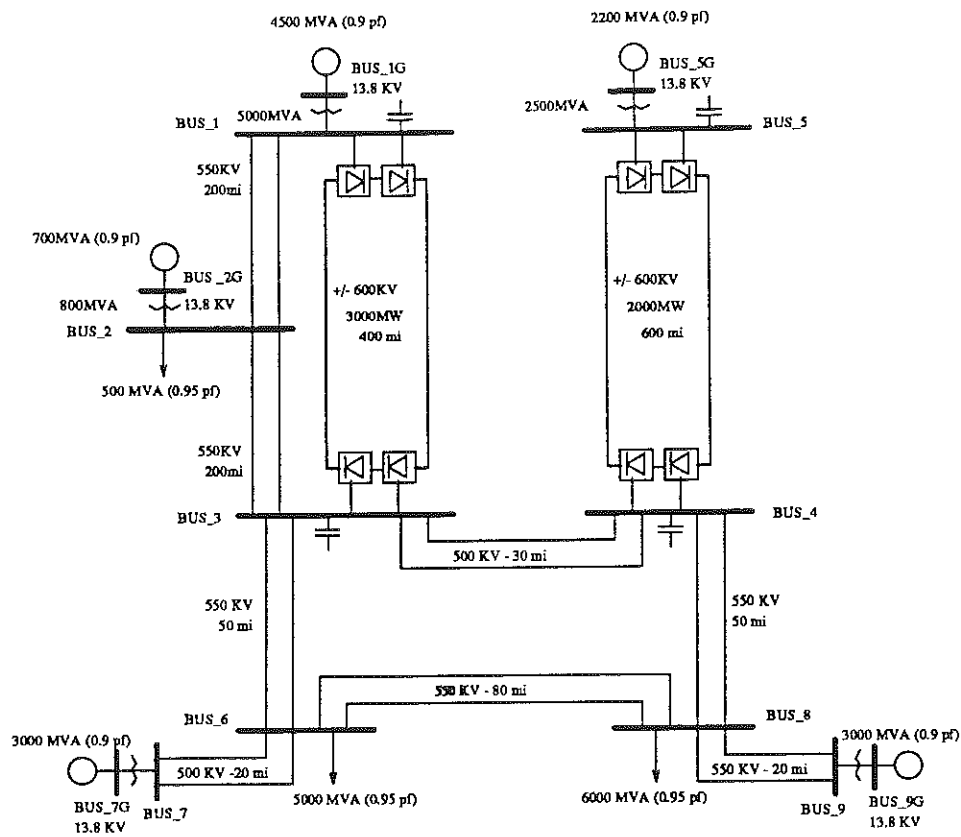


Fig. 5: 14 bus ac/dc system.

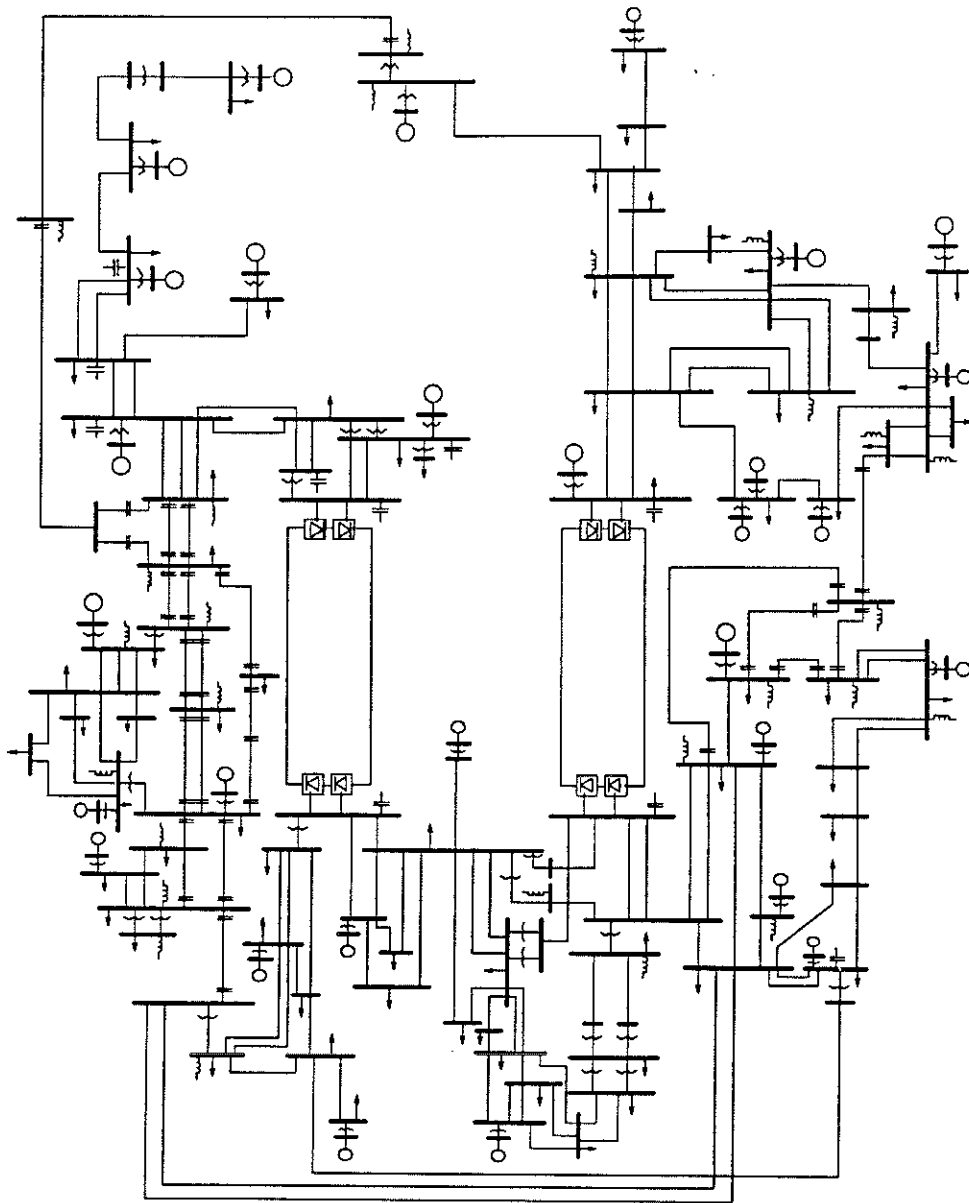


Fig. 6: 173 bus ac/dc system.

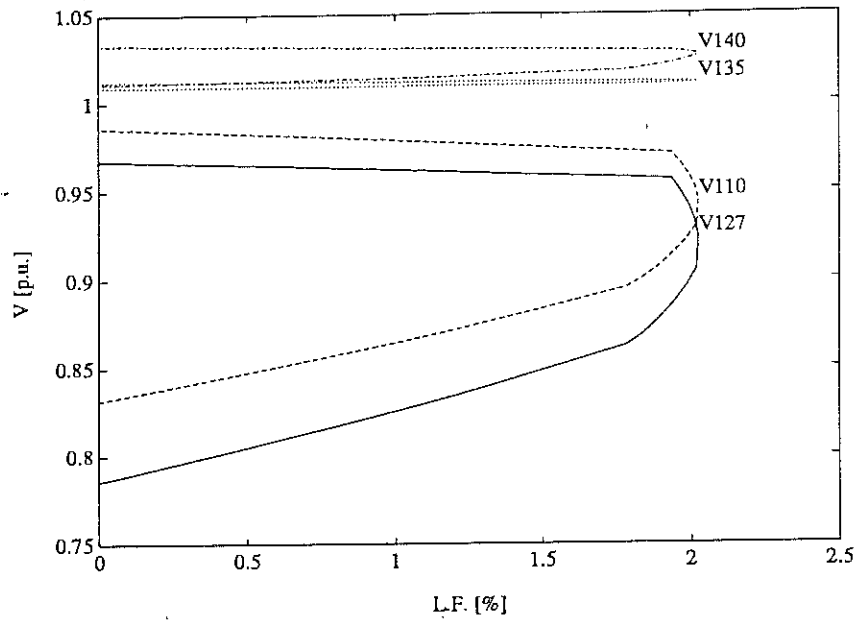


Fig. 7: Voltage Profiles for 173 bus ac/dc system. Sharp corners correspond to generator Q limits.

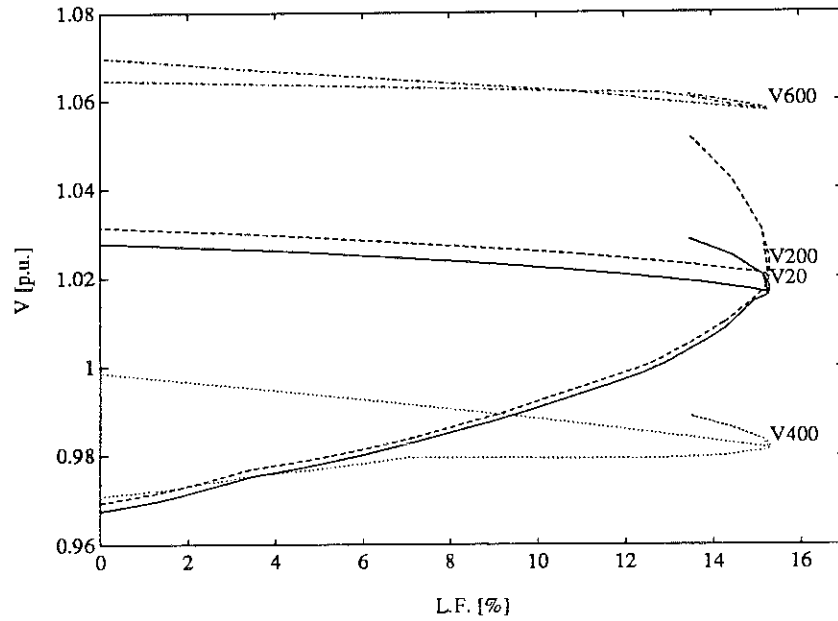


Fig. 8: Voltage Profiles for 2158 bus ac/dc system. Both Q limit immediate instability and PoC points are shown.

DISCUSSION - SESSION ONE

Edited by Robert A. Schlueter, Michigan State University

Nizam U. Ahmad (Oklahoma Gas & Electric Co.):

I am a typical transmission planning engineer. I don't have any special comments about any of the papers, but some general comments. What I learned from John Zaborsky's paper is that there is a one line diagram in there that I understood very well. I also understood that we are living in a state space sea which looks like an amoeba. From Ian Hiskens' paper, I felt like the amoeba turned into a half moon. Then Dr. Dobson told me that I am in a very difficult position, that I may fall off that place very instantly. What I am getting at is (the adequacy of) technology transfer between the academics and the people who are actually using the technology. There is a great difference in understanding of the technology. If you folks are not creating something that I can understand and translate into a two line justification, we'll never have the system built properly so that it will operate properly. Essentially, there is a gap. At every IEEE meeting that I have been to, I've seen the same gap between the users and the producers of technology. We tend to use a lot of Greek letters and things like that, which is fine, but do translate them into English so that an average transmission planner understands. I come from a region, the Southwest Power Pool, which is about four or five states in the U.S. There are forty utilities. Only three of the operators are engineers, so you can imagine how they are, and they have their hands on the controls. My concern is that if we do not translate the technology into plain English, and do it properly, we are going to be a vanishing breed.

Christopher L. DeMarco (University of Wisconsin):

My question goes back to the first two speakers, but then to the issue of modeling. Both of the first two papers made use of a model where we had differential equations and then some algebraic constraints, and usually those algebraic constraints came from power balance (equations). Now I'll admit that there may be others that have to do with generator modeling; those I don't think I will argue with, but I have some skepticism about the algebraic constraints, even for load balance, from an intuitive point of view. I plug my toaster in the outlet; I know voltage drops very suddenly, but it does not drop instantaneously. Somewhere there are some dynamics governing how that transition takes place, and if we put in algebraic constraints we ignore that. Most of the time, in normal operation, that's fine: it's

a good approximation. But I think there are ranges we get into where we have to worry about those. Dr. Zaborsky raised some skepticism about the use of singular perturbation techniques to relax that, and I agree that is a very crude approximation. But, even in regions where the quasistatic behavior seems good, I would caution on the use of algebraic constraints. I'll raise this question for the first two speakers. Take an overly simple example of a PV bus feeding a load (Figure 1). I arbitrarily am saying here

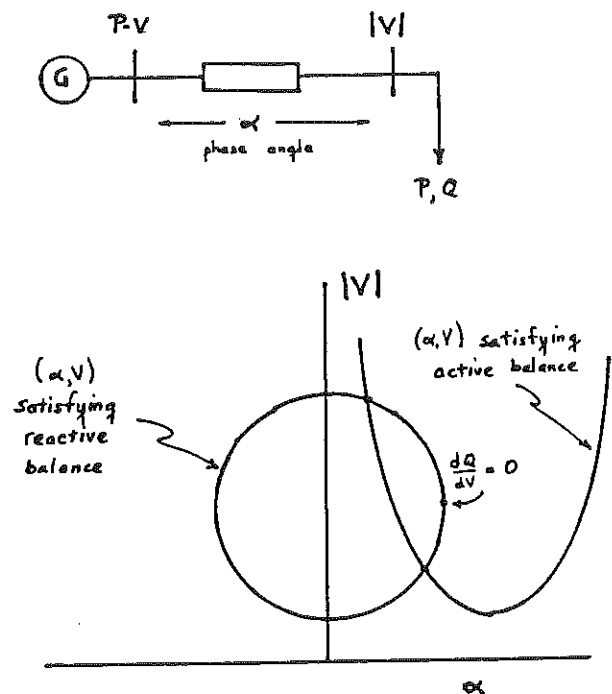


Figure 1

constant power plus constant impedance, a load, plus some frequency dependence; take your pick. But I'm going to draw a different kind of diagram that I like a little better: No parameters are changing in this. Right now the load parameters are constant, but we are back in the state space. The state space, hopefully, is something we understand, at least a portion of it is the phase angle across the line and the voltage magnitude at the load bus. What we get is a set of curves that tell us all the angles and voltages that satisfy the reactive power constraint, all the angles and voltages that satisfy the active power constraint. This is really just another coordinate system. You have operating points or equilibrium points. We know that we don't want to operate at the low voltage one, but I ask this question: If we have the algebraic

constraints, we are telling ourselves this is stable, at least mathematically. If we go back to Ian's (Hiskens) presentation, he had those two curves that formed the half moon (Hiskens' paper, Figure 2), where he said there were two regions of attraction and a transition between them. In Figure 1, we have to stay on the reactive constraint, we can move off the real constraint because we have generator dynamics. What Ian was describing would be a transition to push you along the reactive constraints, passing through just one point that's singular. Remember the algebraic equations for reactive power are constraints; where they become singular we pass through. If we believe the algebraic equations, that point is stable. I raise the question, do we really believe that? Do we believe that should be a stable operating point? I don't think I do in most situations.

Carson W. Taylor (Bonneville Power Administration):

The first session this morning had to do with our definition on how voltage stability phenomena occur and are organized. I found useful a time frame diagram (Figure 2),

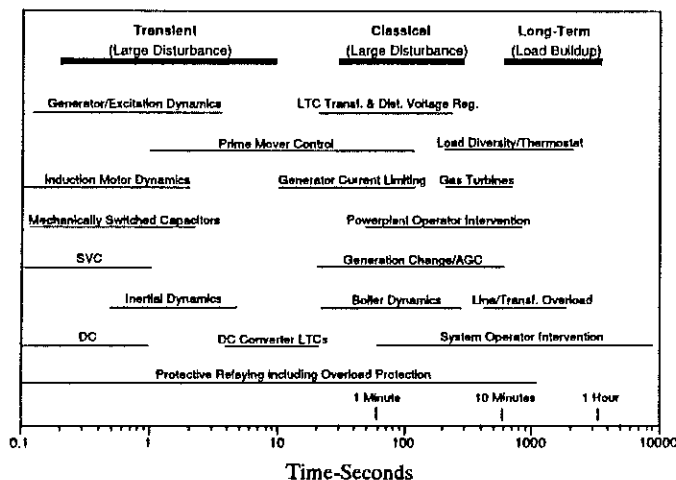


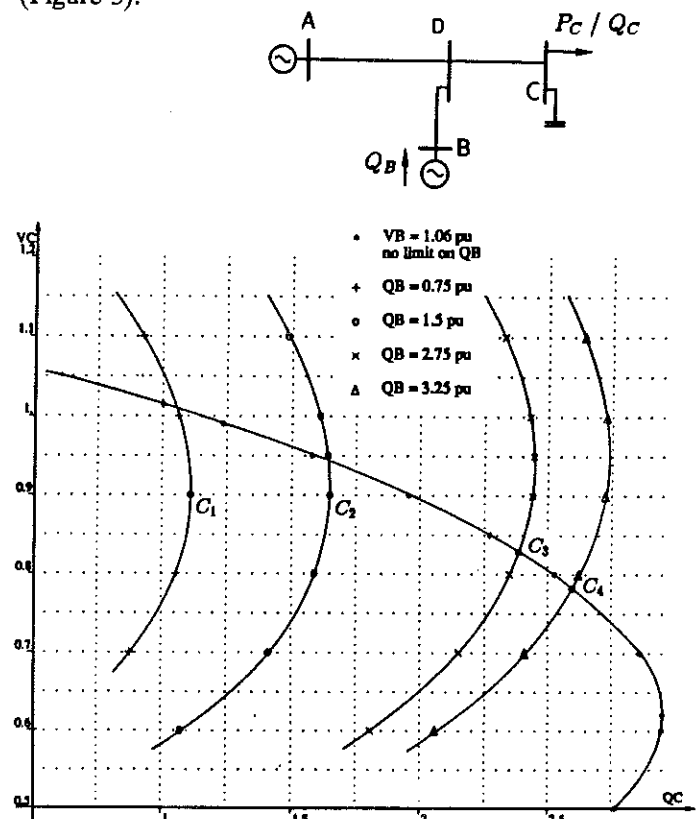
Figure 2

and three definitions as far as voltage stability time frames. First are the fast phenomena I call transient voltage stability, involving usually induction model dynamics or the reactive loading of dc lines (dc links). The middle one is what I call classical voltage stability, involving tap changing and generator current limiting. The final one, for lack of a better name, I call long term, usually involving a buildup like a morning pick up, or in case of the Tokyo blackout, an afternoon pickup (the other two involve large disturbances). Depending on the type of disturbance and the nature of the system, many other factors can be involved, everything from protective relaying to boiler dynamics, and then there are

interactions between these different phenomena. For instance, tap changing can regulate the load side voltage and prevent loss of load diversity on the constant energy type loads with thermostatic controls. Let's say that you have some of our classical voltage instability: the final phases will sort of move down into the transient time frame, or the final stages of the collapse will involve induction motor dynamics.

Thierry Van Cutsem (University of Liege):

Just a simple example to confirm what Dr. Dobson said. This can be done very easily using this four bus system (Figure 3).



Effect of generator limitations on the P-V and Q-V

Figure 3

What is seen here is that, for different levels of reactive power capability of the machine at bus b, we obtain different curves after the machine has gone into limitation. The collapse points are, in fact, C1, C2 which are collapse points with maximum load, and C3, C4 which are of the type you mentioned. You can obtain this curve with a very standard technique using a fictitious generator at bus C to obtain the lower part; so it is very easy and it doesn't involve very intricate concepts. One possible answer to the

question about when does it occur is that if you have a region in a power system with a relatively large amount of reactive reserve, normally you could reach higher load in this part of the system (C_3, C_4). When the system is in trouble in this part, then maybe you could get this kind of phenomena (C_1, C_2). Also, I do agree with you: when evaluating the load margin, of course, the collapse point is the one with QB limit and not the one without QB limit. For example, in the method which I used, based on the evaluation of the maximum load, first the algorithm will go to this point and then if you enforce the limit it will go to this point, and at this point you will have a rise in the generator voltage which is easily detected.

Adam Semlyen (University of Toronto):

I would like to discuss a topic which is related to almost all the papers. There was a general formulation of the voltage problem in the form $\dot{x} = f(x, y)$ and $g(x, y) = 0$, and a parameter which I do not write. There was a lot of discussion about the algebraic constraint when there is a singularity problem. The idea was that y could be eliminated. According to the inverse function theorem you'll consider these equations as being linearized

$$\Delta \dot{x} = A \Delta x + B \Delta y \quad \text{and} \\ 0 = C \Delta x + D \Delta y$$

According to this theorem, in order to eliminate Δy you must have D be nonsingular. If D would be singular, then it would have an eigenvalue μ going to zero. Assuming that it (μ) is not yet zero you could conceptually eliminate Δy . I make just a side remark that this is based on the Inverse Function Theorem, indicating that replacement can be done. This is a sufficient condition for replacement, but it is not necessary. You can conceptually replace Δy , even if this is not satisfied. So I do not consider it equal to zero. I do the replacement and I get $\Delta \dot{x} = [A - BD^{-1}C] \Delta x$. When small, then this is reflected in the inverse. You can take a spectral decomposition, and from all the eigenvalues that are present, only that eigenvalue which goes to zero becomes dominant in the end result. You will get here an eigenvalue λ of $[A - BD^{-1}C]$ and λ will vary as, but will not be equal to, $1/\mu$. When μ goes to zero, λ goes to infinity. When you get singularity in the algebraic equation it is reflected in the reduced dynamic equation by one eigenvalue going to infinity. I remember one paper (Rajagopalan, Lesieutre, Sauer, Pai: Dynamic Aspects of Voltage/Power Characteristics - 91SM419) which (in my opinion) puts exaggerated emphasis on the load flow Jacobian, and an other (of the Sauer, Pai papers) which comes almost 180° back and takes the problem as being

dynamic, and then the load flow Jacobian has a lesser role. In that paper, which takes this formulation $[A - BD^{-1}C]$, eigenvalues are presented and eigenvalues move like this (Figure 4). This is a complex plane and here is σ and ω .

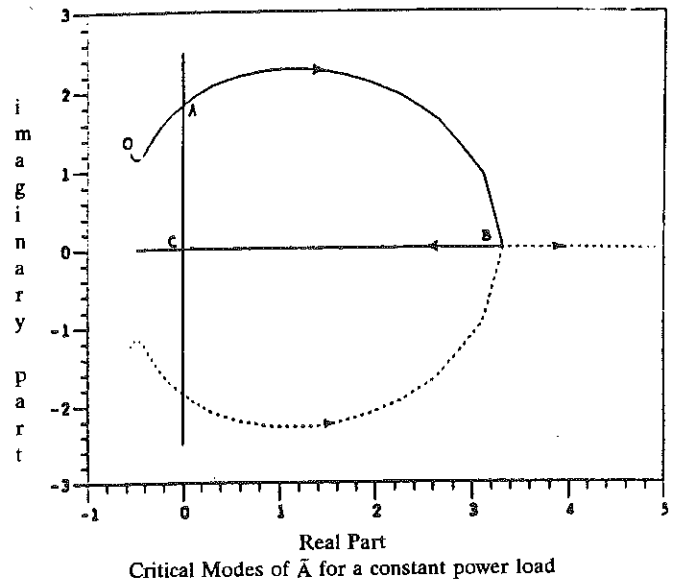


Figure 4

The eigenvalues are here (I do not know what they are originally) and move out here. So Hopf bifurcation occurs and then here they split and one eigenvalue goes to a plus infinity and then it comes back to a minus infinity. The second eigenvalue much later goes and crosses back to the negative part. My final point is that what happens in this condition is a bifurcation at infinity, and bifurcation at infinity happens when the algebraic system becomes singular. Bifurcation at infinity is responsible for the phenomenon that on a nose curve (Figure 5) you have two points which represent transitions between stable and unstable. One transition, bifurcation at infinity, is the one which reflects phenomena in the algebraic constraint.

Harry Kwatny (Drexel University):

To get to the issue of the validity of algebraic equations in these models and differential algebraic systems: I think it is always appropriate to challenge the models, and to go back and see whether or not the results we are getting make any sense with respect to the systems that we are dealing with. In this particular case (Figure 1), I am especially suspicious, just as John (Zaborszky) mentioned, of the differential algebraic model in the region of the point at which the Jacobian becomes singular. Outside of that region (except at that point), for exactly the reasons that John (Zaborszky) had mentioned, I think that the

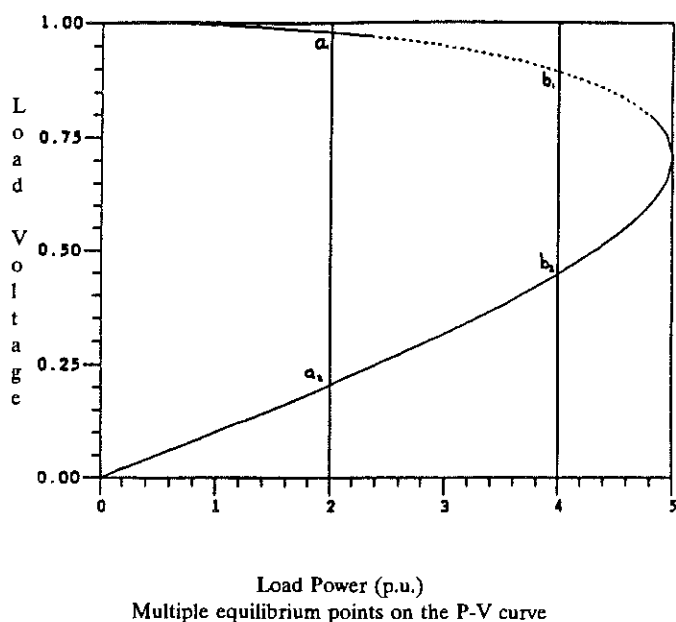


Figure 5

models are valid. Even though other operating constraints may preclude you from wanting to operate at this point, still I think the basic model is valid, although I think challenges should be made. But from a mathematical point of view this is an extremely important point, even though the dynamical equations may not be correct.

The way I like to think about this is that the constraint equations define a manifold on which the motion must take place. Then the equilibrium equations identify the equilibrium points. Now the fact is that we can have a situation of a one dimensional manifold (which actually might be the appropriate model for a system) which would be a cross section of a cylinder, but motion must take place on that cylindrical manifold. The fact that there are two equilibria that are both stable means that we have an extremely extraordinary situation, because if we have a stable equilibrium, where do the flows come from? Usually they originate at an unstable equilibrium and terminate at a stable equilibrium. Here we have two stable equilibria, so you've got to have something strange that is happening. If you examine the algebraic equations that define the manifold, you probably find that this is only one piece, that there are other disconnected branches and it is quite possible that something is happening at one point in which you can actually transition from one segment of the manifold to another. I don't know. There are lots of questions you could ask about what is happening, but the very fact that we have two stable rather than one stable equilibrium emphasizes the significance of this point. As we change certain parameters we might envision situations

in which the manifold stays the same, but the equilibrium curve changes. I would claim that just from geometric arguments, when that happens, one equilibrium point will change its stability characteristics and wind up elsewhere, so there are lots of interesting things (about) what is happening right in here. I agree we should ask questions about the model in order to get a finer picture of what's going on in this region precisely, but this point does organize and make sense of the mathematical structure of the overall problem.

Robert A. Schlueter (Michigan State University):

There are two points that I would like to make (concerning the model mentioned by Adam Semlyen in his discussion). One is that as the eigenvalue μ of D approaches zero there is a case where the matrices B and C , in a sense, can prevent any change in eigenvalue λ , of $A-BD^{-1}C$ (Figure 4). If the row dependence is strictly in the algebraic equations and matrix $[C D]$, there is no coupling into the dynamics when the D matrix is going singular. There is no effect on eigenvalue λ of $A-BD^{-1}C$. We have computational results where we have been able to show that. If fact, it would be my view that, in many cases, this is the kind of collapse you have.

Now let me talk about the collapse a moment. You understand very well what an active power transfer limit is in the algebraic or power balance equations. If you have a PV model at every bus and you increase transfers, the physical limit across a boundary is an active power transfer limit. That's one of the ways you can get dynamics to have instability problems, but you can also not have any dynamic instability problems but just have an active power transfer limit. Now there is another kind of problem in the algebraic equation which is caused by reactive losses. What inability to supply reactive power to a bus or to a cluster of buses. Physically you have eaten up all your reactive supply. Your voltages have gone down to the point where shunt capacitive, line charging, and I^2X losses just choke off your supply. That is physically what is happening, in almost all cases, whether the dynamics or the algebraic equations experience bifurcation. Now there is a third situation (similar to the second situation I talked about) where algebraic equations experience bifurcation. In the second situation, you have voltage change but have angles not having experiencing any change whatsoever. The third situation, where algebraic equations bifurcate, you have voltage and angles having significant change together. This is the generic algebraic bifurcation that results in voltage collapse. What is happening is your active power transfer limit and your reactive transfer limit

are decreasing simultaneously. At the point where that bifurcation occurs you have got both problems happening simultaneously. Now you still need to look at the models of the dynamic and algebraic equations at that point to understand what is going on with dynamics. If you have coupled the algebraic bifurcation problem into the dynamics, then the bifurcation may occur in the dynamics. Most utility people are suspicious that a small percentage of the problems that we identify as collapse are going to be associated with dynamics. They acknowledge that there are some and there is work going on at this point to see how many.

Peter W. Sauer (University of Illinois):

I would like to comment on Chris's (DeMarco) slide and also Harry's (Kwatny) comment, because it relates to a thing that made me very nervous about a week ago, while presenting a paper that talked about the stability of these equilibrium points. We call these multiple equilibrium points, and it's fairly easy to show that for reasonably correct models they are both dynamically stable. Hsiao Dong Chiang came up to me at San Diego and said, "How can that be? How can these stable equilibrium points coalesce and then disappear?" He accused me of violating some sacred law, and I wasn't sure and I had to think about it for a minute. Then it occurred to me that there is a problem, and that is, if we look at the parameters which Adam Semlyen didn't write, but which are very important:

$$\begin{aligned}\dot{x} &= f(x, y, u) \\ 0 &= g(x, y, p)\end{aligned}$$

where u is the control reference set, there are u 's that we keep constant in the dynamic simulation, but then we also have parameters p , and those might be powers that are not constant; they are merely quantities that are specified for load flow. It is for constant p , in the $g(x, y, p)$ equation, that gives us two roots, but from that p if you take the solution on the upper portion of the P-V curve and work backwards into the dynamic model, you get one control set point u . If you take the solution on the lower portion and go backwards into the dynamic model you get a different view. So these are not two equilibrium points in the sense of $x = f(x, u)$. If u is fixed you might have multiple equilibrium points for $f(x, u) = 0$. My point is that I'm not sure that, when we say these are multiple equilibrium points we are saying that they are multiple equilibrium points in the sense of constant control inputs. There is a big difference, because it is for constant u that you look at multiple solutions. These are usually the equilibrium points that Harry (Kwatny) is talking about, I think, and you do

get this type of a thing in a classical model; however, for detailed models normally the relationship between the things that we specify in load flow are different from the things that we keep constant in the dynamic model, and that makes all the difference in the world. Then there is nothing wrong with having these two be stable, because these are not multiple equilibrium points in a dynamic sense, they are only multiple load flow solutions. They have different controls so they don't both satisfy this equation. It's a little bit of trickery on some people's parts to call those multiple stable equilibrium points.

John (Zaborszky), why don't you come up here because I wanted to ask you a question. In your analysis, when you talk about the multiple x low and x high, do the x low and x high correspond to constant control inputs?

John Zaborszky (Washington University):

Yes. That quantity u you refer to in our notion should generally be secondary control and the primary controls are part of the dynamics.

Marija Ilic (Massachusetts Institute of Technology):

My question refers further to what Pete Sauer was talking about; it's related to Ian Dobson's presentation also. As he was presenting his slides he always had the lower part of the PV curve in red, which means it was a no-no region of operation. I think it is related also to the multiple stable equilibrium points that Pete (Sauer) was talking about. My question may be an engineering related question also. Can the system operate on the lower part of the PV curve? Have we cleared this?

Ian A. Hiskens (Queensland Electricity Comm.):

In terms of the multiple stable equilibrium points, it can operate at a stable equilibrium point on the bottom part. That was demonstrated accidentally by a bicycle dynamo feeding an electronically controlled constant power load, and it kept going to the wrong equilibrium point, and the low voltage wasn't really what they were after.

Marija Ilic:

I think if we look through different references in the power system literature, there is no one example from the operating side which spells out the scenario that the system

is operating in that region. I think that it may be one of the fundamental questions.

U. George Knight (Imperial College, London):

Just for what it is worth I once heard Prof. Mortloff of Mortloff and Davis fame say that back in the 1950's he heard of a power system in Canada that settled down to operate at 80% volts quite normally, from some other higher voltage. It might just be heresay.

Adam Semlyen (University of Toronto):

We have reproduced in the lab a power system of five generators operating in a circle in two modes, stably.

Carson W. Taylor:

I'm convinced that we can operate on the lower part of the curve. I've done it in simulations, and I've seen it in other cases. There was an incident in France in 1987 when they operated at a low voltage that I suspect was on the lower part of the P-V curve.

Colin Christy (Commonwealth Edison Co.):

I work in system planning, and the one thing that I want to make sure that you don't do is have me go back and tell the operator that when he is running into problems, he needs to increase his load in order to improve his voltage; I don't think that is our conclusion. One of the things that we have run into, in this whole question, is that it is important to know what your actual independent parameter is. Our actual independent parameter is not load as such or demand as such, instead it might be the amount of equipment that you connect. If I look at a curve of voltage versus the amount of equipment I connect, it could be that we have two stable equilibrium points. If I look at power, it could be that I have two stable equilibrium points for a given amount of demand. However, that demand is not derived necessarily from the same amount of equipment being connected to the system. What we found in our work, and I will present it tomorrow night, is that instead that same amount of demand would be derived from a different amount of equipment connected, so that as we hit a peak demand, we could continue to increase the amount of equipment that is connected. The decline of voltage might have a dominant affect on the system, so that as we connect more toasters to the system, or whatever, voltage dominates and our actual demand would decrease, so that

the lower equilibrium point might, in fact, have the same amount of demand as such, but we have a different amount, a greater amount of equipment connected. Actually, in a physical sense on the system, there is a peak amount of equipment that you can connect. So what? The actual independent parameter is equipment connected and not demand. That way I can go back home and tell the operator to disconnect equipment, not to increase demand.

Lester H. Fink (ECC, Inc.):

Colin are you making the same point that Pete did?

Colin Christy:

We do formulate the problem using a continuation algorithm with the parameter lambda, which is the amount of equipment connected. When we trace the solution manifold, what we find is a maximum in that parameter; we reach the maximum and then decline. So that would say that physically there is a maximum.

Lester H. Fink:

No one has reacted yet to Mr. Ahmad's comment and I think it deserves a comment; any of you academics may feel free to disagree with me completely. When I was a student, and we were struggling with our first exposure to physics, there was a cartoon that appeared in the Philadelphia paper. I think it was a character named Ching Chow who showed up with a pithy saying every day. His comment one day (at a particularly appropriate time) was "simplicity is for him who understands". It has stuck with me ever since. I think part of the problem that you are putting your finger on is that exactly what we are struggling with at a seminar like this *is* to try to understand some of these phenomena which are just coming into prominence, to the point where we can start to make them simply explainable. We haven't gotten there yet. The important part of having a mix of utility engineers and academic engineers is that the utility people, hopefully, can keep the academic people from spiralling off on an unstable orbit into the blue sky yonder. But still we are trying together to understand these phenomena that are happening on the system. It is a little much to ask for a simple explanation until we have thoroughly arrived at the understanding we are looking for.

Maria Ilic:

I was fairly triggered by the comment also. These are not simple concepts; you know we cannot transfer them in fifteen minutes. We all have to work on it. Dr. Zaborszky was talking about a much more global picture of what we were struggling for twenty years with, transient stability and frequency angle, (and) that technology transfer has not been made fully yet. So I think it would be very, very unrealistic to expect to absorb this concept in no time. The bottom line is that we really don't have a mechanism to do that in the utility academic relationship in the states. I was very, very impressed by what I saw in Konsite Power, Korean Electric Power, and Taiwan Power. These people put 1% of the business they do into R&D. So we all should think very seriously about that if you want to address the issue. Otherwise we'll take our two separate paths. I was very glad that the question was asked. Thank you.

Lester H. Fink:

I also think that it is worth pointing out that these studies and this research are driven by what has happened and has been happening on the system. As I have pointed out at other times, in the earlier years of the industry when they were just struggling with first operating machines in parallel, then operating stations in parallel, the dominant problem was one of stability: how could these things operate stably in parallel; all the academic interest went into studying angle stability. Now we are in this new world where these systems are operating against their capability limits. The phenomena that are occurring are different in kind, and we have a whole new class of problems we are trying to struggle to understand, but this interest is driven by actually what is happening out on your system.

Nizam U. Ahmad:

I'm so happy to be here and I'm also so sad to be here, because there are only 70 people from basically the whole world looking at this in the academic world. We should have been 700 today, at least one from every utility should have been here to soak in all this. That's why I'm sad. I didn't mean to be disrespectful or facetious. I sounded like that I know, because of the fact that I'm looking for something that I do not understand. And I do want to understand.

John Zaborszky:

Well, I have to confess something to the gentleman talking

there. I may look like an abstract academic. Some time ago, however, I was a very real system operator, operating a very real system; the man who in electrical emergency situations said, "do this," "do that," "do the following," "now check that," and so on. I was directly operating the system and I was successful. They still tell me that I was the only one whose name was not connected with any major goofup. Of course, that is only because I didn't stay with it very long. The point is that I am very much motivated with the matters of the real system, and I think I still understand them. In working on this problem, for instance, our aim may sound abstract, but it is to develop a basic central theorem which can be used to solve those specific problems which arise in practice on a power system, and provide the basis for it. We are trying to present the results in such a way that they are accessible to people who want to use them without actually getting deeply into the mathematics. The other thing is that, of course, the work is general in nature. It is not tied to any particular model. It includes a component of the algebraic constraint, but we are not claiming that there is always an algebraic constraint. However we do claim that if there are, then of course there are some real physical events connected with these kinds of singularity which result from that.

Vaithianathan Venkatasubramanian (Washington Univ.):

Can I just get that slide by Prof. Semlyen (Figure 4). There are two questions here. Suppose you consider the case when you are at the equilibrium and the static equilibrium somehow reaches the singularity, then as Prof. Adam (Semlyen) said, "you can indeed see that some eigenvalues of the system are going towards infinity." But like Prof. Schlueter pointed out, it is not always the case that one eigenvalue is going to infinity. In fact, you can construct examples where none go or even two diverge to infinity. So you need to know exactly when you have the case when one eigenvalue goes to infinity. In fact, we have presented all the conditions and we have it as a theorem, what we call the singularity bifurcation theorem. We also show that when an equilibrium is at the singularity, that is indeed the most generic situation. That is typically what happens. Typically, you have one eigenvalue which will cross from C^+ to C^- , or from stable to unstable, or unstable to stable, by diverging to infinity. This is the most generic situation and that is what happens most often in practice.

The other point is with respect to Prof. Sauer's question. Suppose you have a power equation, and you have two stable equilibrium points. When you consider the case near the saddle node bifurcation, one of them has to be

unstable; that is very clear. Then what's happening in between? In between we claim there is a singularity induced bifurcation. This is a rough sketch of what's happening. This is something (like) what Prof. Chris (DeMarco) had drawn (Figure 1). When the two equilibria actually merge, one of them has to cross the singularity and when they cross, stability is lost. So one eigenvalue is going to diverge to infinity and it becomes unstable. This is precisely what is happening, and we describe it in detail in the other paper which we had in the morning. We have two eigenvalues, two stable equilibrium points, that is true, but one of them becomes unstable in another bifurcation which is a singularity induced bifurcation. It is not true that the two correspond to two different control values. Thank you.

Ian A. Hiskens:

I am just going to respond to a few of the comments. The first comment was Chris' (DeMarco) and I think that has been fairly well covered, but I think that, in general, we can believe the constraints in most of the places at around these points. As John (Zaborszky) has said, there is some difficulty in knowing what goes on there. We were never interested in the actual mechanism around those points, just that once it got there, there was an opportunity for it to jump from one region to another. Once it had made that jump, then depending where it ended up, it could settle to two different points (Figure 2 of Hiskens' paper). I also agree with the analysis where you have two stable points. What we have found is exactly the same phenomena, that somewhere in there you get the singularity of what we call the JLL, so that at that point you get a swapping of an eigenvalue from being positive to negative or negative to positive. In terms of Prof. Kwatny's comments about changing parameters causing a change in stability, that can be quite well demonstrated using these curves (Figure 5 of Hiskens' paper). If you think in terms of a load level, consider the case where the load is twenty, then you have the traditional case of a stable and an unstable point. However, it's an interesting phenomena, and I think it is related to the comment made by Bob Schlueter, that as you increase the load this actually shrinks. You can see that it is shrinking, and somewhere there is going to be a situation where the constraint manifold is shrunk to a point. At that point, at the top of each of these, is the usual angle instability bifurcation, so it is a singularity bifurcation. So when it shrinks to a point, you end up with both of them occurring at the same place. In my paper, I have shown that describes the traditional voltage collapse type mechanism where you get angle and voltage dynamics doing strange things.

Kwang Y. Lee (Penn State University):

Much of the talk has been on the voltage collapse based upon static or quasi-static analysis of the power system. However, the voltage collapse phenomena are dynamic and dynamic models need to be included in the analysis.

Since the voltage collapse is often associated with bulk power transfer, between two weakly connected areas, the receiving-end may be modeled as an equivalent synchronous machine rather than the induction machine model often used. By using a third-order synchronous machine model we were able to explain the voltage collapse mechanism dynamically. Another point that most papers have missed is the dynamics of relative angle between the two areas. For example, if we look at the power transfer expression

$$P = (E_q^1 V / X_d^1) \sin z$$

The voltage is a function of both the internal voltage E_q^1 and the angle z . Our analysis shows that both E_q^1 and z changes drastically near the voltage collapse point.

The key to identify the voltage collapse in our model is a stability index $G(E_q^1, Z)$. The voltage collapse is inevitable when the index G is greater than unity.

Details of this discussion can be found in B.H. Lee and K.Y. Lee, "A Study on Voltage Collapse Mechanism in Electric Power Systems," IEEE Transactions on Power Systems, Vol. 6, no. 3, pp. 966-994, August 1991.

SPECIAL PRESENTATION

Dynamic Simulation of Voltage Phenomena

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DYNAMIC SIMULATION OF VOLTAGE PHENOMENA

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ABSTRACT

Most of the time the voltage collapse phenomena are studied by means of computer programs designed for the calculation of steady state conditions. But in the real world, the simultaneous occurrences of losses of synchronism, of AVR dynamics or of transformer tap changes call for a full dynamic simulation of voltage phenomena. The present paper shows four examples of dynamic simulations of voltage collapses using a new general purpose stability program (EUROSTAG) covering in a continuous way the classical fields of transient, mid-term and long-term stability, and also the quasi steady state conditions of a power system.

INTRODUCTION

Voltage instability appears when the "electrical distance" between voltage regulating devices and loads increases. High flows of active and reactive power from remote sources induce high reactive losses and low voltages. A voltage collapse takes place if the imbalance between reactive supply and demand cannot be corrected by a limited voltage decrease.

Primary phenomena which can induce voltage collapse can be for instance :

- the loss of a generating unit (loss of active and/or reactive power);
- the loss of a line or a transformer;
- a load pick-up with remote (reactive) sources...;

Factors which can detrimentally affect the voltage stability under high load conditions are :

- transformers equipped with automatic under load tap-changers (ULTC);
- protective devices, particularly on generating units like rotor current limiters which limit or reduce the reactive capability of sources, or like under-voltage relays ...

Voltage instabilities under light load conditions are influenced by similar factors.

Transient or dynamic instabilities can appear now and again during the voltage slower transient phenomena.

Four examples constitute the base of this paper. They are prepared using EUROSTAG program (see description in appendix).

The first example considers the case of an "infinite" bus feeding a reactive load through a purely reactive line. With this simple example, it is possible to show the basics of voltage instability.

The second example, on a very simple but more realistic system, shows the influence during a load pick-up, of the gradual limitation of reactive generation by rotor current limiter action.

The third example introduces the detrimental influence of automatic ULTC which masks the load voltage sensitivity. It also shows that transient phenomena can take place during a quasi steady state phase.

The fourth example illustrates the behavior of a heavily loaded complex power system during the transient and the mid-term periods following the loss of a generating unit.

FIRST EXAMPLE

A purely reactive load is fed through a purely reactive line by an infinite bus. The voltage of this bus is V_0 (p.u.), the reactance of the line is X_1 (p.u.). The load is modeled by :

$$Q = Q_0 |V|^a$$

where

- Q is the actual reactive power (p.u.);
- Q_0 is the reactive power (p.u.) at $V=1$ p.u.;
- $|V|$ is the modulus of the voltage at the load feeding point;
- a is the voltage sensitivity exponent.

Three cases are considered, testz, testi and testp, which correspond to a equal 0, 1 or 2. The load is respectively voltage independent, a current sink or an admittance.

The equivalent reactance of the load X_{ch} is :

$$X_{ch} = \frac{|V|^2}{Q} = \frac{|V|^{2-a}}{Q_0}$$

So for $a=2$ (testz)

$$X_{ch} = \frac{1}{Q_0} \text{ and } V = \frac{V_0}{1 + X_1 Q_0}$$

when Q_0 is growing to infinite, the voltage at the load terminals falls gradually to zero.

For $a=1$ (testi)

$$X_{ch} = \frac{V}{Q_0} \text{ and } V = \frac{V_0 V}{V + X_1 Q_0} \text{ or } V = V_0 - X_1 Q_0$$

Q_0 cannot exceed the value which corresponds to $V=0$, $Q_0 = \frac{V_0}{X_1}$. It equals the p.u. value of the short circuit current.

For $a=0$ (testp)

$$X_{ch} = \frac{V_0^2}{Q_0} \text{ and } V = \frac{V_0 \pm \sqrt{V_0^2 - 4X_1Q_0}}{2}$$

Q_0 cannot be greater than $\frac{V_0^2}{4X_1}$, which corresponds to the maximum transmissible power on the line X_1 , for the voltage V_0 .

In all cases, this maximum is of course the same and corresponds to the well known condition $X_{ch} = X_1$. In the simulation, Q_0 is linearly growing with time. The load behavior will be related to the exponent a . For $a = 0$ or 1 the computation will stop when Q_0 reaches respectively $\frac{V_0^2}{4X_1}$ and $\frac{V_0}{X_1}$.

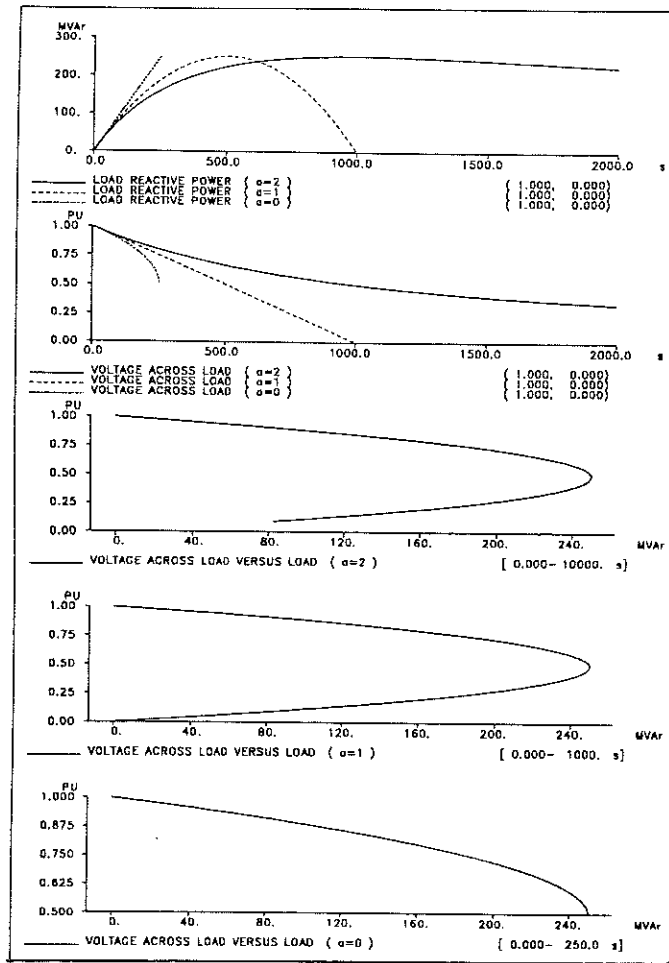


Figure 1

Numerical values :

$X_1 = 0.1$ p.u. 100 MVA base

$V_0 = 1$ p.u.

For $a=0$, the maximum values for Q_0 is :

$$\frac{V_0^2}{4X_1} = \frac{1}{4 \times 0.1} = 2.5 \text{ p.u.} \quad (250 \text{ MVar})$$

For $a=1$, the maximum value for Q_0 is :

$$\frac{V_0}{X_1} = \frac{1}{0.1} = 10 \text{ p.u.} \quad (1000 \text{ MVar})$$

The reactive load for all the cases is simulated by a true varying admittance computed as :

$$y(t) = Q_0(t) / |V(t)|^{2-a}$$

Where $Q_0(t)$ is linearly growing with time at a rate of 0.01 p.u. $\cdot$ s $^{-1}$.

With such a rate the simulation will stop at 250 s for $a=0$ and at 1000 s for $a=1$. The test ($a = 2$) is limited to 10 000 s.

Figure 1 shows the behavior of bus voltage and the load:

- as a function of time, for $a = 0, 1$ or 2
 - the load in MVar
 - the voltage at the load terminals
- the curve V versus Q or a part of it drawn respectively for $a = 2, 1$ and 0 .

On the last curve, the voltage drops to 0.5 p.u., value corresponding to maximum transfer capability of 250 MVar. This voltage versus reactive power curve is a feature of source (V_0, X_1) . With $a = 0$ it is only possible to get the upper part of it and with $a = 2$ a greater rate of change would be necessary to get values near $V = 0$.

SECOND EXAMPLE

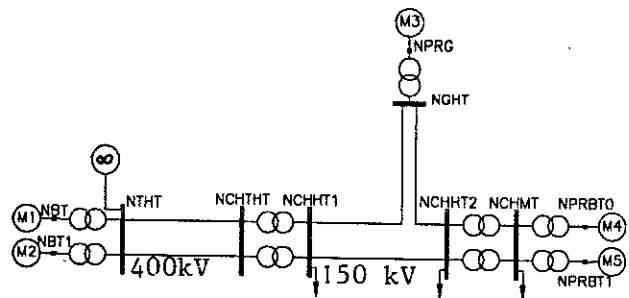


Figure 2.1

A three voltage level system is considered : (see figure 2.1)

- "THT" level at 400 kV;
- "HT" level at 150 kV;
- "MT" level at 70 kV.

On the 100 MVA base the lines NTH - NCHT have impedance of $0.006 + j 0.060$ p.u. . The bus NTH is connected to an "infinite" bus through a same impedance.

The short circuit impedances of transformers are between 10 and 15 %.

NGHT is weakly connected to NCHHT1.

Generating units :

- M1 and M2 produce each 250 MW at 1 p.u. (20 kV);
- M3 produces 100 MW at 1 p.u. (15 kV);
- M4 produces 100 MW at 1 p.u. (15 kV);
- M5 produces 50 MW at 1 p.u. (15 kV).

The system imports 300 MW from NTHT bus. In the dynamic simulation, the loads are modeled as constant impedances, except a load simulating a load pick-up and connected to the bus NCHMT. This load is voltage independent, has a ratio of active to reactive power equal to 1 and grows linearly with time.

The five generating units and the "infinite" bus keep voltage under control. Moreover the infinite bus holds the frequency at its nominal value.

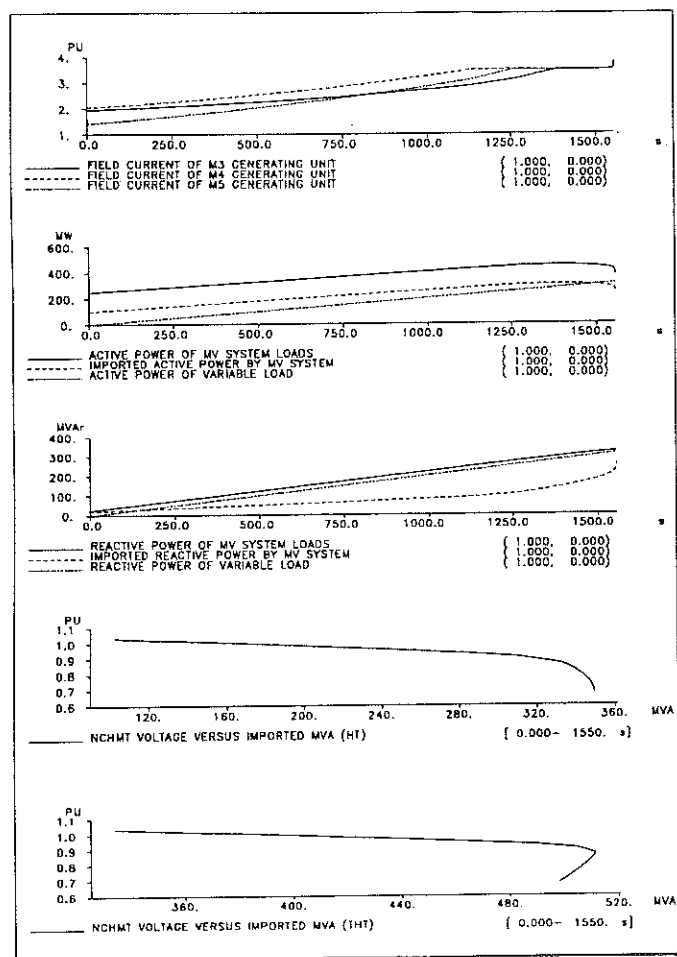


Figure 2.2

Generators are equipped with a voltage controller of PI type with active power feedback used as a stabilizing signal.

Excitation limits are roughly simulated by an Efd limiter. In quasi steady-state the synchronous machines are limited to their capability curves. In case of transients on the power system, the excitation can work at ceiling for a limited period of time (threshold overshoot behind a reset filter).

Generators M3, M4 and M5 have the same characteristics. M4 and M5, operating in parallel on high voltage side, are not equally loaded so their reactive reserves are different. The variable load grows at a rate of $(0.2 + j0.2)$ MVA.s⁻¹. After about 25 minutes, a voltage collapse occurs.

Figure 2.2 shows clearly key instants of this progressive loss of voltage control. These moments are directly related to the limitation of field currents on machines M4 at t=1140 s, M5 at t=1260 s and finally M3 at t=1380 s.

The imported active power in the MT sub-system does not grow as fast as the variable load because the initial part of the load is sensitive to the square of the voltage. When MT generators stop controlling their terminal voltage due to rotor current limitation, the voltage of NCHMT decreases more rapidly and the imported active power increases more and more slowly. The behavior of the imported reactive power is completely different due to a sharp increase of reactive losses when the voltage is falling down. The last two curves of this figure 2.2 show the voltage of NCHMT bus versus the imported apparent power on HV side of HT/MT transformers and versus the imported apparent power on HV side of THT/HT transformers.

The first one has the well known shape, not the second one.

Even after generators M4 and M5 have reached their excitation limits, the slope of the voltage versus power characteristic remains limited. When generator M3 reaches its excitation limit, the slope of the characteristic becomes more and more negative until collapse. On the second characteristic (Voltage NCHMT versus imported apparent power of THT system), the positive slope branch corresponds to the operation of machine M3 under rotor current limiter. This positive slope section remains stable thanks to the presence of loads with impedance characteristic.

THIRD EXAMPLE

The third example shows transient phenomena merged with mid-term voltage dynamics.

The simplified system of figure 3.1 looks like a part of the 400 kV and 150 kV Belgian Systems as it was ten years ago and after tripping of a tie line. The external system is simulated by means of two infinite busses. Most loads are fed by 400/150 kV autotransformers and simulated as passive loads behind subtransmission transformers fitted with automatic ULTC.

Unit M1 has a very low excitation limit of 1.5 p.u. (no load nominal voltage as base).

- Under-voltage protection on generating units : tripping of the unit if the terminal voltage remains under a set value during 3 seconds;
- loss of synchronism protection : tripping of the unit after 1 or several pole slippings;
- over-current protection on lines;
- Tap-changer automatons regulating the low voltage side of HV/MV transformers. The first tap change after 30 s, the followings after 16 s (1 step = 1% Vnom, ± 12 steps).

The simulation scenario is initiated by the tripping of unit M2.

This limiter is ill tuned. As a consequence it drastically reduces the excitation, provoking the loss of synchronism of the unit M1 (around 0 MVAR at generator terminals) (at time 120 s approx.).

Finally, the rotor protection of M6 enters into action and causes the loss of synchronism of the unit. The voltage in 150 kV system drops to nearly zero.

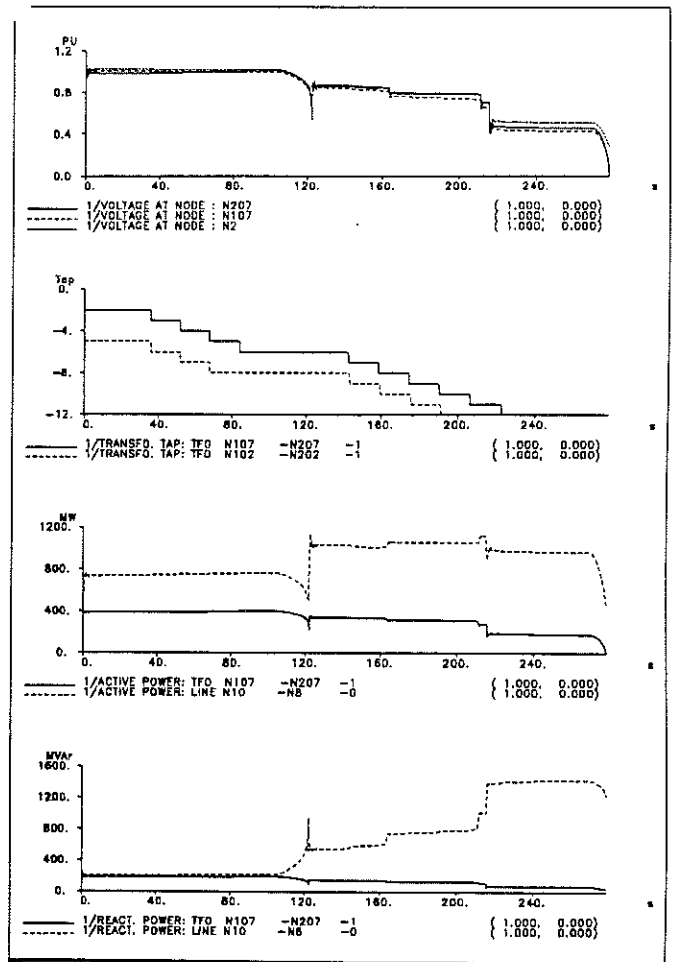


Figure 3.2 shows voltages, transformer tap position, imported active power on a transformer and interconnection line, imported reactive power on the same transformer and the line.

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after disconnection of M1 machine. The last curve shows the active power of M4 generating unit during the same period.

On figure 3.2, one can see the voltage nearly stabilized around 100 s, after some tap-changer movements. Afterwards, the field current limitation of M1 machine which induces its loss of synchronism and its disconnection from the system, drives the network towards collapse.

The introduction at Station N4, for example, of an SVC is adequate to avoid the voltage collapse, M1 generating unit remaining under its rotor current limit.

The study included an assessment of significant incidents that occurred recently in various parts of the world, with a view to identifying the phenomena which would be likely to cause a collapse of the French system.

This resulted in a sample of incident scenarios to be modeled on the French power system. These cases had in common the capacity of being able to "evade" the loss of synchronism protection system.

The modeling of the French power system involved more than 1 100 network nodes, about 2 000 lines and transformers, and some 100 generating units. Various automats and protective relays were introduced : unit islanding relays, line overload protection, rotor protection, ... The modelization of voltage controls took into account the additional angle limiter etc. The load representation comprised the ULTC of HV/MV transformers (synthetic load model with continuously acting equivalent ideal transformer). The total number of state variables was in excess of 6 000.

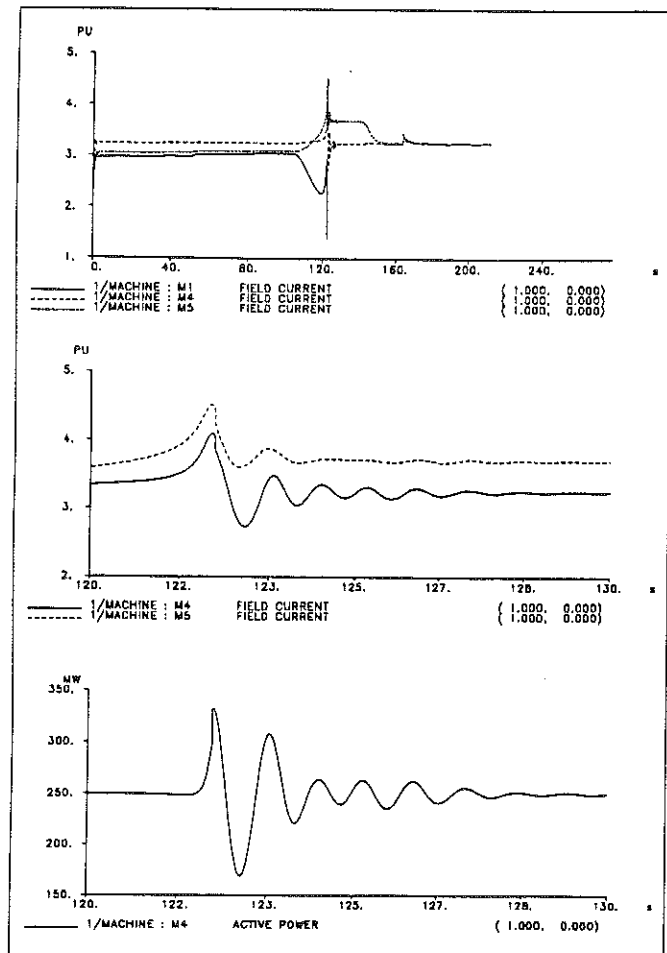


Figure 3.3

FOURTH EXAMPLE

In June 1989, Tractebel started on behalf of Electricité de France (EDF) a study aimed at developing a protection scheme of the system which would reduce the seriousness of the incidents likely to affect the French power system. This study was conducted within the framework of upgrading the EDF's Defence Plan at the 2 000 time-horizon.

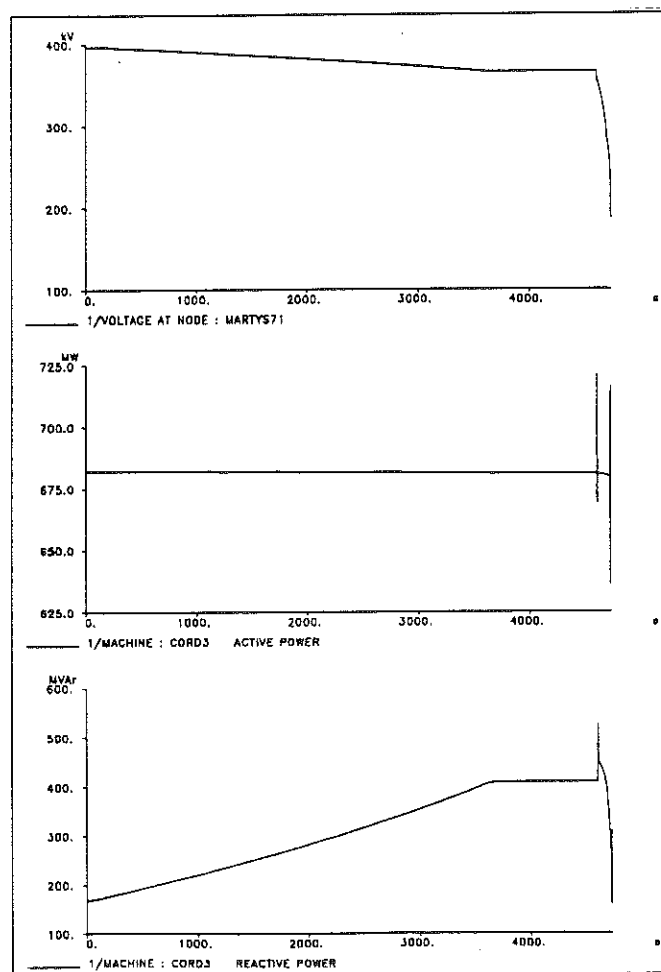


Figure 4.1

The French power system, merged in an equivalent of UCPTE system is considered. In order to study the voltage collapse phenomena, the network of Brittany is imagined at full load. It is then made fragile by opening two interconnection links. Furthermore, an increase in demand (at time $t=0$) of around 15 % in one hour is simulated so as to render the system insecure. In these extreme conditions, the untimely loss (at time $t = 4\ 600$ s) of a generating unit of the Cordemais power station provokes if no measures are taken, an inexorable drop in voltage plus successive islanding of several units.

Figures 4.1, 4.2, 4.3 and 4.4 give some details on the behavior of the system during the two minutes before voltage collapse.

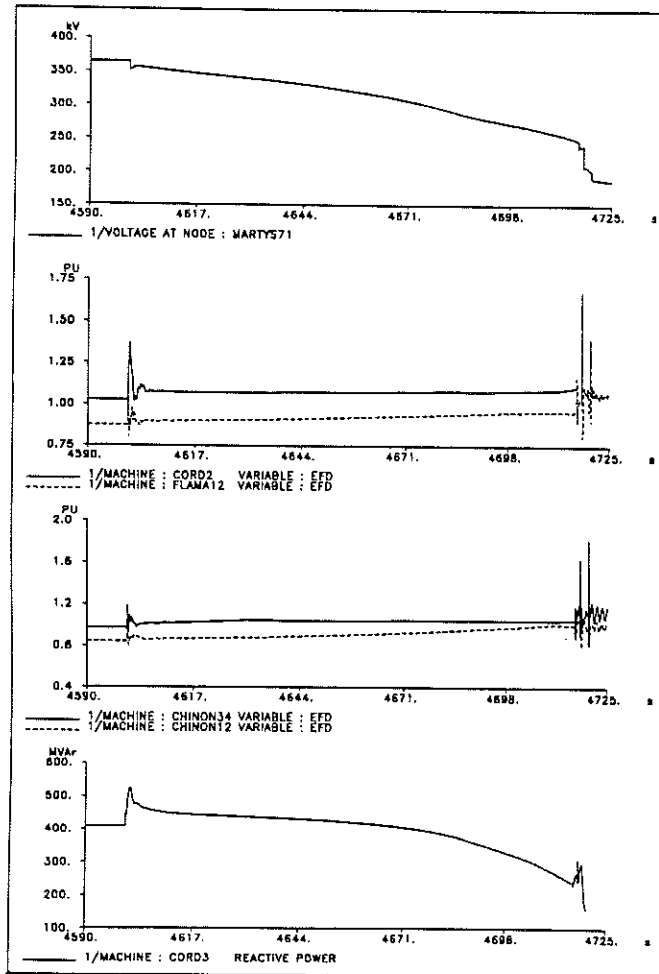


Figure 4.2

Figure 4.1 gives a general overview of the overall transient. At time $t = 0$ begins a load pick-up in Brittany (15 % in an hour). The voltage at La Martyre falls down slowly, the nearest reactive source lies in Cordemais power station. The reactive power of the Cordemais units increases to a high level, the active power remains constant. The units are very close to their full load at the end of the load pick-up. After some 15 minutes, at time $t = 4\ 600$ s, the unit 1 of the

Cordemais power station trips : the voltage collapse begins. The units become progressively field current limited. The voltages at load level are maintained by regulating transformers, loads remain nearly constant but losses increase sharply. The simulation is stopped by the user at time 4725 s with 200 kV at La Martyre 400 kV substation.

Figure 4.2 gives more details on the voltage collapse from 10 second before the trip of Cordemais 1 unit up to the end of simulation. The sudden decreases in voltage just before 4 725 s correspond respectively to the trip by under-voltage relay of unit 2, unit 4 and 5 and finally unit 3 of Cordemais power station.

The figure gives the evolution of the voltage of La Martyre substation, field voltage of Cordemais 2, Flamanville and Chinon units (Flamanville 1 and 2 are grouped in one equivalent unit Flama12; Chinon 1 and 2 are grouped in one equivalent unit Chinon12 and Chinon 3 and 4 are grouped in one equivalent unit Chinon34). The last curve shows the decrease of the reactive power of the Cordemais 3 units due to field current limiter and voltage drop.

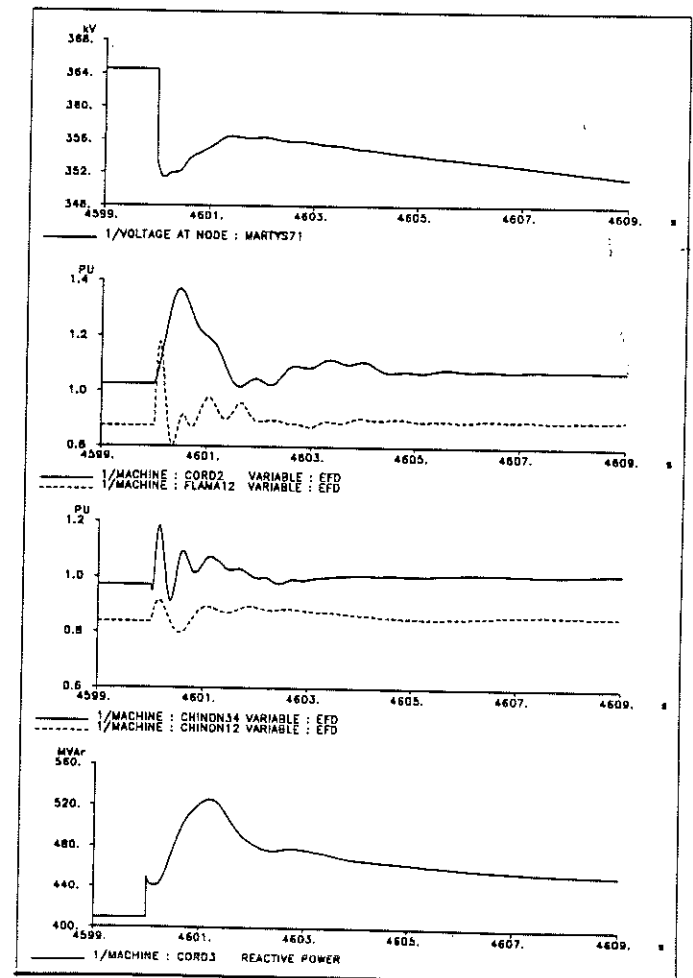


Figure 4.3

Figure 4.3 gives the details of the transient for the same variables after the trip of Cordemais 1 unit.

Figure 4.4 gives the details of the transient for the same variables at the end of the transient, with the loss of all the Cordemais units, successively Cordemais 2, Cordemais 4 and 5 and Cordemais 3.

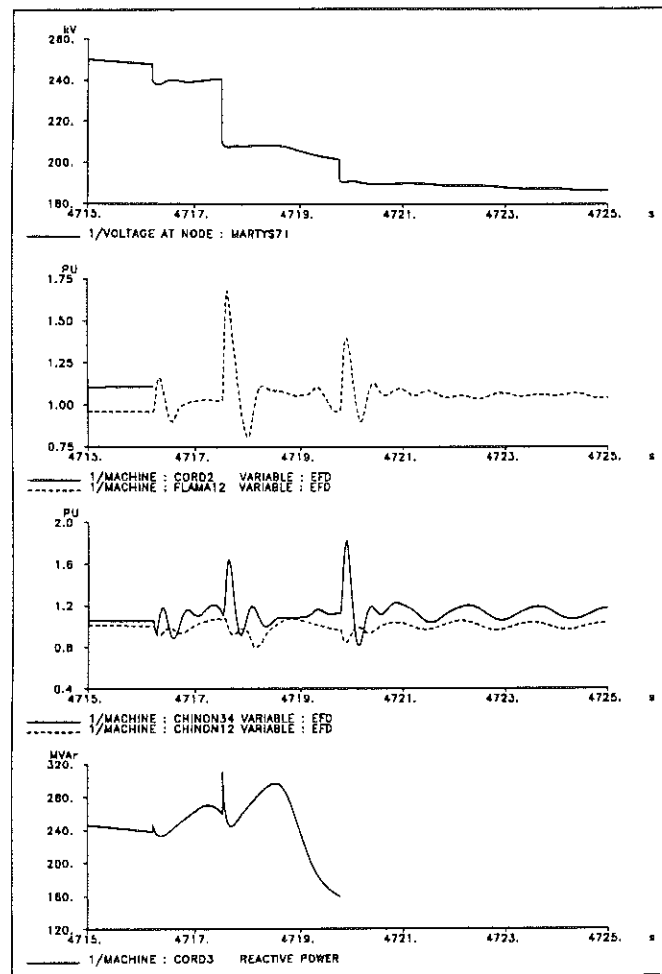


Figure 4.4

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- [3] F.FALCK CHRISTENSEN, A.W. GRAINGER, G. SANTAGOSTINO, M. STUBBE and J. VERSEILLE, "Planning Against Voltage Collapse" - Electra n°111 - pp 55 - 75 - 1987.

APPENDIX

DESCRIPTION OF THE EUROSTAG PROGRAM

All the simulations reported on this paper were obtained by the EUROSTAG program. A brief description of the program follows.

GENERAL

EUROSTAG is a time domain simulation program, developed by Tractebel and Electricité de France for transient, mid-term and long-term stability analysis of large power systems.

The modeling and the algorithm used in the program make it an efficient tool for the simulation of extreme conditions of the system. It allows a detailed study of voltage instability phenomena.

SOLUTION TECHNIQUE

EUROSTAG solves the general short-, mid- and long-term dynamic problem using an implicit integration method. The differential and algebraic equations are solved simultaneously with a variable integration time step [1, 2].

The stepsize varies according to the actual behavior of the system (typically from 1 ms to 100 s) in order to secure a constant accuracy of the calculation process. In fact, the truncation error is calculated at each step for the determination of the exact step length to be used [1].

The program exploits sparsity based techniques and manages, when possible, the numerous discontinuities of the modelization on a local basis, allowing a smooth variation of the stepsize and a high accuracy.

It uses the same modelization of the components whatever the disturbance, the behavior of the system or the duration of the simulation is.

The program is able to reach any steady state conditions of the system and can be used for quasi steady state analysis. An interactive eigenvalue calculation allows the dynamic stability to be studied around any steady state conditions reached during the simulation.

ON-LINE OPERATIONS

Various manoeuvres and operations can be initiated at predetermined moments or during the simulation by user intervention. This is made possible by the graphic monitoring of the changes of the system's main quantities.

The following operations can be carried out :

- opening and closing circuit breakers (even if a separation or resynchronization occurs, the relevant phenomena being simulated);
- shutdown and start-up of generating units;
- shutdown and start-up of induction motors;
- switching loads and compensation means on and off;
- transformer tap-changer operations;
- changes in set-points of controllers.

MODELING CAPABILITY

EUROSTAG has extended and flexible modeling capabilities thanks to graphical input facilities for the direct coding of block-diagrams of new models defined at the user level. This graphical macrolanguage is used to code any classical library of AVR, turbine or governor models, but also to code boiler models, SVC or HVDC models, static or dynamic load models etc...

EUROSTAG has also a flexible capability of simulating automats which are represented as a set of equations determining the conditions and the moment of an operation on the system. All the possible on-line operations can be ordered by an automaton model. This concept is mainly used to model relays, ULTC and centralized control systems.

Beside these facilities, the following models are available :

- synchronous machines :the model allows two rotor windings in each axis with a mutual coupling between field winding and damper in the d axis, and separate saturation characteristics for d and q axis;
- standard models of AVR (comprising rotor current limiter), turbine, governor and boilers;
- standard Static Var Compensators models;
- relays and protection systems, defined as automats;
- models of AGC and AVC;
- model of load sensitive to voltage and frequency;
- models of dynamic loads;
- induction motors :
 - simplified model neglecting rotor transients;
 - detailed model which comprises a 2 winding rotor transients calculation;
- detailed ULTC and transformer models.

Further developments are currently in progress. They covers in particular the following topics :

- unsymmetrical conditions;
- extended HVDC multiterminal systems with DC unsymmetrical conditions
- flexible AC Transmission Systems.

The program has been designed to be run on powerful workstations and is currently used for systems having up to 5000 - 10 000 state variables (typically 1000 - 2000 nodes and a few hundred of machines).

APPLICATIONS

EUROSTAG replaces the classical transient, long-term and dynamic stability programs and allows higher accuracy than classical programs thanks to its extended modelization and its ability to simulate simultaneously fast and slow transients.

The robustness of the algorithm and the modelization capabilities allow the simulation of voltage collapse scenarios, even if they are very complex (simulation of load increase, operator actions, cascading effect, automatic tap-changer actions, loss of synchronism due to low voltage, ...) and very long (one hour or more).

EUROSTAG includes an interactive output analysis program for displaying and printing simulation results. The observation of any quantity of the system is possible without any prior declaration. The post-processing program computes and displays any internal variable or observable quantity (f.i. MVAR on a line, impedance as seen from a line extremity or output of any block of a block-diagram of controller ...), after the simulation and at the request of the user.

Session 2: Monday Evening

SESSION 2:

Types of Voltage Problems

C. C. Liu (University of Washington)

Bifurcation Analysis

A Scenario of Voltage Collapse in a Power System with Induction Motor Loads with a Cascaded Transition of Bifurcations

Y. Tamura (Waseda University)

Stability Enhancement vis Secondary Voltage Regulation

H. G. Kwatny (Drexel University)

Using an Iterative Method to Compute a Closest Saddle Node Bifurcation in the Load Power Parameter Space —and—

Computing an Optimum Direction in Control Space to Avoid Saddle Node Bifurcation and Voltage Collapse

I. Dobson, L. Lu (University of Wisconsin)

Security Analysis and Control - Dynamic

Integrating Voltage Security Measures:

Connections Between Steady State, Small Disturbance Stability, and Nonlinear Dynamic Criteria

C. L. DeMarco (University of Wisconsin), and T. J. Overbye (University of Illinois at Urbana-Champaign)

Analysis and Control of Static and Dynamic Voltage Stabilities

B. H. Lee, K. Y. Lee (Pennsylvania State University) and Y. M. Park (Seoul National University)

On the Condition for Inverse Control Action of Tap-Changers

N. Yorino, H. Sasaki, A. Funahashi (Hiroshima University),

F. D. Galiana (McGill University), M. Kitagawa (Chugoku Electric Power)

Discussion

TYPES OF VOLTAGE PROBLEMS

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Abstract

This is a tutorial paper concerning the causes of voltage collapse in power systems. The causes considered here include generator excitation limit, on-load tap changing, and load dynamics. Each item here is discussed individually and then their interaction will be analyzed. The discussion starts from a simple 2-node power system, followed by generalization to the N-bus interconnected power systems.

1. Introduction

The voltage collapse (VC) has been a very active area of research during the last several years. Good summaries of the basic phenomena, available analytical tools and industry experience are given in [1-2]. In the early stage, the VC is often described as a problem caused by an operating condition that exceeds the transfer limit. The transfer limit of an electrical network is the maximal MW, MVAR or MVA amount that the system can deliver from the generation sources to a substation or area. Specifically, the transfer limit is the maximal amount of power that corresponds to at least one power flow solution. This line of thoughts leads to the argument that a voltage collapse occurs when the maximal power transfer limit is exceeded. A question naturally arises:

If a transfer limit is the maximal amount the power system can deliver, how can the limit be exceeded?

From the well-known P-V, Q-V and S-V curves, one can observe that the voltage value of one of the power flow solutions gradually decreases as the power transfer amount is increased. After the maximal amount is exceeded, the power flow solution does not exist any more. If the steady state of the power system does not exist, the system can not move to an equilibrium. In other words, the power system behavior must be governed by the system dynamics.

The next question is what "dynamics" can cause a voltage collapse. A power system trajectory can become unstable, but, loosely speaking, instability simply means that the system state variables can not settle down to an equilibrium; it does not guarantee that the power system voltage will collapse. Therefore, more analysis must be performed to answer the question, "What dynamics can cause the system voltage to collapse?"

Traditionally, power system stability is often associated with the rotor dynamics. In the detailed generator models, various flux linkage components are modeled, while in simplified swing equations the voltages are simply assumed constant. The instability phenomena caused by a "swinging" rotor angle may be power flow oscillations, or voltage fluctuations. In a previous work [3], for example, the reactive

power and voltages are incorporated in the linearized swing equation model. The linearized model considers the voltage variation as a dependent variable which "follows" the angle swings. In other words, the angle swing induces the voltage fluctuation. In a detailed computer simulation for stability analysis, one should observe that the angles vary and the voltages also fluctuate; the mutual interaction can not be separated easily.

In a voltage collapse, it is possible (at least for a period of time) that the rotor does not swing and the system frequency remains unchanged. Then, the swing equation would be "inactive" in the mathematical model. What other mechanisms can cause the voltage to collapse then? We have attempted to answer this question by analyzing various dynamic mechanisms of a power system [4-7]. Each mechanism can cause a different type of voltage problem and the mechanisms can interact to create a voltage collapse. In this paper, we will discuss the types of voltage problems through various dynamic models related to the power system voltage behavior and control.

2. On-Load Tap Changers

Our analysis started with the first suspect - on load tap changers (OLTCs). The basic control of an OLTC can be described by the following equation[8]:

$$n_{k+1} = n_k - d \cdot f(V - V_0) \quad (1)$$

where n_{k+1} is the tap position at time $k+1$, d is the size of each tap change, V_0 is the reference voltage value, and $f(x) = -1, 0, \text{ and } 1$ if $x < -e, |x| < e, \text{ and } x > e$, respectively. The OLTC changes the tap position when the (secondary) voltage moves away from the reference voltage V_0 by more than a tolerance. Now place the OLTC in the simple power network in Figure 1. Assume that a disturbance occurs, such as a load increase. The post-disturbance power system has at most two power flow solutions, given by the tap positions n_{10} and n_{20} (Figure 2). It can be shown that n_{10} is stable and n_{20} is unstable. The tap position does not change instantaneously. Rather, if the position falls below n_{20} right after the load increase, the tap position will converge to the new equilibrium n_{10} and the load voltage returns to V_0 . If, unfortunately, the disturbance is large enough that the tap position falls outside n_{20} , then the load voltage magnitude will decay monotonically until the physical maximal limit of the tap position is reached. Practically, however, the value of n_{20} is large and hence it is unlikely to create a VC by this inappropriate control alone. Also, in this case, the "stability region" covers the interval up to the unstable equilibrium n_{20} .

The above discussion is based on the 1-tap changer case. Now suppose N tap-changers are interconnected in a power system, some automatic and others manual. We can use the continuous-time approximate tap-

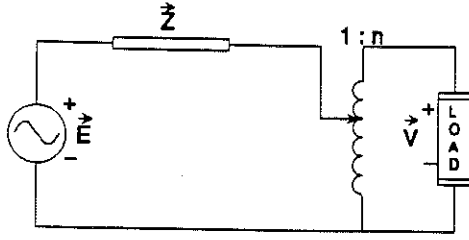


Figure 1. A Simple Power System

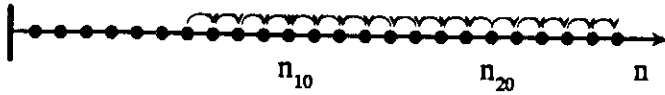


Figure 2. Equilibria and Dynamic Behavior

changer model and represent the interconnected power system by the power flow equations. To make the analysis tractable, we used the decoupled reactive power/voltage relation [4]. The de-stabilizing behavior of tap-changers can be observed from the interconnected power network model, since tap-changing is the only "cause" under consideration.

Suppose the state-space is spanned by the OLTC tap positions. It is shown that an equilibrium (in terms of tap settings) is stable if the associated Jacobian is non-singular. Now that an equilibrium is found, one can try to identify the stability region around the equilibrium. We also developed a method to construct stability regions based on hyperboxes. A stability region or its approximation is useful since it is desirable not to operate the system outside the region. Figure 3 shows the exact stability region for a 2-tap changer case. For the N-OLTC case, we use the approximate region using hyperboxes. Any trajectory falling outside the exact stability region at the initial time of the post-disturbance system will cause at least one voltage to fall monotonically, just as in the 1-tap changer case.

The above results are not sufficient to help understand the VC completely. It is mentioned earlier that tap-changing alone should not cause a voltage collapse since each tap-changing transformer has a physical upper limit. When this limit is reached, the dynamic model for tap-changers can no longer be applied. Even though the resulting voltage can drop monotonically, it does not necessarily mean a collapse. In the following, we will discuss other mechanisms or causes.

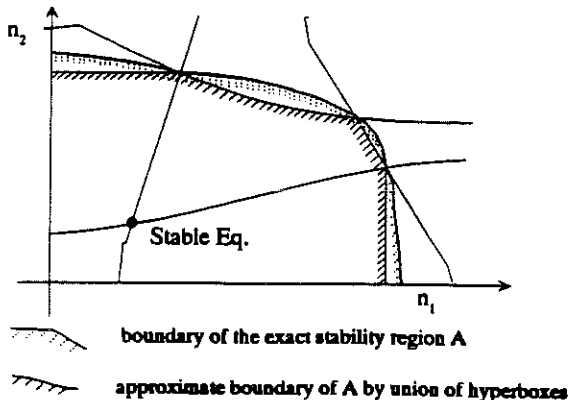


Figure 3. Stability Regions in Space of Tap-Settings

3. Generator Excitation Limits

When a low load voltage is encountered, the reactive line loss can be high, which places a heavy burden on nearby generators. Should these generators reach their reactive limits, their terminal voltage is reduced. Hence, the reactive burden is felt by generators further away. An example of such an event in which the generators reach their limits one after another is the 1982 VC incident in Belgium [9].

We model the generator with a voltage source E' behind a reactance X , where E' varies in order to maintain the terminal voltage at a constant level E_0 during a normal operating condition. The rotor current has an upper limit; the stator current also has an upper limit [10]. When one of the limits is reached, the generator has to operate at a reduced terminal voltage. This is not a truly dynamic model; a truly dynamic model would involve the excitation system, etc. The justification is that the excitation system has a relatively short time constant as far as the voltage collapse phenomenon is concerned. However, our model does consider the rotor current and stator current limits, which are important in the VC analysis.

4. Dynamic Load Model

Next, the load behavior over a wide range of voltages should be considered. In particular, when the voltage falls, the reactive demand might fall initially but as the voltage continues declining the demand eventually increases. This means that at low voltages not only that the reactive line loss can be high but also the reactive load demand can further add to the burden of generating units. Therefore, the interaction between the load-voltage relation and generator reactive capability should be considered in a voltage collapse study.

In general, the existing knowledge of the load-voltage relation is incomplete. Load behavior can be studied by connecting a load device or model to a varying voltage source and then observing changes in the demand [7, 10]. It is observed that the load demand is dynamic since it depends on how (in which direction) the voltage is changing. Therefore, in a voltage collapse analysis, the system model should include not only the tap-changing control but also generator limits and the load dynamics.

We model the load with two components - a static component which depends on the magnitude of the applied voltage V , and a dynamic component which is a function of the rate dV/dt [7, 10].

$$P = P_s(V) + k_p(V) \frac{dV}{dt} \quad (2)$$

$$Q = Q_s(V) + k_q(V) \frac{dV}{dt} \quad (3)$$

Note that the static component can be a polynomial function of V . For example the ZIP model (combination of constant impedance Z , constant current I and constant power P terms) is a quadratic function of V . The coefficients can be determined with a computer program such as EPRI LOADSYN. The second term which depends on dV/dt is dynamic. In [7], we will demonstrate how this is related to the full-fledged induction motor dynamic equations. For our purpose here, it is noted that the dynamic term adds positive (negative) load to the system

as the voltage moves up (down). The effect of the dynamic load is quite significant for the Q part and not as important for the P load.

5. Voltage Collapse Analysis

A simple 2-node power system was shown in Figure 1, which consists of a generator, a transmission line, an OLTC and a dynamic load. Models for the devices have been discussed in Sections 2-4. The state space is the plane of (load) voltage magnitude V and the OLTC turns-ratio n . We will reconstruct the voltage collapse and observe different types of voltage trajectories on the plane.

The state space can be partitioned based on each mechanism. In Figure 4, the region above Curve I corresponds to a decreasing tap since voltage is higher than the reference value V_0 . On the other hand, the tap position tends to increase in the region below Curve I. The region above the curve 'normal' corresponds to normal rotor and stator currents of the generator; when the system trajectory crosses this curve at least one limit (rotor or stator current) is reached and, as a result, the terminal voltage is reduced. Curve II is obtained by setting the dV/dt terms in the load expressions to zero. The region enclosed by Curve II corresponds to voltage recovery ($dV/dt > 0$), i.e., as long as the system state is inside this region, the load voltage increases toward a steady-state S which lies on Curve II. Outside the curve, the voltage tends to drop, or $dV/dt < 0$. For ease of reference we refer to this region enclosed by Curve II as the "leaf."

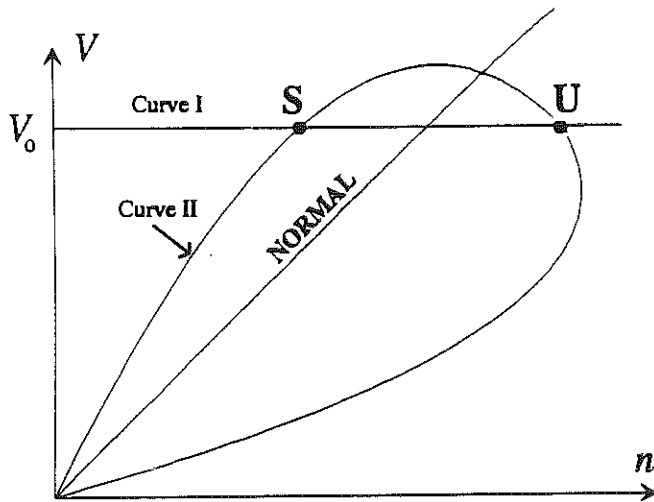


Figure 4. Partition of the State Space

The system reaches an equilibrium state when the load voltage is in a steady-state and at the level V_0 . As shown in Figure 4, there are two equilibria S and U , the intersections of Curves I and II. The equilibrium S is stable, whereas U is unstable and also causes a violation of generator rotor and/or stator current limit(s).

Suppose that the system has been operating at an equilibrium A as shown in Figure 5 when a (large) disturbance such as a load increase occurs. The new equilibria are S and U as shown. Depending on the location and size of the region enclosed by Curve II, (which is dictated by the load characteristics P_s and Q_s), we may have different scenarios.

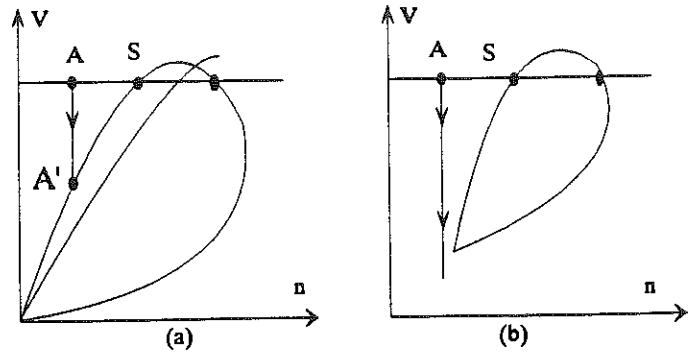


Figure 5. Voltage Trajectories (Fixed Tap)

If tap-changing is not automatic, then the system trajectory would move along a vertical line (fixed turns-ratio n), as given in Figure 5. If the leaf extends sufficiently below A as shown in Figure 5(a), then the voltage decays to a new steady-state value given by point A' . At A' no generator limits are violated. Even though A' is a post-disturbance steady state, the associated voltage is lower than the reference value V_0 . A trajectory of this type simply shows that a load increase caused a voltage drop. Nevertheless, if the leaf is in a different location due to the load characteristics, then the voltage would collapse as shown in Figure 5(b).

If load is considered to be static (i.e., k_p and k_q are set to 0 in the load model), then the movement of the system trajectory is restricted to the boundary of the leaf. Figure 6(a) reveals that with a "long" leaf, the system reaches a stable operating point by tracing the path from A' to S . The new equilibrium S is reached at which the load voltage is restored without violating the generator rotor or stator current limits. However, if the leaf is as in Figure 6(b) then the system voltage following the disturbance can not be determined (since there exists no load voltage for the given value of n), and the model fails to explain what will happen to the system.

Now suppose the generator excitation and stator current limits are ignored. It is possible to create a VC. For example, in Figure 7(a) the system trajectory leaves the voltage recovery region (at point A'') and, beyond this point, the voltage drops monotonically. Whether or not this scenario takes place depends on the relative speed between load dynamic (vertical movement) and the tap-changing (horizontal movement). It is clear that with slower tap-changing, the system has more time to move to the steady-state located on the boundary of Curve II.

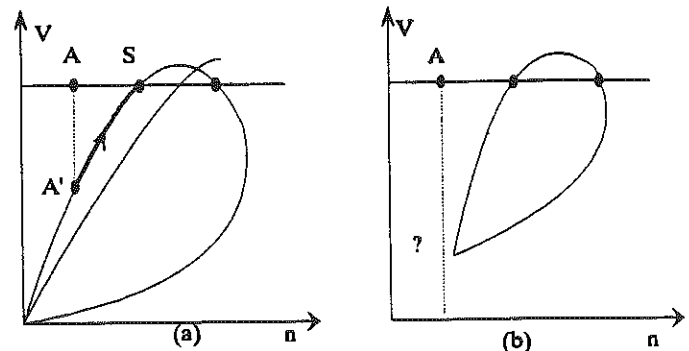


Figure 6. Voltage Trajectories (Static Load)

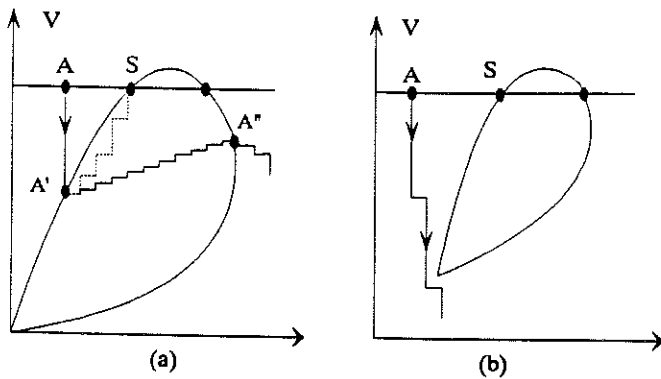


Figure 7. Voltage Trajectories (Unlimited Rotor & Stator Currents)

Hence it is less likely for the system trajectory to “sweep across” the region. In other words, when the system is inside the leaf voltage collapse is less likely with slow tap-changing. When the tap-changing is slow enough, a possible system trajectory is as shown by the dashed curve in Figure 7(a).

Slow tap-changing, however, is not always a condition to avoid a collapse. To illustrate this point, consider the case of a smaller leaf as shown in Figure 7(b). If tap-changing is slow in the beginning then the system state would “miss” the leaf, and hence voltage cannot recover. In other words, fast tap-changing is desirable in the case of Figure 7(b). (Reference [7] provides an example in a higher dimensional space in which fast tap-changing can help avoid voltage collapse while slower tap-changing cannot. See Fig. 7 of [7].)

A rule can be obtained from the above observation - it is important for the system state to get into the voltage recovery region (leaf) and stay in there. To get inside the leaf, fast tap-changing might be needed, whereas to stay inside may require slow tap-changing.

We explain the interaction among the mechanisms in a voltage collapse with Figure 8. From the post-disturbance initial point A, the system trajectory moves down in space since it is outside the voltage recovery region. Eventually, the system reaches the boundary of the leaf at point B. From point B there are two possibilities for the trajectory. If the tap-changing is relatively slow as compared to load dynamic then the system would stabilize, tracing a dashed curve as shown in Fig. 8(a). Otherwise, the system state may cross the ‘normal’ curve at point C. The generator then operates at a reduced terminal voltage E and it can be shown that the leaf shrinks with a reducing E. The system state can exit the leaf at a point D in Fig. 8(b), and the voltage drops monotonically afterward. It is seen that the voltage recovers for a period of time but eventually collapses.

6. Generalization to Interconnected Power Systems

The results of the generalization of the interaction among mechanisms are reported in [6] where the analysis is based on the reactive-power/voltage relationship. The main points are summarized and discussed here. For illustration, we will use the power system in Figure 9, which is taken from [11]. The system has a structure similar to the power system in the Puget Sound area of the Pacific Northwest. The modified

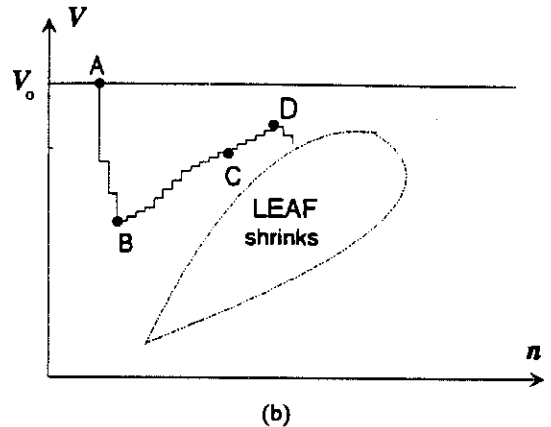
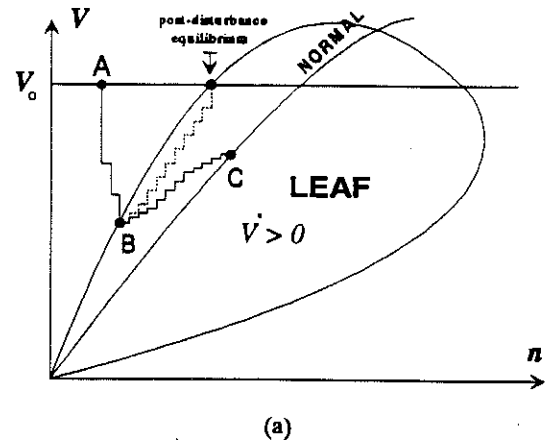


Figure 8. Voltage Collapse

system parameters used for simulation are given in Table 1. Note that the Q model includes the dynamic dV/dt term. The excitation voltage and stator current limits are denoted by E_{lim} and I_{lim} , respectively.

We obtained the following results with dynamic load models.

A special equilibrium v

It can be shown that there exists an equilibrium (in the voltage space) which has coordinates higher than those of any other equilibria. Furthermore, this equilibrium is stable if the associated Jacobian is non-singular. However, in general this may not be the only stable operating point. (When a special class of load characteristic (Qs's) is considered such as quadratic loads, this special equilibrium is the only stable operating point.) The existence of the special equilibrium v confirms the observation that the power flow solution usually reaches the solution with highest voltage magnitudes (normal solution).

The state space is defined in V_s, V_r , i.e., the load voltages. Prior to $t=0$, the system operates at a steady state. The disturbances are: (1) $t=2$ secs. partial loss of reactive support at node 7 (C, from 0.3 to 0.1) and (2) $t=3.5$ secs. double line outage (X_{74} changed). The new stable equilibrium exists and is given by $V_s=V_r=1.003$. Since the quadratic load is used in this case, this is also the special equilibrium.

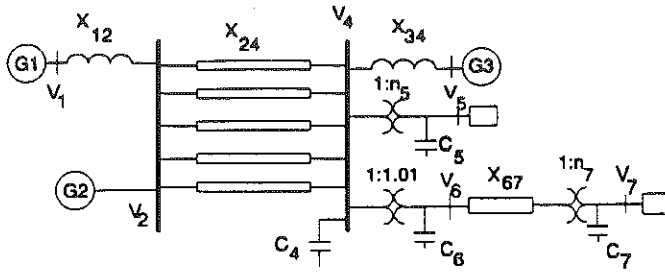


Figure 9. 7-bus Power System [11]

Table 1 System Model and Parameters

Node no.	Data
1	slack node, $V_1=1.02$, $X_{12}=0.2$
2	$V=1.01$ (normal), $P=0.3$ Internal: $X=0.2$, $E_{lim}=1.110$, $I_{lim}=0.602$
3	$V=1.01$ (normal), $P=0.2$ Internal: $X=0.15$, $E_{lim}=1.075$, $I_{lim}=0.502$
4	$X_{24}=0.2$ (pre-disturbance) $X_{34}=0.2$, $C_4=0.65$
5	$C_5=0.0$ $P=0.70 \cdot (0.10V^2 - 0.16V + 1.06)$ $Q=0.45 \cdot (0.20V^2 - 0.34V + 1.14) + 5 (dV/dt)$
6	$C_6=0.0$, $X_{67}=0.1515$
7	$C_7=0.3$ (pre-disturbance) $P=0.60 \cdot (0.10V^2 - 0.15V + 1.05)$ $Q=0.5 \cdot (0.25V^2 - 0.425V + 1.175) + 5 (dV/dt)$
OLTC (nodes 5,7)	time delay = 5 secs., deadband $V_0 + \epsilon = 1.0 + 0.005$, size of tap-change $d=0.005$, tap-range= $0.8-1.2$

Partial ordering of vector field in the voltage space

The use of the dynamic load model in Sec. 4 implies that the load voltages are state variables. The evolution of the system voltage is therefore dictated by a set of dynamical equations. We found that the vector field in the voltage space exhibits a partial ordering [6]. Physically, this means that if there are two known voltage trajectories, one high and one low, which converge to the same stable steady state then any voltage profile in between also converges to the same steady-state operation. This is useful in obtaining the stability regions.

Figure 10 shows a "leaf" with a label P . Inside the leaf, both V_5 and V_7 recover, i.e., $dV/dt > 0$. Hence a point in P will converge to v , the stable equilibrium. Across from v , a trajectory will converge to v , but from an opposite direction, i.e., $dV/dt < 0$. Hence every point in between such a pair will also converge to v .

Voltage stability regions

Approximate voltage stability regions (i.e., subsets of the true region of attraction to the special equilibrium v) can be built from the union of hyperboxes. Building a hyperbox requires two vertices (corners) in the state space: upper and lower. By taking the union of all such hyperboxes, one obtains a larger region, region B_p . Note that this method has been used in [4] to build a region of tap-settings which guarantees stable operation of tap-changers.

The voltage stability region for the sample power system is shown in Figure 10. The boundary of the voltage stability region passes through the unstable equilibrium u . An approximate hyperbox is also found, which is indicated by B_p . The exact region is computed by solving the system equations backward in time with appropriate initial points near the saddle point u [12].

A voltage collapse region

When a large disturbance is considered, the existence of a stable equilibrium does not necessarily mean that the system will stabilize. In fact, it is found that there exists a region C in the voltage space which is a subset of the exact voltage collapse region. That is, if the initial operating point lies inside C , at least one bus voltage will decay toward 0. In Figure 10(a), the region C is a voltage collapse region.

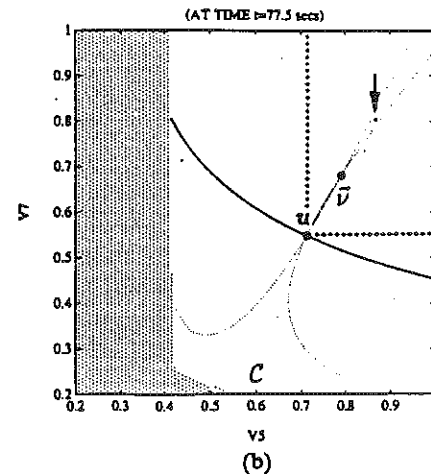
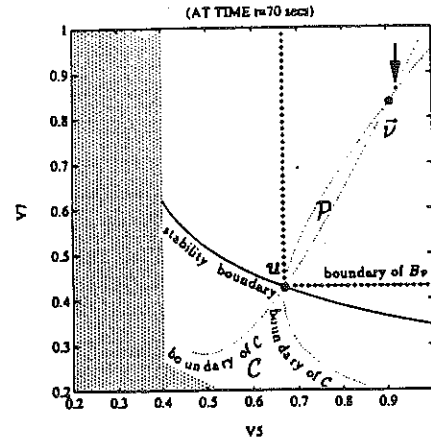


Figure 10. Shrinking regions: (a) $t = 70$, (b) $t = 77.5$. [6]

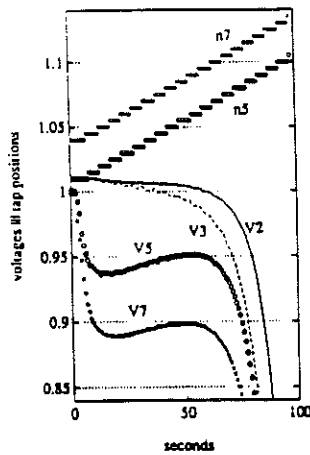


Figure 11. Evolution of voltage [6]

Loss of Equilibrium

Static methods in voltage collapse analysis are concerned with the existence of an equilibrium. For example, in the PV-curve method, the collapse is said to be due to the vanishing of a stable operating point when the power transfer exceeds a certain threshold. However, it is not clear how the voltage would evolve when such a condition is encountered. A dynamic system modeling is required to resolve this issue. The study reported in [6] reveals that in case of no equilibrium, at least one bus voltage decays toward 0, and therefore proves the argument used in static methods.

As parameters such as loading vary, power flow solutions may merge and disappear. The special property of v implies that v is the last solution to disappear in any circumstance. Therefore, it makes sense to track this equilibrium, which is done in static techniques. When this point vanishes, voltage collapse will take place.

In Figure 10(b), a snapshot of the state space at $t=77.5$ secs. is shown. After 7.5 seconds, the voltage stability region shrinks, the leaf P shrinks, and the collapse region C expands. The two equilibria move closer to each other. At a later time, the two solutions collide and then disappear. The voltage collapse is shown in Figure 11.

The presence of tap-changing and generator limits in the system modeling has the following effects on voltage stability analysis[6]:

Effect of tap-changing

The normal excitation region refers to the region in the voltage space which corresponds to normal operation of all generators (i.e., operation within limits). This region depends on the tap-settings. In fact, it shrinks with each tap increment. This means that the effort of tap-changers in improving a low voltage profile may in fact bring one or more generators to their rotor or stator current limit(s).

Effect of generator reactive capability

When one or more generators reach their current limit(s), either as a result of a severe disturbance or the adverse effect of tap-changers, the

generator terminal voltage is reduced. It is shown that the (approximate) collapse region C expands and the stability region Bp shrinks. Furthermore, the special equilibrium v shifts to lower voltage values.

Acknowledgment

This research is sponsored by National Science Foundation under grant ECS 86-57671.

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on "A Study of dynamic load models for voltage collapse analysis"

by K. Jimma, A. Tomac, K. Vu and C. C. Liu

Some clarification results in the load modeling area by making the following distinction:

1. When talking about the *system dynamics*, that is the dynamics on the system side of the bus (or all connections to the bus except the load), then the load is indeed the input and the bus voltage is the output or the state variable.
2. When talking about the *load dynamics*, however, then the bus voltage is the input which generates the dynamic component of the reactive and active powers as an output or state variable through some dynamic equations like

$$\begin{aligned}\dot{Q}_{D_i} &= f_{Q_i}(V_i, \omega_i, Q_{D_i}, z_i) \\ \dot{z}_i &= f_{z_i}(V_i, \omega_i, Q_{D_i}, z_i)\end{aligned}$$

where Q_{D_i} is the dynamic reactive power, V_i, ω_i are the bus voltage and frequency respectively and z_i are internal states of the dynamic load. Similar equations hold for the dynamic real power as well. So, $\dot{V}$ does not appear.

For instance, in this paper, when the dynamic load is modeled as an induction motor (3rd order model i.e. equations (1)-(3)), then the internal states of the dynamic load correspond to the internal voltage e' and the slip s . The dynamic load then corresponds to $P_D + jQ_D = V i^*$ where i is the current as defined in equation (3) of the paper. So, again $\dot{V}$ does not appear.

The general power system model can be written in the form of a differential algebraic equation,

$$\dot{x} = f(x, y), \quad 0 = g(x, y)$$

where x denotes the dynamic states and y denotes the algebraic or the instantaneous states. So normally the load dynamics are absorbed in the f equation and are tied algebraically to the g equation.

The authors indeed recognize that the 3rd order model has the correct structure, that is V acts as the input and the load is the output in studying the induction motor models (the discussion of Figures 1 and 2). But the authors seem to contradict themselves by reversing their roles (Q becomes the driving term and V is the output) in the justification of the $\dot{V}$ model that follows equation (8). When approximating the system behavior, one must be careful to preserve the structure of the actual system so as to avoid wrong conclusions.

The paper proposes an approach to model the external behavior of the induction motor model as the $\dot{V}$ model. We note that the full model which includes the internal dynamics such as the third order model, results in a differential algebraic system. But an approximate model based on the 'black-box approach' such as the $\dot{V}$ model gives us a differential system. The physical behavior of the *actual system* is *not predictable* when the actual system is near a singularity [1]. Now, how can we justify the use of the *approximate model* near this singular region when the system itself is unpredictable? So, the use of the approximate model which seemingly has no singularities is

truly questionable. Such efforts to hide the physical reality can lead to grossly incorrect conclusions.

Let us take a closer look at the physical basis for a possible $\dot{V}$ term. The bus voltage V and the frequency ω are time varying inputs here so that their first (or higher order) derivatives act through the overall dynamics and are not normally expressed explicitly in the dynamic load such as Q_D , that is a kV term does not normally arise.

A large capacitor bank connected to a bus *directly*, comes close to form a special exception from the norm. This is because capacitance is one of the basic circuit elements and because its current intake is proportional to the first derivative of the voltage applied to it i.e.

$$i(t) = C\dot{e}(t)$$

However, there is considerable complexity involved when one looks beyond this instantaneous equation into its implications for the modulated AC signals.

$$\begin{aligned}e(t) &= V_0 \sin(\omega t + \delta) \\ i(t) &= C\dot{V}_0 \sin(\omega t + \delta) + CV_0\omega \cos(\omega t + \delta)\end{aligned}$$

Assuming that δ is constant although rotor angles will also be involved in the actual case, the instantaneous power (which can only be 'real') is

$$S(t) = e(t)i(t) = CV_0\dot{V}_0 \frac{1 - \cos 2(\omega t + \delta)}{2} + \omega CV_0^2 \frac{\sin 2(\omega t + \delta)}{2}$$

The first term here has a nonzero average and hence it is a dynamic component to the *real power*

$$P_D = \frac{CV_0\dot{V}_0}{2}$$

The second term has zero average and it contributes no active power. Consequently, its amplitude must be considered the reactive dynamic power of the capacitor with varying bus voltage

$$Q_D = \frac{\omega CV_0^2}{2}$$

When we switch from peak voltages to RMS voltages $V_0 = \sqrt{2}V$ these equations become

$$P_D = CV\dot{V}, \quad Q_D = \omega CV^2$$

This is a surprising, though a logical result.

1. The reactive power is simply ωCV^2 even when the bus voltage V is time varying. So, there is no $k\dot{V} = g$ model even with a large capacitor connected to the bus.
2. If V is time varying, then there is a contribution proportional to the first derivative $\dot{V}$ in the form of $CV\dot{V}$ to the *active power*. This seems surprising, but if one thinks of it, the real energy swinging in and out of the capacitor is increasing with increasing V so the result is logical. The increased actual energy determining the amplitude of the reactive power has to be supplied through real power.
3. The quasi stationary assumption requires that

$$CV\dot{V} \ll \omega CV^2 \text{ or } \dot{V} \ll \omega V,$$

that is the $\dot{V}$ term can be neglected compared to the ω term. Hence, the P_D term is wiped away by the quasi stationary assumption and the capacitor reactive power

becomes simply ωCE^2 , the form that everyone is familiar with, for constant or slowly varying (i.e. within the quasi stationary range) bus voltage. This eliminates any chance for a $kV = g$ model for the bus capacitors.

Finally, one should mention some mathematical wishful thinking related to some recent field records taken in the US and Japan [2]-[5]. These record the response Q to step changes in the load bus voltage V . For the most part, they are indistinct enough, and the characteristics of the instrumentation are non-specific enough to allow varied interpretation. These look like quasi linear transient responses with varying levels of damping. Some researchers try to read them as approximate delta functions and argue that this implies a derivative of the step function in V and hence it approximates $\dot{V}$, so they incorporate a $\dot{V}$ term in the algebraic equation. The hitch is that the time scale of these events is around 0.01-0.1 sec which may be short enough to dream of delta functions. It also is so short that the quasi stationary model would have disintegrated. Hence seeing a step-delta relationship within this range would be under the rule of a different dynamics and would not justify the inclusion of a quasi stationary $\dot{V}$ [1] for studying slower events such as motor loads. In quasi stationary terms, the only reasonable interpretation is a smooth transient response of the output Q , possibly somewhat distorted by the problems of violating the quasi stationarity.

We conclude that a quasi stationary power system model is of the differential algebraic form. The load dynamics makes this model more detailed but does not change the structure of the model. The stability theory for analysing such differential algebraic systems is proposed in [1].

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CLOSURE

by

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We would like to thank J. Zaborszky, H. Schattler and V. Venkatasubramanian for their comments. We agree that the (physical) power system involves dynamics coupled with algebraic constraints. (Generators are dynamic devices and the

KCL and KVL constraints are algebraic.) In an analytical study, one would be interested in a model which is detailed enough to provide physical insights and yet sufficiently simplified so that the problem is tractable analytically or computationally. It is our goal in this paper to evaluate a specific dynamic load model that is physically justified and mathematically tractable.

Our comments on the relevant issues are given below:

1. Power system models for stability studies always have the differential-algebraic form.

Not necessarily. Typically, the algebraic part of the model is associated with the load buses. Let's look at two cases:

Case 1: All loads modelled by impedances

The algebraic equations can be solved explicitly for the dependent variables (such as load bus voltages) and the result can be absorbed into the differential equations. Thus, no algebraic equations need be present in the model description.

Case 2: Induction motors represented by 3rd-order model

In this case, it can be shown that Eqs. (1-3) in our paper represent a varying voltage source e_l behind an impedance $R_s + jX'$. Specifically, Eq. (3), which may give an impression that the model requires an explicit algebraic equation, represents the current flowing from the terminal node V to the internal node e_l . In other words, there are two buses/nodes associated with each load: terminal, and internal whose voltage phasor e_l is a state variable. Since Eq. (3) can be absorbed into Eqs. (1-2), it is not necessary to represent the algebraic equations explicitly in the system description.

The use of the 1st-order induction motor model implies that the load is modeled by impedances - some varying with the slip; thus, similar to Case 1, no algebraic equations need be given explicitly.

One can argue about the practicality of these models, but most likely there will never be a complete model that covers every aspect of a power system.

2. Validity of the Vdot modal

The motivation of the Vdot model is to reduce inaccuracy introduced by simple induction motor models. This statement is supported by the simulation results with the 3rd order induction motor model. We do not claim that the Vdot model is a theoretical equivalent of the 3rd-order model; we use this model to approximate the external (or terminal) behavior of the full model.

In the simulation results of Figs. 1, 3-4, the applied voltage is considered an input. This is to demonstrate that there is a relation between the (changing) applied voltage V and the f flow into the load. However, when this load is connected to a network, the role of V would be different. How V varies depends on the network as well as the internal voltage e_l of the load. That is, the f flow into the load dictates how V can change. This flow- V relation is an external relation since measurements can be made at the terminal of the load.

It is noted that the difference in reactive power load between the static load model and the Vdot model is quite significant, as shown in Fig. 4. In other words, without the term, the static model would give wrong results for prediction of the voltage behavior. The choice of Vdot is based on judgement; other techniques may be used to correct the discrepancy.

The introduction of Vdot makes V a state variable, which means that V cannot change instantaneously; however, in the 3rd-order model, V may be discontinuous when a disturbance occurs in the network. That is, the two models are not equivalent from a mathematical point of view.

The load behavior of a large power system is not very well understood in general. Despite the difficulty, the following qualitative relation between load and voltage is widely accepted - lack of reactive power implies voltage decay and surplus of reactive power results in a voltage increase. The Vdot model is consistent with this load voltage relation.

We disagree with the discussers on their comment that replacing the 3rd-order model with the Vdot model means replacing a differential-algebraic system by a purely differential system. The Vdot model does involve algebraic equation because one still has to solve for the bus angle, i.e., the real power flow balance at the node must be accounted for. However, our analytical approach in [A], which utilizes the Vdot model, would fail when a singularity case arises.

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A SCENARIO OF VOLTAGE COLLAPSE IN A POWER SYSTEM WITH INDUCTION MOTOR LOADS WITH A CASCADED TRANSITION OF BIFURCATIONS

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ABSTRACT-

Voltage collapse generally occurs due to dynamic interactions among generators, loads including IM's and controllers including OLTC's. To better understand the dynamic interactions, a simple yet well-designed model has to be chosen carefully and studied in detail. In this paper, a basic system with a generator (7-dimensional Park model) with and without AVR and an IM (3-dimensional Brereton model) connected through a transmission line was chosen and systematically investigated using the bifurcation theory approach to present a scenario of voltage collapse starting from a stable state, via states of transition to be governed by the Hopf bifurcation featuring the stable limit cycle with a very long period, and finally leading to a voltage collapse due to the occurrence of the fold (saddle node) bifurcation.

Key words:

voltage collapse, scenario, Park model, Brereton model, Hopf bifurcation, fold bifurcation, slip vs torque characteristics of an IM

1. INTRODUCTION

It can be said that a power system, consisting of various elements, such as transmission lines, generators, transformers, etc. is a nonlinear and time-dependent system. Usually the power system state initially moves in the stable operating region or specifically moves along a trajectory to be affected by the gradual change of parameter value, and sometimes leads to the loss of stability. Depending on the situation, either oscillatory or nonoscillatory behaviour are observed.

The system voltage during the voltage collapse in Tokyo area in 1987 has been observed to behave slowly and in fairly aperiodic manner.

Apart from this, during a series of system tests conducted on a tie line in the North-Eastern 50 Hz area, aiming at the determination of P vs f relationship, a sustained oscillation of P and Q of a limited amplitude was observed with a fairly long period on an ac tie line.

The existence of the voltage swing mode coupled with the power swing mode has been discussed in recent years.

In this paper, a relationship of voltage swing mode and voltage collapse is discussed with the view mentioned above using the bifurcation theory, and a scenario of voltage collapse is proposed.

2. BIFURCATION THEORY

As a system parameter changes, the vector field changes topologically. This phenomenon is called bifurcation. There are several kinds of bifurcations (Fig.1). In this section, two types of bifurcations are described.

(i) Hopf bifurcation

It is called Hopf bifurcation when stable equilibrium points (SEP's) bifurcate into the periodic orbit, depending on the value of parameter μ closely related with the system stability. And this bifurcation is a kind of dynamic bifurcation. Hopf bifurcation occurs when only a pair of complex conjugate eigenvalues crosses the imaginary axis.

(ii) fold bifurcation

It is called fold bifurcation (or saddle-node bifurcation) when both stable equilibrium points (SEP's) and unstable equilibrium points (UEP's) appear or disappear together. And this bifurcation is a kind of static bifurcation. Fold bifurcation occurs when only a real eigenvalue crosses the imaginary axis transversally.

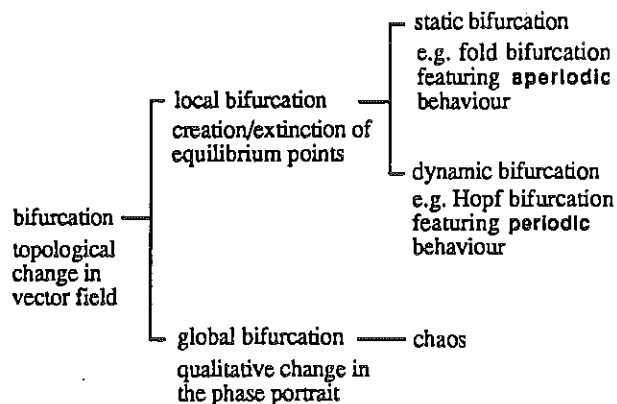


Fig.1 Type of bifurcation

3. A SYSTEM WITH A GENERATOR AND AN IM

Fig.2 shows a system with a generator (7-dimensional Park model) with AVR and an IM (3-dimensional Brereton model) connected through a transmission line. A block diagram of AVR is shown in Fig.3.

Table 1 and Table 2 shows machine parameters of a generator and an IM respectively.

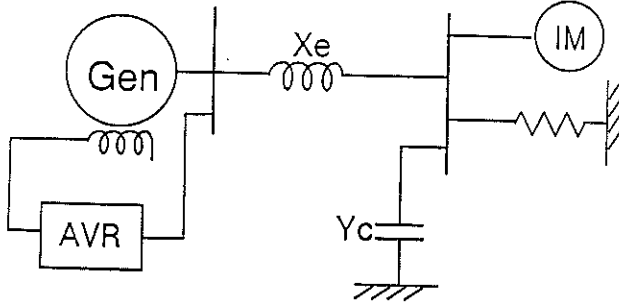


Fig.2 A system with a generator and an IM

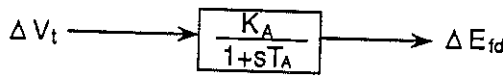


Fig.3 Block diagram of AVR

Table 1 parameters of a generator

X_d	1.661
X_q	1.661
X_d'	0.2739
X_d''	0.2068
X_q''	0.2068
X_l	0.143
T_{do}'	5.900
T_{do}''	0.04
T_{qo}''	0.16
r	0.00
D	2.0
H	5.45

Table 2 parameters of an IM

X_1	0.12
r_2	0.0116
X_2	0.051
X_m	3.904
X	4.024
X'	0.170342
T_o	1.08527
H	1.5

4. VOLTAGE SWING MODE

Table 3 shows eigenvalues of the system with AVR ($K_A=10.0$, $T_A=0.1$) and without AVR. Here, the power of an IM; $P_{IM}=0.95$ [PU].

In case with AVR, there are three pairs of complex conjugate eigenvalues (①, ②, ③).

The contribution factors of eigenvalues ① and ② are shown in Table 4.

From Table 4, it can be said that mode ① is voltage swing mode and mode ② is power swing mode. In case without AVR, there is no mode corresponds to mode ①.

Table 3 Eigenvalues of the system

	eigenvalues (with AVR)	eigenvalues (without AVR)	
①	$\begin{Bmatrix} -0.299+j0.281 \\ -0.299-j0.281 \end{Bmatrix}$	$\begin{Bmatrix} -4.249+j1.990 \\ -4.249-j1.990 \end{Bmatrix}$	④
②	$\begin{Bmatrix} -4.287+j1.986 \\ -4.287-j1.986 \end{Bmatrix}$	$\begin{Bmatrix} -12.35+j168.9 \\ -12.35-j168.9 \end{Bmatrix}$	⑤
③	$\begin{Bmatrix} -12.34+j168.9 \\ -12.34-j168.9 \end{Bmatrix}$	$\begin{Bmatrix} -28.15+j0.0 \\ -22.27+j0.0 \end{Bmatrix}$	
	$-28.32+j0.0$	$-1.171+j0.0$	
	$-22.21+j0.0$	$-0.881+j0.0$	
	$-9.508+j0.0$	$-0.136+j0.0$	
	$-0.146+j0.0$	$0.0+j0.0$	
	$0.0+j0.0$		

Table 4 Contribution factors of eigenvalues ① and ②

	contribution factors of eigenvalues ①	contribution factors of eigenvalues ②
λ_d	0.001625	0.006693
λ_F	1.000000	0.027512
λ_D	0.050694	0.050656
λ_q	0.007570	0.011841
λ_Q	0.100604	0.116334
ω	0.018303	0.226719
δ	0.024245	0.224496
ΔE_{FD}	0.249062	0.008244
e_R	0.958855	0.206261
e_I	0.386855	1.000000
S	0.094270	0.869923

The existence of the voltage swing mode in addition to the power swing mode has been discussed in recent literatures. The voltage swing mode can be understood as a mode of oscillation likely to occur in a closed loop including generator terminal voltage(V_t), AVR, field circuit, terminal voltage of an IM, rotor circuit of the IM, and back to V_t , but not including the inertia of generator nor IM.

5. OCCURRENCE OF HOPF BIFURCATION

When the power of an IM (P_{IM}) is chosen as a bifurcation parameter in the system shown in fig.2 (with AVR), the eigenvalues of the system changes as follows.

In the region P_{IM} is at a low level, eigenvalues of the system are all located in the left half plane. As P_{IM} increases, a pair of eigenvalues associated with the voltage swing mode moves toward right half plane. At $P_{IM}=1.0375$ [PU], the pair of eigenvalues crosses the imaginary axis. This is the point Hopf bifurcation occurs. A stable equilibrium point bifurcates into the periodic orbit (limit cycle), and the radius of the limit cycle grows larger as the parameter increases.

The movement of an eigenvalue associated with the voltage swing mode is shown in Fig.4.

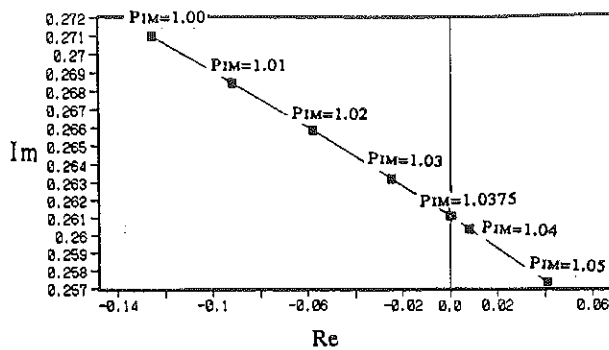


Fig.4 The movement of an eigenvalue associated with the voltage swing mode

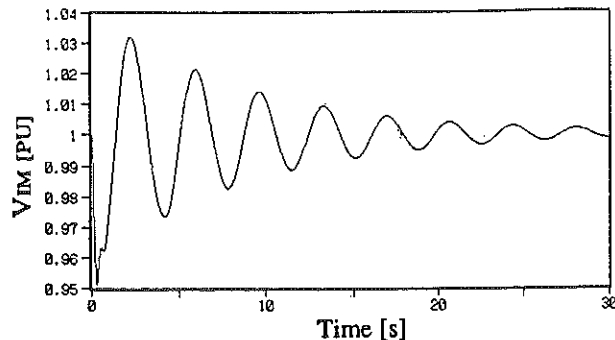


Fig.5 Response to a small disturbance in a pre-bifurcation state ($P_{IM}=0.95$)

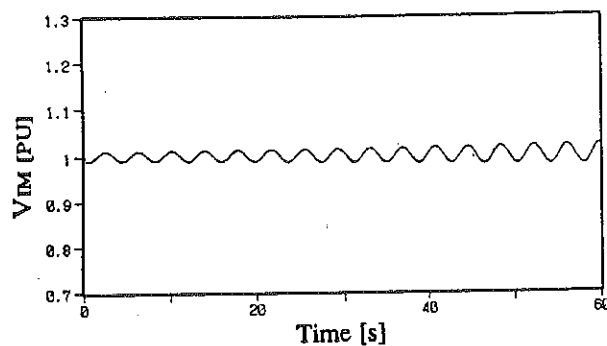


Fig.6 Response to a small disturbance in the Hopf bifurcation state ($P_{IM}=1.04$)

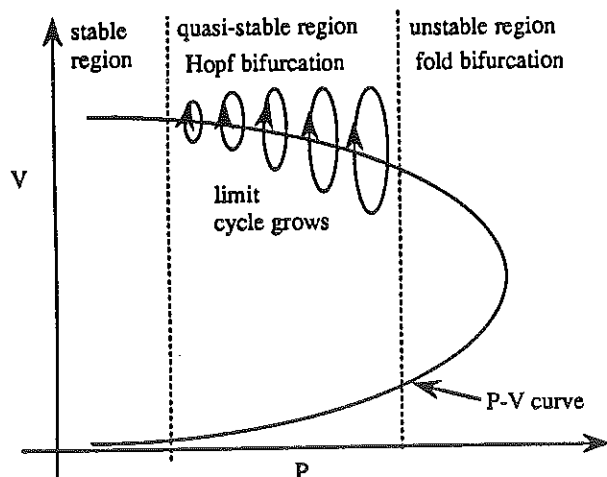


Fig.7 Transition of system state from stable region, via quasi-stable, to unstable region (a concept)

In a pre-bifurcation state, when a small disturbance is applied to the system, the state of it goes as shown in Fig.5. It can be identified that the oscillation of terminal voltage of an IM (V_{IM}) converges as time passes.

In the Hopf-bifurcation state, when a small disturbance is applied to the system, the state of it goes as shown in Fig.6. Sustained oscillations are observed.

6. SCENARIO OF VOLTAGE COLLAPSE

Suppose that the shaft torque of an IM load is increasing in the system. The transition of system states starting from a stable region then moving toward the voltage collapse is interpreted and proposed as shown in Fig.7. Theoretical interpretation of a voltage collapse scenario is as follows:

Region I : Stable region.

Eigenvalues of the system are all located in the left half plane.

Region II : Quasi-stable region where the Hopf bifurcation dominates.

Only a pair of complex conjugate eigenvalues is located in the right half plane.

Sustained oscillations (a stable limit cycle with a long period) are observed. In this region, oscillation energy is dissipated during the excursion of a stable-equilibrium point (SEP) along the slope of the slip vs torque characteristics of the IM.

Region III : Unstable region where the fold (saddle-node) bifurcation dominates.

Further increase of IM shaft torque beyond the maximum torque (T_{max}) will move the operating point of the IM out of the normal range, when both stable and unstable equilibria of the slip vs torque characteristics of the IM disappear together (occurrence of fold bifurcation).

Fig.8 shows transition of system voltage (V_{IM}) with increasing shaft torque of IM load. The oscillation of V_{IM} becomes large as time passes, and finally, an aperiodic sudden drop of system voltage is observed.

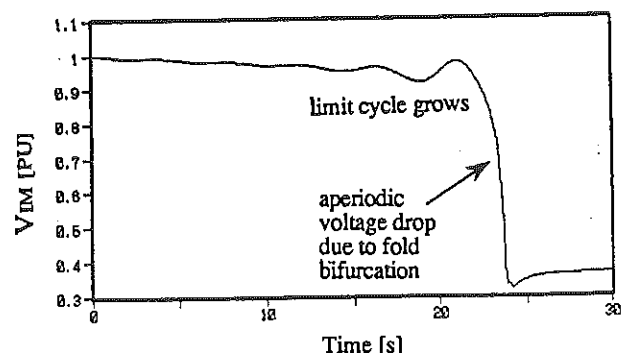


Fig.8 Transition of system voltage with increasing shaft torque of IM load

7. THE EFFECT OF AVR

In this section, P_{im} is fixed to 1.02[PU] and the effect of AVR is examined.

(a) The effect of K_A

When the T_A , time constant of AVR, is fixed to 0.10, and the K_A , gain, is chosen as the bifurcation parameter, the stability of the system is examined.

At the point $K_A=10.0$, eigenvalues of the system are all located in the left half plane. As K_A decreases, a pair of eigenvalues associated with the voltage swing mode moves toward the right half plane. This event indicates that the system damping is weakened by the gradual change of parameter K_A . Fig.9 shows the movement of an eigenvalue.

(b) The effect of T_A

When the K_A is fixed to 10.0, and the T_A is chosen as the bifurcation parameter, the stability of the system is examined.

Fig.10 indicates that as T_A increases, a pair of eigenvalues associated with the voltage swing mode moves toward the right half plane.

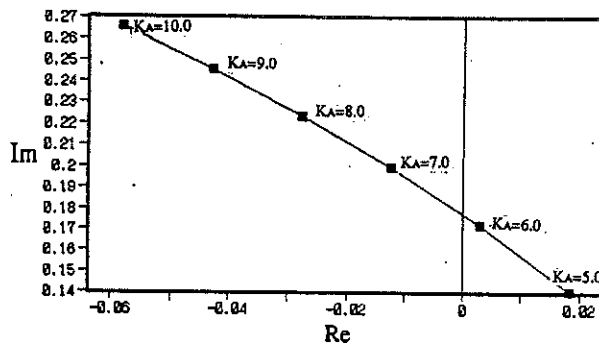


Fig.9 The movement of an eigenvalue associated with the voltage swing mode (K_A is chosen as bifurcation parameter)

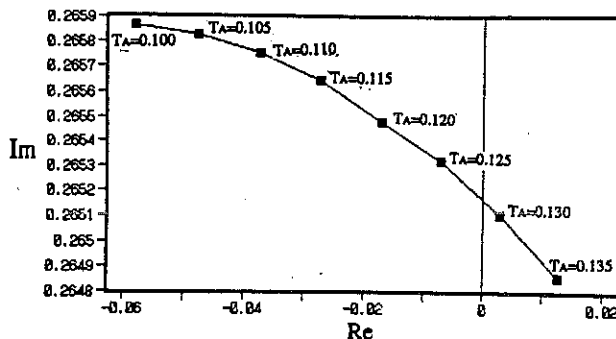


Fig.10 The movement of an eigenvalue associated with the voltage swing mode (T_A is chosen as bifurcation parameter)

Fig.11 shows transition of system voltage with decreasing gain of AVR, after a small disturbance. Note that T_A is fixed to 0.10 and the initial states of K_A , P_{im} are 10.0, 1.02[PU], respectively.

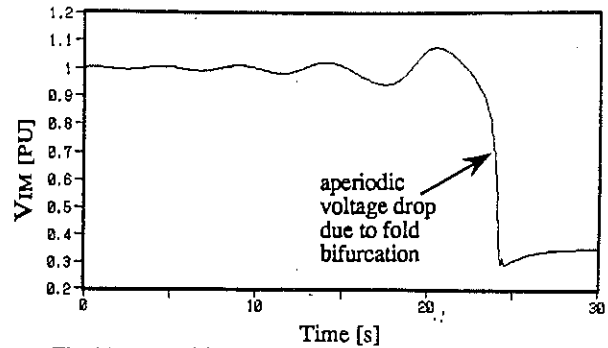


Fig.11 Transition of system voltage with decreasing gain of AVR

8. CONCLUSION

In this paper, a relationship between voltage swing mode and voltage collapse is discussed from a viewpoint of Hopf bifurcation and fold bifurcation theory.

Confirmation of the proposed scenario of voltage collapse with a series of simulations was made with success. The algorithm used is applicable to power systems with a practical size.

ACKNOWLEDGEMENT

The author is very grateful to Mr.T.Motoyoshi, presently with TEPCO, for his research efforts, and to Messrs K.Sakashita and N.Umeda for their compilation work.

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Stability Enhancement via Secondary Voltage Regulation

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Abstract: It is commonly believed that the danger of voltage instability and collapse in heavily loaded power networks can be significantly reduced by secondary voltage control. Such voltage regulators must alter the bifurcation structure of the network. Issues involved with the design of regulators which do so are discussed and illustrated.

1 Introduction

Utilization of electrical energy is intrinsic in American society and continues to grow at a rate which exceeds GNP growth. Yet, the high risks associated with electrical utility capital investments inhibit the construction of new generating plants and transmission facilities. Consequently, energy transfers between utilities have dramatically increased along with transmission line loadings. The result is that the national grid routinely operates unprecedently close to stability limits. In fact, DOE's Russell Eaton sees the US transmission and distribution system as a national security risk [9]. Unquestionably the most immediate consequence of increased stress on the transmission network, in North America and abroad, is the pervasive occurrence of voltage stability problems and the increased risk of voltage collapse. Furthermore, these problems are not about to go away [6] and their solution will require innovative new approaches to voltage control. Indeed, as noted by Lachs [17], the considerable extra amount of reactive power compensation required to prevent collapses by conventional techniques would impose an unacceptably high cost on utilities.

Voltage stability has been the focus of considerable academic and industry inquiry during the past several years. Much of this effort has been devoted to developing an understanding of the mechanics of voltage collapse, to the assessment of system proximity to instability, and to the development of broad planning and operating strategies to deal with system voltage related emergencies. The usual characterization of power system voltage instability and collapse begins with the occurrence of a severe disturbance (Lachs [17], Clark [6], etc.). However, voltage collapse is particularly insidious because it can occur in apparently healthy systems without a significant triggering event (e.g., Clark [6]). It is now understood that this is because the underlying cause is typically a bifurcation associated with the underlying nonlinear power system dynamics and which may be initiated by a small change in system parameters (e.g., [2,12]).

Less attention has been given to the development of feedback control mechanisms which might enhance stability margins and prevent the development of critical situations. The control of voltage on electric power networks involves a hierarchical structure with as many as three layers referred to as primary, secondary and tertiary control. Generally, *primary control* consists of generator automatic voltage regulators (AVR) and under-load tap-changing (ULTC) transformers. These devices respond automatically to local measurements (e.g., generator terminal bus voltage) and operate on a time scale of seconds. *Secondary control* involves determination of the reference or set-point values for the primary controllers. It is a regional control function, often, but not always, automatic and it operates on a time scale of minutes. This level of control may also manage capacitor switching, load shedding and/or other network topology changes. *Tertiary control* is multiregional. Its purpose is to coordinate (perhaps optimize) regional actions and usually functions manually on a time scale of tens of minutes or longer.

In recent years there has been considerable interest in secondary voltage control as a means of enhancing voltage stability. One approach, pioneered by Electricité de France [18,19,21], is the real time regulation of voltage at preselected load busses called *pilot nodes*. In this approach, the network is divided into *control zones*, and a single pilot node is specified in each zone. The adjustable reactive power resources in each zone are used to control its pilot node voltage. Various studies have focused on the identification of control zones and pilot nodes [11,18,19,21,23]. Others have examined related and important issues of controllability [22]. In all of these investigations the analysis is based on a static model of the power network, i.e., the load flow model, and when

voltage collapse is an explicit issue the underlying paradigm is the saddle-node bifurcation.

Very little is known about the dynamical behavior of the closed loop voltage regulated system when operating near voltage stability limits. Indeed, very little is known, in general, about feedback control near bifurcation points although this is an area of current research [1,13,14]. Recent results by Kwatny, et al [13,14] show that feedback regulation can change the (local) static and dynamic bifurcation structure of the controlled system. Thus, the potential for expanding voltage stability limits by feedback regulation is clearly present. However, the changes to the system stability characteristics are quite complex. For example, it is possible that while relaxing a stability limit associated with a saddle-node bifurcation, the regulated system stability boundary may then be characterized by a Hopf bifurcation and the stable region in the parameter space may actually shrink. Such a situation could not be predicted by an equilibrium analysis.

In Section 2 we describe the secondary voltage control problem. The essential function of secondary voltage control is to alter the static bifurcation behavior of the power network in order to extend the static stability region. An example clearly illustrates how this can be accomplished. However, it will also be seen that secondary voltage control profoundly influences dynamical stability and we show that in the regulated system the static stability limits (static bifurcation points) no longer correlate with dynamic stability limits. In Section 3 we summarize and extend recent results on the design of linear and nonlinear regulators for parameter dependent nonlinear systems which are relevant to the voltage control problem. In Section 4 we give an interpretation of static bifurcations in terms of system zeros and give some important results on simultaneous stabilizability which provide a useful insight into the extent of the (parameter space) stability domain. Some concluding remarks constitute Section 5.

2 Automatic Voltage Regulation

For simplicity of exposition, we limit our discussion of voltage control to the case in which generator excitation is the only control means. Moreover, we formulate the voltage regulation problem without an *a priori* distinction between primary and secondary voltage control. However, it will be evident that such a structure is one natural solution to the general regulator problem provided that certain stabilizability conditions obtain.

2.1 The Power System

We will consider a fairly general *classical* power system model composed of $n+m+l$ buses where buses $i=1,...,n$ are the internal buses of n generators, buses $i=n+1,...,n+m$ are m (locally voltage controlled) PV load buses and buses $i=n+m+1,...,n+m+l$ are l (constant power) PQ load buses. The interconnecting network is considered to be the equivalent reduced network resulting from the elimination of the constant admittance loads and network internal buses. Although we consider constant P and Q load models for simplicity, nonlinear voltage dependent load models involving smooth functions $P(V)$, $Q(V)$ are easily accommodated in our analysis. Obviously, as we admit PQ buses, it is not necessary to eliminate all internal buses or constant admittance loads.

The dynamical equations of motion of the classical power system model may be written as, e.g., [12]:

$$M\ddot{\delta} + D\dot{\delta} + f_1(\delta, \phi, V, u, \mu) = 0 \quad (2.1a)$$

$$f_2(\delta, \phi, V, u, \mu) = 0 \quad (2.1b)$$

where M denotes the diagonal matrix of generator rotor inertias, D the damping matrix, δ the n -vector of generator internal bus angles, ϕ the $m+l$ -vector of load bus angles, V the l -vector of PQ load bus voltage magnitudes, u the n -vector of generator internal bus voltages (the controls), and μ a k -vector of network and load parameters. The

functions $f_1: \mathbb{R}^{2n+m+2l+k} \rightarrow \mathbb{R}^n$ and $f_2: \mathbb{R}^{2n+m+2l+k} \rightarrow \mathbb{R}^{m+2l}$ are the usual load flow relations.

Let $(\delta^*, \phi^*, V^*, u^*, \mu^*)$ be an equilibrium point of (2.1). Suppose that the equilibrium point is *strictly causal* in the sense that there exist unique functions $\phi(\delta, u, \mu)$, $V(\delta, u, \mu)$ satisfying $f_2(\delta, \phi(\delta, \mu), V(\delta, \mu), u, \mu) = 0$ in a neighborhood of $(\delta^*, \phi^*, V^*, u^*, \mu^*)$ with $\phi(\delta^*, u^*, \mu^*) = \phi^*$ and $V(\delta^*, u^*, \mu^*) = V^*$. Under these circumstances (2.1) is locally equivalent to the system

$$M\ddot{\delta} + D\dot{\delta} + \phi(\delta, u, \mu) = 0 \quad (2.2)$$

where $\phi(\delta, u, \mu) = f_1(\delta, \phi(\delta, \mu), V(\delta, u, \mu), u, \mu)$. Moreover, the linearized dynamics of (2.1) reduce to the form

$$M\Delta\ddot{\delta} + D\Delta\dot{\delta} + K\Delta\delta = G\Delta u \quad (2.3)$$

where

$$K = \left\{ D_{\delta}f_1 - [D_{\phi}f_1 \ D_{Vf_1}] \begin{bmatrix} D_{\phi}f_2 \\ D_{Vf_2} \end{bmatrix}^{-1} D_{\delta}f_2 \right\}^* \quad (2.4a)$$

$$G = \left\{ D_u f_1 - [D_{\phi}f_1 \ D_{Vf_1}] \begin{bmatrix} D_{\phi}f_2 \\ D_{Vf_2} \end{bmatrix}^{-1} D_u f_2 \right\}^* \quad (2.4b)$$

and $\Delta\delta = \delta - \delta^*$, $\Delta u = u - u^*$. In general, M is positive definite and D is positive semi-definite. However, the matrix K need not be symmetric nor have real positive eigenvalues. The equilibrium point is open loop ($\Delta u = 0$) stable only if all of the eigenvalues of K are real and positive. Note that the matrices K and G depend on the parameter μ , both explicitly through the Jacobian matrices in (2.4) and implicitly through the variation of the equilibrium point with μ .

Near a strictly causal equilibrium point the nonlinear power system dynamics are locally characterized by (2.2), or in state space form

$$\dot{x} = f(x, u, \mu), \quad x := \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix}, \quad f(x, u, \mu) := \begin{bmatrix} \dot{\delta} \\ -M^{-1}D\dot{\delta} - M^{-1}\phi(\delta, u, \mu) \end{bmatrix} \begin{bmatrix} \delta \\ \dot{\delta} \end{bmatrix} \rightarrow x \quad (2.5)$$

Assumption 1 - Causality: In the following we restrict the discussion to regulation near a strictly causal equilibrium so that (2.5) is a valid local characterization of nonlinear power system behavior. Because of this assumption some interesting and potentially significant behavior is not addressed in our analysis. For further discussion of the significance of causality see [7,10,12].

2.2 The Voltage Regulation Problem

The voltage regulation problem is to design a feedback controller which continuously determines the control values $u(t)$ from the evolving time history of a set of measurements $y(t)$ such that (i) the closed loop system is stable and (ii) the voltage magnitudes at a preselected set of network buses ultimately approach specified values. The measurement vector, y , is composed of various bus angles and voltage magnitudes, and possibly their rates of change. Consequently, by using the locally defined functions $\phi(\delta, u, \mu)$, $V(\delta, u, \mu)$, we can always write

$$y = g(x, u, \mu) \quad (2.6)$$

We assume that the voltage is to be regulated at a subset of the PQ busses and denote the index set of these busses by I_c . Since any internal bus may be treated as a PQ bus, this assumption poses no significant restriction. Let z denote the vector of voltage errors associated with the subset of busses chosen for regulation. Then, we have

$$z = h(x, u, \mu) \quad (2.7)$$

where the elements of h are of the form $V_i(\delta, u, \mu) - V_i$ where $i \in I_c$ and V_i is the desired voltage at bus i .

2.3 An Example

We consider a simple example in order to motivate and illustrate the concepts developed herein. Consider the system illustrated in Figure 2.1.

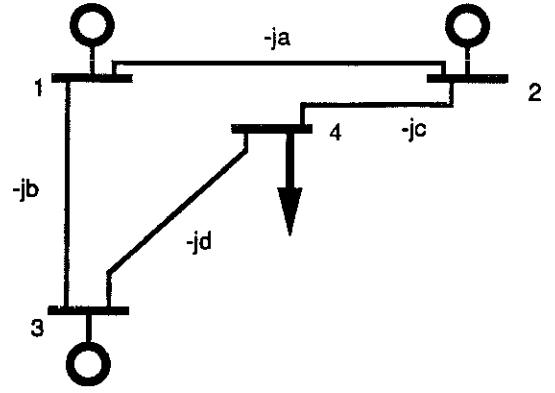


Figure 2.1. A four bus example consisting of three generators and a constant power load.

The admittance matrix is defined by

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} -j(a+b) & ja & jb & 0 \\ ja & -j(a+c) & 0 & jc \\ jb & 0 & -j(b+d) & jd \\ 0 & jc & jd & e+jg \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

where $g = f - c - d$. Let us assume $J_i = 1$, for $i = 1, 2, 3$ and $V_1 = u$, $V_i = 1$, for $i = 2, 3$. If we take bus 1 as a swing bus and define the variables $\theta_1 = \delta_2 - \delta_1$, $\theta_2 = \delta_3 - \delta_1$, $\theta_3 = \delta_4 - \delta_1$ then the swing equations can be reduced to

$$\ddot{\theta}_1 + 2u \sin(\theta_1) - V_4 \sin(\theta_3 - \theta_1) + .5774u \sin(\theta_2) = \Delta P_1 \quad (2.8a)$$

$$\ddot{\theta}_2 + 1.1548u \sin(\theta_2) - V_4 \sin(\theta_3 - \theta_2) + u \sin(\theta_1) = \Delta P_2 \quad (2.8b)$$

$$V_4 \sin(\theta_3 - \theta_1) + V_4 \sin(\theta_3 - \theta_2) = P_4 \quad (2.8c)$$

$$-V_4 \cos(\theta_3 - \theta_1) + V_4 \cos(\theta_3 - \theta_2) + 2V_4^2 = Q_4 \quad (2.8d)$$

where $\Delta P_1 = P_2 - P_1$ and $\Delta P_2 = P_3 - P_1$.

To analyze the system equilibrium point structure near a given operating point, we identify the closest static bifurcation point, classify it, determine which system dependent variables are relevant, identify a minimal set of physical parameters required to characterize the bifurcation, locally characterize the bifurcation surface in parameter space (the *bifurcation set*), and graph the equilibrium surface as appropriate. This is achieved by performing the following sequence of steps:

1. specify the control value u (voltage at bus 1)
2. specify initial values for the four parameters ΔP_1 , ΔP_2 , P_4 , Q_4
3. locate the closest (Euclidean norm in parameter space) static bifurcation point of load flow equations; the result is $[\theta_1, \theta_2, \theta_3, V_4, \Delta P_1, \Delta P_2, P_4, Q_4] = [1.082, 1.977, 1.172, 1.298, 2.180, 2.880, -0.819, 1.180]$
4. identify principle dependent variable in null space spanning vector: the result is V_4
5. from Liapunov-Schmidt reduction
 - (a) identify codimension of bifurcation: result is 1;
 - (b) determine universal unfolding and identify the most sensitive physical parameter: result is P_4
6. plot equilibrium surface V_4 vs. P_4 with all other parameters constant

The result of these computations is Figure 2.2. We can analyze the effect of the control voltage V_1 by:

7. specify another control value and generate equilibrium surface V_4 vs. P_4
8. repeat 7 as many times as desired

In this way Figure 2.3 is obtained.

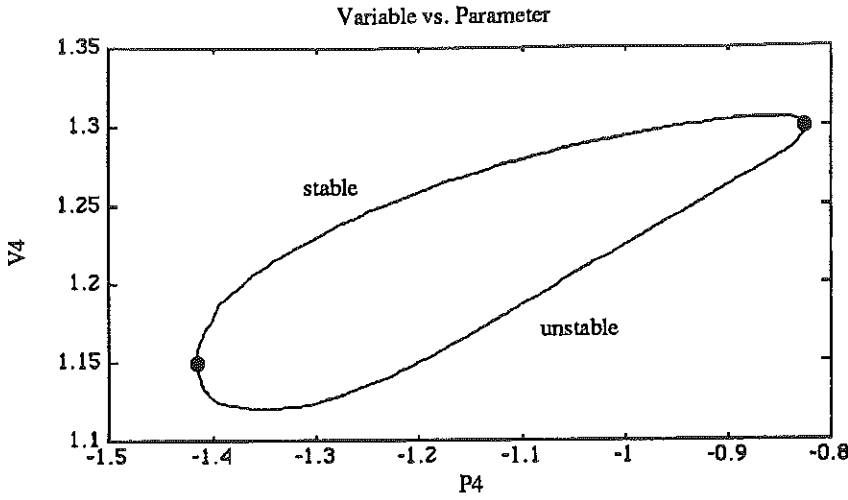


Figure 2.2. The equilibrium surface illustrates two saddle node bifurcation points. The one on the upper right is that originally determined as closest to the initial operating point. The second was discovered during the computation of the equilibrium surface. Near the bifurcation points, a variant of the Newton-Raphson-Seydel method is used [15,24]. Otherwise, a standard Newton-Raphson is employed. Open loop system eigenvalues are computed at each equilibrium point from which it is determined that the upper branch corresponds to open loop stable equilibria and the lower branch to open loop unstable equilibria.

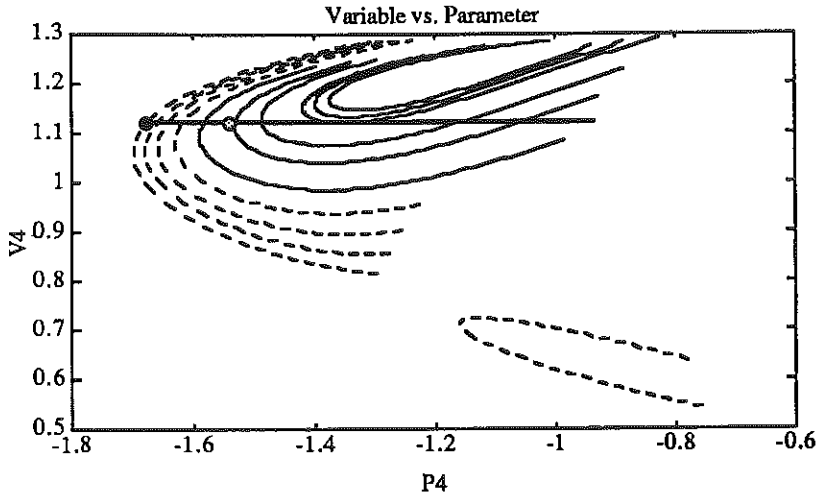


Figure 2.3. A family of equilibrium surface for various values of the control voltage V_1 is illustrated. The curves on the top from right to left represent $V_1=0.98, 0.99, 1.0, 1.05, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6$. The curve at bottom represents $V_1=1.65$. The solid line represents operating points which correspond to a load bus voltage $V_4 = 1.05$. Note that as load increases (real power becomes more negative), the excitation voltage, V_1 , increases. A bifurcation occurs at about $V_1 = 1.61$ (the solid dot). Load voltage cannot be maintained with further load increases if V_1 is the only control. Note that we have not imposed any limits on excitation voltage, so that this phenomenon is intrinsic to the system. The gray dot corresponds to a stability limit. Equilibria between the two dots are open loop stable. To the right of the gray dot the equilibria are open loop unstable.

In the following sections, we will discuss the nature of the bifurcation point observed in the attempt to regulate load voltage and we will develop and clarify the issues it raises with respect to regulator design.

3 Regulator Design

In this section we discuss the general formulation and solution of regulator problems. As will be seen, our view of regulation involves stabilization as well as the ultimate removal of errors from selected outputs. The nonlinear problem described herein is a minor generalization of that previously reported in [13,14] and constitutes a direct extension of linear regulator theory [8,16].

3.1 Definition of the Local Regulator Problem

Consider the nonlinear dynamical system

$$\dot{x} = f(x, u, \mu) \quad (3.1a)$$

$$y = g(x, u, \mu) \quad (3.1b)$$

$$z = h(x, u, \mu) \quad (3.1c)$$

where, $x \in \mathbb{R}^n$ is the system state, $u \in \mathbb{R}^m$ is the control, $y \in \mathbb{R}^q$ is the measurement, and $z \in \mathbb{R}^p$ is the regulated output, $\mu \in \mathbb{R}^k$ is a parameter vector which may be composed of plant parameters, exogenous constant disturbances and/or set points. We assume that the functions f , g and h are smooth (sufficiently differentiable) and our objective is to design a feedback regulator which stabilizes a desired equilibrium point corresponding to $z=0$. A triple (x^*, u^*, μ^*) is an *equilibrium point* of the open loop dynamics if

$$F(x^*, u^*, \mu^*) = \begin{bmatrix} f(x^*, u^*, \mu^*) \\ h(x^*, u^*, \mu^*) \end{bmatrix} = 0 \quad (3.2)$$

We typically obtain equilibria by specifying μ^* and solving equation (3.2) for x^* , u^* . Then $y^* := g(x^*, u^*, \mu^*)$. Typically, it is expected that (3.2) will have solutions only if $m \geq p$. Since the number of controls can always be reduced, we assume henceforth that $m=p$.

In the following paragraphs we consider two types of regulators, either state feedback

$$u = k(x, \mu) \quad (3.3)$$

with $k(x^*, \mu^*) = u^*$, or dynamic output feedback

$$u = \eta(v, y) \quad (3.4a)$$

$$\dot{v} = \phi(v, y) \quad (3.4b)$$

where $v \in \mathbb{R}^r$ with $\phi(v^*, y^*)=0$ and $\eta(v^*, y^*)=u^*$.

Assumption 2 - Solvability: We assume that the composite relation $u = \eta(v, g(x, u, \mu))$ is at least locally solvable for u , i.e., there exists a subset $\Omega \subset \mathbb{R}^r \times \mathbb{R}^n \times \mathbb{R}^k$ and a function $\kappa(v, x, \mu)$ such that

$$\kappa(v, x, \mu) = \eta(v, g(x, \kappa(v, x, \mu), \mu)) \text{ for } (v, x, \mu) \in \Omega$$

Hence $u = \kappa(v, x, \mu)$ on Ω . This assumption allows us to resolve the implicit (algebraic) loop present in the feedback system. Note that,

ordinarily, when using separation principle design with a full state observer solvability is trivially (and globally) obtained because y is absent from (3.4a). On the other hand, when using reduced order observers or fixed order compensation y is typically present in (3.4a).

Correspondingly, the closed loop dynamics with state feedback are of the form

$$\dot{x}_L = f_L(x_L, \mu) := f(x_L, k(x_L, \mu), \mu), \quad x_L = x \in \mathbb{R}^n \quad (3.5)$$

or in the case of dynamic feedback, the closed loop dynamics are defined (on Ω) by

$$\dot{x}_L = f_L(x_L, \mu) := \begin{bmatrix} f(x, \kappa(v, x, \mu), \mu) \\ \phi(v, g(x, \kappa(v, x, \mu), \mu)) \end{bmatrix} \begin{bmatrix} x \\ v \end{bmatrix} \rightarrow x_L, \quad x_L = [x^T, v^T]^T \in \mathbb{R}^{n+r} \quad (3.6)$$

The regulator problem is defined as follows.

The Local Regulator Problem: Determine a feedback control law of type (3.3) or (3.4) so that the following two conditions obtain

1.) *stability:* for each $\mu \in U$, a neighborhood of μ^* , the closed loop has an exponentially stable equilibrium point characterized by

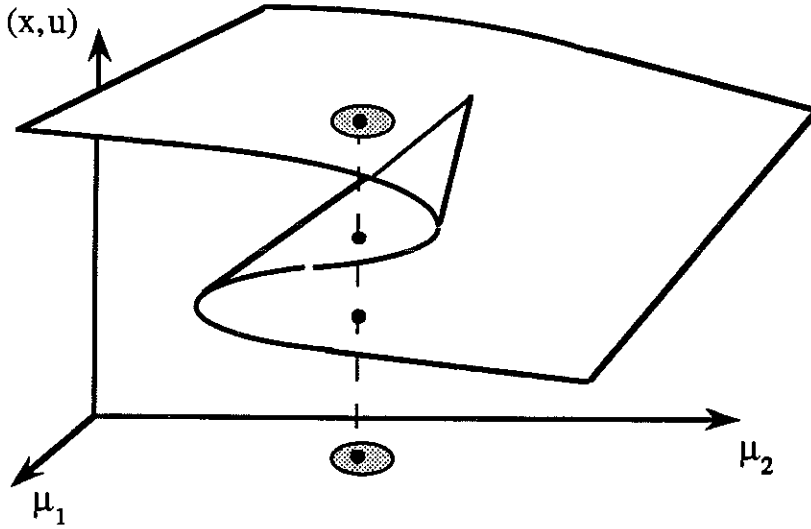


Figure 3.1 A typical equilibrium manifold is illustrated with the nominal (design) parameter value and three corresponding equilibria identified by the dots. The actual parameter value will lie in a neighborhood of the nominal as illustrated by the shaded region in the parameter plane. A local regulator will steer the system to the appropriate point in a neighborhood of the design point as depicted by shaded region in the equilibrium manifold.

The following proposition illustrates the intimate connection between stability and the existence of equilibria under parameter perturbations.

Theorem 3.1: Suppose x_L^* denotes an exponentially stable equilibrium point of the closed loop dynamics at the parameter value $\mu = \mu^*$. Then there exists a function $\hat{x}_L(\mu)$ on a neighborhood U of μ^* with $\hat{x}_L(\mu^*) = x_L^*$ and which satisfies the relation

$$f_L(\hat{x}_L(\mu), \mu) = 0 \quad (3.8)$$

Proof: By hypothesis we have an exponentially stable equilibrium point which satisfies

$$f_L(x_L^*, \mu^*) = 0. \quad (3.9)$$

Notice that exponential stability implies that the Jacobian $D_{x_L} f_L(x_L^*, \mu^*)$ is nonsingular. The result follows from application of the implicit function theorem.

The theorem implies that the stability requirement of the local regulator problem reduces to that of exponential stability of the equilibrium point at the single parameter value $\mu = \mu^*$.

the function $\hat{x}_L(\mu)$ with $\hat{x}_L(\mu^*) = x^*$ and $k(x^*, \mu^*) = u^*$ in the case of state feedback, or $\hat{x}_L(\mu^*) = [x^{*T}, v^{*T}]^T$ and $\eta(v^*, y^*) = u^*$ in the case of dynamic feedback.

3.) *regulation:* $z(t) \rightarrow 0$ as $t \rightarrow \infty$ for all $\mu \in U$, $x_L(0) \in X_L$, a neighborhood of $\hat{x}_L(\mu^*)$.

Remark 3.1 About the local regulator Problem: Consider the set of points in $\mathbb{R}^{n+m+k}$ that satisfy $F(x, u, \mu) = 0$ (equation (3.2)):

$$\mathcal{E} = \{ (x, u, \mu) \in \mathbb{R}^{n+m+k} \mid F(x, u, \mu) = 0 \} \quad (3.7)$$

We assume that $\text{rank}[D_x F \ D_u F \ D_\mu F] = n+m$ on $\mathcal{E}$. Then $\mathcal{E}$ is a regular manifold of dimension k in $\mathbb{R}^{n+m+k}$ and we refer to $\mathcal{E}$ as the open loop equilibrium manifold. $\mathcal{E}$ is the manifold of (output regulated) equilibria of the system (3.1). Figure 3.1 depicts a typical equilibrium manifold for a problem with two parameters. The nominal parameter value illustrated is associated with three equilibria. Suppose the uppermost is chosen as the equilibrium at which the regulator is to be designed. The actual parameter value will belong to a sufficiently small but unspecified neighborhood of μ^* as illustrated in the parameter plane. Our objective is to design a regulator which will identify and stabilize the corresponding equilibrium point in a neighborhood of the nominal equilibrium point.

We employ the concepts of exponential stabilizability and detectability of (3.1) as characterized by Byrnes and Isidori [5]:

Definition 3.1: The system $\dot{x} = f(x, u)$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ with $f(x^*, u^*) = 0$ is *exponentially stabilizable* at (x^*, u^*) if there exists a function $u = k(x)$, defined on a neighborhood of x^* in $\mathbb{R}^n$ with $u^* = k(x^*)$ so that the equilibrium point, $x = x^*$, of

$$\dot{x} = f(x, k(x)) \quad (3.10)$$

is exponentially stable.

Definition 3.2: The system $\dot{x} = f(x, u)$, $y = g(x, u)$, $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^q$ with $f(x^*, u^*) = 0$ and $y^* = g(x^*, u^*)$ is *exponentially detectable* at (x^*, u^*) if there exists a system

$$\dot{\xi} = \gamma(\xi, \psi), \quad \xi \in \mathbb{R}^n, \quad \psi \in \mathbb{R}^q \quad (3.11)$$

where the function γ is defined on a neighborhood of $(x^*, 0)$ in $\mathbb{R}^n \times \mathbb{R}^q$ and the following conditions are satisfied:

$$\gamma(x^*, 0) = 0, \quad (3.12a)$$

$$\gamma(x, g(x, u^*) - y^*) = f(x, u^*), \quad (3.12b)$$

and the equilibrium point $\xi = x^*$ of the system

$$\dot{\xi} = \gamma(\xi, 0) \quad (3.12c)$$

is exponentially stable.

Remark 3.2 Observer construction: Exponential detectability implies that the system

$$\dot{v} = \gamma(v, y - g(x^*, u)) + f(v, u) - f(v, u^*) \quad (3.13)$$

is a local observer in the sense that $\|x(t) - v(t)\| \rightarrow 0$ as $t \rightarrow \infty$ provided $x(t)$ remains sufficiently close to x^* . To see this, consider the coupled (system and observer) equations

$$\begin{aligned} \dot{x} &= f(x, u) \\ \dot{v} &= \gamma(v, g(x, u) - g(x^*, u)) + f(v, u) - f(v, u^*) \end{aligned}$$

First notice that, in view of (3.12a), $x = x^*$, $u = u^*$ and $v = x^*$ defines an equilibrium point. Define

$$A := \frac{\partial f}{\partial x}(x^*, u^*), \quad B := \frac{\partial f}{\partial u}(x^*, u^*)$$

so that the perturbation equations can be written

$$\begin{aligned} \Delta \dot{x} &= A \Delta x + B \Delta u \\ \Delta \dot{v} &= \frac{\partial \gamma}{\partial \xi}(x^*, 0) \Delta v + \frac{\partial \gamma}{\partial \psi}(x^*, 0) \frac{\partial g}{\partial x}(x^*, u^*) \Delta x + B \Delta u \end{aligned}$$

Subtracting the first of these from the second yields

$$\Delta \dot{v} - \Delta \dot{x} = \frac{\partial \gamma}{\partial \xi}(x^*, 0) \Delta v + \frac{\partial \gamma}{\partial \psi}(x^*, 0) \frac{\partial g}{\partial x}(x^*, u^*) \Delta x - A \Delta x$$

From (3.12b) we obtain

$$\frac{\partial \gamma}{\partial \xi}(x^*, 0) = A - \frac{\partial \gamma}{\partial \psi}(x^*, 0) \frac{\partial g}{\partial x}(x^*, u^*)$$

so that the estimation error, $\varepsilon := v - x$, satisfies the perturbation equation

$$\Delta \dot{\varepsilon} = \frac{\partial \gamma}{\partial \xi}(x^*, 0) \Delta \varepsilon$$

which is asymptotically stable by assumption of exponential detectability.

Remark 3.3 Stabilization by linear feedback: If a system is exponentially stabilizable at (x^*, u^*) , the equilibrium point (x^*, u^*) can be stabilized by linear state feedback. If it is exponentially stabilizable and detectable at (x^*, u^*) , the equilibrium point (x^*, u^*) can be stabilized with linear dynamic feedback.

Assumption 3 - Stabilizability & Detectability: Let (x^*, u^*, μ^*) satisfy (3.2). We make the following standing assumptions:

- The number of regulated outputs is the same as the number of controls, $m=p$.
- The system $\dot{x} = f(x, u, \mu^*)$ is exponentially stabilizable at (x^*, u^*) .
- The composite system $\dot{x} = f(x, u, \mu)$, $\mu = 0$, $y = g(x, u, \mu)$ is exponentially detectable at (x^*, u^*, μ^*) .

3.2 Existence of Solutions & Construction of Regulators

We now provide the basic necessary and sufficient conditions for the existence of solutions to the local regulator problem and in doing so provide an explicit method for regulator design. Our development parallels well known constructions for linear systems as described by Kwatny and Kalnitsky in [16] and the numerous references in that paper.

In particular, many of our arguments follow those of Francis [8] for the linear case in which stronger conditions can be obtained.

Theorem 3.2: Let

$$\dot{x}_L = f_L(x_L, \mu) \quad (3.14a)$$

$$z = h_L(x_L, \mu) \quad (3.14b)$$

denote a closed loop system. Let (x_L^*, μ^*) be an equilibrium point satisfying

$$f_L(x_L^*, \mu^*) = 0 \quad (3.15a)$$

$$h_L(x_L^*, \mu^*) = 0 \quad (3.15b)$$

and suppose the equilibrium point (x^*, μ^*) is exponentially stable. Then the output z is regulated only if

$$\begin{bmatrix} D_\mu f_L(x_L^*, \mu^*) \\ D_\mu h_L(x_L^*, \mu^*) \end{bmatrix} \in \text{Im} \begin{bmatrix} D_{x_L} f_L(x_L^*, \mu^*) \\ D_{x_L} h_L(x_L^*, \mu^*) \end{bmatrix} \quad (3.16)$$

Proof: As in Theorem 3.1, exponential stability implies the invertability of $D_{x_L} f_L^*$ and the existence of a function $\bar{x}_L(\mu)$ which satisfies

$$f_L(\bar{x}_L(\mu), \mu) = 0$$

with $\bar{x}_L(\mu^*) = x^*$ on a neighborhood U of μ^* . Regulation implies that $\bar{x}_L(\mu)$ also satisfies

$$h_L(\bar{x}_L(\mu), \mu) = 0$$

Differentiation of these conditions leads to

$$-D_{x_L} h_L^* [D_{x_L} f_L^*]^{-1} D_\mu f_L^* + D_\mu h_L^* = 0$$

which is equivalent to (3.16).

We are now in a position to establish necessary conditions for state feedback solution to the regulator problem. First, let us establish some notation. The perturbation equations associated with (3.1a&c) at an equilibrium point (x^*, u^*, μ^*) may be written

$$\delta \dot{x} = A^* \delta x + E^* \delta \mu + B^* \delta u \quad (3.17a)$$

$$\delta \dot{z} = C^* \delta x + F^* \delta \mu + D^* \delta \mu \quad (3.17b)$$

where

$$A^* := \frac{\partial f}{\partial x}(x^*, u^*, \mu^*), \quad B^* := \frac{\partial f}{\partial u}(x^*, u^*, \mu^*), \quad E^* := \frac{\partial f}{\partial \mu}(x^*, u^*, \mu^*) \quad (3.18a)$$

$$C^* := \frac{\partial h}{\partial x}(x^*, u^*, \mu^*), \quad F^* := \frac{\partial h}{\partial \mu}(x^*, u^*, \mu^*), \quad D^* := \frac{\partial h}{\partial u}(x^*, u^*, \mu^*) \quad (3.18b)$$

Theorem 3.3: The local regulator problem at (x^*, u^*, μ^*) has a state feedback solution only if

$$\begin{bmatrix} E^* \\ F^* \end{bmatrix} \in \text{Im} \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \quad (3.19)$$

proof: Let $u = k(x, \mu)$ with $k(x^*, \mu^*) = u^*$ be a solution, so that the closed loop equations are

$$\dot{x} = f(x, k(x, \mu), \mu) \quad (3.20a)$$

$$z = h(x, u, \mu) \quad (3.20b)$$

Now applying Theorem 3.2 to equations (3.20) we obtain

$$\begin{bmatrix} D_{\mu}f^*+D_u f^*D_{\mu}k^* \\ D_{\mu}h^*+D_u h^*D_{\mu}k^* \end{bmatrix} \in \text{Im} \begin{bmatrix} D_x f^*+D_u f^*D_x k^* \\ D_x h^*+D_u h^*D_x k^* \end{bmatrix} \quad (3.21)$$

so that

$$\begin{bmatrix} D_{\mu}f^*+D_u f^*D_{\mu}k^* \\ D_{\mu}h^*+D_u h^*D_{\mu}k^* \end{bmatrix} \in \text{Im} \begin{bmatrix} D_x f^* & D_u f^* \\ D_x h^* & D_u h^* \end{bmatrix} \quad (3.22)$$

which implies (3.19).

Theorem 3.4: The local regulator problem at (x^*, u^*, μ^*) has a dynamic feedback solution only if it has a state feedback solution.

proof: Apply Theorem 3.2 to the closed loop equations (3.6) and (3.1c) to obtain

$$\begin{bmatrix} D_{\mu}f^*+D_u f^*D_{\mu}k^* \\ D_y \phi^* \{D_{\mu}g^*+D_u g^*D_{\mu}k^*\} \\ D_{\mu}h^*+D_u h^*D_{\mu}k^* \end{bmatrix} \in \text{Im} \begin{bmatrix} D_x f^*+D_u f^*D_x k^* & D_u f^*D_v k^* \\ D_y \phi^* \{D_x g^*+D_u g^*D_x k^*\} & D_v \phi^*+D_y \phi^*D_u g^*D_v k^* \\ D_x h^*+D_u h^*D_x k^* & D_u h^*D_v k^* \end{bmatrix} \quad (3.23)$$

This requires that

$$\begin{bmatrix} D_{\mu}f^*+D_u f^*D_{\mu}k^* \\ D_{\mu}h^*+D_u h^*D_{\mu}k^* \end{bmatrix} \in \text{Im} \begin{bmatrix} D_x f^*+D_u f^*D_x k^* & D_u f^*D_v k^* \\ D_x h^*+D_u h^*D_x k^* & D_u h^*D_v k^* \end{bmatrix} \quad (3.24)$$

and hence,

$$\begin{bmatrix} D_{\mu}f^*+D_u f^*D_{\mu}k^* \\ D_{\mu}h^*+D_u h^*D_{\mu}k^* \end{bmatrix} \in \text{Im} \begin{bmatrix} D_x f^* & D_u f^* \\ D_x h^* & D_u h^* \end{bmatrix} \quad (3.25)$$

and this implies (3.19).

Now we give constructive sufficient conditions for regulator design. First we define a regular equilibrium point.

Definition 3.3: An equilibrium point (x^*, u^*, μ^*) of (3.2) is *regular* if there exists a neighborhood of μ^* on which there exist unique, continuously differentiable functions $\bar{x}(\mu)$, $\bar{u}(\mu)$ satisfying

$$F(\bar{x}(\mu), \bar{u}(\mu), \mu) = 0 \quad (3.26)$$

with $x^* = \bar{x}(\mu^*)$, $u^* = \bar{u}(\mu^*)$.

Notice that the implicit function theorem implies that an equilibrium point is regular if

$$\det [D_x F \ D_u F] \neq 0 \quad (3.27)$$

Theorem 3.5: If the equilibrium point (x^*, u^*, μ^*) is regular, then

(i) there exist functions $k_0: R^n \rightarrow R^n$, $\bar{x}: R^k \rightarrow R^n$, and $\bar{u}: R^k \rightarrow R^m$ so that the regulator problem has a state feedback solution in the form

$$u = k(x, \mu) := \bar{u}(\mu) + k_0(x - \bar{x}(\mu)) \quad (3.28)$$

(ii) there exist functions k_0 , $\bar{x}$, $\bar{u}$ as in (i) and functions $\gamma_1: R^n \times R^k \times R^q \rightarrow R^n$, $\gamma_2: R^n \times R^k \times R^q \rightarrow R^k$ so that the regulator problem has a dynamic feedback solution in the form

$$u = \bar{u}(v_2) + k_0(v_1 - \bar{x}(v_2)) = \eta(v_1, v_2) \quad (3.29a) \quad \text{and}$$

$$\dot{v}_1 = \gamma_1(v_1, v_2, \bar{y}) + f(v_1, \eta(v_1, v_2), v_2) - f(v_1, u^*, v_2) \quad (3.29b)$$

$$\dot{v}_2 = \gamma_2(v_1, v_2, \bar{y}), \quad (3.29c)$$

where $\bar{y} := y - g(x^*, u, \mu^*)$

proof: (i) We construct a state feedback compensator with the desired properties. Since the equilibrium point is regular, there exist functions $\bar{x}(\mu)$, $\bar{u}(\mu)$ which satisfy (3.26) and have the property $x^* = \bar{x}(\mu^*)$, $u^* = \bar{u}(\mu^*)$. Now, let $u = u^* + k_0(x - x^*)$ be a feedback controller with $k_0(0) = 0$ which exponentially stabilizes the equilibrium point (x^*, u^*, μ^*) and which exists by assumption. In fact $k_0(\xi) = -u^* + k(\xi + x^*)$, where $k(\cdot)$ is the stabilizing feedback of Definition 3.1. The closed loop dynamics with controller (3.28) are

$$\dot{x} = f(x, \bar{u}(\mu) + k_0(x - \bar{x}(\mu)), \mu) \quad (3.30)$$

and for each fixed μ (3.2) has a solution $(\bar{x}, \bar{u})$. The perturbation equations are

$$\delta \dot{x} = A(\mu) \delta x, \quad A(\mu) = [D_x f + D_u f D_x k_0]_{x \rightarrow \bar{x}} \quad (3.31)$$

Since A is a continuous function of μ and exponentially stable at μ^* , it is exponentially stable on a neighborhood of μ^* . Regulation follows from the fact that (3.26) implies $h(\bar{x}(\mu), \mu) = 0$ for μ on a neighborhood of μ^* , so that $z = 0$ at equilibrium.

(ii) Exponential observability of the composite system implies the existence of functions $\gamma_1(v_1, v_2, y)$ and $\gamma_2(v_1, v_2, y)$ with the properties

$$\gamma_1(x, \mu, g(x, u^*, \mu) - y^*) = f(x, u^*, \mu), \quad \gamma_2(x, \mu, g(x, u^*, \mu) - y^*) = 0 \quad (3.32)$$

and so that the dynamical system

$$\dot{v}_1 = \gamma_1(v_1, v_2, 0), \quad \dot{v}_2 = \gamma_2(v_1, v_2, 0) \quad (3.33)$$

has an exponentially stable equilibrium point $(v_1, v_2) = (x^*, \mu^*)$.

Now, the closed loop dynamics are

$$\dot{x} = f(x, \eta(v_1, v_2), \mu) \quad (3.34a)$$

$$\dot{v}_1 = \gamma_1(v_1, v_2, g(x, \eta(v_1, v_2), \mu) - y^*) + f(v_1, \eta(v_1, v_2), v_2) - f(v_1, u^*, v_2) \quad (3.34b)$$

$$\dot{v}_2 = \gamma_2(v_1, v_2, g(x, \eta(v_1, v_2), \mu) - y^*) \quad (3.34c)$$

Let us define a state transformation $(x, v_1, v_2) \rightarrow (x, \epsilon_1, \epsilon_2)$ where

$$\epsilon_1 = v_1 - x, \quad \epsilon_2 = v_2 - \mu$$

so that the loop equations become

$$\dot{x} = f(x, \eta(x + \epsilon_1, \mu + \epsilon_2), \mu) \quad (3.35a)$$

$$\begin{aligned} \dot{\epsilon}_1 &= \gamma_1(x + \epsilon_1, \mu + \epsilon_2, g(x, \eta(x + \epsilon_1, \mu + \epsilon_2), \mu) - y^*) \\ &\quad + f(x + \epsilon_1, \eta(x + \epsilon_1, \mu + \epsilon_2), \mu + \epsilon_2) \\ &\quad - f(x + \epsilon_1, u^*, \mu + \epsilon_2) - f(x, \eta(x + \epsilon_1, \mu + \epsilon_2), \mu) \end{aligned} \quad (3.35b)$$

$$\dot{\epsilon}_2 = \gamma_2(x + \epsilon_1, \mu + \epsilon_2, g(x, \eta(x + \epsilon_1, \mu + \epsilon_2), \mu) - y^*) \quad (3.35c)$$

It is easy to verify that an equilibrium point of (3.35) is $(x, \epsilon_1, \epsilon_2) = (\bar{x}(\mu), 0, 0)$ by making use of equations (3.32). Now some lengthy calculations show that the perturbation equations associated with (3.35) are of the form

$$\begin{bmatrix} \dot{x} \\ \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \end{bmatrix} = A(\mu) \begin{bmatrix} x \\ \epsilon_1 \\ \epsilon_2 \end{bmatrix}, \quad \text{with } A(\mu) = A(\mu^*) + O(\mu - \mu^*) \quad (3.36)$$

$$A(\mu^*) = \begin{bmatrix} [(D_x f) + (D_u f)(D_x k_0)]^* & ** & ** \\ 0 & \Gamma_{11} & \Gamma_{12} \\ 0 & \Gamma_{21} & \Gamma_{22} \end{bmatrix}$$

where

$$\Gamma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} = \begin{bmatrix} D_{v_1} \gamma_1(x^*, \mu^*, 0) & D_{v_2} \gamma_1(x^*, \mu^*, 0) \\ D_{v_1} \gamma_2(x^*, \mu^*, 0) & D_{v_2} \gamma_2(x^*, \mu^*, 0) \end{bmatrix}$$

Since $k_0(x)$ is an exponentially stabilizing state feedback controller, it follows that $[(D_x f) + (D_u f)(D_x k_0)]^*$ has all of its eigenvalues in the open left half plane. Similarly, exponential observability implies that Γ (the perturbation matrix associated with (3.33)) has all of its eigenvalues in the open left half plane and this property is therefore inherited by $A(\mu^*)$. It follows from continuity of the eigenvalues of $A(\mu)$ that there is a neighborhood of μ^* on which $A(\mu)$ is exponentially stable.

3.3 Linear Solution of the Local Regulator Problem

Theorem 3.5 prescribes a general nonlinear compensator which solves the local regulator problem under the assumptions of exponential stabilizability and detectability and regularity of the equilibrium point. In view of the remark 3.2, we might ask if the local regulation problem can be solved by linear feedback. Such a result might be immediately anticipated by someone familiar with the linear robust regulator problem [8, 16]. We require, however, somewhat stronger conditions. It will be assumed that:

$$1) \text{Im} \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} = \mathbb{R}^{n+p} \quad (3.37)$$

2) z is readable from y , i.e., there exists a matrix Q such that $z = Qy$

We refer to 1) as the *strong regularity condition*. Now, we design a (constant) disturbance accommodating regulator for the surrogate perturbation system

$$\delta \dot{x} = A^* \delta x + G\omega + B^* \delta u \quad (3.38a)$$

$$\dot{\omega} = 0 \quad (3.38b)$$

$$\delta \dot{z} = C^* \delta x + H\omega + D^* \delta u \quad (3.38c)$$

where $\omega \in \mathbb{R}^p$ (recall $p = \dim(z)$) and G, H are chosen (as they always may be) so that the composite state is detectable in δz . Such a compensator is constructed as follows.

(i) Determine matrices X, U , respectively $n \times p, m \times p$, which satisfy the matrix equations

$$A^* X + B^* U + G = 0 \quad (3.39a)$$

$$C^* X + D^* U + H = 0 \quad (3.39b)$$

(ii) Determine an $m \times n$ matrix K_0 such that $(A^* + B^* K_0)$ is stable.

(iii) Determine an $(n+p) \times p$ matrix $L = [L_1^T \ L_2^T]^T$ such that

$$\left\{ \begin{bmatrix} A^* & G \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} C^* & H \end{bmatrix} \right\}$$

is stable.

The required compensator is then

$$u = u^* + Uv_2 + K_0(v_1 - Xv_2) \quad (3.40a)$$

$$\dot{v}_1 = (A^* + L_1 C^* + B^* K_0)v_1 + (G + L_1 H + B^*(U - K_0 X))v_2 - L_1 Q \delta y \quad (3.40b)$$

$$\dot{v}_2 = (L_2 C^*)v_1 + (L_2 H)v_2 - L_2 Q \delta y \quad (3.40c)$$

$$\delta y := \delta y - D^* \delta u = y - y^* - D^*(u - u^*)$$

Lemma 3.1: Suppose that the equilibrium point (x^*, u^*, μ^*) satisfies the strong regularity condition (3.37). Then there exists matrices G, H such that the composite state of (3.38), $[\delta x^T, \omega^T]^T$ is detectable in the output δz and there exists matrices X, U satisfying (3.39). Moreover, for such G and H , $\text{Im}[U] = \mathbb{R}^m$.

proof: Exponential detectability implies that (C^*, A^*) is a detectable pair. In an even more general context Francis [8] proves that when this is the case, $H=0$ and almost every G renders the composite system detectable. The strong regularity condition insures that (3.39) is solvable for X, U for every G, H . It remains to show that $\text{Im}[U] = \mathbb{R}^m$. Assume that this is not the case. Then there exists a vector v having the property that

$$v \in \text{Im} \begin{bmatrix} A^* \\ C^* \end{bmatrix} \text{ and } v \in \text{Im} \begin{bmatrix} G \\ H \end{bmatrix} \quad (3.41)$$

Now recall that that detectability of the composite system implies that

$$\text{Ker}([C^* \ H]) \cap \text{Ker} \begin{bmatrix} A^* & G \\ 0 & 0 \end{bmatrix} - \lambda I_{n+p} = 0 \text{ for all } \lambda \text{ in the closed right half plane} \quad (3.42)$$

But in view of (3.41), the matrix

$$\begin{bmatrix} A^* & G \\ C^* & H \end{bmatrix}$$

is singular so that (3.42) fails with $\lambda=0$. Hence, we have a contradiction.

Now, we can establish the following result.

Theorem 3.6: If the equilibrium point (x^*, u^*, μ^*) satisfies the strong regularity condition and if z is readable from y with $z = Qy$, then there exist matrices X, U, K_0, L as required for the linear compensator (3.40) and this compensator is a solution of the local regulator problem.

proof: We only sketch the essential elements. First note that the assumptions of exponential stabilizability and detectability assure the existence of the compensator parameters K_0, L . The existence of X, U is established by Lemma 3.1.

Now we must show that the closed loop has an equilibrium point corresponding to $z=0$ for each μ on a neighborhood of μ^* , and that this equilibrium point is exponentially stable. Let us write the loop equations in the form

$$\dot{x} = f(x, u, \mu) \quad (3.43a)$$

$$z = h(x, u, \mu) \quad (3.43b)$$

$$u = u^* + Uv_2 + K_0(v_1 - Xv_2) \quad (3.43c)$$

$$\dot{v}_1 = (A^* + L_1 C^*)v_1 + (G + L_1 H)v_2 + B^*(u - u^*) - L_1 \delta z \quad (3.43d)$$

$$\dot{v}_2 = (L_2 C^*)v_1 + (L_2 H)v_2 - L_2 \delta z \quad (3.43e)$$

where $\delta z := z - h(x^*, u, \mu^*)$

and hence $\delta \dot{z} := \dot{z} - \dot{z}^* = Q\{\delta \dot{y} - D^* \delta u\}$

First we argue that that there exist functions on $x(\mu), u(\mu), v_1(\mu), v_2(\mu)$ a neighborhood of μ^* with $x(\mu^*) = x^*, u(\mu^*) = u^*, v_1(\mu^*) = 0, v_2(\mu^*) = 0$ which satisfy the equilibrium equations

$$0 = f(x, u, \mu) \quad (3.44a)$$

$$0 = h(x, u) \quad (3.44b)$$

$$\dot{u} = u^* + Uv_2 + K_0(v_1 - Xv_2) \quad (3.44c)$$

$$0 = (A^* + L_1 C^*)v_1 + (G + L_1 H)v_2 + B^*(\dot{u} - u^*) \quad (3.44d)$$

$$0 = (L_2 C^*)v_1 + (L_2 H)v_2 \quad (3.44e)$$

Existence of functions $\bar{x}(\mu)$, $\bar{u}(\mu)$ satisfying (3.44a&b) is guaranteed by the assumption that (x^*, u^*, μ^*) is a regular equilibrium point. Since, by Lemma 3.1, $\text{Im}[U] = \mathbb{R}^m$, there exists $v_2(\mu)$ such that $\bar{u}(\mu) - u^* = Uv_2(\mu)$. Thus, (3.44c,d&e) are satisfied with $v_1(\mu) = Xv_2(\mu)$. It remains only to show that this equilibrium point is exponentially stable. We omit these calculations which proceed as in the proof of Theorem 3.5.

Remark 3.4 The application of linear regulators: An early application of this type of linear regulator to a nonlinear plant was in the control of electric power plants (see, for example, McDonald and Kwatny [20]). Although the local regulator problem may be solvable with a linear regulator, in general, it is to be expected that better performance will be achievable with the nonlinear regulator of Theorem 3.5. Indeed, it is widely understood that essential nonlinear plant characteristics, such as control saturation, see [20], must be included in an observer to obtain satisfactory performance.

Remark 3.5 Regulation mechanisms: The nonlinear and linear regulators achieve output regulation via two different mechanisms. In the nonlinear case, the closed loop equilibrium point is defined by the functions $\bar{x}(\mu)$, $\bar{u}(\mu)$ which represent a local characterization of the equilibrium surface, whereas, in the linear case "integral action" provides regulation.

4 Stability, Bifurcation & Zero Dynamics

4.1 Bifurcation Points

It is to be anticipated that feedback regulation at operating conditions with multiple equilibria which are in close proximity may prove troublesome. We have already seen, in Theorem 3.1, that a necessary condition for exponential stability of an equilibrium point of a closed loop system at a parameter value μ^* is the existence of an isolated equilibrium for each μ in a neighborhood of μ^* . Our synthesis procedures of Theorem 3.5 assume an isolated equilibrium point of the open loop dynamics. Subsequently, we will develop a perspective of multiple equilibria from the point of view of the open loop equilibrium equations (3.2) and establish their significance to regulator design.

Definition 4.1 : An equilibrium point (x^*, u^*, μ^*) of (3.2) is a (static) bifurcation point with respect to $F(x, u, \mu)$ if in each neighborhood of (x^*, u^*, μ^*) there exists (x_1, u_1, μ) and (x_2, u_2, μ) with $(x_1, u_1) \neq (x_2, u_2)$ and $F(x_1, u_1, \mu) = 0$, $F(x_2, u_2, \mu) = 0$.

An equilibrium point is a bifurcation point if and only if it is not regular. Thus, we see the significance of the implicit function theorem. The key analytical tools for the analysis of static bifurcation are the implicit function theorem and the reduction method of Liapunov-Schmidt [12,15].

4.2 Bifurcation & Zero Dynamics

We can give a useful interpretation to static bifurcation for systems defined by state equations (3.1a) and output equations (3.1c).

Equation (3.27) is equivalent to

$$\det \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \neq 0 \iff \text{Im} \begin{bmatrix} -A^* & B^* \\ -C^* & D^* \end{bmatrix} = \mathbb{R}^{n+p} \quad (4.1)$$

Thus, we have the following conclusion.

Theorem 4.1 : An equilibrium point (x^*, u^*, μ^*) is a (static) bifurcation point only if

$$\text{Im} \begin{bmatrix} -A^* & B^* \\ -C^* & D^* \end{bmatrix} \neq \mathbb{R}^{n+p}$$

It follows that conditions for static bifurcation reduce to the following two possibilities.

1) if for typical λ ,

$$\text{rank} \begin{bmatrix} \lambda I - A^* & B^* \\ -C^* & D^* \end{bmatrix} = n+p \quad (4.2)$$

then a static bifurcation point corresponds to an invariant zero at the origin. This is the *nondegenerate* case.

2) otherwise, a static bifurcation point corresponds to the condition

$$\det(C^*[\lambda I - A^*]^{-1}B^* + D^*) = \det(G(\lambda)) = 0 \quad (4.3)$$

which implies insufficient independent controls or redundant regulated outputs. This is the *degenerate* case.

Remark 4.1 Bifurcation and nonlinear zero dynamics: When it is possible to associate with (3.1a&c) nonlinear zero dynamics in the sense of Byrnes and Isidori [4], then the nondegenerate case corresponds to a static bifurcation of the zero dynamics (e.g., generically a saddle node bifurcation if $k=1$). The degenerate case corresponds to "structural instability" of the relative degree.

4.3 Simultaneous Regulation

It has been shown that the local regulator problem is solvable with a linear compensator at open loop equilibria which satisfy the strong regularity condition. This means that the closed loop system with a fixed linear regulator designed at a nominal parameter value μ^* will be exponentially stable and output regulation will obtain for all actual parameter values on a neighborhood of the nominal. But to what extent can a single linear compensator provide stability and regulation? We give some insight into the limitations of a single linear compensator.

In Remark 3.1, we defined the equilibrium manifold $\mathcal{E}$ associated with the output regulated open loop system (3.1). Now, consider the closed loop system (3.43) which is the system (3.1) plus a linear regulator designed at a point $(x^*, u^*, \mu^*) \in \mathcal{E}$. The closed loop equilibrium manifold is the set of points

$$\mathcal{E}_{cl} = \{ (x, v, u, \mu) \in \mathbb{R}^{2n+2m+k} \mid \text{such that } (x, v, u, \mu) \text{ satisfies (3.44)} \} \quad (4.4)$$

The structure of $\mathcal{E}_{cl}$ is quite simple. It is a k dimensional manifold whose projection on any $\mathbb{R}^{n+m+k}$ subspace of $\mathbb{R}^{2n+2m+k}$ defined by $v = \text{constant}$ is precisely $\mathcal{E}$. There is a one-to-one correspondence between points in $\mathcal{E}$ and $\mathcal{E}_{cl}$.

The bifurcation points themselves form codimension-1 manifolds in $\mathcal{E}$ or $\mathcal{E}_{cl}$ which divide them into open sets which we call *sheets*. The boundaries of these sets consist of the bifurcation points. Two different sheets are *contiguous* if they share common boundary points.

Theorem 4.2: A single linear compensator will generically fail to simultaneously solve the local regulator problem at two equilibria, one on each of two contiguous sheets of $\mathcal{E}$.

proof: Let A and B denote two equilibria, one on each of two contiguous sheets of $\mathcal{E}$. There exist corresponding points A' , B' in $\mathcal{E}_{cl}$. We assume A' is a stable equilibrium of the closed loop system and show that B' is generically unstable. Choose a path $\mathcal{P}$ connecting A' and B' which transversely crosses a codimension one (in $\mathcal{E}_{cl}$) bifurcation surface at point C' . This is always possible because such paths are generic. Moreover, $\mathcal{P}$ can be chosen so that C' is the only bifurcation point which it contains. C' is a codimension one (static) bifurcation point and generically corresponds to the coalescence of two hyperbolic closed loop equilibria with the number of right half plane eigenvalues differing by precisely one. C' itself corresponds to a closed loop equilibrium point with precisely one zero eigenvalue. As the path $\mathcal{P}$ is traversed from A' to B' , the only real eigenvalue crossing of the imaginary axis occurs at C' and this corresponds to a single eigenvalue. Hence, the number of right half plane eigenvalues of A' and B' generically differs by an odd number. It follows that if A' is stable, B' must be generically unstable.

5 Conclusions

In this paper we have discussed the design of secondary voltage control systems for electric power networks. Such regulators are intended to change the bifurcation structure of the network in order to increase voltage stability margins. We have described how the secondary voltage control problem can be precisely formulated in terms of the classical power system model. With a simple example we have illustrated some of the issues which arise in the design such regulators. The two most

important of these being a new type of static bifurcation, which represents the achievable stability margin limit, and the significant changes (reduction) in the parameter space stability domain when load voltage regulation is enforced. The latter observation implies that stabilization by feedback is likely to be required as an integral part of secondary voltage control.

We have extended and described recent results concerning the design of feedback regulators for nonlinear parameter dependent dynamics [13,14]. Under the assumptions of exponential stabilizability and detectability for the plant, the regulator problem fails to be solvable only at static bifurcation points of the open loop equilibrium equations. We have shown that static bifurcation is associated either with the presence of an invariant zero of the linearized (error) dynamics at the origin or with a degenerate transfer matrix. Thus, a key observation is that in feedback regulator design static bifurcations are linked to changes in the zero dynamics structure rather than stability structure. Our results have been developed in the spirit of the theory of linear disturbance accommodating regulators (Kwatny and Kalnitsky [16]) but in essence restricted to the constant disturbance case. Our focus on constant disturbances allows us to readily establish the connection with bifurcation theory.

We have noted that the stability changes in the open loop dynamics under parameter variation are of little consequence so far as solvability of the local regulator problem is concerned. Only changes in the zero dynamics are significant. The design engineer sets up the bifurcation behavior when the regulated outputs are selected. Thus, it is essential to consider the parametric dependence of the zero dynamics at an early stage of the control design process. On the other hand, the actual size of the achieved domain of dynamic stability of the equilibrium in parameter is crucial. We have shown that this domain is necessarily bounded by the static bifurcation set, because the desired stable equilibrium points reside on a sheet, but the stable points need not fill that sheet. Methods for designing regulators which maximize the size of the stability domain in parameter space are under investigation, for example, Bennett, Glover & Kwatny [3].

Although, we have given two regulator design processes, one linear and the other nonlinear, in quite general terms, there are many important design choices to be made in their application which have not been discussed. We have focused on the regulation function as opposed to the stabilization function of the controller, although we hope that we have made it clear that these are strongly coupled. Because, we have not addressed dynamic stabilization in any depth, we feel justified in the power system model used herein to describe relevant phenomenon. However, we also feel that any detailed discussion of dynamic stabilization with respect to power system secondary voltage control will require the inclusion of the dynamics of excitation systems and other primary local voltage controllers.

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USING AN ITERATIVE METHOD TO COMPUTE A CLOSEST SADDLE NODE BIFURCATION IN THE LOAD POWER PARAMETER SPACE OF AN ELECTRIC POWER SYSTEM

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Abstract: Voltage collapse and blackout can occur in an electric power system when load powers vary so that the system loses stability in a saddle node bifurcation. We use a new iterative method to compute the load powers at which bifurcation occurs and which are locally closest to the current operating load powers. The computation is illustrated in the 6 dimensional load power parameter space of a 5 bus power system.

1. INTRODUCTION

We model an electric power system by load flow equations of the form

$$f(x, \lambda) = 0, \quad x \in \mathbb{R}^n, \quad \lambda \in \mathbb{R}^m \quad (1)$$

where x is the system state vector at a stable equilibrium and λ is a parameter vector of real and reactive load powers. We assume current operating load powers λ_0 at which the corresponding equilibrium x_0 is stable. As the load powers λ vary from the current load powers λ_0 , the equilibrium x will vary in the state space. At critical load powers λ_* the system can lose stability by x disappearing in a saddle node bifurcation and we denote the set of such λ_* in the load power parameter space $\mathbb{R}^m$ by Σ . The instability at λ_* can cause catastrophic collapse of the system voltages and blackout [1,2]. Σ is part of the bifurcation set and typically consists of hypersurfaces in $\mathbb{R}^m$ and their intersections. The instability can be avoided by monitoring the position of the current load powers λ_0 relative to Σ and taking corrective action if λ_0 moves too close to Σ . In particular it is useful to calculate a λ_* in Σ for which $|\lambda_* - \lambda_0|$ is a local minimum of the distance from λ_0 to Σ [3,4,5,6,7]. Then the line segment $\lambda_0\lambda_*$ represents a worst case load power parameter variation and $|\lambda_* - \lambda_0|$ measures the proximity to saddle node bifurcation [3,4,5,6,7]. We call the bifurcation at λ_* 'a closest saddle node bifurcation' with the understanding that the distance to bifurcation is measured in parameter space relative to the fixed value λ_0 . Note that we do not require the pattern of load increase to be predicted. We expect the distance to a closest saddle node bifurcation in load power parameter space to be a useful measure of proximity to voltage collapse.

The idea of computing a closest instability to the current load powers is due to Jarjis and Galiana [3]. The distance measure of [3] is not Euclidean and it is minimized using Fletcher-Powell optimization. Jarjis and Galiana also consider parameter spaces which include

generator voltages as well as load powers. Jung et al [5] suggest a different optimization method to compute the closest instability and Sekine et al [4] attempt to compute the closest instability by gradient descent on the determinant of the Jacobian. Dobson et al [6,7] present a direct method for computing a closest bifurcation. Mori has also worked on closest instabilities of power systems but we have not yet obtained this reference.

This paper explains and applies a new iterative method from [7] to compute $|\lambda_* - \lambda_0|$ and a closest saddle node bifurcation. The iteration exploits a simple formula [8,9] for the normal vector to Σ . We compute a closest saddle node bifurcation in the 6 dimensional load power parameter space of a 5 bus power system.

2. PRELIMINARIES

The iteration to compute a closest saddle node bifurcation has two main ingredients: the formula for the normal vector to Σ and standard methods for finding saddle node bifurcations with a one dimensional parameter variation. We consider these in turn before describing the iteration.

Given a current operating point (x_0, λ_0) we want to find a closest saddle node bifurcation (x_*, λ_*) ; that is, $\lambda_* \in \Sigma$ gives a saddle node bifurcation of f at x_* and the distance $|\lambda_* - \lambda_0|$ is a local minimum. At a saddle node bifurcation the equilibrium x_* is degenerate and the Jacobian $f_x|_{(x_*, \lambda_*)}$ is singular. Throughout this paper, we make the generic assumptions that $f_x|_{(x_*, \lambda_*)}$ has a unique simple zero eigenvalue with a corresponding left eigenvectors w_* and that certain transversality conditions (see [8,7,9]) are satisfied. (We write w_* as a row vector.) It follows that Σ is a smooth hypersurface near λ_* and that a normal vector to the hypersurface at λ_* is

$$N(\lambda_*) = w_* f_\lambda \quad (2)$$

where f_λ is the Jacobian of f with respect to λ [8,7,9]. Note that since the load powers λ appear linearly in the load flow equations (1), f_λ is a constant $n \times m$ matrix.

Suppose we specify a particular linear pattern of future load increase from the operating load powers λ_0 . That is, we specify a ray in parameter space based at λ_0 with a unit vector n so that the load powers λ along the ray are given by

$$\lambda = \lambda_0 + kn \quad (3)$$

as the loading factor k assumes positive real values. There are several methods to compute the closest saddle node bifurcation *assuming* this ray of load increase. That is, we can compute a critical loading factor k_* so that $\lambda_* = \lambda_0 + k_*n \in \Sigma$. For example, this computation is done by [10,11,12] using homotopy methods and [13,5,11,14] using direct or optimization methods. The text by Seydel [15] gives an entry to the numerical analysis literature on homotopy and direct methods.

For this paper we use a variant of the direct method in which solution of the following equations by Newton's method yields the bifurcating equilibrium x_* , loading factor k_* and the left eigenvector w_* :

$$f(x_*, \lambda_0 + k_*n) = 0 \quad (4.1)$$

$$w_* f_x|_{(x_*, \lambda_0 + k_*n)} = 0 \quad (4.2)$$

$$w_* c - 1 = 0 \quad (4.3)$$

Equation (4.1) states that x_* is an equilibrium at parameter $\lambda_* = \lambda_0 + k_*n$ and equation (4.2) states that the Jacobian f_x evaluated at (x_*, λ_*) is singular with left eigenvector w_* . $c \in \mathbb{R}^n$ is a fixed vector and (4.3) ensures that the left eigenvector w_* is nonzero. The only difference between equations (4) and the standard methods [13,5,11,15] is that we use a left eigenvector w_* in place of the corresponding right eigenvector. This is convenient since computing $\lambda_* = \lambda_0 + k_*n$ and x_* by solving (4) yields as a byproduct the left eigenvector w_* needed to compute the normal vector to Σ with formula (2).

3. ITERATION TO COMPUTE THE CLOSEST BIFURCATION

Now we describe how computation of the bifurcation along a given ray and computation of the normal vector of Σ may be iterated to compute the direction n_* and parameter value λ_* of a locally closest saddle node bifurcation. The procedure is as follows:

- (0) Let n_0 be an initial guess for the direction n_* .
- (1) Given n_i , compute the saddle node bifurcation along the ray given by n_i ; that is compute k_i, λ_i, x_i so that $\lambda_i = \lambda_0 + n_i k_i \in \Sigma$.
- (2) Compute the left eigenvector w_i of $f_x|_{(x_i, \lambda_i)}$ corresponding to the zero eigenvalue.
- (3) Set $n_{i+1} = N(\lambda_i) = w_i f_\lambda$.
- (4) Iterate steps 1,2,3 until convergence of n_i to a value n_* . Then $\lambda_* = \lambda_0 + k_*n_*$ and x_* is the corresponding equilibrium.

The quickest way to grasp how the iteration works is to try it with pencil and paper in the case of Σ an ellipse and λ_0 an interior point of the ellipse. Note that the iteration converges in one step if Σ is a hyperplane.

The following claims are proved in [7]:

- If the iteration converges exponentially to a fixed point n_* then the parameter $\lambda_* = \lambda_0 + n_*k_*$ yields a locally closest bifurcation.

- If λ_* is the parameter of a locally closest bifurcation and Σ is 'not too concave' at λ_* then the direction $n_* = N(\lambda_*)$ is an exponentially stable fixed point of the iteration. (The precise meaning of 'not too concave' is that the minimum principal curvature of Σ at λ_* must exceed $-|\lambda_* - \lambda_0|^{-1}$.)

Note that when the iteration converges, it converges to a locally closest instability which is not necessarily the desired globally closest instability. This is a potential problem in practice, particularly if the hypersurfaces of Σ are corrugated or if λ_0 is close to several hypersurfaces of Σ (see section 5). One problem in analyzing the performance of the iteration is the lack of knowledge of the geometry of Σ ; all that seems to be known are formulas for the normal vector [8] and curvature of Σ [7,6]. A conjecture of Jarjis and Galiana [3] implies that the interior of Σ is convex.

4. EXAMPLES WITH 2 AND 5 BUSES:

Consider a simple two bus power system model of a generator slack bus, lossless line and a load with real power P and reactive power Q . The system state $x = (\alpha, V)$ where $Ve^{i\alpha}$ is the load voltage phasor. The load flow equations are

$$0 = -4V \sin \alpha - P$$

$$0 = -4V^2 + 4V \cos \alpha - Q$$

(The generator voltage is 1.0 and the line admittance is 4.0. All quantities are in per unit except that angles are in radians.) The parameter vector $\lambda = (P, Q)$ contains the real and reactive powers consumed by the load. Note that the parameters enter linearly and that f_λ is the negative of the 2×2 identity matrix. The operating point is given by $x_0 = (\alpha_0, V_0) = (-0.138, 0.908)$ and $\lambda_0 = (P_0, Q_0) = (0.5, 0.3)$ (see figure 1). The closest bifurcation at $\lambda_* = (P_*, Q_*) = (0.703, 0.877)$ was computed using the iterative method (see table 1). At each step of the iteration, the direct method (4) was used to compute the closest bifurcation along a ray based at λ_0 . The normal vector to Σ at λ_* is $N(\lambda_*) = (0.331, 0.943)$ and the minimum distance to bifurcation is $|\lambda_* - \lambda_0| = 0.611$. As claimed in section 3, the convergence of the iterative method to the solution demonstrates that λ_* is a locally closest bifurcation.

Consider the five bus system from [5,13] shown in figure 2. This system has generators at buses 1 and 3 (bus 1 is slack), and loads at buses 2, 4 and 5. The line parameters are given in table 2 and the generator parameters are $V_1 = 1.04$, $\alpha_1 = 0$, $V_3 = 1.02$ and $P_3 = -1.1$. We choose the real and reactive powers consumed by the loads as the parameter vector $\lambda = (P_2, Q_2, P_4, Q_4, P_5, Q_5)$. The state vector $x = (\alpha_2, \alpha_3, \alpha_4, \alpha_5, V_2, V_4, V_5)$. The operating point x_0 , bifurcating equilibrium x_1 along the ray of direction n_0 , a closest bifurcating equilibrium x_* and the corresponding left eigenvectors w_0, w_1 and w_* are shown in table 3. The corresponding load powers λ_0, λ_1 and λ_* and the progress of the iteration of n_i are shown in table 4. Inspection of the components of $\lambda_* - \lambda_0$ in table 4 shows

that Q_4 is the load bus power most influential on the load power margin so that bus 4 can be interpreted as the weakest system bus. All calculations were done using Mathematica [16] and the results for both the 2 bus and 5 bus systems check with the results obtained by direct methods in [6].

Our preliminary experience suggest that the iterative method of computing a closest instability is more robust to choice of initial conditions than the direct method of [6] which suffers from the sensitivity to initial conditions typical of Newton type methods. Moreover, the iterative method only converges to locally closest instabilities and does not require the curvature of Σ at the solution to be checked as in the direct method [6]. However, the asymptotic convergence of the iterative method is only geometric and the direct method is faster to compute for the examples above.

5. INITIAL RAY DIRECTIONS AND MULTIPLE HYPERSURFACES

We describe a method from [6] for obtaining the initial ray direction n_0 required by the iteration and give a simple example of the multiple hypersurface structure of Σ . The initial ray direction n_0 may be calculated as follows from information available at the current operating point (x_0, λ_0) : If λ_0 is close enough to one of the hypersurfaces of Σ , then the algebraically largest eigenvalue of the Jacobian f_x at (x_0, λ_0) is the eigenvalue which will increase to zero as λ_0 moves towards $\lambda_* \in \Sigma$. The corresponding left eigenvector w_0 of $f_x|_{(x_0, \lambda_0)}$ approximates the left eigenvector w_* at (x_*, λ_*) so that $w_0 f_\lambda$ approximates $w_* f_\lambda$ which is the direction of the line segment $\lambda_0 \lambda_*$ [8].

This may be illustrated in the case of the two bus power system: The operating point is given by $x_0 = (\alpha_0, V_0) = (-0.138, 0.91)$ and $\lambda_0 = (P_0, Q_0) = (0.5, 0.3)$. $f_x|_{(x_0, \lambda_0)}$ has eigenvalues $(-3.994, -2.904)$ and the left eigenvector corresponding to -2.904 is $w_0 = (0.585, 0.811)$ so that $n_0 = w_0 f_\lambda = (0.585, 0.811)$.

In Figure 1 the bifurcation set Σ consists of only one hypersurface which in this case is a curve. When Σ consists of several hypersurfaces, say $\Sigma = \Sigma^1 \cup \Sigma^2$, they generally intersect in surfaces of lower dimension and λ_0 may be approximately equidistant from Σ^1 and Σ^2 as indicated in Figure 3. Bifurcation at Σ^1 corresponds to a real eigenvalue μ^1 of f_x increasing to zero and bifurcation at Σ^2 corresponds to a different real eigenvalue μ^2 of f_x increasing to zero. In the situation of Figure 3 it would be desirable to search for locally closest bifurcations on both of Σ^1 and Σ^2 . This could be attempted by computing initial ray estimates $n_0^1 = w_0^1 f_\lambda$ and $n_0^2 = w_0^2 f_\lambda$ where w_0^1 and w_0^2 are the left eigenvectors of f_x at (x_0, λ_0) corresponding to μ^1 and μ^2 . In general we suggest searching for hypersurfaces Σ^i using the initial ray directions $n_0^i = w_0^i f_\lambda$ corresponding to the real eigenvalues μ^i closer to zero than some threshold. (An additional complication not treated in this paper is that of Hopf bifurcations; there are also Hopf bifurcation hypersurfaces which are encountered in some contexts, e.g. [17,18,19]. However [7] gives an iterative method of computing a closest Hopf bifurcation.)

It is straightforward to demonstrate multiple bifurcation hypersurfaces Σ^1 and Σ^2 in a simple power system model. Consider a 3 bus power system consisting of an infinite bus connected by two identical, lossless transmission lines to two PQ load buses. The load buses are not directly connected to each other so that the system is essentially two decoupled infinite bus and PQ load systems. Then the load power parameter vector $\lambda = (P_1, Q_1, P_2, Q_2)$ and the parameter space $\mathbb{R}^4$ is the Cartesian product of Figure 1 with itself. There are two 3 dimensional bifurcation hypersurfaces $\Sigma^1 = \Sigma \times \mathbb{R}^2$ and $\Sigma^2 = \mathbb{R}^2 \times \Sigma$ where Σ is the curve in Figure 1. Σ^1 and Σ^2 intersect in the 2 dimensional set $\Sigma \times \Sigma$. Σ^1 corresponds to bifurcation of load bus 1 and Σ^2 corresponds to bifurcation of load bus 2. The situation of λ_0 approximately equidistant from Σ^1 and Σ^2 arises for any approximately equal loadings of the two loads. In this ideal case of perfectly decoupled areas within the same power system the suggested method to obtain initial ray directions will work well because the two left eigenvectors w_0^1, w_0^2 are confined to $\mathbb{R}^2 \times \{0\}$ and $\{0\} \times \mathbb{R}^2$ respectively.

6. CONCLUSIONS

The distance to a closest saddle node bifurcation in load power parameter space is expected to be a useful measure of proximity to voltage collapse. We propose a new iterative method for computing a locally closest saddle node bifurcation in load power parameter space and illustrate its use in simple power system examples. The larger example has 5 buses and a 6 dimensional load power parameter space. The load power variation need not be predicted in this approach; instead, the worst case load power variation is computed. The iterative method is easy to implement if any of the standard methods for finding bifurcation instabilities *assuming* a given load power variation are given. The method applies generally to finding closest saddle node instabilities of a general dynamical system with arbitrary parameters. In particular, more general power system parameters such as controls and generator powers may be considered. The iterative method seems simpler and more robust than the direct method of [6] but may be slower to compute. The iterative method of computing a closest instability is promising and further analytic and numerical work is needed to establish its effectiveness in computing the proximity to voltage collapse and in other applications.

Support in part by NSF Research Initiation Grant ECS-9009079 is gratefully acknowledged.

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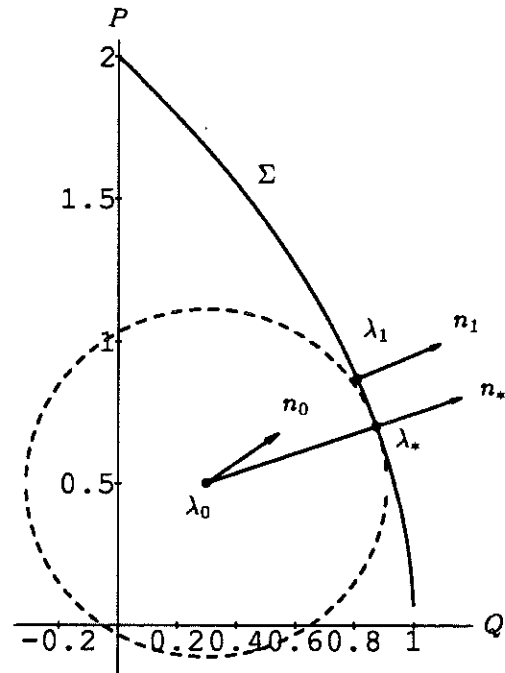


Figure 1. Load power parameter space

i	n_i	λ_i
0	(0.585, 0.811)	(0.869, 0.811)
1	(0.398, 0.917)	(0.744, 0.862)
2	(0.349, 0.937)	(0.713, 0.873)
3	(0.336, 0.942)	(0.705, 0.876)
4	(0.333, 0.943)	(0.703, 0.876)
5	(0.332, 0.943)	(0.703, 0.877)
6	(0.331, 0.943)	(0.703, 0.877)

Table 1. 2 bus iteration

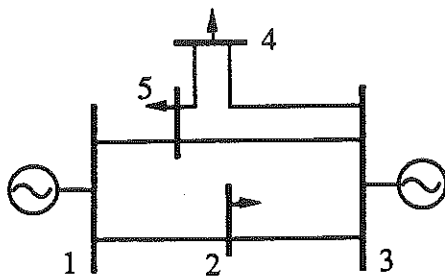


Figure 2. 5 bus system

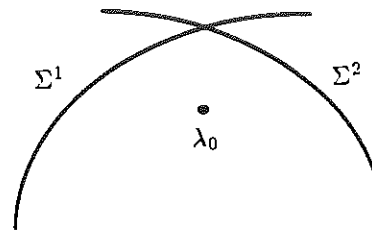


Figure 3. Sketch of multiple hypersurfaces

Line(bus to bus)		$G(pu.)$	$B(pu.)$
1	2	1.40056	-5.60224
1	5	1.84118	-7.48352
2	3	1.84118	-7.48352
3	4	0.70028	-2.80112
3	5	1.12985	-4.47675
4	5	0.93372	-3.43483

Table 2. Line data for 5 bus system

	α_2	α_3	α_4	α_5	V_2	V_4	V_5
x_0	-0.1043	-0.0545	-0.1759	-0.0916	0.9603	0.9151	0.9681
w_0	0.3513	0.4851	0.7160	0.3570	0.0055	-0.0326	0.0074
x_1	-0.3667	-0.4189	-0.9350	-0.3743	0.9161	0.6038	0.8214
w_1	0.0379	0.0581	0.7595	0.0983	0.0083	0.6143	0.1770
x_*	-0.1665	-0.1622	-0.4328	-0.1593	0.9567	0.5253	0.8524
w_*	0.0094	0.0157	0.5213	0.0550	0.0009	0.8385	0.1477

Table 3. State space vectors for 5 bus iteration

	P_2	Q_2	P_4	Q_4	P_5	Q_5
λ_0	1.1500	0.6000	0.7000	0.3000	0.7000	0.4000
λ_1	1.7919	0.6101	2.0080	0.2404	1.3522	0.4136
n_0	0.3513	0.0055	0.7160	-0.0326	0.3570	0.0074
n_1	0.0379	0.0083	0.7595	0.6143	0.0983	0.1770
n_2	0.0130	0.0015	0.5957	0.7857	0.0607	0.1535
n_3	0.0103	0.0011	0.5425	0.8246	0.0563	0.1490
n_4	0.0097	0.0010	0.5272	0.8347	0.0553	0.1480
n_5	0.0095	0.0009	0.5229	0.8375	0.0551	0.1478
n_6	0.0095	0.0009	0.5217	0.8382	0.0550	0.1477
n_*	0.0094	0.0009	0.5213	0.8385	0.0550	0.1477
λ_*	1.1584	0.6008	1.1659	1.0495	0.7491	0.5320
$\lambda_* - \lambda_0$	0.0084	0.0008	0.4659	0.7495	0.0491	0.1320

Table 4. Load power parameter space vectors for 5 bus iteration

COMPUTING AN OPTIMUM DIRECTION IN CONTROL SPACE TO AVOID SADDLE NODE BIFURCATION AND VOLTAGE COLLAPSE IN ELECTRIC POWER SYSTEMS

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Abstract: This note computes an optimum direction of controls to avoid power system voltage collapse. That is, given a load power forecast, we compute the sensitivity with respect to controls of a load power margin measuring proximity to voltage collapse. The computation is simple enough to contribute to the practical planning and control of a power system to avoid voltage collapse blackouts. The computation applies to avoiding saddle node bifurcation instability of a general dynamical system.

1. INTRODUCTION

Voltage collapse is an instability of heavily loaded electric power systems which leads to declining voltages and blackout. It is associated with bifurcation and reactive power limitations of the power system. Power systems are expected to become more heavily loaded in the next decade as the demand for electric power rises while economic and environmental concerns limit the construction of new transmission and generation capacity. Heavily loaded power systems are closer to their stability limits and voltage collapse blackouts will occur if suitable monitoring and control measures are not taken. Progress has been made in understanding voltage collapse and monitoring the proximity to voltage collapse with various indices (most approaches are covered in [1]). The sensitivity to controls of some of the indices have been computed so that control action may be taken to optimally improve the index and the voltage stability of the system. For example, Tiranuchit and Thomas [2] computed the sensitivity of the minimum singular value of the system Jacobian and Overbye and DeMarco [3] computed the sensitivity of an energy function index. Another popular index is load power margin which may be computed by direct methods [4,5,6], optimization [7,8,9,10], continuation [5,6,11], and continuation with linear predictor and no corrector [12,13]. This paper computes the first order sensitivity of a load power margin index to any controls or parameters appearing in the load flow equations.

If system parameters vary slowly and continuously, then voltage collapse can be explained as a dynamic consequence of a saddle node bifurcation instability in which the system operating equilibrium disappears [14]. Discrete contingencies such as loss of a power line can also weaken the system stability and contribute to voltage collapse; if this occurs, or is hypothesised to occur when assessing security, we use the saddle node bifurcation explanation *after* the discrete contingency. This paper computes an optimum control direction which steers the system away from a saddle node bifurcation and the associated voltage collapse.

We exploit the geometry of a control space Λ , the vectors of which contain real and reactive powers of the loads and generators and other control parameters such as settings of tap changing transformers and shunt capacitor devices. (The settings of tap changing transformers and shunt capacitor devices are discrete but we compute the control assuming they are continuous and then approximate the control with the nearest discrete setting [15].) Note that the "controls" include load powers which vary freely in normal operation and are only controlled in emergencies by selectively shedding loads. It is also useful to include design parameters in Λ when planning changes to a power system.

The set of all points of Λ at which there is a bifurcation is called the bifurcation set and it generally contains points other than Σ . These points correspond to saddle node bifurcations not involving the stable equilibrium point, Hopf bifurcations, and other bifurcations not generic in one parameter systems. We do not consider these bifurcations here: Saddle node bifurcations not involving the stable operating equilibrium do not affect its local stability. Hopf bifurcations cause oscillatory instabilities, but are probably not important in voltage collapse because voltage collapses are not observed to be oscillatory. The non-generic bifurcations are not expected to occur in practice.

Define Σ to be the control values in Λ at which the stable operating equilibrium has a saddle node bifurcation. Σ typically consists of curved hypersurfaces and their intersections. We denote the current control parameters by λ_0 ; the position of λ_0 relative to Σ is the key to monitoring and avoiding voltage collapse. We assume that a load power forecast is given in the form of a linear increase from the current load powers. This forecast is described by a ray in Λ based at λ_0 ; controls other than load powers are constant along the ray. The ray will intersect Σ at the point λ_* at which the system bifurcates and loses stability and the proximity to voltage collapse can be measured by the distance $|\lambda_0 - \lambda_*|$. Since the controls other than load powers were fixed in the load forecast ray, $\lambda_0 - \lambda_*$ lies in the load power subspace of Λ and $|\lambda_0 - \lambda_*|$ is a load power margin. If $|\lambda_0 - \lambda_*|$ is too small, then control action should be taken to increase $|\lambda_0 - \lambda_*|$ in order to avoid or reduce the risk of voltage collapse. Control action is represented by a displacement of λ from λ_0 in Λ . This paper computes $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$, the gradient with respect to λ of the index $|\lambda - \lambda_*|$ evaluated at λ_0 . $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ is the sensitivity of the load power margin with respect to control and its direction gives the optimum direction for control. We prove that $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ is parallel

to the normal vector n_* to Σ at λ_* using generic assumptions and a dynamic power system model and show that it is straightforward to compute n_* and $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ from the load flow equations.

This paper adopts the objectives of Kumano, Yokoyama and Sekine [16] and we are indebted to these authors for the geometric idea of computing an optimum direction in a load power space which is augmented with other controls. This augmented space also appears in the framework of Mercede et al [17]. Our computation of the optimum direction is based on the left eigenvector formula in Dobson [18] for the normal vector n_* to Σ . The formula for the normal vector in [16], which uses a right eigenvector, is only valid for power systems with symmetric Jacobians so that the left and right eigenvectors coincide. The computation in this paper is much simpler and more direct than that of [16]. The idea of using a normal vector to determine which loads to shed appeared in [18].

2. POWER SYSTEM STATIC AND DYNAMIC MODELS AND THE NORMAL VECTOR n_*

Consider the load flow equations

$$0 = f(x, \lambda) \quad \text{or} \quad \begin{cases} 0 = f_1(x, \lambda) \\ 0 = f_2(x, \lambda) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$ is a state vector describing the bus voltage phasors and $\lambda \in \Lambda$ is a control vector. f_1 describes real power balance at the generators and f_2 describes real and reactive power balance at the loads.

We seek to compute the sensitivity $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ from the static equations (1) but want the results to be consistent with the sensitivity for a general dynamic model extending (1) to which the voltage collapse theory of [14] applies. A dynamic model [18] which extends (1) by including generator swing dynamics and general load dynamics is

$$\dot{z} = h(z, \lambda) \quad \text{or} \quad \begin{cases} \dot{\omega} = f_1(\delta_G, y, \lambda) - \Delta \omega \\ \dot{\delta}_G = \omega \\ \dot{y} = g(f_2(\delta_G, y, \lambda), \omega) \end{cases} \quad (2)$$

The state vector $z = (\omega, x) = (\omega, \delta_G, y)$ where δ_G are the generator voltage angles in x and Δ is a diagonal matrix of generator dampings. g defines any smooth dynamic load model which depends on frequency ω and the real and reactive power balance at each load (see [14] for an example). Little is known about the large signal behaviour of such load dynamics and a convincing function g is hard to obtain. For our purposes we require the existence of g and the assumptions that g has a unique zero at $(0, 0)$ and that Dg , the Jacobian of g with respect to its first argument, is globally invertible. We also assume the technical condition that the operating region of (2) is contained in a compact positively invariant set; see [14].

We assume a load forecast in which the load powers increase as a linear function of a loading parameter t :

$$\lambda(t) = \lambda_0 + t\mu \quad (3)$$

where μ is a fixed vector of unit length defining the direction of the load forecast ray in Λ . We write $\lambda_* = \lambda(t_*)$ for the first intersection of the load forecast ray with Σ and t_* for the loading parameter at the bifurcation. (The disappearance of the operating equilibrium and the existence of λ_* is guaranteed by the absence of any equilibria for absurdly high loadings.) If the load increase is slow enough to be modelled as a quasistatic variation of (2) and the resulting system $\dot{z} = h(z, \lambda(t))$ parameterized by t is assumed to be a generic one parameter system, then the theory of [14, 19] applies and we obtain the following conclusions about the structure of the bifurcation at (z_*, λ_*) : If we write $h_z|_*$ for the Jacobian of h with respect to z evaluated at (z_*, λ_*) , then $h_z|_*$ has a single, simple zero eigenvalue and corresponding right and left eigenvectors v'_* , w'_* so that $h_z|_* v'_* = w'_* h_z|_* = 0$. In addition, the bifurcation is a saddle node bifurcation satisfying the transversality conditions [20, 21, 19]

$$w'_* h_t|_* = w'_* h_\lambda|_* \mu \neq 0 \quad (4.1)$$

$$w'_* h_{zz}|_*(v'_*, v'_*) \neq 0 \quad (4.2)$$

Moreover, Σ is a smooth hypersurface near λ_* and has a well defined normal vector n_* at λ_* . Note that we write w'_* , n_* and $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0}$ as row vectors.

The formula from [18] for the normal vector n_* to Σ at λ_* is simply

$$n_* = w'_* h_\lambda|_* \quad (5)$$

where $h_\lambda|_*$ is the Jacobian of h with respect to λ evaluated at the bifurcation.

Now we demonstrate that the saddle node bifurcation of the dynamic model (2) at $(z_*, \lambda_*) = (0, x_*, \lambda_*)$ induces a bifurcation of the solutions of the static equations (1) at (x_*, λ_*) and the Jacobian $f_x|_*$ has a zero eigenvalue with corresponding left eigenvector w_* . Moreover the normal vector n_* of (5) is also given by

$$n_* = w_* f_\lambda|_* \quad (6)$$

This allows n_* to be calculated from the static equations (1) assuming only the general form, but not the details, of the dynamical model (2). To derive (6), first note that the unique zero of g at $(0, 0)$ implies that (1) has an equilibrium at (x_0, λ_0) if and only if (2) has an equilibrium at $(0, x_0, \lambda_0)$. The respective Jacobians of (1) and (2) are

$$f_x = \begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix} \quad \text{and} \quad h_z = \begin{pmatrix} -\Delta & f_{1x} \\ I & 0 \\ g_\omega & Dg f_{2x} \end{pmatrix}$$

Writing $w'_* = (w'_{1*}, w'_{2*}, w'_{3*})$, observe that

$(w'_{1*}, w'_{2*}, w'_{3*}) h_z|_* = 0$ implies that

$$0 = (w'_{1*}, w'_{3*}) \begin{pmatrix} f_{1x} \\ Dg f_{2x} \end{pmatrix} = (w'_{1*}, w'_{3*} Dg) \begin{pmatrix} f_{1x} \\ f_{2x} \end{pmatrix}$$

so that $w_* = (w'_{1*}, w'_{3*} Dg)$ is the left eigenvector of f_x corresponding to a zero eigenvalue. (w_* is unique since if $w_* f_x = (w_{1*}, w_{3*}) f_x = 0$ then $(w_{1*}, w_{1*} \Delta -$

$w_{3*}(Dg)^{-1}g_w, w_{3*}(Dg)^{-1}h_z|_* = 0$ and $w_{1*} = w'_{1*}$ and $w_{3*} = w'_{3*}Dg$ since w'_* is unique.) Moreover,

$$\begin{aligned} n_* &= w'_* h_\lambda|_* = (w'_{1*}, w'_{2*}, w'_{3*}) \begin{pmatrix} f_{1\lambda}|_* \\ 0 \\ Dg f_{2\lambda}|_* \end{pmatrix} \\ &= (w_{1*}, w_{3*}) \begin{pmatrix} f_{1\lambda}|_* \\ f_{2\lambda}|_* \end{pmatrix} = w_* f_\lambda|_* \quad (7) \end{aligned}$$

It is straightforward to check that $w'_* h_{zz}|_*(v'_*, v'_*) = w_* f_{xx}|_*(v_*, v_*)$ where $v'_* = (0, v_*)$ and v_* is the right eigenvector of $f_x|_*$ corresponding to the zero eigenvalue. This, together with (7), shows that the bifurcation of (1) inherits from (2) the transversality conditions (4).

3. COMPUTATION OF n_* AND THE LOAD POWER MARGIN SENSITIVITY TO CONTROL

Given a load forecast (3), the system will bifurcate and collapse at $\lambda_* = \lambda_0 + t_* \mu$. We compute λ_* by solving a variant of the extended system equations [6,4,5]:

$$\left. \begin{aligned} f(x_*, \lambda_*) &= 0 \\ w_* f_x|_* &= 0 \\ w_* u - 1 &= 0 \end{aligned} \right\} \quad (8)$$

(The last equation of (8) ensures that w_* is nonzero; in practice u can be chosen to be any nonzero constant vector.) Equations (4) can be solved for (x_*, w_*, t_*) using Newton-Raphson iteration. Then $\lambda_* = \lambda_0 + t_* \mu$. Note that equations (8) are a variant of the extended system equations in that they use left eigenvectors in place of the usual right eigenvectors. This has the advantage of computing the left eigenvector w_* required in the formula (6) for n_* as a by-product of the computation of (x_*, λ_*) . The optimization methods of computing the load power margin also share this advantage since w_* is the Lagrange multiplier vector [7]. Continuation methods [6, 11, 12, 5] could also be used but would require a further computation of w_* from $f_x|_*$.

There is a sign ambiguity in the formulas (5) and (6) for n_* since both n_* and $-n_*$ are normal vectors to Σ . It is convenient to assume that the normal n_* is pointing "outward" so that changing λ in the direction n_* leads to the disappearance of the equilibrium x_* . Formula (6) becomes

$$n_* = \pm w_* f_\lambda|_* \quad (9)$$

Strictly speaking, the sign of n_* should be determined by checking that the equilibrium x_* disappears when $\lambda = \lambda_* + \epsilon n_*$ for small positive ϵ . In practice the sign of n_* should be apparent from the expectation that n_* should correspond to an increase in power for most loads. f_λ is easy to compute and evaluate at the bifurcation.

Now we prove that the load power margin sensitivity

$$\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = -(n_* \mu)^{-1} n_* \quad (10)$$

; it follows from (10) that the optimum direction for control is $-n_*$. λ_* and t_* are well defined smooth functions of the current control parameters λ near λ_0 because Σ is a smooth hypersurface near λ_* and

(4.1) implies that $n_* \mu \neq 0$ so that the load forecast ray intersects Σ transversally at λ_* . $\lambda_* = \lambda + t_* \mu$ and $|\mu| = 1$ imply that $\nabla_\lambda(|\lambda - \lambda_*|)|_{\lambda_0} = \nabla_\lambda t_*|_{\lambda_0}$. Specify Σ near λ_* as the zero set of a smooth real function α . Then $\alpha(\lambda_*) = 0$ and t_* satisfies $0 = \alpha(\lambda + t_* \mu)$ and $0 = (\nabla_\lambda \alpha)|_{\lambda_*} (I + \mu \nabla_\lambda t_*)|_{\lambda_0}$. Hence $0 = n_* (I + \mu \nabla_\lambda t_*)|_{\lambda_0} = n_* + (n_* \mu) \nabla_\lambda t_*|_{\lambda_0}$ and $\nabla_\lambda t_*|_{\lambda_0} = -(n_* \mu)^{-1} n_*$, yielding (10). The geometric content is clear: the optimum direction to increase the distance in a given direction μ of a point λ_0 to a hypersurface Σ is antiparallel to the outward normal n_* to Σ .

It may not be desirable to use all the possible controls to improve voltage stability. For example, load powers can be controlled by shedding loads but this is only done in an emergency. In less extreme cases, the objective is to move further from voltage collapse using controls other than load shedding. The optimum control direction is then the projection of $-n_*$ onto the subspace of Λ containing the controls other than load powers.

Reactive power limitations of generators have a strong effect on the the load power margin and should be accounted for when the load power margin is computed [7,12,8,9]. Since a generator reaching a reactive power limit changes the system (for example, generator voltage magnitude and reactive power output may interchange their roles as state and control variables), care must be taken that λ_* is computed for the system consistent with the generator reactive power limits. This system will often differ from the more lightly loaded system at λ_0 .

4. EXAMPLE

We illustrate the computation of the sensitivity of the load power margin to controls with the Ward & Hale 6 bus system with a modification to bus 6 as shown in Figure 1 (see [22] for a similar modification). The state vector x contains the voltage angles and magnitudes of buses 3,4,5,6,2; bus 1 is the slack bus. The control vector $\lambda = (P_3, Q_3, P_4, Q_4, P_5, Q_5, P_6, Q_6, P_2, V_2, n_4, n_7, C_4, C_6)$ contains real and reactive bus powers, bus 2 generator voltage magnitude, transformer tap ratios, and the capacitances at bus 4 and 6. λ_0 at the operating point x_0 is given by the values in Figure 1. The load forecast is given by linearly increasing the load powers at buses 3,5,6 so that

$$\begin{aligned} \lambda(t) &= \lambda_0 + t \mu = \lambda_0 + t(0.3207, 0.0758, 0, 0, 0.1749, \\ &\quad 0.1049, 0.2915, 0.8746, 0, 0, 0, 0, 0, 0)^T \quad (11) \end{aligned}$$

We solve equations (8) for (x_*, w_*, t_*) (see table 1) and compute $\lambda_* = \lambda(t_*)$ from (11). The load power margin $|\lambda_* - \lambda_0| = t_* = 0.162$. The first 9 columns of the f_λ matrix correspond to load powers and are a 9×9 identity matrix. The remaining columns correspond to the other controls. It is straightforward to compute the normal vector n_* using formula (9) (see table 2).

The optimum direction for parameter control is $-n_*$. For example, inspection of the components of $-n_*$ shows that Q_6 is the load bus power most influential

on the load power margin and bus 6 is the load which can most effectively be shed. Bus 6 can also be interpreted as the weakest system bus [12]. If no loads are to be shed, then the optimum control direction is given by the negative of the second row of n_* in table 2, showing, as might be expected, that increasing the capacitor values and decreasing the tap ratios steers the system away from voltage collapse. The relative sizes of the components of $-n_*$ give the relative proportions of each control to most effectively increase the load margin.

5. DISCUSSION AND CONCLUSIONS

When the load power margin is computed by a constrained optimization technique, the sensitivity of the load power margin to the controls such as load powers on the right hand side of the constraints is given by the Lagrange multiplier vector at the bifurcation [7,8,9]. This is useful because weak buses or areas may be identified as those with large voltage magnitude sensitivities to generator power variations [13,8,9]. The Lagrange multiplier result follows from our computation because the left eigenvector w_* of the Jacobian is the Lagrange multiplier vector at the bifurcation and the f_λ matrix is simply the identity matrix in this case so that the Lagrange multiplier vector is n_* . Our computation confirms the Lagrange multiplier results and generalizes them to any controls or parameters appearing in the load flow equations.

Overbye [3] and Kumano [16] suggest using the optimum control direction for real time avoidance of voltage collapse: Once the optimum control direction $-n_*$ is computed, a term can be included in the optimal power flow which has minimum cost when selected controls are changed in the direction $-n_*$. In an emergency, it would be appropriate to neglect cost and use all available controls according to the direction $-n_*$, including shedding selected loads. Note that the optimum combination of loads to be shed that we compute is strictly valid only for small load changes.

Liu and Vu [23] have demonstrated examples in which it can be advantageous to lock tapchangers to improve voltage stability; we suggest that the best direction and relative proportions of tapchanges to avoid voltage collapse may be deduced from the sensitivities of the load power margin to the tap changer settings.

This paper uses an index of voltage collapse which assumes a load forecast. If the forecast is unavailable, or a measure of system robustness independent of load variation is wanted, then the sensitivity computation also works with the load power margin index of [24] which measures the minimum distance (worst case load power variation) from λ_0 to Σ .

Basic to our computation is the smooth slow variation of the controls λ which allows quasistatic analysis of (2). This is an idealization; in practice, the system state and controls are subject to disturbances and variations which may be discrete or not slow compared to the dynamics of (2). A consequence is that the system state will not exactly track the stable equilibrium of (2) and may be displaced outside the region of attraction of

the stable equilibrium just before bifurcation (see [3] for further discussion). Nevertheless, we suggest that it is still sensible to control the real system with the controls described above because steering the idealized system away from the bifurcation is a precondition for stability of the real system and will tend to increase the minimum size of disturbance which can destabilize the real system.

We have computed the sensitivity to controls and parameters of a load power margin index of voltage collapse. The computation applies to a dynamic load model with a general form of load dynamics but may be conveniently carried out on the load flow equations. The controls and parameters may appear anywhere in the load flow equations. The computation is simple, with the main computational burden being the computation of a left eigenvector at the bifurcation. However, two of the methods of computing the load power margin produce the required eigenvector as a by-product. The computation has several applications to power system planning and security.

The computation applies to control or parameter changes to steer a general parameterized dynamical system away from a saddle node bifurcation if the formula (5) is used to calculate the normal vector n_* .

The authors thank C.L. DeMarco and F.L. Alvarado for useful comments. This work was supported in part by NSF Research Initiation grant ECS-9009079.

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	α_3	V_3	α_4	V_4	α_5	V_5	α_6	V_6	α_2
x_0	-0.216	1.106	-0.173	1.035	-0.225	1.101	-0.226	1.170	-0.019
x_*	-0.263	0.979	-0.203	0.910	-0.266	0.895	-0.253	0.892	-0.114
ω_*	0.181	0.258	0.145	0.275	0.303	0.473	0.298	0.632	-0.017

Table 1.

	P_3	Q_3	P_4	Q_4	P_5	Q_5	P_6	Q_6
n_*	0.114	0.162	0.091	0.172	0.190	0.296	0.187	0.395
	P_2	V_2	C_4	C_6	n_4	n_7		
	-0.010	-0.616	-0.314	-0.143	0.309	0.117		

Table 2.

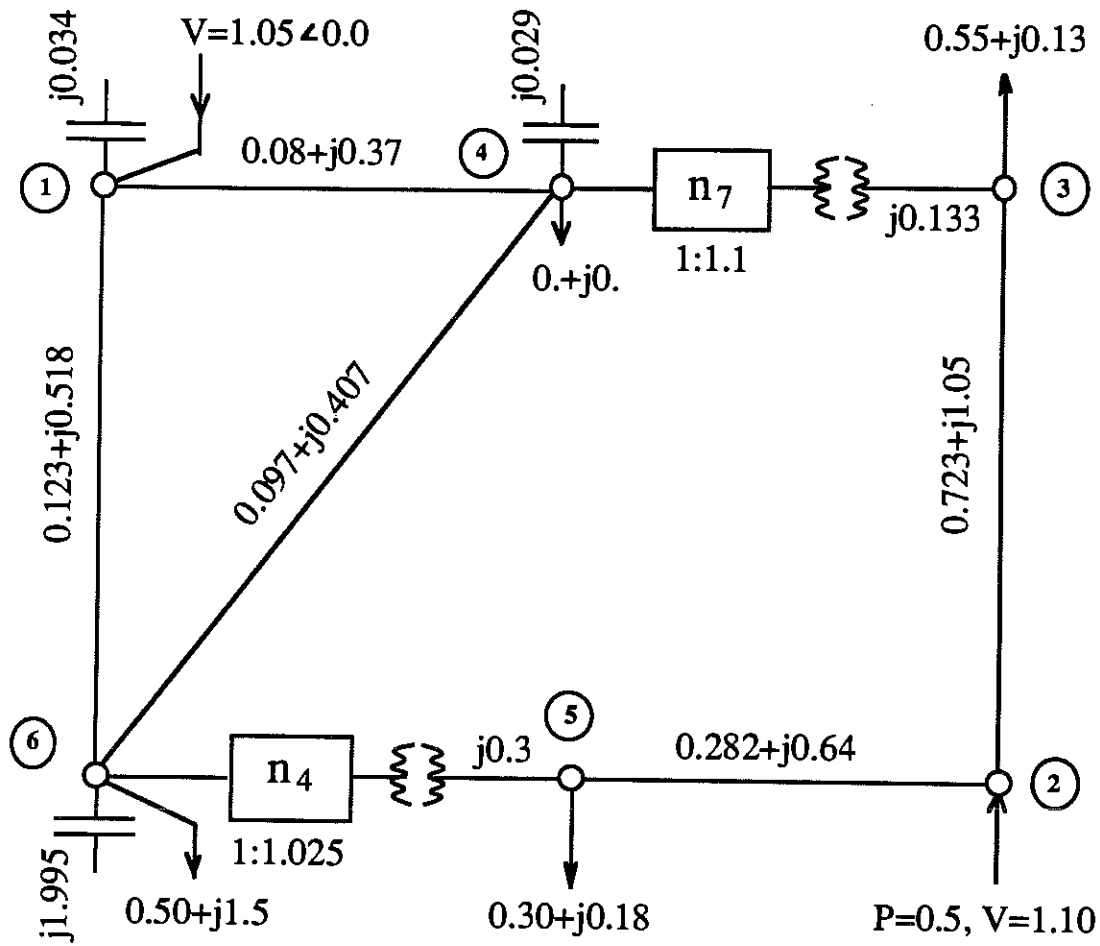


Figure 1. Modified Ward & Hale 6 bus system

Integrating Voltage Security Measures: Connections Between Steady State, Small Disturbance Stability, and Nonlinear Dynamic Criteria

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ABSTRACT

Research literature examining the phenomena of voltage collapse has increasingly converged towards an explanation involving quasi-static evolution of the system operating point towards a condition where that operating point loses asymptotic stability. However, many works have also recognized the need to supplement such studies with information reflecting the nonlinear dynamics of the system. This paper will review a range of existing results on voltage collapse security measures, and will show a range of power system models where the relationship between quasi-static and dynamic security measures can be clearly established. It will be shown that the similarities and differences of the resulting security measures can best be understood by examining the time scale of the external load and parameter variations that drive the evolution of the power system.

I. Introduction and Background

The threat of voltage collapse and voltage instability phenomena has attracted considerable attention in the power system community over the past decade, as indicated by a tremendous volume of research literature that has been produced on the topic. With the accumulating experience of the power system community, there is growing recognition of the various aspects of the problem, combining as it does both slow and fast time scale phenomena, and quasi-static analysis methods with dynamic models. This paper will contrast various methods of analysis in a consistent system model.

One of the main goals of voltage stability analysis for power system operation is the identification of a useful, computationally tractable security index that can be used to judge a system's vulnerability to voltage collapse, and to evaluate the effectiveness of various corrective strategies when the security of the system is threatened. Different modelling assumptions, and the mix of static and dynamic criteria has often made it difficult to effectively compare various voltage security measures. One of the most comprehensive attempts at such comparison is the work reported

in [1], where various measures are compared in terms of a common criteria based on statistical decision theory. In that work various methods are compared in terms of their ability to reliably classify operating conditions as secure or insecure with respect to voltage collapse. Among the conclusions of that work was the observation that "one needs at least one static and one dynamic criterion to identify the voltage collapse problem," implying that there is fundamentally different information obtained from the two classes of indices. However, as the understanding of voltage collapse and the related system modelling required to represent the problem has evolved, it has become clear that there are close connections between static and dynamic criteria. This paper will attempt to further illustrate these connections, and will argue that for a wide class of models the information obtained from each type of measure can be self consistent (predicting the same ultimate operational limits to load and parameter changes), and complementary (with quasi-static measures naturally capturing slower time scale parameter variations, dynamic measures capturing the effect of faster time scale parameter variations). Many researchers have previously addressed connections between static and dynamic approaches to voltage collapse studies, so little in this paper will be truly new, but the authors hope that collection of a range of results in a single consistent presentation (as well as an updated interpretation of the authors' own work) will prove useful. This paper is not, however, intended as a comprehensive review. The range of papers on voltage collapse is so extensive that it is unfortunately inevitable that many relevant works will not be adequately discussed here. To the researchers responsible for these works, the authors offer their apologies. The field will await a comprehensive survey at some future date.

This paper will be organized as follows. A general structure of equations common to power system models will be discussed first, and the role of both slow and fast time scale parameter and load variations in these models will be highlighted. With this structure established, three major "schools of thought" regarding voltage collapse analysis will be reviewed. These will include bifurcation analyses, sensitivity analyses, and nonlinear dynamic analyses based on energy functions. For the last of these

approaches, a new interpretation in terms of an optimal control problem will be introduced, which clarifies the role of fast time scale disturbances in this analysis. The discussion of nonlinear dynamic measures will also draw connections to voltage collapse proximity indicators which make use of multiple power flow solutions, which play an important role in the optimal control problem. The final section of the paper will then restrict attention to a specific power system model (though *not* specific network data), and will show that in this model the various analysis methods offer results that are very closely related, with differences arising only from the time scale of parameter or load variation assumed.

II. General Modeling for Integrated Analysis

To facilitate an examination of both quasi-static bifurcation and dynamic analyses, as well as more classical sensitivity analyses, it is useful to look first at a general structure of equations that encompasses the nonlinear dynamic model. Working just with the general structure of equations, it is possible to illustrate many of the necessary assumptions that underlie sensitivity and bifurcation analyses, and to contrast these with Lyapunov based nonlinear stability analyses. However, a general abstract description of equation structure is not sufficient to examine all of the practical modeling issues that influence voltage stability. Therefore, following the general structure of equations to be presented below, a specific power system model that captures many of the characteristics relevant to voltage collapse will be constructed in Section VI.

The most commonly used structure of equations used in power system dynamic simulation programs is that of differential equations with algebraic constraints [2]. For voltage collapse analysis, it will be useful to "pull out" the dependence of these equations on time varying load parameters, which will be denoted here as $L(t)$. In simple constant P-Q load models, these parameters will simply be variations in active and reactive load. For voltage dependent loads (referred to as "nonlinear loads" by some authors [3]), these might be time varying coefficients in a polynomial, reflecting a time varying change in the composition of a particular load. In this later case, these time varying coefficients may multiply system variables such as voltage, so the vector $L(t)$ may be premultiplied by a state dependent matrix. The resulting system model will have the following structure.

$$\dot{z} = f(z, y) + B^1(z, y)L(t) \quad (1a)$$

$$0 = g(z, y) + B^2(z, y)L(t) \quad (1b)$$

For example, consider a very simple case of this general structure: a single generator, represented by a classical model, feeding a load that is a sum of a constant P-Q term and a time varying inductive term. The dynamic vector z would have two components, corresponding to the generator angle and frequency; the single algebraic variable y would correspond to the load bus voltage magnitude. The model would have two first order differential equations, defining derivative of frequency and angle, and a single algebraic constraint imposing reactive power balance at the load bus. The reactive power balance equation would contain the single time varying $L(t)$ term, representing the varying inductive load, and this term would be premultiplied by voltage magnitude squared (i.e., $B^2(z, y) = y^2$).

As noted above, this type of mixed system of differential equations with algebraic constraints is very commonly used in power systems simulations. However, the behavior of trajectories in such systems can be markedly different from that of pure systems of differential equations alone. For example, it is possible for trajectories to exist over some interval of time, but disappear at some later point in the trajectory. This is phenomena has been labelled "impasse point" behavior in circuit simulations [4], [5], where it usually indicates that the model is deficient because important parasitic dynamics have been neglected. Also, when comparing two systems that have the same steady state equations for equilibria, the stability of equilibrium points can change wildly depending on whether or not some the defining equations remain as algebraic constraints in the full dynamic model. Not surprisingly, techniques for estimating regions of attraction must also be modified to accommodate such mixed differential/algebraic systems. These issues have begun to be addressed in the power systems literature in such works as [6] and [7], but deserve further scrutiny in voltage stability analyses. This point will be raised again in reviewing bifurcation analyses.

To resolve the difficulties inherent in differential/algebraic representations one may seek physically plausible *dynamic* models for load that admit the same equilibria as predicted by the algebraic load model. One approach to introducing such dynamics is reported in [8], using a load model described in [9]. In these works, the steady state load model used in powerflow analysis is modified. An alternative approach that keeps the standard steady state powerflow model is described in [7], where the reactive power balance equations that force a system to always have zero mismatch at each bus are "perturbed" to a dynamic model that allows instantaneous mismatch to exist. The new dynamics are assumed to have a fast time constant that moves voltage magnitude back to an equilibrium with zero mismatch. This involves modifying the reactive power mismatch equations by introducing a term dependent on the derivative of bus voltage magnitude. With either of these approaches, the result is a new system model composed purely a state space

model, much like the result of adding of parasitic dynamics in the circuit case of [5]. In this case, (1) is condensed to the form:

$$\dot{x} = f(x) + B(x)L(t). \quad (2)$$

A first step in comparing various analysis techniques lies in identifying the assumed time scales of the parameter variations that enter models such as (1) and (2). Load studies in power systems have shown that aggregate variation in demand at a distribution substation level can often be decomposed into two major components: a relatively large magnitude "moving average" component, slowly varying on a time scale of minutes to hours, and a smaller magnitude, zero mean component, varying on a time scale of seconds [10], [11]. This separation of time scales is represented by decomposing the $L(t)$ term into a slow time scale component, and a small magnitude, zero mean component, as indicated below.

$$L(t) = L_{\text{slow}}(t) + L_{\text{small}}(t) \quad (3)$$

The first component of (3) captures the familiar hourly variation in the "load curve" through the course of 24 hours or a week, and is used to determine loading levels in most types of power system studies. The second component models the inherently random behavior that results from aggregating thousands of individual switching actions in customer loads. Once the slowly varying average component is removed, this remaining fast time scale variation is typically quite small in magnitude (a few percent of nominal load), and is often considered negligible in standard power system stability studies. However, operating conditions associated with voltage collapse typically display very high sensitivity to small variations in load. Hence, later in this paper we will argue that it may be advisable to keep even such small variations explicitly represented in the model. However, as a first step, consider the effect of including only the term $L_{\text{slow}}(t)$ from (3) in (2), and assume that the time rate of change of this term is indeed slow relative to the time constants of the dynamics. Intuitively, one expects that the evolution of the system state, $x(t)$, will tend to be close to the trajectory, $\hat{x}(t)$, predicted by solving the quasi-static equilibrium equation below:

$$0 = f(\hat{x}(t)) + B(\hat{x}(t))L_{\text{slow}}(t). \quad (4)$$

This should remain valid provided that the dynamics of (2), when linearized about $\hat{x}(t)$, are stable with time constants much faster than the variation of $\hat{x}(t)$. Rigorous conditions for $\|x(t) - \hat{x}(t)\|$ to remain bounded by a given constant are easily obtained in the case when the system has a suitable Lyapunov function, and are

derived in [12]. Voltage collapse analysis may be viewed as an attempt to efficiently predict (or control against) operating conditions that ultimately fail to satisfy these conditions, since collapse is associated with $x(t)$ (particularly bus voltages) diverging from the desired operating point predicted by $\hat{x}(t)$.

III. Bifurcation Analysis

The discussion of this section will assume that small magnitude random variations in load are negligible, and that $L_{\text{slow}}(t)$ defines the only parameter variation of interest. For simplicity of notation in this section, this load variation will be denoted only as $L(t)$. One expects that trajectories $x(t)$ satisfying (2) will diverge if (4) ceases to have a solution altogether, or if $\hat{x}(t)$ no longer defines a (sufficiently) stable equilibrium for (2) when the load term is "frozen" at $L(\hat{t})$. This is the viewpoint adopted in voltage stability bifurcation analyses such as [13] and [14]. These approaches typically assume that a suitable load prediction exists to define the function $L(t)$, with a starting value $L(0)$ that yields an acceptable, stable operating point. The goal then becomes one of determining if there exists a time $\hat{t}$, or equivalently, a load level $L(\hat{t})$, at which the assumed operating point $\hat{x}(\hat{t})$ loses small disturbance stability. For such bifurcation analyses, stability of the operating point is judged by examining the linearization of the dynamics about the points predicted by $\hat{x}(t)$. However, considerable information regarding the *local* behavior of the true nonlinear trajectories can be obtained from such linearized analysis. For example, [14] shows that when the true state $x(t)$ diverges from $\hat{x}(t)$, its initial direction of motion can be predicted by an eigenvector obtained from the linearization about the critical value of $\hat{x}(\hat{t})$.

Identification of critical loading levels $L(\hat{t})$ is facilitated by the fact that for many simplified dynamic models, the critical loading is associated with an operating point where the power flow Jacobian, denoted by $J(x)$, is singular. Relating this observation to (2), this property arises if the linearization $\partial f/\partial x$ has a zero eigenvalue when $J(x)$ (which often makes up a sub-block of $\partial f/\partial x$) is singular. Section VI will examine a specific class of system models relevant to voltage collapse for which this property holds. The critical role of power flow Jacobian has been recognized for many years, and motivates a number of security measures that measure proximity to voltage collapse by the distance of the power flow Jacobian from singularity [15]. From a computational standpoint, so long as the load variation is function of one parameter (such as time t in the $L(t)$ model here), it is a straightforward matter to augment steady state power flow equations with constraints that force the power flow Jacobian to be singular. In this way, the critical parameter value $\hat{t}$ can be solved with very little more computation than a standard power

flow. This general computational approach is described in [16], with the power systems application developed in [17]. In many models, the critical value $\hat{t}$ typically predicts a point where two power flow solutions coalesce, and for further increase in t , the powerflow no longer has a solution in the neighborhood of $\hat{x}(\hat{t})$.

For more detailed dynamic power system models, the linearized dynamic about an operating point predicted by $\hat{x}(t)$ may lose stability when complex eigenvalues cross the imaginary axis, rather than by a real eigenvalue crossing through zero. In the former case, one expects the physical instability to be manifest by growing oscillations¹; in the latter, the divergence of state would be an exponential with positive real exponent. Hopf bifurcation associated with imaginary axis eigenvalue crossings was studied in [18], where it was shown that variation in exciter gains could yield this type of transition to oscillatory instability. More recent work in [19] has suggested that oscillations associated with generator exciter and field dynamics may be responsible for some types of voltage instability. However, the field reports on voltage collapse incidents [20], [21] suggest that the divergence of voltage magnitudes from the operating point displayed a monotonic decline, rather than growing sinusoidal oscillation. This admittedly limited evidence would seem to indicate that the instability of voltage collapse should be modelled by a purely real eigenvalue crossing through the origin.

To summarize, the general bifurcation approach may be outlined as follows:

- i) A model of the form (2) is assumed, with $L(t)$ a known function of a single variable.
- ii) The quasi-static equilibrium $\hat{x}(t)$ is defined implicitly via solutions to (3).
- iii) The linearization of (2) evaluated at $x(0)$, $\partial f/\partial x(x(0))$, is assumed stable with all eigenvalues strictly in the left half plane.
- iv) The critical value $\hat{t}$ is identified as the smallest positive value of t for which $\partial f/\partial x(\hat{x}(t))$ has eigenvalues crossing the imaginary axis.

To obtain a numerical proximity indicator from this approach, a "distance" to collapse can be defined by the norm size of the load change necessary to cause loss of stability in the linearization, i.e., by $\|L(\hat{t}) - L(0)\|$. A Euclidean norm is typically used, but other choices, such as a ℓ_∞ norm, may have more engineering significance. The traditional Q-V curves that define loading limits in a single radial line (sometimes extended to analyze tie line limits in larger systems [22]) are simple single parameter versions of this type of measure. This distance to collapse is expressed in the load

parameter space; a similar distance to collapse may be defined in the state space, by calculating $\|\hat{x}(\hat{t}) - \hat{x}(0)\|$.

Alternative approaches may relax the assumption of known dependence of L on t , and seek a test to indicate whether any load variation in a set of possible load levels yields a transition to instability for the linearization about $\hat{x}$ (the dependence of $\hat{x}$ on the single parameter t can no longer be assumed). In this case, load levels at individual buses are treated as independent variables, and a set of possible load levels would be defined by examining load vectors within a given weighted norm bound. One then has a corresponding family of possible linearized system matrices, and the goal is to test if there are any unstable matrices in this set. Because the dependence of the linearized matrices on load changes is nonlinear, the set of matrices obtained may be quite complex. However, in [23] it is argued that for some models (similar to those that will be described in Section VI), the set of linearized matrices obtained may be well approximated by a "matrix polytope," that is, a matrix generalization of a polyhedral set. For the case when the transition to instability must take place with an eigenvalue passing through the origin, this problem is very similar to that of finding singular matrices in an interval matrix family. Recent work on the general interval matrix singularity problem [24] has indicated that the best known algorithms for this problem have computational cost that grows exponentially with the dimension of the matrices and number of free parameters. This suggests that the problem may formally belong to the class of computationally intractable problems that computer science classifies as "NP-hard."

If the problem of deciding whether or not an interval family of matrices has a singular element is NP-hard, a practical algorithm to decide (with complete certainty) whether or not a multiparameter family of power system operating points is guaranteed stable is probably computationally intractable. However, this "curse of dimensionality" is often confronted in power system computations, and sufficiently accurate and reliable approximate algorithms with much lower computational cost can often be found. Hence research into this type of multiparameter bifurcation/small disturbance stability problem will undoubtedly continue.

IV. Sensitivity Approaches

Most of the early, classic works on voltage stability relied on steady state sensitivity analyses, observing that voltage instability could be associated with an operating point at which the ratio of incremental changes in bus voltage to changes in reactive demand approaches infinity [25], [26]. Interestingly, in many models a generalization of this approach yields the same prediction of critical load values as bifurcation analysis, and several more

¹It is also possible that the growing oscillations predicted by the linearized, small disturbance stability analysis capture only local behavior, and that the actual trajectory enters a limit cycle further away from the old operating point. This aspect of Hopf bifurcation analysis is considered as an explanation for power systems oscillations in [18] and [44].

modern approaches have exploited sensitivity type approaches [27], [28]. To describe such approaches, consider again (4), which implicitly defines the motion of the quasi-static equilibrium as the load parameters evolve in time. Sensitivity approaches typically treat incremental changes in the load vector ΔL as varying independently about a known operating point x^0 and nominal load level L^0 . The resulting form of (4) may then be rewritten as:

$$0 = f(x^0 + \Delta x) + B(x^0 + \Delta x)(L^0 + \Delta L). \quad (5)$$

Linearizing the relation above about (x^0, L^0) the implicit function theorem predicts the sensitivity of Δx with respect to ΔL as being given by:

$$\Delta x = - \left[\frac{\partial f}{\partial x}(x^0) + \frac{\partial B}{\partial x}(x^0)L^0 \right]^{-1} B(x^0)\Delta L \quad (6)$$

When the load parameters are simply P-Q load levels, the matrix B is constant, so the $\partial B/\partial x$ term disappears, leaving

$$\Delta x = - \left[\frac{\partial f}{\partial x}(x^0) \right]^{-1} B\Delta L \quad (7)$$

From (7) it is clear that the state variables, such as voltage magnitudes, can become infinitely sensitive to load levels (and in particular, to reactive load levels) as the system approaches an operating point x^0 where $\partial f/\partial x(x^0)$ is singular. But singularity of $\partial f/\partial x(x^0)$ will of course imply that the linearization of (2) about this operating point is on the boundary of stability, with an eigenvalue at the origin. Therefore, while a sensitivity analysis might appear unrelated to the system dynamics, one sees that under standard assumptions on the types of load parameters considered, identifying operating points with infinite sensitivity of state to load changes will simultaneously identify operating points on the boundary of small disturbance stability.

In many cases, such sensitivity analyses can be further simplified to examination only of the powerflow Jacobian, $J(x)$. For example, consider again a simple model that includes only generator swing dynamics and algebraic constraints for power balance at P-Q load buses. The structure of the resulting system of equations will be that of (1), a mixed system of differential equations with algebraic constraints of the form:

$$\dot{\omega} = M^{-1} \{ -D\omega - h(\delta, V) \} \quad (8a)$$

$$\dot{\delta}_g = \omega \quad (8b)$$

$$0 = g(\delta, V) \quad (8c)$$

where M represents generator rotational inertias, D generator damping, $h(\delta, V)$ represents active power mismatch at generator buses, and $g(\delta, V)$ represents both active and reactive power balance constraints at load buses. Note that δ_g denotes the subset of phase angles at generator buses, and δ the complete set of bus phase angles. Then (8) may be rewritten as

$$\begin{bmatrix} \dot{\omega} \\ \dot{\delta} \\ 0 \end{bmatrix} = \begin{bmatrix} -M^{-1}D & -M^{-1} & 0 \\ I & 0 & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} \omega \\ h(\delta, V) \\ g(\delta, V) \end{bmatrix} \quad (9)$$

It is clear that the linearization of (9) about an operating point will have a zero eigenvalue if the Jacobian of the vector function $[\omega^T, h^T(\delta, V), g^T(\delta, V)]^T$ becomes singular. Defining $x := [\omega^T, \delta^T, V^T]^T$, one has

$$\frac{\partial}{\partial x} \begin{bmatrix} \omega \\ h(\delta, V) \\ g(\delta, V) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & J(x) \end{bmatrix} \quad (10)$$

and it follows that in this type of model, singularity of the powerflow Jacobian at x^0 is equivalent to singularity of $\partial f/\partial x(x^0)$, which in turn is equivalent to infinite sensitivity of state variable with respect to certain incremental changes in load parameters, as per (7). This type of relation will be examined in more detail in Section VI.

It is important to note that some recent work employing sensitivity analysis expands these concepts to consider a type of steady state "controllability" of voltage by changes in reactive injections. From an operational standpoint, it is often necessary not only that sensitivity of voltage to changes in injections be bounded, but that these changes have an appropriate sign. For example, it is desirable that an increase in reactive injection from a generator should increase load bus voltages in an associated control area. These requirements are more stringent than simply avoiding singularity of the powerflow Jacobian. Work reported in [27] suggests a variety of computational algorithms for testing this kind of steady state controllability, several of which require the Jacobian to maintain a M-matrix structure.

V. Nonlinear Dynamic Approaches

In describing approaches to voltage collapse that consider nonlinear dynamics, it is important to note that bifurcation analyses do in fact identify transitions in the stability of equilibria for a nonlinear model. However, since bifurcation analysis is based on behavior of the linearization of the dynamics, one expects that this approach will determine primarily local behavior

of the trajectories². In examining the behavior in nonlinear dynamic systems with a stable equilibrium point, it is often useful to obtain more global information, characterizing the set of all initial conditions that are attracted back to that equilibrium, the so-called region of attraction. Intuitively, one might associate the "size" (in an appropriate norm or metric) of this region of attraction with the ability of the stable operating point to recover from fast time scale perturbations which tend to disturb the state away from equilibrium. Several of the nonlinear dynamic analyses of voltage collapse which have appeared in the literature have been concerned with identifying regions of attraction.

In [29], a nonlinear model is postulated that focuses exclusively on the dynamics of tap changing transformers, with a decoupled power flow model representing the interchange of power between buses. In this work, time varying parameters of the form $L(t)$ are not explicitly considered in the dynamic equations, but it is clear that the regions of attraction estimated by [29] will vary with system loading levels. It is also interesting to note that while this model is naturally formulated as differential equations with algebraic constants in the form of (1), assumptions of the decoupled power flow approximation and pure impedance load models makes it possible to solve the algebraic constraints (1b) explicitly for y in terms of dynamic variables $x:=z$. Therefore the authors obtain a set of purely differential equations with the structure (2). This type of model is globally well posed, with no possibility of ill defined trajectories or impasse point behavior. Using only the tap changer dynamics, the authors identify a feasible stable equilibrium in the state space of tap positions, and to construct hyper-rectangles in this state space that approximate the equilibrium's region of attraction. It is interesting to note that the authors of [29] comment that the equilibrium of interest is stable if the associated Jacobian matrix for the dynamic equations, $\partial f/\partial x$, is nonsingular. Hence this system model again has the property that transition to instability must take place with an eigenvalue of the linearized dynamics passing through the origin, rather than with complex eigenvalues crossing the imaginary axis. While the question is not explored in [29], it would be interesting to examine whether or not this condition for loss of stability of the equilibrium can be related to singularity of the power flow Jacobian $J(x)$, as is the case in many other dynamic models.

An alternate nonlinear dynamic approach is an energy based voltage collapse security measure introduced by the authors in [30]. The present paper will review that work by presenting a somewhat different development from that in [30], exploiting an optimal control derivation of energy that makes explicit the role of both slow and fast time scale disturbances, $L_{\text{slow}}(t)$ and $L_{\text{small}}(t)$. This analysis will assume that a suitable dynamic load model or

singular perturbation has been employed to obtain a system of the form (2), having no algebraic constraints. An illustration of the singular perturbation approach will be presented in Section VI.

The analysis assumes the system is at a known load level and operating point; for simplicity, let these correspond to time $t=0$, so the operating point is defined by $(L(0), x(0))$. Furthermore, assume that this initial operating point has a stable linearization, i.e., $\partial f/\partial x(x(0))$ has all eigenvalues strictly in the left half plane. To isolate changes in load, (2) may be re-written as:

$$\dot{x} = \{ f(x) + B(x)L(0) \} + B(x)\Delta L(t) \quad (11)$$

with $\Delta L(t)=L_{\text{small}}(t)$ treated as a zero mean, fast time scale load disturbance input to the system. Hence the effect of the load disturbance will not be to change the equilibrium point, but rather to temporarily perturb the state away from the equilibrium $x(0)$. If $\Delta L(t)=0$, the system would remain at the stable equilibrium $x(0)$. Sufficiently small "impulses" in $\Delta L(t)$ will result in a state trajectories that asymptotically return to $x(0)$, but sufficiently large load variations will produce state trajectories that leave the region of attraction about $x(0)$ and diverge.

As discussed above, the "size" of the region of attraction offers a relative measure of the operating point's ability to recover from such load disturbances. Using an optimal control formulation, one can make this concept much more precise. The impact of load disturbances clearly depends both on their instantaneous magnitude and their duration. Therefore, to measure the size of load disturbances on a time interval $[0, T]$, the following "cost function" is proposed:

$$C_T(\Delta L(\cdot)) := \int_0^T \|\Delta L(t)\|_2^2 dt \quad (12)$$

The optimal control problem is then be formulated as follows: for all possible final times T , and all possible piecewise smooth controls $\Delta L(\cdot)$ that steer the state from $x(0)$ to a point $x(T)$ on the boundary of the region of attraction for $x(0)$, what is the minimum value achieved by $C_T(\Delta L(\cdot))$? The underlying assumption is that once the state is driven to the boundary of the region of attraction for $x(0)$, the deterministic dynamics will tend to dominate, and cause the state to diverge. For this phenomena to serve as a plausible description of voltage collapse, one expects that the form deterministic dynamics after the state reaches $x(T)$ should be such that the voltage magnitudes tend to rapidly decline.

As shown in [31], and subsequently exploited in [30], [32], there exists a class of power system models for which this optimal control problem is solvable in closed form. Indeed, rather than defining only the lowest cost of steering the state from stable equilibrium to points on the boundary of the region of attraction, it

²Though again we note that Hopf bifurcation can go somewhat further, and predict nonlinear limit cycle behavior in the vicinity of the old operating point.

is feasible to define a function $\vartheta(x)$ that measures the minimum cost of steering from stable equilibrium $x(0)$ to any other x . Moreover, it is easy to see that the resulting function has the properties of a Lyapunov function with respect to the unperturbed dynamics with $\Delta L(\cdot) \equiv 0$. For example, since the state will not move from $x(0)$ if the control $\Delta L(\cdot) \equiv 0$, there will always be some nonzero cost associated with moving the system off its equilibrium to a neighboring non-equilibrium state, $x \neq x(0)$. Therefore, $\vartheta(x)$ must be locally positive definite about $x(0)$. Also, the function must be nonincreasing along trajectories of the unperturbed system. Suppose there exist an $x(t_1)$ and an $x(t_2)$, $t_1 < t_2$, such that a trajectory of the unperturbed system initiated at $x(t_1)$ passes through $x(t_2)$. The cost of steering from $x(0)$ to $x(t_2)$ must be less than or equal than that of steering to $x(t_1)$. To reach $x(t_2)$, one can use the control that steers $x(0)$ to $x(t_1)$, set $\Delta L(t)$ to zero thereafter. The state will eventually reach $x(t_2)$ with no additional control cost. It follows that $\vartheta(x(t_1)) \geq \vartheta(x(t_2))$, i.e., the function $\vartheta(x(t))$ is nonincreasing along trajectories $x(t)$ of the unperturbed system.

As shown in [31], the minimum control cost defined by this optimal control approach is equal to a standard Lyapunov function for a system model that include voltage magnitude variation and reactive load models, such as that introduced in [33]. It is also interesting to note that the problem of finding the lowest cost of control to steer the state from stable equilibrium $x(0)$ out of $x(0)$'s region of attraction is closely related to a classic problem in Lyapunov analysis. In particular, the "lowest cost" path from $x(0)$ out of the region of attraction passes through what is termed the "closest unstable equilibrium point," or closest u.e.p. This closest u.e.p may be defined geometrically as the (unstable) equilibrium first encountered by expanding constant contours of $\vartheta(x)$ from $x(0)$. In the optimal control interpretation, a given constant contour represent the set of all points that can be reached with the same cost of control. Using the properties of $\vartheta(x)$ as a Lyapunov function, it is easily shown that these contours must remain closed, bounded, and contained in $x(0)$'s region of attraction until they first encounter another equilibrium point. This equilibrium will prove to be unstable³, and defines the lowest saddle point on $x(0)$'s region of attraction. Moreover, for the operating conditions and loading patterns associated with voltage collapse, this u.e.p. is typically one that displays very low voltage magnitudes. These low voltage u.e.p.'s are precisely the low voltage power flow solutions that have been exploited in other approaches to detecting the onset of voltage collapse conditions [34], [35], [36]. Trajectories initiated near a low voltage u.e.p., but outside $x(0)$'s region of attraction, will show very rapid

decline in voltage magnitude as the state diverges. This will be examined in more detail in the specific system model of Section VI.

As a final comment on modeling in the Lyapunov/optimal control analyses, it is important to note that the closed form solution and true Lyapunov function are only rigorously established for system models with zero transfer conductance and no voltage dependent terms in the active load model. The issue of Lyapunov functions for systems with nonzero conductances has received tremendous attention in the literature, but actually proves a minor point in practical analyses. The controversy in the literature has focused on the question of whether or not it is possible to obtain a *global* Lyapunov function for a system with transfer conductances; that is, a Lyapunov function that remains strictly nonincreasing along *all possible* trajectories, from any initial condition in the state space. This remains a difficult question, with some evidence [37] suggesting that such a global Lyapunov function is impossible to construct for models with transfer conductance. However, in the practical case when one begins with an operating point $x(0)$ whose linearization is strictly stable, it is a trivial matter to construct a *local* Lyapunov function that has the desired properties in a neighborhood of $x(0)$. In [38], a methodology is developed to identify a simple additive correction for the idealized $\vartheta(x)$ that results from the exact, path independent case. The result yields a *local* Lyapunov function that satisfies the required conditions in a fairly large neighborhood of the stable equilibrium $x(0)$. While not true *global* Lyapunov functions, these energy⁴ functions display behavior that is approximately nonincreasing along trajectories initiated in $x(0)$'s region of attraction, and prove quite reliable in estimating this region of attraction [32]. Alternative approaches to deriving practically useful energy functions for for systems models lacking a path independent integral have been in use for many years [39].

To relate this type of energy analysis back to steady state and bifurcation analysis, the role of fast versus slow time scale load variations becomes important. Bifurcation analyses examine load variations that occur on time scales slower than the underlying system dynamics, and identify how far a given load increase can be "pushed" until the operating point loses stability. The energy analysis begins with a stable operating point obtained when the slow load variation is frozen at a value below the critical threshold. For that operating point $x(0)$, the energy function evaluated at the closest u.e.p identifies the smallest fast time scale load disturbance that can push the state out of $x(0)$'s region of attraction, causing the state to then diverge. This energy margin essentially measures the size of the region of attraction, but does so using a metric that is closely related to the dynamics of the system: the solution to the

³This statement is true for systems of the form assumed in [D&B], but again, care is necessary in analyzing mixed systems of differential equations with algebraic constraints. The equilibrium first encountered by expanding contours of the same function $\vartheta(x)$ can prove to be stable in mixed differential/algebraic models.

⁴The convention here will be to use the term energy function when the resulting expression can not be proven to be nonincreasing along all possible trajectories

optimal control problem. The limit of the energy margin agrees with the limits predicted by a bifurcation analysis in that as the slow load variation pushes the operating point towards a value where it loses small disturbance stability, the energy margin goes to zero. The bifurcation analysis for this type of model predicts that the operating point $x(0)$ must merge with an unstable saddle point (a type-one u.e.p.) at the critical loading level. This unstable equilibrium is precisely the closest u.e.p. in energy analysis; when the closest u.e.p. and operating point merge, it is obvious that the energy margin between the two must shrink to zero.

As a final note on the relation of slow and fast time scale phenomena in examining voltage collapse, the recent work of [43] is noteworthy. In that work, the authors do not separate time scales of exogenous load variations, as has been done here, but rather separate time scales in the dynamic equations themselves. This leads to a partition of the state variables (possibly in a new coordinate system) into slow and fast sets, and an approximate decoupling of the slow and fast dynamic subsystems. Then separate stability analyses can be performed for the decoupled slow and fast subsystems, allowing separate examination of instability associated with slower dynamics such as tap changing transformers, and faster dynamics of generator excitation dynamics coupled to load characteristics. This type of integrated analysis may ultimately allow coupling between the type of model described here, and the tap changing transformer dynamic analysis of [29].

VI. Sample System Model and Comparison of Proximity Measures

To illustrate the relationships between analysis methods in more detail, it is not necessary to examine specific numerical examples, but rather the class of models must be considered. To this end, this section will develop a model of the structure of (1) that has many of the features considered in voltage stability analyses, and will allow explicit comparisons between methods of the types described in preceding sections. We will then discuss a methodology for modifying this mixed system of differential equations with algebraic constraints to obtain a system of purely differential equations. Also, the nature of ill-posed trajectories in the mixed differential/algebraic systems will be reviewed, and the relation of this problem to load modelling discussed. Finally, the relation between sensitivity, bifurcation and energy analyses will be illustrated in detail for the differential equation model.

It is also important to note the types of analyses that are not captured in such models. Only an approximate representation of reactive output saturation of generation is represented; no detailed exciter dynamics are included. Therefore, approaches to voltage stability improvement through improved nonlinear control

schemes for excitation, such as that proposed in [40], is beyond the scope of this model. Similarly, loss of small disturbance stability via complex eigenvalues associated with exciter dynamics migrating across the imaginary axis, as discussed in [18] and [19], is also not considered.

To illustrate the structure discussed in the preceding sections for a specific model, consider the following possible representation. The differential equations represented will arise from simple swing dynamics for generators, and from active loads have a component that is proportional to bus frequency, as originally developed for Lyapunov analysis in [41]. Linear frequency dependent loads offer a simple approximation of behavior of various types of motor loads. Reactive loads are allowed to be an arbitrary polynomial or exponential function of bus voltage magnitude; denote this function $Q_i(V_i)$. The expression for reactive power absorbed by the network can be found in any standard text treating power flow analysis, and can be written as a function of the vector $\alpha \in \mathbb{R}^{n-1}$ of phase angles (relative to a reference bus), and the vector $V \in \mathbb{R}^n$ of bus voltage magnitudes; denote this expression at bus i as $\tilde{g}_i(\alpha, V)$. The resulting reactive power balance equation becomes:

$$0 = Q_i(V_i) - \tilde{g}_i(\alpha, V) \quad (13)$$

In some types of analyses, it proves convenient to first normalize (13), dividing the equality by V_i^{-1} . The resulting Jacobian of the associated power balance equations becomes nearly symmetric, which can be advantageous both for certain numerical algorithms and for energy type analyses. With this normalization, the reactive power balance equations then have the structure

$$0 = V_i^{-1} \{ Q_i(V_i) - \tilde{g}_i(\alpha, V) \}, \quad (14)$$

which then make up the algebraic constraints shown by (1b) in the general model. The $L(t)$ terms would appear as variations in the terms of $Q_i(V_i)$; these can be made explicit when necessary.

Many authors have commented on the importance of reactive limits on generators in the voltage collapse phenomenon. The structure of equation in (14) may be easily adapted to model this effect. Assume that the generator excitation system at bus i is normally in a voltage control mode, with voltage setpoint V_i^0 . A simple representation of the exciter's behavior would increase the generator's MVar output when bus voltage drops below V_i^0 , and decrease its output when bus voltage rises above V_i^0 . However, the reactive output has both upper and lower limits, so if the reactive power absorbed by the network exceeds the maximum output of the generator, the bus voltage will go out of its "control band" about V_i^0 . This effect can be approximately represented by choosing $Q_i(V_i)$ for the generator as shown in Figure 1 below.

With this assumed form of reactive power balance at a generator buses, the effect of reactive limits can be included in an algebraic constraint of the same form as (14). It should be clear that the characteristic given in Figure 1 is really just a "smoothing" of the familiar switching function between PV bus behavior and PQ bus behavior that is typically used to represent reactive limits in power flow analysis. In the limit of a step change between $Q_{\max}$ and $Q_{\min}$, one recovers the PV/PQ switching behavior. However, for small disturbance stability analyses that require differentiation of the characteristic, the smoothed representation in Figure 1 appears a more tractable and equally plausible approximation. With the smoothed approximation, the generator terminal bus voltage magnitude remains a variable in all ranges of operation, so the dimension of the power flow Jacobian is not changed as the generator reaches its reactive limit.

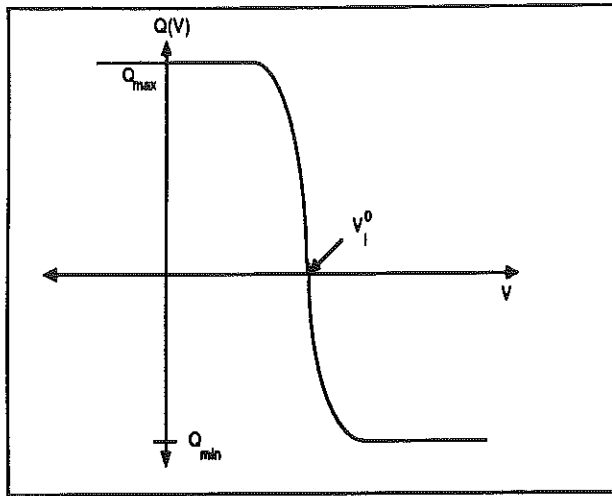


Figure 1: Generator Reactive Output versus Terminal Voltage

With the individual component models described above, the overall system model can be assembled, with variables given by:

$$\begin{aligned} \omega &\in \mathbb{R}^m, \text{ generator deviation from synchronous frequency,} \\ \alpha &\in \mathbb{R}^{n-1}, \text{ bus voltage phase angles relative to reference bus,} \\ V &\in \mathbb{R}^n, \text{ bus voltage magnitudes.} \end{aligned}$$

The differential/algebraic equations for this power system model are then written as:

$$\dot{\omega} = -M^{-1}T_1^T f(\alpha, V) \quad (15a)$$

$$\dot{\alpha} = T_1 \omega + T_2 D_1^{-1} T_2^T f(\alpha, V) \quad (15b)$$

$$0 = g(\alpha, V) \quad (15c)$$

where:

$$f(\alpha, V) := \bar{f}(\alpha, V) + P^0$$

$$\bar{f}_i(\alpha, V) := \text{active power absorbed by network at bus } i$$

$$P_i^0 := \text{nominal active power demanded by load at bus } i$$

$$g(\alpha, V) := (\text{diag}[V])^{-1} \{\bar{g}(\alpha, V) + Q_D(V)\}$$

$$\bar{g}_i(\alpha, V) := \text{reactive power absorbed by network at bus } i$$

$$Q_{D,i}(V_i) := \text{reactive power demanded by load at bus } i \text{ (or -generated } Q)$$

and M , D_1 , D_V are constant diagonal matrices describing system parameters, and T_1 and T_2 are constant matrices describing network topology.

As mentioned above, differential equations with algebraic constraints may not produce well defined trajectories from all initial conditions. In particular, they may produce trajectories that exist over some time interval $[0, \hat{t}]$, but become ill-defined for $t > \hat{t}$. A trivial example of this occurs in a two dimensional (non power system) example of $dx/dt = x+y$; $0=x^2+y^2-1$. Initiating this system at $(x=0, y=1)$, it is easily verified that the trajectories evolve on the circle defined by the algebraic constraint, until the point $(x=1, y=0)$ is reached. At this impasse point, x can no longer increase while still remaining on the circle, and the trajectory can not be continued. In a numerical integration that alternates between updating dynamic variables and solving algebraic constraints, this problem would show up as a new value of x being generated for which no y solution exists. Reference [7] verifies that this type of impasse behavior is easily constructed for simple power system examples when the reactive power balance is modelled as an algebraic constraint, and the load has some non-zero constant Q component (though it need not be purely a constant Q load).

The impasse point problem disappears if *all* loads are modelled as purely constant impedances, since in this case it is always possible to explicitly solve for the algebraic variables in terms of dynamic variables. This problem is tacitly acknowledged in numerical simulation approaches; for example, the review paper [2] comments that for load models containing constant power terms, solving the algebraic constraints may become numerically ill conditioned (a sign of the onset of impasse point behavior) when voltages dip. In [2] it is suggested that the simulation instantaneously "switch" to a constant impedance model. While such an approach may be physically motivated by the phenomena of stalling motors, it is clear that a more precise description of such transitions would be needed for reasonable voltage stability analysis. Moreover, it is clear that analytically studying the behavior of large systems of differential equations with algebraic

constraints can be even more challenging than numerically solving for specific trajectories.

To eliminate the problem of algebraic constraints while maintaining consistent behavior in the vicinity of the operating point, the algebraic constraints may be singularly perturbed to produce a differential equation for the algebraic variables. In the case of the bus voltage magnitudes and reactive power balance equations, this approach produces equations of the form:

$$\dot{V} = \frac{1}{\epsilon} V_i^{-1} \{ Q_i(V_i) - g_i(\alpha, V) \} \quad (16)$$

From an engineering standpoint, (16) may be interpreted as follows. The demand term is taken as the "independent input," and the voltages respond to this input to maintain reactive power balance. The right hand side of (16) is the difference between reactive power delivered by the network and reactive power absorbed by the load (or vis versa for the generator case). When the load instantaneously demands more reactive power than is being delivered, (16) predicts that the bus voltage drops until power balance is re-established. When excess reactive power is delivered by the network, bus voltage magnitude will increase. The rate of this change varies with ϵ ; for ϵ sufficiently small, it is essentially instantaneous and behavior is nearly identical to the original static model. Clearly, the equilibria of (16) are identical to the solutions of (14). To obtain consistent behavior between a model based on (14) and that of a model based on (16), standard results in singular perturbations (described in [7]) require that the linearization of (16) be stable in a neighborhood about the solution of interest for (14) (i.e., the solution of the full powerflow). Satisfaction of this condition depends in part on the exact form of the voltage dependence of the load. This analysis to confirm stability for a particular load model is left to the reader; here we will simply state without proof that experience with a wide range of load models shows the linearization of (16) about reasonable (per unit voltage magnitude above 0.7) operating points is typically stable. However, behavior of (16) differs from the model with algebraic constraints if one examines stability in the vicinity of "low voltage" power flow solutions. The algebraic constraint combined with standard small disturbance stability models would predict these equilibria to be stable; the differential model (16) predicts such points to be unstable. Unfortunately, because such low voltage solutions are usually not operable for other reasons, it is difficult to confirm which prediction best matches observed physical behavior. The last point concerning this model relates to the choice of ϵ . Clearly, this time constant should be small (probably less than 0.01 second) to match observed voltage behavior in power systems; fortunately, the energy function does not depend on the choice of ϵ .

For differential equations to define generator terminal voltage under the reactive generation limit model described above, the

form of (16) is kept, with $Q(V)$ reinterpreted as the reactive power supplied by the generator. For this interpretation of (16), ϵ should be chosen as the time constant associated with the response of the generators reactive output with respect to terminal voltage regulation errors. Again, because ϵ does not enter the energy function explicitly, the exact choice of this parameter is not critical.

With the singular perturbation assumptions above, the modified state model is composed purely of differential equations, and takes the form

$$\dot{\omega} = -M^{-1}T_1^T f(\alpha, V) \quad (17a)$$

$$\dot{\alpha} = T_1 \omega + T_2 D_1^{-1} T_2^T f(\alpha, V) \quad (17b)$$

$$\dot{V} = -D_V g(\alpha, V) \quad (17c)$$

with the new parameter matrix D_V simply being a constant diagonal matrix composed of the ϵ_i terms.

For a network with zero transfer conductances, and for $f(\alpha, V)$ defined with active loads having no voltage dependent component, the composite vector function $[(M\omega)^T, f^T(\alpha, V), g^T(\alpha, V)]^T$ is exact. Hence the value of a path integral of this function from $(0, \alpha^0, V^0)$ to (ω, α, V) will depend only on the endpoints. Therefore one may define

$$\vartheta(x^0, x) := \int_{(\alpha^0, V^0)}^{(\omega, \alpha, V)} [(M\lambda)^T, f^T(\xi, \mu), g^T(\xi, \mu)] [d\lambda^T, d\xi^T, d\mu^T]^T \quad (18)$$

which under the stated assumption on the network and active power loads may be re-expressed as

$$\begin{aligned} & \frac{1}{2} \omega^T M \omega - \frac{1}{2} \sum_{i=0}^n \sum_{k=0}^n B_{ik} V_i V_k \cos(\alpha_i - \alpha_k) \\ & + \frac{1}{2} \sum_{i=0}^n \sum_{k=0}^n B_{ik} V_i^0 V_k^0 \cos(\alpha_i^0 - \alpha_k^0) \\ & + \sum_{k=0}^n \int_{V_k^0}^{V_k} \frac{Q_{D,k}(\mu)}{\mu} d\mu - [P^0]^T (\alpha - \alpha^0) \end{aligned} \quad (19)$$

In the case where the network is not lossless, and/or the active load does have a voltage dependent component, the integral given in (18) is not path independent. In this case, a simple linear correction term added to $\vartheta(x^0, x)$ makes the function satisfy the criteria formal criteria for a Lyapunov function in the vicinity of the stable equilibrium, as described in [32]. The discussion to follow will assume the exact case for illustrative purposes. To illustrate the close connection between the energy function and the dynamic model, we define

$$A = \begin{bmatrix} 0 & -M^{-1}T_1^T & 0 \\ T_1^T M^{-1} & -T_2 D_1^{-1} T_2^T & 0 \\ 0 & 0 & -D_V^{-1} \end{bmatrix} \quad (20)$$

With these definitions, the power system dynamics of (17) may be rewritten

$$\dot{x} = A \nabla_x \vartheta(x^0, x) \quad (21)$$

By construction, A is nonsingular and negative semi-definite. In particular, A is the sum of two matrices: a diagonal matrix of non-positive terms, and a skew symmetric matrix. Roughly speaking, the right hand side of (21) can be thought of as a sum of a gradient like term (diagonal matrix times $\nabla \vartheta$) and a Hamiltonian like term (skew symmetric matrix times $\nabla \vartheta$). Given that A is nonsingular, equilibria of (21) occur only at points where $\nabla_x \vartheta(x^0, x) = 0$. Furthermore, if the Hessian of ϑ about an equilibrium is positive definite, using LaSalle's theorem with ϑ as a candidate Lyapunov function it may be shown that that equilibrium must be asymptotically stable.

The relation between the gradient of energy and the equilibria of the dynamic equations provides a first link between energy analysis and steady state criteria. Suppose that a slow time scale variation in load parameters is considered, of the general form suggested in (4), but with the load variations appearing as additive P-Q terms. Then (17) is modified to become

$$\dot{\omega} = -M^{-1}T_1^T f(\alpha, V) \quad (22a)$$

$$\dot{\alpha} = T_1 \omega + T_2 D_1^{-1} T_2^T (f(\alpha, V) + P(t)) \quad (22b)$$

$$\dot{V} = -D_V g(\alpha, V) + Q(t) \quad (22c)$$

The equation for the quasi-static evolution of the equilibrium, $\hat{x}(t)$, can then be written as:

$$0 = A \{ \nabla \vartheta(x^0, \hat{x}(t)) + BL(t) \} \quad (23)$$

where

$$B := \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}; \quad L(t) := \begin{bmatrix} 0 \\ P(t) \\ Q(t) \end{bmatrix}.$$

Again noting that the matrix A is nonsingular, the quasi-static evolution of the equilibrium is entirely determined by

$$0 = \nabla \vartheta(x^0, \hat{x}(t)) + BL(t) \quad (24)$$

Moreover, when partitioned into three blocks consistent with (ω, α, V) , the second and third blocks of $\nabla \vartheta$ are simply the power balance equations. Hence one has the unsurprising result that the only parts of the equilibrium that evolve with changing load are the bus phase angles and voltage magnitudes, as predicted by the power flow.

The close association of the power flow Jacobian to the linearized dynamics can also be easily illustrated in this model. The linearization of the dynamics about an equilibrium $\hat{x}(t)$ is determined by the Hessian of ϑ , and is given by

$$\dot{x} = A \frac{\partial^2}{\partial x^2} \vartheta(\hat{x}(t), \hat{x}(t)) \quad (25)$$

But from the construction of the energy function ϑ , it follows (for the idealized exact case) that

$$\frac{\partial^2}{\partial x^2} \vartheta(\hat{x}(t), \hat{x}(t)) = \begin{bmatrix} M & 0 \\ 0 & J(\hat{x}(t)) \end{bmatrix} \quad (26)$$

Recalling that M is a diagonal matrix of generator inertia constants, the Hessian of the energy is positive definite if and only if the Jacobian of the power flow is positive definite. Moreover, from (25) and (26) it is clear that a singular power flow Jacobian must produce a zero eigenvalue of the linearized dynamics. One concludes that the linearization is stable if and only if the power flow Jacobian is positive definite, and is on the boundary of small disturbance stability when the power flow Jacobian becomes singular. Hence, a quasi-static bifurcation analysis to determine when the equilibrium $\hat{x}(t)$ loses small disturbance stability is reduced to testing when $J(\hat{x}(t))$ loses strict positive definiteness and becomes singular.

Specializing the relation (7) to this model also confirms that the quasi-static solution for the operating point becomes infinitely sensitive to changes in P-Q load levels as $J(\hat{x}(t))$ approaches singularity. Hence, for the model examined here, tests for infinite sensitivity of voltages to loads predict the same critical loading value as a bifurcation analysis to detect when the operating point loses small disturbance stability. The energy/optimal control based security measure also goes to zero as the operating point approaches the condition of singular $J(\hat{x}(t))$.

The above discussion confirms that the three basic methods all predict the same critical loading level for the quasi-static load variation, $L_{slow}(t)$. However, when the nominal load has not yet reached a level where small disturbance stability is lost, the manner by which different methods measure the proximity to collapse can vary widely. As noted earlier, some methods use the smallest singular value of the power flow Jacobian. If applied to models that allow discontinuous switching of generators from PV

to PQ representations, the Jacobian will display a change of dimension when the switching occurs, and the smallest singular value will then be discontinuous with respect to changes in load, as was illustrated for an IEEE 30 bus test system example in [42]. While the continuous model for reactive saturation suggested in Figure 1 would eliminate the discontinuity, eigenvalues and singular values remain highly nonlinear functions of the elements of $J(\hat{x}(t))$, so this proximity measure would still be expected to display very nonlinear behavior. Sensitivity based measure that attempt to track the magnitude of certain dV/dQ terms as $J(\hat{x}(t))$ evolves will also tend to display very nonlinear behavior, and may vary widely from bus to bus. The plots of such measures provided in [42] are very informative in this regard. As an alternative, approaches that use the norm margin of load variation until collapse, of the form $\|L(\hat{t}) - L(0)\|$, will tend to be smoother, more nearly linear function of t . However, these measures may be very sensitive to the assumed functional dependence of L on t . In physical terms, this is the assumed pattern of load increase. It is clear that a pattern of load increase that puts greater load on buses with weak reactive support will push the system to collapse more rapidly, and lead to a smaller margin. This is a desirable characteristic for the proximity measure, *provided the predicted pattern of load increase is sufficiently accurate*. In an operating environment where the load projections may have significant margins of error, and operators wish to determine the risk of voltage problems on a time horizon of several hours, this could prove problematic. The obvious alternative is to look for a worst case pattern of load increase (perhaps with some constraints to keep the studied load patterns realistic), and judge proximity to collapse by the minimum, worst case distance. However, from a computational standpoint, this reduces again to the potentially intractable problem of testing for singularity in a multiparameter family of Jacobian matrices.

The energy/optimal control approach offers a more computationally tractable "worst case" proximity measure, but it is able to do so because of a different assumption on the nature of the load variation. The quasi-static measures seek to identify the worst case variation in $L_{\text{slow}}(t)$ such that the solution to (4) yields an operating point on the boundary of small disturbance stability, or equivalently, with infinite sensitivity of state to further load variations. The energy measure assumes that the current operating point defines a *fixed* equilibrium for the dynamic model, and the load variations of interest are on a fast time scale that excites the dynamics and moves the state temporarily away from equilibrium. In this formulation, the "worst case" fast time scale load variation $\Delta L(t)$ is in fact the solution to the optimal control problem that drives the system state out of the equilibrium point's region of attraction. As described in Section V, the energy based proximity measure is then an integral of the norm squared of this $\Delta L(t)$ load

variation with respect to time. It is important to stress that once the operating point and load level under study are set, *no assumed pattern of quasi-static load evolution ($L_{\text{slow}}(t)$) is necessary for the energy calculation*. This is perhaps the most critical distinction between the quasi-static measures and the energy based measure, and again stresses the need for careful understanding of the assumed time scales when comparing various voltage collapse proximity measures.

VII. Conclusions

This paper has compared the a range of voltage collapse analysis techniques that include both quasi-static bifurcation and sensitivity measures, and nonlinear dynamic measures. For a reasonable power system model incorporating many of the characteristics relevant to voltage collapse, and an assumption of slow time scale, quasi-static evolution of loads, it is shown that all such measures predict the same ultimate loading limit. However, if the load variations of interest include both a slow moving "average" term, and a faster time scale zero mean "disturbance" term, it is argued that a measure of proximity to collapse offered by nonlinear analyses will contain additional information. It is hoped that the close connections between various analysis techniques can be exploited, so that the various security measures can be used in complementary fashion in actual applications.

VIII. Acknowledgements

The support of this work by the National Science Foundation grant ECS-8857019 is gratefully acknowledged.

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ABSTRACT

The static and dynamic voltage stabilities are integrated. To consider a shunt compensator and/or a more general load which is dependent on voltage, a criterion for the static voltage stability is developed and the system is controlled to satisfy this static stability criterion. The dynamic voltage stability model includes excitation systems, tap-changers, capacitors and power system stabilizers in addition to network equations. A unique parameter optimization technique with a modal performance measure is developed to determine optimal control parameters to improve dynamic voltage stability.

INTRODUCTION

Abnormal voltages and voltage collapse pose a primary threat to power system stability, security and reliability. Voltage problems are now as important as electromechanical stability or thermal limits in many power systems. When a local reactive power supply is limited or is some distance from customer loads, voltage drop becomes excessive and troublesome. If a long transmission line to a region which has a potential voltage problem is lost, or a loss of local generation occurs, then a local reactive demand may not be met by a local reactive power reserve and the voltage stability problem may occur. Excessive voltage decline can occur following some severe system contingencies and this situation could be aggravated, possibly leading to voltage collapse, by further tripping of more transmission facilities, var sources, or generating units due to overloading.

Kwatny et al. [1] studied the problem by applying the bifurcation analysis to the load flow equations. They showed that a static bifurcation associated with voltage collapse exists and at that point the load voltages are infinitely sensitive to parameter variations. Liu et al. [2] presented a dynamic description of the voltage collapse by characterizing the voltage stability region in terms of the continuous tap-changer model. Schlueter et al. [3,4] introduced PQ and PV stabilities and controllabilities for the static voltage stability and presented a unified theoretical foundation for determining tests for voltage security conditions. Lee et al. [5] used these conditions for PQ stability and controllability as security constraints for the economic operation and demonstrated their usefulness for security voltage stability. Dobson et al. [6] and Chiang et al. [7] explained the dynamics of voltage collapse as a dynamic consequence of the bifurcation by using a simple model, including a load of a dynamic induction motor. Rajagopalan et al. [8] primarily considered a brief dynamic voltage stability of a system with excitation model by investigating the eigenvalues of the linearized system matrix. Lee and Lee [9] presented a mechanism of the dynamic phenomenon

of voltage collapse from the physical point of view and showed that an iterative reaction of voltage drop between a dynamic load and the system network can cause voltage collapse.

The dynamic voltage stability and the static voltage stability problems are integrated in this paper. For more accurate analysis of the dynamic voltage stability, the system model includes excitation systems, under-load-tap-changers, capacitors and power system stabilizers in addition to network equations. When considering a more general load which is dependent on voltage, a modified criterion for the static voltage stability is developed and the system is controlled to satisfy this static stability criterion. Then, the system is further controlled for dynamic voltage stability. For dynamic voltage stability enhancement, a parameter optimization technique with a modal performance measure is used to determine optimal control parameters.

STATIC VOLTAGE STABILITY

It is well known that load dynamics greatly affect the voltage stability. In [3], only the constant PQ loads that are independent on bus voltage were considered. Since the voltage dependent loads play very important role in voltage stability, more suitable constraints need to be developed. The voltage dependent load model can also be used in representing the static var compensators (capacitive or reactive), which are the controls used in enhancing the static voltage stability.

The mismatch active and reactive powers at system buses, except the swing bus, are given by

$$\Delta P_i = F_P(\theta, V) = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \theta_{ij}) - P_{Gi} - P_{oi} - P_{Li}(V_i) \quad (1)$$

$$\Delta Q_i = F_Q(\theta, V) = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \theta_{ij}) - Q_{Gi} - Q_{oi} - Q_{Li}(V_i) \quad (2)$$

where V_i and θ_i are the magnitude and phase angle of the voltage at the bus i , respectively; Y_{ij} and θ_{ij} are the magnitude and phase angle of the ij -th element of Y_{bus} matrix, respectively; P_{Gi} and Q_{Gi} are real and reactive powers of a generator at bus i , respectively; P_{oi} and Q_{oi} are constant real and reactive power loads, respectively; and $P_{Li}(V_i)$ and $Q_{Li}(V_i)$ are real and reactive loads, respectively, which depend on the voltage magnitude. The loads are negative in numerical values. For a fast load flow the modified Jacobian J can be obtained as

follows:

$$J = \begin{pmatrix} \frac{\partial F_P}{\partial \theta} & V \frac{\partial F_P}{\partial V} \\ \frac{\partial F_Q}{\partial \theta} & V \frac{\partial F_Q}{\partial V} \end{pmatrix}, \quad (3)$$

where $J_1 \equiv V \frac{\partial F_Q}{\partial V}$ is the matrix with $V_j \frac{\partial F_{Q_i}}{\partial V_j}$ as its i -th row and j -th column entry. Then, the diagonal elements of J_1 are rewritten as follows:

$$V_i \frac{\partial F_{Q_i}}{\partial V_i} = Q_i - V_i^2 B_{ii} - V_i \frac{\partial Q_{Li}(V_i)}{\partial V_i}, \quad (4)$$

where

$$Q_i = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \theta_{ij})$$

$$B_{ii} = Y_{ii} \sin(\theta_{ii}).$$

STATIC VOLTAGE STABILITY CRITERION

Following Schlueter [3] for PQ stability and controllability, J_1 for PQ(load) buses should be an M-matrix in order for the system to be PQ stable. For J_1 to be an M-matrix, the diagonal entries of J_1 need to be positive and J_1 is required to be strictly diagonally dominant, i.e., the diagonal term in equation (4) is larger than the sum of the absolute values of the off-diagonals. Hence, the following constraints are imposed:

$$Q_i - V_i^2 B_{ii} - V_i \frac{\partial Q_{Li}(V_i)}{\partial V_i} > \sum_{\substack{j \in L \\ j \neq i}} |Q_{ij}|, \quad (5)$$

where Q_i is reactive load injection at bus i , L is the set of load(PQ) buses and Q_{ij} are off-diagonals of J_1 , which can be expressed as follows:

$$Q_{ij} = V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \theta_{ij}), \quad (6)$$

where Q_{ij} is negative in a general power system since $90^\circ \leq \theta_{ij} \leq 110^\circ$. Thus, equation (5) is rewritten as follows:

$$Q_i - V_i^2 B_{ii} - V_i \frac{\partial Q_{Li}(V_i)}{\partial V_i} + \sum_{\substack{j \in L \\ j \neq i}} Q_{ij} > 0. \quad (7)$$

The minimum value of equation (7) is a lower bound on eigenvalues of J_1 , and thus when it is equal to zero a static bifurcation exists to cause a voltage collapse. The minimum eigenvalue of J_1 also needs to be greater than a certain threshold to assume the convergence of a load flow program, hence a positive constraint constant k is imposed. Thus, the constraint can be expressed as follows:

$$Q_i - V_i \frac{\partial Q_{Li}(V_i)}{\partial V_i} - V_i^2 B_{ii} + \sum_{\substack{j \in L \\ j \neq i}} Q_{ij} \geq k > 0, \quad (8)$$

where k is defined as the *stability margin*. The constraint proposed in [3] does not include the variable load term, $V_i \cdot \partial Q_{Li}(V_i)/\partial V_i$, in the above equation (8). If the left-hand side of equation (8) can be increased above the stability margin k , the voltages in the network will be more controllable. The value of k is a positive threshold, which can be determined arbitrarily

by the decision maker depending on the condition of a power system and the weight on the static voltage stability. The above *voltage stability criterion* will play an important role in designing controls for static voltage stability.

STATIC VOLTAGE STABILITY CONTROL

To improve the static voltage control stability the capacitive compensator and the tap-changer are the ones most often used control devices. The above voltage stability criterion, equation (8), is directly applicable in modeling the roles of these equipments for control purpose.

Shunt capacitor

The reactive power of a capacitance C is $\omega C V^2$, hence with this load alone the constraint for static voltage stability in equation (8) can be expressed as follows:

$$\begin{aligned} \omega C V_i^2 - V_i \cdot \frac{\partial}{\partial V_i} (\omega C V_i^2) - V_i^2 B_{ii} + \sum_{\substack{j \in L \\ j \neq i}} Q_{ij} \\ = -V_i^2 B_{ii} - \omega C V_i^2 + \sum_{\substack{j \in L \\ j \neq i}} Q_{ij} \geq k > 0. \end{aligned} \quad (12)$$

Here, the term $-V_i^2 B_{ii}$ is the largest term and dominates the left-hand side of the inequality. The term $-V_i^2 B_{ii}$ is positive because B_{ii} is negative in a general power system. As the value of the capacitor increases, the bus voltage increases and so does the value of $-V_i^2 B_{ii}$ in proportion to the square of the voltage magnitude. Therefore, the value of the left-hand side of the above inequality will be increased as the capacitor increases. Thus, a shunt capacitor can be used to control static voltage stability.

Tap-changer

The under-load-tap-changer (ULTC) is also a control equipment for static voltage stability. Since bus voltage can be increased very effectively by ULTC, the term $-V_i^2 B_{ii}$ increases in the constraint equation (8) and, thus, the static voltage stability is improved. Since it controls voltages relatively, when the voltage of one bus increases other bus voltage decreases. Thus, the static voltage stability may be improved at one bus at the expense of another bus.

THE POWER SYSTEM MODEL

The power system model for dynamic voltage stability study includes the following components: the nonlinear machine model with a 2-axis representation of the generator, the IEEE type 1 excitation system, the power system stabilizer of two cascaded lead stages, and the continuous model for the tap-changing transformer.

The direct axis internal transient voltage E'_d is very small compared to the quadrature axis internal transient voltage E'_q , and the armature resistance R_a is very small compared to the direct and quadrature axis synchronous reactances X_d and X_q . Thus, the effects of these can be neglected for the simplicity of analysis. Therefore, the complicated stator algebraic equations are simplified as follows [9]:

$$I_{di} = \frac{E'_{qi} - V_i \cos(\delta_i - \theta_i)}{X'_{di}} \quad (13)$$

$$I_{qi} = \frac{V_i \sin(\delta_i - \theta_i)}{X'_{qi}}, \quad (14)$$

where δ is the rotor angle, I_d and I_q are respectively the direct and quadrature axis stator currents, and X'_d and X'_q are respectively the direct and quadrature axis transient reactances. These current equations are substituted into the differential equations and the network equations in order to decrease the dimension of the overall equations. Then the complete dynamic voltage stability model is obtained as follows:

Generator model:

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s, \quad i = 1, \dots, m \quad (15)$$

$$M_i \frac{d\omega_i}{dt} = P_{Mi} - \left[\frac{E'_{qi} V_i \sin(\delta_i - \theta_i)}{X'_{di}} - \frac{1}{2} V_i^2 \frac{X'_{qi} - X'_{di}}{X'_{di} X'_{qi}} \sin 2(\delta_i - \theta_i) \right] - D_i(\omega_i - \omega_s), \quad i = 1, \dots, m \quad (16)$$

$$T'_{doi} \frac{dE'_{qi}}{dt} = -E'_{qi} - (X_{di} - X'_{di}) \frac{E'_{qi} - V_i \cos(\delta_i - \theta_i)}{X'_{di}} + E_{fdi}, \quad i = 1, \dots, m \quad (17)$$

where ω_i and ω_s are respectively the velocity of machine i and the synchronous velocity; M and D are respectively the inertia and damping constants; and T'_{do} and E_{fd} are respectively the direct axis transient short circuit time constant and the stator emf corresponding to the d.c. field voltage.

Exciter model:

$$T_{Ei} \frac{dE_{fdi}}{dt} = -(K_{Ei} + S_{Ei}(E_{fdi}))E_{fdi} + V_{Ri}, \quad i = 1, \dots, m \quad (18)$$

$$T_{Ai} \frac{dV_{Ri}}{dt} = -V_{Ri} + K_{Ai} R_{fi} - \frac{K_{Ai} K_{Fi}}{T_{Fi}} E_{fdi} + K_{Ai}(V_{refi} - V_i) + K_{Ai} U_{Ei}, \quad i = 1, \dots, m \quad (19)$$

$$T_{Fi} \frac{dR_{fi}}{dt} = -R_{fi} + \frac{K_{Fi}}{T_{Fi}} E_{fdi}, \quad i = 1, \dots, m \quad (20)$$

where V_R , V and V_{ref} are the regulator output, generator terminal and reference voltages, respectively; U_E is the stabilizing signal from PSS; R_f is the regulator stabilizing circuit output; K_A and K_F are the regulator gain and the regulator stabilizing circuit gain, respectively; K_E is an exciter constant; T_E , T_A and T_F are the exciter, the regulator and the regulator stabilizing circuit time constants, respectively; and $S_{Ei}(E_{fdi})$ is a saturation function of the DC generator-exciter.

PSS model:

$$\frac{dX_{pssi}}{dt} = -\frac{1}{T_{pi}} X_{pssi} + \frac{d\omega_i}{dt}, \quad i = 1, \dots, m \quad (21)$$

$$\begin{aligned} \frac{dU_{E2i}}{dt} = & -\frac{1}{T_{2i}} U_{E2i} + \frac{K_{Ci}}{T_{2i}} X_{pssi} \\ & + \frac{T_{1i} K_{Ci}}{T_{2i}} \frac{dX_{pssi}}{dt}, \quad i = 1, \dots, m \end{aligned} \quad (22)$$

$$\frac{dU_{Ei}}{dt} = -\frac{1}{T_{2i}} U_{Ei} + \frac{1}{T_{2i}} U_{E2i} + \frac{T_{1i}}{T_{2i}} \frac{dU_{E2i}}{dt}, \quad i = 1, \dots, m \quad (23)$$

where U_E is the stabilizing signal as an output of the PSS, and X_{pss} and U_{E2} are the state variables of the PSS system.

ULTC model:

$$-T_{ti} \frac{da_i}{dt} = (V_{refi} - V_i), \quad i = 1, \dots, n \quad (24)$$

where the model is a continuous approximation of the discrete under-load-tap-changer [2], a is the turns ratio, V and V_{ref} are the secondary voltage measurement and the reference voltage, respectively, and T_t is the time constant of the ULTC.

Network model:

$$P_i = V_i \left[\frac{E'_{qi} - V_i \cos(\delta_i - \theta_i)}{X'_{di}} \sin(\delta_i - \theta_i) + \frac{V_i \sin(\delta_i - \theta_i)}{X'_{qi}} \cos(\delta_i - \theta_i) \right] + P_{0i} + P_{Li}(V_i), \quad i = 1, \dots, m \quad (25)$$

$$Q_i = V_i \left[\frac{E'_{qi} - V_i \cos(\delta_i - \theta_i)}{X'_{di}} \cos(\delta_i - \theta_i) - \frac{V_i \sin(\delta_i - \theta_i)}{X'_{qi}} \sin(\delta_i - \theta_i) \right] + Q_{0i} + Q_{Li}(V_i), \quad i = 1, \dots, m \quad (26)$$

where the net injected real and reactive powers are given by

$$P_i = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \theta_{ij}) \quad (27)$$

$$Q_i = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \theta_{ij}). \quad (28)$$

DYNAMIC VOLTAGE STABILITY CONTROL

When dynamic components, such as generator, exciter, PSS, and ULTC, are included in the power system, the system may go into dynamic voltage instability and voltage collapse [9]. In order to prevent this, parameters of these dynamic components need to be selected in a judicious way. One approach to this is the optimization of parameters so that the oscillations are minimized around a stable equilibrium point. The problem is first formulated using the new concept of modal performance measure, and then the parameter optimization scheme is developed.

PROBLEM FORMULATION

The power system model for dynamic voltage stability in equations (15)–(28) can be shown to be of the following differential-algebraic type:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y}) \quad (29)$$

$$0 = \mathbf{g}(\mathbf{x}, \mathbf{y}), \quad (30)$$

where $\mathbf{x}$ represents the state variables of machines, excitation systems and power system stabilizers, and $\mathbf{y}$ represents network variables, such as the voltages and angles at each of the network buses. Using the equilibrium point $\mathbf{x}_0$, equations (29) and (30) are linearized to give

$$\Delta \dot{\mathbf{x}} = \mathbf{A} \Delta \mathbf{x} + \mathbf{B} \Delta \mathbf{y} \quad (31)$$

$$0 = \mathbf{C} \Delta \mathbf{x} + \mathbf{D} \Delta \mathbf{y}, \quad (32)$$

where $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are matrices of appropriate dimension defined by $\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}} \Big|_{\mathbf{x}_0}$, $\mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{y}} \Big|_{\mathbf{x}_0}$, $\mathbf{C} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}} \Big|_{\mathbf{x}_0}$, and $\mathbf{D} = \frac{\partial \mathbf{g}}{\partial \mathbf{y}} \Big|_{\mathbf{x}_0}$.

When $\mathbf{D}$ is singular, it can be shown that the Jacobian $\mathbf{J}$ in equation (3) is singular and thus a static bifurcation exists to cause voltage collapse. Therefore, the static voltage stability enhancement must be performed first before the dynamic voltage stability enhancement. Thus, we assume now $\mathbf{D}$ is nonsingular, then the incremental algebraic variables can be eliminated and the final dynamic system is

$$\Delta \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B} \mathbf{D}^{-1} \mathbf{C}) \Delta \mathbf{x} = \tilde{\mathbf{A}} \Delta \mathbf{x}, \quad (33)$$

$$\Delta \mathbf{y} = -\mathbf{D}^{-1} \mathbf{C} \Delta \mathbf{x} = \tilde{\mathbf{C}} \Delta \mathbf{x}, \quad (34)$$

where $\Delta \mathbf{x} = \mathbf{x}(t) - \mathbf{x}_s$, $\Delta \mathbf{y} = \mathbf{y}(t) - \mathbf{y}_s$, and $\mathbf{x}_s$ and $\mathbf{y}_s$ are the steady state values of $\mathbf{x}(t)$ and $\mathbf{y}(t)$, respectively.

The control variables are model parameters in the system matrix $\tilde{\mathbf{A}}$, hence the control problem becomes non-conventional and the usual linear optimal control theory cannot be applied. Since the objective of dynamic voltage stability can be achieved by minimizing oscillations of the state and network variables, we define a new performance measure which will limit the magnitude or envelope of oscillations.

If we let $\mathbf{z}(t) = \mathbf{y}(t) - \mathbf{y}_r$, where $\mathbf{y}_r$ is the reference value of $\mathbf{y}$. Then the objective of the control problem is to minimize the error $\mathbf{z}(t)$ with respect to system parameters, where from equation (34), $\mathbf{z}(t)$ can be expressed as the output vector:

$$\mathbf{z}(t) = \tilde{\mathbf{C}} \Delta \mathbf{x} + \mathbf{y}_s - \mathbf{y}_r. \quad (35)$$

Since the envelope of this error trajectory can be decomposed into a number of system modes, the error can be equivalently minimized by minimizing the envelopes of individual modes in a way similar to the work of Jung et al. [10]. Without the loss of generality, we assume that the $\tilde{\mathbf{A}}$ matrix has distinct eigenvalues. (Since we are in the process of changing eigenvalues this is a realistic assumption). Then $\Delta \mathbf{x}$ is expressed by modes as follows:

$$\Delta \mathbf{x}(t) = \sum_{j=1}^n (\mathbf{v}_j \Delta \mathbf{x}_0) \mathbf{u}_j \cdot \exp(s_j t), \quad (36)$$

where s_j is the j -th eigenvalue of $\tilde{\mathbf{A}}$, $\mathbf{u}_j$ and $\mathbf{v}_j$ are the corresponding $n \times 1$ right and $1 \times n$ left eigenvectors, respectively, $\Delta \mathbf{x}_0$ is the initial value of $\Delta \mathbf{x}$, and n is the dimension of $\Delta \mathbf{x}$.

Let $\Delta \mathbf{x}_j(t)$ be the j -th state of the vector $\Delta \mathbf{x}(t)$; $\mathbf{z}_j(t)$ be the component of $\mathbf{z}(t)$ that depends on the j -th state of the vector

$\Delta \mathbf{x}(t)$; $\mathbf{z}_{j,i}$ be the i -th mode of $\mathbf{z}_j(t)$; $u_{i,j}$ be the i -th entry of the j -th right eigenvector $\mathbf{u}_j$; and $\tilde{\mathbf{C}}_j$ be the j -th column vector of the matrix $\tilde{\mathbf{C}}$.

Then the output in equation (35) becomes

$$\mathbf{z}(t) = \sum_{j=1}^n \tilde{\mathbf{C}}_j \Delta \mathbf{x}_j(t) + \mathbf{z}_0 = \sum_{j=1}^n \mathbf{z}_j(t) + \mathbf{z}_0, \quad (37)$$

where $\mathbf{z}_0 = \mathbf{y}_s - \mathbf{y}_r$, and

$$\mathbf{z}_j(t) = \sum_{i=1}^n \mathbf{z}_{j,i}(t) = \tilde{\mathbf{C}}_j \sum_{i=1}^n (\mathbf{v}_i \Delta \mathbf{x}_0) u_{i,j} \cdot \exp(s_i t). \quad (38)$$

Since the error trajectory $\mathbf{z}(t)$ is now decomposed, its minimization can be achieved by minimizing the envelopes of its component trajectories. The area under each envelope can be represented by the *modal performance measure*, defined as follows:

$$J(\mathbf{p}) = \sum_{j=0}^n J_j \quad (39a)$$

where

$$J_j = \int_0^T \sum_{i=1}^n \mathbf{z}_{j,i}^t W_j \mathbf{z}_{j,i}^* dt \quad \text{for } j - \text{th state}, \quad (39b)$$

$$J_0 = \int_0^T \mathbf{z}_0^t W_0 \mathbf{z}_0^* dt \quad \text{for constant term}, \quad (39c)$$

where T is an integration time interval, W_j is a weighting matrix for j -th state, and superscripts t and $*$ denote transpose and conjugate, respectively. The minimization of the performance measure is to be achieved with respect to system parameters which are contained in the matrix $\tilde{\mathbf{A}}$.

PARAMETER OPTIMIZATION

The modal performance measure $J(\mathbf{p})$ can be minimized by evaluating its sensitivity with respect to the parameter vector $\mathbf{p}$. The gradient ∇J can be expressed by applying the chain rule as

$$\nabla J = \frac{\partial J}{\partial \mathbf{p}} = \sum_{j=0}^n \frac{\partial J_j}{\partial \mathbf{p}} = \sum_{j=0}^n \sum_{k=1}^n \sum_{l=1}^n \left(\frac{\partial J_j}{\partial \tilde{a}_{k,l}} \right) \left(\frac{\partial \tilde{a}_{k,l}}{\partial \mathbf{p}} \right) \quad (40)$$

where $(\frac{\partial J_j}{\partial \tilde{a}_{k,l}})_{k,l}$ is the (k,l) entry of the sensitivity matrix $(\frac{\partial J_j}{\partial \tilde{\mathbf{A}}})$ and $\tilde{a}_{k,l}$ is the (k,l) entry of matrix $\tilde{\mathbf{A}}$.

From equations (39b) and (38), we have

$$J_m = \sum_{i=1}^n (\tilde{\mathbf{C}}_m^t W_m \tilde{\mathbf{C}}_m^*) (\mathbf{v}_i \Delta \mathbf{x}_0)^t (\mathbf{v}_i \Delta \mathbf{x}_0)^* \cdot (\mathbf{u}_{m,i}) (\mathbf{u}_{m,i})^* \cdot f(\sigma_i), \quad (41)$$

where

$$f(\sigma_i) = \frac{1}{2\sigma_i} (\exp(2\sigma_i T) - 1) \quad (42)$$

and σ_i is the real part of the eigenvalue s_i .

The sensitivity of the performance J_m , with respect to elements of the matrix $\tilde{\mathbf{A}}$, can be derived by linearizing the performance

measure J_m . Suppose that ΔJ_m , $\Delta \sigma_i$, $\Delta u_{m,i}$ and Δv_i are the increments of J_m , σ_i , $u_{m,i}$ and v_i , respectively, due to the increments of the elements of the matrix $\tilde{A}$. Then the incremental performance is defined by

$$\Delta J_m = J_m(\tilde{A} + \Delta \tilde{A}) - J_m(\tilde{A}). \quad (43)$$

From equation (41),

$$J_m(\tilde{A} + \Delta \tilde{A}) = \sum_{i=1}^n (\tilde{C}_m^t W_m \tilde{C}_m^*) (f(\sigma_i) + f'(\sigma_i) \Delta \sigma_i + O(\Delta \sigma_i^2)) \cdot (v_i \Delta x_0 + \Delta v_i \Delta x_0)^t (v_i \Delta x_0 + \Delta v_i \Delta x_0)^* \cdot (u_{m,i} + \Delta u_{m,i}) (u_{m,i} + \Delta u_{m,i})^* \quad (44)$$

where

$$f'(\sigma_i) = \frac{\partial f(\sigma_i)}{\partial \sigma_i} = \frac{2\sigma_i T \exp(2\sigma_i T) - \exp(2\sigma_i T) + 1}{2\sigma_i^2} \quad (45)$$

and $O(\Delta \sigma_i^2)$ are the terms with the second and higher orders of $\Delta \sigma$.

Eliminating the higher order terms of all increments except Δx_0 , from equation (43) the incremental performance ΔJ_m can be expressed by

$$\Delta J_m = \sum_{i=1}^n (\tilde{C}_m^t W_m \tilde{C}_m^*) \left\{ f(\sigma_i) (v_i \Delta x_0) (v_i^* \Delta x_0^*) u_{m,i} \Delta u_{m,i}^* + f(\sigma_i) (v_i \Delta x_0) (v_i^* \Delta x_0^*) \Delta u_{m,i} u_{m,i}^* + f(\sigma_i) (v_i \Delta x_0) (\Delta v_i^* \Delta x_0^*) u_{m,i} u_{m,i}^* + f(\sigma_i) (\Delta v_i \Delta x_0) (v_i^* \Delta x_0^*) u_{m,i} u_{m,i}^* + f'(\sigma_i) \Delta \sigma_i (v_i \Delta x_0) (v_i^* \Delta x_0^*) u_{m,i} u_{m,i}^* \right\}. \quad (46)$$

Here it is required to express $\Delta u_{m,i}$, Δv_i and $\Delta \sigma_i$ in terms of $\Delta \tilde{a}_{k,l}$ or $\Delta \tilde{A}$. These incremental values are evaluated following the eigenvalue and eigenvector sensitivities of Crossley and Porter [11].

By substituting these incremental values into equation (46), the incremental performance becomes

$$\Delta J_m = \sum_{i=1}^n (\tilde{C}_m^t W_m \tilde{C}_m^*) \cdot \left\{ f(\sigma_i) v_i \Delta x_0 v_i^* \Delta x_0^* 2 \operatorname{Re} \left(\underline{U}_m^t D_i V \Delta \tilde{A} u_{m,i} \right) + f(\sigma_i) u_{m,i} \Delta u_{m,i}^* 2 \operatorname{Re} \left(v_i \Delta \tilde{A} U D_i V \Delta x_0 v_i^* \Delta x_0^* \right) + f'(\sigma_i) \operatorname{Re} \left(v_i \Delta \tilde{A} u_i \right) v_i \Delta x_0 v_i^* \Delta x_0^* u_{m,i} u_{m,i}^* \right\} \quad (47)$$

where Re means a real part, and

$$D_i = \operatorname{diag}[1/(s_i - s_1), \dots, 1/(s_i - s_{i-1}), 0, 1/(s_i - s_{i+1}), \dots, 1/(s_i - s_n)]$$

$$U = [u_1 \quad u_2 \quad \dots \quad u_n]$$

$$V = [v_1^t \quad v_2^t \quad \dots \quad v_n^t]^t$$

$$\underline{U} = U^t = [\underline{U}_1 \quad \underline{U}_2 \quad \dots \quad \underline{U}_n].$$

In order to obtain the sensitivity matrix $\frac{\partial J_m}{\partial \tilde{A}}$, the following trace properties are used:

$$\operatorname{Tr}(BC) = \operatorname{Tr}(CB) = CB, \quad (48)$$

$$\Delta J = \operatorname{Tr}(M \Delta A) \implies \frac{\partial J}{\partial A} = M^t, \quad (49)$$

where B and C are column and row vectors, respectively; A and M are square matrices; and Tr means the trace. Using these properties,

$$\Delta J_m = \sum_{i=1}^n (\tilde{C}_m^t W_m \tilde{C}_m^*) \cdot \left\{ f(\sigma_i) |v_i \Delta x_0|^2 2 \operatorname{Re} \left(\operatorname{Tr} \left[\underline{U}_i \underline{U}_m^t D_i V \Delta \tilde{A} \right] u_{m,i}^* \right) + f(\sigma_i) |u_{m,i}|^2 2 \operatorname{Re} \left(\operatorname{Tr} \left[\Delta \tilde{A} U D_i V \Delta x_0 v_i \right] v_i^* \Delta x_0^* \right) + f'(\sigma_i) |v_i \Delta x_0|^2 |u_{m,i}|^2 \operatorname{Re} \left(\operatorname{Tr} \left[\Delta \tilde{A} u_i v_i \right] \right) \right\} \quad (50)$$

and, the sensitivity matrices are obtained as follows:

$$\frac{\partial J_m}{\partial \tilde{A}} = \sum_{i=1}^n (\tilde{C}_m^t W_m \tilde{C}_m^*) \cdot \left\{ f(\sigma_i) |v_i \Delta x_0|^2 2 \operatorname{Re} \left(\left[\underline{U}_i \underline{U}_m^t D_i V \right]^t u_{m,i}^* \right) + f(\sigma_i) |u_{m,i}|^2 2 \operatorname{Re} \left(\left[U D_i V \Delta x_0 v_i \right]^t v_i^* \Delta x_0^* \right) + f'(\sigma_i) |v_i \Delta x_0|^2 |u_{m,i}|^2 \operatorname{Re} \left(\left[u_i v_i \right]^t \right) \right\}. \quad (51)$$

Then, the gradient of the performance measure with respect to the parameter vector p is expressed as follows:

$$\frac{\partial J_m}{\partial p} = \sum_{k=1}^n \sum_{l=1}^n \left(\frac{\partial J_m}{\partial \tilde{A}} \right)_{k,l} \left(\frac{\partial \tilde{a}_{k,l}}{\partial p} \right), \quad m = 1, \dots, n \quad (52)$$

It remains to evaluate the gradient for the steady-state performance $\frac{\partial J_0}{\partial p}$ in equation (40). From equation (39c),

$$J_0 = \int_0^T z_0^t W_0 z_0^* dt = (y_s - y_r)^t W_0 (y_s - y_r)^* T. \quad (53)$$

Here y_s is a steady state value of nonlinear algebraic equations which can be solved only by an iterative numerical method, and mathematical expressions of $\partial y_s / \partial \tilde{A}$ and $\partial y_s / \partial p$ are not known. Using equation (53) the steady state term in the equation (40) can be expressed as follows:

$$\sum_{k=1}^n \sum_{l=1}^n \left(\frac{\partial J_0}{\partial \tilde{A}} \right)_{k,l} \left(\frac{\partial \tilde{a}_{k,l}}{\partial p} \right) = \frac{\partial [(y_s - y_r)^t W_0 (y_s - y_r)^* T]}{\partial p}. \quad (54)$$

The value of this partial derivative can be determined numerically by the load flow about the small perturbation of parameter vector p .

Thus, the gradient of the modal performance measure ∇J in equation (40) has been evaluated. Then one of the hill-climbing or gradient methods can be used to determine the minimum of

the performance measure J . In this paper, the steepest descent method is used for the parameter optimization problem and the results are the optimal values of parameters that not only decrease the dynamical oscillations of the output $y(t)$, but also make the values of $y(t)$ close to its reference value y_r . The optimal parameter vector p can be found by the method of steepest descent defined by the iterative algorithm

$$p_{k+1} = p_k - \alpha_k \nabla J(p_k) \quad (55)$$

where α_k is a nonnegative scalar and can be chosen optimally by searching for the minimum of $J(p_k - \alpha \nabla J(p_k))$ along the direction of the negative gradient $-\nabla J(p_k)$.

ANALYSIS AND CONTROL OF STATIC AND DYNAMIC VOLTAGE STABILITY

The analysis and control methodologies for both static and dynamic voltage stabilities are applied to a six-bus power system model shown in Figure 1. Buses 1 and 2 are generator buses, and others are load buses. A shunt capacitor, in the form of static var controller (SVC), is at bus 5 and a tap-changer transformer is also connected at bus 5 through the transmission line from bus 5 to bus 6 and its turns ratio of bus 5 side is 1 : a . The line data and the initial data for generation and load of the power system are given in Table 1 and Table 2, respectively. It is assumed that constant power, constant current and constant impedance loads at each PQ bus occupy 90%, 5% and 5% of the total load at its bus, respectively.

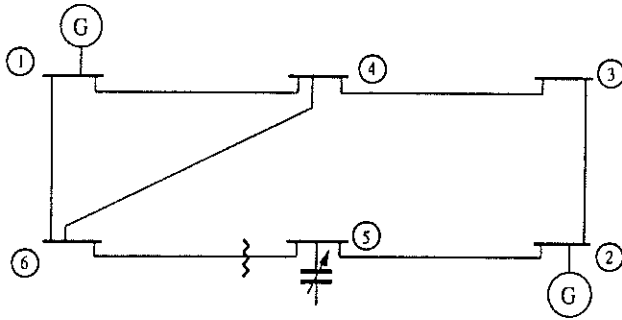


Figure 1. Six-bus power system model

Table 1. Line data

Line number	From bus number	To bus number	Line impedance (p.u.)	
			R	X
1	1	6	0.102	0.413
2	1	4	0.110	0.480
3	4	6	0.120	0.516
4	5	6	0.190	0.820
5	2	5	0.368	1.490
6	2	3	0.115	0.510
7	3	4	0.080	0.360

Table 2. Initial data for generation and load of the system

Bus number	Voltage magnitude	Voltage angle	(in p.u.)	
			P	Q
1	1.0	0.0		
2	1.0		0.4	
3			-0.38	-0.05
4			-0.10	-0.02
5			-0.30	-0.05
6			-0.30	-0.03

STATIC VOLTAGE STABILITY

When there is no control, i.e., the capacitor and tap-changer not being utilized, the results of load flow and the *stability margins* for static voltage stability at each bus are given in Table 3. Here, the voltage of bus 5 is the lowest and the stability margin at bus 5 is also the lowest. Bus 5 is the weakest node that can cause voltage collapse and, thus, the main objective of control is to improve the voltage of this bus. The *minimum stability margin* k can be assigned as an arbitrary positive value and 0.5 is selected as its value so as to maintain the bus voltage within 10% voltage drop.

Table 3. The results of load flow without control of static voltage stability

Bus number	Voltage magnitude (p.u.)	Voltage angle (rad)	P(p.u.)	Q(p.u.)	Value of static voltage stability constraint
1	1.0	0.0	0.718	0.218	
2	1.0	-0.079	0.400	0.175	
3	0.929	-0.198	-0.380	-0.050	1.570
4	0.936	-0.144	-0.100	-0.020	1.725
5	0.852	-0.329	-0.300	-0.050	0.390 *
6	0.915	-0.175	-0.300	-0.030	1.902

*no tap control: $a=1$.

Case 1: With tap-changing transformer

In order to improve the static voltage stability, the tap-changer is controlled since it is very effective in controlling the bus voltage. If the control of the tap-changer is not sufficient, the capacitor can be also controlled for the enhancement of the static voltage stability. It is assumed that the minimum and maximum tap-settings of tap-changer are 0.85 and 1.15, respectively. The results of the control action are given in Table 4. When the turns ratio of the tap-changer is controlled to be 0.9, the voltage magnitude at bus 5 is increased from 0.852 p.u. to 0.902 p.u., which is within 10% voltage drop. The tap-changer relatively controls the voltage magnitudes of two buses connected at each end. In Table 4, it is shown that the voltage magnitude at bus 5 is increased from 0.852 p.u. to 0.902 p.u., while the voltage magnitude of bus 6 decreased from 0.915 to 0.905. After the tap-changer is controlled, the voltage magnitudes of all

Table 4. The results of load flow with control of static voltage stability ($a=0.9$)

Bus number	Voltage magnitude (p.u.)	Voltage angle (rad)	P(p.u.)	Q(p.u.)	Value of static voltage stability constraint
1	1.0	0.0	0.740	0.223	
2	1.0	-0.093	0.400	0.152	
3	0.924	-0.214	-0.380	-0.050	1.566
4	0.931	-0.151	-0.100	-0.020	1.718
5	0.902	-0.337	-0.300	-0.050	0.509 *
6	0.905	-0.178	-0.300	-0.030	1.785

*tap control: $a=0.90$

buses are within 10% voltage drop. Furthermore, the stability margin for static voltage stability of bus 5 was improved from 0.39 to 0.509. It is shown that the stability margins for static voltage stability of other buses decrease by a small amount since the voltage magnitudes of other buses decrease somewhat after the tap-changer is controlled. However, it is important to note that the stability margin of the weakest node, bus 5, which can cause voltage collapse, is improved with the increase of its voltage magnitude after the control.

Case 2: With shunt capacitor

The capacitor is also controlled in order to improve the static voltage stability. The control value of capacitor is 0.030 p.u. reactance with $a=0.92$ and the results are shown in Table 5. Similarly with the above case 1), the stability margin for static voltage stability of bus 5 was improved from 0.39 to 0.501 and the voltage magnitudes of all buses are within 10% voltage drop. The results in Table 4 and Table 5 show that the increase of the minimum stability margin improves the overall voltage profile of the power system preventing the severe voltage decline.

It was shown that the stability margin due to the values of tap-setting or capacitor varies almost linearly within the range of their practical values, and thus, both the tap-setting and capacitor can be used effectively for static voltage stability enhancement.

Table 5. The results of load flow with control of static voltage stability ($C=0.030$ p.u., $a=0.92$)

Bus number	Voltage magnitude (p.u.)	Voltage angle (rad)	P(p.u.)	Q(p.u.)	Value of static voltage stability constraint
1	1.0	0.0	0.739	0.232	
2	1.0	-0.091	0.400	0.141	
3	0.926	-0.213	-0.380	-0.050	1.568
4	0.934	-0.150	-0.100	-0.020	1.724
5	0.914	-0.335	-0.300	-0.050	0.501
6	0.912	-0.178	-0.300	-0.030	1.818

*capacitor control: $C=0.030$ p.u. ($a=0.92$)

DYNAMIC VOLTAGE STABILITY

It is assumed that the data for the generators at buses 1 and 2 are same and the data for dynamic voltage stability study are shown in Table 6. The parameter optimization technique developed is applied for dynamic voltage stability. The three cases are considered as follows:

Table 6. Data for dynamic voltage stability of the system

Variable	Value	Variable	Value
M	0.0498 p.u.	T_F	0.35 sec
D_c	0.0053 p.u.	T_i	60. sec
X_d	0.22 p.u.	K_A	25. p.u.
X_q	0.38 p.u.	K_E	-0.0582 p.u.
T_{do}	1.28 sec	K_F	0.105 p.u.
T_E	1.65 sec	A_E	0.0015
T_A	0.02 sec	B_E	1.5833

Case 1: Generator and excitation systems without PSS

The performance measure is on bus voltages such that the oscillation of the bus voltages is small and bus voltages remain close to desired values.

From the control for the static voltage stability, the turns ratio of the tap changer, a , is set to 0.90. Through the optimization method, the optimal value of the capacitor is found to be 0.0337 p.u. reactance. Assume that the system is disturbed by the small load change at bus 3 from -0.36 per unit to -0.38 per unit, where the negative power represents load.

The dynamic stability model with the optimal capacitor value was simulated using the 5-th order Runge-Kutta method and the trapezoidal rule. The trajectories of states and bus voltages for different values of the capacitor are also shown in Figures 2-4 for comparison. In Figure 2, it is shown that the steady state value of the rotor angle increases as the value of the capacitance C increases, but change in oscillation is very small. Figure 3 shows that the steady state value of the internal voltage E'_q decreases as the value of the capacitance C increases. In Figure 4, the trajectories of V_s are compared according to the values of the control parameter C . The steady state value of V_s changes significantly by the value of C . The change in oscillation is relatively small compared to the steady state value. However, the close examination shows that oscillation is minimum when the value of C is optimum, and the further increase in C tends to increase the oscillation.

Case 2: PSS included in the system

In this case the power system stabilizer (PSS) is added to the dynamic voltage stability model of the above system. The parameter optimization method is used to determine the parameter values of PSS and the optimal parameter values found are

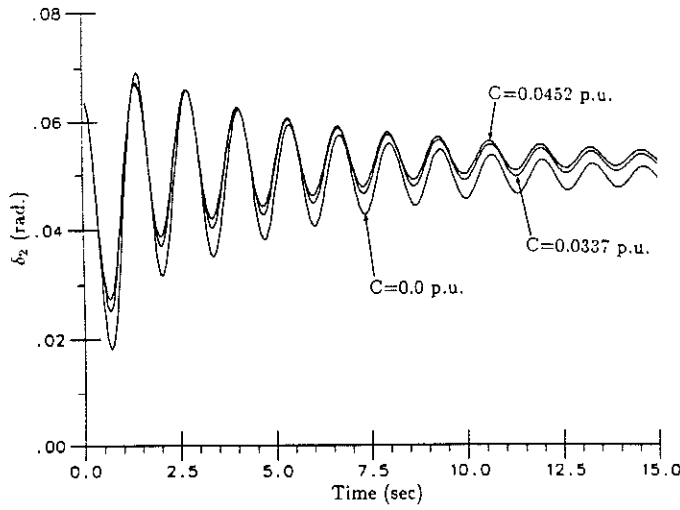


Figure 2. Comparison of trajectories of δ_2 for different values of C ($a=0.90$)

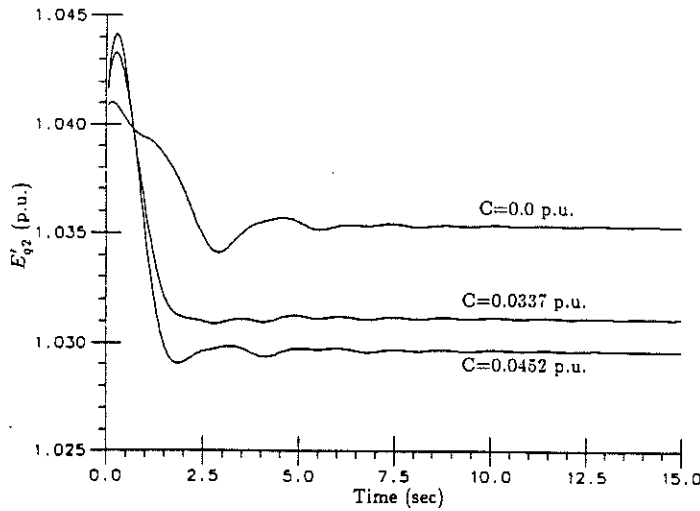


Figure 3. Comparison of trajectories of E'_q for different values of C ($a=0.90$)

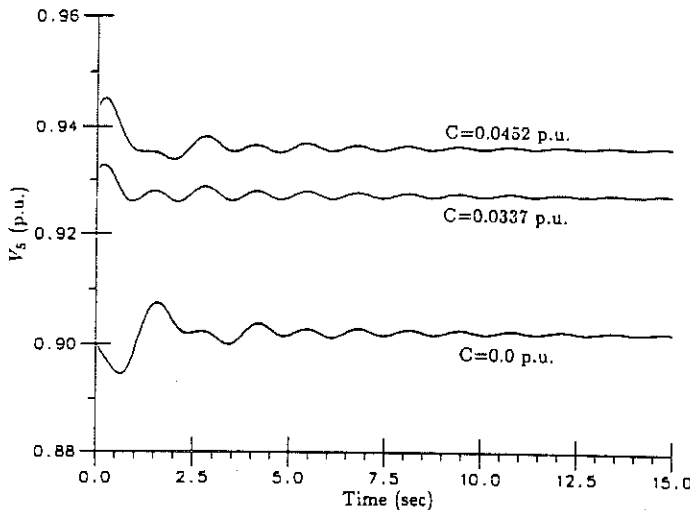


Figure 4. Comparison of trajectories of V_s for different values of C ($a=0.90$)

$T_r=5.0$, $K_c=0.0654$, $T_1=0.951$, and $T_2=0.088$. In Figure 5, the trajectories of δ_2 are compared according to the different values of PSS parameters.

In order to see the effects of PSS, the state trajectories of the system with PSS are drawn and compared to the case without PSS. The oscillation of the rotor angle has been decreased drastically due to the optimal parameter values for PSS. However, this resulted in a slight increase of the oscillations of the internal voltage. The supplementary signal of PSS damps the sustained low frequency oscillation of rotor speed, but it can give a side effect on the voltage control of an excitation system since this supplementary signal is not introduced for the purpose of controlling voltage. However, it is shown that the oscillation of bus voltage V_s for the system with PSS is decreased slightly. This is because the bus voltage depends on the rotor angle, which is heavily damped by the PSS, and, furthermore, the modal performance measure is designed to minimize the oscillation of the bus voltage.

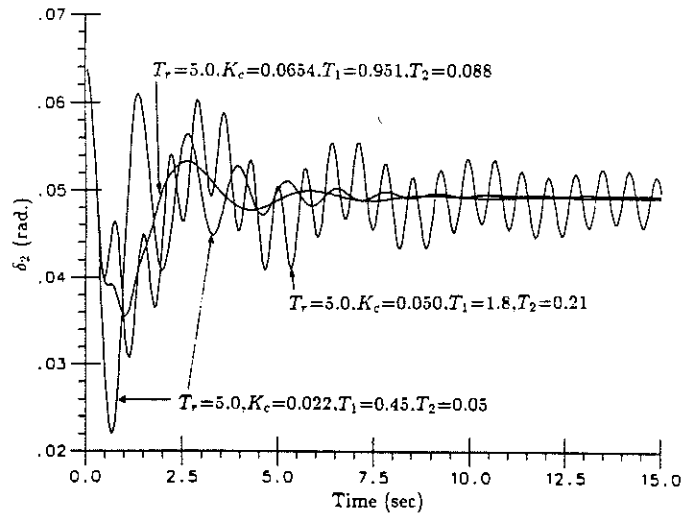


Figure 5. Comparison of trajectories of δ_2 for different PSS parameters ($a=0.90$, $C=0.0$ p.u.)

When considering the PSS, the performance measure includes both voltage magnitude and rotor angle. In this case, the optimal value of the capacitor is 0.0277 p.u. reactance. The trajectories of states and bus voltages at the optimal value are shown in Figures 6–7 along with those for the case of non-optimum values of the capacitor for comparison. In Figure 6, the oscillation of the rotor angle δ is the minimum at the optimum capacitor value, but its steady state value increases as the value of the capacitance C increases. Similarly, the steady state value of the internal voltage E'_q was shown to decrease as the capacitance C increases; however, the oscillations of these internal voltages are minimized at the optimum capacitor value, but increase over the certain value of C . The trajectories of V_s are shown in Figure 7 for different values of the capacitor. The steady state value of the voltage magnitude V_s increases significantly as the capacitance C increases. However, here again its oscillation is minimized at the optimum capacitor value; and the further increase of capacitor makes the dynamic voltage stability worse.

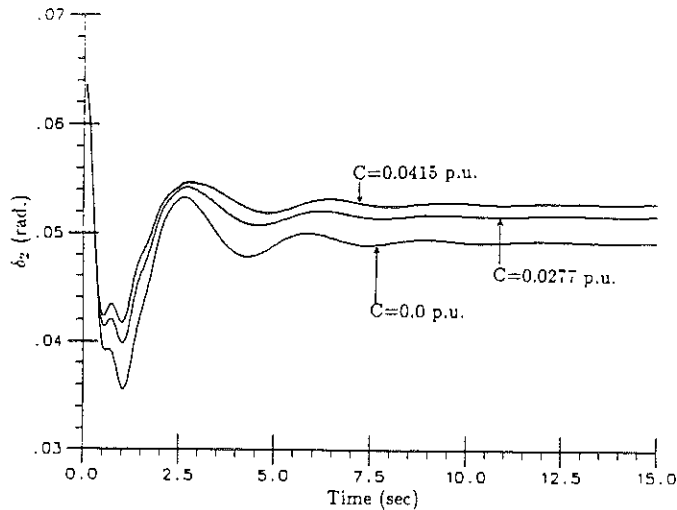


Figure 6. Comparison of trajectories of δ_2 for different values of C in case 2 ($a=0.90$)

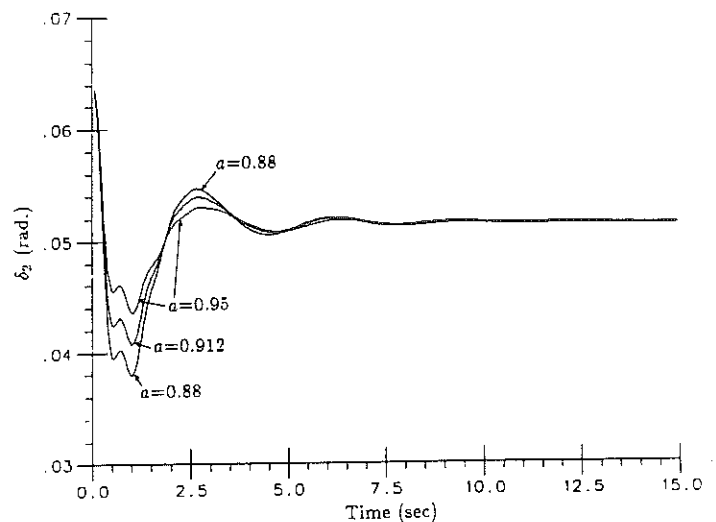


Figure 8. Comparison of trajectories of δ_2 for different tap settings in case 3 ($C=0.0277$ p.u.)

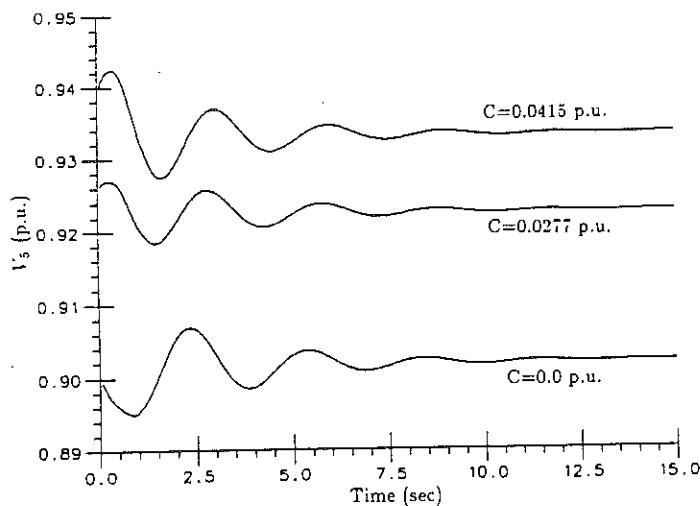


Figure 7. Comparison of trajectories of V_s for different values of C in case 2 ($a=0.90$)

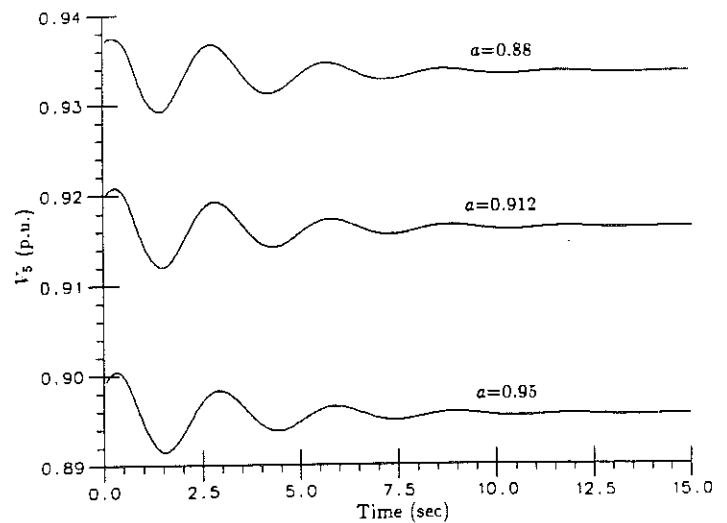


Figure 9. Comparison of trajectories of V_s for different tap settings in case 3 ($C=0.0277$ p.u.)

Case 3: ULTC included in the system

Discrete tap settings: The parameter optimization technique is applied to this case in a similar way, with a tap-changer as an additional controller for dynamic voltage stability. The reference voltage of the tap-changer, in this case, is assumed to be 0.92 p.u. Through the optimization method, the optimal value of tap-setting is found to be 0.912. In order to compare the effects of tap-setting, the trajectories of the state variables for different tap-settings, $a=0.88$, 0.912 or 0.95, are shown in Figures 8-9. In Figure 8, the oscillation of the rotor angle δ increases as the tap-setting decreases from 1 to its minimum value. This means that the tap-changer gives a negative effect to the conventional power system stability. The steady state value of the internal voltage E'_q was shown to decrease as the tap-setting decreases, but the oscillations of these variables are about the same with

very little changes. The trajectory of V_s is shown in Figure 9. The oscillations of voltage V_s are similar to one another, but its steady state values are different significantly. Its steady state value increases as the tap-setting decreases from 1 to its minimum value. The steady state value of V_s is near 0.92 p.u. when the tap-setting is the optimum value of 0.912.

Continuous tap settings: The taps of ULTC are in discrete steps. However, in practice, a step size in the tap position contributes a relatively small amount of voltage correction. The continuous model of the ULTC [2] was often used to study voltage stability. With the use of the continuous ULTC model, which operates automatically with a time constant of 60 sec, the trajectories of the states are shown to be almost identical to the case of discrete tap-settings. This is because the time constant of ULTC is very long compared to the time constants of the excitation system.

CONCLUSION

A static voltage stability criterion is developed and the shunt capacitor and tap-changer were used for control of static voltage stability. From the simulation result, it was shown that the control of the shunt capacitor and the tap-changer can increase the stability margin for static voltage stability and the increase of the stability margin improves the overall voltage profile of the power system, preventing a severe voltage decline. It is shown that the static voltage stability criterion developed in this paper is a useful tool in preventing a severe voltage drop.

A parameter optimization technique with a new performance measure is developed to determine optimal control parameters for dynamic voltage stability enhancement. For the control of dynamic voltage stability, a capacitor and a tap-changer were also used in addition to PSS. The control of the capacitor is very efficient in increasing the steady state value of the voltage magnitude, but some care should be given since excessive value of the capacitor can cause much oscillation of the voltage magnitude. PSS reduces the sustained low frequency oscillations of the rotor, but it can give a side effect on an excitation system. The steady state values of both the rotor angle δ and the voltage magnitude V_s increase as the capacitance C increases; however their oscillations are minimized at the optimum value of C . When the tap-changer is controlled, the steady state value of the bus voltage can be increased significantly, but its oscillation changes very little. However, the oscillation of the rotor angle δ increases giving a negative effect on the conventional power system stability as the tap-setting decreases. Through numerical simulations, it is shown that the parameter optimization method is a very useful technique in reducing the oscillation and enhancing dynamic voltage stability.

ACKNOWLEDGEMENT

This work was supported in part by the Korea Electric Power Corporation and Allegheny Power System.

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ON THE CONDITION FOR INVERSE CONTROL ACTION OF TAP-CHANGERS

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Abstract - Voltage instability is concerned with various dynamic factors in a power system. The authors have classified possible voltage instability patterns based on the time scale separation of these dynamic factors, and questioned the instability of some of multiple load flow solutions [16]: The lower voltage portion of the P-V curve is stable in some cases where on-load tap-changing (OLTC) transformers are activated during the variation of loads.

This paper investigates power system stability in the presence of OLTC transformers based on the results obtained in the above study. To assess this type of voltage instability, two kinds of sensitivity criteria are used: One is the sensitivity of load voltage to the tap position, which is exact when exact load characteristic data are used in the computation. The other is an equivalent criterion calculated from load flow data only without knowing the load characteristics. It is demonstrated that stability can occur when the operating point moves to the lower portion of the P-V curve under the conditions defined by the above criteria. Theoretical analysis of the latter criterion shows that line and transformer reactance values govern the stability. The validity of these results is verified through numerical simulations in a three bus system where dynamic characteristic of OLTC and changing loads with static nonlinear characteristics are taken into account.

1. INTRODUCTION

Recently, due to the tendency of increasing loads on long transmission lines, voltage stability has been one of the major topics among power system engineers. This problem has long been studied by many researchers through various approaches which are roughly divided into two groups: One is the so called static approach in which algebraic equations such as load flow equations are used to study this problem [1-5]. The voltage stability limit is treated as the singular point of the load flow Jacobian corresponding to the tip of the P-V curve in the static approaches. The other approaches are to investigate the dynamic mechanisms by using differential equations [6-16], in which various dynamic models have been used. They are, for example, analyses of induction motor loads [6,7], OLTC transformer dynamics [8-10] and inclusion of generator dynamics [11,12], etc. Specially the relationship between static and dynamic approaches has been studied in [13-15] which have clarified that the critical condition detected by the static approaches corresponds to one of the stability limits in dynamic systems, that is, the static bifurcation point. By extending these works, the authors have studied possible voltage instability mechanisms and classified them based on the singular perturbation method [16]. It has been suggested in that study that the stability of slow and fast dynamics should be treated separately and that the stability region should be determined as the intersection of the slow and fast stability regions.

Concerning the stability of the multiple load flow solutions, it is widely believed that the high voltage solution is always stable and the low voltage solutions unstable. This is theoretically correct under the assumption that every load bus has a dynamic mechanism to keep its power constant and that the equilibrium point is fixed. This assumption is valid in cases such as voltage controlled buses or induction motor loads under fixed loadings. It is however questionable whether on-load tap-changing (OLTC) transformers may be treated in the same way. This is because their dynamics are almost as slow as load changes in actual power systems. In fact, interesting simulation results have been obtained in [16] where the stability region extends to the lower portion of the P-V curve. Thus, the stability condition of OLTC needs to be examined more deeply in such situations.

Inverse control action of tap-changers is a mechanism through which the voltage under OLTC control behaves as a function of the tap position in a manner opposite to the normal behaviour.

This paper investigates the stability of OLTC based on the results obtained in [16], where a power system has been divided into slow and fast subsystems by taking into account the dynamic responses of the power system constituent factors. Analysis of OLTC is performed in the slow subsystem where fast dynamics, such as generators and loads, are treated as static algebraic equations. This treatment is based on the singular perturbation theory. Load variation with time is taken into consideration as well as OLTC dynamics.

In the first half of this paper, two stability criteria are used: One is the sensitivity of load voltage to the tap position. This criterion is used to check the validity of the analysis, whose computation requires exact load characteristic behaviour data. The other is a equivalent criterion calculated from load flow data. This can be obtained without load characteristic data, therefore seems more applicable to real power systems. It is demonstrated that this type of stability can occur when the operating point moves to the lower portion of the P-V curve under the conditions defined by the above criteria.

In the latter half of this paper, through theoretical investigation of the above conditions, factors which contribute to this stability are studied. It is clarified that the stability tends to occur as the ratio of (line reactance / transformer reactance) decreases. The validity of these results are verified through numerical simulations in a three bus system where the dynamic characteristic of the OLTC and the static characteristic behaviour of load are taken into account.

2. BACKGROUND

2.1 Classification of Voltage Instabilities [16]

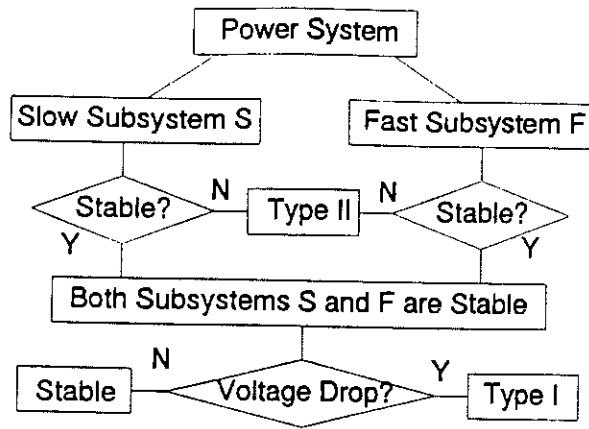


Fig. 1 Slow and Fast Subsystems

In the approach of [16], a power system is divided into slow and fast subsystems by using the singular perturbation method as shown in Fig. 1. The slow subsystem, S, approximates only the slow dynamics of the power system such as tap-changers, system frequency, etc, while the fast subsystem, F, represents the remaining fast dynamics such as generator dynamics, controllers, loads, etc. The stability of the original system is governed by the stability of these subsystems. Voltage instabilities have been classified based on the stability conditions of these subsystems as follows:

- Type I: voltage drop in the stable region
- Type II-1: instability of the slow dynamics subsystem, S
- Type II-2S: static instability (or static bifurcation) of the fast dynamics subsystem, F
- Type II-2D: dynamic instability of the fast dynamics subsystem, F

The voltage drop of type I occurs under steady state operation. In practice, if the voltage drop of this type is in excess of some acceptable limit, then it would be considered unstable. Even when a voltage drop of type I has occurred, the system still has the capability to remain stable. The power system is operable unless it hits any one of the stability limits of type II. These stability limits are explained below.

The inverse control action of OLTC belongs to the type II-1 instability since the dynamic response of tap-changers is in general quite slow (of the order of minutes). In order to analyze the slow subsystem, S, only the slow dynamics are considered. That is, the fast dynamic variables are treated as static. The main purpose of this paper is to study the nature of and the conditions for this type of instability.

The fast subsystem, F, characterizes only the fast dynamics in the original system. The slow dynamic variables treated as constants. This treatment exactly corresponds to the conventional power system models which are widely used in transient stability studies. The stability condition for the fast subsystem is that all the eigenvalues of the subsystem F have negative real parts:

$$\text{Re } \lambda(A_{FF}) < 0 \quad (1)$$

where A_{FF} denotes the system matrix of subsystem F derived

by linearization, which is a function of slow variables such as tap position, mean frequency as well as load changes. Equation (1) represents the stability condition for both types II-2S and II-2D. That is, the case when the critical eigenvalue of A_{FF} is real, which means a zero eigenvalue, corresponds to the static bifurcation of type II-2S, while a pair of imaginary eigenvalues corresponds to the dynamic instability of type II-2D.

The critical condition of the static bifurcation, which is written as $\lambda(A_{FF})=0$, is equivalent to $\det(A_{FF})=0$ since the determinant is equal to the product of all the eigenvalues. Therefore the static bifurcation can be detected by the determinant without carrying out eigenvalue analysis of A_{FF} . Furthermore, as for the characteristic of $\det(J_s)$, the following relationship has been derived [16]:

$$\det(J_s) = H \det(A_{FF}) \quad (2)$$

where the matrix J_s represents the extended load flow Jacobian in which the static characteristics of loads and generators are added to the load flow equations. (This matrix, J_s , also corresponds to the Jacobian matrix when the N-R method is used to solve all the algebraic equations existing in the slow subsystem S at a given time; a part of these algebraic equations comes from the fast differential equations by neglecting the fast derivatives.) H is a scalar which changes depending on the slow variables. From eq. (2), it is clear that $\det(J_s) = 0$ when the critical eigenvalue of A_{FF} becomes zero, that is, when $\det(A_{FF}) = 0$. This means that $\det(J_s)$ changes its sign when the real eigenvalue crosses zero, that is, when the static bifurcation of type II-2S occurs. Therefore, the following condition can be used to detect type II-2S stability.

"If the sign of $\det(J_s)$ changes when type II-2S stability has occurred"

$$\det(J_s) > 0 \quad (< 0) \quad (3)$$

It is noted that the condition $H = 0$ also leads to $\det(J_s) = 0$. This situation occurs sometimes, when we use algebraic equation models for loads. However, this phenomena has not yet been investigated sufficiently.

It should be note that dynamic instability of type II-2D cannot be detected by eq. (3). Eigenvalue analysis of A_{FF} is needed to assess this type of stability.

2.2 Power System Model

As mentioned in the previous section, the singular perturbation method is adopted in analyzing the slow subsystem. That is, in this paper, only the slow dynamic factors are treated as dynamic, while the remaining fast dynamic factors are dealt with in algebraic equations. The following assumptions are made here to render the analysis more efficient:

- (A1) Only tap-changer dynamics are considered as slow dynamics, while the other slow dynamics such as load frequency characteristics are neglected here.
- (A2) As a static characteristic of the generating units, a constant terminal voltage within a limited var generation is assumed. Further, we consider the following type of load characteristics here,

$$P_i = P_{0i} V_i^\alpha, \quad Q_i = Q_{0i} V_i^\beta \quad (4)$$

Then, a power system model corresponding to the slow subsystem can be represented as follows:

$$\text{OLTC:} \quad n^{k+1} = f_n(n^k, V, V_{ref}) \quad (5a)$$

$$\text{Gen \& Load:} \quad y = f_{GL}(V, L) \quad (5b)$$

$$\text{LF eqn:} \quad y = f_{LF}(n^k, V) \quad (5c)$$

where

$$\begin{aligned} n^k &: \text{tap position vector} \\ v &: \text{node voltage vector} = [V, \delta] \\ y &: \text{node injection vector} = [P, Q] \text{ or } [P, V] \\ L &: \text{control vector by consumers} = [P_0, Q_0] \end{aligned}$$

Eq. (5a) represents the tap-changer dynamics, in which the tap position n^k is changed as a function of voltage. Eq. (5b) stands for the generator and load characteristics which are mentioned in the previous assumption (A2). Eq. (4) is included in (5b). Eq. (5c) represents the load flow equations.

The above equations, (5a) to (5c), stand for the slow subsystem in Fig. 1 in which only the slow dynamics are expressed as dynamic equations (5a). The other characteristics including fast dynamics are represented by the static equations (5b) and (5c). The extended Jacobian, J_s , can be expressed in terms of eqs. (5b) and (5c) as follows:

$$J_s = J_{LF} - J_{GL} \quad (6)$$

where

$$\begin{aligned} J_{LF} &: \text{load flow Jacobian } (= \partial f_{LF} / \partial v) \\ J_{GL} &: \text{Jacobian for generators and loads } (= \partial f_{GL} / \partial v) \end{aligned}$$

2.3 Type II-2S Stability Region

When the response of generators and loads is fast enough, and when their static characteristics are approximated by the above expressions, the type II-2S stability region may be obtained by applying condition (3) with J_s as defined by eq. (6). Fig. 2 describes this stability region for a single load example system. Solid lines describes the region where condition (3) holds. It is important for the reader to note that this stability region tends to extend to the lower portion of the P-V curve as the exponent value, α , becomes large. As seen from this example, the stability region could include the lower portion in some cases, as far as type II-2S stability is concerned. It is, therefore, important to investigate the stability of the lower part of the P-V curve from the viewpoint of type II-2D. This type of analysis has been carried out in reference [16]. This paper emphasizes analysis of type II-1 instability, in particular affect of tap changers of this type of stability. stability criteria.

3. ANALYSIS OF INVERSE CONTROL ACTION OF TAP-CHANGERS

When studying stability of tap-changer dynamics, it may be important to take into account load changes. This is because the load rate of change is comparable to the response of the tap-changers in actual situations. That is, instability can occur if the movement of tap-changers fails to follow the moving system equilibrium point which then escapes from its stability region, resulting in voltage collapse. In such situations, linearization around the equilibrium point, which is an approach seen in the existing literature, may not always be suitable since

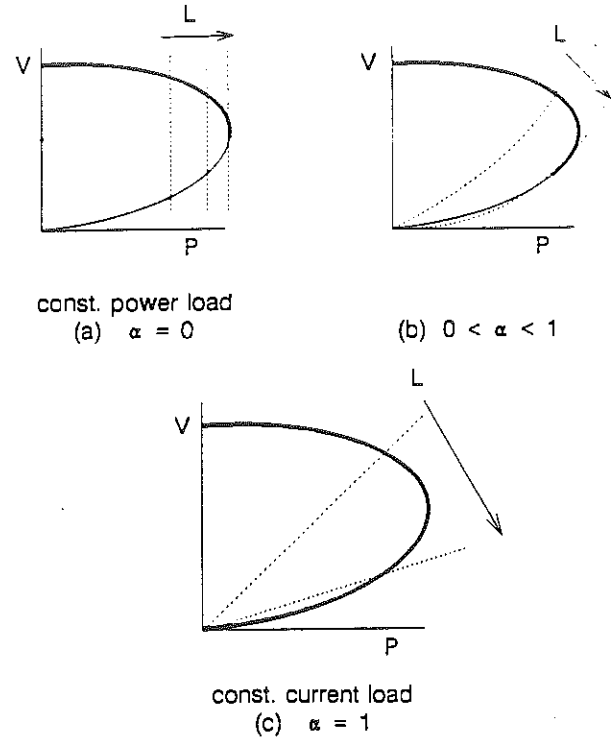


Fig. 2 Type II-2S Stability Region determined by eq. (3)

the trajectory of the operating point is no longer close to the equilibrium point. The inability to use linearization is a major difficulty assessing this type of voltage stability. In this section, therefore, we only focus our attention on the trajectory of the operating point, then examine a stability criterion by applying sensitivity analysis to the trajectory. In this analysis, the following two situations are assumed for the same power system:

- situation (i): load characteristics are known.
- situation (ii): load characteristics are unknown.

The former represents an ideal situation where one can analyze an exact stability condition by using eqs. (5a) to (5c). On the other hand, the latter stands for an actual case where constant P and Q are assumed for purpose of analysis instead of the existing real load characteristics. Stability criteria will be studied for both these situations.

3.1 Stability Criteria for situation (i)

First we consider a stability condition at some operating point when the transformer tap changes from n^k to n^{k+1} governed by eq. (5a). The bus voltage may change while satisfying eqs. (5b) and (5c). That is,

$$0 = f_{LF}(n^k, v) - f_{GL}(v, L) \quad (7)$$

If we denote the tap deviation as $\Delta n = n^{k+1} - n^k$, the voltage change, $\Delta v = [\Delta V, \Delta \delta]$, can be estimated through linearization of eq. (7) around the operating point. That is,

$$0 = J_s \Delta v + J_n \Delta n \quad (8)$$

where

$$J_n = \partial(f_{LF} - f_{GL})/\partial v$$

$$J_s = \text{defined in (6).}$$

Eq. (8) can be rewritten as follows:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} J_{s11} & J_{s12} \\ J_{s21} & J_{s22} \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \delta \end{bmatrix} + \begin{bmatrix} J_{n1} \\ J_{n2} \end{bmatrix} \Delta n \quad (9)$$

Eliminating $\Delta \delta$ from eq. (9), we obtain the following equation.

$$\Delta V = S_1 \Delta n \quad (10)$$

where

$$S_1 = \frac{\partial V}{\partial n} = -(A J_{n1} + B J_{n2})$$

$$\begin{bmatrix} A & B \\ * & * \end{bmatrix} = J_s^{-1}$$

Eq. (10) prescribes how much the node voltages change when the taps vary. Now, we pay attention to a particular bus voltage which is controlled by OLTC transformer. For stability, this voltage has to increase as the tap is raised, which means that the factor of S_1 corresponding to the studied bus has to be positive. Thus, for stability, the above condition has to hold for all the voltage controlled buses. We represent this condition as follows.

$$S_1 > 0 \quad (11)$$

In the above equation, ">0" is defined as all the factors corresponding to the voltage controlled buses by the tap changers are positive.

This stability criterion based on S_1 , however, requires exact knowledge about load characteristics. Another criterion is examined in the following.

3.2 Stability Criteria for situation (ii)

The tap changing action in the example system of Fig. 3 is analyzed in Fig. 4. The solid lines describe the network characteristics corresponding to eq. (5c), where the tap-position changes from n_1 to n_2 . The dashed lines show load characteristics of eqs. (4) and (5b) for different $L = [P_0, Q_0]$, in which constant current characteristics are assumed for simplicity. When the load is light, the operating point, A, lies on the upper portion of the curve. It is seen that the voltage increases as the tap changes from n_1 to n_2 . On the other hand, when the operating point moves to the lower portion, B, the voltage decreases due to the tap change. It may be understood from the above observation that a necessary condition for this instability is that the operating point is on the lower portion, that is, the low voltage solution of the multiple load flow solutions.

This instability, however, does not always occur whenever the operating point moves to the lower portion. It follows from Fig. 4 that, when the network characteristic curves have an intersection, C, for different tap ratios, the lower

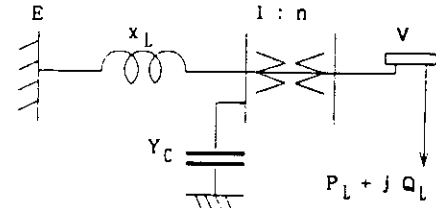


Fig. 3 A Power System Model

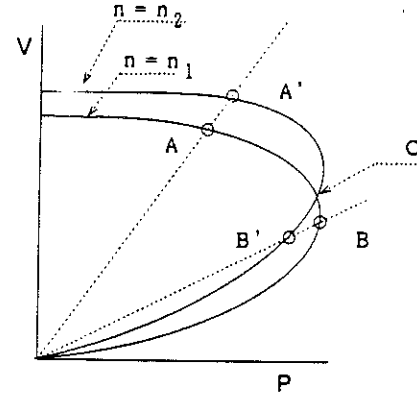


Fig. 4 Process of Inverse Control Action of OLTC

portion becomes unstable. Thus, this type of stability depends on whether or not the intersection exists. This condition is also equivalent to whether or not the lower portion of the curve is raised as n increases.

Whether or not the lower portion of the P-V curve increases can be verified in terms of the sensitivity of the voltage at the lower portion obtained from only the network characteristics of eq. (5c). This sensitivity matrix can be derived through a similar procedure in eqs. (7) to (10) where the load characteristics of eq. (5b) are no longer considered, as shown below. First, the linearization corresponding to eq. (8) is derived from eq. (5c) as follows.

$$0 = J_{LF} \Delta V' + J_n' \Delta n \quad (12)$$

where

$$J_n' = \partial f_{LF} / \partial v$$

$$J_{LF} = \text{defined in (6).}$$

In the above equation, "." is used to denote the conditional situation where the existing load characteristics of eq. (5b) are neglected. That is, in general, $\Delta V'$ contains error due to the omission and is therefore not equal to Δv for the same change in Δn . From the above equation, we obtain the following equation which corresponds to eq. (10).

$$\Delta V' = S_2 \Delta n \quad (13)$$

where

$$S_2 = -(A' J_{n1}' + B' J_{n2}')$$

$$\begin{bmatrix} A' & B' \\ * & * \end{bmatrix} = J_{LF}^{-1}, \quad \begin{bmatrix} J_{n1}' \\ J_{n2}' \end{bmatrix} = J_n'$$

The above derivation shows that the sensitivity matrix, S_2 , is obtained from the standard load flow Jacobian, J_{LF} . To investigate the stability of the lower portion of the P-V curve, S_2 is obtained at the low voltage solution obtained by the multiple load flow calculation. Then, the factor of S_2 corresponding to the studied bus is examined. If this factor is positive, the voltage of the lower portion of the P-V curve increase as n is raised as shown in Fig. 4, and therefore an intersection appears, resulting in instability. On the other hand, if the factor of S_2 is negative, the voltage at the lower portion decreases as n increases. This means that no intersections appear and the system is stable. That is, for stability, the following condition holds.

$$S_2 = \frac{\partial V^{(i)}}{\partial n} < 0 \quad (14)$$

where

$V^{(i)}$: low voltage load flow solution

The above criterion means that the inverse control action occurs if the operating point moves to a low voltage load flow solution and if eq. (14) does not hold at this point. Note that the inequalities are opposite to each other in conditions (11) and (14).

3.3 Analysis of Condition (14)

Condition (14) is examined here by using the example of Fig. 5. In this system, the load bus voltage is controlled by a tap-changing transformer. The line reactance existing in the primary side is denoted as x_L and the secondary reactance including leakage reactance is written as x_T . Then the lower voltage solution at the load bus is represented as:

$$V^{(i)} = \sqrt{(A - \sqrt{B}) / 2} \quad (15)$$

where

$$\begin{aligned} B &= A^2 - 4S^2(x_T + n^2x_L)^2 \\ A &= n^2E^2 - 2(x_T + n^2x_L)Q \\ S &= \sqrt{P^2 + Q^2} \end{aligned}$$

Substituting (15) into (14), we obtain the following condition:

$$V^{(i)} > V_{cr} \quad (16)$$

where

$$V_{cr} = \sqrt{\frac{2S^2(x_T + n^2x_L)x_L}{E^2 - 2x_L S \sin \theta}}$$

Or, the above condition can be rewritten as follows.

$$S > S_{cr} \quad (17)$$

where

$$S_{cr} = \frac{-n^2 \sin \theta + \sqrt{D}}{2(x_T + n^2x_L) \cos^2 \theta} E^2$$

$$D = n^4 - (x_T \cos \theta / x_L)^2$$

$$\sin \theta = Q/S, \cos \theta = P/S$$

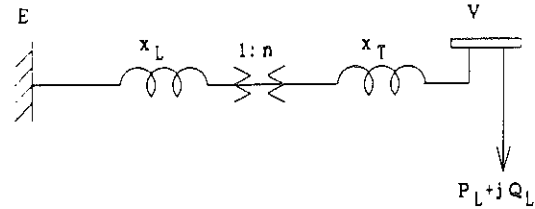


Fig. 5 A Power System Model

The conditions (14), (16) and (17) are equivalent to each other in the example system. That is, if the operating condition moves into the region of (16) or (17), the inverse control action will occur. The necessary condition for this instability to occur is obtained under the condition of positive S_{cr} with positive D . That is:

$$n^2x_L > x_T \quad (18)$$

When the above condition holds, there exists an instability region on the network characteristic curve. It should be noted that condition (11) only depends on reactance values, that is the inherent values of the studied network. This means that the stability criterion proposed in section 3.2 can be used to check such an inherent characteristic of a network.

4. NUMERICAL SIMULATIONS

In this section, the theoretical results developed in the previous sections are verified through numerical simulations. The simulations are carried out by using an example system as shown in Fig. 3, where the following parameters are used:

$$\begin{aligned} E &= 1.10 \text{ pu} \\ r_L + jx_L &= 0.02 + j0.35 \text{ pu} \\ x_T &= j0.05 \text{ pu} \\ L &= [P_o, Q_o] = [0.4, 0.25] : \text{base load} \end{aligned} \quad (19)$$

4.1 Preliminary examination by means of sensitivity S_2

Before carrying out the numerical simulations, we have examined whether or not there is a possibility of the occurrence of type II-1 voltage instability in this example system. Namely, load flow calculations have been carried out to obtain the low voltage solution and its sensitivities for different loading conditions. Then, condition (14) has been checked. Fig. 6 shows these sensitivities in the example system, where the tap

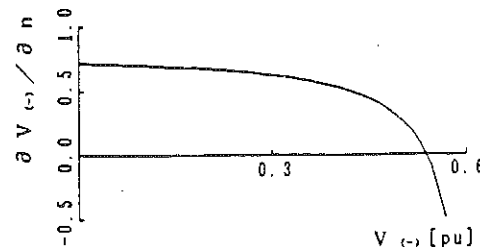


Fig. 6 Behaviour of Sensitivity S_2 ($dV^{(i)}/dn$)

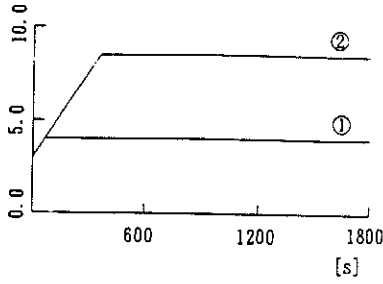


Fig. 7 Load Increasing Patterns

ratio n is fixed to 1.0 to simplify the examination. It is clear from the Figure that $dV^{(i)}/dn$ remain positive when the load voltage is less than the critical value of 0.53 pu. This means that the instability can occur in this region if the initial tap position is 1.0. (It is clear from condition (16) that the critical voltage will increase when the tap ratio is raised.) In any case, it is demonstrated that there exists the possibility of the inverse control action in this system.

4.2 Method of Numerical Simulations

Numerical simulations are carried out taking into account OLTC dynamics as well as increasing load. The OLTC model adopted here, which corresponds to eq. (5a), is explained first: The initial state of the tap ratio is selected as 1.0 and its unit step size is assumed to be 0.01. The maximum and minimum values of the tap ratio are neglected in order to clarify the characteristics of the OLTC control action. The tap changes by

one step when the load voltage continues to exceed the dead band of 0.01 p.u. for 30 seconds around the reference voltage 1.0 p.u. The following two cases are set up to investigate the effect of leakage reactance.

Case 1: leakage reactance 5% is taken into account.
Case 2: leakage reactance is neglected.

We assume two different increasing load patterns for the load model in eq. (4), where $L = [P_0, Q_0]$ is changed with respect to time as shown in Fig. 8. In this Figure, the abscissa shows the multiplier of the base load which is listed in (19). Furthermore, $\alpha = \beta = 1$ is assumed in eq. (4), that is, a constant current load. For the increasing load pattern 1, var compensation of 0.7 pu is added to the load bus three times at 100, 200 and 300 seconds.

To obtain the numerical solution, first we solve the algebraic equations (5b) and (5c), then check the criterion for changing the tap position. This procedure is repeated every second. This simulation method is based on reference [17].

4.3 Results of Numerical Simulations

Figs. 8(a) and 8(b) show the behaviour of the load voltages and the tap ratios for load patterns 1 and 2, respectively, in which solid lines correspond to case 1, dashed lines to case 2. Referring to the waveforms in Fig. 8(a), the voltage decreases initially, but recovers finally due to the tap control action. It is clear that this control action becomes efficient after the load stops increasing. When cases 1 and 2 are compared to each other, no major differences are seen.

On the other hand, in Fig. 8(b) where the load becomes heavier, the waveform of the voltage initially behaves like a step shape due to the var compensations which are added three

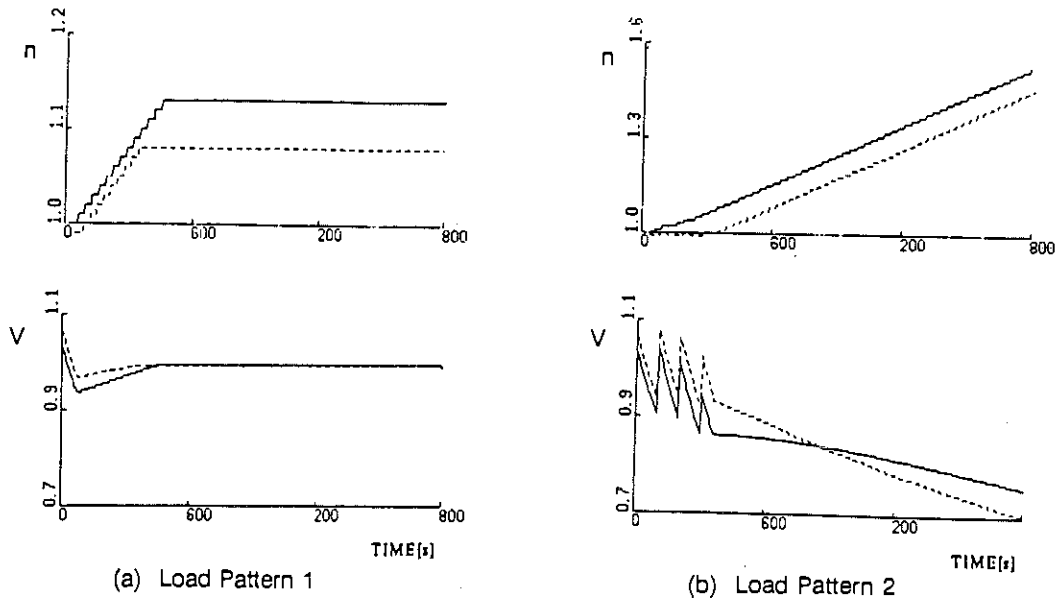
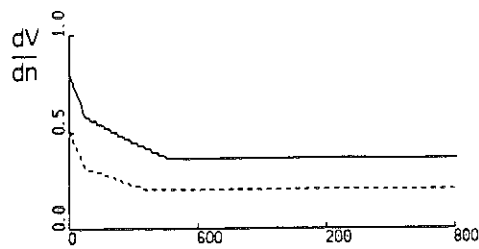
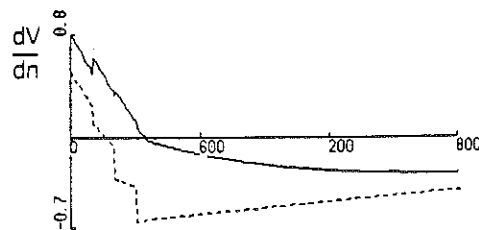


Fig. 8 Behaviours of Tap Position n and Load Voltage V



(a) Load Pattern 1



(b) Load Pattern 2

Fig. 9 Behaviours of Sensitivity S_1 (dV/dn)

times. The voltage increases as the vars are injected. However, if we observe the waveform after the load stops increasing, the inverse control action of the tap changer can be observed clearly. The load voltages decrease with respect to time for both the cases because of the tap changer. When these cases are compared, a more severe voltage drop can be seen concerning the dashed line of case 2. This observation agrees with the theoretical investigation in that the leakage reactance of the transformer has an important role for this type of voltage instability.

4.4 Investigation by Means of Sensitivity S_1

Figs. 9(a) and 9(b) show the behaviour of dV/dn for load patterns 1 and 2, respectively. For load pattern 1 in Fig. 9(a), both the lines keep positive values, meaning that the instability never occurs under these conditions. On the other hand, Fig. 9(b) shows negative values after 350 and 160 [s] for cases 1 and 2, respectively. This observation means that the inverse control action occurs after these times for both the cases, which agrees with the simulation results in Fig. 8.

5. CONCLUSION

The influence of the inverse control action of OLTC transformers on power system stability is investigated through sensitivity analysis as well as numerical simulations. Inverse control action of tap changers is a mechanism through which the voltage under tap changer control behaves as a function of the tap position in a manner opposite to the normal behaviour. The results obtained in this paper are summarized as follows:

- (1) A necessary condition for slow dynamics stability is that the P-V curves at buses whose voltage is controlled by OLTC transformers, should not intersect for different tap ratios.
- (2) This type of instability can occur only on a low voltage solution of the multiple load flow solutions, that is, the lower portion of the P-V curve.
- (3) Instability can be detected through the sensitivity condition

$$S_1 = dV/dn > 0$$
 when load characteristics are exactly known, or through

$$S_2 = dV^{(i)}/dn < 0$$
 when unknown load characteristics are treated as constant power loads.
- (4) An important factor concerning this type of stability is the ratio of transformer reactance to line reactance.

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DISCUSSION - SESSION TWO

Edited by Colin Christy, Commonwealth Edison Company

Maria Ilic (Massachusetts Institute of Technology):

On the importance of changing secondary control setpoints and so forth, I just want to emphasize regarding terminology, for the record, that in the past, secondary control assumed the steady state concept. The considerable work done by EDF on pilot points assumes the generator dynamics to be asymptotically stable and only deals with quasi steady state concepts of regulating terminal voltage at generators, so that you regulate steady state load voltages. In the approach that Harry [Kwatny] discussed we are deviating from that terminology. I think that we should keep them straight, so that we don't have that confusion.

Harry Kwatny (Drexel University):

I think the right way to think about the secondary control problem is as a regulation problem. By regulation I mean two things. One, you have to drive a certain prescribed set of variables to specific values in steady state, but I also want to allow the possibility for adding feedback compensation to help do that if the system is not stable. In the classical papers I don't recall whether or not any allowance was made for feedback stabilization, but it is true that the key idea was to change that static bifurcation structure, and that is done through the regulation function. So the regulation function replaces the idea of static because regulation has to do with what happens in steady state; it's like adding integral control action in the feedback regulator. We want integral action, in a multivariable sense, because we want certain equations to be satisfied in steady state, namely, that certain variables go to precise values. This is basically an attempt to change the static structure, but I'd like to allow the possibility of adding stabilizing compensation if necessary.

Maria Ilic:

I think it's very important, because on the EDF regulation there was regulation of setpoints and the only quasi-dynamics came from the PI regulator which regulates the reference points. I think what you have added is the very fast dynamics of generators, so I think we are mixing time scales very considerably here. I think there will be confusion in interpretation in the future.

Harry Kwatny:

Confusion or not, I think it's the right way to think about it. The other thing I'd like to point out: I said absolutely nothing in this talk (although there are some things in the paper) about how the dynamic stabilizing compensator is to be designed. I do mention in the paper, for example, that one thing that I ignore in this model is any primary voltage controls at all. As a matter of fact, the primary voltage controllers and stabilizers can be thought of (the traditional ones) as a decentralized stabilizer on top of the regulation function. So I don't think that there's any inconsistency at all.

Claudio Cañizares (Univ. of Wisconsin, Madison):

I have a couple of questions for Professor Liu and Professor Yorino. One was, I noticed that in Professor Liu's work, they included the dynamic voltage dependence of the reactive load, which I guess is similar to the singular perturbation suggested by Chris DeMarco. I wonder whether this actually resolves that problem of having two unstable equilibrium points. In our case it has happened before. Including these perturbations seems resolve that problem. The other question is: the limits in the LTC tap, from what I saw in Professor Liu's presentation, seem to help in keeping the system stable when it is collapsing. I was wondering whether they were included in those dynamic simulations that you presented. The last question is pretty much open for discussion. In these two papers regarding tap-changers, and some other papers I have seen, the angle dynamics are neglected. However, while voltage collapse could be related to a voltage problem, you can also have a voltage collapse by mechanisms of angle bifurcation. At a certain point that bothers me, neglecting the full dynamics of your system and just presenting parts of it.

Chen-Ching Liu (University of Washington):

The first question is: if the reactive model includes the voltage dependency does that give you more than one stable equilibrium point? We still have two stable equilibrium points; we did not eliminate that possibility. But if you start using the quadratic form of a static load, then you have a unique stable equilibrium point. Then for

that special class it is resolved; one stable and one unstable. The other point is on tap limits. In our case, we actually disabled the LTC in the beginning of the whole scenario, so it is not automatic at all; it is all constant.

Claudio Cañizares:

The question was whether those are actual tap limits in your LTC's or just something you locked at a certain value.

Chen-Ching Liu:

When you lock, you don't lock it at the upper limits, you lock it when you are in the leaf. So whichever position you are taking when you enter the leaf, that's the chance. Which means you need some hardware to do that. There is a third questions I think.

Claudio Cañizares:

Yes, it was open for discussion: on neglecting angle dynamics.

Khoi Vu (University of Washington):

In the study of voltage collapse, we don't include the variation of frequency. In other words, we don't model the angle dynamics. Now the reason for that is that we observe that there are cases where there is a considerable time period where there is no significant change in system frequency and our analysis only applies for that time period. Of course there are cases where we have both angle and voltage problems. That is still something we want to study in the future.

Chen-Ching Liu:

The angles are modeled implicitly, so those terms we assume are absorbed in the reactive power equations. It is not a complete model, but it assumes that the angles are constant in that sense.

John Zaborszky (Washington University):

I would like to make a brief comment on the last presented paper by the Japanese authors and Galiana. It's interesting that they came up with the same criterion for what we call the boundary of the feasibility region connected with tap-

changer operation that we are using in our paper. Apparently this is a parallel development.

Isao Iyoda (Mitsubishi Electric Corporation):

I direct a very primitive question to the lecturers on security. As Professor Liu pointed out, dynamics are very important in deciding security, but there are various characteristics of load. One is a saturable reactor, where if the voltage collapses, power is constant and only reactive power is reduced. For some rectifiers with a capacitor, once the voltage collapses, no current goes through the rectifier because the rectified voltage is less than the capacitor voltage. Thirdly, for a controlled rectifier, when the voltage collapses, active and reactive power at once decrease and after that the active power recovers to its original value, and reactive power recovers, but to a different value. Because of nonlinearity, the operating point becomes the lower point. For instance, for the on-load-tap-changers, it becomes limited, so then it operates stably. In security analysis, how do you manage these kind of nonlinearities in dynamics?

Chen-Ching Liu:

I guess your question is how do we take care of these kind of loads in the model. I can put a constant plus some polynomial approximation, because we can allow a polynomial function. In the polynomial function you can have a constant, first order, second order, or any exponential, so I think it's possible. But whether that would lead to a simple picture of the security region is a big question. If you use a quadratic equation, you can get a very simple picture like I showed you, but if you have voltage raised to 0.5, maybe that would screw up the shape.

Christopher DeMarco (Univ. of Wisconsin, Madison):

I would say two points come in play here and maybe Chen-Ching [Liu] was being too modest in answering, because I think these kind of characteristics are precisely where you're going to want some dependence on the V dot term. Obviously you're getting behavior that's not a straight algebraic equation between how the real and reactive power are behaving and how voltage behaves. You get some time dependence, so I think some of that would come into play in getting a dynamic model with respect to voltage. The other hypothesis I would like to put up as a matter of thought: the sort of P-V curve (coming back to a comment Colin Christy made earlier today) may not be appropriate of these types of loads. Really the free

parameter you want to vary may not be a P versus V . You may see the shape of this curve change in such a way that you really do get a clearer division (between the area in which you're stable and the area in which you're unstable) if you pick your parameterization to say how these kinds of loads are changing with time, or what have you.

Adam Semlyen (University of Toronto):

I have a question for Chris DeMarco. He ended by stating that there exists a Jacobian which has to be positive definite, and a zero eigenvalue would of course defeat this requirement. I believe that was the at end of the presentation. My questions would be: what is the meaning of an eigenvalue being big enough, and what is small?

Christopher DeMarco:

The point I was trying to make was only that the limiting case of the margin we were working on would agree with the limiting case of some more traditional measures. When you have a Jacobian having a zero eigenvalue, that corresponds to the energy opening up and having no margin at all. Frankly, from our experience, I don't think I would recommend using eigenvalues associated with that matrix as a measure, because they tend to be highly nonlinear with respect to loading parameters. Instead, this sort of optimal control solution, this integral if you will, yields an energy and is instead what we would propose as an alternate measure. My point in bringing up the eigenvalue was only to show that the limiting case, when you have no margin at all, agreed with some other traditional measures.

Kwang Lee (Penn State University):

I just want to make a brief comment on the angle aspect that the gentlemen asked about a moment ago. Actually, angle and voltage are closely related if you look at load. Suppose we look at a very simple constant load. The active power is constant. The voltage magnitude is now a function of E , the internal voltage of the generator or machine, and also the angle. So both will affect change in the voltage magnitude. The way we have analyzed using the same dynamic model, we have found that when we have a voltage collapse, E changes the way voltage changes, in a very similar way. At the same time, the angle also changes. So when we have a voltage collapse, both angle and voltage change. The way we were able to find out was, we have developed some kind of simplified index which is called G . When the magnitude of G is slightly more than one, then eventually we end up with voltage collapse. However, when

G is less than one, then we don't have this problem. The case when we have this phenomenon happening is when we have a very critical voltage collapse phenomena. The stability index we developed in our previous paper on voltage collapse phenomena is functional with multi-variables, so I cannot really describe it here. According to this stability index, you are able to determine before we wade too far that the system is unstable.

Maria Ilic:

I want to comment on Professor Yorino's paper, on his criteria for stable voltage with respect to tap-changer position dynamics. In his reference [9] at the end of his paper there is a much more general criterion, which is for the multidimensional case when you have more than one tap changer. I guess the criterion, to determine if the voltage behaves in a quasi steady state stable manner with respect to tap-changer position, is something like the partial of reactive power with respect to voltage inverted, times the partial of reactive power with respect to n , where all these are vectors. It is only in the case where you have a single load that these are simple scalar quantities. In the multidimensional case, determining if voltage-load dynamics are going to be stable relative to tap-changer position changes is much more complex. One of the sufficient conditions is that this matrix should have an M -matrix property, and that is much harder to monitor. One would somehow have to determine something about the characteristics of this Jacobian. This is a simple criterion, but it only holds when you have a single load. These criteria also assume that there is decoupling between reactive power and voltage. There is even further generalization, and if you don't assume the decoupling between real and reactive power, then I think Prof. Begovic has a more general starting point. Thus, this is a very simplified story in the single load case.

Naoto Yorino (Hiroshima University):

I agree with Professor Ilic.

Vaithianathan Venkatasubramanian (Washington Univ.):

I actually would like to disagree with Maria in what she just said. In our paper we derived a similar condition, and it comes out that even if you have a multiple number of tap-changers you can show that the Jacobian generalizes to a similar Jacobian. I'm not being precise here, so you will have to refer to the paper to get the exact condition, but the point I would like to make is that this condition is

indeed the condition for even the case where you have multiple tap-changers. You can show that the dynamics become unstable or stable when this condition is satisfied. This is only for the case when you are dealing with traditional tap-changers. That is, we are assuming they are slow so that the quasi-static assumption is satisfied.

Session 3: Tuesday Morning

SESSION 3:

Dynamic vs. Static Aspects of Voltage Problems

P. W. Sauer, B. C. Lesicutre, M. A. Pai (University of Illinois)

Security Analysis and Control - Steady State

Decision Trees for Preventive Voltage Stability Assessment

L. Wehenkel, Th. Van Cutsem, M. Gilliard, D. Pavella (University of Liege),
B. Heilbronn, M. Goubin (Electricité de France)

Decision Trees for Detecting Emergency Voltage Conditions

Th. Van Cutsem, L. Wehenkel, M. Pavella (University of Liege),
B. Heilbronn, M. Goubin (Electricité de France)

Method and Test Results for the MIT Approach to Steady State Voltage Monitoring and Corrective Actions

D. Bertagnolli (NEPEX), A. Catelli (NUS), T. Gentile (EUA), C. Jayko (Commonwealth Electric),
M. D. Ilic, J. Lacalle-Melero, W. Stobart (Massachusetts Institute of Technology),
J. Mattson (Boston Edison), R. Dunlap, D. Pantalone (NEES), V. Tsolios (REMVEC)

On-Line Voltage Collapse Monitor

A. O. Ekwue, R. M. Dunnett, N. T. Hawkins, W. D. Laing (National Grid Co., UK)

Discussion

Dynamic vs. Static Aspects of Voltage Problems

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1 Abstract

This paper discusses several of the practical and theoretical issues associated with the dynamic vs. static issues of voltage problems. It focuses mainly on the issues of maximum loadability and the avoidance of nonphysical models. It concludes with the proposition that static analysis normally produces optimistic upper bounds on practical questions about loadability. Tighter bounds on loadability require small disturbance stability analysis, and even tighter bounds require large disturbance stability analysis.

2 Introduction

The problem of maintaining acceptable voltage levels clearly extends back to the first days of Edison and undoubtedly even further back. In modern times, the term voltage instability has been used to describe many situations which result in systems where voltages are not automatically restored to acceptable levels. These situations have virtually all been associated with reactive power capabilities. In one of the earlier descriptions, voltage instability meant the loss of synchronism due to a reduction of generator real power output caused by low terminal voltage. In his discussion of [1], Mr. F.H. Birch said [2],

"Reference is made in the paper to voltage instability. This process, as I understand it, is rapid and cumulative. It may occur when the generating plant, loosely tied to the main system, is subjected to an additional reactive demand which it cannot meet. The ensuing drop in voltage reduces the reactive output capacity of the generating plant and the active load imposed upon it, and the speed of the machines tend to rise. If the impedance between the machines and the main system is high, insufficient synchronising power will be exchanged to hold the machines in step. On loss of synchronism, the interconnector will be subject to heavy oscillatory transfers of active and reactive power, and a shut down will follow. Do the authors agree with this outline of the process?"

Messrs. Casson and Sheppard replied with [2],

"We agree with Mr. Birch's outline of the process of voltage instability, which we believe will be one of the greatest hazards which the system design and operating staff will have to face in this country, in the future."

This close tie between voltage control capability and loss of synchronism set the stage for interpretations which mix real power, reactive power, voltage magnitude and angle. This mix

is interesting because it defies the usual "decoupling" of $P - \theta$ and $Q - V$. It is precisely this defiance of decoupling that makes these phenomena so interesting and yet so difficult to analyse and control. The singularity of load flow Jacobians can often be traced to a condition where the matrix loses its diagonal dominance and reflects a total coupling rather than the decoupling between real and reactive power. For this reason, we feel that it is inappropriate to try to formulate a "pure reactive power" analog to the classical "pure real power" transient stability problem.

Weedy and Cox discussed long term voltage decays which they said eventually lead to voltage "collapse" which is non-synchronous in form. In these slow cases, they stated that the voltage collapse process may be classified as being steady-state rather than transient [3]. They also examined the use of criteria proposed by Venikov in the Russian literature involving the sensitivity of sending end voltage to receiving end voltage. Weedy has several sections on voltage stability in his book [4]. He notes the critical dependence on load modeling and tap changing under load action.

T. Nagao supported the concept of voltage collapse proceeding very slowly [5]. He stated

"It would be very natural to consider that the voltage collapse phenomena are closely related to the dynamic performances of the power system."

He then proceeds to rule out dynamic performance of

- (a) Stator/network electrical transients
- (b) Generator shaft inertia
- (c) Automatic voltage regulator
- (d) Speed governor
- (e) Fault and protection
- (f) Dynamic response of load

and focus on the dynamic performance of the following, which he considers to be of primary importance:

- (g) Transformer tap control
- (h) Switching of shunt capacitance
- (i) Consumer voltage regulation
- (j) Load variation

with transformer tap control the most important. Nagao gives criteria for stable tap control based on voltage sensitivity at the receiving end of a transmission line (also the transformer location). He uses a discrete time dynamic model, illustrating responses over a period of minutes.

In the same year, Venikov et. al. proposed a criteria using the load flow Jacobian to detect steady-state instabilities [6]. A clarification of this criteria from a dynamics viewpoint and its limitations were recently given in [7].

Between the period of 1975-1991 there have been many papers which address the voltage collapse/instability problems. A large group of these papers are discussed or presented in [8]. Many of these papers support the idea that most voltage collapse/instability phenomena are dynamic in nature only through the time variation of the loads and/or tap changing transformers and capacitor switching. In virtually every case, the phenomena being studied are situations which lead to unacceptable sensitivity.

One thing is probably true about the dynamic vs. static aspects of voltage stability/collapse. If a system reaches a sensitive voltage condition, it will probably eventually result in a complete dynamic instability. We say probably because there are many counter examples where operation in a voltage sensitive condition does not result in a complete dynamic instability.

If the objective is to avoid a voltage sensitive condition, then this can normally be done using either purely static models with due consideration of limits on voltage control devices. In our opinion, this is a somewhat narrow view of things from a power system operator or planners view. First, this voltage sensitivity issue is only one of many conditions which must be avoided. Second, the voltage usually causes a complete coupling of phenomena which are normally separated for analysis purposes. It is only natural then that this coupling should extend to the static and dynamic aspects of analysis.

Our conclusion at this point is that a pure static approach to anything associated with power system analysis is an optimistic estimate of things like loadability, simultaneous interchange capability, or the current state of system security. Only a complete dynamic approach can give a realistic assessment of the existing or future state of the system. By complete dynamic approach, we mean that the model must reflect all dynamic phenomena which have the potential to cause unacceptable operating conditions. Since critical loading brings the system to a point where almost all phenomena are coupled, it is not clear which dynamic models can be ignored. Attempts to simplify or reduce dynamic models run the risk of overlooking a critical response.

The recent paper from Japan [9] proposes a time scale decoupling and classification of voltage instabilities. In principle, this concept is good, and has the potential to break a huge problem down into several smaller problems. Furthermore, this approach is applicable to phenomena other than "voltage" instabilities. Unfortunately, the actual implementation of dynamic security assessment remains a formidable task [10],[11].

3 Practical Issues

Power system planners design new facilities to serve forecasted load. They design them so that the system performance is acceptable and economical for normal operation and for contingencies. Power system operators (and operation planners) are tasked with determining operation procedures which give acceptable system performance at minimum cost on a shorter forecast schedule. In determining the acceptability of a system's performance, they must analyse many things. Some of the fundamental issues are:

1. Is there enough local generation to serve the forecasted load? If not, do neighbors have enough to give in order to serve the forecasted load?
2. If the answers to 1 are yes, can the network transmit the generation to the load?
3. If the answers to 1, 2 are yes, will the network voltages and currents be at acceptable levels?
4. If the answers to 1, 2, 3 are yes, can the system survive small disturbances?
5. If the answers to 1, 2, 3, 4 are yes, can the system survive large disturbances?
6. If the answers to 1, 2, 3, 4, 5 are yes, is the system sufficiently far away from having a no answer to any of them?
7. If the answers to 1, 2, 3, 4, 5, 6 are yes, will the system be operating at minimum cost?
8. If any answer to 1, 2, 3, 4, 5, 6, 7 is no, can the system be changed so that the answer is yes?
9. If the answers to 1, 2, 3, 4, 5, 6, 7, 8 are yes, then repeat all of this for another likely forecasted load distribution!

Where do voltage collapse/instability problems enter into these questions? Clearly the maximum power transfer problem deals with question 2 (and of course 6, 8). This problem is purely static in nature and does not depend on the actual load characteristic (for a given forecasted power and power factor). If low voltage is considered a voltage collapse/instability problem, then clearly this deals with question 3 (and of course 6, 8). This problem is also purely static in nature and does not depend on the actual load characteristic (for a given forecasted power and power factor). By evolution, questions 4 and 5 (and of course 6, 8) have come to include a concern for voltage phenomena. These problems are purely dynamic in nature and depend on the actual load characteristics.

This concern for voltage phenomena has stimulated an interest in more detailed dynamic modeling to capture things which were traditionally ignored. A major concern here is the minimum experience with the design of "stabilisers" for "voltage oriented" instabilities. By "stabilisers", we mean the coordinated use of all equipment and controls. We sort of know how to avoid "mechanical oriented" instabilities with proper use of relays, power system stabilisers (PSS), braking resistors, high voltage DC, and other "real power" equipment. Experience with Var sources, tap changing under load transformers, switchable capacitors and their coordinated control for system integrity is much more limited.

4 The Ambiguity of Power

Perhaps the greatest source of problems (and challenging research issues) in power system analysis is the desire to study systems at specified power levels. This desire injects a physical ambiguity which leads to multiple mathematical solutions. For example, when power system engineers want to study the "no load" case, they usually mean the injected power is zero ($P + jQ = 0$). In addition, they usually mean that they want to study the open circuit case ($I = 0$). Unfortunately, this zero injected power condition can also be satisfied by the short circuit case ($V = 0$). This phenomena is complicated by the fact that sometimes they do want to study the short circuit case. While the difference between the two solutions is obvious for these two extremes, the difference is not always so obvious for cases where the "load" is specified by a non zero power.

In many cases, power system engineers do not care what the load actually is. They simply want to study the system condition at a specific level of power. This is understandable since levels of power are normally more physically meaningful than levels of current or voltage. We do however usually specify the voltage level when we want to study the short circuit case (no load just doesn't sound right). While our ambiguity has not created too many serious problems in the past, it is slowly forcing us to become more precise in our work.

5 Maximum Loadability

Recognising our ambiguity, consider the elementary concept of single source, single line, single load maximum power transfer. Since "power" is ambiguous, we will use pure resistors to model the load as shown in Figure 1.

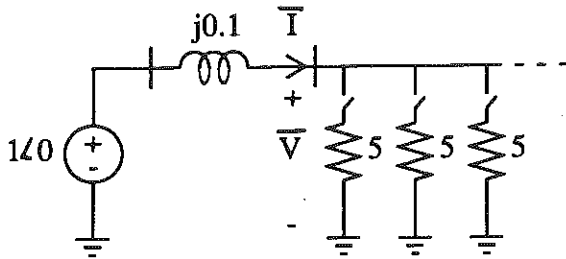


Figure 1: Single bus maximum power transfer

For N resistors switched in, Table 1 gives the solutions for load voltage, current and power

Table 1: Single bus loading conditions

N	V	I	P
0	1	0	0
5	0.995	0.995	0.99
10	0.981	1.961	1.924
15	0.958	2.873	2.753
30	0.857	5.145	4.409
50	0.707	7.071	5.000
160	0.298	9.54	2.844
400	0.124	9.923	1.230
1000	0.050	9.988	0.500
∞	0	10.00	0

This data of course gives the infamous PV curve of Figure 2. One purpose for going through this simple example was to make the point that it is physically possible to operate at all points on this PV curve. The physical significance of operation of course ranges from the open circuit case ($N = 0$) to the short circuit case ($N = \infty$). It is straightforward to replace the ideal source with a synchronous machine modeled in full detail and show that all equilibrium points on the PV curve are stable equilibrium points.

Another purpose for going through this simple example was to comment on the notion of static voltage instability. System behavior on the lower portion of the PV curve has been

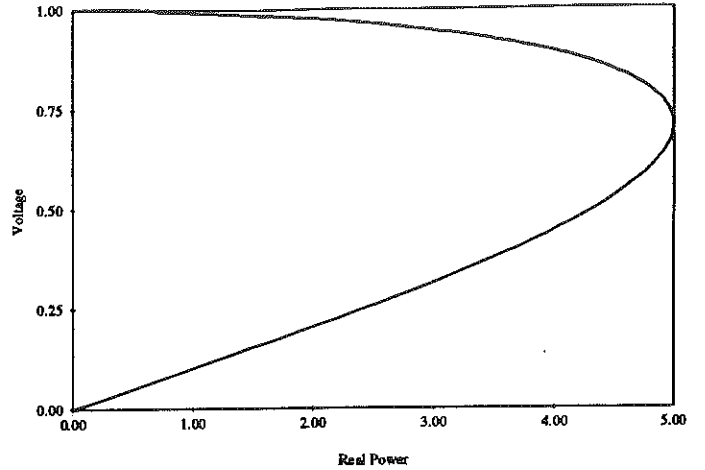


Figure 2: PV curve

termed a voltage instability because an "increase in load" results in a decrease in power transmitted. The increase in load refers to the switching in of another resistor. The term voltage instability is not particularly descriptive since the voltage does decrease as "load is increased" which is not unusual. What is unusual is that the power transmitted decreases as "load is increased." In addition, since the actual system may well be dynamically stable while operating on this portion of the curve, it also seems inappropriate to use the term instability. As such, it is more precise to describe operation on the lower half of the PV curve as "operation on the lower half of the PV curve."

Regardless of the terminology, there is interest in knowing the maximum value of power which can be supplied to a load. More importantly, there is interest in knowing how "far" a given system is from this point of maximum loadability. For this particular example, the formal problem of maximum static loadability is the solution to:

$$\begin{aligned} &\text{Maximise: } P \\ &\text{Subject to: } 0 = f(X, P) \end{aligned} \quad (1)$$

where X contains the circuit variables (i.e. $\bar{V}$ and/or $\bar{I}$). A classical solution of this problem is to introduce a Lagrange multiplier λ and solve the unconstrained problem,

$$\text{Maximise } P + \lambda f(X, P) \quad (2)$$

The solution is obtained by solving

$$1 + \lambda \frac{\partial f}{\partial P} = 0 \quad (3)$$

$$f(X, P) = 0 \quad (4)$$

$$\lambda \frac{\partial f}{\partial X} = 0 \quad (5)$$

for X , λ and P . In the multi-bus case, the maximum power transfer problem can be formulated in many different ways. The most restrictive formulation (but perhaps also the most meaningful one) introduces a scalar multiplier (p) which replaces P in the above problem. Working from a base case ($p = 1$), it is possible to find the maximum possible p for which a solution exists. This was formulated by Brucoli et. al. in [12]. Essentially the same formulation was later analysed

in [13]. Similar results for scalar p using a matrix singularity calculation were presented by Alvarado in [8], and Ajjarapu in [14]. An important extension of this concept is the computation of points where complex eigenvalues of a matrix cross the imaginary axis [15]. This allows the approach to be used to determine maximum loadability with consideration of steady-state stability.

The maximum p will of course depend on the assumed generation scheduling and load power factor. The scalar p formulation is nice because it also represents a simple margin to maximum power transfer automatically. The disadvantage of the scalar p formulation is that it calculates maximum power transfer in "one direction" only. Variations in load in other directions can lead to higher or lower values of total load and different margins to maximum power transfer. There are a very large number of options in the vector p formulation. For example, an alternate scalar p problem could solve for the maximum increase in real power at one bus while all others remain fixed. Alternatively, vector p problems can allow two, three or n combinations. This includes the option to find the minimum distance from the $p = 1$ case to the maximum power transfer condition. While the approach is quite different, the energy based methods of [16]-[18] offer a similar "minimum distance" solution with the potential to account for detailed system dynamics. An important issue which is not clearly settled is which formulation of p is the most physically meaningful. To be meaningful, this calculation must represent a forecast of likely future loading and likely generation availability.

The concept of maximum static loadability is the basic subject of a large number of publications. It is the most common interpretation of the voltage collapse/voltage instability terminology. For this paper, the important point is that this calculation is normally concerned only with determining the power levels for which an equilibrium point exists. If maximum dynamic loadability is to be properly addressed, techniques such as [15]-[18] offer two different potentially useful methods. They offer two extremes in the issue of vector p formulations.

The concepts of maximum simultaneous interchange and maximum loadability added line and generation constraints in a linear programming solution with vector p being either all available generation or interchange powers [19]-[22]. The linear inequality constraints were formulated from line and generator loading limits. For the line flow constraints, the justification for a single limit which considers thermal, voltage, and steady-state stability margin was based on composite ratings such as St. Clair curves [23]. The steady-state stability limit was based on an angle difference across an equivalent system. A margin was built in by lowering the critical angle. This margin of steady-state stability was shown to be critically dependent on Var supply capability [24]. These early papers recognised that critical loadability may be constrained by thermal limits, voltage drop limits, or steady-state stability limits. As such they properly attempted to consider all possible phenomena in one calculation. Transient stability constraints were considered in the loadability problem in [25].

The main conclusion in this brief summary of maximum loadability is that for a given problem, the critical loading of a system may be constrained by a static phenomenon or a dynamic phenomenon. There is still a significant challenge in determining which phenomenon is binding for a given loadability problem. Once the binding constraint is known, then either a static or dynamic approach is needed to determine margins of loadability and possible corrective strategies.

6 Dynamic Aspects of Voltage/Power Characteristics

While the concept of maximum power transfer and the search for the tip of the nose curve offer an upper bound on loadability, the critical loading level may be restricted by dynamic phenomena. As such, the question of maximum loadability need not involve voltage phenomena at all. Over the years, most dynamic analysis has focused on shaft oscillation and loss of synchronism. Evidently this was because these types of dynamic problems were classical in nature and relatively easy to comprehend in terms of traditional energy laws. The wide use of the classical model is an indication of the extent to which voltage dynamics have been ignored.

Perhaps the easiest dynamic concept to examine and analyse is steady-state stability. Unfortunately, the computation of eigenvalues for large scale systems is an extremely time consuming task. The key in this type of analysis is finding the critical eigenvalue or pair of eigenvalues. One logical approach to achieve this goal is the elimination of the dynamic states which do not contribute to the critical eigenvalues. If the critical eigenvalue is known to be associated with shaft dynamics, then perhaps only shaft dynamics need to be modeled. If the critical eigenvalue is known to be associated with the voltage control system, then perhaps only the voltage control system dynamics need to be modeled. While there has been extensive work in this area to identify the critical shaft dynamics, there is very little on any other type.

Examination of the single machine/infinite bus system does provide considerable insight into these issues. It can be shown that as the loading on a generator without voltage control (fixed excitation) is increased (in any direction), the first point of instability is normally associated with the field winding flux dynamics [26]. When a voltage regulator is added, the first point of instability is normally associated with either the shaft dynamics or the interaction of the regulator with the field flux. In principle, this supports the reports that voltage collapse phenomena can occur when generators hit their Var limits (actually fixed excitation rather than fixed Vars). Unfortunately, it is not always so simple, and systems with voltage regulators are not always limited by critical eigenvalues associated with shaft dynamics. Even in the single machine, single load case, fascinating dynamic phenomena associated with voltage can occur. To see this, consider the following two-axis dynamic model [27]

$$T_{d0}' \frac{dE_q'}{dt} = -E_q' - (X_d - X_d')I_d + E_{fd} \quad (6)$$

$$T_{q0}' \frac{dE_d'}{dt} = -E_d' + (X_q - X_q')I_q \quad (7)$$

$$\frac{d\delta}{dt} = \omega - \omega_s \quad (8)$$

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = T_M - E_d'I_d - E_q'I_q - D(\omega - \omega_s) \quad (9)$$

$$T_B \frac{dE_{fd}}{dt} = -(K_B + S_B(E_{fd}))E_{fd} + V_R \quad (10)$$

$$T_F \frac{dR_f}{dt} = -R_f + \frac{K_F}{T_F} E_{fd} \quad (11)$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_f - \frac{K_A K_F}{T_F} E_{fd} + K_A (V_{ref} - V_1) \quad (12)$$

$$T_{RH} \frac{dT_M}{dt} = -T_M + \left(1 - \frac{K_{HF} T_{RH}}{T_{CH}}\right) P_{CH} + \frac{K_{HF} T_{RH}}{T_{CH}} P_{SV} \quad (13)$$

$$T_{CH} \frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \quad (14)$$

$$T_{SV} \frac{dP_{SV}}{dt} = -P_{SV} + P_C - \frac{1}{R} \frac{\omega}{\omega_s} \quad (15)$$

Equations (6)-(9) model the synchronous machine (with $X'_d = X'_q$). Equations (10)-(12) model the excitation and voltage regulator. Equations (13)-(15) model the system turbine plus governor. The stator/network/load model is given in Figure 3

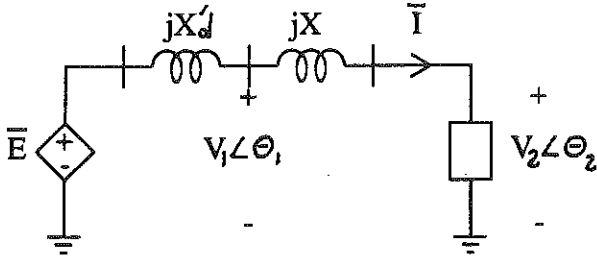


Figure 3: Single machine/single load

where

$$\bar{E} = (E'_d + jE'_q)e^{j(\delta - \pi/2)} \quad (16)$$

$$\bar{I} = (I_d + jI_q)e^{j(\delta - \pi/2)} \quad (17)$$

With this model, the algebraic equations needed to eliminate I_d , I_q , V_1 from (6)-(15) are,

$$0 = V_1 e^{j\theta_1} + jX'_d (I_d + jI_q) e^{j(\delta - \pi/2)} - [E'_d + jE'_q] e^{j(\delta - \pi/2)} \quad (18)$$

$$0 = -V_1 e^{j\theta_1} + jX (I_d + jI_q) e^{j(\delta - \pi/2)} + V_2 e^{j\theta_2} \quad (19)$$

$$V_2 e^{j\theta_2} (I_d - jI_q) e^{-j(\delta - \pi/2)} = P_L(V_2) + jQ_L(V_2) \quad (20)$$

The stator plus network plus load algebraic equations can be combined to give,

$$V_2 e^{j\theta_2} \frac{((E'_d - jE'_q)e^{-j(\delta - \pi/2)} - V_2 e^{-j\theta_2})}{-j(X'_d + X)} = P_L(V_2) + jQ_L(V_2) \quad (21)$$

$$(I_d + jI_q) e^{j(\delta - \pi/2)} = \frac{(E'_d + jE'_q) e^{j(\delta - \pi/2)} - V_2 e^{j\theta_2}}{j(X'_d + X)} \quad (22)$$

$$V_1 e^{j\theta_1} = jX (I_d + jI_q) e^{j(\delta - \pi/2)} + V_2 e^{j\theta_2} \quad (23)$$

We remark that the most important algebraic equation is (21). For given E'_d , E'_q , δ , it must be solved for V_2 , θ_2 so that I_d , I_q , V_1 , θ_1 can easily be found from (22), (23). The ability to solve (21) depends critically on $P_L(V_2)$ and $Q_L(V_2)$. This will be discussed in detail later. For now, we examine the steady-state stability for various levels of load modeled as constant power ($P_L(V_2) = \text{const.}$, $Q_L(V_2) = \text{const.}$). With each operating point having the terminal voltage magnitude at 1 p.u. and the reactance of the transmission line at 0.1 p.u., the voltage vs. power curve is easily obtained for a unity power factor load. This is shown in Figure 4. From an electrical network point of view, each point on the curve in Figure 4 represents a valid operating point.

The regions of this curve over which the dynamic system is stable/unstable were examined by observing the eigenvalues of the linearized synchronous machine model at each point. In the case of a constant power load, there exists stable and unstable regions. Using eigenvalue sensitivity analysis it was seen that the variables associated with the unstable eigenvalues were the E'_q , R_f pair. Parametrically changing the load from 0.0 (no load) to 2.36 p.u., the system is stable. In Figure 4 this corresponds to the portion of the curve from 'O' to 'A'. The system loses steady-state stability at point 'A'. The system becomes stable again at a loading of 4.79 p.u. (point 'B') which is before the point of maximum power transfer (power level of 5.0 p.u.). It was also observed that the entire lower portion of the curve in Figure 4 was dynamically stable.

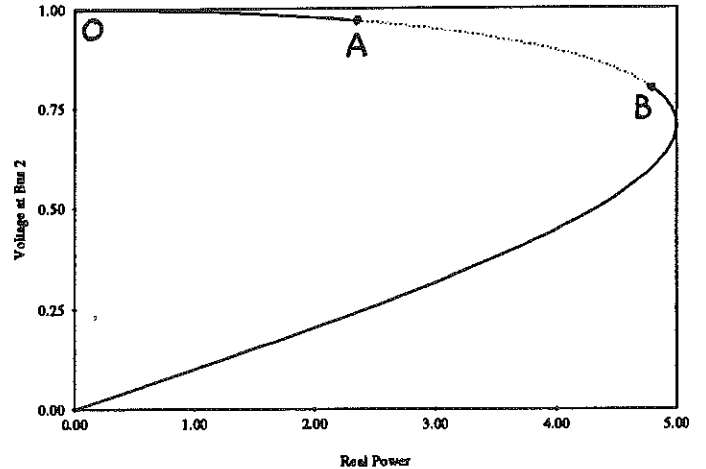


Figure 4: P-V curve for a single machine case

Thus, the point of maximum stable loading is $P_L = 2.36$, although if it could be reached, the system is stable for a loading at the maximum power transfer level of 5.0. It is interesting to also note that if voltage control for $V_2 = 1$ was available at the load, the maximum power transfer would be 10.0. Further analysis of this example is given in [28].

It is interesting to perform a nonlinear simulation of the response of the system when it begins from a stable point and is subjected to a load change which would place its equilibrium in the unstable region. For example, suppose the system is operating at the stable point corresponding to a load of 2.0 p.u. A sudden change in loading to 2.4 p.u. results in a dynamic simulation (using trapezoidal rule with time step $h = .001$) which fails to solve at $t = 0.268$ sec. This failure to solve can be explained by noting that for each E'_d , E'_q , δ from the dynamic model, there is another PV curve showing the maximum real

power which can be transferred with the value of $\bar{E}$. PV curves at two instants of time are shown in Figure 5. For a specified P_L (constant), there is a critical value of E that can supply this load. In this simple example, the critical value of E is,

$$E_{cr} = \sqrt{2P_L(X'_d + X)} \quad (24)$$

For this example, the value of E varied as shown in Figure 6 (V_1 and V_2 are shown also).

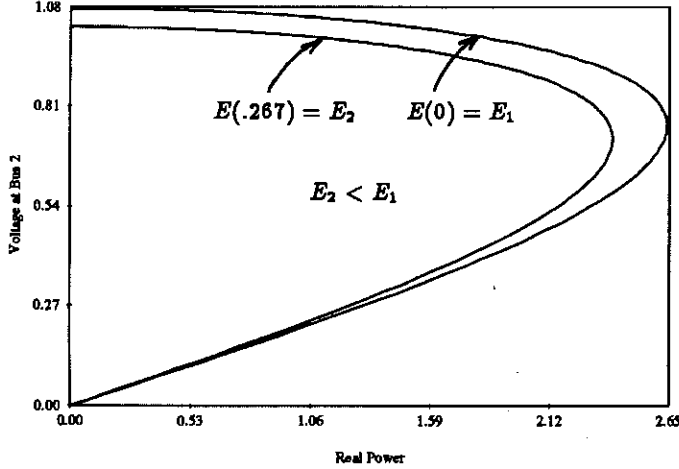


Figure 5: PV curves during transient

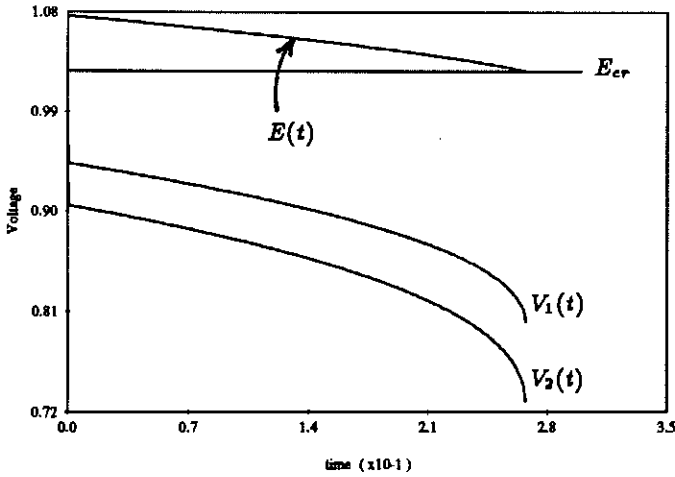


Figure 6: Voltages during transient

The simulation failed to solve when E dropped below E_{cr} at $t = 0.268$ sec. When the simulation was restarted at $t = 0.267$ with a time step of $h = .0001$, it continued for a while, but failed to solve at $t = 0.2675$ sec. When the simulation was restarted at $t = 0.2674$ with a time step of $h = .00001$, it continued for a while, but failed to solve at $t = 0.26746$. Continuing this process forced the system to a point where the Jacobian of the trapezoidal rule formulation became singular (between 0.2674 and 0.26746). An investigation of this singularity is given in the following sections.

7 Loss of Existence of A Solution

In the static load model, P_L and Q_L were assumed to be algebraic functions of the load voltage magnitude. It is fairly

straightforward to include load dynamics (such as induction motors) by introducing one or more of the possible load dynamic properties. For example, it is possible to include the shaft dynamics of an induction motor and introduce a speed dependent resistor in the load model using the quasi-steady-state equivalent circuits. When the induction motor is modeled in this way with series impedances, it introduces a maximum torque (pullout) which depends on the square of the terminal voltage. The importance of this maximum torque depends on the mechanical load model. If the mechanical load is modeled as a constant torque, then it is possible that the terminal voltage will drop to a point where the load torque exceeds the pullout torque and the machine can cause high currents which produce even lower voltages. An excellent analysis of this phenomenon is given in [29]. If the mechanical load torque is modeled as a speed dependent torque, then its behavior should be considerably less destructive. The investigation into the electrical dynamics of induction machines has raised another source of potential problems [30]. It is still not obvious at this point which induction motor dynamics actually introduce conditions which aggravate voltage problems.

A key issue in voltage collapse/instability problems is the question of bifurcation and/or causality [31]. In examining the dynamic vs. static aspects of voltage problems there is an important question which should be asked. Is the model physically correct? To see why this question is important, consider a mathematical model of a power system in the form:

$$\frac{dx}{dt} = f(x, y) \quad x(0) = x^0 \quad (25)$$

$$0 = g(x, y) \quad (26)$$

This structure may or may not have arisen due to a singular perturbation of the y dynamics. In any case, it is based on some assumptions about the physical system. In order for this model to be physically realistic, the algebraic equations ($g(x, y)$) must be solvable for y as a function of x . By solvable, we mean that a real solution for y must exist. If a real solution for y does not exist for some $x(t)$, then the dynamic model is nonphysical for that value of x [32]. By nonphysical we mean some critical phenomenon has not been modeled correctly.

To see what this means from a numerical point of view, consider the solution of (25) and (26) by the trapezoidal rule. For given values of t , $x(t)$, $y(t)$, the solution at time $t + h$ is the solution of

$$x(t+h) = x(t) + \frac{h}{2} [f(x(t+h), y(t+h)) + f(x(t), y(t))] \quad (27)$$

$$0 = g(x(t+h), y(t+h)) \quad (28)$$

for $x(t+h)$, $y(t+h)$. Using Newton's method, the iterative sequence is

$$\begin{bmatrix} x(t+h)_{(i+1)} \\ y(t+h)_{(i+1)} \end{bmatrix} = \begin{bmatrix} x(t+h)_{(i)} \\ y(t+h)_{(i)} \end{bmatrix} - J_{TR(i)}^{-1} \begin{bmatrix} F_{(i)} \\ G_{(i)} \end{bmatrix} \quad (29)$$

where

$$F_{(i)} = -x(t+h)_{(i)} + x(t) + \frac{h}{2} [f(x(t+h)_{(i)}, y(t+h)_{(i)}) + f(x(t), y(t))] \quad (30)$$

$$G_{(i)} = g(x(t+h)_{(i)}, y(t+h)_{(i)}) \quad (31)$$

and

$$J_{TR(i)} = \begin{bmatrix} -I + \frac{h}{2} \frac{\partial f}{\partial x} & \frac{h}{2} \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \bigg|_{\substack{x=x(t+h)(i) \\ y=y(t+h)(i)}} \quad (32)$$

For a given integration time step h , the convergence to a solution at time $t + h$ requires a nonsingular $J_{TR(i)}$. Presumably the initial guess for $x(t + h)$ and $y(t + h)$ is taken as $x(t)$ and $y(t)$. In order to assure a close initial guess and rapid convergence, it is customary to take a small time step h . If h is made arbitrarily small, the J_{TR} matrix approaches,

$$J_{TR} |_{h=0} = \begin{bmatrix} -I & 0 \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix} \quad (33)$$

The singularity is clearly dependent on the singularity of the Jacobian of the algebraic equations,

$$J_{AB} = \frac{\partial g}{\partial y} \quad (34)$$

Thus if the time step is sufficiently small (an accurate numerical approximation), a singularity in the trapezoidal rule iterative process would be due to a singularity of the algebraic equation Jacobian. Since the algebraic equations are normally the stator/network plus load models, these are crucial for good simulation. The following section focuses on methods to avoid the loss of existence of a solution.

8 Avoiding Nonphysical Models

In mathematical modeling, we strive to reproduce the fundamental properties of physical systems without undue complexity. If a dynamic simulation stops because of a loss of existence of a solution then the model is nonphysical. Consider the following static load model

$$P_L + jQ_L = P_o V^{k_p} + jQ_o V^{k_q} \quad (35)$$

where V is some defined "bus voltage magnitude" and P_o , k_p , Q_o , k_q are constants. For this discussion, we also classify any $V = 0$ solutions as nonphysical. Numerous simulations that we have performed indicate that a $V \neq 0$ solution always exists provided

$$\begin{aligned} k_p &> 1, & k_q &> 1 \\ P_o &\geq 0, & Q_o &\geq 0 \end{aligned} \quad (36)$$

To investigate this, consider the problem of the last section where equation (21) needs to be solvable for V_2 , θ_2 , given $\bar{E}$. For $\bar{E} = E \angle \gamma$, this algebraic equation with resistance added is,

$$\begin{aligned} V_2 \angle \theta_2 (E \angle -\gamma - V_2 \angle -\theta_2)(G - jB) = \\ P_L(V_2) + jQ_L(V_2) \end{aligned} \quad (37)$$

For a given static load model

$$P_L(V_2) = P_o V_2^{k_p} \quad (38)$$

$$Q_L(V_2) = Q_o V_2^{k_q} \quad (39)$$

it is useful to know the range of values that k_p , k_q can be assigned so that the model is physical (will ensure that (37)-(39) can always be solved for $V_2 \neq 0$, θ_2 for any P_o , Q_o , $E \neq 0$,

γ). We first note that for $k_p = 0$, $k_q = 0$, the equations are the normal constant power load model which is known to be nonphysical in many cases. Thus we search for values of $k_p > 0$, $k_q > 0$. For $k_p > 0$, $k_q > 0$, equations (37)-(39) will always be satisfied by the $V_2 = 0$, $\theta_2 = \text{anything}$ solution. To search for solutions which have $V_2 \neq 0$, we require the existence of solutions to the "current" equation,

$$\begin{aligned} 1 \angle \theta_2 (E \angle -\gamma - V_2 \angle -\theta_2)(G - jB) = \\ P_o V_2^{k_p-1} + jQ_o V_2^{k_q-1} \end{aligned} \quad (40)$$

or

$$\begin{aligned} g_p(V_2, \theta_2) = -P_o V_2^{k_p-1} + EG \cos(\theta_2 - \gamma) \\ + EB \sin(\theta_2 - \gamma) - V_2 G = 0 \end{aligned} \quad (41)$$

$$\begin{aligned} g_q(V_2, \theta_2) = -Q_o V_2^{k_q-1} + EG \sin(\theta_2 - \gamma) \\ - EB \cos(\theta_2 - \gamma) + V_2 B = 0 \end{aligned} \quad (42)$$

We now look for ranges of $k_p > 0$, $k_q > 0$ which guarantee that (41)-(42) can be solved for $V_2 \neq 0$, θ_2 for any $G \geq 0$, $B < 0$, $P_o \geq 0$, $Q_o \geq 0$, $E \neq 0$, γ . To gain insight into this question, consider the unity power factor load ($Q_o = 0$) with the following parameters,

$$E = 1, \gamma = 0, G = 0, B = -10, \quad (43)$$

giving

$$g_p = -P_o V_2^{k_p-1} - 10 \sin \theta_2 = 0 \quad (44)$$

$$g_q = 10 \cos \theta_2 - 10 V_2 = 0 \quad (45)$$

To see which k_p allow unlimited selection of P_o , look at the family of plots showing V_2 vs. P_o in Figure 7,

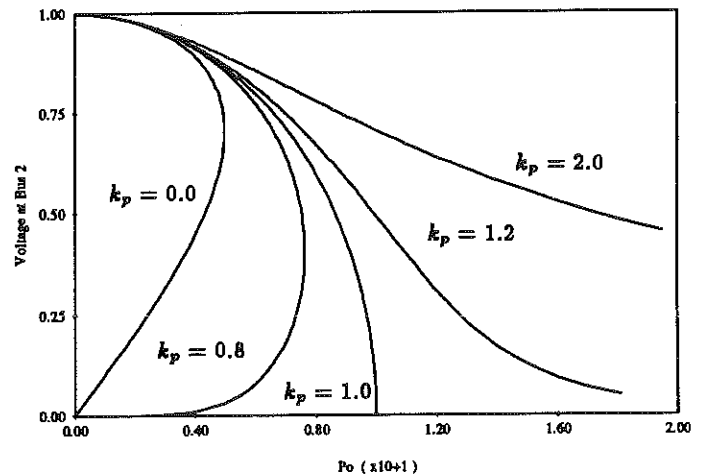


Figure 7: Load voltage vs. power coefficient

These plots indicate that choices of $0 < k_p \leq 1$ can result in nonphysical models. To illustrate this for the constant current case, use the data of this example and try to solve the algebraic equations for $V_2 \neq 0$, θ_2 when $k_p = 1$ and $P_o = 11$. The original power equations are,

$$V_2 \angle \theta_2 (1 \angle 0 - V_2 \angle -\theta_2)(j10) = 11 V_2 \quad (46)$$

This power equation clearly has the $V_2 = 0$ solution. The $V_2 \neq 0$ solution must satisfy

$$-10 \sin(\theta_2) = 11 \quad (47)$$

$$10 \cos(\theta_2) - 10V_2 = 0 \quad (48)$$

These equations cannot be solved for $V_2 \neq 0$, θ_2 .

To formally show that $k_p > 1$, $k_q > 1$ avoids nonphysical models, we examine the Jacobian matrix of (41)-(42),

$$J_{AB} = \begin{bmatrix} \frac{\partial g_p}{\partial V_2} & \frac{\partial g_p}{\partial \theta_2} \\ \frac{\partial g_q}{\partial V_2} & \frac{\partial g_q}{\partial \theta_2} \end{bmatrix} \quad (49)$$

with

$$\frac{\partial g_p}{\partial V_2} = -(k_p - 1)P_o V_2^{k_p-2} - G \quad (50)$$

$$\frac{\partial g_p}{\partial \theta_2} = -EG \sin(\theta_2 - \gamma) + EB \cos(\theta_2 - \gamma) \quad (51)$$

$$= -Q_o V_2^{k_q-1} + V_2 B \quad (52)$$

$$\frac{\partial g_q}{\partial V_2} = -(k_q - 1)Q_o V_2^{k_q-2} + B \quad (53)$$

$$\frac{\partial g_q}{\partial \theta_2} = EG \cos(\theta_2 - \gamma) + EB \sin(\theta_2 - \gamma) \quad (54)$$

$$= P_o V_2^{k_p-1} + V_2 G \quad (55)$$

The determinant of J_{AB} is,

$$\det J_{AB} = -(k_p - 1)P_o^2 V_2^{2k_p-3} - k_p G P_o V_2^{k_p-1} - G^2 V_2 \\ - (k_q - 1)Q_o^2 V_2^{2k_q-3} + k_q B Q_o V_2^{k_q-1} - B^2 V_2 \quad (56)$$

For given $E \neq 0$, γ , $G \geq 0$, $B < 0$, the solution $V_2 = E$, $\theta_2 = \gamma$ always exists for the case $P_o = 0$, $Q_o = 0$, and has a determinant of J_{AB} which is either positive or negative. If it can be shown that the determinant of J_{AB} cannot change signs (and is nonzero) when other values of $E \neq 0$, γ , $G \geq 0$, $B < 0$, $k_p > 1$, $k_q > 1$, $P_o \geq 0$, $Q_o \geq 0$ are specified, then a $V_2 \neq 0$ solution always exists [33]. To show that this is the case, we first show that V_2 cannot be zero for these conditions. Rewrite (41)-(42) as

$$[C] \begin{bmatrix} \cos(\theta_2 - \gamma) \\ \sin(\theta_2 - \gamma) \end{bmatrix} = \begin{bmatrix} P_o V_2^{k_p-1} + V_2 G \\ Q_o V_2^{k_q-1} - V_2 B \end{bmatrix} \quad (57)$$

where

$$C = \begin{bmatrix} EG & EB \\ -EB & EG \end{bmatrix} \quad (58)$$

For $E \neq 0$, $G \geq 0$, $B < 0$,

$$\det C \neq 0 \quad (59)$$

For $k_p > 1$, $k_q > 1$, this means that $V_2 = 0$ requires

$$\cos(\theta_2 - \gamma) = 0 \quad (60)$$

$$\sin(\theta_2 - \gamma) = 0 \quad (61)$$

Since this is a contradiction, there can be no $V_2 = 0$ solution to (41)-(42). This means that the determinant of J_{AB} cannot

change signs (and cannot be zero) for $k_p > 1$, $k_q > 1$, $G \geq 0$, $B < 0$, $P_o \geq 0$, $Q_o \geq 0$.

This restriction on k_p , k_q does not mean that a system cannot encounter the maximum power condition, or avoid a condition of sensitive voltage. It does however ensure that a dynamic simulation using any P_o , Q_o load characteristic coefficients will not fail to have a solution due to a nonphysical load model.

The results in [34] also discuss the issue of ensuring causality and give virtually the same restrictions on k_p and k_q . The issue of causality can in some cases also be avoided by the use of dynamic load models which allow the algebraic equations to be converted into differential equations [35], [36]. We see this as a very promising approach to the modeling issues of voltage instability problems. The issue of causality and its impact on voltage sensitivities is also analysed extensively in [37].

9 Tap-Changing-Transformers

The role of tap changing under load (TCUL) transformers in voltage problems has been the subject of considerable discussion. In evaluating the dynamic vs. static aspects of these devices, we offer several opinions. First, as voltage regulators, they are clearly important in determining steady-state voltage levels at loads. With this affect, they certainly impact the closeness to a static maximum loadability condition.

For a system operating in a stable condition, the controlled voltage value will normally lie within the dead band of the controller [38]. Unless the sensed voltage coincides with the critical level needed to initiate tap changing action, small disturbance stability should be unaffected by TCUL transformers. That is, eigenvalue analysis of the traditional continuous time dynamic model using the initial tap position should be valid. By this we mean, that if all eigenvalues found using a fixed tap value are stable, then for small enough disturbances, the system will remain stable. However, it is possible that the tap changing control logic is such that a large enough disturbance could cause a tap change in a direction which eventually aggravates the system voltage condition resulting in some dynamic instability. Strictly speaking, this seems to be a large disturbance stability issue. However, if a single tap action initiates such an event, then it seems logical that a continuous small disturbance stability model of the tap changing control logic would be appropriate. The results of [39] - [42] address this issue and propose promising criteria for evaluating stability.

10 Conclusions

Loadability (or equivalently transferability) of power systems seems to be the primary concern of power system operators and planners. Evaluation of an existing or proposed operating condition requires the determination of the limiting phenomenon. The limiting phenomenon may be static levels such as thermal or voltage. This limit can be determined from static analysis, and may be the most restrictive for many systems. The limiting phenomenon may be dynamic stability such as loss of synchronism or other unacceptable transient behavior. Clearly static level limits and dynamic stability limits arise from two distinctly different phenomena. Steady-state stability is normally considered a subset of the second phenomenon.

Maximum power transfer is an upper bound on loadability. This limit is important regardless of whether it results in a

dynamic instability or not. Since it is computable from static modeling, it is related very closely in computational approach to the static level limit. Since it can arise in the middle of dynamic analysis, it is also an important phenomenon to understand and characterize. It is the basic idea behind avoiding nonphysical models. The characterization is difficult because traditional decoupling normally does not apply.

As an upper bound, maximum static loadability should be considered an optimistic limit. In progressing to a more realistic loadability limit, we would place steady-state stability limits as the next less optimistic approach. This is somewhat obvious since this analysis requires the existence of an equilibrium point. Since steady-state stability analysis is a subset of large disturbance stability analysis, it would seem correct that large disturbance stability limits would produce the most realistic loadability limits. This brings us to the conclusion that all of the issues associated with voltage instability/collapse can be evaluated using dynamic security assessment.

In all cases, the key problem in assessing loadability limits is the assumed distribution of load, generation, interchange, and the assumed disturbance. Even for the more optimistic static case, the maximum loadability is critically dependent on these assumptions. After these assumptions are made, the next most important problem in assessing loadability is the assumed dynamic property of the system. If all dynamics are ignored, the loadability result will be optimistic. The extent to which the dynamic loadability limit is realistic depends on the extent to which the limiting dynamic phenomenon is captured.

11 Acknowledgements

This work was supported in part by the National Science Foundation Grant NSF ECS 87-19055, EPRI Contract RP 8010-21, and the Grainger Foundation Endowments to the University of Illinois.

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DECISION TREES FOR PREVENTIVE VOLTAGE STABILITY ASSESSMENT

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Abstract. A new approach to preventive voltage stability assessment is proposed. It consists of using on-line, decision trees built off-line. This building relies on an inductive inference method which automatically extracts information contained in a sample of "learning states" and organizes it, while compressing it, in a tree fashion. This is a hierarchical organization, where the extracted information is expressed in terms of salient system parameters thus found to govern the phenomena of concern. On the other hand, the "learning states" are system normal operating states, preclassified with respect to a preassigned contingency. Once constructed, a decision tree may be used to assess the robustness vis-à-vis this contingency of any new operating state, in terms of the values of its parameters with respect to those appearing in the tree. The method is illustrated by means of a real system. Trees are constructed in the context of preventive voltage stability assessment, their salient features are discussed and their potential highlighted.

Keywords. Preventive voltage security assessment, real-time indicators, artificial intelligence, decision tree method.

1 INTRODUCTION

Voltage instability can be caused by unacceptable load increases and/or by contingencies. Voltage collapse proximity indices have thus been developed to help anticipate incidents caused by load increases. As concerning contingencies, one generally resorts to time-consuming post-contingency simulations, for various load levels, system configurations, compensation schemes, etc. However, no method is as yet available which provides a global view of system robustness.

The decision tree methodology offers an attractive alternative. This artificial intelligence method belongs to the family of "learning from examples" methods: decision trees are automatically built on the basis of a "learning set" and a list of candidate attributes. The "learning set" is composed of system operating states, preclassified with

respect to a preassigned contingency. The list of candidate attributes, on the other hand, are parameters of the system in its precontingency state, presumed to drive the disturbed system behaviour, i.e. to carry sound information about it. The tree building consists of selecting the most informative among them and hierarchically ordering them according to their relevance to the problem, on the basis of information theory measures. The tree thus uncovers and clearly describes the underlying mechanism. Subsequently, it suitably informs about the robustness of unseen situations with respect to the contingency of concern.

The methodology follows a pattern similar to that developed in the context of transient stability. Initiated about five years ago [1,2], this latter method has provided very interesting results [3 to 6]. This quite naturally motivated us to adapt and apply it to voltage stability assessment.

The proposed method is supported and illustrated by means of a real-world example. It concerns the Brittany region of the French system which has already experienced voltage stability problems. It is modelled by a 366-bus system, comprising the 400 kV grid of the French system and the 225 kV level in its western part. Decision trees are constructed with respect to three contingencies, various salient parameters are assessed, and their main features are pointed out. In particular, the trees are found to be simple and reliable, and the overall method flexible and robust for preventive voltage stability assessment.

2 METHOD DESCRIPTION

2.1 General automatic construction of decision trees

Decision tree methodology in general is a nonparametric inductive learning technique, able to produce classifiers

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about a given problem in order to assess new, unseen situations and/or to uncover the mechanisms driving this problem [7 to 10]. The building of a decision tree relies on a learning set (LS), i.e. a set of preclassified states¹: starting at the top node of the tree with the entire LS, one progresses by recursively creating successor nodes, i.e. by splitting the LS into subsets of increasing classification purity. The procedure is stopped when all the newly created nodes are "terminal" ones, containing "pure enough" learning subsets. Below we summarize the tree building, its essential parameters and relating formalism. A more detailed description may be found in [6].

Notation

Let the state s_k be characterized by a certain number, say n , of attributes²; to simplify, we will assume for the time being that these are ordered numerical attributes a_i (the same number for each state). We accordingly define the learning set (LS) by the collection of a number, say N , of preclassified states and characterized by its attributes and its class³.

Also, we define the test set (TS), by another set of, say M , preclassified states, obtained in a similar but independent way.

In what follows, learning sets will be used to build decision trees, test sets to evaluate their ability to classify unseen states.

Remarks. 1.- The above sets are considered to be statistical samples drawn from the population of possible states. Their proper size is a question of great concern and essentially depends upon the particular application domain (e.g. see [6]).

2.- The extension to categorical attributes in addition to ordered ones may be easily obtained [8]. Both this and the generalization to more than two classes will currently be used below for building trees in the context of voltage stability.

Building

A decision tree (DT) is a tree structured upside down, comprising test and terminal nodes. Each test node is associated with a test on the attribute values of the states, to each possible outcome of which corresponds a successor node. The terminal nodes carry the information required to classify states. Such a tree is for example portrayed in

¹In our case the states are system operating points; they are preclassified with respect to a preassigned contingency relevant to preventive voltage security assessment, such as opening of an important tie-line.

²In our case these attributes will be parameters of the system operating point in its normal (precontingency configuration), selected among those presumed to drive voltage phenomena, as for example voltage magnitudes, reactive power production/consumption, etc.

³For the sake of simplicity, we will restrict ourselves to the construction of two-class trees. The generalization to more than two classes is however quite straightforward.

Fig. 3, built for the purpose of preventive voltage stability assessment, illustrated in Section 3. It is composed of 23 nodes: 11 test nodes labelled T for Test, 12 terminal nodes (labelled L or D for Leaf or Deadend, defined below, as appropriate). Note that when only numerical attributes are considered (whatever continuous or discrete), the tests are *dichotomic*, and the resulting tree is *binary*: each test node is split into *two* successors. In the sequel, only binary trees will be considered.

The building of a decision tree necessitates selecting a priori relevant attributes; we will henceforth refer to them as the *candidate attributes*. These are parameters of the system readily available and presumably carrying sound, essential information about the problem of concern. Given a LS and a list of candidate attributes, the automatic building of a tree conforms to the following procedure [9]:

- Starting at the *top node* of the tree, with the list of candidate attributes and with the entire learning set of preclassified states, analyze these states in order to select a test which achieves their "optimal" splitting into two subsets, i.e. which yields a maximum amount of information about the states classification (see § 2.2). The selection proceeds in two steps:

- (i) for each attribute, say a_i , find its "optimal" value in the sense specified above, by scanning the values it assumes for all learning states; this provides an optimal threshold value v_{i*} and defines the test

$$a_i < v_{i*} ? \quad (1)$$

- (ii) among the different candidate attributes, choose the best one, a_* , along with its optimal value, v_{**} , to split the node.

In short, step (ii) defines the optimal attribute, step (i) its optimal threshold value.

- The selected test thus applied to the learning set of the node, splits it into two subsets, corresponding to the two successors of the node.

Starting with the top node of the tree and the entire LS, the two subsets:

- LS_1 containing all states for which $a_* < v_{**}$
- LS_2 containing all states for which $a_* \geq v_{**}$

correspond to the two successors of the top node.

- The successors are labeled *terminal* or not on the basis of the stop splitting criterion described below. For the nonterminal nodes, the overall procedure is called recursively, to build the corresponding subtrees. For the terminal nodes, the class probabilities p_+ and p_- are estimated on the basis of the corresponding subset of learning states there stored, and the class label of the majority class is attached, possibly along with a weighting factor on the class probability ratio.

The crux of the entire tree construction lies in the selection of the splits and the decision whether to declare a node terminal, or to continue splitting. These questions, considered in [6,11], are summarized below.

“Optimal” splitting

The above strategy consists of considering the best test to be the one which separates at most the states of the two classes in the local learning subset, i.e. which provides the purest *direct successors*. It is therefore *locally*, rather than *globally optimal*.

This locally optimal test selects at each node the *test attribute* along with its threshold value having the best “score”, in the above, local sense. This score is assessed via the normalized information gain defined by eq.(6) in § 2.2.

Stop splitting

The strategy consisting of stop splitting at a node only if the corresponding learning subset is completely class pure was found to build overly complex trees, with terminal nodes generally containing only a very small and unrepresentative sample of learning states. This contributes to increase the complexity of the tree thus decreasing its interpretability of the phenomena, and also to weaken its ability to correctly classify unseen states (i.e. states not belonging to the learning set). The criterion proposed in [5] circumvents this difficulty by stop splitting a node as soon as one of the following two conditions is met.

1. The local subset of learning states is “sufficiently” class pure; we henceforth refer to such a terminal node as a *leaf*. The degree of required class purity is fixed via a parameter of the algorithm specified by H_m , the maximum residual entropy (see § 2.2). In practical investigations, we use same constant value of H_m for all nodes (generally $H_m = 0.1$ bits). Generally, this very low value yields very detailed trees; the stop splitting criterion then relies on the second condition, outlined hereafter.
2. There is no possibility to enhance the tree accuracy in a statistically significant way by splitting a node further : this is then considered to be a *deadend* terminal node. The criterion for stop splitting is formulated as a statistical hypothesis test : *given the local subset of learning states and the optimal split, can we accept the hypothesis that the apparent⁴ increase in accuracy, measured in this subset, as provided by the split, is a purely random effect ?*

The answer to this question is the χ^2 -based criterion developed in [5,6] : fixing the α -risk of not detecting these situations, and testing the value of the apparent increase

⁴ Apparent as opposed to real increase in accuracy, refers to that measured in the LS as opposed to that measured for unseen states.

against the corresponding threshold allows one to detect with a probability of $(1-\alpha)$ the cases where the apparent increase in accuracy is a random effect.

The value of α has indeed drastic effects on the resulting tree characteristics as will be illustrated in Section 4.

2.2 A collection of useful information theory expressions

The above short description uses notions such as “amount of information” or “purity increase”. Below we give a collection of information theory measures useful to construct trees, to choose their attributes and to evaluate their overall classification ability. The mathematical derivation of these expressions may be found e.g. in one of the references [6,10,11].

Information gain

Consider a tree node and let S be the subset of learning states at it. Moreover denote by $p(+/S)$ (resp. $p(-/S)$) the probability of a state of S to belong to the class $+$ (resp. $-$).⁵

According to the information theory, the *information gain* (or entropy decrease) in S provided by the test T expressed by eq. (1) may be quantified by :

$$I_C^T(S) \triangleq H_C(S) - H_{C/T}(S) . \quad (2)$$

Indeed, in eq. (2), $H_C(S)$ is the *prior classification entropy* of S , measuring the impurity of S ; $H_{C/T}(S)$ is the *mean posterior classification entropy* of S given the outcome of T , measuring the residual impurity should S be split into S_{YES} and S_{NO} .

$H_C(S)$ is expressed by :

$$H_C(S) \triangleq -[p(+/S) \log_2 p(+/S) + p(-/S) \log_2 p(-/S)] . \quad (3)$$

Similarly, $H_{C/T}(S)$ is expressed by :

$$H_{C/T}(S) \triangleq p(S_{YES}/S) H_C(S_{YES}) + p(S_{NO}/S) H_C(S_{NO}) . \quad (4)$$

$H_C(S)$ ranges in between 0 and 1 : $H_C(S) = 0$ corresponds to a perfectly pure subset (since then $p(+/S) = 1$ or 0 and correspondingly $p(-/S) = 0$ or 1), while $H_C(S) = 1$ corresponds to a perfectly impure subset (since then $p(+/S) = p(-/S) = \frac{1}{2}$).

⁵ Note that these are *apparent* probabilities, in that they are assessed on the basis of the sample of learning subsets, as opposed to real probabilities on all the possible states which however are generally unknown.

Similarly, the *information gain provided by the entire DT* readily derives from (2) (replace sample S by LS and test T by DT) :

$$I_C^{DT}(LS) = H_C(LS) - H_{C/DT}(LS) . \quad (5)$$

Another way of quantifying the ability of test T to produce purified successors is the *normalized information gain* proposed in [5] :

$$G_C^T(S) \triangleq \frac{2I_C^T(S)}{H_C(S) + H_T(S)} \quad (6)$$

where $H_T(S)$ measures the uncertainty of the outcome of T in S :

$$H_T(S) \triangleq -[p(S_{YES}/S) \log_2 p(S_{YES}/S) + p(S_{NO}/S) \log_2 p(S_{NO}/S)] . \quad (7)$$

Expression (6) is more efficient than (2) thanks to its normalized properties; in particular, it is dimensionless and varies in between 0 (no gain of information is provided by the test T) and 1 (the test provides the maximum of information and yields perfectly pure subsets S_{YES} and S_{NO}). (At the same time, $I_C^T(S)$ ranges respectively from 0 to $H_C(S)$.)

Information quantity

The *information quantity* provided by a test attribute at a tree test node derives from (2) and equals

$$I_Q^T = N_{node} * I_C^T(S) \quad (8)$$

where N_{node} denotes the number of learning states contained in S . Thanks to the additivity property of the above information quantity, the overall information quantity provided by a test attribute selected at different test nodes (with different threshold values) amounts to the sum of partial information quantities provided by this attribute at the different test nodes. Similarly, the overall *information quantity provided by a tree* amounts to the sum of the information quantities provided at all its test nodes.

Usefulness and uses

Let us specify the context within which the above various measures show to be particularly convenient.

(i) The normalized information gain or score (see eq. (6)) allows to objectively compare the ability of the various candidate attributes to produce purified successors at a *given test node* of the growing tree, and hence to select the one furnishing the local optimum test.

(ii) The information quantity expressed by (8) allows to quantify and compare the overall purification ability of the various test attributes *at the various tree test nodes*. (Also, remember that the overall information quantity of

a given test attribute selected at more than one test nodes equals the sum of its partial information quantities.)

On the other hand, the overall information quantity provided by a tree (totalling the partial ones provided by all the tree test nodes), as compared with that potentially contained in the learning set, allows to conveniently assess the classification purity provided by this tree.

2.3 Salient features of a tree

Size

The description of § 2.1 suggests that parameter α plays a fundamental role on the overall shape and properties of a tree. In general, its choice should be guided by the observation that too large a tree will yield a higher misclassification rate and hide relevant relationships among more or less random details, whereas too small a tree will not make use of some of the information available in the LS . Indeed, the terminal nodes of a too small tree have not been expanded enough and this prevents from getting purer subsets and the corresponding insight about the role that the attributes would have played in this expansion; a too large tree, on the other hand, results from the splitting of statistically unrepresentative subsets; therefore, it is likely to cause an increase in the misclassification rate when classifying states not belonging to the LS . Stated otherwise, a tradeoff appears between the two following sources of misclassification : *bias* (overlooking significant information in the LS) and *variance* (badly interpreting the randomness in the LS) : too large a tree will suffer from variance whereas too small a tree will present bias.

Effectiveness and its evaluation

The reliability of a tree is its ability to properly classify seen as well as unseen cases. Among the various criteria proposed to assess this or equivalently the misclassification rate, we mention the test sample estimate [12]. It is obtained by considering all M states belonging to the test set, dropping them through the tree, and computing the ratio of misclassified states ⁶ over M . This is the most reliable and unbiased estimate; it is easy to compute, and has low variance, provided that M is sufficiently large. Another advantage is that its variance can be easily estimated; therefrom, confidence intervals can be easily computed. Hence, in the sequel we will be using it as the benchmark for assessing accuracy (the reciprocal of misclassification rate).

Accuracy or *reliability* ⁷, together with size measured by its *complexity* ⁸, determine the *tree effectiveness*.

⁶In the sequel the terms "test set error rate", "misclassification rate", or "error probability" are used interchangeably.

⁷These two terms will be used interchangeably.

⁸Intuitively, complexity of a tree refers to its number of nodes. Depending on the formulation, it may take on more specific definitions.

2.4 Decision trees applied to preventive voltage stability. Principle

Whatever the specific power system security aspect, the principle of the decision tree method follows the same general pattern : *"for the security problem at hand, build DTs on the basis of "relevant" information, to subsequently assess any new, unseen operating state in terms of the values of its test attributes"*.

In the context of preventive voltage stability assessment the relevant information concerns :

- (i) relevant operating states, preclassified with respect to contingencies endangering voltage stability;
- (ii) a list of relevant candidate attributes, i.e. likely to drive the phenomena.

Information (i) is obtained by :

- collecting "precontingency" operating states; this is achieved by considering plausible scenarios of consumption-generation-topology schemes, and running a load flow program to generate the corresponding operating states; note that the plausible scenarios consider both normal and weakened system conditions;
- considering a relevant contingency (such as loss of generation, line tripping, etc.), and classifying each operating state with respect to this preassigned contingency. The sample of preclassified operating states thus obtained constitutes the "data base". Subsequently, it will be decomposed into learning and test sets.

The classification relies on the load flow convergence, in addition to voltage limits and to the sign of the CAR indicator (see in § 3.2.3).

Information (ii), concerned with parameters of the system likely to drive the phenomena, must be easily available to the operator, so as to make the use of DTs independent of any post-contingency simulation.

Human expertise is particularly helpful for appropriate choice of contingencies, precontingency operating states and candidate attributes altogether.

3 CASE STUDY

3.1 System description

The considered system is Brittany. This "peripheral" region of France characterized by lack of local generation under heavy load periods, has experienced voltage problems in the past [13].

The one-line diagram of Fig. 1 represents the 400 kV and 225 kV parts of the Brittany region of the French grid. The system thus modelled comprises 366 buses, 55 generators and 614 lines and transformers. The overall load of the base case of the study is of 61,000 MW; of them, 10,200 MW concern the West France and in particular 3,477 MW the Brittany area. The main power plant in this latter area is Cordemais (COR.P6/P7 in the figure); Flamanville (close to MENU7) and Chinon (close to DISTR7) are two other important power plants of the West of France. We also mention the existence of two gas turbines at the extreme West of Brittany (denoted BRENN6-G, DIRIN6-G in the figure).

3.2 Generating samples of instances

3.2.1 Precontingency operating conditions

They are generated from the base case load flow, by considering random changes of key parameters like topology and load, together with operating rules, as specified below.

• Topology.

40 % of the normal, "n" type of situations, and 60 % of the "n-1" type are considered; these latter concern loss of :

- one 400 kV circuit in Brittany
- one 400 kV circuit on tie-lines between Brittany and the remaining West region
- one 400/225 kV autotransformer
- two long 225 kV lines of Brittany.

• Load.

The base case active load (3,477 MW) is randomly modified within the range [2,500 3,900]. Its reactive power (1575 MVar, in the base case) is modified homothetically.

• Shunt compensation.

In the MV level it follows the load in order to keep a constant power factor, unless installed capacity hit (this occurs for the base case load).

In the HV level, five zones of compensation are considered; the amount of compensation is modified zone by zone, homothetically within a zone, according to the rule :

$$\text{total MVar} = \text{base case MVar} * \frac{\text{load}}{\text{base case load}} * v$$

v is a random number ranging in between 0.6 and 1 (0.6 results from trial and error, monitoring voltages and Q productions).

In the above, installed capacity limits are also taken into account.

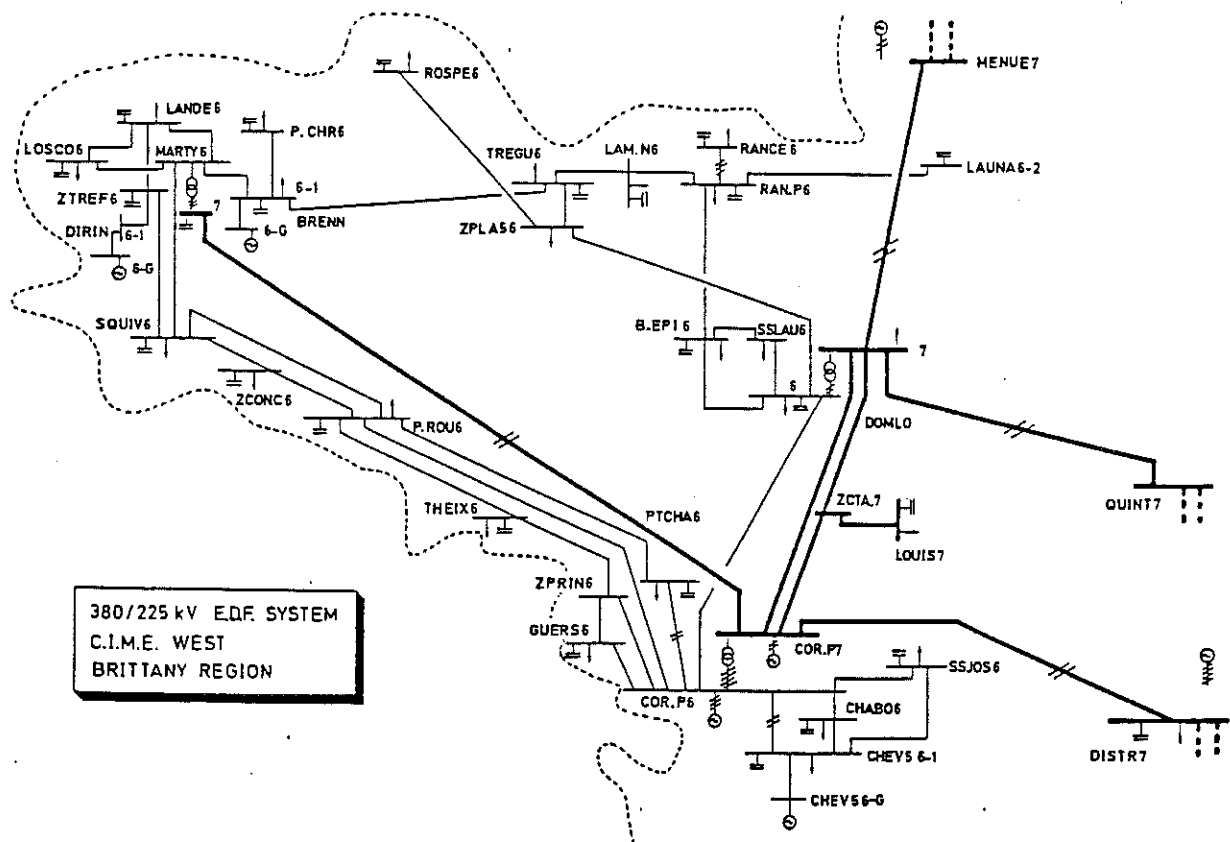


Figure 1: One line diagram of the Brittany region of the French system

- Active generation (re)scheduling.

It consists of reproducing various possible generation schemes while avoiding combinatorial explosion. More specifically,

- 3 sites (listed below) have their production varied
- in each site :
 - * consider all possible combinations of units
 - * except that two similar units are not differentiated, and that
 - * a min and max number of units is obeyed (min often = zero)
 - * a committed unit is set to its nominal power.

To fix ideas, this gives

- * 8 combinations at Cordemais
- * 5 combinations at Chinon
- * 3 combinations at Flamanville
- setting up the global production scheme :
 - * consider all combinations between sites
 - * but keep global production within min and max limits (load dependent)

(for example, this gives 63 production schemes between 2480 and 5680 MW);

- one of them is selected randomly
- the load/production imbalance is compensated by machines outside of Brittany.

- Reactive generation scheduling.

- fix V at PV buses
- make V vary randomly but within limits and constraints :
 - * define groups of generators (5 groups; includes more production sites than the active scheduling)
 - * all generators of the same group have their V changed in the same way
 - * a global V level is selected for all groups $V \in [0.98 \ 1.05]$
 - * add a random ΔV in each group; $\Delta V \in [-0.01 \ +0.01]$
(idea : avoid unrealistic Q power transfer between generators)
 - * this includes a synchronous condenser (CHEV56-G on Fig.1 is one group in itself)

- Computing a precontingency operating condition or state.

Given the above topology and/or load variation and generation rescheduling, the corresponding precontingency operating state is obtained from the base case by running a new load flow and verifying whether this program converges "fast enough" (within 30 iterations). If so, the provided solution will be considered to be a precontingency operating state provided that, in addition, conditions (ii) and (iii) of § 3.3.2 are also met.

Remark. The set of configurations which are generated may vary with the considered contingency. For example, if the latter concerns the loss of a given unit, the precontingency states must all have this unit in operation.

3.2.2 Contingencies

Three contingencies have been considered, namely :

1. Loss of Unit 4 at Cordemais (600 MW, 225 kV) (loss of generation located close to the load center)
2. Loss of 400 kV line between Domloup et Manuel by opening :
 - - one circuit
 - - two circuits.

3.2.3 Classification

In a two-class classification scheme, a precontingency operating state will be classified "secure" or "insecure" with respect to a given contingency : secure if, following this contingency, it still meets the three requirements :

- (i) the load flow converges "fast enough" (within 30 iterations)
- (ii) the CAR index ⁹ is non negative (the enforcement of the generator Q limits can make -in some rare cases- the load flow "jump" on an unstable solution)
- (iii) all voltages must be above limits at buses feeding nuclear plant auxiliaries.

Otherwise, it is classified insecure.

In a three-class classification scheme, a more refined assessment will be used to distinguish "vulnerable" and "alert" states among these found to be insecure : vulnerable, if the rapid action of gas turbines is able to drive back a state from insecure to secure, alert otherwise.

⁹for "Coefficient d'Acheminement du Réactif" in French : it assesses the ratio of the incremental total machine generation to the incremental load increase [14].

3.2.4 Learning and test sets

About 2000 preclassified instances (states) have been generated according to the above procedure. A certain amount, say $N = 1000$, will be used as the learning set to build a tree, the remaining, say $M = 1000$, will be used as the test set to appraise the tree ability to classify cases other than the learning ones. The choice of the learning and test states is random.

3.2.5 Candidate attributes

Their selection requires some previous knowledge of voltage problems in the area of concern. Some are load flow INPUT data; the others are OUTPUT results. Besides, some combined attributes are also considered, chosen so as to preserve simplicity.

A total of 101 candidate attributes have thus been proposed during the tree automatic construction, namely :

- (i) 21 voltages
- (ii) 8 dealing with load level and compensation levels
- (iii) 34 line power flows or cut-set power flows
- (iv) 13 dealing with generator production
- (v) 13 power flows through 400/225 kV transformers
- (vi) 12 dealing with reactive power reserves

We specify the following.

- (i) Voltage attributes are taken at 225 or 400 kV buses in Brittany or are generator V set-points
- (ii) these attributes concern :
 - total active load
 - total reactive load : taking MV compensation into account but not the HV compensation
 - amount of HV shunt compensation : in each of the 5 regions plus total in Brittany
- (iii) the power flow types of attributes concern :
 - active and reactive power flows at one end of each 400 kV line
 - the same for two "key" lines at the 225 kV level
 - the same but referred to a single circuit for double-circuit lines
 - active and reactive import of the Brittany region from the remaining of the French system (sum over three 400 kV lines)

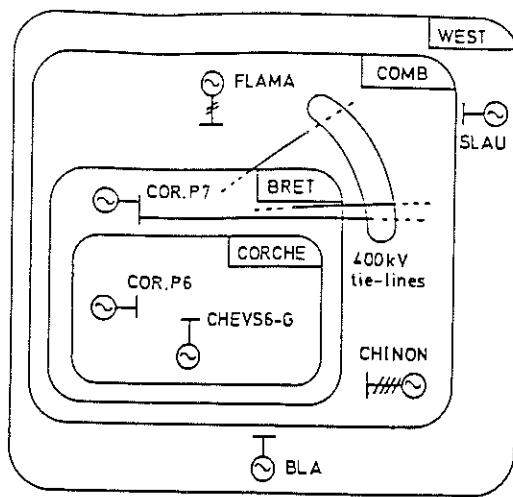


Figure 2: Nested sets of power plants

- the same but divided by the number of circuits in service
 - sum of active and reactive power flows over a cut-set including three "key" lines at 225 kV level
 - the same but divided by the number of circuits in service
- (iv)
- active power production of the plants which participate in the active generation scheduling
 - the gross (i.e. at machine LV bus) reactive production of the plants which participate in the reactive generation scheduling
- (v)
- active and reactive power flows through the auto-transformers :
COR.P7/LOR.P6 (see one-line diagram)
DOMLO7/DOMLO6
MARTY7/MARTY6
 - the same but divided by the number of transformers in parallel
 - sum of active and reactive power flowing from the 400 kV to the 225 kV level
- (vi) reactive reserve of one plant is taken as the reactive power capacity minus the (precontingency) reactive power production (the first term varies according to the unit commitment scheme). This takes into account the lost unit(s) (where applicable)
- individual reactive reserves
 - local reserves defined on nested sets of plants (see Fig. 2 - bus names refer to one-line diagram)
BRET = reserve in Brittany region
COMB = reserve over plants participating in the active power scheduling
WEST = reserve over plants participating in the active and reactive power scheduling

In addition to those using the above 101 simple attributes, other DTs have been built on the basis of 31 CAR indices at load buses of the Brittany region [15].

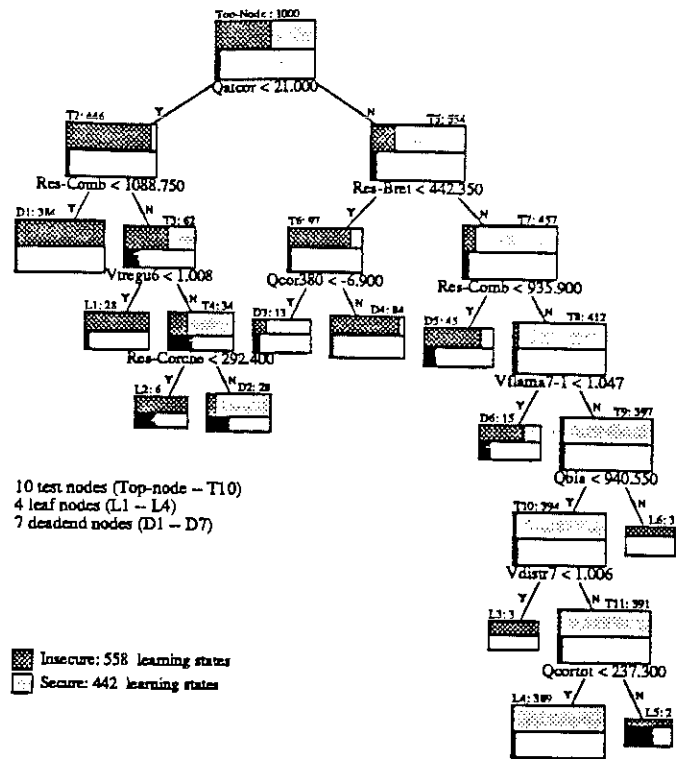


Figure 3. Contingency 1: loss of 600MW of generation at Cordemais
 $N = 1000$ $M = 1000$ $\alpha = 5 \times 10^{-5}$ $P_e = 5.3\%$

3.3 Sample of typical decision trees

3.3.1 Decision tree description

Let us first describe the notation, consistently used in the sequel, on the basis of Fig. 3 which portrays a representative two-class tree obtained for the contingency "loss of 600 MW generation in the Brittany region".

The caption specifies the overall number of states of the learning set (N) and of the test set (M), as well as the misclassification rate (P_e in %) assessed via this latter. Also listed is the number of secure and insecure learning states, as defined in § 3.2.3. Another information given in the caption concerns the number of test nodes (labelled T) and terminal nodes, subdivided into leaves (L) and deadends (D); their sum gives the total number of the tree nodes.

Other characteristics are specified in the design of the tree itself. Thus, each node is represented by a box. Above the box appears the node name, labelled T , D or L as appropriate, along with the number of its learning states. The node box itself is subdivided into upper and lower parts¹⁰; this latter is further subdivided into black and white areas, proportional to the incorrectly vs. correctly classified test states. Finally, below each test node is indicated the *test attribute*¹¹ along with its threshold value of

¹⁰The relative height of the upper and lower parts is proportional to the relative size of the learning and test sets corresponding to this node.

¹¹A test attribute is a candidate attribute selected by the automatic tree building.

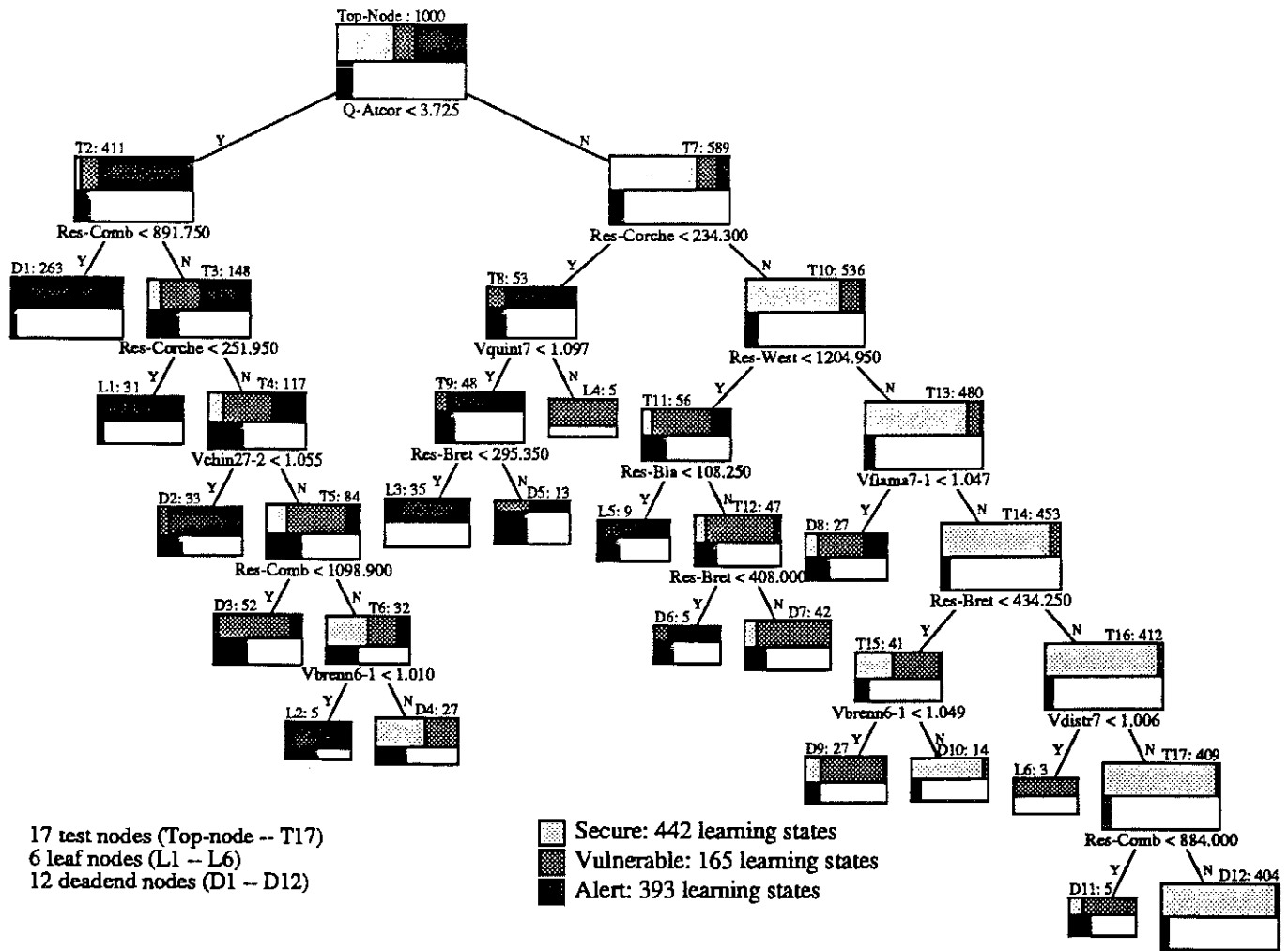


Figure 6. Contingency 1 : 3-class decision tree
 $N = 1000$ $M = 1000$ $\alpha = 5 \times 10^{-5}$ $Pe = 12.6\%$

sum of the partial quantities obtained at the two nodes.

The total information quantity provided by the tree is obtained by summing up the above partial quantities and amounts to 818.8. On the other hand, the total information quantity contained in the learning set is calculated via eq.(4) (replace S by LS) and amounts to 990.3. Hence, the tree classification succeeds in decreasing the impurity (entropy) by 82.7 %.

A further interesting exploratory tool is provided by "masking" an important test attribute. The tree of Fig. 5 helps to describe and illustrate this "masking effect". It concerns the attribute "Qatcor" of Fig. 3 : eliminating it from the list of candidate attributes leads to the automatic selection at the top test of the attribute "Res-Comb". Figure 5 portrays the obtained tree under the otherwise same conditions as for tree of Fig. 3. Observe that masking Qatcor hardly affects the overall tree accuracy. The explanation is given by the scores (normalized information gain expressed by eq.(6)) obtained by the test attribute at the top node of the tree of Fig. 3. We find Qatcor : 0.3856 ; Res-Comb : 0.3830 , i.e. very similar scores. Another interesting comparison concerns the information quantity

provided by the higher (top) test nodes of each tree. Thus :

- for the tree of Fig. 3 : Qatcor : 382.1 ; Res-Comb : 154.6 ; Res-Bret : 134.3 ; total information quantity provided by the tree : 818.8 ;
- for the tree of Fig. 5 : Res-Comb : 378.1 ; Res-Bret : 188.0 ; Res-Bla : 99.1 : total information quantity provided by the tree : 818.4 .

Hence, from the viewpoint of their total information gain both trees appear to be almost equivalent.

3.3.3 A three-class decision tree

The tree of Fig. 6 has been constructed under the same conditions as that of Fig. 3, but on the basis of the three-class classification defined in § 3.2.3. Observe that complexity as well as misclassification error rate are both higher for the three-class tree. This misclassification increase is however more apparent than real; indeed, the errors of the three-class tree mostly concern adjacent classes

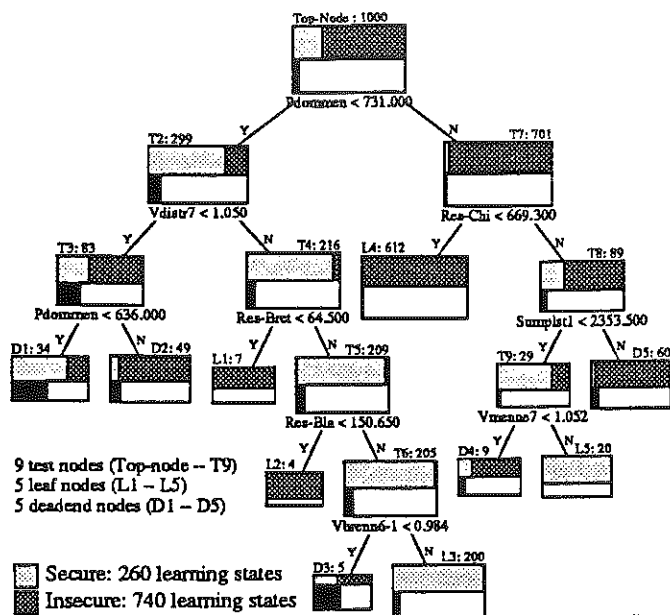


Figure 7. Contingency 2 : loss of 2 circuits Domloup-Mennel
 $N = 1000$ $M = 1000$ $\alpha = 5 \times 10^{-5}$ $Pe = 5.2\%$

(alert vs. vulnerable or vulnerable vs. secure) : the security diagnostic is therefore much less harmful than in the two-class tree.

An interesting advantage of three-class over two-class trees is the insight they allow on possible corrective actions (such as the gas turbines considered here).

As expected, most of the test attributes selected in the tree of Fig. 3 appear also in Fig. 6.

3.3.4 Other contingencies

Figures 7 and 8 are samples of two-class trees obtained for the contingency #2 defined in § 3.2.2. The larger complexity exhibited by the less severe contingency might be explained by the fact that the security boundaries become less neat; this makes the security diagnostic less clear.

4 CONCLUSION

The objective pursued in this paper was to explore the feasibility of the decision tree method as an indicator of preventive voltage stability. The results reported go beyond this feasibility objective : trees obtained on a real world power system have shown to be simple, straightforward, reliable and flexible. They are able to provide various types of information about the phenomena of concern, and suitable for on-line diagnostic; even more important is their ability to suggest remedial actions, i.e. to give a sample of possible practical answers to the "what if" question, whenever it arises.

Admittedly, a good deal of investigations is still needed to make the method truly practicable. In particular, research efforts should be devoted to a systematic exploration of tree parameters, to the data base generation, to the choice of suitable types of attributes, to classification aspects. The results obtained so far suggest that these efforts are worthwhile.

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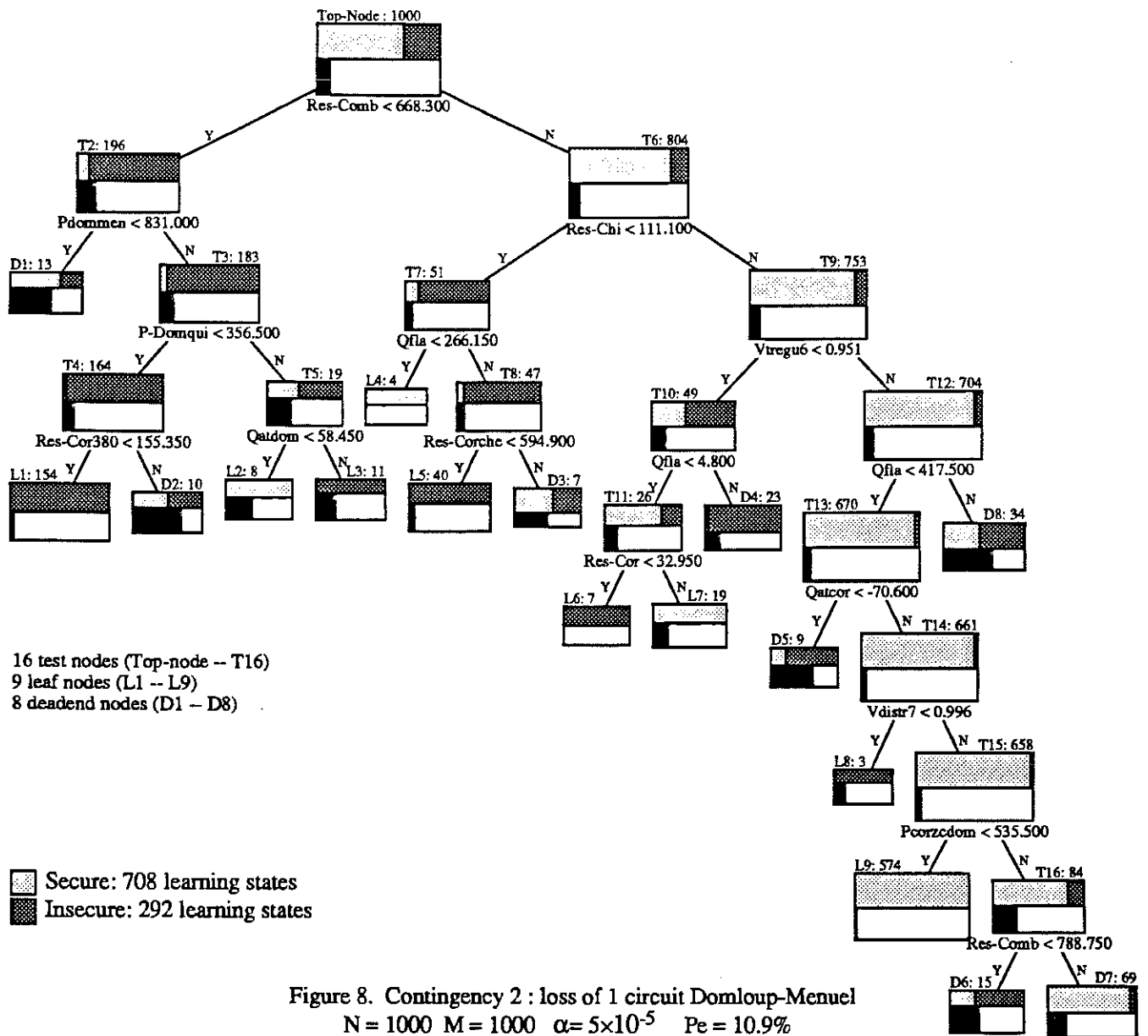


Figure 8. Contingency 2 : loss of 1 circuit Domloup-Manuel
N = 1000 M = 1000 $\alpha = 5 \times 10^{-5}$ $P_e = 10.9\%$

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DECISION TREES FOR DETECTING EMERGENCY VOLTAGE CONDITIONS

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Abstract. Many voltage collapse incidents throughout the world have originated from severe disturbances. While of paramount importance, preventive security analysis against these incidents has some limitations, especially when dealing with multiple contingencies. Hence it needs to be complemented by adequate emergency control. An essential characteristic is that, in most cases, following a system disturbance, voltage instability takes some time to develop. Hence a short time interval is left to initiate counter-measures, provided that the decrease in security is detected soon enough and the corrective action is well prepared. The early detection of critical states, i.e. system states likely to evolve towards instability is essential. To this purpose, however there is a need nowadays to develop fast and reliable indicators, which only involve simple calculations. The Decision Tree (DT) method offers a promising possibility to detect the above critical states. This artificial intelligence method belongs to the family of "learning-by-examples" methods: DTs automatically built on the basis of a "learning set" can subsequently be used to assess unseen situations. Within the particular context of critical state detection, we propose to elaborate learning sets by simulating a collection of system states resulting from various operating conditions and various disturbances, further classified into stable and unstable. The candidate attributes are easily observable quantities which characterize the system just after the disturbance. An academic type example, based on an 8-bus test system is thoroughly examined, in order to illustrate the proposed method and point out its potential and specifics.

Keywords. Voltage stability, emergency control, real-time indicator, artificial intelligence, decision tree method.

1 Introduction

Voltage stability is more and more often quoted as a limiting factor to the secure operation of modern power systems [1].

There are basically two types of incidents likely to endanger the voltage stability of a power system.

The first one is a load increase, which cannot be faced with the available generation and compensation equipments. This may result from an unexpectedly large load increase and/or a previous weakening of the system.

The second one is a major incident affecting the generation or transmission system in such a significant way that the pre-fault load level cannot be satisfied any longer. The main actors of this catastrophe are the transformer On-Load Tap Changers (OLTCs), the machine protecting devices bringing back the reactive power production within limits and those resulting incidents such as undervoltage generator trippings. All of them contribute to a dramatic voltage decline taking place within up to a few minutes after the triggering event [2].

Voltage Collapse Proximity Indicators (VCPIs) have been devised to deal with the first risk [1, 3]. These indicators rank from simple sensitivity calculations [4] to more elaborate margin determination. Among indicators able to anticipate the effects of generator reactive power limitations and to identify the weak points of the system let us mention that proposed in [5].

If properly devised and regularly monitored, VCPIs can help the operators of a control center to anticipate the effects of an unacceptable load increase. Moreover, this anticipation capability along with the inherent delay in the load increases give the operators some time to take a remedial action (such as switching in capacitors, changing generator voltage set-points, starting fast units, etc.).

For voltage instabilities following major incidents, the picture is quite different. Indeed, there is a comparatively much smaller time left to take corrective actions (such as tap changers' blocking and in the last resort load shedding). This time appears even shorter considering the pressure put on the operators in such circumstances... However, although small, this time interval exists in most cases, thanks to the delayed action of the OLTCs and the delayed reductions in the machine excitation levels. This contrasts with transient angle stability for instance whose fast phenomena fall well below the time scope of a control center decision.

The above considerations motivate both the preventive and the curative analysis and control of the voltage security.

Preventive control involves contingency evaluation. Measures of the system robustness also play an important role

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within this context [6]. The preventive analysis of voltage stability with respect to contingencies is dealt with in a companion paper, Reference [7], using a similar decision tree methodology, in order to obtain stability limits in terms of the pre-contingency values of easily observable quantities. However, although essential, preventive security analysis is inherently limited to a list of "credible" contingencies. Even if prefiltering methods allow to sweep a large number of them, it is rarely acceptable from an economical viewpoint (or even possible) to operate a power system in such a way that it can stand more than single or a few double contingencies. Now, many experienced voltage collapses resulted from more than a single triggering incident and there has been very often "at least one event which should never occur" [8]!

Therefore, an adequate emergency control is needed to face such events. However, relatively few results have been obtained so far, with the exception of the work reported in [9] and the particular schemes implemented by some companies. Among them, the blocking of OLTCs is more and more widely used but it is recognized that there is a need for a more satisfactory triggering criterion than the usual one.

As already quoted, some emergency control is possible thanks to the "slow" dynamics of the phenomena. It must be emphasized however that time is critical. Therefore, the successful salvaging of a collapsing system requires : (i) the early detection of a *critical state*, i.e. a state which is going to evolve towards instability if no action is taken; (ii) a good (off-line) preparation of the corrective actions, and (iii) an automatic -to the extent possible- handling of the latter. This paper mainly focuses on the first aspect, i.e. the fast detection of critical states, a prerequisite step of any emergency control of voltage stability.

Because of the time constraint, what is needed is a simple "indicator", i.e. a one which would not involve elaborate calculations but would rather rely on quantities readily available in a control center from real-time measurements. However, devising a reasonably reliable indicator of emergency conditions while still preserving simplicity is a real challenge. Indeed, the detection of critical states is made difficult by the complexity of the phenomena (non-linearities, discontinuities, etc.), by the a priori large number of parameters influencing the system voltages, by the wide-range of possible operating conditions and, last but not least, by the important consequences of the decisions to be taken therefrom.

These constraints have motivated the use by the authors of the so-called Decision Tree (DT) methodology to obtain the sought indicators. This artificial intelligence method [10] belongs to the family of "learning-by-examples" methods. Generally speaking, it is particularly well suited to the automatic identification of the boundary between two (or more) classes of states in a multi-dimensional, complex space of "attributes". Specifically, in our problem, we propose to use it in order to find the boundary between critical and non-critical states in the space of the readily available, real-time measured quantities which characterize the system just after a (major) disturbance. The DT method

relies on an off-line elaborated learning set out of which it automatically builds decision trees. The latter only involve simple tests on the attributes and are therefore easy to use in real-time to assess unseen situations.

This methodology has been applied to other power system security problems : successfully to the preventive analysis of transient angle stability [11], promisingly to the preventive analysis of voltage stability [7].

This paper is organized as follows. Section 2 deals with voltage phenomena and states the problem to be solved. Section 3 applies the DT method to the problem of concern. The procedure to follow is then explained step-by-step on a simple example, in Section 4, while the corresponding obtained results are discussed in Section 5. The paper ends with concluding remarks.

Please note that the fundamentals of the DT method and the related notation used in the sequel are briefly described in Section 2 of the companion paper of Ref. [7] and are not recalled here.

2 Critical state detection

This section describes the problem to be solved from a physical viewpoint. In particular, it defines the basic system states on which the off-line building of the DTs will rely.

Unless otherwise specified, all descriptions refer to a voltage instability following a major incident, generally affecting the EHV transmission and/or the generation system. It is referred to in the sequel as the "disturbance".

2.1 Voltage phenomena and system states

Basically the system is viewed as going through three states :

- the *pre-disturbance state* is the set of pre-existing operating conditions. For the sake of simplicity, the system is supposed to be at the equilibrium;
- the *state "Just After Disturbance" (JAD)* characterizes the intermediate, "short-term", equilibrium reached after electromechanical transients have died out, i.e. roughly speaking from 10 to 20 seconds after the disturbance inception. At that time, the primary frequency control has acted as well as the fastest protecting devices reducing machine excitation levels. On the other hand, neither the delayed OLTCs nor the slower machine protections (including possibly plant operators) have acted yet on the voltage profiles. In this state, the system is at the equilibrium from the viewpoint of its fast-acting part, i.e. machines, regulators, compensators, etc. but not from the viewpoint of the OLTCs which are going to respond to the deviations in sub-transmission and distribution voltages. This state is much influenced by the behaviour of loads with voltage. The disturbances generally result in a depressed voltage and hence a lower

demand level (except for the reactive demand of some strongly compensated loads);

- the *post-disturbance state* is the “mid-term” state reached by the system when it is voltage stable. All loads fed through automatic OLTCs are restored close to their pre-disturbance level. The impossibility to restore this level owing to the system weakening and the adverse effect of the OLTCs is precisely the very mechanism of voltage collapse.

Clearly, in practice, the JAD state might not exist or the system might not succeed in reaching it, which would denote either a transient angle instability (loss of synchronism) or a transient voltage instability (caused e.g. by stringent, fast load dynamics). Both problems fall below the scope of a centralized action (the second problem requires for instance fast, local compensation means like SVCs) and hence are not handled by the emergency control approach described in this paper.

Similarly, the post-disturbance state could not be viable in the long-term, for instance because of line overloads, but it is supposed here that either time is left to the operators or other emergency tools exist to deal with this problem. Note however that the reaching of a post-disturbance state which is stable but fragile from the voltage viewpoint should be rightfully considered as an emergency situation as well. Cases falling into this category are for instance :

- when insufficient post-contingency reactive power margins are left to the system,
- or when the latter exhibits high VCPI values;
- when the computed post-contingency voltages of some generators are below their admissible threshold values, with the associated risk of generator tripping.

2.2 Problem statement

As quoted in the introduction, the final sought objective is to depict whether, after a disturbance, the system is in a critical state, in order to fast trigger an adequate remedial action. More precisely stated :

A fast indicator is sought, to be used when the system has experienced a disturbance and is in the corresponding JAD state, in order to detect whether this state is going to evolve towards instability. This indicator should be simple enough and should preferably rely on readily available real-time measurements, relative to the JAD state of concern.

2.3 Discussion

The above statement stresses upon several requirements of our problem, which we believe the DT methodology is particularly well suited to meet. These requirements are briefly discussed hereafter.

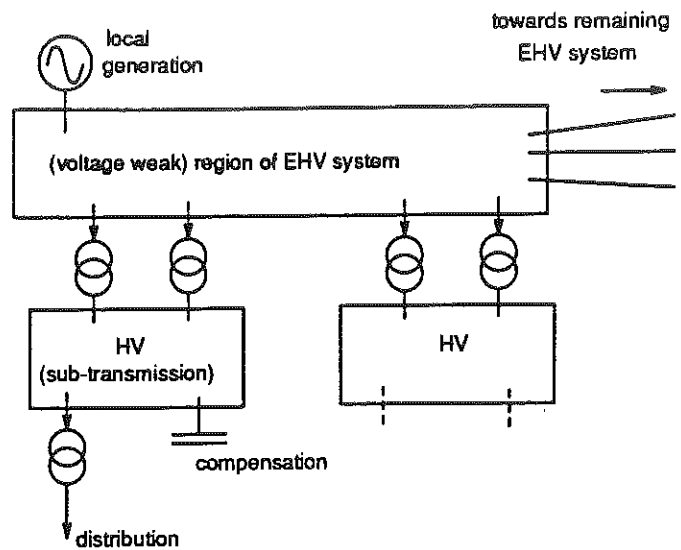


Figure 1: General structure of an EHV/HV system

For already mentioned reasons, time is critical in this problem and the sought critical state indicator should be kept simple and fast to use. This implies for instance that :

1. *No elaborate power system calculations should be involved.* Indeed, presently available simulation tools (time integration or simplified simulation) cannot be run in such a stringent real-time context. As is discussed in Ref. [7], one important feature of the DT method is precisely that all heavy computations are performed off-line in the learning phase. More specifically, one could argue that, by running a load flow with the system in its post-disturbance configuration and the injections set at their pre-disturbance level, most critical cases could be diagnosed through the divergence of the load flow algorithm. However, this calculation would require to first run a state estimation and to retrieve the values of the bus injections, as estimated before the disturbance occurred. In any case, this scheme would be much more time consuming and complex to implement than the DT solution proposed in this paper. Moreover, it would only give an isolated, yes/no type diagnosis while DTs can offer a global, explicit although concise characterization of the critical states. Finally, DTs can take advantage of more refined (off-line) diagnosis tools than just a load flow as suggested above.
2. *The sought indicators should rely on quantities usually measured from a control center.* This involves primarily voltage magnitudes, active and reactive power flows and bus injections as well as simple functions of them such as generator reactive power reserves, amounts of shunt compensation, interface flows, etc. A question of practical interest is from how low in voltage measurements should be considered. Generally speaking, bulk power voltage security is the concern of an (at least) regional control center, mainly in charge of monitoring the EHV grid and hence having a relatively small need of information from the HV subtransmission levels (Figure 1). This is the case for the Brittany region of the French

system on which the method is currently being tested [7, 12, 13], and it is likely to be so for most systems. Few information from the HV levels should therefore be required by the sought indicators. Two exceptions : the amount of HV shunt compensation and the measurements around the EHV/HV transformers equipped with automatic OLTCs will be used. In particular, voltages at the HV extremity of the EHV/HV transformers are likely to play an important role, as explained later on.

3. *The sought indicators should preferably use data relative to the JAD state of concern and preferably not rely on pre-disturbance data.* These requirements go along the line of the simplicity of implementation. Similarly but perhaps more critically, the indicators should not require the explicit nature and location of the disturbance. These aspects are further discussed in Section 3.

3 Generating samples of classified states and building the DTs

Both the learning and the test phase of the DT method require to generate representative states and to subsequently classify the latter (into critical or non critical) [7,10,11]. This section is devoted to these aspects.

3.1 The overall procedure

Figure 2 gives a diagrammatic overview of the procedure proposed for an emergency condition detection problem.

The procedure starts from a base case, a set of standard load flow data originating generally from planning or operational planning studies.

A relatively large number of variants are then generated, which are representative of possible system operating conditions. Roughly speaking, these variants are obtained by introducing random modifications in the base case situation. This mainly involves :

- variations in the load level and distribution
- changes in network topology
- corresponding rescheduling of the active generation
- various reactive power generation schemes
- as well as variations in the amount of (shunt) compensation.

Each variant is treated by a standard load flow to obtain a corresponding pre-disturbance operating point. This load flow uses a constant power load representation. If the HV levels are represented explicitly the load flow has to adjust the ratios of the EHV/HV transformers equipped with OLTCs to satisfy their HV voltage control rule, since a pre-disturbance state is supposed to correspond to a complete equilibrium of the system. In the course of running this load flow, divergence of the algorithm can be met because

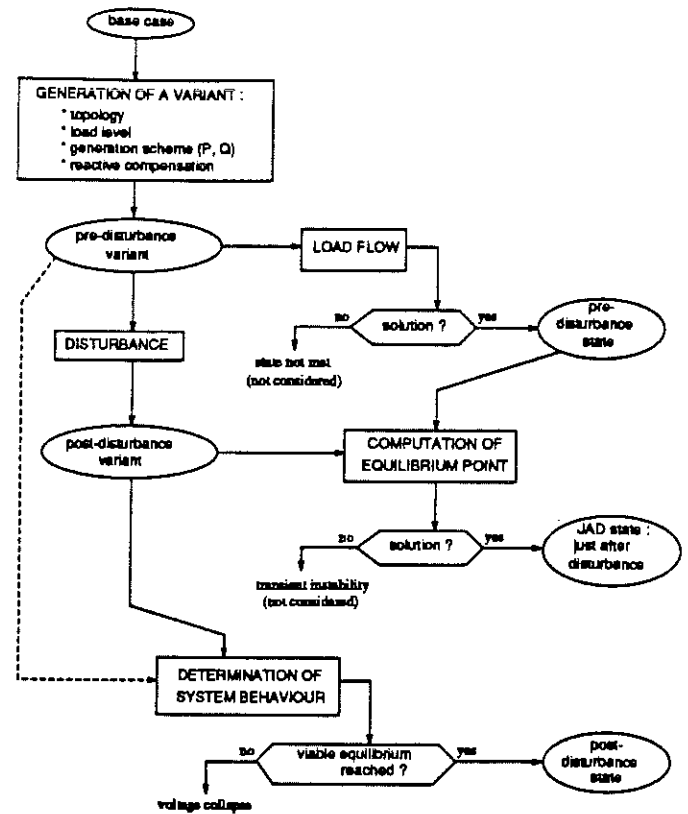


Figure 2: The overall procedure.

the generated data already correspond to voltage collapse : the variant is then simply discarded as corresponding to a pre-disturbance state which could not exist.

Next, disturbances are imposed to the system : the latter are outages of EHV transmission and/or generation equipments which are relevant to the voltage stability problem of concern. For each contingency case, the JAD state is determined, by means of a system equilibrium point calculation. This requires a load flow modified so as to :

- take into account the voltage dependence of the load active and reactive powers. The latter dependence indeed strongly influences the JAD state. The simplest, widely used modelling is

$$P = P_0 \left(\frac{V}{V_0} \right)^\alpha \quad Q = Q_0 \left(\frac{V}{V_0} \right)^\beta \quad (1)$$

where (P_0, Q_0, V_0) refer to the corresponding pre-disturbance state and (P, Q, V) to the JAD state;

- take into account generation rescheduling after primary regulation;
- tolerate machine overloads or impose excitation limitations, as the case may be (taking time into account).

Again, one might theoretically fail to find the above equilibrium point, especially when the load modelling of Eq.(1) uses α and β exponents lower than 2. This case requires a careful checking of the load model validity, as it would

normally correspond to a transient (i.e. fast) voltage instability. If so, as already mentioned, these cases are out of scope of the emergency detection which we are dealing with and therefore they are ignored in the sequel.

Finally, an analysis is made of whether each JAD state is critical or not. This classification can rely on various analysis tools, ranging from a load flow to a full mid-to-long-term dynamic simulation. The choice depends on the required refinement of diagnosis but it must be emphasized that the proposed method accepts a simple load flow as well. In this case, loads have to be represented as constant power injections, set at their pre-disturbance values, while imposing the outage(s). The divergence of the algorithm indicates that the JAD state is critical, for the pre-disturbance load level cannot be restored, and conversely. As already mentioned, one can require the post-disturbance state to pass some more tests before classifying the case as "stable". On the other hand, a full dynamic simulation - as is used for the simple example of the next section - potentially offers more information than just a yes/no type of answer. For instance, it would allow to classify the unstable cases according to some severity scale, e.g. by observing the time left to take a corrective action, an important notion for this application of the DTs.

3.2 Building up and testing the DTs

At this stage, the set of JAD states is divided into two parts; one for learning, one for testing purposes. In addition, let us recall that all the JAD states have been classified critical or non-critical, according to the above described procedure.

Everything is ready for the automatic building of the DTs. To this purpose, a list of candidate attributes is drawn up; the latter are variables whose values are *directly or easily obtained from the JAD states*. The DT testing obviously uses the same variables.

3.3 Discussion

The method described in this paper significantly departs both from the existing approaches of the voltage security problems and from the applications of the DT methodology to preventive security problems, as discussed hereafter.

Like transient (angle) stability, the voltage stability of a power system following a major incident depends : (i) on the pre-disturbance operating conditions as well as (ii) on the disturbance nature, location, importance, etc.

As far as *preventive* security is concerned, the attributes treated by the DT method originate necessarily from the system *pre-disturbance* states. Hence, it is required to specify the disturbance of concern when considering a DT. Alternatively, in practice, a DT can be built for a set of related disturbances or the DT can involve the disturbance as an attribute.

The present *emergency* detection problem departs from this in the sense that we seek to obtain DTs valid for a large

number of contingencies. This is made possible by the fact that the learning (or test) set is composed of scenarios combining various pre-disturbance operating conditions *as well as various contingencies*. In other words, the used attributes implicitly reflect the disturbance undergone by the system; the DT method appears in fact as a clever way to extract a characterization of the critical states out of the huge and complex information contained in the simulated JAD states.

Whether it will be possible to effectively treat a large number of disturbances through the *same* DT is still an open question which receives much attention in the ongoing applications to real-life systems. One may conjecture that several DTs will be needed to cover a whole region in order, among others, to preserve both the simplicity and the accuracy of each DT, e.g. by avoiding to mix "non-interacting" phenomena. In any case, the comforting fact is that the information on the disturbance exists in the used JAD attributes.

Also, the simple example of the next section allows to reasonably hope that attributes from the pre-disturbance states are not required to reach the objective. This and the absence of an explicit description of the disturbance satisfies the practical requirements outlined in Section 2.3.

Another aspect is the generation of a representative sample of instances, a necessary condition to the success of the method. There is no general rule guaranteeing the appropriateness of a data base but the following guidelines may be of help :

- the data base should cover all foreseen plausible situations;
- as well some "abnormal" ones (owing to the possibility of a previous system weakening);
- however one should avoid biases resulting from too many odd situations;
- the objective is *not* to generate all the possible combinations of the parameters influencing voltage stability but rather to obtain a good *sample* of this huge set of combinations;
- this holds in particular for the disturbances : we do not intend to apply each disturbance to each pre-disturbance state. Rather, we intend to obtain a representative example of the huge set of possible JAD states;
- obtain the test states independently from the learning ones; consider a relatively large number of them to reliably evaluate the DT.

However, our experience shows that generating a "good" data base is not so difficult after all : thanks to its robustness, the tree itself is able to detect anomalies and to suggest corrections. Often, as overlooked states are undersampled and insufficiently influence the DT building, omissions are revealed when testing the tree, by abnormal, isolated errors. From the latter, the designer can get his own expertise quite soon.

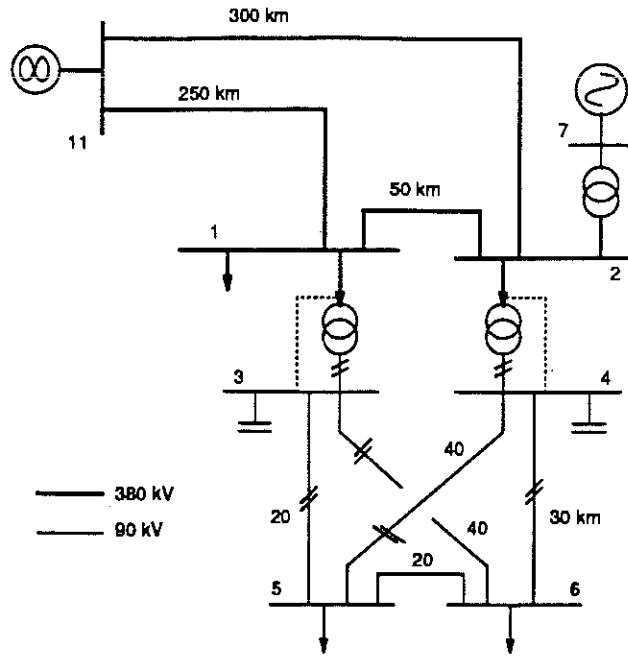


Figure 3: One-line diagram of the test system.

4 A detailed illustrative example

This section is devoted to the treatment of a simple 8-bus system. The objective is to illustrate in detail the proposed method; it is certainly not to draw premature conclusions about its success when applied to real-life systems, in spite of the very promising results already obtained.

Note however that if this simple example is free from the complexity linked to the real system sizes, on the other hand, as far as voltage phenomena are concerned, it has all realistic ingredients which contribute to make the critical state detection a difficult problem!

4.1 The system

The one-line diagram of the simulated system is shown on Figure 3. This system is basically made up of a "load center" (buses 1 to 6) fed to the great extent by an infinite bus (bus 11) through two relatively long EHV (380 kV) lines. Some active and reactive power support is however provided close to the loads by the plant located at bus 7. The latter is made up of up to three identical units.

Among the three loads, one is connected to the 380 kV grid directly (bus 1) while the other two are connected to a HV (90 kV) sub-transmission system. The latter is also equipped with compensation capacitors at buses 3 and 4.

Sub-transmission voltages are controlled at buses 3 and 4 by the automatic OLTCs of the EHV/HV transformers 1-3 and 2-4, respectively.

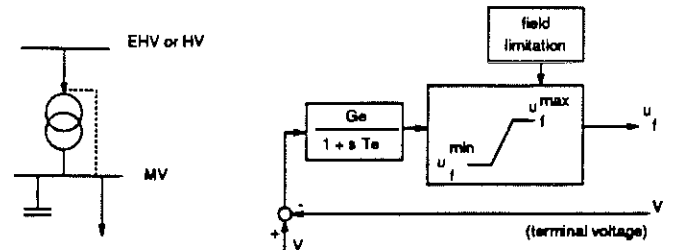


Fig. 4: Load modelling

Fig. 5: AVR modelling

4.2 Dynamic behaviour of the components

The system behaviour has been obtained by numerically integrating the differential-algebraic equations describing the system trajectory after the disturbance is imposed.

Each load is represented as shown on Figure 4. The load itself, voltage dependent as in (1), is compensated by a capacitor connected to the same MV bus. The voltage of the latter is controlled by the automatic OLTC of the (EHV/MV or HV/MV) transformer, whose leakage reactance is taken into account.

Table 1 hereafter summarizes the main characteristics of the loads, which, as can be seen, differ from one load to another.

Table 1 - Load characteristics

bus	α	β	Base case power factor ($\cos \phi$)			V_2^0 set-point
			without capacitor	with capacitor	including transformer	
1	1	1	0.9	0.96	0.93	1.0
5	1	1	0.9	0.95	0.92	1.0
6	2	2	0.9	0.95	0.92	1.0

The two EHV/HV transformers as well as the three ones feeding the loads are thus equipped with automatic OLTCs. When a disturbance causes the secondary (low) voltage V_2 to deviate by more than a tolerance ϵ from its set-point value V_2^0 , the OLTC reacts, but after an initial dead-time τ_1 only. It then changes the corresponding ratio by $\pm \Delta r$ each τ_2 seconds until V_2 approaches V_2^0 by less than ϵ , which only occurs in stable cases. The EHV/HV transformers react earlier ($\tau_1 = 30$ s, $\tau_2 = 10$ s) than the other transformers ($\tau_1 = 60$ s, $\tau_2 = 10$ s).

The (equivalent) machine is represented by a third order model describing the rotor motion and field flux decay. The Automatic Voltage Regulator (AVR) is represented by the simple first order model of Figure 5. In addition a rotor protection is simply modelled which, once u_f has exceeded a threshold value u_f^{per} for more than 40 s, causes u_f^{max} (and hence u_f) to be decreased to u_f^{per} . This threshold value corresponds to the maximum permanent admissible excitation level and is obtained from the machine capability curves.

4.3 The pre-disturbance operating conditions

In a first step, a set of 500 representative pre-disturbance states has been generated. To this purpose, variants are first obtained by randomly choosing the values of a few key quantities within pre-specified intervals. Uniform a priori distributions have been supposed for the sake of simplicity but for real-life systems such random variations should be combined with system operating rules.

More precisely, each variant has been obtained in the following way :

- the infinite bus voltage has been varied between 1.02 and 1.10 pu;
- the active power generation at bus 7 has been taken between 0 and 350 MW. From one to three identical (133 MVA - 113 MW) units are then committed, in order to just cover the above generation level;
- the reactive power generation at bus 7 has been taken between a lower and an upper bound, equal respectively to the absorption (-20 MVar) and production (+ 64 MVar) capability of one unit multiplied by the number of committed units. The upper bound yields $u_j^{per} = 2.64$ pu for the equivalent machine;
- the total active load has been varied randomly between 900 and 1350 MW. A random fraction p_1 is taken by bus 1, p_5 by bus 5 and $(1 - p_1 - p_5)$ by bus 9. The power factor of each load also varies between two bounds. The choices are summarized in Table 2 below.
- Finally, the number of capacitor banks (50 MVar each) at buses 3 and 4 is varied randomly between two and six.

Table 2 - Bounds on random load variations

bus	P_{min}	P_{max}	power factor (at EHV or HV bus)	
			min	max
1	0.28	0.38	0.912	0.944
5	0.28	0.38	0.902	0.935
9	0.24	0.44	0.902	0.935

These data are treated by a standard load flow which adjusts the ratios of transformers 1-3 and 2-4 to satisfy a 1.05 ± 0.01 pu voltage at bus 3 and 1.09 ± 0.01 pu at bus 4. Cases of divergence are discarded. Moreover, bus 7 is treated as a PQ-bus, hence yielding V_7 as a result. The latter (machine bus) voltage is required to fall within a $[0.95 \ 1.05]$ pu interval, which further filters out the pre-disturbance variants.

4.4 The disturbances and the corresponding JAD states

Five disturbances have been applied to the system, which are listed in Table 3 below. About the 3rd and 4th disturbances, note that the loss of 2 units amounts to the loss of a single unit, if a single one is in service in the corresponding configuration. The same holds for 3 units.

Table 3 - Overview of disturbances (2500 states)

#	nature	nb. of stable cases	nb. of unstable cases		
			with final equil.	no final equil.	total
1	tripping of line 11-1	159	167	174	341
2	tripping of line 11-2	360	110	30	140
3	loss of 2 units at bus 7	500	0	0	0
4	loss of 3 units at bus 7	500	0	0	0
5	tripping of line 11-2 and of 2 units at bus 7	74	375	51	426
		1593			907

In all cases, the JAD state is obtained by taking a snapshot of the system at time $t = 20$ s. At that time, electromechanical transients have completely died out while the OLTCs and the machine protection have not acted yet. It must be emphasized that the latter two facts can make the EHV voltages (at buses 1 and 2) appear normal and hence make it more difficult to detect whether the JAD state is critical or not.

4.5 Classification of the JAD states

Up to five minutes of the system behaviour have been simulated (such a long time is required in some cases to reach a post-disturbance, voltage stable equilibrium).

A case is labelled stable if all voltages controlled by OLTCs are successfully brought back at their set-point values (within the ϵ tolerance).

Instability on the other hand can take on various forms from the simulation viewpoint :

- either the system finally installs itself at the equilibrium but the voltage profile is dramatically low, the controlled secondary voltages have been moved away by a reversed action of the OLTCs and the loads are obviously not restored. The final settling down only results from the tap hitting their limits;
- or a dynamic instability has occurred which manifests itself by voltages plunging down to zero, by the machine going out of step (insufficient synchronizing torque) or by a singularity (caused by the instantaneous response of the loads). In such a case, the 5-minute horizon is not reached.

The system stability is very much dependent on the pre-disturbance state as well as on the disturbance. Table 3 summarizes the effects of the five disturbances. As can be seen, for instance, the loss of line 11-1 leads rather to instability and the loss of line 11-2 rather to stability. The difference lies in the fact that the former is shorter than the latter. Also, disturbances 3 and 4 are mild; note however that this information will enrich the learning set used by the DT method.

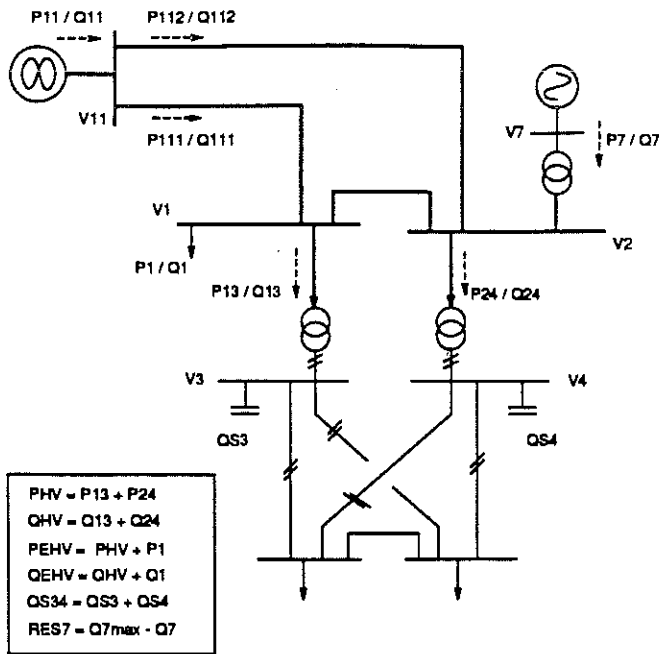


Figure 6: Attributes for the DT building

4.6 The attributes

The basic set of attributes proposed to the DT building algorithm is made up of 28 quantities, which can be decomposed into :

- (i) 22 simple quantities : the latter appear on the one-line diagram of Figure 6; they involve voltage magnitudes, active and reactive power flows and injections;
- (ii) 6 combinations or simple functions of the above ones : they are listed at the bottom of Figure 6. Among them, RES7 is going to play an important role : this is the reactive power reserve available on the machine of bus 7. Note that this reserve refers to the permanent admissible reactive limit and hence can exhibit negative values owing to the overload capability of the machine, at the time the values of the attributes are taken ($t = 20\text{ s} < 40\text{ s}$ duration of field forcing).

Note also that, except for the V3 and V4 voltages, no attribute is taken on the HV part of the system, in order to meet Requirement 2 of § 2.3.

4.7 The learning and test sets

Considering the 5 disturbances imposed to the system and the 500 available pre-disturbance states, 2500 JAD states are then available. The latter are randomly decomposed into 1250 learning states and 1250 test states. Among the former, 454 are critical and 796 non-critical.

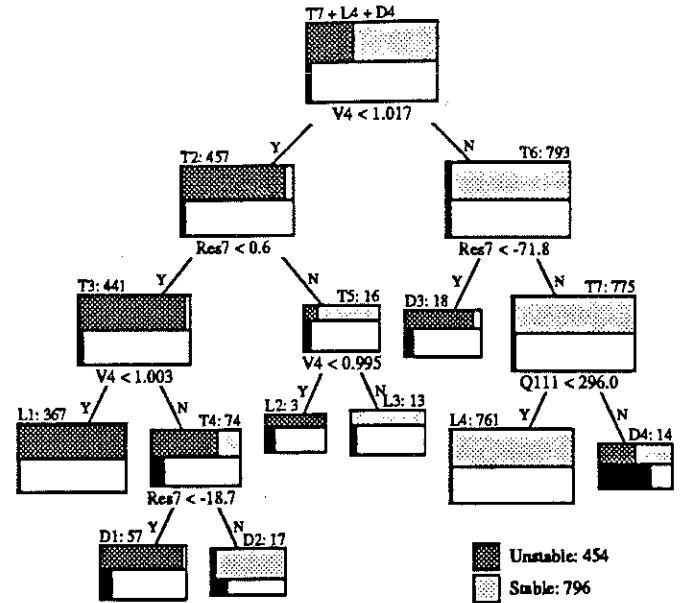


Figure 7: 5 Disturbances, 28 cand. attrib. from JAD states
N = 1250 M = 1250 Alfa = 0.0001 Pe = 3.8%

5 Simulation results on the simple system

5.1 The obtained DT

Figure 7 shows the DT built from the 1250 learning states as well as the results of the tests performed on the remaining 1250 states.

This figure must be interpreted as follows.

Each node of the tree is represented by a box. Above the box appears the node's name, labelled T (for test), D (for deadend) or L (for leaf), as appropriate, along with the number of its learning states. The node box itself is subdivided into upper and lower parts; the relative height of the upper and lower parts is proportional to the relative size of the learning and test sets corresponding to this node. The upper part is further subdivided into two (differently shaded) parts, to indicate the ratio of stable and unstable learning states. Similarly, the lower part of the box relates to the test states directed to this node; the black and white areas inside are proportional to the incorrectly vs. correctly classified test states. Finally, below each test node are indicated the attribute of the locally selected test along with its threshold value. The node's learning states passing the test are directed to the left, the remaining ones to the right.

This DT corresponds to the set of 28 candidate attributes shown on Figure 6 and computed from the JAD states (only). It relies on $H_m = 0.1$ and $\alpha = 10^{-4}$, which are the two key parameters of the algorithm defined in §2 of Ref. [7]. The latter choice (justified in Section 5.5) results in four deadends in the obtained tree (D1 to D4); D1, D2 and D4 are labelled unstable and D3 stable, according to the majority class.

The first salient result is the simplicity of the tree : it only includes 15 nodes and 7 tests on attributes. Within the limit of validity of this simple example, this indicates the adequacy of the DT method to deal with the problem of concern.

Simplicity also characterizes the use of such a DT. Within a real-time context, the gathered values of the attributes would be merely dropped through the tree and the reached deadend or leaf would provide the conclusion. In our example, observing for instance $V4 = 1.0$ pu and $Res7 = -1.0$ MVar just after a disturbance, leads to Leaf L1, i.e. to declare the system state critical (and to trigger emergency control!). In other words, the DT is truly an efficient emergency indicator, in the sense defined in Section 2.

5.2 Interpreting the DT

In order to further illustrate the DT building procedure, it is interesting to analyze the score obtained by the various attributes, for each step of the iterative tree building procedure (see § 2.1 of Ref. [7]). In practice, this complements the information contained in the DT itself (which only focuses on the best score).

These results are summarized in Table 4 which shows at each test node of the tree, the few best current tests as well as the corresponding normalized information gain $G_C^T(S)$ given by Eq.(6) of Ref. [7]. It must be remembered that the attribute selection only obeys a local optimality criterion; therefore the obtained scores can only be interpreted at a given node of the tree. As an example, at node T3, V4 is the best attribute for the learning states relative to T3, i.e. for the 441 learning states such that $V4 < 1.017$ and $Res7 < 0.6$ (see Figure 7).

A global view of the contribution of each selected attribute is provided by the information quantity I_Q^T computed according to Eq.(8) of Ref. [7]. Table 5 gives the results corresponding to the tree of Figure 7 ($\alpha = 10^{-4}$) as well as to its completely developed version ($\alpha = 1$).

Table 5 - Information quantities

$\alpha = 10^{-4}$		$\alpha = 1$	
attribute	I_Q^T	attribute	I_Q^T
V4	872.4	V4	875.1
Res7	153.0	Res7	153.0
Q111	28.7	Q111	28.7
		V3	19.0
		P7	7.1
		P24	7.0
		P1	5.5
		Q1	4.1

These results inspire the following comments :

- (i) two attributes do really emerge from the list of candidates : V4 (a sub-transmission voltage controlled by an OLTC) and Res7 (the machine reactive power

Table 4 - Scores of the best attributes during DT building

Test node	Attribute and v_{i*}	$G_C^T(S)$	Test node	Attribute and v_{i*}	$G_C^T(S)$
Top node	$V4 < 1.017$	0.6917	T4	$Res7 < -18.7$	0.6484
	$V3 < 0.973$	0.6908		$Q7 < 89.5$	0.4962
	$V1 < 0.973$	0.5618		$P7 < 3.3$	0.3679
	$Q11 < 266.3$	0.5476		$P24 < 244.9$	0.2432
	$V2 < 0.981$	0.5322		$Phv < 601.2$	0.2023
	$Q111 < 303.5$	0.3798		$V7 < 0.938$	0.2004
	$Res7 < -46.2$	0.3233		$Q24 < -66.4$	0.2004
	$V7 < 0.897$	0.2750	T5	$V4 < 0.995$	1.0000
	$Pehv < 856.5$	0.2338		$V1 < 0.943$	0.6546
	$Q112 < 217.7$	0.2338		$Q112 < 279.8$	0.5991
	$Phv < 545.6$	0.2220		$V3 < 0.947$	0.5991
	$P24 < 241.8$	0.2095		$Q7 < 145.9$	0.4934
T2	$Res7 < 0.6$	0.3036		$V2 < 0.956$	0.4934
	$V3 < 0.973$	0.2686		$V7 < 0.966$	0.3892
	$V4 < 1.003$	0.2569		$V11 < 1.097$	0.3195
	$Q11 < 217.7$	0.2054		$P1 < 402.6$	0.3195
	$V1 < 0.971$	0.1905		$Qehv < 237.7$	0.3195
	$V2 < 0.981$	0.1594		$P11 < 623.2$	0.3195
	$Q24 < -87.1$	0.1139		$P7 < 271.7$	0.3148
	$Q111 < 356.4$	0.0956	T6	$Res7 < -71.8$	0.4666
T3	$V4 < 1.003$	0.2103		$Q111 < 294.3$	0.3146
	$V3 < 0.973$	0.2009		$V3 < 0.898$	0.2596
	$Q11 < 239.1$	0.1827		$P111 < 803.9$	0.2261
	$Q24 < -87.1$	0.1826		$V4 < 1.030$	0.2084
	$V1 < 0.971$	0.1331		$Q11 < 294.3$	0.2077
	$Qehv < -26.0$	0.1272		$Q7 < 193.8$	0.1671
	$V2 < 1.020$	0.1154		$V1 < 1.001$	0.1422
	$P11 < 1045.8$	0.0843	T7	$Q111 < 296.0$	0.2765
	$Q111 < 356.4$	0.0981		$Res7 < -28.2$	0.2682
	$Q13 < -110.6$	0.0653		$P111 < 905.6$	0.1833
				$Q11 < 297.3$	0.1684
				$V3 < 0.986$	0.1504
				$V4 < 1.030$	0.1402
				$Q7 < 92.4$	0.0973
				$V1 < 1.000$	0.0973

reserve); they are present in all but one tests of the DT;

- (ii) V3 and V4 are equivalent attributes (compare the respective G_C^T values in Table 4) and if V4 was "masked", V3 would replace it while leaving the tree unchanged; this indicates that V4 should not be interpreted as a particular bus voltage but rather as a measure of the disturbance impact on the HV sub-transmission system;
- (iii) Res7 has apparently no equivalent substitute;
- (iv) although superseded by other attributes, Q11 also carries much information; this is the reactive support provided by the infinite bus through lines 11-1 and 11-2, an obviously significant variable for the problem of concern. Note that, unlike Q7, Q11 was not subject to physical limitations;

- (v) very often Q11, the sum of Q111 and Q112, contains more information than these latter two attributes. This is most likely because when line 11-1 (resp. 11-2) has been lost, Q111 (resp. Q112) has a zero value, whatever the system behaviour (stable or not); this makes this attribute less discriminant;
- (vi) tests on EHV voltages (V1 and V2) also appear in Table 4 but they do not appear as good as the HV voltages. This indicates that DTs are able to provide sound criteria to *early* trigger the blocking of OLTCs instead of relying on an already installed deterioration of the EHV voltage profile to take such an action, after the fact.

The large amount of information brought by V4 (or V3) and Res7 is a very meaningful result. Indeed, let us recall that the validity of the proposed approach rests in particular on the fact that the used (JAD) attributes implicitly reflect the undergone disturbance. V4 (or V3) and Res7 contain very much of this information, since both quantify :

- how far, just after the disturbance, the system has moved away from its previous equilibrium;
- how much both the OLTCs and the machine reactive power limitations will subsequently stress the system.

For instance, as regards V4, while its pre-disturbance value always equals the OLTC voltage set-point (within the ϵ tolerance), the stronger the disturbance, the larger the deviation of V4 (in the JAD state) with respect to its pre-disturbance value and the stronger the subsequent stress of the EHV voltage profile. Similarly, the smaller the value of the machine reactive reserve (Res7), the larger the initial stress of the system (larger XI^2 losses, smaller ωCV^2 support, etc.) and the more negative this reserve, the more important the impact of the subsequent reactive limitation.

One can easily verify on Figure 7 that low values of V4 and Res7 lead indeed to declare a case unstable, and conversely. Note however that the indicator (automatically) proposed by the DT is not a simple test on V4 or Res7 but rather adequately "interlaced" tests on the latter, which take into account, for instance those cases with a "rather low" V4 but a "rather large" Res7 (Leaf L3 of the DT : stable).

5.3 The accuracy of the DT

The accuracy (or reliability) of a DT is its ability to properly classify unseen cases. For the DT of Figure 7, it has been evaluated by following the procedure described in Section 2.3 of Ref. [7]. The obtained ratio of misclassified states is 3.8 %, which means that 47 states have been misclassified out of the ($M =$)1250 independently generated test states. Their distribution on the tree is shown on Figure 7.

While avoiding to dangerously extrapolate this result to other systems, it is very encouraging to observe such a

good reliability of the DT, which confirms the validity of the proposed method.

Also, a careful analysis of the classification errors deserves further attention. The latter are of two types : (i) false alarms (32 cases out of the 47); (ii) non detection of a critical state (the remaining 15 cases).

It is required to further analyze the nature and the importance of the errors, since an error of type (i) above does not have the same consequences as one of type (ii); similarly, the non detection of a critical state is less dangerous if more time is left to take a corrective action...

More generally speaking, it would be of great interest to quantify the degree of stability or instability of the system (instead of just a critical/non critical answer). This would result in several attractive applications, namely :

- a refined analysis of the classification errors, as mentioned above;
- the possibility of considering more than two classes; this would help to avoid large errors;
- the possibility to define distances in the attribute space which would reliably relate to the security or insecurity level; therefrom the DT would not only allow to classify an unseen case but also to complement this diagnosis by an estimate of how far the system is from the "boundary of criticality".

The above methodology has been successfully tested within the context of the preventive transient angle stability, where the critical clearing time of the fault provides the sought measure of the system robustness [11, 16]. In the case of voltage stability, measures of robustness for both the stable and unstable cases are presently investigated.

Within a similar context, investigations are also performed to analyze how the quality of the DT diagnosis evolves with time, e.g. by building DTs from learning sets originating from successive system snapshots.

5.4 DTs obtained from other attributes

As already quoted, an attractiveness of the DT of Figure 7 lies in the fact that it only involves attributes relative to the JAD states (see Section 2.3). Tests have been carried out however, in which other candidate attributes were proposed to the DT building algorithm, namely :

- the 28 attributes of Figure 6, taken from the JAD states, along with the ratios of transformers 1-3 and 2-4;
- the same 30 attributes along with the 28 attributes of Figure 6 *taken from the pre-disturbance states*.

It has been very significant to observe that the DT building algorithm did not select any of the added candidate attributes, hence yielding the same tree as in Figure 7. This confirms that most of the information is contained in the JAD states.

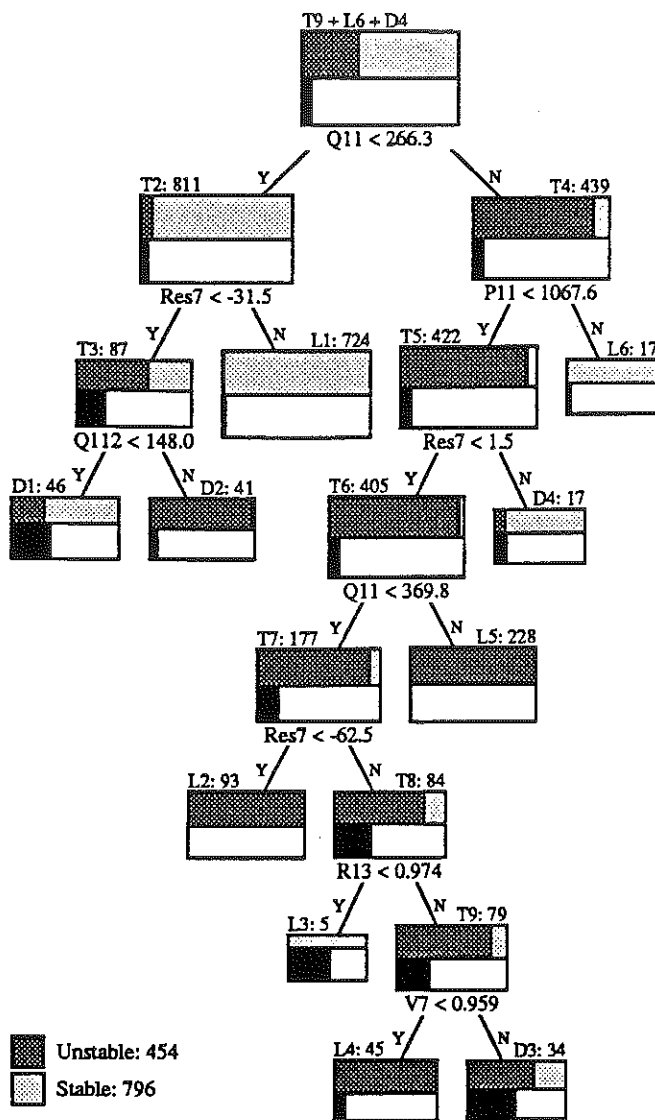


Figure 8. 54 cand. attrib. from pre-disturb. and JAD states
 $N = 1250$ $M = 1250$ $\text{Alfa} = 0.0001$ $\text{Pe} = 6.0\%$

In order to further see how pre-disturbance attributes could emerge, a case has been run with the above 30 JAD and 28 pre-disturbance attributes, except the V1, V2, V3 and V4 JAD voltages (the best attributes as revealed by Table 4) which have been eliminated from the candidate attributes. The obtained DT is shown on Figure 8. As can be seen :

- it is more complex;
- the P_e percentage of misclassified test cases increases from 3.8 to 6.0 %;
- no pre-disturbance attribute has been selected however;
- the transformer ratio R13 has been selected but its role is marginal.

5.5 On the influence of the α -risk

The DTs of Figures 7 and 8 correspond to an α -risk equal to 10^{-4} . For comparison purposes, Figure 9 shows what the DT of Figure 7 would have looked like when setting $\alpha = 1$, i.e. when fully developing the DT.

The so obtained DT clearly illustrates the developments of Section 2.3 of Ref. [7], i.e. the tree has become complex, hiding the most relevant relationships among details.

In fact the DTs of Figures 7 and 8 have been "pruned", i.e. the α -risk has been adjusted so as to obtain *a posteriori* the minimal test set error rate. Pruning strategies are further discussed in [14, 15].

6 Conclusions

The emergency control of voltage stability is still in its infancy. It is however a highly needed control center function, complementary to the preventive assessment of voltage security. One important aspect of this control is the early detection of critical states in order to reliably trigger automatic corrective actions.

Devising criteria to this purpose is a delicate task, generally speaking, for decisions might have heavy consequences and for it requires experts to perform impressive amounts of off-line simulations.

The DT methodology is attractive for both organizing this off-line analysis and devising efficient on-line criteria. Indeed, among others, it succeeds in extracting the relevant, compact and meaningful information out of a large set of off-line simulations and the indicator it provides is a collection of simple tests involving real-time measurements.

Obviously, several important questions have to be answered as regards the practical application of the method : among others, issues such as the building up of different trees for different sub-regions, the deeper assessment of the classification errors, the possibility of suggesting corrective actions start being addressed now. The already obtained results are however very promising.

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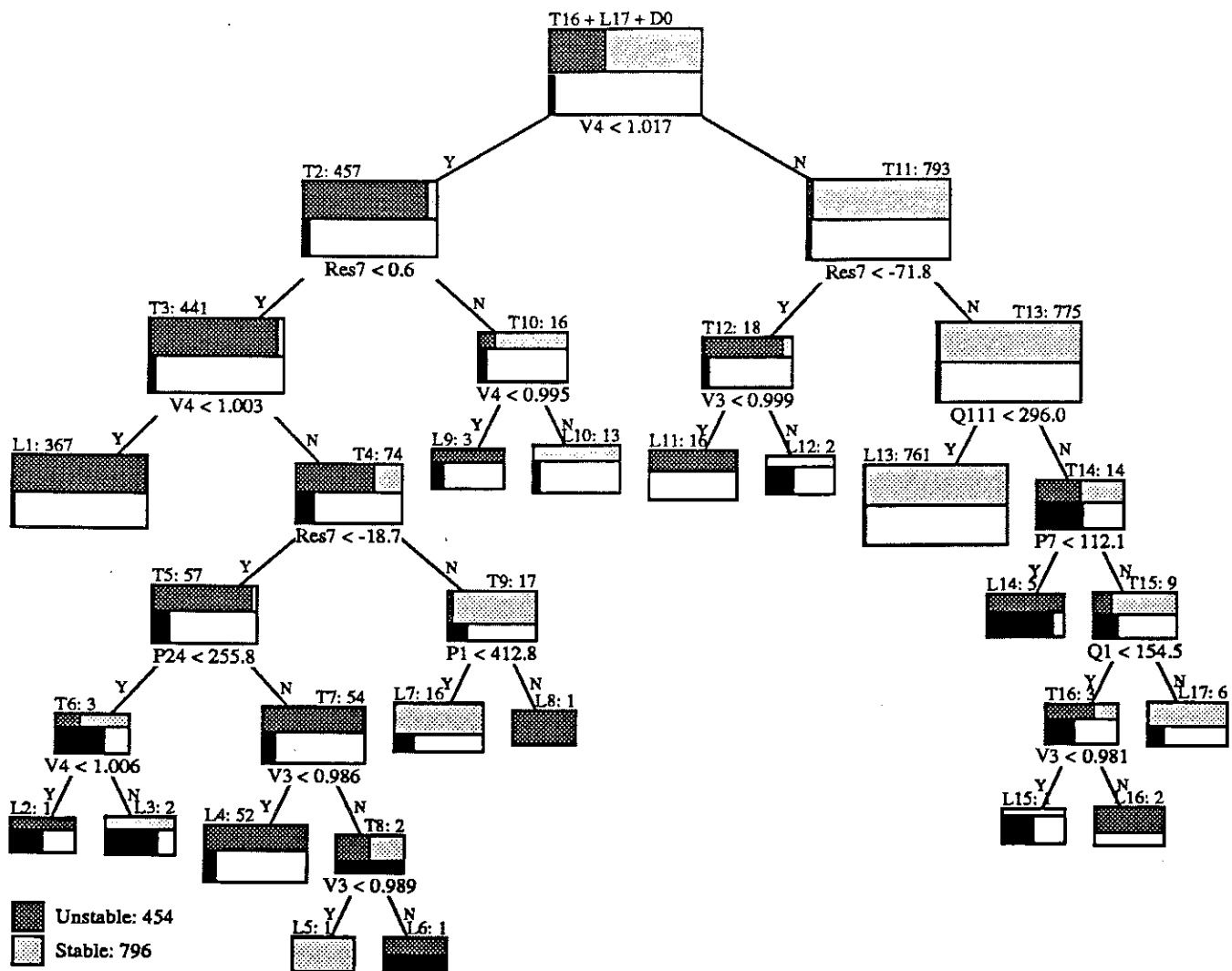


Figure 9. 5 Disturbances, 28 cand. attrib. from JAD states
 $N = 1250$ $M = 1250$ $\text{Alfa} = 1.0$ $\text{Pe} = 3.6\%$

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Method and test results for the MIT approach to steady state voltage monitoring and corrective actions

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Abstract

While the present state-of-the-art in computer based monitoring and corrective action taking at the Energy Management System (EMS) level offers excellent analysis tools for on-line evaluation of postcontingency steady state voltages and phase angles, not many tools are available for the on-line corrective strategies to prevent violating prespecified operating limits. Optimal power flow techniques are under development to meet this need. This paper describes the theoretical foundation, computer algorithm and software for steady state voltage monitoring and control developed at MIT for the New England Power Pool (NEPOOL) system. This new algorithm that reschedules real and reactive power following a given contingency so that voltage profile and phase angle differences remain within prespecified bounds is illustrated on one scenario with voltage problems in part of New England.

1. INTRODUCTION

The method is intended primarily to monitor voltage and reactive power conditions, to identify potential problem situations, and to make decisions regarding their severity. However, since large system changes also involve significant real power changes, the method has the capability to incorporate their effects on voltage changes. A system-wide vector measure of voltage violations outside the prespecified limits is used as the main indicator of reactive power problems, and their propagation through the system. If, for a given contingency, this measure indicates that voltages are outside the operating limits, the method is extremely effective in rapidly identifying control actions for improving the voltage profile. The control actions consist of changing terminal voltages at generators and under load tap changing transformers, as well as switching capacitor banks and reactors. If no priorities are specified, the algorithm uses controls so that those most effective for remedying specific voltage violation are explored first. If particular control priorities are specified, most likely due to economic and environmental constraints, the specifications are easily implementable. When system conditions change such that the specified lower bound on target voltage cannot be met, the algorithm will automatically reduce the target lower bound until it permits an attainable voltage, still above the absolutely lowest voltage prespecified. In the most drastic situations a partial load reduction is done, as the control option with the absolute lowest priority. In actual implementation of this method both target voltages and the absolute lowest

acceptable voltage values are specified by the human operator. The lowest bound is usually determined by protection settings which activate below this value, while the target voltage is set for most economic operation under normal conditions.

The method has been derived using nonlinear analysis and therefore has the advantage of being valid for large changes in reactive power or voltage. As the method and simulations are presented, it is important to keep in mind that the exact load flow is not solved. Real and reactive power rescheduling is computed so that the resulting load voltages and phase angle differences are respectively above and under prespecified target values. The payoff from monitoring for bounds rather than exact voltages is a computational technique that is extremely effective.

It should be noted that the method is effective only if certain conditions on the system matrix are satisfied. This eigenvalue condition is one of the ways of expressing that common sense remedial actions are effective, namely that an increase in reactive power demand causes a decrease in voltage, and vice versa. The eigenvalue condition for a small one machine/one load system means that the method is fully reliable only if a postfault operating point is on the upper part of the P-V curve. The software for checking this condition is available. Our experience with the method suggests it be used under the assumption that the eigenvalue condition is met. For any case, a parallel execution of the eigenvalue test can be made, which is considerably slower. A warning will be given that our method is not effective, only if the eigenvalue condition is not met. Test results of this report show that this condition is consistently met even in the most severe cases studied.

To summarize, an effective method is presented for monitoring steady state security limits, without actually computing load flow. Since the method does not involve any model linearization, it is applicable for monitoring voltage changes due to large changes in transmission network topology (line outages), as well as in generation and loading changes. The method has a unique feature of identifying corrective actions so that voltages and phase angles are maintained within the given limits. In this sense, it has a function similar to an optimal power flow algorithm. The method is computationally much more attractive than an optimal power flow. It is amenable to development of an expert system generated in-pnt. This is primarily because the computational time does not increase significantly with increase of system size, since no matrix inversion is needed.

The method is intended for establishing on-line steady state

security monitoring and corrective actions. The method has a significant potential to assist the operator in on-line monitoring and corrective actions. It gives the operator information on system response to actual events rather than to hypothetical events from a predetermined contingency list. It may permit corrective actions to be less stringent than the action mandated by the contingency list. Therefore, it is likely that more conservative preventive actions could be relaxed by responding to actual evolving rather than hypothetical events from the given contingency list. This is particularly important when deciding on partial load reduction in order to preserve system security, since load shedding should be undertaken only if absolutely necessary. Our method offers technical measures which indicate when, where and how much load reduction needs to be done. As a result it offers an interactive way of evaluating this highly unpopular action. Experience in testing the method on several large NEPOOL test cases is described and illustrated on one such large example. Finally, it should be stressed that the method provides a convenient off-line tool for planning application.

2. THEORETICAL FOUNDATION OF THE DEVELOPED METHOD

In order to understand the path taken in this work, the following observations are important: Coupled load flow equations have considerably more complex structure than the decoupled real power-angle and reactive power-voltage problems. In the past, both decoupled problems have been successfully formulated as nonlinear network problems, which has brought analysts closer to understanding conditions for existence of solution to these problems. General conditions under which the real power-angle problem has a solution are directly understood only for the simplest two-bus systems. The concept of a steady state stability limit can be directly related to the maximum loadability conditions only for such simple systems. Similar developments for the reactive power-voltage decoupled problem have introduced network based interpretation of this problem, and of the conditions under which the unique voltage solution above the prespecified limits exists.

Generalizations of similar conditions for multi-bus systems, although feasible, typically lead to quite conservative conditions for steady state stability limits expressed in terms of a minimum singular value of the system matrix, for example. It has been only recently that we have shown that network theoretic based sufficient conditions for solution existence could be exploited in a careful way with an end result of non-conservative remedial strategies which bring postcontingency voltages to within prespecified limits[1]. This work forms the theoretical basis for the method reported here for voltage monitoring and control under the decoupling assumption. Keeping in mind the fact that it is not known how to interpret the fully coupled load flow problem (and consequently questions of corrective strategies for bringing system variables to within the prespecified limits) from the network viewpoint, the following approach is taken. Since most realistic system contingencies involve both real and reactive power changes, an iterative strategy is proposed. A new method dual to our reactive power rescheduling method is developed for scheduling real power to a near optimum following the real power imbalance due to a given contingency, assuming the precon-

tingency voltages. Next, based on the sufficient conditions for the existence of a voltage solution within the prespecified limits, a set of actions is developed for bringing voltages within the limits following a major contingency by exploring existing reactive power resources. The effectiveness of this method comes from being able to recover both voltages and angles to within the given limits by iteratively solving two decoupled problems, whose solutions are well understood.

It is further important to understand a qualitative difference between the method introduced here and the solution to the decoupled or coupled load flow problem. The proposed iterative method results in a near optimal voltage and real power rescheduling, while a typical load flow problem solves for other variables once real power and terminal voltages at generators are given.

If inadequate resources are available, the method allows for relaxing operating limits, again in a systematic way at busses where this is unavoidable. One application could be fine tuning of large energy transfers in order for voltages not to fall below the operating limit, while approaching steady state limits, often for economic reasons.

The proposed method has yet another unique characteristic. Real power losses are not required to be compensated from a fictitious slack bus; changes in postcontingency losses are distributed in a near optimal way relative to presumably optimal precontingency losses. In fact, the concept of a slack bus is fully avoided. Recall that this is not so even in computationally very complex optimal corrective methods, such as optimal power flow methods.

Notation

- Bus enumeration: $1, 2, \dots, n$ - load buses; $(n+1), \dots, (n+k)$ - generators(including slack).
- $V_j - \delta_j$ - $j = 1, 2, \dots, (n+k)$. load and generator voltages and angles respectively
- $V_{lb_j}^t$ - lower bound on the "target" voltage at load j ; $j = 1, 2, \dots, n$.
- $\delta_{ub(i,j)}^t$ - upper bound on the "target" phase angle difference between the directly connected nodes i and j ; $i, j = 1, 2, \dots, (n+k)$.
- $\delta_{ub(i,j)}^{abs}$ - absolute upper bound on the phase angle difference between the directly connected nodes i and j ; $i, j = 1, 2, \dots, (n+k)$.
- $V_{lb_j}^{abs}$ - absolute lower bound voltage at load bus j . $j = 1, 2, \dots, n$.
- P_j, Q_j , net power generated at node j ; $j = 1, 2, \dots, n$.
- P_j^{min}, P_j^{max} , lower and upper active net power limits at generator j ; Q_j^{min}, Q_j^{max} , lower and upper active net power limits at generator j ; $j = (n+1), 2, \dots, (n+k)$.
- $G_{i,j} - jB_{i,j}$ - admittance of a branch connecting buses i and j . For a typical branch, $G_{i,j}, B_{i,j} > 0$; $j = \sqrt{-1}$.
- jb_j - shunt admittance at bus j ; $b_j > 0$ for capacitive shunts.

The method is based on the use of sufficient conditions for the existence of the unique solution to the nonlinear reactive power-voltage problem. The theory emphasized in this work is biased towards low voltage problems only. An analogous technique could be developed for high voltage problems.

To review the theory, let us consider an electric power system. With the notation introduced above, the equations defining the nonlinear reactive power-voltage problem are simply the decoupled load flow equations of the form:

$$\begin{aligned} -V_i \sum_{\substack{j=0 \\ j \neq i}}^{n+k} V_j (B_{i,j} \cos \theta_{i,j} + G_{i,j} \sin \theta_{i,j}) + \\ + V_i^2 \left[\left(\sum_{\substack{j=0 \\ j \neq i}}^{n+k} B_{i,j} \right) - b_i \right] - Q_i = 0 \end{aligned} \quad i = 1, \dots, n \quad (1)$$

The decoupled reactive power-voltage problem is the problem of finding voltages which satisfy eq. (1), given load reactive power demand Q_i and nominal voltage phase angle differences $\theta_{i,j}$. Depending on the application of the developed method, phase angles could be taken as corresponding to the normal operating conditions, computed only once, or updated from an alternative program.

The system of equations (1) is quadratic in voltages and in general does not have a unique solution. This can easily be seen from the Q - V curve of a one generator one load system. The problem could be also viewed as the question of finding a new steady state solution for a given (power) network as the inputs Q_i and its parameters $\theta_{i,j}$, $B_{i,j}$, $G_{i,j}$ and b_i change. The existing literature on the uniqueness of the solution for certain classes of nonlinear networks, does not directly apply to the problem studied here. Conditions are typically conservative for the unique solution to exist, regardless of its numerical value. From the experience with the Q - V problem defined above, it does not come as a surprise that the power network does not possess such properties for an arbitrary voltage value.

A restating of the Q - V problem as a nonlinear network problem, such that the network satisfies required mathematical properties only in the region of secure voltage operation, i.e. above a prespecified V_{lb}^t , has led to the main theoretical results which form the basis for the proposed algorithm. First of all, in order to represent the Q - V equations (1) in the nonlinear network form for which solution existence conditions have been formulated, they are rewritten in matrix form:

$$(H - \hat{\beta}I)V + g(V) = c \quad (2)$$

where the $n \times n$ matrix H is defined by;

$$\begin{aligned} H_{i,i} &= B_{i,i} - b_i + \tilde{\beta}_i \\ H_{i,j} &= -B_{i,j} \cos(\theta_{i,j}) - G_{i,j} \sin(\theta_{i,j}) \quad \text{for } i \neq j \end{aligned} \quad (3)$$

and where,

$$\beta_i = \frac{|Q_i|}{(V_{lb}^t)^2} ; \quad \hat{\beta} = \max_i \beta_i ; \quad \tilde{\beta}_i = \hat{\beta} - \beta_i \quad (4)$$

Vector c represents the known contribution of the generator (PV) busses to the reactive power balance. Its i^{th} component

is:

$$c_i = \sum_{\substack{j=n+1 \\ j=0}}^{n+k} (B_{i,j} \cos \theta_{i,j} + G_{i,j} \sin \theta_{i,j}) V_j \quad (5)$$

where V_j are here the generator voltages.

The vector function, $g(V)$ reflects the impact of the reactive power load at node i , Q_i on the reactive power flow. Its i^{th} component is:

$$g_i(V_i) = \beta_i V_i - \frac{Q_i}{V_i} \quad (6)$$

Nonlinear network theory is applied to establish mathematical conditions under which the problem defined by eq. (2) has a unique voltage solution always larger than V_{lb}^t . Details of the theoretical results are available in [2,3,4]. Earlier a scalar criterion for existence of the unique voltage solution was reported based on the theoretical formulation of the problem in [4]. This scalar criterion is stated here for the sake of completeness and to give the reader a better understanding of qualitative differences between this earlier result and the follow-up results presented below.

Result 1 *If the inequality (7) holds, there is an α^* such that eq. (1) has a unique solution with load voltages greater than the scalar V_{lb}^t , provided that the generator voltages are all greater than α^* .*

$$\max_{i=1, \dots, n} |Q_i| < \frac{(V_{lb}^t)^2}{2} \lambda_{\min}[H] \quad (7)$$

where $\lambda_{\min}[H]$ is the minimum eigenvalue of the matrix H .

This result was intended to be the basis for developing an automated technique to monitor voltage problems. As long as the inequality in eq. (7) is satisfied, the power system is guaranteed to have the unique voltage solution $V_i > V_{lb}^t$ at any load bus, i . Shortly after the work on the project started it was found that such a scalar criterion based on the single minimum eigenvalue's reflecting properties of the entire network is extremely conservative, in that it typically requires unacceptably high settings, α^* at generator voltages. For the rest of the paper, a vector bound V_{lb}^t is considered.

Results based on monitoring only the minimum eigenvalue of the network (or equivalently its minimum singular value) are conservative and may not be useful for designing effective voltage monitoring and control algorithms. At this point the real challenge became to define a *vector criteria* reflecting reactive power imbalance at each load bus. Again, starting from the same basic concepts as in [2], the following is proposed here:

Result 2 *For load voltages $(V_1, V_2, \dots, V_n)$ to be above the prespecified lower bound V_{lb}^t , it is sufficient that, as the operating conditions on the system change:*

1. Matrix $(H - \hat{\beta}I)$ remains positive definite and
2. The vector $D < 0$, or

$$D = (H - \hat{\beta}I)V_{lb}^t - c + g(e) < 0 \quad (8)$$

where,

$$e = (H - \hat{\beta}I)^{-1}c \quad (9)$$

For the proof of this result, refer to lemma 2 of [3]. To understand the physical meaning of the vector e , a closer look at the theory in [2] (p. 14) reveals that e is an upper bound on the load voltages and therefore is relatively close to 1.

In order to appreciate the full potential of this result for effective voltage monitoring and control, it is important to recognize that the condition stated in eq. (8) *does not involve matrix inversion*. Namely, as the loading conditions and network parameters (line, capacitor banks, etc.) change, the matrix H and the parameter $\hat{\beta}$ also change. However, computing their impact on the value of vector D is simple, because it involves relatively few multiplications and additions.

As the generator voltage settings change, vector c changes and computing this effect is also simple, using equation (5).

In principle, the only significant computational effort involved in monitoring D would be recomputing the impact of the component $g(e)$. However, this is not needed, since the following approximation holds:

Result 3

$$g(e) \approx g(1) \quad (10)$$

The justification for this approximation is that the vector e is always close to the vector 1. To make result 3 clearer, $g(e)$ is expanded in a Taylor series. Combining eqs. (6) and (4):

$$g_i(e_i) = \frac{|Q_i|}{(V_{lb,i}^t)^2} e_i - \frac{Q_i}{e_i} \quad (11)$$

Taking the partial derivative with respect to e_i ,

$$\frac{\partial g_i(e_i)}{\partial e_i} = \begin{cases} \frac{Q_i}{(V_{lb,i}^t)^2} + \frac{Q_i}{(e_i)^2} & \text{for } Q_i > 0 \\ -\frac{Q_i}{(V_{lb,i}^t)^2} + \frac{Q_i}{(e_i)^2} & \text{for } Q_i < 0 \end{cases} \quad (12)$$

Furthermore, the second derivative is

$$\frac{\partial^2 g_i(e_i)}{(\partial e_i)^2} = -\frac{2Q_i}{e_i^3} \quad (13)$$

Hence, a Taylor series expansion of $g_i(e_i)$ in powers of $(e_i - 1)$ follows,

$$g_i(e_i) - g_i(1) = \left(\frac{-Q_i}{(V_{lb,i}^t)^2} + \frac{Q_i}{1^2} \right) (e_i - 1) - \frac{2Q_i}{1^3} (e_i - 1)^2 + \dots + \frac{(-1)^{n+1} n! Q_i}{n! (1)^{n+1}} (e_i - 1)^n \quad (14)$$

assuming all reactive loads at load busses are negative (inductive). From this Taylor series, it can be concluded that the error in the proposed approximation is of the order of the square of voltage changes, which is negligible, relative to the order of D , for all practical purposes.

The vector $g(1)$ is defined by:

$$g_i(1) = \frac{|Q_i|}{(V_{lb,i}^t)^2} - Q_i \quad (15)$$

Hence, although the dependency of $g(e)$ on the actual voltages has been avoided, $g(1)$ is still dependent on the voltage bound defined by V_{lb}^t . However, $g(1)$ is nevertheless simple to calculate, using eq. (15). The approximation is derived for the

most typical load model which has constant reactive power; a similar approximation is possible for any other static load model.

Result 2 is used as the kernel for the proposed computer algorithm. Its unique properties will be illustrated later in the paper. It suffices for now to say that this vector criteria is in some sense a measure of the gap between the load flow solution and the lower bound on this solution, V_{lb}^t . In this light, important properties of the vector criteria of eq. (8) are:

1. When $V_{lb}^t = V^{lf} \Rightarrow D = 0$.
2. For any load i , $D_i > 0$, is an indicator of a trouble location, with regard to voltage.
3. Having $D_i \ll 0$ is an indication that node i has more than enough reactive power support to maintain the prespecified load voltage.

Note: The first condition of result 2, which stated that the matrix $(H - \hat{\beta} I)$ remain positive definite, is always satisfied in realistic scenarios. When tests were run with the IEEE 39 bus system, no single outage caused change of this property. As a result, it is recommended that the vector D be monitored in "real time", while checking the positive definiteness of $(H - \hat{\beta} I)$, which is a far more computationally involved procedure, is handled by a supporting routine which runs intermittently. The supporting routine might generate a flag if the matrix loses positive definiteness, warning of very severe voltage problems to which the method discussed here is not applicable. Such a scenario has yet to be discovered by the authors. While considering this question, it was found that none of the recommended changes on the IEEE 39 bus system reported in [5] caused the network matrix to lose positive definiteness.

3. COMPUTER ALGORITHMS FOR MONITORING AND CORRECTIVE ACTIONS

Using the theoretical results of the previous section, first the basic algorithm for reactive power-voltage rescheduling under decoupling assumption is presented. Next, the decoupled algorithm for rescheduling real power to provide post contingency phase angle differences is briefly discussed. This method is in many ways dual to the one already explained above for voltage rescheduling. Finally an iterative method for linking the real and reactive power rescheduling is introduced.

The D vector is used to show where reactive power shortages occur after a fault. The formulation of the D vector is such that the effects of raising voltages on reactive power balance are quickly and clearly identifiable. Hence, the algorithm is able to use knowledge of the available shunts and the reactive power limits of generators to suggest control actions which will restore reactive power balance and thereby recover voltages at or above prefault levels. No proof is made as to the optimality of the suggested control strategies.

Methodology in Brief

Initially, V_{lb}^t is set equal to load voltages from the most recent load flow, V^i . Hence, all of the D vector's elements

start out at zero, indicating perfect reactive power balance. After a fault, the resulting shortage of reactive power is recognized through the D vector. The algorithm then uses the numerical system data to determine whether the load voltages which existed before can be recovered through available control actions. If the prefault voltages cannot be recovered, the algorithm then suggests that some load voltages be reduced (lowered) by a small amount. The possible control actions which the computer algorithm is able to suggest are:

- Switching in capacitive shunt at nodes where it is available.
- Raising generator voltages or modifying the ratio of ULTC's.
- Lowering the voltage at those load busses where the desired voltage cannot be maintained. This is referred to as reducing the voltage.
- Shedding the load as an ultimate action.

It should be noted at the outset that the order in which events occur internally to the algorithm is in no way meant to reflect the chronological occurrence of the physical events being simulated. The description of the algorithm's internal flow is for the edification of the reader only. Usage of the algorithm entails entering a scenario and receiving a list of suggested control actions.

Localized Response

Immediately after a fault is cleared, only the nodes which were directly connected to the fault are flagged by the D vector as having a reactive power shortage. That is, in the case of a line outage, there is a reactive power shortage at most at both ends of the line. In the case of a generator outage, only those nodes which are directly connected to the generator are seen as having a reactive power shortage. Clearly, if these directly affected nodes have reactive power shortages, some nodes which are directly connected to them will also experience a shortage of reactive power. *This is an inherent property of power systems and not a result of heuristic approximations.* However, the algorithm always begins with those controls which are most local to the fault. So, the algorithm first attempts to restore reactive power balance to those nodes directly affected by the fault.

Algorithm both Predicts and Suggests

It should be mentioned that the algorithm in its present form has both a predictive and suggestive nature. In order to recommend control actions, the effects of these actions must be predicted. In addition, since, in general, no single action can completely restore reactive power balance, the algorithm must be able to predict how a series of actions will affect system voltages. This is a simple task when load flows are run between control actions, but without running load flows, it becomes a non-trivial, highly nonlinear problem to suggest actions which will restore at least prefault voltage levels. Furthermore, the algorithm accomplishes both the predictive and suggestive roles without inverting any matrices. Hence, the

algorithm works without requiring conventional computationally intensive procedures used by load flow and optimal power flow packages.

Urgency Determined by D Magnitude

From the formulation of the D vector described in the previous section, it can be seen that a shortage of reactive power is indicated by a positive D vector element. The magnitude of the D element is within $\approx 10\%$ of the magnitude of the reactive power shortage at that node. Assuming a serious single or multiple contingency, the algorithm determines which node is in most urgent need of reactive power support by selecting the node whose corresponding D element is most highly positive. The algorithm will continue to try to restore reactive power balance to this node until it is no longer the node whose D element is largest. Then the most urgent reactive power mismatch is somewhere else, so the algorithm suspends consideration of the node where it started and moves on to the new center of urgency. It should be stressed that until all elements of the D vector are once again at or below zero, the V_{ib}^t voltages are not an accurate representation of the current system load voltages. One large D element might correspond to a whole region of low voltages. It is only when the algorithm has suggested actions - which internally it automatically takes - which drive all of the D elements back down to zero, that the V_{ib}^t voltages can be taken as an accurate prediction.

Controls are Located using Tiers

Once the algorithm has decided on which node to work first, it next finds the closest controls. Any system can be redrawn with a designated node, say node i , as its "center" point. The nodes which are directly connected to i are said to be on the first "tier" [6] away from i . All of the nodes which are directly connected to the nodes on the first tier, excluding other nodes on the first tier and i , are on the second tier, and so forth. When the system has been represented in this way, the tiers can be used to approximate which of the controls are closest. Clearly, a node which is on the third tier away from i could be closer in electrical distance to i than a node which is on the second tier. However, a node which is on the fifth tier away from i is not likely to be closer to i in electrical distance than a node which is on the first or the second tier. Once a list of the closest controls has been found using the tiers, a smart routine is used to determine which of the closest 10 generators, for example, is electrically closest to i . Finally, it should be remembered that electrical distance is not the only measure of a control's effectiveness; the amount of reactive power support available at that node must also be considered.

At present the algorithm does not try to determine which of the controls are closest across different tiers. The potentially inaccurate assumption is made that the controls which are on tiers closest to i should be applied first. Additionally, no attempt has been made to determine which of several controls on the same tier should be applied first. Neither of these issues is thought to be of critical importance because of the complexities, political and otherwise, which will always go into determining which generator voltages must be raised directly after a fault.

Reducing Target Load Voltages

If the reactive power cannot be restored by raising the nearest generator voltages, by switching in the nearest shunt capacitors and adjusting load tap changers, then it is claimed that the voltage at i is not maintainable at the level indicated by $V_{ib,i}^t$. When this has been determined, $V_{ib,i}^t$ is lowered by the amount necessary to drive $D @ i$ to zero.¹ An absolute lower bound on all load voltages, V_{ib}^{abs} , is assumed to exist. If all control actions near i have been exhausted and voltage reduction $V_{ib,i}^t$ does not restore $D @ i$ to zero without violating the bound V_{ib}^{abs} , then the algorithm instructs the user that $V_{ib,i}^t$ cannot be maintained without shedding load.

Voltage cannot be maintained above the lowest limit under certain conditions. In these cases the algorithm allows for a partial load reduction. The algorithm suggest the minimum necessary reduction and at its most effective locations. If the suggested load reduction is not acceptable due to environmental, economic or other constraints, a new combination of suggested actions could be generated. The process could be repeated until the solution meets these constraints.

The importance of recognizing when load shedding has to be done is far reaching, since there are no other algorithms with this option. If for no other reason, this is an extremely important decision for political reasons. The algorithm provides the necessary technical evidence to evaluate such an unpopular action.

Flow of the Algorithm

After the system numerical data has been read in and V_{ib}^t has been set equal to V^i , the algorithm is ready for a contingency simulation. When a contingency has been selected, the algorithm begins to find control actions for recovering load voltages. Once the matrices H and $H - \hat{\beta}I$ and the vectors β , c , and $g(1)$ have been built, $\hat{\beta}$ can be determined and the D vector can be calculated. The D vector is then reordered so that D_1 is the largest of the D elements, D_2 is next largest etc. A second vector keeps track of the nodes to which each D element corresponds.

Let D_1 be located at i . The algorithm proceeds by representing the portion of the system directly surrounding i as a set of tiers. Because control actions taken at a great distance from i are unlikely to restore reactive power balance at i , it is only necessary to represent the system around i to a depth of several tiers. At this point, the nearby shunts and generators can be found, as described earlier, and the control actions can be tried.

The algorithm proceeds by selecting, one by one, the nearest locations at which there are shunt capacitances available. At each bus that has a capacitor bank, shunt is added incrementally until the location of D_1 changes. After all of the shunt at a particular bank has been used up, if neither the location of D_1 has changed nor the magnitude of D_1 has been reduced by at least some specified percentage, then that capacitor bank is marked as being relatively useless at restoring reactive power to i . To maintain reactive power reserve, the shunts at that node are switched back out and the next shunt location is

considered. Once all nearby shunts have been either switched in or marked as useless, the algorithm proceeds on to raising generator voltages.

As with the shunt capacitors, the generators nearest to i are found and one by one each generator's voltage is raised until either the location of D_1 changes or the generator reaches its reactive power limit. Because raising a generator voltage is not inherently an incremental procedure, the process of raising voltages exactly the right amount is more involved. Initially, the algorithm raises the voltage by a very small increment. If that voltage increase is not sufficient to decrease D_1 at all, the generator voltage is further raised until there is some change in D_1 . Given the amount by which the generator voltage increase has decreased D_1 it is possible to estimate the voltage increase required to drive D_1 to zero. As a rough approximation, this desired voltage increase, ΔV_{gen}^{new} can be estimated using an extrapolation formula:

$$\Delta V_{gen}^{new} = \Delta V_{gen}^{old} \frac{D_1^{new} - D_1^{old}}{D_1^{new}} \quad (16)$$

As might be expected, the voltage change calculated in this way tends to be more than enough to drive D_1 to zero. It is preferable not to attempt a larger generator voltage increase than is necessary, thereby driving D_1 below zero, because of the reactive power limits of the generator. Hence, the generator voltage is increased by a fraction of the calculated amount and the procedure is repeated until either the generator hits its reactive power limit or the location of D_1 changes. Once again, when the generator's reactive power limit is reached, if the location of D_1 has not changed and D_1 has not been reduced by at least some specified percentage, then that generator is marked as useless at restoring reactive power. In this case, the generator voltage is returned to its previous level.

Spread of Reactive Support

It remains to describe the means by which the reactive power support spreads out from the control location to i , the location of the greatest reactive power imbalance. Recall that a positive $D @ i$ indicates a shortage of reactive power at i , meaning that there is currently insufficient reactive power support to recover $V_{ib,i}^t$. Similarly, a negative $D @ j$, where j is any load node, indicates an excess of reactive power at j , meaning that there is more than enough reactive power support to recover $V_{ib,j}^t$. This excess reactive support will drive up the voltage at j . Let $V_{ib,j}^{new}$ be the new voltage at which $D @ j$ is zero. To calculate $V_{ib,j}^{new}$, eq. (8) is rewritten as,

$$D_j = (H - \hat{\beta}I)_{j,j} V_{ib,j}^t + \sum_{\substack{i=1 \\ i \neq j}}^n (H - \hat{\beta}I)_{i,j} V_{ib,i}^t - c_j + g_j(1) \quad (17)$$

(Note that when the D elements are calculated, the j^{th} element corresponds to bus j , because the D elements have not yet been put into descending order.) If only $V_{ib,j}^t$ is adjusted², eq. (17) becomes:

$$\Delta D_j = (H - \hat{\beta}I)_{j,j} \Delta V_{ib,j}^t \quad (18)$$

¹ The D vector is ordered largest to smallest immediately after it has been calculated, so D_i refers to the i^{th} largest element of the D vector. Whereas, $D @ i$ refers to the D element corresponding to the i^{th} node.

² Throughout this paper an attempt has been made to use the word *adjust* to refer to a correction made to an inaccurate number in order to better represent reality, while the word *raise* signifies an allowable control action.

Because the term $(H - \beta I)_{j,j}$ is positive, a positive ΔD_j implies a positive adjustment, ΔV_{ibj}^t . Also, in this case, desired $\Delta D_j = 0.0 - D_j = -D_j$. Thus,

$$V_{ibj}^{t_{new}} = V_{ibj}^{t_{old}} - \frac{D_j^{old}}{(H - \beta I)_{j,j}} \quad (19)$$

where D_j^{old} is negative.

Upward adjustment of voltages at nodes with D elements which are negative, in order to drive the corresponding D elements to zero, will also supply further reactive support for the nodes which are directly connected to these nodes. Hence, suppose the algorithm encounters a shortage of reactive power at i ($D @ i$ is positive). Further suppose that there are no shunt capacitors available and the closest generator is in the third tier away from i . As the voltage at this generator is slowly raised, the D elements corresponding to load busses directly connected to the generator become negative. The voltages at these busses, the j busses, are adjusted upward to drive their D elements back to zero, using eq. (19). If load busses which are directly connected to these load busses now have negative D elements, the voltage at these load busses are also adjusted upward. This process is repeated until the adjustments made to surrounding load voltages are negligible. At this point, the evaluation is made as to whether the D_1 has changed location. If it has not, the generator voltage is raised again. The whole process is repeated until the reactive power limit of the generator is reached or the D_1 changes location.

4. DECOUPLED REAL POWER-PHASE ANGLE RESCHEDULING ALGORITHM

As it was mentioned above, the main purpose of this method is to solve the following problem in an iterative way: Reschedule real power generation so that the postcontingency phase angles are within the prespecified limits. The decoupled assumption is made.

This method, similarly to the algorithm for rescheduling voltage problems is iterative and was motivated by the localized nature of power system response to real power changes; *localized*, meaning that changes in voltage phase angles at buses close to the location of change from the normal operation are the largest in absolute value; and the values decrease going away from the deviation. This property has been explored earlier for very effective solving of the load flow problem with a technique known as the concentric relaxation method [8], and it was proved rigorously shortly afterwards [9,10]. Here a further generalization in exploring this property is proposed for effective solving of the real power rescheduling problem. The following departures from the standard decoupled load flow formulation are taken:

- Real power at generators can change during the iterative process of finding new schedule. This holds for the slack bus as well. In this sense, the notion of a slack bus as a large generator capable of producing infinite real power is fully eliminated. This change leads to solutions more optimal with respect to losses than the conventional load flow solution, in particular when the designated slack bus is electrically far from the troubled area.

- The proposed iterative numerical scheme is in some sense similar to the Gauss-Seidel approach to load flow, with the difference that a *particular ordering in solving system equations that obey the real power balance for phase angles and generated real power is dictated by the localized property. Moreover, the real power scheduling approach is biased to compensate for changes via electrically closest sources of energy.*

For details about the theory and algorithm, a separate reference is available [14].

5. ITERATIVE ALGORITHM FOR COUPLING REAL POWER AND VOLTAGE REMEDIAL ACTIONS

A new iterative method is introduced in this section to define remedial actions for contingencies that cause significant voltage and real power changes. The method consists of two steps: First, the real power rescheduling algorithm discussed in [14] is used to compute real power generation throughout the system that will assure phase angle differences within the prespecified limits $\delta_{u_{b(i,j)}}^{ab}$, assuming precontingency voltages. Next, with the new phase angle differences due to the new generation the decoupled reactive power-voltage method [1] is used to reschedule reactive power resources that will assure all voltages above the prespecified limit V_{ik}^{ab} . Theoretical conditions under which this iterative scheme converges are omitted here.

Depending on the accuracy required for the coupled real-reactive power balance, this process could be repeated. Our experience has been that this is not necessary because high accuracy is achieved without repeating the process. The example in the following section illustrates the accuracies achieved.

From a numerical viewpoint, the proposed iterative method for real power rescheduling could be assessed as a generalization of the concentric relaxation method for basic load flow solution. The new method computes real power rescheduling at all generators affected by the contingency, so that the postcontingency angle differences result in smallest transmission losses, instead of solving the load flow problem, which for given power generation and demand at every bus (except at the slack bus) evaluates phase angles at each bus (except at the slack bus),

6. NUMERICAL SIMULATIONS

The test results illustrating the method on small systems up to 40 buses can be found in [1], [14]. Due to space limitations only one numerical example illustrating the method on large test systems suggested by the sponsors is presented. The example is one scenario with voltage problems in part of New England.

The scenario of interest is documented in the load flow data for a 1729 New England bus system. Two cases are given: MITBASE and MITCAPIN. They represent the same sequence of events except that in MITCAPIN case, available capacitors are connected to the system prior to the studied outage.

Case 1: MITBASE

Scenario has initially off line the following units:

Bus name	Bus	Power(MW)
Mystic 7	830	565
Mystic 6	2455	144
Canal 1	776	556
Pilgrim	832	670

and a total of 700 MVAR capacitor shunts available for switching in

Bus name	Bus	Power(MVAR)
Woonkst	546	70
Drumrock	562	140
Mink F18	582	40
Millbry	590	240
Tewksbury	632	63
Bourne	788	60
W.Walpol	912	90

Our algorithm indicates that the nearest generators with reactive power reserve are initially

Bus name	Bus	Tier
Mill 1	229	6
Mill 2	230	6
Mill 3	231	6
N H Hbr	467	8

Case 2: MITCAPIN

This scenario has initially off line the same units as the former one, but shunt capacitors have been already incorporated to the grid. This results in an initial condition where more generators have initial reactive power reserve. These are:

Bus name	Bus	Tier
Brayt1+218	2460	1
Med J1+23.8	844	4
Mill 1	229	6
Mill 2	230	6
Mill 3	231	6
Man/Sost	8586	6
Nfld 1,2	218	7
Nfld 3,4	219	7
Somst	2471	7

The same outage has been simulated for for both initial scenarios (failure of the unit BR PT 3 23.0, unit 516). Each outage was solved in two different ways, with and without the real power decoupled rescheduling algorithm. It was attempted to evaluate the influence of the real power decoupled rescheduling algorithm on the results of the reactive power decoupled algorithm:

1. Scenario: MITBASE

outage : BR PT 3 23.0, unit 516

1.a Method : decoupled reactive rescheduling power method.

1.b Method : coupled real and reactive power rescheduling method

2. Scenario: MITCAPIN

outage : BR PT 3 23.0, unit 516

2.a Method : decoupled reactive rescheduling power method.

2.b Method : coupled real and reactive power rescheduling method

The reactive power rescheduling actions presented in Table 1 and Table 2 give the locations of the capacitors switched in as part of the corrective actions.

Table 1: Generators rescheduled to bring voltages at least above the precontingency values

Gen	2460	844	8586	229	230	231	2471	218	219	232	674
Case											
1.a	0	0	0	X	-	-	0	0	0	-	0
1.b	0	0	0	X	X	-	0	0	0	-	0
2.a	X	X	X	X	X	X	X	-	-	-	-
2.b	X	X	X	X	X	X	X	X	X	X	X

X indicates the generators whose terminal voltage was changed and 0 indicates the generators initially at their reactive power limit

Table 2: Capacitors rescheduled to recover the system (only available for MITBASE)

Cap	546	562	582	590	632	788	912
Case							
1.a	-	X	X	-	-	-	X
1.b	-	X	X	-	-	-	X

X indicates capacitor connected

Locations of load voltages which had to be lowered are listed in Table 3 for all four cases.

Table 3: Locations of load voltages which had to be lowered

Bus	516	518	515	573	3873
Case					
1.a	X	X	X	X	X
1.b	X	X	X	X	X
2.a	X	X	X	-	-
2.b	X	X	-	-	-

X Bus where voltage was reduced

Analysis of case 1(MITBASE)

For this scenario there was very little reactive power reserve in the affected area. The tierwise organization of buses actively participating in rescheduling is presented in Fig 1. In addition, initial angle differences were slightly higher compared to the ones of the second scenario (MITCAPIN).

Note that the initial voltage profile of the load flow solution MITBASE is lower than the solution MITCAPIN. The initial target voltages are therefore lower. As a result, a thorough examination of the differences between these two cases is not possible. Nevertheless interesting conclusions can be obtained from a careful comparison of both results.

Results of the decoupled Q-V algorithm, MITBASE case

The units Mill 1 229 and Mill 2 230 are not electrically close to Br Pt 3 516. For this reason the target voltages at loads around Br Pt 3 516 have to be lowered.

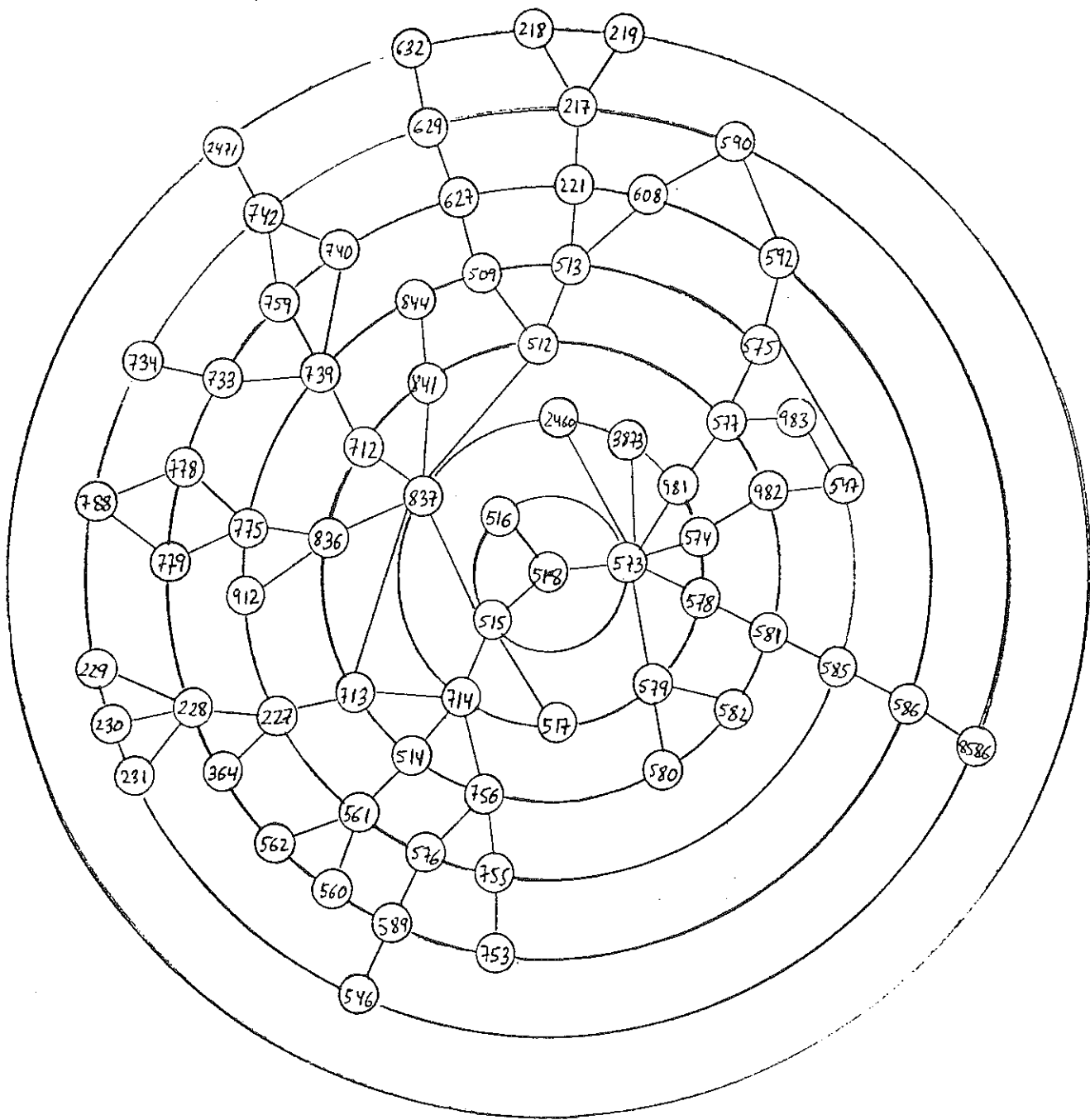


Figure 1: 1729 bus System

A larger voltage gradient is created between the nodes with positive deviation and the sources of reactive power when the voltage profile is modified through voltage reduction. At the same time, the initial mismatch affecting at first only one node, spreads within the affected area. This brings the worst positive deviation closer to possible sources of reactive power.

The largest target voltage reduction reached almost 4.2% at Br Pt 3 516. It was never larger than 1.2% for the other nodes.

The program shows slow convergence for this case since the worst deviation at the 25 th iteration is slightly high (0.41 pu.)

Only capacitors were used to correct the voltage.

Results of the decoupled real-reactive power algorithm, MITBASE case

The index to decide which generators should be used is the ratio of the reactive power increment available at the candidate generator and the reduction of deviation at the node with maximum deviation. Units 229 Mill 1 and 230 Mill 2 have this ratio over 0.01. They were considered as "closest" generators for this case. Shunt capacitors had a different criterion; it was required to have a reduction above 10.0%, in order to accept their connection as a control action. Only the shunt capacitors allocated in the first tiers are used.

The worst reactive power mismatch for the 25 th iteration (0.2 pu) is lower because the system has received more reactive power injection. The maximum target voltage reduction occurs at node 516 Br Pt 3. This reduction is smaller (3.8%) compared to the former simulation. The other nodes were reduced less than 1%. It seems that the corrections given by the real power algorithm help in this case to recover the voltage profile. However, it cannot be concluded that the real power rescheduling is always going to have the same effect.

Analysis of case 2 (MITCAPIN)

Results of the decoupled Q-V algorithm

The closest generators are immediately used. As these units had high reactive power reserve the target voltage reduction is lower. About 3.3% for node 516 Br Pt 3, and less than 0.5% for the next in magnitude.

The largest positive deviation in the last (25th) iteration is now 0.25.

Results of the decoupled real-reactive power algorithm

The voltage profile was enhanced further than in the former case; again the real power rescheduling actions seem to be helpful to recover the reactive power balance. The largest reduction was 3.1% at node 516 Br Pt 3 followed by 0.1% at node 518 Bray M34

Results of the decoupled real power-phase angle rescheduling algorithm; influence of the coupling

For the data received, almost all units were at their real power limit. The maximum generating capacity was set equal to the actual power generated for the units which belong to the area outside REMVEC. If the information about the maximum capacity was not available for units within REMVEC, their maximum limit was set equal to its present output. Since the cases studied involve significant power mismatches, constraints on real power were relaxed. The rescheduling allowed the units to work up to 10.0% above the maximum specified capacity.

The total amounts of power generated in pu. are given for the generators used in the rescheduling process are given in

Table 4.

Table 4: Total amounts of power generated in pu. are for the generators used in the rescheduling process

		initial	final	maximum	increment
MITBASE	1.b	97.14	101.45	100.23	4.31
MITCAPIN	2.b	105.80	112.30	109.65	6.41

The total power lost initially was 585 MW. MITBASE showed a worst real power mismatch on the last iteration of 5.2 MW and MITCAPIN a value of 8.6 MW. There is a group of generators used for MITCAPIN that was not used for MITBASE. These were the following Units:

Table 5: Units rescheduled for the MITCAPIN scenario which were not used for MITBASE

21 Yarm 4	417 Stevenso	2453 MystG4	18.0
57 Gulf Isl 115	531 Comerfd	2454 MystG5	18.0
83 Prides-P	675 Vernon h.	8304 Cxmish	
84 Loudn/So	783 Blaks/k	8337 Smead5,6	

The results for MITBASE are given in the app. Only 430 MW are added to the network(see Table 4; the initial power lost was 585 MW. This could be due to either of two different reasons; numerical in case we had a large number of buses with positive deviation values in the last iteration, but all smaller than the threshold imposed, or simply to the fact that the system has less losses.³

The influence of the coupling could be seen by analyzing the initial deviations vector D, before and after the real power rescheduling. The difference between these two values is an indicator of the decoupling error for the set of proposed corrective actions. In general these values are higher on nodes near strong transmission lines (very small impedance).

The largest initial deviations appeared at the buses given in Table 6.

Table 6: Largest initial reactive power deviations appearing after the real power rescheduling

Bus	227	518	713	714	837	777	8941
MITBASE	0.13	-0.24	0.13	-0.16	-0.29	0.19	0.123
MITCAPIN	0.12	-0.21	0.11	-0.14	-0.26	0.08	-

7. CONCLUSIONS ABOUT THE METHOD

The algorithm converges very fast in the first few iterations. Then, it slows down due to the basic nature of this localized Gauss-Seidel technique. Standard acceleration factors for the Gauss-Seidel technique will be incorporated to avoid this problem.

³The program finishes either when the largest deviation is smaller than a certain threshold or when the maximum number of iterations is exceeded.

When a generator is very far from the problem node (the one with the maximum positive deviation), its index for correcting voltage is very low. (The index of a resource is measured as the ratio between the increment of active or reactive power at the generator candidate and the reduction of deviation or power mismatch at the trouble node). Extensive tests have shown that the value of this measure is not constant with fixed step increments of power for a given node.

It is important to establish a good knowledge on the relationship between the numerical threshold defined for the D vector and the engineering relevance of maintaining D below this threshold at the solution. This information is most likely system dependent.

Numerical oscillations can appear when the increment of power is lower than a certain threshold at a given step of the program. This threshold depends on the transmission network, previous corrective actions and system loading. For very large changes in the network it is important to couple the effects of real power rescheduling on reactive power rescheduling algorithm.

In the case of generator outages, the decoupled reactive power method gives good accuracy and has been shown to be a conservative criterion in all the cases simulated. Critical lines can result in large changes of angle differences which have to be estimated by the real power algorithm. In all test cases it was shown that the minimum eigenvalue remains positive.

8. APPENDIX: Some specific numerical results for large test systems

MITBASE 1.a

Shunts added

Bus name	Bus	Power(MVAR)
Drumrock	562	140
Mink F18	582	40
W.Walpol	912	90

Generators used

Bus name	Bus	Qini	Qfin	Qmax	Vini	Vfin
Mill 1	229	2.15	2.39	2.39	1.0073	1.0113

Buses where voltage was reduced

Bus name	Bus	Vini	Vfin	inc(-)
Brt Pt 3	516	1.0054	0.9635	0.0419
Bray M34	518	0.9733	0.9618	0.0114
Brtnpt34	515	0.9932	0.9850	0.0082
Brayt pt	575	0.9910	0.9896	0.0013
Bray Dls	3873	0.9910	0.9897	0.0013

MITBASE 1.b

Shunts added

Bus name	Bus	Power(MVAR)
Drumrock	562	140
Mink F18	582	40
W.Walpol	912	90

Generators used

Bus name	Bus	Qini	Qfin	Qmax	Vini	Vfin
Mill 1	229	2.15	2.53	2.39	1.0073	1.0168
Mill 2	230	2.15	3.75	4.10	0.9996	1.0186

Buses where voltage was reduced

Bus name	Bus	Vini	Vfin	inc(-)
Brt Pt 3	516	1.0054	0.9665	0.0388
Bray M34	518	0.9733	0.9641	0.0091
Brtnpt34	515	0.9932	0.9871	0.0061
Brayt pt	575	0.9910	0.9904	0.0004
Bray Dls	3873	0.9910	0.9906	0.0003

MITCAPIN 2.a

Generators used

Bus name	Bus	Qini	Qfin	Qmax	Vini	Vfin
Brayt1+218	2460	1.44	2.36	2.23	1.0026	1.0332
Med J1+23.8	844	0.12	0.84	0.84	1.0186	1.0340
Mill 1	229	1.71	2.39	2.39	1.0017	1.0133
Man/Sost	8586	0.06	1.44	1.20	1.0000	1.0358
Somst	2471	0.65	0.82	0.82	0.9840	0.9976
Mill 2	230	1.71	2.05	4.10	1.0060	1.0011

Buses where voltage was reduced

Bus name	Bus	Vini	Vfin	inc(-)
Brt Pt 3	516	1.0422	1.0092	0.0330
Bray M34	518	1.0110	1.0069	0.0041
Brtnpt34	515	1.0329	1.0315	0.0013

MITCAPIN 2.b Generators used

Bus name	Bus	Qini	Qfin	Qmax	Vini	Vfin
Brayt1+218	2460	1.44	2.21	2.23	1.0026	1.0320
Med J1+23.8	844	0.12	0.86	0.84	1.0186	1.0362
Mill 1	229	1.71	2.38	2.39	1.0017	1.0270
Man/Sost	8586	0.06	1.67	1.20	1.0000	1.0492
Nfld 1,2	218	1.29	2.01	1.98	1.0202	1.0536
Mill 2	230	1.71	4.08	4.10	1.0060	1.0360
Nfld 3,4	219	1.29	1.99	1.98	1.0199	1.0519
Somst	2471	0.65	0.84	0.82	0.9840	0.9999
Mont 6	232	1.58	2.00	2.00	1.0146	1.0258
Mill 3	231	1.71	1.46	0.94	0.9950	1.0056

Buses where voltage was reduced

Bus name	Bus	Vini	Vfin	inc(-)
Brt Pt 3	516	1.0422	1.0104	0.0317
Bray M34	518	1.0110	1.0097	0.0013

Aknowledgements

The authors greatly appreciate the continuous help of Dr. Derek Teare, Director of the MIT Electric Utility Program in coordinating the project and preparing this manuscript.

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ON-LINE VOLTAGE COLLAPSE MONITOR

- by -

A. O. EKWUE, R. M. DUNNETT, N. T. HAWKINS and W. D. LAING

National Grid Company, U.K.

ABSTRACT

A voltage collapse monitor has been installed as part of the on-line security analysis suite of the National Grid Company. Its run time is comparable with that of standard security analysis functions. Accuracy is adequate for the management of voltage collapse situations and improves when the margin to collapse is small. Information is presented to the control engineer as a set of structured displays in a form which is directly useful for management of the system.

1 INTRODUCTION

Automatic control actions in a power system are designed to counter any drop in voltage which may result from an increase in demand or a circuit failure. Heavily loaded systems can reach a state where sensitivities are reversed so that control actions actually make the situation worse. Control is then lost and the voltage falls progressively, a situation known as voltage collapse. A system is prone to collapse if there are heavy power flows into an area with insufficient local reactive reserves. A feature of this phenomenon is the speed with which it can occur. In contrast to thermal overload situations, there is insufficient time for post-fault corrective action. Consequently the control engineer must manage the bulk power transfers in preventive mode to maintain adequate margin to collapse.

Conventional security analysis predicts post-fault flows, voltages and injections of real and reactive power. It does not give a good indication of proximity to voltage collapse, because a heavily compensated system can maintain acceptable voltage levels even when it is dangerously close to collapse. There is therefore a need for a separate indicator of voltage collapse margin. In practical system operation, the short-term defence against collapse is to reduce power flows into the area at risk either by increasing local generation or by reducing demand. The most useful indicator is therefore the amount by which the flow across a defined boundary could increase before collapse would occur. This may be evaluated for a choice of boundaries, both in the intact case and after defined faults. The precise answer depends on exactly how the flow is caused to increase, i.e. the distribution of generation change and the change in power factor. However, to be useful in an on-line situation, the calculation must be rapid, and it is sensible to make a central assumption. In the algorithm to be described here we increase all demands on one side of the boundary at constant power factor and all active generation on the other side in pre-specified proportions. Exhaustion of reactive reserves is then checked individually, and leads to a prediction of the collapse point and hence the margin to collapse.

The algorithm used on this work has been described in detail by Hawkins et al [1]. The margin can be defined as the increase in bulk power transfer across a pre-defined boundary which can be sustained before voltage collapse occurs. Some of the computational advantages of this method over previous techniques include:-

- simulation of the changes brought about by load increases;

- simulation of circuit outage effects;
- consideration of reserves of reactive power of the generators;
- consideration of the discontinuities which occur as voltage collapse is approached combined with linearized calculations; this leads to fast solutions and makes the method attractive for on-line applications.

Similar methods based on sensitivity analysis have also been described by Flatabo et al [2,3] and Lemaître [4]. The method used here incorporates additional features which improve robustness. Specifically, (a) the transfer is calculated as a function of voltage, and (b) an interpolation procedure estimates the possibility of collapse at points other than those at which generator reactive reserves become exhausted.

This paper describes the successful interfacing of the algorithm with the National Grid Company's on-line database and network analysis suite [5], permitting on-line use. The method was validated previously on a small system [1] and on a section of the NGC system. The interfacing of the algorithm with the full NGC system has confirmed the accuracy of the technique.

2 VOLTAGE COLLAPSE MARGIN CALCULATION

The function of the voltage collapse margin calculation (VCM) is to determine the limit of active power transmission across defined boundaries before voltage collapse would occur. This algorithm described in [1] starts with the results of the on-line state estimation of the intact network or from critical contingency cases evaluated by the network analysis suite and based on on-line data [5].

An increase in consumer demand in an area, or an increase in power transfer over the system brought about by a change in the pattern of load and generation, is accompanied by a change in:

- (a) system voltages, and
- (b) reactive power output from voltage regulated reactive sources.

With a knowledge of the sensitivities of (a) and (b) to system demand (or transfer), and a knowledge of the magnitudes of generator reactive power reserves, it is possible to estimate which generator would first reach its reactive power limit as demand increased, and to estimate the system voltage profile when this would occur.

Under the new voltage profile, and with a reduced set of voltage controlled busbars, the load flow equations are re-linearized to determine the new sensitivities.

This procedure is repeated; the demand is increased in stages while more and more generators limit, and finally voltage collapse is indicated. This is assumed to occur when either

- there is a change in sign (from negative to positive) of the sensitivity of voltage to reactive demand at any load busbar, or
- the reactive reserves in the system are exhausted.

The voltage collapse margin is then given by the increase in transfer calculated as a function of the voltage profile estimated at that point.

3 PRACTICAL IMPLEMENTATION DETAILS

VCM calculations involve the flow across a boundary, defined by

- (a) the boundary circuits, and
- (b) the direction of flow increase across the boundary.

Responsibility for choosing the boundaries rests with the user, but this is not difficult given that normal operation applies transmission limits defined in this way. There is no theoretical limit to the number of boundaries that may be considered, but each boundary takes a few seconds on a VAX 8700 computer to evaluate for each circuit configuration.

Further details of the implementation are:

- For each boundary, there is a list of cases comprising the intact case and/or selected contingencies for which the transfer limit will be assessed. The contingency list is a subset of the manually specified contingencies [5]. The minimum transfer will then be displayed as the limit for that boundary.
- For each boundary, there is a nodal injection vector associated with it and this determines the pattern of change in load and generation used to effect an increase in transfer for the calculation of the voltage collapse margin.
- The definition of a boundary divides the system into importing and exporting sides. This classification of nodes is done automatically using network connectivity principles described in [6]. The loads on the importing side are increased at constant power factor with corresponding increase in the active power generation on the exporting side. The generation is scaled such that the total active power generation change is matched by the total change in the active power load.
- The nodes on the importing side of the boundary become candidates for checking for voltage collapse. The first node to collapse, i , is identified as the first node to observe a change of sign (from negative to positive) in the inverse of $\frac{dV_i}{dQ_i}$.

3.1 Displays

A number of new displays have been written for the National VAX on-line display suite to minimize operator input. Displays allow control of the VCM algorithm and display of the output. The form and the tree structure of the displays are shown in Fig. 1.

- VCM SUMMARY is the top level display given on entry to VCM from the existing ENHANCED SECURITY ASSESSMENT DISPLAYS menu. It provides a comparison between present transfer and transfer limit calculated by the VCM algorithm for each boundary, and allows selection of a boundary for the VCM OUTPUT FOR 'boundaryname' display.
- VCM OUTPUT FOR 'boundaryname': gives the transfer limit for each contingency or intact case considered for that bound-

ary. It allows selection of the contingency for VCM CALCULATION FOR 'boundaryname' 'contingencyname' display.

- VCM CALCULATION FOR 'boundaryname' 'contingencyname' lists each generator which reached a reactive limit and the transfer at which this occurred in the calculation for the boundary selected.
- VCM BOUNDARY SELECTION depicts the names of boundaries already set up for active power transfer summation and highlights those selected for the VCM calculation.
- VCM CONTINGENCY SELECTION FOR 'boundaryname' lists all contingencies in the AC State Estimation and Security Assessment list, and highlights those selected for the VCM calculation for the chosen boundary.
- DEFINE NATIONAL CONSTRAINTS permits boundaries to be created or re-defined.

4 FURTHER DEVELOPMENTS

The algorithm represents a major advance in the on-line Energy Management Systems providing control engineers with indicators for proximity to voltage collapse. To obtain results consistently on the required timescale, certain assumptions are made:-

- Megawatt flows across the boundary studied are increased without regard to the limits of individual generators or to the loading distribution that will result through an economic dispatch.
- Megavar limits are assumed constant as active power is increased.

These assumptions are considered reasonable in the context in which they are currently used. Nevertheless, there is scope for research into ways of making the model more realistic without unduly compromising the execution time.

5 CONCLUSIONS

The algorithm described in this paper has been integrated with the on-line database of the National Grid Company. Run time is comparable with that of the standard security analysis. Accuracy is adequate for the management of voltage collapse situations and improves when the margin is small. Information is presented to the control engineer as a set of structured displays in a form which is directly useful for management of the system

6 ACKNOWLEDGEMENT

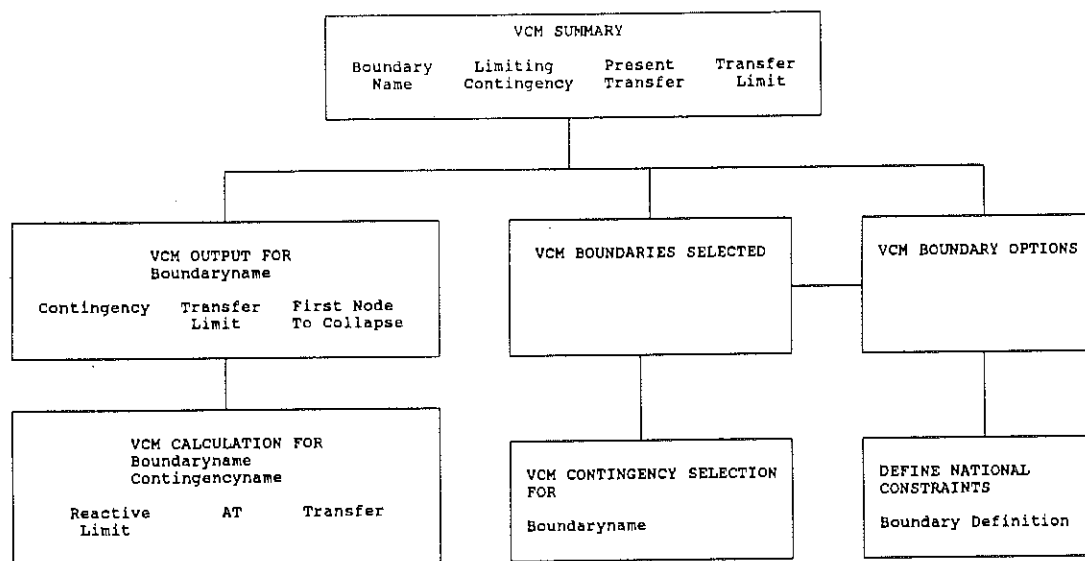
The work was done in various parts of the National Grid Company PLC and is published with the Company's permission.

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Figure 1: VCM DISPLAY TREE



DISCUSSION - SESSION THREE

Edited by Chen-Ching Liu, University of Washington

John Zaborszky (Washington University):

The PV curve has existed for many years. Pete Sauer's PV curve, Figure 1, is different from the conventional PV curve. Also, I would like to stress the importance of the singular boundary. Singularity is useful in a local area where what is neglected becomes important. Arbitrary addition of a dE/dt term to the model is not valid. Pete Sauer proposed a load model in an exponential form, but the model is an approximation.

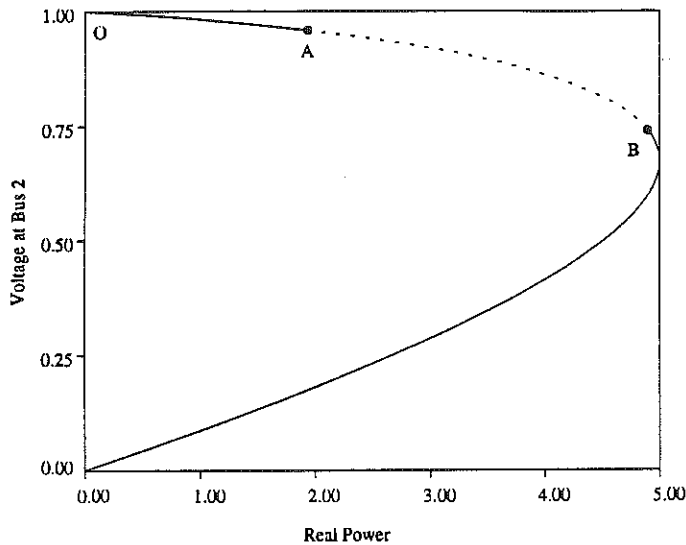


Figure 1

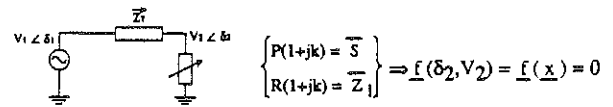
Claudio A. Cañizares (Univ. of Wisc., Madison):

(He showed the transparency in Figure 2 and discussed the relation between bifurcation and changes in various parameters.)

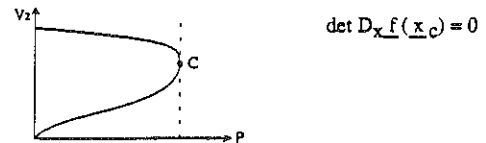
Lester H. Fink (ECC, Inc.):

I have been trying to get a feel of what is dynamic and what is non-dynamic. I started out trying to formulate a broadly based question, but I could not exactly bring it into focus, so I narrowed the question down to Pete Sauer's transparency (Figure 1). The dashed portion indicates a region of instability, which pertains when we are dealing

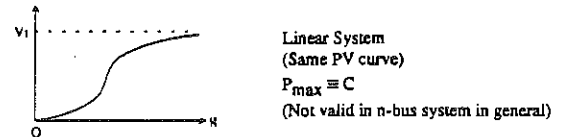
1)



Bifurcation Diagram for P changes:



B.D. for R changes:



2)

$$\begin{aligned} \dot{\underline{x}} &= \underline{f}(\underline{x}, \underline{\mu}, p) \\ &\quad \text{Control variables} \\ \text{Options: } \left\{ \begin{array}{ll} \text{a} & \underline{g}(\underline{x}, \underline{\mu}) = \underline{\mu} \quad (\text{closed loop control}) \\ \text{b} & \underline{\mu} = \underline{\mu}_0 \quad \text{constant} \end{array} \right. \\ \Rightarrow \text{For a} \rightarrow &\left[\begin{array}{l} \bullet \underline{f}(\underline{x}, \underline{\mu}, p) = 0 \quad \text{P.F. equations} \\ \bullet \underline{\mu} \text{ changes with } \underline{x}, p \text{ (implicit } \underline{g}) \\ \bullet \text{e-values must be calculated including } \underline{\mu} \end{array} \right] \\ \Rightarrow \text{For b} \rightarrow &[\underline{f}(\underline{x}, \underline{\mu}_0, p) = 0 \quad \text{P.F. equations} \\ \Rightarrow \text{"P.V." curves (bifurcation diagrams) with right } p \end{aligned}$$

Figure 2

with a constant power load model. On the other hand, if we have a constant impedance load, then the dashed portion disappears and the whole curve represents a stable continuum or region. The distinction seems to be between the case when you have a pure constant power load model versus a pure constant impedance load model. However, the power system is not pure; as the load changes, the load mix changes. If the load is changing, how can the power or impedance be constant? Certainly if one looks at different points on this curve, one might say that the point is constructed with a constant power load model. But if it represents a condition on the system, then it means as one

traverses the curve the power is not constant. So, the question relates to a more realistic model. Does it mean that no matter what else pertains to the load model there is still some region of instability, or where do you switch from one to the other? Is it possible that the system can be operating in a stable region and, with a change in the complexion of the load, the region suddenly becomes unstable. In other words, what happens with the pure models when we face up to the impurity of the system?

Peter W. Sauer (Univ. of Illinois-Urbana):

I agree that there are more parameters varying than power.

John Zaborszky (Washington University):

The conventional way is to have all parameters constant, except the load. If your other parameter is varying, then you draw a series of curves. I have an example in that people did not work with the PV curve but used it for demonstration.

Peter W. Sauer:

I want to comment on John's question of neglected dynamics. There is something we haven't checked: to include network stator transients to see whether those fast dynamics can become a problem. If we have to determine our boundary based on those transients, the simulation will be very difficult.

John Zaborszky:

You can write the equation, but you will never get the constant. Fortunately the other model pinpoints where this trouble lies, precisely by the singularity, which is a mathematical singularity we will never find physically, but that is the neighborhood where you do have those problems. So that becomes a barrier around the area, we call it a component and others call it a causal area. It pinpoints the location of the boundary beyond which you cannot predict trajectories.

Peter W. Sauer:

Even the concept of constant power especially with full stator transients is unrealistic.

John Zaborszky:

Your model can be constant power/constant impedance, a mixture of things. There may still be a singularity around.

Claudio A. Cañizares:

You mentioned that you kept the voltage terminal constant and allowed the internal generator variables to change according to that, but you don't have that control loop inside.

Peter W. Sauer:

We study one equilibrium point. We choose to study a certain point and find the control needed. In the dynamic analysis the control is kept constant and then we move on and study a different point.

Claudio A. Cañizares:

You should have kept those points constant.

Peter W. Sauer:

Perhaps that would be an interesting thing to do. I'm not sure what that would mean then, because then the PV curve is completely curve which has constant voltage.

About Les Fink's comment on purity, the constant power model has always bothered me. You can do an exhaustive analysis of this type with all known load models. In fact, Bill Price of General Electric mentioned that is what they are doing. It really drives home the point that load modeling is one of the last holdouts for where there really is a need for something to be done.

Carson W. Taylor (Bonneville Power Administration):

There really is no such thing, at least at present, as constant power load in a fast time frame. What is normally considered a constant power load is an induction motor, but over the first few seconds it is actually an impedance and then it turns itself into a constant power load in a couple of seconds. If you are going to do transient cases you ought to model the motor as a dynamic device.

Also, there is a comment on the field control setting. There have been a lot of studies on root locus plots, etc., of stability as you adjust the feedback constant of the excitation system. Some of these things could be explained in simple manners. One of the convenient ways is to compare the system characteristics. A line can be developed using standard constant power loads. Then, the operating point is the intersection of the system characteristic with the load characteristic. In Figure 3, a line shows an impedance load and another line a constant current load. You can modify the characteristic to get a line that will always intersect even on the bottom part of the curve and form a stable operating point.

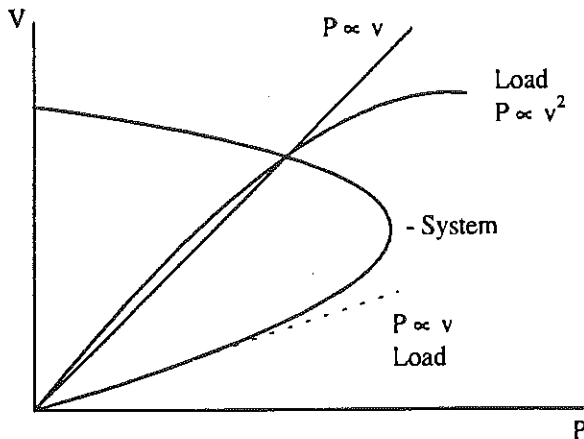


Figure 3

Adam Semlyen (University of Toronto):

Suppose I have a constant Q model and λ is a parameter representing the load. The vertical lines correspond to λ_1 , λ_2 , λ_3 , and the critical point is reached at the turning point. (See Figure 4.) This

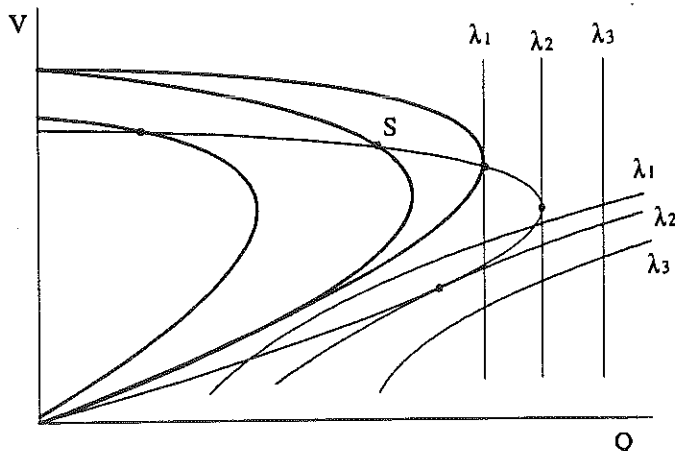


Figure 4

singularity is related to the type of input. If the type of input is fixed Q , then the singularity appears at the intersection with the vertical line. If I have a different load characteristic as these curved lines, λ_1 , λ_2 , λ_3 , representing an increased loading, then the point of tangent would be the point of bifurcation.

I would like to insist on one fact. I understood here several times mention of loss of causality and the possibility of leaving this algebraic curve, traced in terms of one parameter. That being a problem, I feel that there is no need to insist that we have Q or P as a parameter; we reach a tip and therefore a singularity. We can trace a curve in continuation with some other parameters.

Harry G. Kwatny (Drexel University):

I think the point that is being made here is that the PV diagram is not necessarily a bifurcation diagram.

Ian A. Hiskens (Queensland Electricity Commission)

From the viewpoint of a planning engineer, it would be useful to get back to the real system dynamics. There are three issues: one that has been mentioned is the load model, another is the actual load and measuring system behavior to determine parameters for that load models, and the third is related to the robustness of that model, because parameters keep changing. There are a lot of unanswered questions and I'd be interested to know of any actual utility information on going out and looking at the behavior of load; not just fitting curves to the traditional exponential type models, but fitting parameters to other load models as well.

Marija Ilic (Massachusetts Inst. of Technology):

The nonlinear network formulation approach we take in voltage monitoring and control can actually absorb more realistic quasi steady-state models or polynomial models. It does not have to be constant PQ or constant impedance. You can have different models and still establish sufficient conditions for the multi-dimensional system to have a unique solution with all the voltages above the limits.

U. George Knight (Imperial College London):

First of all, on the decision tree approach, Dr. Van Cutsem mentioned the possibility of extending this to an on-line application. I wonder whether he would anticipate any

difficulty in constructing a decision tree for a system whose configuration might change: for instance, the generation schedule might change. How dependent is the structure of the decision tree on those configurations and generation schedules?

On the question of keeping capacitors in and other control mechanisms, it seems that this is almost certainly a good idea. There is no guarantee that a contingency is going to be a credible contingency. If you have capacitors already in there, it is more hopeful that you would survive a non-credible contingency. I do think there is one word of caution. That is, the more you compensate a system, the flatter the voltage curve may become. Therefore if trouble does develop, it is most likely to develop with less warning. So if you have more capacitors and mechanisms, it probably means that the operator has to be more alert against the possibility of a voltage collapse.

Marija Ilic:

A power system can have a lot of capacitance to support the voltage. Then, it appears that the voltage is one per unit for a very long time, but things can change suddenly when you have an incremental increase in large power transfer. It is one of the points engineers debate all the time - Do you put the controls first before the contingency occurs or not?

Thierry Van Cutsem:

At the basic level, there is no restriction with respect to generation. Different generation schemes have been considered in the pre-contingency state of the system, especially in the preventive mode. Topology was varied and the generation scheme was also varied. So for the emergency detection tree, the problem is that the attributes we take are the attributes just after the disturbance. What we intend to do is to include topology attributes as well, and we will see whether the method automatically selects those attributes as being relevant to the problem.

Robert A. Schlueter:

If you look at a PV curve at a fixed point of observation, you can add transfer or loading and get a collapse 200 - 400 miles away. This is due to the transfer increase, but the collapse is not in the area where you monitor the voltage.

On the PV curve, you are choosing to look at one output as a function of the transfer level. What you really need to do

is to understand what controls are exhausted in each geographical area. As a function of that transfer increase and in each of those geographical areas there is a certain level of control, which when exhausted the system will collapse. So ignoring control changes, which has been suggested, is really looking at a fictitious system. It is not really what is collapsing if you fix the controls you have to monitor. Keep the control activated in order to get a picture of the true collapse. In summary, one has to understand structurally what is having reactive problems.

Lester H. Fink:

I was not objecting to constant power or constant anything else load models. When we set up a model to investigate or explore a question, the investigator is free to specify whatever model he wants. You are abstracting from the real world in order to focus on a key issue and then get an answer that you can relate back to the real world. My question was to the latter point. I was asking for help in making that translation: given that is the curve that your model gives, how do I interpret it in terms of the real system?

Carson W. Taylor:

We have a serious voltage stability problem in the Puget Sound area. We went through a lot of studies on our load modeling. Specifically, we used the EPRI LOADSYN software, putting in a very detailed representation of the subtransmission over a thousand load buses in the Seattle/Tacoma area. For dynamics that is representing equivalents for induction motors: approximately 50-60 induction motors modeled dynamically. We have also done some system tests where we have switched reactors on and capacitors off to lower the voltage by 5% on the EHV level and monitored the load response at a number of locations.

On the question of capacitor application, it is interesting to take the extreme case of whether or not you spend a lot of money for a static VAR compensator. If you do that, you are certainly going to hold the voltage at 1 per unit until the SVC hits its limit. Then the system would probably collapse. If you are worried about the voltage criteria being able to monitor voltage, you would not want to apply a static VAR compensator. That would make your voltage stability problem worse. The bottom line is that voltage in many cases is not a good indication of impending voltage collapse, and one really needs to look at things like the reactive reserve.

Kwang Y. Lee:

I commented on the Jacobian being an important matrix, with a role in determining the stability of the system. What we want to do is to make this block matrix an M matrix. By requiring it to be an M matrix we can obtain the stability criteria. This is not a vector equation; it is a scalar equation for each bus i . If you want to do it in a vector framework, then we can solve everything simultaneously, where this is really coupled. However, our experience shows that by assigning reasonable stability margins for each bus, you are able to maintain the stability profile.

The load model is very important. There is a particular term for the load connected to bus i . Therefore, if you have a description for the load at a particular bus, then you can add that into the term. On reflecting the stability margin, I showed that our method can also be utilized for a control purpose. For example, if you have a capacitor bank, that is a capacitive load that can be controlled to enhance the stability. (Refer to K. Lee's paper in the proceedings.)

Robert K. Johnson (Western Area Power Administration):

One reason to introduce a constant power load is to facilitate the computation. Another reason is that in many cases, at least in the long term, constant power is really what the system sees. For example, when we talk about a constant power load, what we are trying to describe is what happens to the voltage across the terminals of the load in the system. If the voltage browns a little bit and the lights dim, what do I do? I turn more lights on. If my load is an oven and I lower the voltage, the thermostat starts sucking up more power. To the system the load looks constant, at least in the long time frame.

Most loads will react to varying system voltages to produce constant loads. The power system can not see you plug in your toaster or turn on your lights. The levels are too small. Overall to the transmission system it looks like that load is constant. If the transmission system lowers the bus voltage for some reason then the current goes up. That is what happens and that is why one models it that way. With respect to the PV curve, at least for the one generator/one load example, what it means is that the voltage at the device terminals is completely dependant on how much power you draw.

Peter W. Sauer:

I would like to reply to Les Fink's question about what the instability point A means in the physical system. Much of my answer is that I really do not have a clue, because first of all the model was non-physical. If you start at a point and create a disturbance to go to another point, you do hit a point where the constant load power model will cause the trapezoidal rule to fail. Therefore, that is why we sought models that do admit solutions.

The other thing I wanted to comment on is stabilizers. There are no stabilizers in the model. The stabilizers were designed to stabilize the system. For the electromechanical mode of instability, people have been able to eliminate regions of instability with power system stabilizers. That was geared toward the problems of shaft dynamics. In this particular case, the mode that is unstable is not electromechanical shaft dynamics, so it is not obvious whether a power system stabilizer would help. But there may be a need for other stabilizers to be designed that would eliminate this instability region.

Marija Ilic:

I think it is pretty much an identical question that Carson Taylor and Kwang Lee had. The voltage is not a good measure of voltage problems. I fully agree that the vector D that we introduced is in some way a measure of the reactive reserve. That is a much better measure, and that measure is almost an identical measure that Kwang Lee had. The difference is that with the nonlinear approach one can handle more than can be handled with the linearized model.

Session 4: Tuesday Evening

SESSION 4:

Dynamic/Static Voltage Stability Security Criteria

R. A. Schlueter, T-Y. Guo (Michigan State University), I-P. Hu (General Electric Co.)

Security Analysis and Control - Steady State

Applications of the Optimal Power Flow to Analysis of Voltage Collapse Limited Power Transfer

R. R. Austria, N. D. Reppen (PTI), J. A. Uhrin, M. Patel, A. Galatic (Allegheny Power Service, Corp.)

Voltage Stability Margin Enhancement via Optimal Power Flow

M. M. Begovic, X. Y. Chao, A. P. S. Meliopoulos (Georgia Institute of Technology)

An Approach to Study Steady-State Voltage Stability of Large Scale Power Systems

C. Christy (Commonwealth Edison Co.), V. Ajjarapu, B. Srinivasu (Iowa State University)

A Newton-Raphson Method for Steady State Voltage Stability Assessment

E. Carpaneto, G. Chicco, R. Napoli, F. Piglione (Politecnico di Torino)

An Artificial Neural-Net Based Approach to Monitoring Power System Voltage Instability

H. Mori (Meiji University), Y. Tamaru (Tokyo Electric Power Co.)

Overvoltage Control During Restoration

M. M. Adibi (IRD Corp.), R.W. Alexander (PP&L Co.), B. Avramovic (ECC, Inc.)

Discussion

by

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1. INTRODUCTION

Security criteria for voltage stability have been based on:

- (a) load flow models rather than transient midterm stability models;
- (b) point of collapse methods that attempt to establish a bifurcation point by increasing loads or transfers. A margin is then provided in terms of the additional load or transfer that can be added before loss of voltage stability will occur;
- (c) specification of a voltage stability criterion in terms of one or more of the following:
 - (i) requirement that voltage at the bifurcation or point of collapse be below $V_k^0 = .95$ pu,
 - (ii) requirement that the voltage $\hat{V}_k$ after the first contingency be a certain percentage above V_k^0 or a certain percentage below the precontingency voltage V_k^* . The bus or buses k where the criterion is specified are those that are most sensitive to voltage drop or have the smallest margin in terms of reactive load stress,
 - (iii) specification that the margin expressed in terms of additional load (real or reactive) or transfer stress to the point of collapse be above a certain threshold after the first contingency. This threshold can be a specified level and/or a percentage above the value at the bifurcation point.

These voltage stability criteria can not explicitly be assured of protecting the system against dynamic bifurcations of the transient midterm stability model since they were developed based on load flow model. This paper develops security criteria that can protect the system against three dynamic bifurcations. One of these three dynamic bifurcations, saddle node bifurcations, has been investigated in the literature on voltage stability [2]. Two additional dynamic bifurcations of the transient stability model, flux decay and Hopf bifurcation of the mechanical dynamics, are shown to occur as a result of approaching bifurcation in the network algebraic equations of the transient stability model. A second type of bifurcation of the transient

stability model is identified to be algebraic bifurcation, as opposed to dynamic bifurcation (saddle node, Hopf, flux decay). Algebraic bifurcation occurs when the jacobian of the network real and reactive power balance equations at generator terminal, generator high side transformer buses, and load buses are row dependent. If the rows of the jacobian at the generator internal buses belong to the set of network jacobian rows that are row dependent, then one of the three dynamic bifurcations may occur before algebraic bifurcation occurs. There are three different subclasses of algebraic bifurcation identified:

- voltage bifurcation, where the rows of the network jacobian associated with reactive power balance equations at generator terminal, generator transformer high side, and load buses are row dependent;
- angle bifurcation, where the rows of the network jacobian associated with real power balance equations at generator terminal, generator transformer high side, and load buses are row dependent;
- voltage and angle bifurcation, where the rows of the network jacobian associated with real power and reactive power balance equations at generator terminal, generator transformer high side, and load buses are row dependent.

Angle bifurcation is shown to be associated with reaching real power transfer limits across interfaces or boundaries without observing significant voltage change at network buses. Voltage bifurcation is shown to be voltage collapse at network buses where no significant change in branch angle differences are observed to occur. Voltage and angle bifurcation is shown to be voltage collapse where both bus voltage and branch angle differences change significantly. It should be noted that algebraic bifurcation could be viewed as load flow bifurcation, but results are presented in this paper that show the point of collapse methods applied to a load flow and to the transient midterm stability model can give very different results.

A third type of voltage collapse must be accounted for in voltage stability security criteria in addition to algebraic and dynamic bifurcations. This third type of voltage collapse results in loss of load due to voltage that drops below a threshold

(typically .88 pu) and causes relays on large motors to trip. The loss of load will generally prevent dynamic or algebraic bifurcations from occurring but is certainly of very significant concern to the utility. A voltage collapse security criterion must be able to assure that none of the three types of voltage collapse will occur.

Present voltage collapse security criteria attempt to prevent algebraic bifurcation (Type IIA) and load dropping voltage collapse (Type I) but do not directly attempt address dynamic bifurcation voltage collapse (Type IID) [40]. The types of voltage instability were proposed in a very insightful paper by Tamura [40] and is slightly modified in this paper. An indirect method of attempting to prevent dynamic bifurcation voltage collapse is to provide sufficient margin using current load flow based security criteria. There has been no proof, theoretically or practically, that dynamic bifurcation voltage collapse will be prevented in this manner. Furthermore, there are two major difficulties with current voltage collapse security criteria for preventing Type I and Type IIA voltage collapse:

- (1) a load flow model is used rather than an approximate transient midterm stability model;
- (2) the voltage collapse security criteria do not take into account how many different modes of collapse can occur. Voltage collapse security criteria should be developed only after an exhaustive search has been performed to find the numerous Type I and IIA voltage collapse modes.

Section 2 of this paper develops a transient midterm stability model that has sufficient complexity to be able to analyze all three types of bifurcation. The model includes generator flux decay, exciter, power system stabilizer, governor-turbine, and mechanical dynamics. The generator modeling includes magnetic saturation, load drop compensation, and field current limits based on synchronous generator power capability. Q-V curves are computed based on this general system model using the EPRI Transient Midterm Stability Program. Q-V curves are also computed using the EPRI load flow program. The results are shown to be quite different due to the following factors:

- the load flow model assumes that the exciter gain is infinite and that the generator magnetic saturation can be neglected so that voltage regulation is perfect;
- the load flow model assumes that voltage regulation of the exciter occurs at either the generator terminal bus or at the generator high side bus rather than at a bus or fictitious point in the network dictated by the load drop compensator;

- the load flow model assumes reactive limits that are generally conservative compared to field current limits.

It is shown [32] that the load flow Q-V curve need not always be conservative in predicting voltage collapse compared to a Transient Midterm Stability model Q-V curve. The poorer voltage regulation of this latter model can sometimes cause the margin to the point of collapse predicted by the Transient Midterm Stability model to be less than for the load flow model. A "dynamic" load flow was developed as part of EPRI project RP-3040 that not only computes the dynamic equilibrium by better modeling the generator and exciter combination but allows computation of governor, exciter, tap changer, and automatic generator control load flows [6,42,43].

The relationship between load flow and midterm stability model is discussed in Section 3 along with definitions of jacobian and sensitivity matrices for each. Section 4 defines PQ controllability for both the transient midterm stability model and load flow model required later to justify and relate the various tests for voltage collapse.

The load flow based voltage stability criteria described earlier in this Introduction section, are based on either:

- (a) P-V curves where transfer is increased across some interface and the voltages are monitored at some set of buses that are particularly sensitive to the transfer in terms of voltage decline;
- (b) P-V curves where total system load is increased at some set of buses in a particular pattern and compensated for by a set of generators in some particular participation pattern [33,34,35]; or
- (c) reactive load is added at some bus or at some set of buses in some pattern [33,39,41].

A utility could base it's voltage collapse security criterion on one such stress test (eg: Q-V curve at one bus in the system) or several of the same type (P-V curves for transfers across several different interfaces). The security criterion specifies the conditions discussed earlier on the P-V or Q-V curves given by each of the set of stress tests that is deemed sufficient to prevent collapse for that utility. Research has shown that for the utilities studied there are numerous modes of collapse; i.e. eigenvalues of transient midterm stability model will approach the $j\omega$ axis or zero and lead to one of the three types (Type I, Type IIA, Type IID) of voltage collapse. Each of these modes of collapse is associated with a unique group of buses and their inability to obtain the reactive power required for incremental voltage drop to compensate for the reactive losses due to this voltage drop. Each group

of buses is protected by a unique set of reactive supply or voltage control devices whose reserves are exhausted at the point of collapse (minimum of a Q-V curve or nose of a P-V curve). The reactive supply and voltage control resources that protect a group of buses is called a reactive reserve basin. An exhaustive search procedure for finding feasible modes of collapse and their reactive reserve basins is necessary to assure that a sufficient number of independent stress tests are incorporated in the security criterion developed in Section 6 of the paper.

A review of existing methods [33,34,35,39,41] of assessing proximity to the point of collapse is performed in Section 5. A classification of bifurcations in terms of algebraic and dynamic is performed based on modification of those in [40]. A subclassification of algebraic bifurcations into angle, voltage, and angle and voltage bifurcations is justified for the first time based on the rows of the network jacobian that are row dependent and the kinds of tests used to detect it. Finally, subclassification of dynamic voltage collapse bifurcations into saddle node, Hopf, and flux decay is performed for the first time. Furthermore, this is the first time that flux decay [32] and Hopf bifurcations of mechanical dynamics have been shown to occur for voltage collapse conditions. Each of the various sensitivity, point of collapse, Lyapunov methods for assessing proximity to collapse are reviewed and classified in terms of the collapse types and subclassifications each can identify. The security criteria developed need to protect against each type and subclass of bifurcation. Thus the results of Section 5 are needed to properly formulate security criteria. One also needs to develop point of collapse methods that are convergent, require little computation, and are flexible enough to detect all modes of collapse. Thus review of the various point of collapse methods is essential to developing security criteria.

A voltage collapse security criterion is proposed in section 6 of the paper. This voltage collapse security criterion utilizes a new point of collapse method that detects the closest bifurcation type (Type I, Type IIA, Type IID) along any stress pattern. Stress patterns could include:

- adding reactive load at a single bus or in some pattern at several buses;
- adding real power load at some set of buses in a particular pattern and compensating for it at some set of generators based on a particular participation pattern;

- increasing transfer across one interface or increasing transfers across several interfaces in some pattern.

The new voltage collapse security criterion also utilizes a systematic procedure for searching for all the feasible voltage collapse modes. This systematic procedure has been initially developed for detecting Type I and Type IIA voltage collapse using the Q-V curve point of collapse method. The reactive supply and voltage control resources that are exhausted at each Q-V curve minima are determined. Q-V curves are required at less than 6% of the buses but the method finds numerous modes of collapse that have been undetected in less systematic procedures. This procedure can be enhanced from one that can only detect Type I and Type IIA voltage collapse to all three voltage collapse types if the enhanced Q-V curve based procedure for determining the closest Type I, Type IIA, and Type IID voltage collapse were substituted for the Q-V curve.

Two new dynamic/static voltage collapse security criteria are proposed. Both criteria specify procedure for determining the voltage at broad flat Q-V curve minima, that the minima is above .95 pu, and that the voltage after a first contingency is some percentage of the precontingency voltage or above the voltage at the Q-V curve minima. These specifications are given for each of the feasible modes of collapse (or reactive reserve basins) determined by the systematic procedure. The difference between the two security criteria is that one (Q-V margin security criterion) measures reactive margins in terms of the reactive load stress that is added at the bus where the Q-V curve is computed. The second (reactive reserve basin security criterion) specifies the reactive margin in terms of the percentage and distribution of the reactive supply or voltage control reserves in the reactive reserve basin. The voltage control and reactive supply devices that belong to each reactive reserve basin are computed as part of the systematic procedure for determining the feasible modes of collapse. The two security criteria specify (1) the maximum reactive margin (and it's distribution) expended for the first contingency and (2) the minimum reactive margin (and it's distribution) that must remain to protect the system against any second contingency or operating change after the first contingency.

2. MODEL DEVELOPMENT

2.1 Introduction

A general power system model, which includes mechanical dynamics, flux decay dynamics, and excitation, power system stabilizer, and governor

turbine system dynamics of a generator and real and reactive power balance equations for each network bus, is developed in this chapter. This model can be used to test for every algebraic voltage instability and dynamic voltage instability currently under study. Previous models neglect field current limits, generator saturation, load drop compensator, and governor-turbine dynamics, and stabilizer dynamics. Generally, the load flow equilibrium and the general power system model equilibrium are shown to diverge at the point of collapse. Finally, it is shown that the assumed conservatism of a load flow model, in predicting proximity to loss of voltage stability, may not always be valid. The assumed load flow conservatism is based on its conservative approximation to field current limits. The factors that affect the relative voltage collapse insecurity in the load flow and general power system model is discussed. Finally, a dynamic load flow that has a better approximation to the controls in a general power system model is discussed.

There are two different kinds of nonlinear equations in this general power system model. One is a set of nonlinear differential equations which represents the dynamics of the generator. The other is a set of nonlinear algebraic equations which represents the real and reactive power balance equation for each bus in the network.

Differential Equation Model

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{\lambda}(t)) \quad \mathbf{\lambda}(t) \in [\mathbf{\lambda}_a, \mathbf{\lambda}_b]$$

Algebraic Equation Model

$$\mathbf{0} = \mathbf{g}(\mathbf{x}(t), \mathbf{z}(t), \mathbf{\lambda}(t))$$

where

$\mathbf{x}(t) = (\delta, \mathbf{E}'_q, \mathbf{y})$: state vector of the generator dynamics

$\mathbf{z}(t) = (\mathbf{E}_L, \mathbf{V}_L)$: state vector of bus voltage and angle of terminal buses, high side transformer buses, and load buses

$\mathbf{\lambda}(t)$: the set of slow varying operating parameters that change over time. $\mathbf{\lambda}(t)$ can be used to represent real and reactive power load, generation dispatch, under load tap changers, and switchable shunt capacitors. As $\mathbf{\lambda}(t)$ varies over $[\mathbf{\lambda}_a, \mathbf{\lambda}_b]$, there is assumed to be at least one equilibrium point for each $\mathbf{\lambda}$, $(\mathbf{x}_0(\mathbf{\lambda}), \mathbf{z}_0(\mathbf{\lambda}))$.

A necessary condition for the system to experience system voltage instability is that the jacobian

$$\mathbf{J}^{***} = \begin{bmatrix} \frac{\partial \mathbf{f}}{\partial \mathbf{x}} & \frac{\partial \mathbf{f}}{\partial \mathbf{z}} \\ \frac{\partial \mathbf{g}}{\partial \mathbf{x}} & \frac{\partial \mathbf{g}}{\partial \mathbf{z}} \end{bmatrix}$$

be singular at some $\mathbf{\lambda}_0$ and $(\mathbf{x}_0(\mathbf{\lambda}_0), \mathbf{z}_0(\mathbf{\lambda}_0))$.

Section 2.2 discusses the derivation of each element in the general power system model. The air gap saturation, line drop compensation, field current limit, and excitation system control are discussed in detail. Section 2.3 presents a general power system model which has l generator terminal buses, m generator high side transformer buses, and n load buses.

2.2 Single Bus System Model

The differential equation model includes the mechanical dynamics, flux decay dynamics, excitation system dynamics, and governor turbine dynamics of the generator. The air gap saturation, field current limit, and line drop compensator should be included in this model.

2.2.1 Mechanical Dynamics

$$M\dot{\delta} + D\dot{\delta} = P^e - F_G(\delta, \mathbf{V}, \theta, \mathbf{y}; \mathbf{E})$$

where

M : generator per unit inertia constant

D : generator load damping coefficient

δ : internal bus angle with respect to a synchronous rotating reference frame

P^e : electrical power at the internal bus

F_G : network equation at generator internal bus

The nonlinear differential equation is called the swing equation because it is the same equation which describes the "swinging" of a pendulum in a uniform gravitational field. It describes the "swings" in the power angle δ during a transient.

2.2.2 Flux Decay Dynamics

The flux decay equation for each generator is

$$\begin{aligned} r'_{d0} \dot{\mathbf{E}}'_q &= E_{fd} - E_q - E_{fd} - (E'_q + I_d(x'_d - x'_d)) \\ &= E_{fd} - \left[E'_q + \frac{Q^e}{E'_q} (x'_d - x'_d) \right] \\ &= E_{fd} - \frac{x'_d E'_q}{x'_d} + \frac{(x'_d - x'_d) V \cos(\delta - \theta)}{x'_d} \end{aligned}$$

where

E_q : internal generator voltage proportional to field flux linkage behind steady state direct axis reactance

E'_q : internal generator voltage proportional to field flux linkage behind transient direct axis reactance

E_{fd} : generator field voltage

I_d : projection of terminal current on direct axis

V : generator terminal voltage

x'_d : transient direct axis reactance

x_d : steady state direct axis reactance

Q^e : generator reactive power generation at the internal bus if $x_q = x'_d$

r'_{d0} : generator direct axis transient open circuit time constant

The model that includes air gap saturation is

$$\begin{aligned}\tau'_{d0} \dot{E}'_q &= E_{fd} - (1 + S_D(E_p)) E_q \\ &= E_{fd} - (1 + S_D(E_p)) \left(E'_q + \frac{I_d(x_d - x'_d)}{1 + S_D(E_p)} \right) \\ &= E_{fd} - (1 + S_D(E_p)) E'_q - \frac{Q_{GE}(x_d - x'_d)}{E'_q}\end{aligned}$$

$$S_D(E_p) = \frac{B(E_p - A)}{E_p}$$

$$V_d = V_d - I_q x_p = V_d - \frac{V_d x_p}{x_{qs}}$$

$$V_q = V_q + I_d x_p = V_q - \frac{(E_q - V_q) x_p}{x'_{ds}}$$

$$E_p = \sqrt{V_d^2 + V_g^2}$$

E_p : Voltage behind the potier leakage reactance

$$S_{GL,0} = \frac{B(1.0-A)^2}{1.0} \quad \text{and} \quad S_{GL,2} = \frac{B(1.2-A)^2}{1.2}$$

2.2.3 Excitation System Dynamics

Block diagram of a closed-loop control system for a power system. The reference voltage V_{REF} is compared with the feedback voltage V_1 at a summing junction. The error signal is processed by a controller block $K_A/(1+sT_A)$, then an amplifier (represented by a gain block with a saturation nonlinearity), and a summing junction to produce the reference voltage V_R . V_R is fed into the Exciter block $1/(K_E+sT_E)$ and the Stabilizer block $sK_F/(1+sT_F)$. The Exciter output E_{fd} is fed into the generator block $G(s)$ and also summed with the output of the Stabilizer block. The generator output V is fed back through a Measurement block $1/(1+sT_R)$ to produce V_1 . A feedback path for line drop compensation is also shown, where V is summed with V_1 to produce V_c , which is then fed into a block $V_c = |V + jx_c I|$.

If we write a set of state equations to represent excitation system dynamics we get the following state equations.

$$\dot{V}_1 = \frac{-V_1 + |V + jx_c I|}{\tau_p}$$

$$V_3 = \frac{1}{\tau_F} \left(-V_3 + \frac{K_F V_R}{\tau_F} - \frac{K_F E_{fd} (S_E(E_{fd}) + K_E)}{\tau_F} \right)$$

$$\dot{V}_R = \frac{1}{\tau_A} (K_A (V_{REF} - V_1 - V_3) - V_R)$$

$$\dot{E}_{fd} = \frac{1}{\tau_E} (V_R - (S_E(E_{fd}) + K_E) E_{fd})$$

V_1 :	measured voltage
V_3 :	stabilizer output voltage
V_R :	amplifier output voltage
E_{fd} :	exciter field voltage from exciter
τ_R :	regulator input filter time constant
τ_F :	regulator stabilizing circuit time constant
τ_E :	exciter time constant
τ_A :	amplifier input filter time constant
x_c :	line drop compensation reactance
K_F :	regulator stabilizing circuit gain
K_E :	exciter self-excitation at full load field voltage
K_A :	amplifier circuit gain
S_F :	rotating exciter saturation at ceiling voltage

V_{REF} : exciter voltage reference

The reactive generation limit of the load flow model is decided by the synchronous machines capability curve. The capability curve of a synchronous machine shows the limits placed on the electrical watts and VARs (a) by the permissible temperature rise of the rotor and stator, (b) by stability of the mechanical system connected to the shaft, and (c) assuming operation at rated terminal voltage. Figure 2.3 shows a typical capability curve for a generator, plotted on the S plane, where P is the vertical axis and Q is the horizontal axis. Operation within the boundaries of the curve is safe from the standpoints of heating and stability. The maximum reactive generation is decided by the level of real power generation. Since the real power generation is fixed in the load flow model, the maximum reactive power generation could be decided by the capability curve at that specific real power generation level.

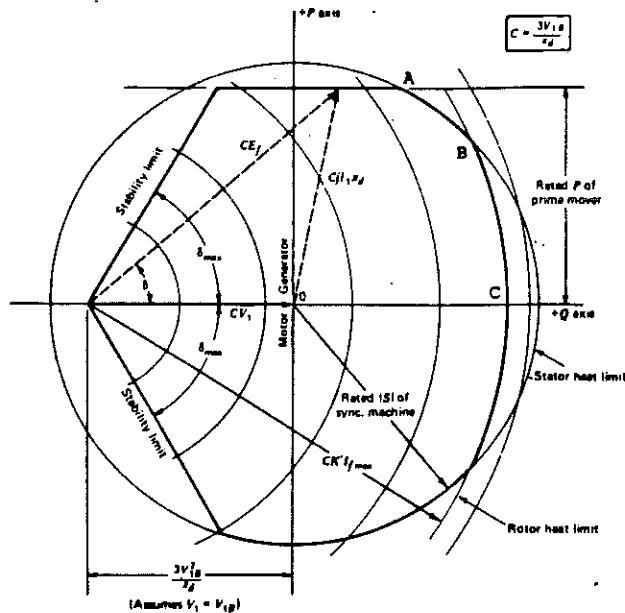


Figure 2.3 Generator capability curve

The actual load flow reactive generation limit is specified as some approximation to the field current limit in the transient stability model. The field current limit which is the rotor heat limit, is specified at the intersection of the stator heat limit and rotor heat limit. This intersection point is point B in Figure 2.3. This point B is specified in terms of the stator heat limit $|S|$ and the power factor of the generator. The stator current phasor can be calculated from this information, and the voltage at the internal bus can be calculated. Since the magnitude of the generator internal voltage is proportional to the field current, the maximum field current can be calculated. It should be noted that

this is the continuous rating level. The field current can exceed this continuous rating level for a short period as long as the field current capability curve shown in Figure 2.4 is not exceeded.

A field current capability curve relates the field current value to the duration of the interval during which the field current exceeds that value.

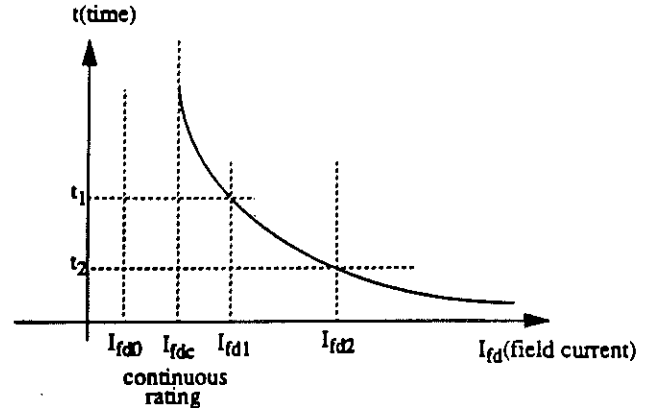


Figure 2.4 Field current limit controller

If the field current exceeds the field current limit, the generator rotor will over heat and a field current limit control automatically disables the excitation system control and brings the field current down to a level (continuous rating) that can be sustained indefinitely without overheating the generator rotor. For example, in Figure 2.4, if the field current is at I_{fd2} , the time period that allows field current to remain at that value is t_2 . After that period of time (t_2), the field current will be brought down to its continuous rating to prevent any damage of the exciter. If the field current is lower than its continuous rating (I_{fdc}), it can remain at that value for infinite time. The action of the field current control will be approximated by setting K_a to zero and setting E_{fd} to the value appropriate to produce the continuous rating of field current. This model of the field current limiter is not available in all transient stability programs and this approximate model is quite adequate for assessing steady state retention or loss of voltage stability. The reactive generation limit in a load flow is generally taken as points A, B, or C in Figure 2.3 depending on the application and operating condition. If point A is chosen, the generator can produce its maximum real power generation without exceeding the stator heat limit or rotor heat limit. However, the generator could produce far more reactive generation if the generator operated at a real power generation level far below the maximum real power generation level. Point A is a conservative choice for reactive generation limit. If the reactive generation limit

was chosen to be at point B, the continuous rating rotor field current limit, the stator heat limit would be exceeded if the generator produced real power above that at point B, $P_B = (S)(\text{Pf}_b)$. If point C is chosen, the stator heat limit and rotor heat limit would be exceeded if real power is generated and the stator heat limit would be exceeded for real power generation levels above P_B . Note that although the transient stability model can exceed the rotor heat limit for short periods without having to modify the excitation control, the load flow must utilize the rotor heat limit since a load flow is a model of the steady state operating condition of the generator. If the rotor heat limit were ever exceeded in the steady state operating condition, physical damage to the generator would result. The load flow reactive generation limit on a generator is often chosen at point A since the rotor heat limit and stator heat limit will never be exceeded. The reactive generation limit was always chosen as point A in this case. Sometimes, point B can be used to specify the rotor heating limit as long as the generator is operating far below P_B . Point C might be used for a synchronous condenser. It should be noted that the transient stability model can produce more reactive power in steady state based on the rotor heat limit in Figure 2.3 than the reactive power limit based on point A in Figure 2.3.

Furthermore, the transient stability model can produce more reactive power than the rotor heat limit as long as the field current level and duration are not above the field current capability curve of Figure 2.4. Thus, the load flow would appear to be conservative. The load flow's conservatism in predicting proximity to voltage collapse depends on:

- how effective the generator excitation control is in controlling the voltage at some point in the network called the control point;
- how the load flow approximates the actual control point determined from the load drop compensator.

The voltage control point is decided by the setting of line drop compensation in the power system. The voltage control point can be at a point between internal bus and terminal bus, terminal bus and transformer high side bus, or out in the network. The voltage control point out in the network is generally less vulnerable as long as the control is maintained but the control will be lost more quickly. Conversely, the voltage control point close to the generator internal bus is more vulnerable (less effective in controlling voltage) but the control is lost more slowly. There are generally two possible control points in the load flow model, which are the generator terminal bus and transformer high side bus. If the actual voltage control point is chosen closer

to the generator internal bus than that is modeled in the load flow, the load flow modeling would make the system appear less vulnerable to voltage problems as long as the voltage control is active since network voltage is more effectively controlled than that in the transient stability model. However, the generator would generally lose its reactive generation reserve in the load flow due to the more effective control of the network than in the transient stability model. Losing the control of voltage due to exhaustion of reactive generation reserves in the load flow and not in the transient stability model generally makes the load flow far more vulnerable to loss of voltage stability than the transient midterm stability model.

2.2.4 Power System Stabilizer Model

The function of power system stabilizer (PSS) is mainly to improve dynamic stability of power system, using other regulator input signals in addition to terminal voltage. These signals are chosen to provide positive damping to oscillations. Figure 2-5(a) shows the general model of such a power system stabilizer. Some common stabilizer input signals are: accelerating power, speed, frequency and terminal voltage. Transferring the lead-lag blocks and washout block into first order lagging blocks, as shown in Fig. 2-5(b), we have the state space form for PSS model:

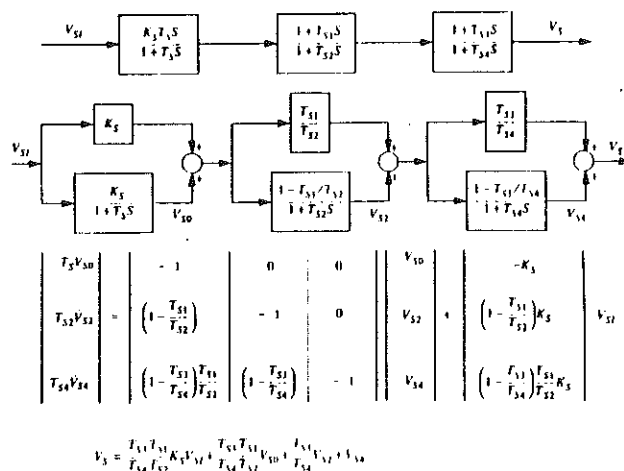
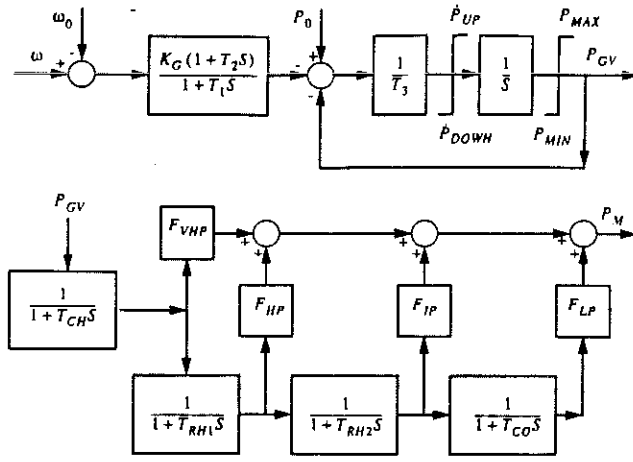


Figure 2.5 General model (2.5a) and state space representation (2.5b) of a power system stabilizer

2.2.5 Speed-Governing-Turbine System Model

The mechanical power P_M is controlled by the speed-governing system through turbine system, using rotor speed as an input signal. Even though there are mechanical-hydraulic control (MHC) and electro-hydraulic control (EHC) types of speed-governing system, basically, their models are very similar. Figure 2-6(a) and Figure 2-6(b) are the general models of speed-governing system for steam-turbine and hydro-turbine, respectively [46].



Steam
Hydro

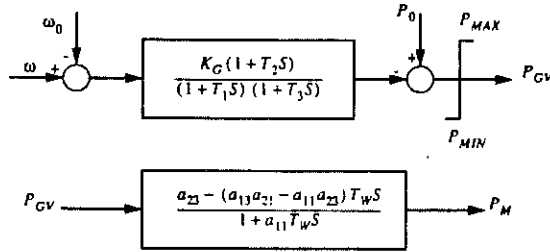


Figure 2.6 Speed-Governing-Turbine Model [46] for a steam (2.6a) and hydro (2.6b) turbine system

P_{GV} is the power at gate or valve and is the input of turbine system. The general models of steam-turbine and hydro-turbine system are shown in Fig. 2-7(a) and Fig. 2-7(b), respectively [46].

The state space model of speed-governing-turbine system can be obtained by combining the models of speed-governing and turbine systems together.

2.2.6 Algebraic Equation Model

The algebraic equation model is the real and reactive power balance equation which represent the power flow at each bus in the network. If the air gap saturation is considered at the generator internal bus and terminal bus, the algebraic equation model is:

$$S_D = \frac{B(E_p - A)^2}{E_p}$$

$$S_Q = \frac{x_q S_D}{x_d}$$

$$x'_{ds} = \frac{x'_d - x_p}{1 + S_D} + x_p$$

$$x_{qs} = \frac{x_q - x_p}{1 + S_Q} + x_p$$

$$P^e = \frac{E_q V \sin(\delta - \theta)}{x'_{ds}} + \frac{V^2}{2} \left(\frac{1}{x_{qs}} - \frac{1}{x'_{ds}} \right) \sin(2(\delta - \theta))$$

$$= \frac{E_q V \sin(\delta - \theta)(1 + S_D)}{x'_d + S_D x_p} +$$

(a) Steam Governor-Turbine

$$\begin{bmatrix} T_1 \dot{P}_1 \\ T_3 \dot{P}_{GV} \\ T_{CH} \dot{P}_{VHP} \\ T_{RH1} \dot{P}_{HP} \\ T_{RH2} \dot{P}_{IP} \\ T_{CO} \dot{P}_{LP} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} P_1 \\ P_{GV} \\ P_{VHP} \\ P_{HP} \\ P_{IP} \\ P_{LP} \end{bmatrix}$$

$$+ \begin{bmatrix} K_G(1 - T_2/T_1)(\omega - \omega_0) \\ -K_G(T_2/T_1)(\omega - \omega_0) + P_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_M = F_{VHP} P_{VHP} + F_{HP} P_{HP} + F_{IP} P_{IP} + F_{LP} P_{LP}$$

(b) Hydraulic Governor-Turbine

$$\begin{bmatrix} T_1 \dot{P}_1 \\ T_3 \dot{P}_{GV} \\ T_W \dot{P}_W \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ 0 & a_{13} a_{21}/a_{11}^2 - 1/a_{11} \end{bmatrix} \begin{bmatrix} P_1 \\ P_{GV} \\ P_W \end{bmatrix}$$

$$+ \begin{bmatrix} K_G(1 - T_2/T_1)(\omega - \omega_0) \\ -K_G(T_2/T_1)(\omega - \omega_0) + P_0 \\ 0 \end{bmatrix}$$

$$P_M = (a_{23} - \frac{a_{13} a_{21}}{a_{11}}) P_{GV} + P_W$$

Figure 2.7 State space representation of a steam (2.7a) and hydro (2.7b) speed governor-turbine system

$$\begin{aligned} & \frac{V^2}{2} \left(\frac{x_d + x_q S_D}{x_d x_q + x_q S_D x_p} - \frac{(1 + S_D)}{x'_d + S_D x_p} \right) \sin(2(\delta - \theta)) \\ Q^e &= \frac{2E_q V \sin(\delta - \theta) - E_q V \cos(\delta - \theta) - V^2 \sin^2(\delta - \theta)}{x'_{ds}} + \\ & \frac{x_{qs}(E_q - V \sin(\delta - \theta))^2}{x'^2_{ds}} \\ &= \frac{(1 + S_D)}{x'_d + S_D x_p} (2E_q V \sin(\delta - \theta) - E_q V \cos(\delta - \theta) - V^2 \sin^2(\delta - \theta)) + \\ & \frac{(x_d x_q + x_q S_D)(1 + S_D)^2 (E_q - V \sin(\delta - \theta))^2}{(x'_d + S_D x_p)^2 (x_d + x_q S_D)} \\ P_G &= \frac{E_q V \sin(\delta - \theta)}{x'_{ds}} + \frac{V^2}{2} \left(\frac{1}{x_{qs}} - \frac{1}{x'_{ds}} \right) \sin(2(\delta - \theta)) \\ &= \frac{E_q V \sin(\delta - \theta)(1 + S_D)}{x'_d + S_D x_p} + \\ & \frac{V^2}{2} \left(\frac{x_d + x_q S_D}{x_d x_q + x_q S_D x_p} - \frac{(1 + S_D)}{x'_d + S_D x_p} \right) \sin(2(\delta - \theta)) \\ Q_G &= \frac{E_q V \cos(\delta - \theta)}{x'_{ds}} - V^2 \left(\frac{\cos(\delta - \theta)^2}{x'_{ds}} + \frac{\sin(\delta - \theta)^2}{x_{qs}} \right) \\ &= \frac{E_q V \cos(\delta - \theta)(1 + S_D)}{x'_d + S_D x_p} - \\ & V^2 \left(\frac{\cos(\delta - \theta)^2 (1 + S_D)}{x'_d + S_D x_p} + \frac{\sin(\delta - \theta)^2 (x_d + x_q S_D)}{x_d x_q + x_q S_D x_p} \right) \end{aligned}$$

$$P_G - P_H^d = \sum_{j=1}^n V_H V_{Lj} Y_{Lj} \cos(\theta - \theta_{Lj} - \gamma_{Lj})$$

$$Q_G - Q_H^d = \sum_{j=1}^n V_H V_{Lj} Y_{Lj} \sin(\theta - \theta_{Lj} - \gamma_{Lj})$$

where

P^g : generator real power generation at internal bus

Q^g : generator reactive power generation at internal bus

P_G : real power injection at generator terminal bus

Q_G : reactive power injection at generator terminal bus

P_H^d : real power load at high side transformer bus

Q_H^d : reactive power load at high side transformer bus

S_D : generator saturation function applied to direct axis reactance

S_Q : generator saturation function applied to quadrature axis reactance

x_p : potier reactance

E_{qi} : internal bus voltage

δ : internal bus angle

V : terminal bus voltage

θ : terminal bus angle

V_H : high side transformer bus voltage

θ_H : high side transformer bus angle

V_L : load bus voltage

θ_L : load bus angle

2.3 General Power System Model

The variable and parameters that are used in this section are defined in LIST OF SYMBOLS in [32].

Indices i are for machines that have not hit field current limits and j are for machines that have hit field current limits.

$$\dot{\omega}_i = \left(\frac{1}{M_i} \right) (P_{Mi} - D_i \omega_i - P_i^g) \quad (2.1)$$

$$\dot{\omega}_j = \left(\frac{1}{M_j} \right) (P_{Mj} - D_j \omega_j - P_j^g) \quad (2.2)$$

$$\dot{\delta}_i = \omega_i \quad (2.3)$$

$$\dot{\delta}_j = \omega_j \quad (2.4)$$

$$\begin{aligned} \dot{E}_{qi} &= \left(\frac{1}{\tau_{d0i}} \right) \left(\left(\frac{-x_{dji} E_{qi}}{x_{dji}} + \frac{(x_{dji} - x'_{dji}) V_{Ti} \cos(\delta_i - \theta_{Ti})}{x'_{dji}} \right) (1 + S_{Di}) + E_{fdi} \right) \\ &= \frac{1 + S_{Di}}{\tau_{d0i}} \left(\frac{-(x_{dji} + S_{Di} x_{pi}) E_{qi}}{x'_{di} + S_{Di} x_{pi}} + \frac{(x_{dji} - x'_{dji}) V_{Ti} \cos(\delta_i - \theta_{Ti})}{x'_{di} + S_{Di} x_{pi}} \right) + \frac{E_{fdi}}{\tau_{d0i}} \\ &= \left(\frac{1}{\tau_{d0i}} \right) \left(E_{fdi} - (1 + S_{Di}) E_{qi} - \frac{Q_i^g (x_{di} - x'_{di})}{E_{qi}} \right) \end{aligned} \quad (2.5)$$

$$\begin{aligned} \dot{E}_{qj} &= \left(\frac{1}{\tau_{d0j}} \right) \left(\left(\frac{-x_{dji} E_{qj}}{x_{dji}} + \frac{(x_{dji} - x'_{dji}) V_{Tj} \cos(\delta_j - \theta_{Tj})}{x'_{dji}} \right) (1 + S_{Dj}) + E_{fdj} \right) \\ &= \frac{1 + S_{Dj}}{\tau_{d0j}} \left(\frac{-(x_{dji} + S_{Dj} x_{pj}) E_{qj}}{x'_{dj} + S_{Dj} x_{pj}} + \frac{(x_{dji} - x'_{dj}) V_{Tj} \cos(\delta_j - \theta_{Tj})}{x'_{dj} + S_{Dj} x_{pj}} \right) + \frac{E_{fdj}}{\tau_{d0j}} \\ &= \left(\frac{1}{\tau_{d0j}} \right) \left(E_{fdj} - (1 + S_{Dj}) E_{qj} - \frac{Q_j^g (x_{dj} - x'_{dj})}{E_{qj}} \right) \end{aligned} \quad (2.6)$$

$$\dot{V}_{1i} = \left(\frac{1}{\tau_{Ri}} \right) (-V_{1i} + |V_{Ti} + j(I_{Ti} x_{ci})|) \quad (2.7)$$

$$\dot{V}_{3i} = \left(\frac{1}{\tau_{Fi}} \right) \left(-V_{3i} + \frac{K_{Fi} V_{Ri}}{\tau_{Ei}} - \frac{K_{Fi} E_{fdi} (S_{Ei} (E_{fdi}) + K_{Ei})}{\tau_{Ei}} \right) \quad (2.8)$$

$$\dot{V}_{Ri} = \left(\frac{1}{\tau_{Ai}} \right) (K_{Ai} (V_{REFi} - V_{1i} - V_{3i}) - V_{Ri}) \quad (2.9)$$

$$\dot{E}_{fdi} = \left(\frac{1}{\tau_{Ei}} \right) (V_{Ri} - (S_{Ei} (E_{fdi}) + K_{Ei}) E_{fdi}) \quad (2.10)$$

$$\begin{aligned} P_i^g &= P_{Ti}^g = \frac{E_{qi} V_{Ti} \sin(\delta_i - \theta_{Ti}) (1 + S_{Di})}{x'_{di} + S_{Di} x_{pi}} + \\ T_{S1} \dot{V}_{S0i} &= -V_{S0i} - K_{S1} V_{S1i} \end{aligned} \quad (2.11)$$

$$T_{S2i} \dot{V}_{S2i} = (1 - T_{S3i}/T_{S4i}) V_{S0i} - V_{S2i} + (1 - T_{S1i}/T_{S2i}) K_{S1} V_{S1i} \quad (2.12)$$

$$T_{S4i} \dot{V}_{S4i} = (1 - T_{S3i}/T_{S4i}) (T_{S1i}/T_{S2i}) V_{S0i} \quad (2.13)$$

$$+ (1 - T_{S3i}/T_{S4i}) V_{S2i} - V_{S4i} \quad (2.14)$$

$$+ (1 - T_{S3i}/T_{S4i}) (T_{S1i}/T_{S2i}) K_{S1} V_{S1i} \quad (2.15)$$

$$V_{S1i} = (T_{S3i}/T_{S4i}) (T_{S1i}/T_{S2i}) K_{S1} V_{S1i}$$

$$+ (T_{S3i}/T_{S4i}) (T_{S1i}/T_{S2i}) V_{S0i}$$

$$+ (T_{S3i}/T_{S4i}) V_{S2i} + V_{S4i}$$

$$T_{1i} \dot{P}_{1i} = -P_{1i} + K_{G1} (1 - T_{2i}/T_{1i}) (\omega_i - \omega_0) \quad (2.16)$$

$$T_{3i} \dot{P}_{GV_i} = -P_{1i} - P_{GV_i} - K_{G1} (T_{2i}/T_{1i}) (\omega_i - \omega_0) + P_{0i} \quad (2.17)$$

$$T_{CHi} \dot{P}_{VHP_i} = P_{GV_i} - P_{VHP_i} \quad (2.18)$$

$$T_{RH1i} \dot{P}_{HP_i} = P_{VHP_i} - P_{HP_i} \quad (2.19)$$

$$T_{RH2i} \dot{P}_{IP_i} = P_{HP_i} - P_{IP_i} \quad (2.20)$$

$$T_{COi} \dot{P}_{LP_i} = P_{IP_i} - P_{LP_i} \quad (2.21)$$

$$P_{M_i} = F_{VHP_i} P_{VHP_i} + F_{HP_i} P_{HP_i} + F_{IP_i} P_{IP_i} + F_{LP_i} P_{LP_i}$$

or

$$T_{1i} \dot{P}_{1i} = -P_{1i} + K_{G1} (1 - T_{2i}/T_{1i}) (\omega_i - \omega_0) \quad (2.22)$$

$$T_{3i} \dot{P}_{GV_i} = -P_{1i} - P_{GV_i} - K_{G1} (T_{2i}/T_{1i}) (\omega_i - \omega_0) + P_{0i} \quad (2.23)$$

$$T_{W_i} \dot{P}_{W_i} = (a_{13i} a_{12i} / a_{11i}^2) P_{GV_i} - (1/a_{11i}) P_{W_i} \quad (2.24)$$

$$P_{M_i} = (a_{23i} - a_{13i} a_{21i} / a_{11i}) P_{GV_i} + P_{W_i}$$

$$\frac{V_{Ti}^2}{2} \left(\frac{x_{di} + x_{qi} S_{Di}}{x_{di} x_{qi} + x_{qi} S_{Di} x_{pi}} - \frac{(1 + S_{Di})}{x'_{di} + S_{Di} x_{pi}} \right) \sin(2(\delta_i - \theta_{Ti})) \quad (2.25)$$

$$P_j^g = P_{Tj}^g = \frac{E_{qj} V_{Tj} \sin(\delta_j - \theta_{Tj}) (1 + S_{Dj})}{x'_{dj} + S_{Dj} x_{pj}} +$$

$$\frac{V_{Tj}^2}{2} \left(\frac{x_{dj} + x_{qj} S_{Dj}}{x_{dj} x_{qj} + x_{qj} S_{Dj} x_{pj}} - \frac{(1 + S_{Dj})}{x_{dj} + S_{Dj} x_{pj}} \right) \sin(2(\delta_j - \theta_{Tj})) \quad (2.26)$$

$$P_{Ti}^* - P_{Ti}^d = V_{Ti} V_{Hi} Y_{TiHi} \cos(\theta_{Ti} - \theta_{Hi} - \gamma_{TiHi}) + V_{Ti}^2 Y_{TiHi} \cos(\gamma_{TiHi}) \quad (2.27)$$

$$P_{Tj}^* - P_{Tj}^d = V_{Tj} V_{Hj} Y_{TjHj} \cos(\theta_{Tj} - \theta_{Hj} - \gamma_{TjHj}) + V_{Tj}^2 Y_{TjHj} \cos(\gamma_{TjHj}) \quad (2.28)$$

$$\begin{aligned} -P_{Hi}^d &= \sum_{s=1}^l V_{Hi} V_{Hs} Y_{HiHs} \cos(\theta_{Hi} - \theta_{Hs} - \gamma_{HiHs}) + \\ &\sum_{j=1}^m V_{Hi} V_{Hj} Y_{HiHj} \cos(\theta_{Hi} - \theta_{Hj} - \gamma_{HiHj}) + \\ &\sum_{k=1}^n V_{Hi} V_k Y_{HiHk} \cos(\theta_{Hi} - \theta_k - \gamma_{HiHk}) + \\ &V_{Hi} V_{Ti} Y_{HiTi} \cos(\theta_{Hi} - \theta_{Ti} - \gamma_{HiTi}) \end{aligned} \quad (2.29)$$

$$\begin{aligned} -P_{Hj}^d &= \sum_{i=1}^l V_{Hj} V_{Hi} Y_{HjHi} \cos(\theta_{Hj} - \theta_{Hi} - \gamma_{HjHi}) + \\ &\sum_{s=1}^m V_{Hj} V_{Hs} Y_{HjHs} \cos(\theta_{Hj} - \theta_{Hs} - \gamma_{HjHs}) + \\ &\sum_{k=1}^n V_{Hj} V_k Y_{HjHk} \cos(\theta_{Hj} - \theta_k - \gamma_{HjHk}) + \\ &V_{Hj} V_{Tj} Y_{HjTj} \cos(\theta_{Hj} - \theta_{Tj} - \gamma_{HjTj}) \end{aligned} \quad (2.30)$$

$$\begin{aligned} -P_k^d &= \sum_{i=1}^l V_k V_{Hi} Y_{kHi} \cos(\theta_k - \theta_{Hi} - \gamma_{kHi}) + \\ &\sum_{j=1}^m V_k V_{Hj} Y_{kHj} \cos(\theta_k - \theta_{Hj} - \gamma_{kHj}) + \\ &\sum_{s=1}^n V_k V_s Y_{ks} \cos(\theta_k - \theta_s - \gamma_{ks}) \end{aligned} \quad (2.31)$$

$$\begin{aligned} Q_{Ti}^* &= \frac{(1 + S_{Di}) V_{Ti} (2E_{qi} \sin(\delta_i - \theta_{Ti}) - E_{qi} \cos(\delta_i - \theta_{Ti}) - V_{Ti} \sin^2(\delta_i - \theta_{Ti}))}{x_{di} + S_{Di} x_{pi}} + \\ &\frac{(x_{qi} x_{di} + x_{qi} x_{pi} S_{Di}) (1 + S_{Di})^2 (E_{qi} - V_{Ti} \sin(\delta_i - \theta_{Ti}))^2}{(x_{di} + S_{Di} x_{pi})^2 (x_{di} + x_{qi} S_{Di})} \end{aligned} \quad (2.32)$$

$$\begin{aligned} Q_{Tj}^* &= \frac{(1 + S_{Dj}) V_{Tj} (2E_{qj} \sin(\delta_j - \theta_{Tj}) - E_{qj} \cos(\delta_j - \theta_{Tj}) - V_{Tj} \sin^2(\delta_j - \theta_{Tj}))}{x_{dj} + S_{Dj} x_{pj}} + \\ &\frac{(x_{qj} x_{dj} + x_{qj} x_{pj} S_{Dj}) (1 + S_{Dj})^2 (E_{qj} - V_{Tj} \sin(\delta_j - \theta_{Tj}))^2}{(x_{dj} + S_{Dj} x_{pj})^2 (x_{dj} + x_{qj} S_{Dj})} \end{aligned} \quad (2.33)$$

$$\begin{aligned} Q_{Ti}^d &= \frac{E_{qi} V_{Ti} \cos(\delta_i - \theta_{Ti}) (1 + S_{Di})}{x_{di} + S_{Di} x_{pi}} - \\ &V_{Ti}^2 \left(\frac{\cos(\delta_i - \theta_{Ti})^2 (1 + S_{Di})}{x_{di} + S_{Di} x_{pi}} + \frac{\sin(\delta_i - \theta_{Ti})^2 (x_{di} + x_{qi} S_{Di})}{x_{di} x_{qi} + x_{qi} S_{Di} x_{pi}} \right) \end{aligned} \quad (2.34)$$

$$\begin{aligned} Q_{Tj}^d &= \frac{E_{qj} V_{Tj} \cos(\delta_j - \theta_{Tj}) (1 + S_{Dj})}{x_{dj} + S_{Dj} x_{pj}} - \\ &V_{Tj}^2 \left(\frac{\cos(\delta_j - \theta_{Tj})^2 (1 + S_{Dj})}{x_{dj} + S_{Dj} x_{pj}} + \frac{\sin(\delta_j - \theta_{Tj})^2 (x_{dj} + x_{qj} S_{Dj})}{x_{dj} x_{qj} + x_{qj} S_{Dj} x_{pj}} \right) \end{aligned} \quad (2.35)$$

$$Q_{Ti}^* - Q_{Ti}^d = V_{Ti} V_{Hi} Y_{TiHi} \sin(\theta_{Ti} - \theta_{Hi} - \gamma_{TiHi}) - V_{Ti}^2 Y_{TiHi} \sin(\gamma_{TiHi}) \quad (2.36)$$

$$Q_{Tj}^* - Q_{Tj}^d = V_{Tj} V_{Hj} Y_{TjHj} \sin(\theta_{Tj} - \theta_{Hj} - \gamma_{TjHj}) - V_{Tj}^2 Y_{TjHj} \sin(\gamma_{TjHj}) \quad (2.37)$$

$$\begin{aligned} -Q_{Hi}^d &= \sum_{s=1}^l V_{Hi} V_{Hs} Y_{HiHs} \sin(\theta_{Hi} - \theta_{Hs} - \gamma_{HiHs}) + \\ &\sum_{j=1}^m V_{Hi} V_{Hj} Y_{HiHj} \sin(\theta_{Hi} - \theta_{Hj} - \gamma_{HiHj}) + \\ &\sum_{k=1}^n V_{Hi} V_k Y_{HiHk} \sin(\theta_{Hi} - \theta_k - \gamma_{HiHk}) + \\ &V_{Hi} V_{Ti} Y_{HiTi} \sin(\theta_{Hi} - \theta_{Ti} - \gamma_{HiTi}) \\ -Q_{Hj}^d &= \sum_{i=1}^l V_{Hj} V_{Hi} Y_{HjHi} \sin(\theta_{Hj} - \theta_{Hi} - \gamma_{HjHi}) + \end{aligned} \quad (2.38)$$

$$\begin{aligned} &\sum_{s=1}^m V_{Hj} V_{Hs} Y_{HjHs} \sin(\theta_{Hj} - \theta_{Hs} - \gamma_{HjHs}) + \\ &\sum_{k=1}^n V_{Hj} V_k Y_{HjHk} \sin(\theta_{Hj} - \theta_k - \gamma_{HjHk}) + \\ &V_{Hj} V_{Tj} Y_{HjTj} \sin(\theta_{Hj} - \theta_{Tj} - \gamma_{HjTj}) \end{aligned} \quad (2.39)$$

$$\begin{aligned} -Q_k^d &= \sum_{i=1}^l V_k V_{Hi} Y_{kHi} \sin(\theta_k - \theta_{Hi} - \gamma_{kHi}) + \\ &\sum_{j=1}^m V_k V_{Hj} Y_{kHj} \sin(\theta_k - \theta_{Hj} - \gamma_{kHj}) + \\ &\sum_{s=1}^n V_k V_s Y_{ks} \sin(\theta_k - \theta_s - \gamma_{ks}) \end{aligned} \quad (2.40)$$

$$S_{Di} = \frac{B_i (E_{pi} - A_i)^2}{E_{pi}} \quad (2.41)$$

$$S_{Dj} = \frac{B_j (E_{pj} - A_j)^2}{E_{pj}} \quad (2.42)$$

2.4 Comparison of Load Flow and General Power System Steady State Simulations

Load flow simulation has been used as a standard tool to analyze voltage instability. The solution of load flow equations is assumed to be the steady state solution (equilibrium point) of the power system. In this section, we show that the solution of the load flow equations is similar to the equilibrium point of the general power system model only under light load conditions. These two solutions tend to diverge as the power system is stressed. Since the voltage instability usually occurs in heavy load conditions, we will not be able to get the correct data to analyze voltage instability problems by using load flow simulation.

Figure 2.8 shows the network diagram of a 12 bus system model [37]. There are three internal buses, three terminal buses (low side transformer buses),

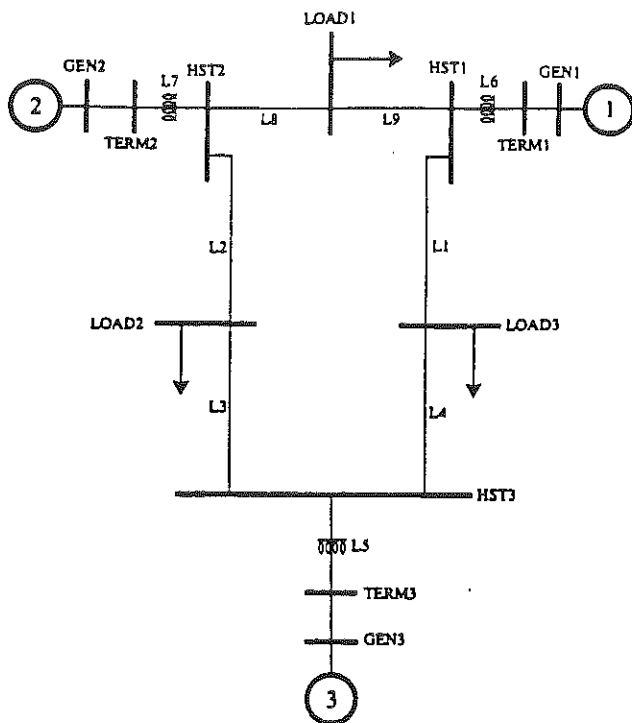


Figure 2.8 A twelve bus test system

three high side transformer buses, and three load buses. In the simulation, we increase the reactive power load at bus LOAD2 and solve for the solutions of a load flow model and general power system model. The reason we choose to increase the reactive power load at bus LOAD2 is that generator bus GEN3 has relatively small reactive generation capacity compared with GEN1 and GEN2. It is easier to exhaust reactive generation capability of GEN3 in the load flow simulation and the field current capability of GEN 3 in a general power system model simulation. In the simulation results of Figure 2.9 and Figure 2.10, we assume the voltage of terminal buses are controlled by the generators in both the load flow model and the general power system model. The only difference between Figure 2.9 and Figure 2.10 is that in Figure 2.9 we use a smaller exciter amplifier gain ($K_A=50$) and in Figure 2.10 we use a larger exciter amplifier gain ($K_A=200$) for all the exciters in the general power system model. In each figure, we show the voltage magnitude differences of load flow model and general power system model solutions for high side transformer buses (HST1, HST2, and HST3) and load buses (LOAD1, LOAD2, and LOAD3). The x axis is the reactive power load at bus LOAD2 and y axis is the voltage magnitude difference of those two different models.

In both Figure 2.9 and Figure 2.10, the solutions of both models are quite similar when the reactive power load at bus LOAD2 is less than 100 MVAR. The solutions start to diverge from each other at the point where the reactive power load at bus LOAD2

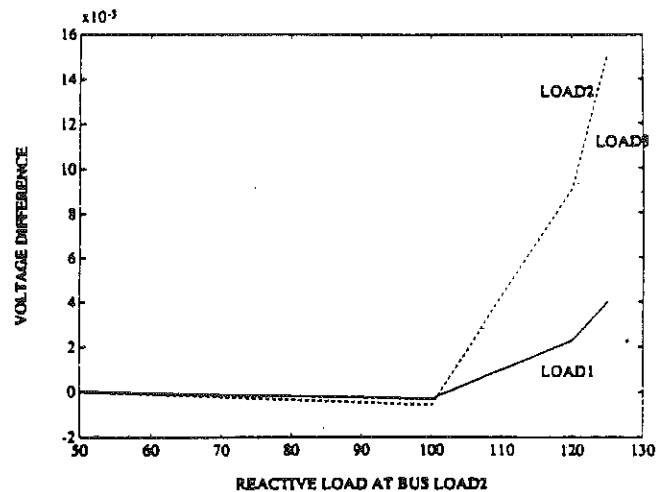
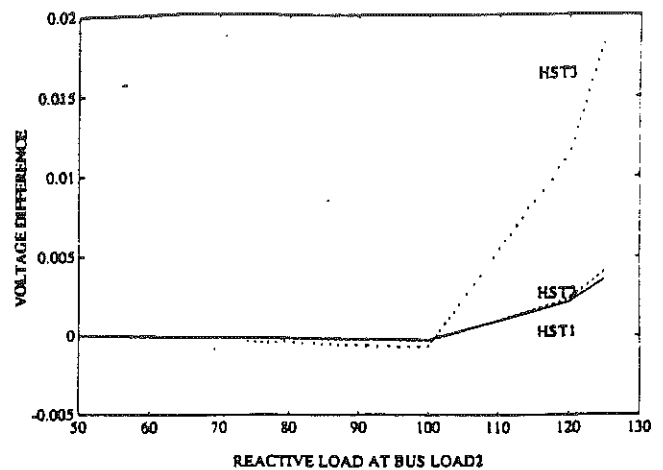


Figure 2.9 Comparison of the solutions of load flow model and general power system model ($K_A=50$)

exceeds 100 MVAR. This is because the load flow model fixed the terminal bus or high side transformer bus voltage and approximates the field current limit by a reactive power limit. The general power system model does not specify the generator terminal or high side transformer bus voltage but attempts to regulate it through the effects of generator flux decay dynamics, generator air gap saturation as well as the exciter loop voltage regulation dynamics. Furthermore, the exciter's line drop compensator generally does not regulate voltage at either the terminal bus or high side transformer bus voltage as with load flow but regulates some fictitious point that can be anywhere between the internal generator bus and some point out in the network. Finally, the reactive power limit in a load flow model is an approximation to the reactive power produced by the generator after the field current capability limit of the generator is hit and the field current limit controller has brought the field current down to its continuous rating level. Thus, the general power system model would allow a much higher field current level than indicated by the

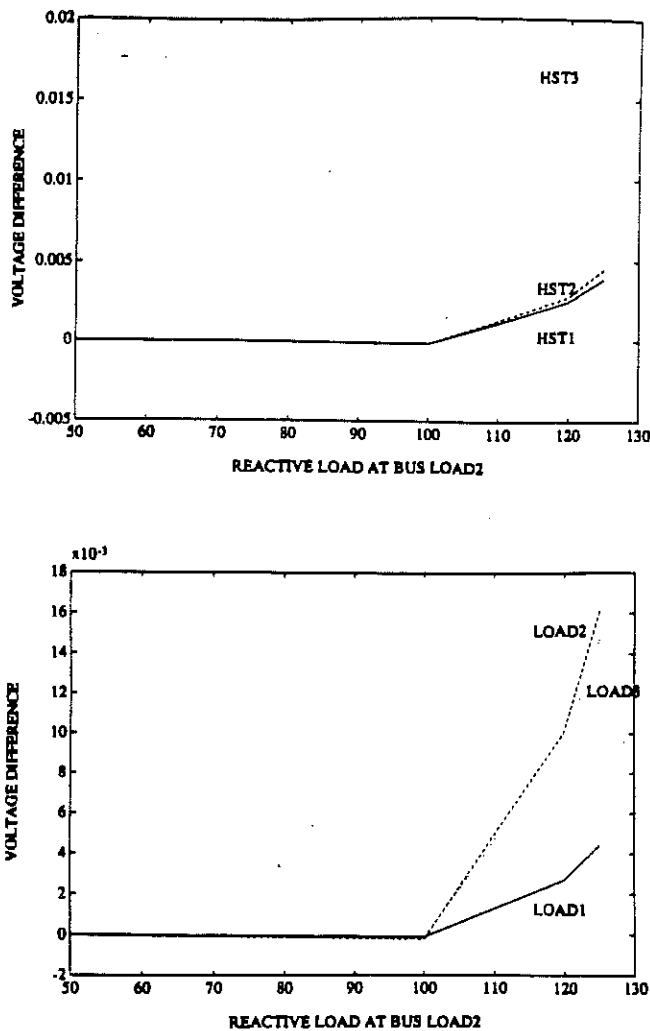


Figure 2.10 Comparison of the solutions of load flow model and general power system model ($K_A=200$)

continuous field current level for durations indicated by a field current capability curve. The computational results presented compare the equilibrium point of the general power system model before field current limits are hit (the exciter is not disabled by the field current limit controller and field current is not reduced to continuous rating level) with that obtained from a load flow that contains a reactive generation limit that is related to continuous rating limit. This result will show a divergence between the general power system model equilibrium point and the load flow power system model equilibrium point even before field current limit is hit in the general power system model; that is when the system has not yet progressed to the point where generators have lost control of voltage.

The simulation results show that the solution of the general power system model could not converge to an equilibrium at 125 MVAR which is much less than the reactive load level of 182 MVAR at which the load flow

did not converge to a solution. This result reflects the poorer convergence properties of the EPRI Transient Midterm Program compared to the EPRI load flow program.

The solutions in Figure 2.10 for an exciter amplifier gain of $K_A=200$ are far closer than the solutions in Figure 2.9 for $K_A=50$ in light load conditions. The reason that there is better agreement between the equilibria of the load flow and general power system model at large exciter gain is that the load flow model assumes the exciter gain is infinite so that the control of terminal voltage has no error. The general power system model has a finite excitation system closed loop gain K_{EX} and an error in regulating voltage proportional to the inverse of K_{EX} (K_{EX}^{-1}). Since K_{EX} is proportional to the exciter amplifier gain K_A and inversely related to air gap saturation, the agreement between the models increases with K_A and decreases as the air gap saturation and reactive generation increases.

It is assumed that the Q-V curve for the load flow model will always be conservative compared to the Q-V curve computed for the general power system model; i.e. the Q-V curve for the load flow model will always sustain lower reactive load stress at each voltage level and will experience bifurcation at a higher voltage level than will the general power system model. This assumption is generally true since the reactive generation limit for load flow are smaller than the reactive generation limit (Point A in Figure 2.3) corresponding to field current limit. The Q-V curve shown in Figure 2.11, where reactances on lines L2 and L3 are multiplied by 5.0, indicates the Q-V curve for the general system model can withstand less reactive load stress than the load flow model. This is due to the poorer voltage control of the general power system model discussed above. Note EPRI Transient Midterm Stability could not compute equilibria for the general power model beyond a reactive load stress level of 60 MVAR. This result clearly shows that the load flow may not always be conservative. A second Q-V curve, shown in Figure 2.12, was computed for the case where the reactances on lines L_2 and L_4 are increased. In this case, the load flow is clearly conservative and the Q-V curve for both the general power system model can be computed out to larger reactive load stress levels.

The reasons for the divergence of the load flow model from the general power system model are summarized in the following:

- (a) the load flow simulation uses reactive power generation to regulate bus voltage and the general power system model uses field current to regulate bus voltage;

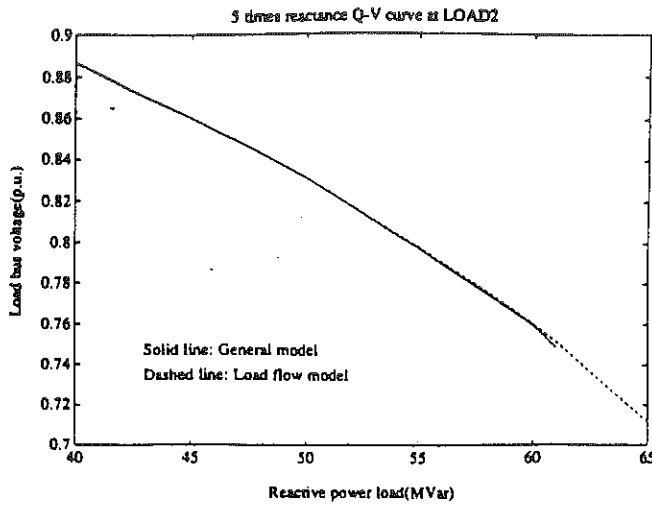


Figure 2.11 Q-V comparison for general and load flow model when reactances L2 and L3 are increased by a factor of five.

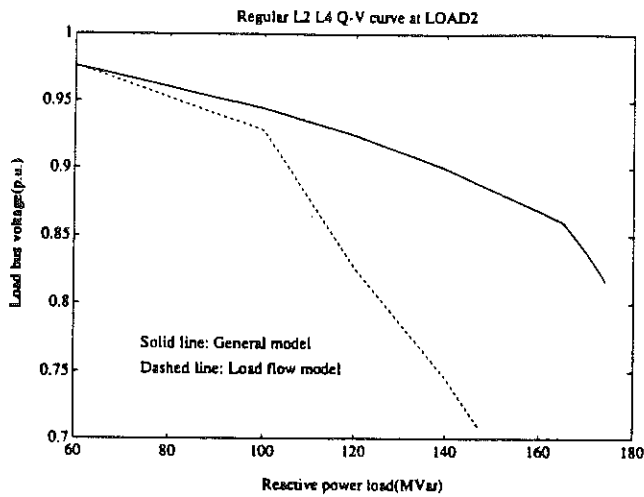


Figure 2.12 Q-V curve comparison on general and load flow model for increase in reactance on lines L2 and L4

- (b) the general power system model takes into account the air gap saturation, which affects the closed loop gain of the exciter control loop, but the load flow model does not;
- (c) the load flow model assumes K_A is infinite but K_A is a finite value in the general power system model; the exciter loop gain K_{EX} is proportional to the exciter amplifier gain and inversely proportional to air gap saturation. The error in regulating voltage is proportional to K_{EX}^{-1} ;
- (d) generators in the load flow model regulate either their high side transformer bus voltage or terminal bus voltage but the general power system model which takes line drop compensation into account allows any point between the generator internal bus and some fictitious point out in the network to be regulated.

A "dynamic" load flow has been implemented as part of the EPRI project RP-3040 awarded based on an

MSU proposal. The effects of exciter gain, field current limit rather than reactive generator limits, load drop compensation have been included in the generator model. The load flow can compute a governor, exciter, and automatic generation control load flow [6,42,43] so that the user can attempt to more accurately establish whether loss voltage stability will occur and the time frame after the contingency when the loss of voltage stability will occur.

3. THEORETICAL BACKGROUND AND MODEL DEVELOPMENT

Loss of voltage stability will be studied as a bifurcation problem of a transient stability model

$$\dot{\mathbf{x}}(t) = \mathbf{f}_1(\mathbf{x}(t), \mathbf{z}(t); \lambda(t)) \quad (3.1)$$

$$\mathbf{0} = \mathbf{g}_1(\mathbf{x}(t), \mathbf{z}(t); \lambda(t))$$

where $\lambda(t)$ is a slowly changing scalar or vector parameter. As $\lambda(t)$ increases very slowly over a range $[\lambda_1, \lambda_2]$, there are equilibrium points, $\mathbf{x}_0^\lambda, \mathbf{z}_0^\lambda$ such that

$$\mathbf{f}_1(\mathbf{x}_0^\lambda, \mathbf{z}_0^\lambda; \lambda) = \mathbf{0}$$

$$\mathbf{g}_1(\mathbf{x}_0^\lambda, \mathbf{z}_0^\lambda; \lambda) = \mathbf{0}$$

for each λ . A necessary condition for a voltage collapse bifurcation to occur at $\mathbf{x}^* = \mathbf{x}_0^{\lambda^*}, \mathbf{z}_0^* = \mathbf{z}_0^{\lambda^*}$ is that the jacobian

$$\begin{bmatrix} \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}}(\mathbf{x}^*, \mathbf{z}^*; \lambda^*) & \frac{\partial \mathbf{f}_1}{\partial \mathbf{z}}(\mathbf{x}^*, \mathbf{z}^*; \lambda^*) \\ \frac{\partial \mathbf{g}_1}{\partial \mathbf{x}}(\mathbf{x}^*, \mathbf{z}^*; \lambda^*) & \frac{\partial \mathbf{g}_1}{\partial \mathbf{z}}(\mathbf{x}^*, \mathbf{z}^*; \lambda^*) \end{bmatrix}$$

has a zero eigenvalue. The slow development of loss of voltage stability and whether the system will remain stable or become unstable as $\lambda(t) > \lambda^*$ must be analyzed based on the dynamics of this model.

If

$$\mathbf{f}_2(\mathbf{x}, \mathbf{z}; \lambda) = \mathbf{0} \quad (3.2)$$

$$\mathbf{g}_2(\mathbf{x}, \mathbf{z}; \lambda) = \mathbf{0}$$

is a loadflow model for each $\lambda \in [\lambda_1, \lambda_2]$, one would also view the bifurcation problem as a multiple loadflow solutions problem [1,3,5,7,10,26,32,33] where the number of solutions change as λ approaches λ^* . $\lambda(t)$ may be used to represent (a) changes in real or reactive load; (b) changes in real or reactive transfer; (c) changes in real generation on generators due to inertial, governor, or AGC, or economic dispatch response to loss of generation contingencies; (d) changes in reactive generation due to field

current limits; or (e) actions of under load tap changers or capacitors in the distribution system and in the transmission system. It should be noted that there is controversy on whether tap changers or capacitors variation should be parameters $\lambda(t)$ or dynamic states $x(t)$. Actions of tap changers are chosen as parameters since their actions cannot occur continuously in time and because once their limits are hit their difference equation model can't explain the continuous decline in voltage observed during loss of voltage stability.

The power system transient stability model to be studied in this paper has the form

$$\dot{\delta} + D\dot{\delta} + F_G(\delta, \theta_L, E'_q, V_L, Y) - P_G = 0 \quad (3.3)$$

$$F_L(\delta, \theta_L, E'_q, V_L, Y) - P_L = 0 \quad (3.4)$$

$$G_G(\delta, \theta_L, E'_q, V_L, Y) - Q_G = 0 \quad (3.5a)$$

$$G_L(\delta, \theta_L, E'_q, V_L, Y) - Q_L = 0 \quad (3.5b)$$

$$\dot{Y} + H(\delta, \theta_L, E'_q, V_L, Y) = 0 \quad (3.6)$$

where

- δ_1 - angle for generator internal bus at swing bus
- δ - $(\delta_2, \delta_3, \dots, \delta_G)^T$ - angle at synchronous machines internal bus excluding the swing bus
- θ_L - $(\theta_1, \dots, \theta_N)^T = (\theta_T^T, \theta_H^T, \theta_{LL}^T)^T$
- θ_L - angle at all generator terminal, generator high side transformer, and load buses, not including the synchronous machine internal buses and swing bus
- θ_T - l vector of angles at generator terminal bus
- θ_H - m vector of angles at generator high side transformer buses
- θ_L - n vector of angles at load buses
- V_L - $[V_1, V_2, \dots, V_N]^T$
- V_L - N vector of voltage at generator terminal, generator high side transformer and load buses
- E'_q - $[E'_1, E'_2, \dots, E'_G]^T$
- E'_q - voltage behind transient reactance at generator internal buses
- y - the state of the electrical dynamics, exciters, power system stabilizer, and governor turbines of synchronous generators and synchronous condensers
- $\hat{M}$ - diag $(M_2, M_3, \dots, M_G)$
- $\hat{D}$ - diag $(D_2, D_3, \dots, D_G)$
- F_G - $(f_2, f_3, \dots, f_G)^T$
- F_L - $(g_1, g_2, \dots, g_N)^T$
- G_G - $(g_{N+2}, g_{N+3}, \dots, g_{N+G})^T$
- G_L - $(g_{N+G+1}, g_{N+G+2}, \dots, g_{N+2N})^T$
- H - $(f_{N+1}, f_{N+2}, \dots, f_{N+k})^T$ represent generator flux decay, exciter, governor-turbine and power system stabilizer dynamics of the $M+G$ generators that have not reached their field current limits
- G - number of synchronous machines
- l - number of generator terminal buses

- m - number of generator high side transformer buses
- n - number of load buses
- $N = l+m+n$ - number of buses other than generator internal buses

- P_G - $G-1$ vector of real power injection at synchronous machine internal buses excluding the swing bus
- Q_G - $G-1$ vector of reactive power injection at synchronous machine internal buses excluding the swing bus
- P_L - N vector of real power injection at generator and load excluding the synchronous machines and the swing bus
- Q_L - N vector of reactive power injection at load buses

The explicit form of these equations can be found in Section 2.

The transient stability model Eqs. 3.3-3.6 can experience a static bifurcation if the jacobian

$$J^{***} = \frac{\partial F}{\partial x} (\delta, \theta_L, E'_q, V_L, E_{REF}, Y)$$

$$x^T = (\delta^T, \theta_L^T, E_q'^T, (V_L/V_L^0)^T, Y^T)$$

is singular at an equilibrium point $(\delta^0, \theta_L^0, E_q'^0, V_L^0, Y^0)$ where

$$F(\delta, \theta_L, E'_q, V_L, Y) = \begin{bmatrix} F_G \\ F_L \\ G_G \\ G_L \\ H \end{bmatrix}$$

The "dynamic" loadflow model Eq. 3.2 is obtained by letting $\dot{\delta} = 0$, $\dot{\theta}_L = 0$ and $\dot{Y} = 0$ in Eqs. 3.3-3.6. This is the dynamic load flow that should be used in voltage stability studies and has been implemented in a recent EPRI research project. However, actual load flow model is obtained from the dynamic load flow model based on four assumptions to be discussed shortly. The actual load flow model has the form

$$\begin{aligned} F_G(\delta, \theta_L, E, V) - P_G &= 0 \\ F_L(\delta, \theta_L, E, V) - P_L &= 0 \\ G_{PV}(\delta, \theta_L, E, V) - Q_{PV} &= 0 \\ G_{PQ}(\delta, \theta_L, E, V) - Q_{PQ} &= 0 \end{aligned} \quad (3.7)$$

M - number of generator PV buses

$$G_{PQ} = (g_{N+M+1}, g_{N+M+2}, \dots, g_{2N})^T$$

$$G_L = [G_{PV}^T \quad G_{PQ}^T]^T$$

$$G_{PV} = (g_{N+1}, g_{N+2}, \dots, g_{N+M})^T$$

Q_{PV} - reactive injections at generator (PV) buses

Q_{PQ} - reactive injections at load (PQ) buses

$$\begin{aligned}\underline{V}_L &= (\underline{E}^T, \underline{V}^T)^T \\ \underline{E} &= (E_1, E_2, \dots, E_M)^T \\ \underline{V} &= (V_{M+1}, V_{M+2}, \dots, V_N)^T \\ \underline{\delta} &= (\delta_H^T, \delta_L^T)^T \\ \underline{\delta} &= \underline{\delta}_T\end{aligned}$$

The loadflow model jacobian has the following form when voltage setpoints at generators and synchronous condensers can change

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_{PV} \\ \Delta Q_{PQ} \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ A_3 & B_3 & C_3 & D_3 \\ A_4 & B_4 & C_4 & D_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta E/E^0 \\ \Delta V/V^0 \end{bmatrix} \quad (3.8a)$$

$$= \hat{J} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta E/E^0 \\ \Delta V/V^0 \end{bmatrix} \quad (3.8b)$$

or when $\Delta E = 0$

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_{PQ} \end{bmatrix} = \hat{J} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V/V^0 \end{bmatrix} \quad \hat{J} = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix}$$

The loadflow model may experience multiple load flow solutions [3] or a voltage collapse bifurcation at an equilibrium $\underline{x}_0$ if $\hat{J}$ is singular. The explicit loadflow model (3.7) and midterm stability model (3.3-3.6) are given in Section 2. $\hat{J}$ is just one of several forms for the load flow jacobian which has been written in literature. The elements of $\hat{J}$ are defined in [3] and [32] and are omitted here due to limitation of space. The transient stability model and loadflow model allow constant power and constant impedance real and reactive load models.

The specific formulation given in Eqs. 3.3-3.7 provides the structure for showing the differences between the dynamic and static approaches to loss of voltage stability for the case where it develops due to slow changes $\lambda(t)$ in operating conditions. Assumptions are necessary for developing the load flow model from the general power system model. The first assumption [1] is that the circuit model of the generator is solely a voltage source and thus the generators' stator resistances, transient reactances, and q axis synchronous reactances are zero. The generator terminal bus or high side transformer bus is thus the generator internal bus and the voltage of this bus is directly controlled by the exciter. Thus, the elements of $\underline{E}$ represent the generator terminal bus voltage on the $M \leq G$ where exciters have not reached

their field current limits. The second assumption [1] is that the exciter gain is so high that the terminal bus voltage is the voltage setpoint V_{REF} when generator field current limits have not been reached and the exciter control loop has not been disabled. Historically, generator terminal buses have also been eliminated from the load flow model so that the generator high side transformer bus was the PV bus. More recently, generator terminal buses are being retained in the load flow model to better approximate the load drop compensator.

The third assumption is that the field current limit is replaced by a reactive power generation limit $Q_{j_{min}}^e \leq Q_j^e \leq Q_{j_{max}}^e$. Assumption 2 and 3 eliminate generator flux decay and exciter dynamics (3.6). The dimension of $\underline{Q}_{PV}$ and $\underline{E}$ are reduced when the generator reaches reactive generation limits in 3.7. Note that when field current limits are reached in a general power system model control at terminal voltage is lost due to disablement of the exciter control loop. The effective PV bus is then the input to the field winding on the generator. No change in the network equation (3.4, 3.5) occurs when a generator hits field current limits in the general power system model. When a generator reaches reactive power generation limits the dimension of $\underline{V}$ and $\underline{Q}_{PQ}$ must be increased in 3.7. The fourth assumption is that bus voltage controlled by tap changers, switchable shunt capacitors, and static var compensators are assumed to be PQ buses in the loadflow. The significant differences in the two models (load flow and the general system model) are very great not only in terms of the equilibria, defined as noted in Section 2, but also in terms of the conditions for voltage collapse bifurcation and the types of bifurcation that can occur.

Loss of voltage stability is principally concerned with reactive power voltage relationships. Thus a sensitivity matrix can be derived from Eq. 3.8 by first setting $\begin{bmatrix} \Delta P_G \\ \Delta P_L \end{bmatrix} = 0$, solving for

$$\begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix} = - \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} C_1 & D_1 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} \Delta E/E^0 \\ \Delta V/V^0 \end{bmatrix} \quad (3.9)$$

and substituting to obtain a reduced model

$$\begin{bmatrix} \Delta Q_{PV} \\ \Delta Q_{PQ} \end{bmatrix} = J^* \begin{bmatrix} \Delta E/E^0 \\ \Delta V/V^0 \end{bmatrix} \quad J^* = \begin{bmatrix} C_3^* & D_3^* \\ C_4^* & D_4^* \end{bmatrix} = \begin{bmatrix} C_3 & D_3 \\ C_4 & D_4 \end{bmatrix}$$

$$\begin{bmatrix} A_3 & B_3 \\ A_4 & B_4 \end{bmatrix} \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} C_1 & D_1 \\ C_2 & D_2 \end{bmatrix} \quad (3.10)$$

A more common sensitivity model (Model 2) has the form

$$\Delta \underline{V}/\underline{V}^0 = S_{Q_L V}^{*-1} \Delta Q_{PQ} + S_{VE}^* \Delta \underline{E}/\underline{E}^0 \quad (3.11)$$

where

$$S_{Q_L V}^* = D_4 - [A_4 B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$$

$$S_{VE}^* = -S_{Q_L V}^{*-1} \left[C_4 - [A_4 B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \right] \quad (3.12)$$

A simplified sensitivity model (Model 1) can be derived assuming the loadflow equations are decoupled. Model 1 based on a decoupled loadflow model has the form

$$\Delta \underline{V}/\underline{V}^0 = S_{VE}^* \Delta \underline{E}/\underline{E}^0 + S_{Q_L V}^{*-1} \Delta Q_{PQ} \quad (3.13)$$

where

$$S_{Q_L V}^* = D_4$$

$$S_{VE}^* = -S_{Q_L V}^{*-1} C_4 \quad (3.14)$$

assuming $\begin{bmatrix} A_3 & B_3 \\ A_4 & B_4 \end{bmatrix} = 0$ and $\begin{bmatrix} C_1 & D_1 \\ C_2 & D_2 \end{bmatrix} = 0$ in Eq. 3.12.

Model 1 (Eqs. 3.13-3.14) and Model 2 (Eqs. 3.11-3.12) are quite important in loadflow sensitivity based tests for loss of voltage stability discussed in Section 5.1. The form of S_{VE}^* (S_{VE}) and $S_{Q_L V}^*$ ($S_{Q_L V}$) are derived in order to establish tests for loss of voltage stability based on PQ controllability property and to relate the resulting tests to the array of tests in the literature based on S_{VE} and $S_{Q_L V}$.

The loadflow can have a bifurcation if J in Eq. 3.8 is singular at $(\underline{\delta}^0, \underline{\theta}^0, \underline{E}^0, \underline{V}^0)$. The transient stability model can have a bifurcation which may result in loss of voltage stability if

$$J^{***} = \begin{bmatrix} & & & & E_1^* \\ & & & & E_2^* \\ & & \hat{J}^{***} & & E_3^* \\ & & & & E_4^* \\ A_5^* & B_5^* & C_5^* & D_5^* & E_5^* \end{bmatrix};$$

$$\hat{J}^{***} = \begin{bmatrix} A_1^* & B_1^* & C_1^* & D_1^* \\ A_2^* & B_2^* & C_2^* & D_2^* \\ A_3^* & B_3^* & C_3^* & D_3^* \\ A_4^* & B_4^* & C_4^* & D_4^* \end{bmatrix};$$

$$\hat{J}^{***} = \begin{bmatrix} A_1^* & B_1^* & D_1^* \\ A_2^* & B_2^* & D_2^* \\ A_3^* & B_3^* & D_3^* \\ A_4^* & B_4^* & D_4^* \end{bmatrix} \quad (3.15)$$

is singular at a point $(\underline{\delta}^0, \underline{\theta}^0, \underline{E}^0, \underline{V}^0)$ and can't have a bifurcation and, thus, isn't experiencing loss of voltage stability if J^{***} is nonsingular at that point. Note that singularity of J^{***} may occur for reasons other than loss of voltage stability.

Further, some researchers would call Hopf bifurcation loss of voltage stability and J^{***} would not be singular when Hopf bifurcations occur. Note that the

submatrix in $\hat{J}^{***}$ and $\hat{J}^{***}$ are quite different than the submatrices in $\hat{J}$ and $\hat{J}$ for load flow. Each submatrix in $\hat{J}$ and $\hat{J}^{***}$ represent a jacobian of a particular set of equations (represented by subscripts 1,2,3,4) with respect to a particular set of variables (represented by matrices A,B,C,D). Although the submatrices in $\hat{J}^{***}$ and $\hat{J}$ are all different (indicated by * on submatrices in $\hat{J}^{***}$) because the sets of equations and the sets of variables, which define the meaning of matrices A_i, B_i, C_i, D_i and $A_i^*, B_i^*, C_i^*, D_i^*$ are different, there is a consistency that is captured by retaining the same letters and numbers. The PV bus in load flow is assumed to be the generator terminal bus and in the general system model it is the internal generator bus. The vectors $\underline{\delta}$ and $\underline{E}$ are used to represent the phase and voltage at PV buses in both models. The vectors $\underline{\theta}$ and $\underline{V}$ are the variables at the non PV buses in both models. In the general power system model, $\underline{\theta}_L$ and $\underline{V}_L$ are the bus angle and voltage at generator terminal, generator high side transformer, and load buses and $\underline{\delta}$ and $\underline{E} = \underline{E}'_q$ represent the angle and voltage at the generator internal buses. In the load flow model, $\underline{\delta}$ and $\underline{E}$ represent the angle and voltage at generator terminal buses and $\underline{\theta}$ and $\underline{V}$ are the angle and voltage at the generator high side transformer and load buses but not at the generator terminal buses used in $\underline{\theta}_L$ and $\underline{V}_L$ for the general power system model; i.e. $\underline{\theta}_L^T = (\underline{\theta}_L^T, \underline{\theta}_L^T)$, $\underline{V}_L^T = (\underline{V}_L^T, \underline{V}_L^T)$.

Thus submatrices with letters A and C represent jacobians with respect to variable $\underline{\delta}$ and $\underline{E}$ buses in both models. Submatrices with letters B and D represent jacobians with respect to non PV buses in both models. Submatrices with subscripts 1 and 3 represent the jacobians of real power and reactive power equations at PV buses in both models. Submatrices with subscripts 2 and 4 represent the jacobians of real power and reactive equations at non PV bus in both models. It should be specifically noted that matrices D_i^* and B_i^* , $i = 1,2,3,4$ are jacobians where partials are with respect to voltages

and angles at all generator terminal, generator high side transformer, and load buses but in the load flow the partials with respect to voltages and angles at generator terminal buses are not in B_i and D_i but in A_i and C_i for $i = 1, 2, 3, 4$. The matrices A_i^* and C_i^* for $i = 1, 2, 3, 4$ in the general power system model represent partial with respect to angle δ_i and voltage E'_{q_i} at generator internal bus and these matrices are not in the load flow model. It should be noted that as long as the row dependence that cause J^{***} to be

singular occurs with rows of $\begin{bmatrix} B_2^* & D_2^* \\ B_4^* & D_4^* \end{bmatrix}$ associated with generator high side transformer and load buses, the

corresponding rows of $\hat{J}$, which lie in $\begin{bmatrix} B_2^* & D_2^* \\ B_4^* & D_4^* \end{bmatrix}$, will

be row dependent since rows of $\hat{J}$ are subvectors of rows of J^{***} . The differences in the models are in how generator terminal buses are modeled, the model of generator dynamics, as well as the model of real and reactive power equations at internal buses. Most tests for loss of voltage stability generally (with the exception of [4]) have depended solely on $\hat{J}$ since $\hat{J}$ is the loadflow jacobian when $\Delta E = 0$. The measures for loss of voltage stability developed in Section 5.1

are based on S_{QLV}^* that depends on $\hat{J}$. The dimension of S_{QLV}^* increases as generator (PV) buses become load (PQ) buses. It should be noted that loss of voltage stability tests should be based on J^{***} and not on $\hat{J}$, $\hat{J}$, or submatrices of $\hat{J}$. Matrices S_{VE}^{***} and S_{QLV}^{***} are defined for the general power system model as they are based on the load flow model (3.12) except that matrices A_i , B_i , C_i , D_i are replaced by matrices A_i^* , B_i^* , C_i^* , D_i^* for $i = 1, 2, 3, 4$. These sensitivity matrices will be shown to indicate loss of voltage stability in the general power system model as they do in the load flow model.

Kwatny [5] has defined causality as a property associated with nonsingularity of matrix.

$$J^{**} = \begin{bmatrix} B_2^* & D_2^* \\ B_4^* & D_4^* \end{bmatrix} \quad (3.16)$$

which is a test condition for uniquely solving the algebraic equations Eqs. 3.4, 3.5b:

$$F_L(\underline{\delta}, \underline{\theta}_L, \underline{E}'_q, \underline{V}_L, \underline{Y}) - \underline{P}_L = 0$$

$$G_L(\underline{\delta}, \underline{\theta}_L, \underline{E}'_q, \underline{V}_L, \underline{Y}) - \underline{Q}_L = 0$$

for $\underline{\theta}_L(\underline{\delta}, \underline{Y}, \underline{E}_{REF})$ and $\underline{V}_L(\underline{\delta}, \underline{Y}, \underline{E}_{REF})$ in a neighborhood of $(\underline{\delta}^0, \underline{\theta}_L^0, \underline{E}'_{q^0}, \underline{V}_L^0, \underline{Y}^0)$. Kwatny [5] defined a system as (locally) causal if Eqs. 3.4, 3.5b

admit a solution $\underline{\theta}_L(\underline{\delta}^0, \underline{Y}^0, \underline{E}'_{q^0})$ and $\underline{V}_L(\underline{\delta}^0, \underline{Y}^0, \underline{E}'_{q^0})$ in a neighborhood of $(\underline{\delta}^0, \underline{\theta}_L^0, \underline{E}'_{q^0}, \underline{V}_L^0, \underline{Y}^0)$ where $\underline{\theta}_L^0 = \underline{\theta}_L(\underline{\delta}^0, \underline{Y}^0, \underline{E}'_{q^0})$ and $\underline{V}_L^0 = \underline{V}_L(\underline{\delta}^0, \underline{Y}^0, \underline{E}'_{q^0})$. A system is (locally) causal if J^{**} is nonsingular. If the system is not causal it may or may not experience a bifurcation. Furthermore, the system Eqs. 3.3-3.6 may experience a bifurcation and still be causal. Causality and its relationship to loss of voltage stability tests is discussed in Section 5.2.

4. VOLTAGE CONTROLLABILITY

Definitions are proposed for PQ voltage controllability based on minimal conditions that would be required to assure voltage security in a load flow model. Similar conditions can be stated for the general power system model where the condition holds for all generator terminal, generator high side transformer, and load buses rather than at PQ buses. PV buses in Transient Midterm Stability model (3.3-3.6) are the exciter reference voltages (or the generator internal buses when the generator model does not include flux decay dynamics).

Definition (1): Consider the system with M PV buses and N PQ buses. If the PV buses and the lines connecting them to other buses are removed, then PQ buses that remain form isolated groups. Each of these groups is called a load center. The set of all PQ buses in the system can be partitioned into load centers based on the following relation: two PQ buses are in the same load center if they are connected either directly or through some other PQ buses. There are no links from PQ buses of one load center to PQ buses of another load center. These load centers are disjoint.

Definition (2): A voltage control area (VCA) is a set of load (PQ) and generator (PV) buses where voltages respond very similarly (coherently) to the reactive load and generation changes outside this voltage control area.

Definition (3): A system is called PQ controllable at any time t along the model (3.7) trajectory or at the equilibrium point if: (i) when $\Delta E(t) = 0$, any nonzero nonnegative disturbance $\Delta Q_{PQ}^*(t)$ causes the PQ state $\Delta V(t)$ to become nonnegative; and for each j there is a nonzero nonnegative disturbance $\Delta Q_{PQ}(t)$ that causes $\Delta V_j(t)$ to become positive when $\Delta V_i = 0$ for $i \neq j$, and (ii) when $\Delta Q_L(t) = 0$, any nonzero nonnegative control $\Delta E(t)$ causes the PQ state $\Delta V(t)$ to become nonnegative, and for each j there is a nonzero nonnegative control $\Delta E(t)$ that causes $\Delta V_j(t)$ to become positive.

* Q_{Li} is a power injection and as such is a negative number for load (PQ) buses. So, $\Delta Q_{Li} > 0$ means that the load decreased at the i th load (PQ) bus.

PQ controllability requires that the network respond in its typical manner. Thus, any increase in reactive load at any bus or combination of buses causes voltage decline or no voltage change at every PQ bus in the network. PQ controllability also requires that any voltage increase at any PV bus or at any combination of PV buses will cause increase or no change in voltage at every PQ bus in the network. Furthermore, PQ controllability requires that there exist a set of positive voltage setpoint changes at PV buses that produce a positive change in ΔV_j at bus j for each bus j . PQ controllability requires that a set of positive reactive injection changes at PQ buses to produce a increase in voltage ΔV_j at only bus j and no change in voltage at any other PQ bus for each load bus j . It is obvious that if the system was PQ controllable for all time, which is the normal property of a voltage secure power system, loss of voltage stability is impossible because one can control voltage at every PQ bus. It should, therefore, not be surprising that every loss of voltage stability test that gives sufficient conditions for preventing loss of voltage stability depends on the sensitivity matrix tests or jacobian matrix tests that assure PQ controllability. It is also obvious that any proximity tests for loss of voltage stability should measure the proximity to violating the sufficient conditions PQ controllability and necessary conditions for voltage instability should indicate the sufficient conditions for PQ controllability are violated.

Having defined PQ controllability for the load flow model, one can derive an array of test conditions so that the system is PQ controllable using

$$\Delta V = S_{QLV}^{*-1} \Delta Q_{PQ} + S_{VE}^* \Delta E$$

Theorem 1: The system is PQ controllable if S_{QLV}^* is an M matrix and S_{VE}^* is nonnegative with no zero rows [3].

The proof of the theorem can be found in [3].

The M matrix condition on S_{QLV}^* also requires elements of S_{QLV}^{*-1} be nonnegative and have non-zero rows. The M matrix condition is stronger and requires off diagonal elements of S_{QLV}^* be nonpositive, diagonal elements be positive and S_{QLV}^* be nonsingular.

The fundamental requirement for PQ controllability is that S_{QLV}^* be an M matrix. It can be shown [3] that for S_{VE}^* to be nonnegative S_{QLV}^* must be an M matrix, and angle constraint

$$-180 + \gamma_{ij} \leq \theta_i - \theta_j \leq 180 - \gamma_{ij} \quad (4.1)$$

must be satisfied for every branch connecting load (PQ) bus i and generator (PV) buses j . γ_{ij} is the angle of the ij th element of the Y_{bus} . Condition 4.1 is a steady state angle stability condition. Thus, PQ controllability assumes each load (PQ) bus j can be affected by at least one disturbance Q_k in Q_L and by at least one control E_i in E . If a load center is not connected to at least one generator (PV) bus, the load center (which is defined as a set of interconnected load (PQ) buses that are isolated when all connections to generator (PV) buses are eliminated) would not be connected to the rest of the system. Since the power system model is assumed to be irreducible, there are no true load centers in this power system model. However, a voltage control area (VCA) can be considered to approximate a load center as it loses reactive generation reserve and all its buses are load (PQ) buses. When all generator (PV) buses become load (PQ) buses, the voltage control area (VCA) can be considered as a load center that is isolated because the transmission boundary of a voltage control area (VCA) is weak. The weakness of the voltage control area boundary is the structural weakness that leads to loss of voltage stability because the boundary [26,29] weakness limits the reactive power flow that crosses it for a specific voltage drop at buses in that voltage control area.

5. LITERATURE REVIEW OF VOLTAGE STABILITY AND INSTABILITY TESTS

The purpose of this section is to associate the sufficient conditions for PQ controllability with the tests for bifurcation [1,3,5,7,10,26,27,32, 33,34], Lyapunov stability tests [8,9], Q-V curve and P-V curve tests [30], loadflow sensitivity [11,12,28,29,35] tests, and linearized dynamic [13,14,16,17] tests and flux decay bifurcation tests [32] for voltage instability. All of these tests are based on establishing whether some conditions hold on $\hat{J}$, $\hat{J}^{***}$, or J^{***} . Our effort is to show that the conditions on each of these test matrices can be derived from or are directly related to the condition on S_{QLV}^* and S_{VE}^* that assure (Theorem 1) PQ controllability holds. The following derivation of these various test conditions on these different test matrices and the related discussion is not intended to indicate any test is superior or inferior but rather than all are testing the same basic property. Computational results in [3,29,30] confirm that all the tests are testing the same properties. Since one purpose of this paper is to unify the literature through a review from a voltage control stability

perspective rather than numerical evaluation and comparisons as are the purpose of [3,29,30], these numerical results and comparisons are omitted in this paper.

5.1 Load Flow Tests

The test conditions for PQ controllability on S_{QLV}^* will now be shown to assure the loadflow jacobian

matrix $\hat{J}$ is nonsingular. Nonsingularity of $\hat{J}$ or $\hat{J}^{***}$ will assure that a loss of voltage stability bifurcation does not occur in the transient stability model (3.3,3.4,3.5b) or loadflow model (3.7) [1,3,5,7,10,26,27,33,34]. If S_{QLV}^* is a M matrix, there are PV buses connected to each voltage control area, and (4.1) holds on all such branches, PQ controllability is assured. S_{QLV}^* or S_{QLV}^{***} is defined as

$$S_{QLV}^* = D_4 - [A_4 B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad (5.1)$$

and is not properly defined unless $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ is

nonsingular. From Schur's formula

$$\det(\hat{J}) = \det(S_{QLV}^*) \det \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \quad (5.2)$$

$\hat{J}$ ($\hat{J}^{***}$) is nonsingular, if and only if, S_{QLV}^* (S_{QLV}^{***})

is nonsingular when $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ is nonsingular. A properly defined matrix S_{QLV}^* (which implies $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$

is nonsingular), that is a M matrix (which implies S_{QLV}^* is nonsingular), is sufficient to assure

nonsingularity of $\hat{J}$. PQ controllability not only requires S_{QLV}^* be a M matrix but also that (4.1) be satisfied on every branch connecting PV and PQ buses, and that PV buses be connected to each voltage control area. Furthermore, a bifurcation of the loadflow (3.7) and transient stability model (3.3-3.5) can not occur, if and only if, S_{QLV}^* (S_{QLV}^{***}), defined in 3.12, exists and satisfies the conditions for assuring PQ controllability. It should be noted that there are three different ways that $\hat{J}$ or $\hat{J}^{***}$ can be singular:

(a) rows of $[A_4 B_4 D_4]$ being dependent is associated with loss of voltage stability since $S_{QLV}^* = D_4 -$

$[A_4 B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$ is row dependent if the same rows of $[A_4 B_4]$ and $[D_4]$ are dependent and can be summed to zero by the same weighting row vector;

(b) rows of $\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ being row dependent is associated with steady state angle stability.

Note that a sufficient condition for the topological Lyapunov function [36] to be locally stable is that

$$\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} - \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 B_4] \quad (5.3)$$

be positive definite since if this matrix is positive definite

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}^T \begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} > 0 \text{ for } y_1, y_2$$

then for $y_2 = -B_2^{*-1} A_2^* y_1$ and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = y_1 A_1^{**} y_1 > 0,$$

$A_1^{**} = A_1^* - B_1^* B_2^{*-1} A_2^*$ is positive definite.

Since A^{**} is necessary and sufficient for local steady state angle stability [36], the condition

that $\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ is positive definite is sufficient for local steady state angle stability. The

matrix $\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ will not be positive definite if

the rows of $J_p = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ are dependent since

rows of J_p that are dependent are the rows of

$$\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} - \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 B_4]$$

that will be dependent due to the fact that these

same rows will be dependent in both $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ and

in $\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 B_4]$ and can be summed to zero

by the same weighting row vector. Requiring row independence of $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ to define the matrix

S_{QLV}^* assures that $\hat{J}$ is not singular due to loss of steady state angle stability due to rows of $\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ being row dependent. Note that

$\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$, defined in 5.3, requires D_4 to have

maximum row rank and prevents singularity of $\hat{J}$ due to row dependence of $[A_4 B_4 D_4]$. Thus, the M

matrix tests for loss of voltage stability on S_{QLV}^* and the positive definite requirement on

$$\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} \text{ and, thus on } A_1^{**}, \text{ for steady state angle}$$

stability assures that $\hat{J}$ is not singular due to

$$\text{voltage instability when the } \begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix} \text{ test for}$$

angle stability is properly defined and is not experiencing loss of angle stability when the S_{QLV}^* test for voltage stability is properly defined. It should be noted that the argument should be made using $A_1^*, B_1^*, C_1^*, D_1^*$ of the model (3.3,3.4,3.5b) (but was made without loss of generality with A_1, B_1, C_1, D_1 of the load flow model (3.7)) since [36] uses a classical transient stability model with actual generator internal buses.

(c) the dependence of rows of $[A_4 \ B_4 \ D_4]$ with rows of

$$\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix} \text{ without row dependence of either is associated with voltage and angle stability. If}$$

$$D_4 \text{ and } \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \text{ are nonsingular so that } [A_4 \ B_4$$

$$D_4] \text{ and } \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix} \text{ have maximum row rank, then singularity of } \hat{J} \text{ must occur due to dependence of rows of } [A_4 \ B_4 \ D_4] \text{ and } \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}.$$

Premultiplying $\hat{J}$ by

$$\begin{bmatrix} \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} & 0 \\ 0 & D_4^{-1} \end{bmatrix} \quad (5.4)$$

and then premultiplying by

$$\begin{bmatrix} I & 0 \\ -D_4^{-1}[A_4 \ B_4] & I \end{bmatrix} \quad (5.5)$$

produces a matrix

$$\begin{bmatrix} I & \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \\ 0 & I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \end{bmatrix} \quad (5.6)$$

Note that the row manipulations on $\hat{J}$ performed by premultiplying $\hat{J}$ by (5.4) and (5.5) assures rows of $J_p = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ are not row dependent due to assumed existence of $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1}$ and rows of $J_q = [A_4 \ B_4 \ D_4]$ are not row dependent due to the assumed existence of D_4^{-1} . Since

premultiplication of $\hat{J}$ by maximum row rank matrices (5.4) and (5.5) will not change the row rank of $\hat{J}$, $\hat{J}$ will be singular if the matrix

$$J_{pq} = I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \quad (5.7)$$

is singular. Since the $[A_4 \ B_4 \ D_4]$ rows of $\hat{J}$ are independent due to nonsingularity of D_4 and the $\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ rows of $\hat{J}$ are independent due to nonsingularity of $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$, then singularity of

$$(5.7) \text{ indicates dependence of rows of } J_p = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix} \text{ with rows of } J_q = [A_4 \ B_4 \ D_4].$$

It turns out that S_{QLV}^* is not only a test of row dependence of J_q but also of joint row dependence of J_p and J_q (tested by J_{pq}) since $S_{QLV}^* = D_4 \left[I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \right] = D_4 J_{pq}$ would be row dependent if either D_4 or $J_{pq} = I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix}$ were row dependent.

Similarly, $\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ is not only a test for row

dependence of J_p but also a test of joint row dependence of J_p and J_q since

$$(a) \begin{bmatrix} A_1^* & B_1^* \\ B_2^* & B_2^* \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} \begin{bmatrix} I - \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 \ B_4] \end{bmatrix} \quad (5.8)$$

(b) the matrix

$$J_{pq}^* = I - \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 \ B_4] \quad (5.9)$$

can be shown to test for joint row dependence of $\hat{J}$ by premultiplying $\hat{J}$ by (5.4) and

$$\begin{bmatrix} I & - \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \\ 0 & I \end{bmatrix} \quad (5.10)$$

to produce a matrix

$$\begin{bmatrix} I - \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} [A_4 \ B_4] & 0 \\ D_4^{-1} [A_4 \ B_4] & I \end{bmatrix} \quad (5.11)$$

$\hat{J}$ is row dependent if (5.9) is row dependent since $\hat{J}$ is multiplied by maximum row rank matrices when $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ and D_4 are nonsingular. Since nonsingularity of $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ guarantees row independence of J_p and D_4 guarantees row independence of J_q , matrix (5.9) is a test for joint row dependence of J_p and J_q . These results suggest that $\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ is a test for row independence of J_p and joint row independence of J_p and J_q when $\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ is written as in (5.8). Thus, joint row dependence of $[A_4 \ B_4 \ D_4]$ and $\begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix}$ appears to cause both loss of voltage stability and loss of angle stability.

5.1.1 Types of Load Flow Bifurcations

Row dependence of J_p would result in very large angle differences, no changes in ΔV in the left eigenvector of the zero eigenvalue of $\hat{J}$ at the bifurcation point. Note that if the left eigenvector has zero elements, the participation factor for that eigenvalue will have zero elements in the same locations as the left eigenvector. Since participation factors indicate those states which have the most influence on a particular mode and since the participation factors for voltage at buses are zero when J_p is row dependent, the row dependence of J_p is associated with bus angle changes. This type of bifurcation is called angle bifurcation since there may be more than one solution $\delta_1 \neq \delta_j$, $\theta_1 \neq \theta_j$, $V_1 = V_j$ that will satisfy the equation (3.7). Similarly, when J_q is row dependent, one would call it a voltage bifurcation since the left eigenvector of the zero

eigenvalue of $\hat{J}$ which occurs at the bifurcation point has large elements in ΔV and zero in $\Delta \delta$ and $\Delta \theta$. This is why it is called a voltage bifurcation since there are several solutions $V_1 \neq V_j$, $\theta_1 = \theta_j$, $\delta_1 = \delta_j$ that will satisfy (3.7). Joint row dependence of J_p and J_q indicated by singularity of either J_{pq} or J_{pq}^* would result in large elements in both ΔV and $[\Delta \delta, \Delta \theta]$ for the left eigenvector of the zero eigenvalue of $\hat{J}$, which occurs at the bifurcation point. Joint row

dependence of J_p and J_q is called a voltage/angle bifurcation because there may be more than one solution of (3.7) with different δ and V values. It should be noted that singularity of J_{pq}^* and J_{pq} are equivalent tests since J_{pq} and J_{pq}^* are nonsingular, if and only if, $\hat{J}$ is nonsingular when $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ and D_4 are nonsingular. Thus, there are only three different types of algebraic bifurcation of (3.7) when E is fixed or of the algebraic equations (3.4, 3.5b). This is quite important since we will show what types of bifurcations are tested using any particular test. Note that S_{QLV}^* is a test for both voltage bifurcation and voltage and angle bifurcation but is not a test for angle bifurcation. Note that a test on singularity of $\hat{J}$ is a test on all three types of bifurcation.

Tamura [27] developed several tests based on the measures $\left[\lambda_{\min}(\hat{J}), \det(\hat{J}), \text{and VIPI} \right]$ which measure how close the jacobian is to be singular. Since $\hat{J}$ is nonsingular if PQ controllability holds then measures of loss of PQ controllability are related to the above measures for proximity to voltage instability. Tests for singularity of $\hat{J}$ do not indicate whether the singularity is due to angle bifurcation (row dependence of J_p), voltage bifurcation (row dependence of J_q), or joint voltage and angle bifurcation (row dependence of J_{pq}).

5.1.2 Load Flow Point of Collapse Tests

A Q-V curve is a common method used by utilities in the United States to assess proximity to loss of voltage stability. A load (PQ) bus k is converted to a generator (PV) bus and the reactive generation injection $Q_k(V_k)$ is plotted as the voltage setpoint is varied. Voltage collapse is said to occur if a reactive load of magnitude $-Q_k(V_{k_{\min}})$ is added at bus k when $Q_k(V_{k_{\min}}) < 0$ and $V_{k_{\min}}$ is the voltage setpoint value at which $Q_k(V_k)$ is minimized. If $Q_k(V_k) > 0$ for all V_k , $Q_k(V_{k_{\min}})$ is the added reactive generation that must be added at bus k to obtain a loadflow solution since there is no load flow solution to the load flow equations without adding reactive generation at bus k .

The slope of the Q-V curve satisfies

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_L \end{bmatrix} = \Delta Q_k(V_k) e_k = \hat{J} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} \quad (5.12)$$

$$\Delta V_k = -e_k^T \Delta V \quad (5.13)$$

where $\underline{e}_k$ is 2N-2 unit vector where only the element associated with voltage at bus k or reactive power injection at bus k is one. It should be noted that a Q-V curve can be calculated by requiring $\Delta P_G = 0$, $\Delta P_L = 0$ and $\Delta Q_L = 0$ except for element k. If $\begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix}$ is solved for in (5.12) and then substituted back into (5.12) to obtain an expression for ΔV , ΔV satisfies

$$\Delta V = S_{Q_L V}^{*-1} \Delta Q_k \underline{e}_k$$

The slope of the Q-V curve equals the reciprocal of the kth diagonal element of $S_{Q_L V}^{*-1}$ since

$$\Delta V_k = \underline{e}_k^T \Delta V = \left\{ \underline{e}_k^T S_{Q_L V}^{*-1} \underline{e}_k \right\} \Delta Q_k$$

$$\Delta Q_k = \frac{1}{\underline{e}_k^T S_{Q_L V}^{*-1} \underline{e}_k} \Delta V_k \quad (5.14)$$

Since the slope of the Q-V curve is positive and approaches zero as the voltage setpoint decreases toward $V_{k \min}$, the minimum of the Q-V curve, the kth diagonal element of $S_{Q_L V}^{*-1}$ must rapidly approach infinity and then become negative infinite. This is exactly what is observed if $S_{Q_L V}^{*-1}$ has a nonpositive diagonal element; $S_{Q_L V}^*$ is no longer an M matrix and is no longer nonsingular and positive definite and PQ controllability would be violated. The computation of the $S_{Q_L V}^{*-1}$ matrix is thus equivalent to computing slopes of Q-V curves at every bus. Thus, the point at which loss of stability occurs on a Q-V curve requires that the loadflow jacobian $\hat{J}$ be singular since

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_L \end{bmatrix} = \hat{J} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} = \underline{0} \quad (5.15)$$

from (5.12) when $\Delta Q_k(V_{k \min}) = 0$, $\Delta P_G = 0$, $\Delta P_L = 0$ and $\Delta Q_{PQ} = 0$ at the Q-V curve minimum. Thus, the point at which loss of voltage stability occurs on a Q-V curve is the exact point where loss of PQ controllability occurs and the point where the necessary conditions for loadflow bifurcation occurs. The computation of a Q-V curve is a point of collapse test for how much reactive load must be added for row dependence of J_q or J_{pq} to occur, which indicates a voltage bifurcation or a voltage and angle bifurcation occurs at $V_{k \min}$.

A P-V curve is another very common point of collapse measure of distance to loss of voltage stability. The P-V curve will be shown to be a test for any of the three types of bifurcation of J . The P-V curve is a plot of the magnitude of the transfer T_{ij} between buses i and j versus the voltage at bus k. The curve is plotted by solving the loadflow after

increasing real power generation at bus i and decreasing real power load at bus j by T_{ij} for several increasing transfer level values T_{ij} . The voltage at bus k can then be plotted for each T_{ij} value as $V_k(T_{ij})$. The P-V or "nose" curve is used to measure distance to collapse in terms of the transfer or system load level change required to reach the point of bifurcation where the slope is negative infinite. The slope of the PV curve at any point required solving

$$\begin{bmatrix} \Delta P_G \\ \Delta P_L \\ \Delta Q_L \end{bmatrix} = \begin{bmatrix} \underline{e}_{ij} \\ \dots \\ 0 \end{bmatrix} \Delta T_{ij} - \hat{J} \begin{bmatrix} \Delta \delta \\ \Delta \theta \\ \Delta V \end{bmatrix} \quad (5.16)$$

for $\begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$ by solving the real power balance equations

$$\begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \left[\underline{e}_{ij} \Delta T_{ij} - \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \Delta V \right]$$

where $\underline{e}_{ij}$ is a vector where element i is one, element j is negative one and all other elements are zero.

Solving for $\begin{bmatrix} \Delta \delta \\ \Delta \theta \end{bmatrix}$ requires that $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ be nonsingular, which is a steady state angle stability condition. Substituting into the reactive power balance equations, collecting terms in ΔV , and solving for ΔV , the following expression is obtained

$$\Delta V = - \left[D_4 - [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \right]^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \underline{e}_{ij} \Delta T_{ij}$$

The slope of the P-V curve satisfies

$$\Delta V_k = \underline{e}_k^T \Delta V = - \underline{e}_k^T \left[S_{Q_L V}^* \right]^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \underline{e}_{ij} \Delta T_{ij} \quad (5.17)$$

Note that existence of $S_{Q_L V}^{*-1}$ requires row independence of J_q and joint row independence of J_p and J_q and that existence of $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1}$ requires row independence of J_p . If the system is operating above the nose of the P-V curve where the slope is not negative infinite, where both $S_{Q_L V}^{*-1}$ and $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1}$ exist, the matrix $\hat{J}$ has not experienced singularity due to row dependence of J_q , J_p and joint row dependence of J_p and J_q . However, the nose of the P-V curve can occur due to row dependence of J_q , joint row dependence of J_p and J_q , or row dependence of J_p since the slope of the P-V curve can be written as

$$\Delta V_k = -e_k^T \left[\begin{bmatrix} I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} e_{ij} \Delta T_{ij} \end{bmatrix} \right] \quad (5.18)$$

Singularity of any of the three matrices

$$\begin{bmatrix} I - D_4^{-1} [A_4 \ B_4] \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \end{bmatrix}, D_4, \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$$

can cause the slope to be negative infinite and thus the nose of the P-V curve to occur. It should be noted that the row dependence of J_q or joint row dependence of J_p and J_q will result in a Q-V curve minimum but that any row dependence of $\hat{J}$ will produce the nose of the P-V curve to occur. It is not clear if the capability to detect loss of row independence of J_p gives the P-V curve any advantage over a Q-V curve. Since Q-V curves and P-V curves are so commonly used to assess violation of operating criteria, that are based on them, a more detailed discussion is in order:

- (a) The Q-V curve is a measure of the reactive load stress that can be added at a particular bus, and thus, the local distribution of reactive generation reserves available to a specific bus k and the capability of the network to draw those reactive generation reserves to this bus k where the Q-V curve is computed. Since the $S_{Q,V}^*$ elements are almost identical for all buses in each voltage control area in [29,30], the Q-V curve is a test for the distribution of reactive reserves available to a voltage control area as well as the capability of the network to supply that reactive power to that voltage control area. The Q-V curve is a test for distance or margin in terms of added reactive load to the point of loss of PQ controllability for a voltage control area, since loss of PQ controllability is guaranteed to occur at the Q-V curve minimum. The Q-V curve is also a test for distance or margin to bifurcation of a voltage control area due to row dependence of J_q or joint row dependence of J_p and J_q since such row dependences occur at the Q-V curve minimum. The row dependence of J_q occurs as reactive generation reserves are progressively exhausted at generator (PV) buses that are electrically further and further from the test bus k until the reactive power obtained from the

remaining reactive generation for voltage drop can't meet the increased reactive losses due to voltage drop at bus k . These reactive losses includes series I^2x losses and the shunt capacitive withdrawal. The joint row dependence of J_p and J_q occurs when a Q-V curve is produced for a radial transmission line connecting a generator to a load bus. The series I^2x losses on the line and shunt capacitive withdrawal at the load bus place a limit on the reactive load that can be supplied at the load bus.

- (b) The computation of a P-V curve is a point of collapse method that measures distance or margin to collapse (static) in terms of added transfer over a branch or interface or added system load in a particular pattern at load buses and compensated for using a particular participation pattern at generators. The P-V curve based transfer limit will often be dictated by a bifurcation that occurs due to power flow across voltage control area boundaries, which are the weakest boundaries in the network with the largest surge impedance load (SIL) levels based on experience on the B.C. Hydro, BPA, Detroit Edison, and Consumers Power systems. Some utilities utilize a Q-V curve to assess reactive stress margin at a specific load bus or buses and other utilities utilize a P-V curve to assess real power transfer limits across one or more interfaces or a loading limit. It is clear that a Q-V curve can not assess transfer limits due to row dependence of J_p , but if it is assumed that steady state angle stability limits are not a problem for a utility, a Q-V curve or a P-V curve could be used to assess row dependence of J_q or joint row dependence on J_p and J_q . Utilities that utilize the Q-V curve justify their operating limits or enhancements in terms of reactive supply limitation and utilities that utilize P-V curve justify their operating limits and enhancements in terms of transfer or loading limitations. Note that as long as J_p is assured

to be nonsingular, P-V and Q-V methods should hopefully, but not necessarily, identify identical points of voltage collapse bifurcation and just specify the operating limits differently. Since both transfer or loading increase and reactive supply limitation cause voltage collapse, and since specification of limits on one does not assure sufficient margin on the other, operating limits should probably be specified in terms of both reactive reserves and on transfer levels.

- (c) P-V and Q-V curves are point of collapse methods that attempt to provide stress measures in terms of particular transfers, real power loading and generation patterns, and reactive load at particular buses. There is at present no systematic manner for identifying a sufficient number of independent real power transfers, loading and generation patterns, and bus reactive load stress tests to assure voltage collapse will not occur on a system. No literature currently exists on how to theoretically specify a sufficient number of tests (and associated constraints) to prevent voltage collapse for all combination line outage and loss of generation contingencies for any particular operating condition. A heuristic procedure has been developed for identifying a set of independent Q-V curve tests where each will test for a uniquely different voltage collapse bifurcation that could occur due to the fact the Q-V curve minima occur at $V_{k_{min}} > .88$. This method has been evaluated

on several systems and shows there are many tests required. Attempting to specify planning or operating criteria on one or a few tests for a particular utility experiencing voltage collapse will generally not prevent collapse from happening since the tests are not of sufficient dimension to constrain the system from experiencing bifurcation for all operating conditions and all first contingencies. The procedure has been tested on several large 3000 bus, 4500 branch data bases. This procedure is discussed in more detail in the Security Criteria section of this paper.

- (d) Loss of nonsingularity of $\hat{J}$ [1,5,7,10,26,27,32,33,34] and sensitivity matrix tests [28,29,35] are global tests for row dependence of J_p , J_q , and joint row dependence of J_p and J_q . The tests do not require the engineer to select specific buses to monitor voltage on a P-V curve for each transfer or combination of transfers or the buses to assess voltage change for Q-V curve type reactive load stress test for each area with insufficient reactive supply. However, the critical transfers and transfer combinations and the areas of specific reactive supply limitation must still be known to perform the proper tests using tests for singularity of $\hat{J}$ or matrix sensitivity tests in order to assess transfer and reactive supply limitation. The procedure mentioned in (c) would provide the set of transfer and transfer combinations (where P-V curves must be run) and the set of areas with

reactive supply limitation (where Q-V curves must be run) to provide a sufficient set of tests to prevent many, if not all, the modes of voltage collapse.

- (e) If a Q-V curve can not be computed due to lack of loadflow convergence, $\hat{J}$ may have become singular due to row dependence at buses in some region other than where reactive supply limitation is being tested using a Q-V curve. A Q-V curve is much easier to compute than a P-V curve since a load (PQ) bus is converted to a generator (PV) bus to compute the Q-V curve by adjusting the voltage setpoint at this PV bus. The conversion of a PQ bus to a PV bus provides reactive supply in the area with reactive supply problems and helps assure loadflow convergence. The loadflow solutions close to the nose of the P-V curve often can not be computed or require significant user intervention to obtain solutions. Since the P-V curve naturally encounters loadflow convergence problems, the precise transfer at the point of voltage collapse can't be determined with normal load flow algorithms.

Alternate point of collapse methods have been proposed. These methods are reviewed because P-V and Q-V methods appear to be inadequate to assure that all the possible modes of voltage collapse are detected. These alternate point of collapse methods offer flexibility in detecting modes of collapse required for the security criteria proposed in Section 5.

Alvarado [33] has developed a method for directly computing the point of collapse by appending the bifurcation conditions

$$\hat{J} \underline{y} = 0 \quad (5.19)$$

$$\underline{y}^T \underline{y} = 1$$

to the normal load flow equation (3.7). To obtain a well posed problem with the same number of equations as variables Alvarado allows real and reactive loads and real generation to depend on a parameter. This variation would allow computation of the point of collapse for

- a P-V curve for any single transfer or complex transfer pattern change;
- a P-V curve for any complex load and generation pattern change;
- a Q-V curve at load stress at any bus or pattern for a set of buses

This variation allows much more complex real and reactive loading change patterns than can be computed by P-V and Q-V curves. The algorithm in [33] could be used to obtain bifurcations (row dependance in $\hat{J}$ of

normal rank $2N-M-1$) that can't be obtained by running $N-M$ Q-V curves at all load buses and single transfer or real power and generation loadings. It was pointed out in [38] that there is a feasible reactive load space of dimension $N-M$. There is a bifurcation in every direction in this space. The Q-V curves run at all $N-M$ buses only provide the bifurcations that occur in the $N-M$ orthogonal directions where reactive load is added at only one of the $N-M$ buses for each Q-V curve. It is clearly impossible to determine all possible bifurcations of $\hat{J}$, one solely desires to find those that can occur for the combination of (a) feasible anticipated operating conditions, (b) a single line outage contingency, and (c) a loss of reactive resource contingency, which is the normal planning criterion for voltage stability.

Cutsem [39] has proposed an optimization algorithm for computing the point of collapse. Reactive load is added in a prespecified pattern and Kuhn Tucker necessary conditions for the optimal total system reactive load that can be added in a prespecified pattern before voltage collapse occurs. The necessary conditions include equations identical to (3.7) and (5.19). Thus, the Cutsem method [39] is a method for computing the point of collapse for Q-V curves where reactive load is added at all buses simultaneously. Computing a Q-V curve by adding reactive load in a pre-specified pattern at all PQ buses would almost certainly not converge near the bifurcation point. Computing a P-V curve by increasing real power load or transfers in some complex pattern would also certainly not converge near the bifurcation point. Both the Alvarado [33] and Cutsem [39] algorithms are claimed to be rapidly convergent. Thus, these algorithms [33,39] can be used to determine the point at which voltage collapse bifurcations occur, which may not be detected using Q-V curves (which allow reactive loads to be added at one bus at a time without load flow convergence problems) and P-V curves (which always have load flow convergence problems).

Reference [34] is a point of collapse method that tests for singularity of $\hat{J}$ as a function of load admittance change patterns. The singularity tests for both real power and reactive power load limits at every bus to determine where the singularity occurs. The method also indicates whether the singularity occurs due to real power load or transfer limits or reactive power load limits. This is a global test of singularity of $\hat{J}$ as a function of increase in a particular load admittance pattern.

A point of collapse method that adds real power load in a particular pattern at buses in a defined

region and compensates for this load by adding generation at certain generators in a particular participation pattern was tested in [35]. The point of collapse was detected using sensitivity matrices. The algorithm eliminates load flow calculations by utilizing the jacobian to project or predict each time a PV bus to PQ bus change occurs as one adds load in this pattern and compensates by picking it up in a particular pattern. At the projected point of PV to PQ bus conversion, the sensitivity tests for voltage collapse is performed and, if no part of the system has suffered collapse, the jacobian is updated and used to project the next point of PV to PQ conversion if additional load is added. If the sensitivity tests indicate collapse occurs at any projected point of PV to PQ bus conversion, the algorithm terminates and indicates the total system load that must be added to obtain collapse.

5.1.3 Sensitivity Tests

The results presented on PQ controllability can now be related to the array of tests given in previous voltage stability literature given the equation

$$\Delta V/V_0 = S_{QLV}^{*-1} \Delta Q_L + S_{VE}^* \Delta E/E_0$$

The early voltage stability criterion of Borreman [12] contained a condition $\left. \frac{dV_1/V_1}{dQ_{Lj}/Q_{Lj}} \right|_{P_L=P_0} < 0$ where Q_{Lj} is

reactive demand rather than reactive injection as used in this paper. The condition required the elements of S_{QLV}^{*-1} to be positive to prevent loss of voltage

stability. The linearized dynamic research [13,14,16,17] placed a stronger condition on S_{QLV}^* based on assuring the under load tap changer controls, switchable shunt capacitor controls, or excitation voltage controls be stable. The condition imposed was that $S_{QLV}^* (S_{QLV})$ is an M matrix, which assures that $S_{QLV}^* (S_{QLV})$ have nonpositive off diagonal elements, that S_{QLV}^* have positive diagonal elements, that $S_{QLV}^* (S_{QLV})$ be nonsingular, and that elements of $S_{QLV}^{*-1} (S_{QLV})$ be positive or zero. The existence of S_{QLV}^* (which assures $\begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix}$ is nonsingular) and the M matrix condition on Model 2 S_{QLV}^* is required to assure

nonexistence of a algebraic bifurcation of the loadflow equations (3.7) when $\Delta E = 0$ and the transient stability model equations (3.3-3.5) (changes in the number of stable and unstable equilibrium points at steady state operating points). The M matrix condition on S_{QLV}^* matrix along with the angle constraint (4.1) and the requirement that every voltage control area be connected to a PV bus prevents loss of PQ controllability. Controllability only

assures that loadflow and transient stability simulations will not have bifurcations but both may have severe numerical problems even though the system is PQ controllable [3].

Simulation results in [3] indicate the elements of $S_{Q_L V}^{*-1}$ ($S_{Q_L V}^{-1}$) are very small when the system is voltage secure but increase rapidly as voltage security is lost. As one approaches loss of voltage stability, elements of $S_{Q_L V}^{*-1}$ ($S_{Q_L V}^{-1}$) approaches infinity and then experimental results [3,29,30] indicate that $S_{Q_L V}^{*-1}$ ($S_{Q_L V}^{-1}$) are negative after loss of voltage stability occurs. Note that the PQ controllability condition only requires that the eigenvalues of $S_{Q_L V}^*$ be positive and jacobian J^* is nonsingular. Note that using matrix $S_{Q_L V}^*$ one can tell the bus or region which is experiencing reactive supply limitation without ever computing a Q-V curve for every bus for that specific operating condition since those $S_{Q_L V}^{-1}$ elements will be zero or negative. Note that [35] is a point of collapse method that uses this test to provide an assessment to loss of voltage stability in a control center.

The only literature on conditions on S_{VE}^* are in Abe [13] and Glavitsch [11]. Abe [13] concludes that elements of S_{VE}^* should be positive and Glavitsch [11] condition (Theorem 1) requires that row sums of S_{VE}^* should not approach zero. Our PQ controllability condition requires elements of S_{VE}^* to be positive similar to Abe [13] and that row sums should not be zero similar to [11]. Results in [3] establishes that S_{VE}^* will be nonnegative and have no zero rows if $S_{Q_L V}^*$ is an M matrix and the angle constraint (4.1) is satisfied on branches connecting load (PQ) and generator (PV) buses.

Experimental results in [3] indicate that row sums of S_{VE}^* (S_{VE}) increase as one loses voltage security and can become quite large as one approaches the conditions for loss of voltage stability. Experimental results indicate row sums of S_{VE}^* (S_{VE}) become negative after one experiences loss of voltage stability. The Glavitsch [11] condition and our controllability condition clearly support that row sums of S_{VE}^* (S_{VE}) are not to approach zero if loss of voltage stability is to be avoided, and that zero row sums of S_{VE}^* (S_{VE}) is apparently mathematically a boundary condition for loss of voltage stability. This condition is difficult to test since the loadflow is not likely to converge at points after S_{VE}^* (S_{VE}) row sums become large and certainly not at points as these row sums approach zero and go negative. This may be why Glavitsch [11] implemented his test in a program that did not require loadflow convergence.

Glavitsch used his tests to indicate buses or areas where loss of voltage stability could occur and thus produced a global test for loss of voltage stability.

PV controllability [3] places conditions on the proper relationship between reactive generation at PV buses Q_G and both the voltage setpoint at PV buses E and the reactive load at PQ buses (Q_L). Conditions are developed on sensitivity matrixes $S_{Q_G Q_L}$ and $S_{Q_G E}$ that assure PV controllability. Reference [28] also extended PQ controllability represented by (4.1) by deriving sensitivity of voltage at PQ buses to real power load, real power generation, and MVAR compensation. Reference [28] extended PV controllability by relating reactive generation Q_G to these same variables. Although these additional sensitivity matrixes were derived, no reliable loss of voltage stability condition tests were obtained from these additional sensitivity matrixes. Reference [4] establishes the same condition on $S_{Q_G Q_L}$ as established by the PV controllability condition.

5.2 Dynamic Tests

The dynamic tests depend on model (3.3-3.6) and jacobians J^{***} , $\hat{J}^{***}$, and sensitivity matrixes $S_{Q_L V}^{***}$ and $\hat{S}_{VE}^{***}$ rather than the load flow model (3.7) jacobian J , and sensitivity matrixes $S_{Q_L V}^*$ and S_{VE}^* .

The results in this section will discuss saddle node and flux decay bifurcations. Saddle node bifurcations have been investigated using Lyapunov methods in [8,9]. It is discussed first because it helps one observe the link between dynamics and the static tests previously discussed. It also helps in differentiating algebraic and dynamic bifurcations and classifying the subclasses of algebraic bifurcations.

5.2.1 Lyapunov Method

The Lyapunov method [8,9] has proposed the use of the minimum height of the potential well around a stable equilibrium point as a proximity measure for loss of voltage stability. Although the method has been confirmed as a measure of loss of voltage stability on different example systems, justification of it's use to measure proximity to loss of voltage stability (row dependence of J_q or joint row dependence of J_p and J_q) may appear difficult to comprehend when the Lyapunov function is solely a measure of angle differences at generator internal buses when a classical transient stability model is used. The Lyapunov based methods for assessing proximity to voltage collapse uses a topological energy function so that the energy function measures not just angle differences and magnetic energy when voltages are fixed but measures both the stored magnetic energy and electric energy of both the

network and load models. Furthermore, the energy of arbitrarily complex load models can be included. The model is a general power system model where generator flux decay, exciter, stabilizer and governor turbine model (3.6) are neglected. Note that E'_q is not a variable and thus G_G in (3.5a) need not be written. The use of such a topological energy function possibly relieves some concern about it's use as a proximity measure for voltage stability. It will now be shown that the height of the potential well of a topological energy function is a proximity measure for:

- (a) loss of voltage and angle stability (joint row dependence of $J_g = [A_1 \ B_1 \ D_1]$ with $J_{pL} = [A_2 \ B_2 \ D_2]$ and $J_q = [A_4 \ B_4 \ D_4]$ when none of the three matrices are row dependent;
 - (b) loss of angle stability (joint row dependence of J_g and J_{pL});
- if causality holds.

The system (3.3,3.4,3.5b) is locally stable, if and only if, the topological energy function is positive definite in a neighborhood $(\underline{\delta}, \underline{\theta}_L, \underline{V}_L)$ around equilibrium point $(\underline{\delta}_0, \underline{\theta}_{L0}, \underline{V}_{L0})$ that satisfies (3.4, 3.5b). This requirement of positive definiteness in the local neighborhood of $(\underline{\delta}_0, \underline{\theta}_{L0}, \underline{V}_{L0})$ satisfying (3.4, 3.5b) is guaranteed, if and only if, $\hat{J}^{***}$, defined in 3.15, is nonsingular and positive definite when causality (nonsingularity of $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$) holds.

Note the * on submatrices of $\hat{J}^{***}$ and J^{***} are dropped for the remainder of Section 5, where J^{***} is defined in (3.15) and J^{**} is defined in (3.16). Note $\underline{V}_L = \begin{bmatrix} \underline{V}_T \\ \underline{V}_H \\ \underline{V}_{LL} \end{bmatrix}$ where $\underline{V}_T$ is the terminal voltage and $\underline{V}_H, \underline{V}_{LL}$ are the high side transformer and load bus voltages. This result is now justified by first noting that for some nonzero $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$, there would be a nonzero value of $\Delta\delta$ from linearization of (3.4, 3.5b), if $\Delta P_L = 0$ and $\Delta Q_L = 0$,

$$\begin{bmatrix} A_2 \\ A_4 \end{bmatrix} \Delta\delta + \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = 0$$

when the system is causal so that $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$ is nonsingular. The requirement that the energy function be positive definite for variations $[\Delta\delta, \Delta\theta_L, \Delta V_L]$ around $(\underline{\delta}_0, \underline{\theta}_{L0}, \underline{V}_{L0})$ satisfying (3.4,3.5b) requires

$$V_P = \begin{bmatrix} \Delta\delta \\ \Delta\theta_L \\ \Delta V_L \end{bmatrix}^T \hat{J}^{***} \begin{bmatrix} \Delta\delta \\ \Delta\theta_L \\ \Delta V_L \end{bmatrix} > 0 \quad (5.20)$$

where variations satisfy

$$\begin{bmatrix} A_2 \\ A_4 \end{bmatrix} \Delta\delta + \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = 0 \quad (5.21)$$

since (5.20) is a linearization of the energy function in a neighborhood of $(\underline{\delta}_0, \underline{\theta}_{L0}, \underline{V}_{L0})$ and (5.21) is a linearization of (3.4,3.5b) for $(\Delta\delta, \Delta\theta_L, \Delta V_L)$ around $(\underline{\delta}_0, \underline{\theta}_{L0}, \underline{V}_{L0})$. Solving for $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$ from (5.21), and substituting in (5.20), the energy function is

$$V_P = \Delta\delta^T A_1^{**} \Delta\delta; A_1^{***} = A_1 - \begin{bmatrix} B_1 & D_1 \end{bmatrix} \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix}$$

The necessary and sufficient condition for local stability is that A_1^{**} be positive definite given that causality holds. It can also be seen that A_1^{**} exists and is nonsingular, if and only if, $\hat{J}^{***}$ is nonsingular if $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$ is nonsingular (causality holds) from Schur's formula

$$\det(\hat{J})^{***} = \det(A_1^{**}) \det \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$$

When causality holds, nonsingularity and positive definiteness of $\hat{J}$ assures that the minimum height of the potential is not zero. When causality is lost, one must also know what happens to the minimum height of the potential well.

When causality is lost, it is clear that $\Delta\delta = 0$ from (5.21) when the variation $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$ lies in the null space of $J^{**} = \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$. The energy function (5.20) then becomes

$$V_P = \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}^T \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix} \begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix} = 0 \quad (5.22)$$

for $\Delta\delta = 0$ which is true when $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$ variations lie in the null space of $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$. Thus, the energy function (5.20) for variation $\begin{bmatrix} \Delta\theta \\ \Delta V \end{bmatrix}$ satisfying 5.21 can not be positive definite when causality does not hold. Thus, the minimum height of the energy function is zero when loss of causality occurs. It will be shown that the loss of causality in itself is not an indication of voltage collapse bifurcation because it is necessary but not sufficient for network bifurcation [8,9].

Thus, local stability requires causality to hold and A_1^{**} be positive definite. It should be noted that dynamical system theory can not describe what happens when loss of causality occurs and, thus, causality not only assures that existing theory can describe what happens, but provides a necessary and sufficient

condition for local stability (i.e. A_1^{**} be positive definite.)

It will now be shown that positive definiteness and, thus, nonsingularity of A_1^{**} is violated if:

- (a) $J_g = [A_1 \ B_1 \ D_1]$ is row dependent, or;
- (b) $J_c = \begin{bmatrix} A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix}$ is jointly row dependent

with J_g but neither J_g or J_c is row dependent by themselves.

The matrix A_1^{**} satisfies

$$A_1^{**} = A_1 - [B_1 \ D_1] \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix} \quad (5.23)$$

A_1^{**} is singular when J_g is row dependent since A_1 and $[B_1 \ D_1] \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix}$ are both row dependent in the same rows as J_g when J_g is row dependent.

It will now be shown that singularity of $I - A_1^{-1} [B_1 \ D_1] \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix}$ is a test of joint row dependence

of $J_c = \begin{bmatrix} A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix}$ and $J_q = [A_1 \ B_1 \ D_1]$ when neither J_c or J_q are row dependent.

Premultiplying $\hat{J}^{***}$ by

$$\begin{bmatrix} A_1^{-1} & 0 \\ 0 & \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \end{bmatrix} \quad (5.24)$$

to produce

$$\begin{bmatrix} I & A_1^{-1} [B_1 \ D_1] \\ \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix} & I \end{bmatrix} \quad (5.25)$$

and premultiplying (5.25) by

$$\begin{bmatrix} I & -A_1^{-1} [B_1 \ D_1] \\ 0 & I \end{bmatrix}$$

produces

$$\begin{bmatrix} I - A_1^{-1} [B_1 \ D_1] \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix} & 0 \\ \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix} & I \end{bmatrix} \quad (5.26)$$

Since A_1^{-1} and $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1}$ exist because J_q and J_c

are assumed to be row independent, singularity of

$$I - A_1^{-1} [B_1 \ D_1] \begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}^{-1} \begin{bmatrix} A_2 \\ A_4 \end{bmatrix}$$

which would cause singularity of $\hat{J}^{***}$ and (5.26) and, thus, must be due to joint row dependence of J_g and J_c .

5.2.2 Saddle Node and Algebraic Bifurcation Tests

Joint row dependence of J_g and J_c or row

dependence in J_g suggests the rows of $\hat{J}^{***}$ associated with real power equations at generator internal buses must be included in the dependent rows of $\hat{J}^{***}$ for Lyapunov methods to assess proximity to loss of voltage stability. Thus, loss of voltage stability, loss of angle stability, or joint loss of voltage and angle stability that do not include rows of $\hat{J}^{***}$ associated with real power equations at generators internal buses, which would result in loss of causality, may not be detected by Lyapunov methods. Lyapunov methods detect the occurrence of saddle node bifurcations when causality holds. Saddle node

bifurcation occurs when (a) $J_g = [A_1 \ B_1 \ D_1]$ is row dependent, or (b) J_g and $J_c = \begin{bmatrix} A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix}$ are

jointly row dependent but neither J_g or J_c are row dependent. Saddle node bifurcation can occur due to angle bifurcation if J_g is row dependent or J_g and J_{PL} are row dependent. Saddle node bifurcation can't occur due to voltage bifurcation since row dependent of J_q which is part of J_c will not cause saddle node bifurcation. Saddle node bifurcation can occur due to voltage and angle bifurcation if J_g and J_q or J_g , J_{PL} and J_q are jointly row dependent but none of these row matrices are row dependent.

Algebraic bifurcation occur when $J_c =$

$$\begin{bmatrix} A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix} \text{ is row dependent, and loss of causality}$$

occurs when $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$ is singular which may or may not

result in algebraic bifurcation since $\begin{bmatrix} A_2 \\ A_4 \end{bmatrix}$ may not be

row dependent with the same row weighting as

$$\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}. \text{ Algebraic bifurcation results in loss of}$$

causality and both algebraic bifurcation and loss of causality, which correspond to load flow based tests

for voltage stability, indicate inability of the

network equations to have unique solution. Saddle

node bifurcation is based on the assumption that J_c

and, thus, $\begin{bmatrix} B_2 & D_2 \\ B_4 & D_4 \end{bmatrix}$ are row independent. Existence

of a saddle node bifurcation can not occur simultaneously with an algebraic bifurcation because the conditions for saddle node bifurcation do not allow J_c to be row dependent, which is exactly the condition for algebraic bifurcation. Saddle node bifurcation, thus, requires that the generator mechanical dynamic equations be involved in the set of equations experiencing bifurcation where algebraic bifurcation (load flow bifurcation) do not involve generator mechanical dynamics. The algebraic or load flow bifurcation involves stress in the network that includes generator terminal, high side transformer and load buses, whereas saddle node bifurcation is loss of steady state angle and transient stability of the model reduced to generator internal buses. Algebraic bifurcation and load flow bifurcation can be due to an angle bifurcation if $J_{PL} = [A_2 \ B_2 \ D_2]$ is row dependent or due to voltage bifurcation if $J_q = [A_4 \ B_4 \ D_4]$ is row dependent, or due to steady state angle and voltage bifurcation if $J_c = \begin{bmatrix} A_2 & B_2 & D_2 \\ A_4 & B_4 & D_4 \end{bmatrix}$ is row dependent but $J_{PL} = [A_2 \ B_2 \ D_2]$ and $J_q = [A_4 \ B_4 \ D_4]$ are row independent. All of these bifurcations cause loss of causality and thus can be observed in loss of positive definiteness of the energy function in a neighborhood around $(\delta_o, \theta_{L_o}, V_{L_o})$ that satisfies (3.4, 3.5b).

Lyapunov methods appear to capture loss of voltage stability, loss of angle stability, or loss of joint voltage and angle stability due to any algebraic bifurcation, or saddle node bifurcation. All of these bifurcations can cause the Lyapunov energy function to lose its positive definiteness property in the neighborhood of $(\delta_o, \theta_{L_o}, V_{L_o})$ that satisfies (3.4, 3.5b). Although the Lyapunov method assess these bifurcations, it can not classify the saddle node bifurcation, those algebraic bifurcations, or loss of causality due to:

- (a) loss of angle stability ($\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ no longer positive definite);
- (b) loss of voltage stability (S_{QLV}^* no longer a M matrix);
- (c) loss of voltage and angle stability ($\begin{bmatrix} A_1^* & B_1^* \\ A_2^* & B_2^* \end{bmatrix}$ no longer positive definite and S_{QLV}^* no longer an M matrix).

Furthermore, the Lyapunov method has the flaw of detecting loss of causality, where it causes no bifurcation of the model (3.3,3.4,3.5b).

Up to this point, generator flux decay, exciter, stabilizer, and governor turbine dynamics have been

neglected. It is clear that if (3.6) and (3.5a) are included, J_g, J_q, J_{PL}, J_c are defined as

$$J_g^{***} = \begin{bmatrix} A_1 & B_1 & C_1 & D_1 & E_1 \end{bmatrix}$$

$$J_q^{***} = \begin{bmatrix} A_4 & B_4 & C_4 & D_4 & E_4 \end{bmatrix}$$

$$J_{PL}^{***} = \begin{bmatrix} A_2 & B_2 & C_2 & D_2 & E_2 \end{bmatrix}$$

$$J_c^{***} = \begin{bmatrix} A_2 & B_2 & C_2 & D_2 & E_2 \\ A_4 & B_4 & C_4 & D_4 & E_4 \end{bmatrix}$$

The row dependence condition for:

- voltage bifurcation on J_g^{***} ;
- angle bifurcation on J_{PL}^{***} ;
- voltage and angle bifurcation on J_c^{***} where angle and voltage bifurcation conditions can't be met; will carry over. Algebraic bifurcation will still encompass all three of these algebraic bifurcations and be tested based on row dependence of any combination of rows of J_c^{***} . Saddle node bifurcation will occur when J_g^{***} and J_c^{***} are jointly row dependent but J_c^{***} is not row dependent. There are at least two other bifurcation types that can occur. One discussed the model (3.3-3.6) and jacobian J^{***} that include J_g^{***}, J_c^{***} , and

$$J_{QG}^{***} = \begin{bmatrix} A_3 & B_3 & C_3 & D_3 & E_3 \end{bmatrix}$$

$$J_y^{***} = \begin{bmatrix} A_5 & B_5 & C_5 & D_5 & E_5 \end{bmatrix}$$

to study two additional dynamic bifurcations that cause voltage instability:

- Hopf bifurcation of mechanical dynamic;
- flux decay bifurcation.

Hopf and saddle node bifurcations are being studied for this more complex model given in Section 2 in [44].

5.2.3 Flux Decay Bifurcation Test

To study flux decay bifurcation, one must utilize the model (3.3-3.6) developed in Section 2 rather than (3.3,3.4,3.5b) used in the study of saddle node bifurcation. The model must include the effects of:

- (a) generator magnetic saturation;
 - (b) field current limitation;
 - (c) line drop compensation;
 - (d) exciter and power system stabilizer models;
- that are crudely approximated in the (3.3,3.4,3.5b) or (3.7) models.

Computational results in [32] indicate that when a generator has not hit field current limit the eigenvalues associated with the mechanical dynamics, the generator flux decay dynamics, and the exciter dynamics are generally complex. Although real parts of these eigenvalues will be shown to approach zero

and possibly result in Hopf bifurcation of mechanical dynamics that can cause field current limit violation as reactive load network and network stress, it does not appear that it will cause voltage instability unless the Hopf bifurcation results in unstable oscillations. The disablement of the excitation system and the reduction of the field current to continuous rating levels caused by the oscillation generally seems to extinguish the Hopf bifurcation. The resultant system after the excitation system is disabled has real eigenvalues rather than complex eigenvalues in the examples in [32]. If the real eigenvalues associated with the generator flux decay dynamics are negative the system is stable and J^{***} remains nonsingular. If the eigenvalues associated with the flux decay dynamics are positive, the system experiences a dynamic voltage collapse where the induced flux, field current, and reactive power out of the generator approach zero forcing the voltage collapse in the system. When the eigenvalues associated with this flux decay dynamics become positive, the determinant of J^{***} changes sign. A specific flux decay bifurcation test is desired which would indicate when the eigenvalues associated with the flux decay dynamics become real rather than being complex and indicate whether the eigenvalue is positive or negative.

It should be noted that in [32] experiments were performed that produced algebraic bifurcation where S_{QLV}^{***} approach positive infinity and became negative and $J_c^{***} = \begin{bmatrix} A_2 & B_2 & C_2 & D_2 & E_2 \\ A_4 & B_4 & C_4 & D_4 & E_4 \end{bmatrix}$ became row dependent and no dynamic collapse occurred. Thus, algebraic bifurcation has been experimentally shown to occur in the (3.3-3.6) model.

An analysis of flux decay bifurcation is now undertaken. To perform this analysis, it will be necessary to assume that algebraic and saddle node bifurcations have not occurred so that the required sensitivity matrices are well defined. Since voltage collapse is assumed to be a static bifurcation, derivatives $\dot{\delta}$ and $\dot{\omega}$ can be set to zero and the swing equations are deleted and replaced by an algebraic real power balance equation for the generator internal buses. The algebraic equation is solved for $\begin{bmatrix} \Delta\delta \\ \Delta\theta_L \end{bmatrix}$ assuming ΔP_G and ΔP_L are zero. After substituting for $\Delta\delta$ and $\Delta\theta_L$, ΔV_L is solved to produce (5.28). $\begin{bmatrix} \Delta\delta \\ \Delta\theta_L \end{bmatrix}$ and ΔV_L are then substituted in the linearized (3.5a) equation to produce (5.27) that relates reactive power generation at the internal bus voltage $\Delta E'_q$ and ΔQ_L .

The sensitivity matrix model has the form from (3.11) and [3]

$$\Delta Q_G = S_{QG}^{***} \Delta E'_q + S_{QGQL}^{***} \Delta Q_L \quad (5.27)$$

$$\Delta V = S_{QLV}^{-1} \Delta Q_L + S_{VE}^{***} \Delta E'_q \quad (5.28)$$

We also set the derivative of exciter and power system stabilizer states belonging to y_2 where $y = (E'_q, y_2)$ to be zero and obtain another set of algebraic equations. We can solve for the field voltage in terms of the terminal bus voltage as

$$E_{fd1} = \text{diag} \left\{ \frac{-K_{A1} K_{R1}}{S_{E1}(E_{fd1}) + K_{E1}} \right\} V_{T1}$$

and thus linearizing

$$\Delta E_{fd1} = \text{diag} \left\{ \frac{-K_{A1} K_{R1}}{S_{E1}(E_{fd01}) + K_{E1} + S_{E1}(E_{fd01}) E_{fd01}} \right\} \Delta V_{T1} \quad (5.29)$$

where E_{fd1} is the vector of generator field voltage and V_{T1} is the vector of terminal bus voltage of

$$\text{generator } i \text{ and } S_{E1}(E_{fd1}) = \frac{\partial S_{E1}}{\partial E_{fd1}}(E_{fd1}).$$

The only differential equations left after setting $\dot{\delta} = 0$, $\dot{\omega} = 0$, $\dot{y}_2 = 0$ are for the flux decay dynamics $y_1 = E'_q$ of $y = (y_1, y_2)$ in (3.6). The linearized flux decay dynamics have the form

$$\Delta \dot{E}'_{q1} = \frac{1}{T_{d01}} \left[\Delta E_{fd} - \Delta E'_{q1} + \frac{Q_{10}^e (x_{d1} - x'_{d1})}{E'^2_{q10}} \Delta E'_{q1} - \frac{(x_{d1} - x'_{d1}) \Delta Q_1^e}{E'^2_{q10}} \right] \quad (5.30)$$

Substituting (5.26-5.28) into (5.29), we obtain in matrix form

$$\begin{aligned} \Delta \dot{E}'_q = & \left[\text{diag} \left\{ \frac{1}{T_{d01}} \right\} \right] \left[\text{diag} \left\{ -1 + \frac{Q_{10}^e (x_{d1} - x'_{d1})}{E'^2_{q10}} \right\} \right. \\ & + \text{diag} \left\{ \frac{-K_{A1} K_{R1} V_{10}}{S_{E1} + K_{E1} + S_{E1} E_{fd10}} \right\} \underline{W} S_{VE}^{***} \\ & \left. - \text{diag} \left\{ \frac{x_{d1} - x'_{d1}}{E'^2_{q10}} \right\} S_{QG}^{***} \right] \Delta E'_q + \eta \Delta Q_L - T \Delta E + \eta \Delta Q_L \quad (5.31) \end{aligned}$$

where $\underline{W}$ is a $G \times N$ matrix with one nonzero element in each row that selects the terminal bus voltage vector out of the vector V_L that includes voltage at terminal, high side transformer, and load buses.

If the field current limit is hit, the excitation system is disabled and the second term

$$\frac{-K_{A1} K_{R1} V_{10}}{S_{E1}(E_{fd10}) + K_{E1} + \dot{S}_{E1}(E_{fd10})E_{fd10}} \frac{WS_{VE}^{***}}{S_{VE}^{***}}$$

becomes zero. This means that a large negative definite matrix term of $\underline{I}$ will be removed. The eigenvalues of matrix $\underline{I}$ will be moved to a point where either it is very close to the origin or even becomes positive. When the excitation system is disabled, the field current is reduced to continuous rating which reduces air gap saturation.

PQ controllability requires S_{VE}^* to be nonnegative [3]. Strong PQ controllability requires row sums of S_{VE}^{***} to be near 1. PV controllability requires that S_{QE}^{***} have positive diagonal elements, be an M matrix, and be α diagonally dominant. Loss of PQ controllability occurs when elements of row sums of S_{VE}^{***} approach infinite and then instantly go negative. Note that if elements of S_{VE}^{***} are negative, it is virtually certain that the eigenvalue of matrix $\underline{I}$ will be positive since the diagonal matrix

$$\text{diag} \left(\frac{-K_{A1} K_{R1} V_{10}}{S_{E1}(E_{fd10}) + K_{E1} + \dot{S}_{E1}((E_{fd10})E_{fd10})} \right) \quad (5.32)$$

multiplying the second term in $\underline{I}$ is so large. Thus, flux decay bifurcation can occur when field current limit have not been reached if PQ controllability is lost, if the second term dominates (5.31) and this term is no longer negative definite due to loss of PQ controllability.

However, if the second term is missing due to field current limits being reached, loss of PV controllability can also cause flux decay bifurcation. Loss of PV controllability results in loss of the M matrix and α diagonal dominance property of S_{QE}^{***} .

When the exciter is disabled due to field current limitation and the second term in equation (5.31) is missing, the loss of PV controllability can cause the eigenvalues of $\underline{I}$ to become positive, since the term

$$\text{diag} \left(\frac{1}{r_{d0i}} \right) \text{diag} \left(\frac{x_{d1} - x'_{d1}}{E_{q10}^2} \right) S_{QE}^{***}$$

is not small compared to the first term. If S_{QE}^{***} is α diagonally dominant, the above term has positive eigenvalues and $\underline{I}$ has negative eigenvalue, but if S_{QE}^{***} is no longer α diagonally dominant or loses its M matrix property, then the eigenvalues of the term will be negative and the eigenvalues of $\underline{I}$ can become positive. Loss of PV controllability can occur [3] before algebraic bifurcation and loss of PQ controllability occur. Thus one can experience flux decay bifurcation before algebraic bifurcation occurs. This is an important result since it proves that this

dynamic bifurcation may occur before algebraic bifurcation when field current limits are hit. A second important result is that in [32] one can prove that this flux decay bifurcation results in a decay of E'_q to zero as well as a decay to zero of reactive power output Q_G . Flux decay bifurcation tears the system apart once it occurs due to the effective shorting of the internal bus of the generators that hit field current limits and experience flux decay bifurcation.

6. SECURITY CRITERIA FOR VOLTAGE COLLAPSE

6.1 Types and Subclasses of Voltage Instability on a Transient Midterm Stability Model

There are three types [40] of voltage stability that must be prevented by security criteria

- (1) Type I - an extremesis condition where relays on large motor loads cause them to drop out due to low voltage. The relays often cause these motors to trip out if voltage remains below .88 pu for a period of time;
- (2) Type IIA - algebraic bifurcation that occurs when J_c^{***} has row dependence. There are three types of algebraic bifurcations;
 - (a) angle bifurcation tested by row dependence of $J_{PL}^{***} = [A_2 \ B_2 \ C_2 \ D_2 \ E_2]$;
 - (b) voltage bifurcation tested by row dependence of $J_q^{***} = [A_4 \ B_4 \ C_4 \ D_4 \ E_4]$;
 - (c) voltage and angle bifurcation tested by joint row dependence of J_{PL}^{***} and J_q^{***} but neither are themselves row dependent;
- (3) Type IID - dynamic bifurcation that occurs when J^{***} are row dependent. There are three types of dynamic bifurcation;

- (a) saddle node bifurcation which occurs when $\hat{J}^{***}$ is row dependent but J_c^{***} is not row dependent

$$\hat{J} = \begin{bmatrix} J^{***} \\ J_g^{***} \\ J_c^{***} \end{bmatrix}$$

$$J_g^{***} = [A_1 \ B_1 \ C_1 \ D_1 \ E_1]$$

$$J_c^{***} = \begin{bmatrix} J_{PL}^{***} \\ J_q^{***} \end{bmatrix}$$

Saddle node bifurcation ignores (3.6) and (3.5a) and is the bifurcation of the generator mechanical dynamics that occur when;

- (1) J_g^{***} and J_{PL}^{***} are row dependent (angle bifurcation),

- (ii) J_g^{***} and J_q^{***} are jointly row dependent but neither J_g^{***} or J_q^{***} are row dependent themselves (voltage and angle bifurcation),
- (iii) J_g^{***} , J_q^{***} , and J_{PL}^{***} are jointly row dependent but none of the three are row dependent by themselves and (voltage and angle stability) and J_g^{***} and J_{PL}^{***} are not jointly row dependent.

Saddle node bifurcation would result in loss of transient stability of the generator mechanical dynamics and occurs when the boundary of the region of attraction approaches the dynamic equilibrium point;

- (b) Hopf bifurcation of the mechanical dynamics that result in oscillations of the mechanical dynamics. A recent thesis [44] has derived conditions on when these oscillations can occur. One of the special circumstances where these oscillations occur is when the system approaches voltage collapse. The unstable Hopf bifurcation can cause field current limits to be exceeded and bring on a flux decay bifurcation [32];
- (c) flux decay bifurcation which will occur when loss of PV controllability occurs. Flux decay bifurcations occur most readily when the generator field current limit is hit. Flux decay bifurcations will result in generator internal voltage, generator terminal voltage, and generator reactive power approaching zero. All of these three affects of flux decay bifurcation will force system blackouts to occur. Most often flux decay bifurcations have not developed and this is why systems recover from near voltage instability. One must perform load shedding or islanding to prevent flux decay bifurcation from causing a blackout.

This classification is different from Tamura's [40] in that algebraic, flux decay, and saddle node bifurcation would be classified as static bifurcations and Hopf bifurcations would be classified as dynamic bifurcations in [40]. A second minor difference is that Tamura [40] does not associate motor load tripping as the cause of Type I voltage collapse. The classification made in this paper is closer to classification of tests by the type of equations that experience bifurcation rather than based on dynamical system theory definitions. The power system engineers can readily relate to algebraic bifurcations that can be studied by his current load flow tools and those

dynamic bifurcations which must be studied using dynamical system tools. Although the model of tap changers and switchable shunt capacitors and their controls are included in this model developed in Section 2 based on [32,44], discussion of the type of voltage collapse in [40] associated with these controls is not discussed. It is an important subject and needs to be included in a consistent manner into the methods developed in this section.

6.2 Closest Dynamic/Static Point of Collapse Methods

At present there are several point of collapse methods [33,34,35,39] that utilize a load flow model that includes generator terminal, generator transformer high side buses, and load buses where generator terminal buses are the PV buses or swing bus and the other buses are PQ buses. These methods are generally limited to Type IIA voltage collapse problems. These methods provide a margin to the point of collapse for any operating condition or any operating condition after single or multiple contingencies. Fink [41] has proposed a point of collapse method for Type I voltage collapse problems. There are at present no known Type IID point of collapse methods for assessing the point of Type IID collapse and the margin that exists between the current operating point and the Type IID collapse point. Potentially, one could attempt to compute eigenvalues of the (3.3-3.6) using the Small Signal Stability Package (SSSP) [47] as one applies stress by (a) adding real power load in some pattern, and compensating for it by specifying a pattern for generation; (b) adding reactive load at a bus (Q-V curve) or at several buses in some pattern; (c) increasing transfer across one or more interfaces. The computation required to determine all eigenvalues that approach the $j\omega$ axis of the complex plane at one operating point for the very large data bases used in voltage collapse is large. Thus, a Type IID point of collapse method that attempts to find the point where dynamic eigenvalues just cross the $j\omega$ axis is feasible but will require significant computation if SSSP is used.

Developing security criteria that prevent any of the three types of voltage collapse from occurring requires three principal tasks.

Task 1 is to develop closest (dynamic/static) point of collapse methods that can indicate the point of collapse, the type of collapse, and the margin to this point of collapse in terms of the stress level of;

- (a) real power loading in some pattern and the associated generation pattern that compensates for the increased real power load;
- (b) the total reactive power load added at a bus or in a pattern at several buses;

(c) the transfer across a single interface or the simultaneous transfer across several interfaces.

The method must be able to check whether a Type I, IIA, or IID voltage collapse has occurred at every stress level as the method searches in a particular direction in parameter space for the closest point of collapse, regardless whether it is Type I, Type IIA, or Type IID. Very fast Type IID point of collapse methods are being evaluated at MSU that would drastically reduce the computation required for a closest (Type I, Type IIA, Type IID) point of collapse method.

6.3 Feasible Mode of Collapse Search Procedure

Task 2 is the development of a search method for identifying all the "modes of collapse" or "reactive reserve basins" where voltage collapse is feasible. Such a search method was developed by the authors for Type IIA and Type I voltage collapse [45]. Our experience indicates that if a Q-V curve is run for each voltage control area [45], a different combination of the generator reactive reserves, switchable shunt capacitive reserves, and load tap changer reserves will be exhausted at the minimum of the Q-V curve for each such voltage control area [29]. A Q-V curve run at different buses in the same voltage control area produces nearly identical Q-V curves since the buses in a voltage control area act so coherently that they can be considered as a single equivalent bus [45]. The synchronous generator reactive reserves, switchable shunt capacitive reserves, and tap changer reserves, that are exhausted at the Q-V curve minima, will be the same for the Q-V curve calculated at any bus in the voltage control area [45]. These reactive resources and voltage control reserves protect that voltage control area from the bifurcation (row dependence of J) that occurs at the Q-V curve minima and are called the "reactive reserve basin" for that voltage control area.

Computing a Q-V curve causes a particular set of rows and columns to be added to the jacobian for each of the generator PV buses where reactive generation reserves are exhausted. Moreover, the jacobian may be modified when tap reserves of under load tap changers and switchable shunt capacitor reserves are acting or exhausted. Each of the jacobian modifications causes the structure and dimension of the jacobian at the point of collapse to be very different from that at the operating point. Thus, a mode of collapse search procedure is not attempting to identify all the eigenvalues or modes of a single jacobian of one particular structure and dimension that can be made to approach zero. The "jacobian eigenvalue that approaches zero" as a Q-V curve is computed from the

operating point to the collapse point is an eigenvalue of a sequence of jacobians of different structure and dimension. Assuming that particular eigenvalue can be found in each jacobian with a modal shape that is focused around the voltage control area where the Q-V curve is computed and assuming the sequence of such eigenvalues approaching zero as the Q-V curve minimum is approached, then one could talk about an eigenvalue of the jacobian approaching zero. Computing Q-V curves in each voltage control area could identify most, if not all, "modes of collapse"; i.e. a sequence of eigenvalues approaching zero with mode shapes centered around the voltage control area. Each eigenvalue in the sequence is computed for a jacobian in a sequence of jacobians produced at points where additional reactive or voltage control resources are exhausted. Note that a "mode of collapse" as just defined is associated with a specific voltage control area where (1) the Q-V curve is computed. A "feasible mode of collapse" is one where the sequence of eigenvalues reaches zero at a voltage $V_{k_{min}}$, where the Q-V curve is minimum, and (2) $V_{k_{min}}$ is below some

threshold V_k^{**} , and (3) where the reactive margin at the Q-V curve minimum $-Q_k(V_{k_{min}})$ is less than some threshold Q^{**} . Not all modes of collapse associated with every voltage control area are feasible since it may require too large a reactive load stress to obtain a Q-V curve minimum and thus no feasible contingency or operation change could exhaust enough reactive supply to cause voltage collapse. Furthermore, Type IIA voltage collapse will not occur if $V_{k_{min}}$ is below .88 since a Type I voltage collapse will occur first in this case and no eigenvalue will approach zero. If one desires to obtain all Type I modes of collapse one would save the reactive resources exhausted at $V_k = .88$ pu if the Q-V curve had a minimum below .88 pu. If the reactive margin of the Q-V curve is very large at $V_k = .88$ for that voltage control area that voltage control area would not have a feasible Type I mode of collapse. The search procedure developed at MSU for obtaining "feasible modes of collapse" only requires computing Q-V curves for less than 10% of the voltage control areas, which may be for 6% of the buses in the model.

The search procedure for identifying feasible Type I and Type IIA modes of collapse has been applied to several large data bases. The results help one understand how complex voltage collapse is because it has been shown that there may be twenty or more feasible modes of collapse. Modes of collapse have different reactive reserve basins with different overlapping combinations of reactive and voltage

control resources and can affect the same region or several overlapping regions. Since the computer program focuses on identifying the unique reactive and voltage control resources that are exhausted at the point of collapse for any mode of collapse computed from a Q-V curve in a particular voltage control area bus cluster, one can utilize the term "reactive reserve basin" rather than "mode of collapse". The use of the term reactive reserve basin was also motivated by the facts that the exhausted resources lie within an encircling basin around the voltage control area and that reactive resources outside this reactive reserve basin are consumed by reactive losses and never reach the load bus where the Q-V curve is calculated at the point of collapse. The reactive reserve basin identified for a voltage control area is to protect that voltage control area against any contingency and is either partially or totally exhausted for any contingency. The set of reactive supply and control devices in a reactive reserve basin decreases as one stresses the system due to exhaustion of reserves on some devices and choking off supply due to increased reactive losses of other devices that belong to the reactive reserve basin under less stressed conditions.

A method for identifying the closest Type I, Type IIA, and Type IID point of collapse discussed under Task 1, could replace the Q-V curve calculation to determine all the feasible "modes of collapse" when one isn't searching solely for Type I or Type IIA points of collapse but rather the closest Type I, Type IIA, or Type IID point of collapse. It should be noted that even if a reactive reserve basin was computed for every voltage control area, one could not be assured of capturing every feasible mode of collapse. Special methods are under development at MSU for seeking every feasible mode of collapse.

It should be noted that having a list of feasible modes of collapse (reactive reserve basins) does not necessarily indicate collapse will occur due to exhaustion of the reactive or control reserves in that reactive reserve basin. One should validate that there are contingencies, among the worst known contingencies, that can significantly reduce the reactive reserves for each feasible reactive reserve basin. Feasible modes of collapse that can experience voltage collapse or near voltage collapse for single contingencies are called critical modes of collapse.

6.4 Dynamic/Static Voltage Stability Security

Criterion

There are two different voltage collapse security criteria that will now be presented. The first is an extension of a common Q-V curve based criterion that

takes into account the fact that one needs to protect the system against the voltage collapse of Type I, IIA, and IID rather than just IIA and that one needs to protect against collapse in every reactive reserve basin. The second security criterion is similar to the first one but does not measure margin in terms of the reactive load stress that can be added at a bus in the voltage control area rather than the magnitude and distribution of reactive reserves on generators, capacitors, and tap changers in the reactive reserve basin that are exhausted at the point of collapse.

Before the Q-V margin criterion is proposed, we define that critical voltage control areas are those voltage control areas where Q-V curves are computed to determine the reactive reserve basins for those areas. It is assumed that there are M critical reactive reserve basins and thus M critical voltage control areas ($m = 1, 2, \dots, M$), which are not adequately protected by sufficient reactive supply or voltage control reserves. The Q-V margin criterion is now stated:

- (a) the minimum of the Q-V curve can be quite narrow for some critical voltage control areas and quite broad for others. To avoid difficulty in defining $V_{k_{\min m}}$ for the Q-V curve computed at bus k in critical voltage control area m when the Q-V curve minimum is broad and flat, $V_{k_{\min m}}$ is defined at a voltage with ΔQ above the absolute Q-V curve minimum as shown in Figure 6.1;

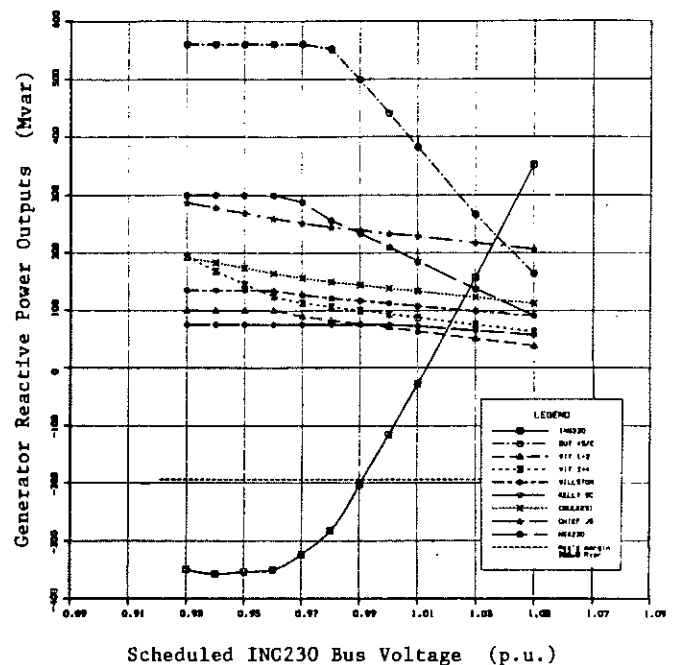


Figure 6.1 Post outage unsupported voltage: 1.013 p.u. Bottom of curve: 0.936 p.u., -360.1 Mvar

- (b) an upper bound is placed on $V_{k_{\min m}} < .95$ for the Q-V curve computed after any single line outage contingency for each critical voltage control area m. This requirement assures loss of voltage stability does not occur in the normal operating voltage range;
- (c) a lower bound on the post contingency operating voltage V_{k_m} after any line outage contingency is imposed in each critical voltage control area m. This lower bound on V_{k_m} assures that the operating condition after any first contingency is sufficiently close to $V_{k_m}^*$, the precontingency voltage at bus k in critical voltage control area m. Another reason for specifying V_{k_m} is that it helps insure that voltages closer to the load do not go below .88 pu causing a Type I voltage collapse either after the initial line outage contingency or after a second contingency loss of reactive resource or operating change. One could also define $\hat{V}_{k_m}$ as a percentage of $V_{k_m}^*$. Other measures similar to lower bound on $V_{k_m}^*$ or an upper bound on $V_{k_m}^* - \hat{V}_{k_m}$ would measure the change due to the contingency in terms of the reactive reserve basin reactive reserves $Q_k(V_{k_m}^*) - Q_k(\hat{V}_{k_m})$;
- (d) a measure of the Q-V curve margin remaining at $\hat{V}_{k_m}$ for each critical voltage control area m is needed to specify the reactive reserve margin that must exist in the reactive reserve basin associated with that critical voltage control area after the first contingency has occurred. The margin for a specific reactive reserve basin would be ΔQ plus the maximum or the weighted sum of;
- (i) the reduction in Q-V curve margin in critical voltage control area m for adding the maximum reactive load at buses in the reactive reserve basin associated with that critical area. The maximum reactive load is the product of the tangent of the angle associated with minimum acceptable power factor times the real load in that reactive reserve basin at any specific time,
 - (ii) the added reactive losses due to anticipated operating changes or emergency actions that affect reserves in reactive reserve basin m,
 - (iii) the reduction of Q-V margin in critical voltage control area m due to loss of the largest reactive resources in the reactive reserve basin.
- The abilities to determine all reactive reserve basins, to quickly evaluate Q-V curves for all critical voltage control areas, and to determine if reserves are exhausted in any reactive reserve basin for all single and double contingencies are implemented as part of the MSU program package. The setting of $\hat{V}_{k_m}$ and the margin in (iii) above could be set based on experience with the MSU program package for various operating conditions.
- The reactive reserve basin security criterion replaces the Q-V curve reactive load stress margin by the reactive supply and voltage control reserves and their distribution. It should be noted that priority control or reactive supply devices are identified a priori for each reactive reserve basin and that these voltage control or reactive supply devices should not be allowed to exhaust their reserves because the loss of priority voltage control or reactive supply reserves will quickly increase the rate of reactive losses within that reactive reserve basin and quickly bring about voltage collapse bifurcation for rather small changes in operating conditions. The reactive reserve basin security criterion is as follows:
- (a) same specification on how to determine $V_{k_{\min m}}$ for Q-V curves with broad flat minima as used in (a) of the Q-V margin security criterion;
 - (b) same specification of $V_{k_{\min m}} < .95$ pu for each critical voltage control area as used in the (b) for the Q-V margin security criterion;
 - (c) same specification of V_{k_m} in terms of $V_{k_m}^*$ as used in (c) for the Q-V margin security criterion. Note V_{k_m} is the post contingency operating voltage at bus k in critical voltage control area m and $V_{k_m}^*$ is the base case voltage at bus k in critical voltage control area m;
 - (d) the worst line outage contingency should not exhaust more than 40% of the reactive supply reserves on the priority control devices in critical reactive reserve basin m. The reactive and voltage control reserves that remain after the worst line outage contingency should be distributed so that no more than 50% of the remaining reactive supply reserves should be associated with total reactive capacity of any reactive supply device;

- (e) a measure of the reactive reserves remaining at $\hat{V}_{k_m}$ for each critical voltage control area m is needed to specify the reactive reserve margin that must exist in that critical voltage control area's reactive reserve basin after the first contingency has occurred. This margin for a specific reactive reserve basin would be ΔQ plus the maximum or the weighted sum of;

- (i) the reduction in voltage control and reactive supply reserves in reactive reserve basin m (for $m = 1, 2, \dots, M$) for adding the maximum reactive load at buses in the reactive reserve basin. The maximum reactive load is the product of the tangent of the angle associated minimum acceptable power factor times the real load in that reactive reserve basin at any specific time,
- (ii) the reduction in voltage control and reactive supply resources in reactive reserve basin m due to anticipated operating changes or emergency actions that affect a reactive reserve basin,
- (iii) the reduction of voltage control and reactive supply reserves in reactive reserve basin m (for $m = 1, 2, \dots, M$) due to loss of the largest reactive resource in the reactive reserve basin.

7. CONCLUSIONS

Security criteria for voltage stability were developed in this paper by:

- (a) developing a general power system model that includes generator flux decay, mechanical, exciter, power system stabilizer, and governor-turbine dynamics. The model includes field current limits and field current capability, generator rotor flux saturation, and the load drop compensator. The network model includes under load tap changers, phase shifters, and switchable shunt capacitors. The model is far more detailed than [1] or others previously used in analyzing voltage stability;
- (b) showing that the load flow and general model equilibria are not identical and that the difference grows as one approaches a point of collapse. It is shown that the load flow is not always conservative in predicting collapse because load flow generator modeling assumes the generator voltage regulation is perfect. Load flow has been assumed to be conservative due to the conservative values of reactive generation limit approximation to rotor field current limits. A better "dynamic" load flow model has

been developed that models the load drop compensator, field current limiter, and finite exciter loop gain. This "dynamic" load flow can produce governor, exciter, tap changer, and automatic generation control load flow [6,42,43] that approximate the effects of power and voltage controls that act in different time frames after a disturbance;

- (c) developing a theoretical framework on voltage collapse based on the classification of voltage collapse as Type I (low voltage causing relays to trip large motors); Type IIA (algebraic bifurcation) and Type IID (dynamic bifurcation) based on modification of the types of collapse given in [40]. A subclassification of algebraic bifurcation was given;

- voltage bifurcation where the voltage experience change but branch angle differences do not experience significant change during the voltage collapse;
- angle bifurcation where branch angle differences experience change but bus voltages do not experience significant change during the voltage collapse;
- voltage and angle bifurcation where both bus voltage and branch angle changes are significant during the voltage collapse.

Three subclassifications of dynamic bifurcations are identified; saddle node, flux decay, and Hopf bifurcation of mechanical dynamics. Flux decay bifurcations were analyzed for the first time and shown to occur before algebraic bifurcations that couple to generator terminal buses could occur. Hopf bifurcation of mechanical dynamics was discussed for the first time. Most, if not all, of the current methods of assessing voltage collapse were classified in terms of the types and subclassifications of voltage collapse they detect. This provided a means of unifying and comparing existing point of collapse methods;

- (d) developing security criteria for voltage collapse based on;

- (i) a new point of collapse method that could detect the closest Type I, Type IIA, and Type IID collapse as a Q-V curve is computed. There are currently no point of collapse methods for Hopf or flux decay bifurcations. Since all of the voltage of collapse types and subclassifications can be approached as a Q-V curve is computed, one only wants to know the closest one that will limit system design and operation,

- (ii) a feasible mode of collapse search procedure that attempts to systematically search for all modes of collapse in a particular system. Methods for assuring that all the modes of collapse found are being developed. Currently a method has been implemented with a Q-V curve point of collapse method that finds the closest Type I or Type IIA mode of collapse. If the new point of collapse procedure described in (i) above is implemented, this procedure could search for all of the closest modes of collapse regardless of whether they are of Type I, Type IIA, or Type IID. The point of collapse can be specified in terms of $V_{k_{min}}$ and the reactive load stress added at $V_{k_{min}}$; $-Q_k(V_{k_{min}})$ where $Q_k(\cdot)$ measures the reactive

injection at bus k. The point of collapse can also be characterized by the reactive generation reserves, switchable shunt capacitive reserves, and under load tap changing reserves that are exhausted at the Q-V curve minima. This set of reactive supply and voltage control resources are called a reactive reserve basin since reactive resources and voltage controls outside of this reactive reserve basin do not allow additional reactive load to be added at the test bus at the Q-V curve minima. The reactive reserve basin provides reactive supply to the voltage control area associated with the reactive reserve basin.

Computational results indicate that there are many undetected feasible modes of collapse in the systems tested to date. Moreover, the closest Type I, IIA, and IID modes of collapse is generally unknown since in most cases Type IID point of collapse tests aren't run, or if run, are only performed for a few modes of collapse.

- (iii) security criteria that provide reactive margin for each feasible mode of collapse. For each mode of collapse, the margin should protect the system against any line outage contingency in combination with the worst loss of reactive supply or operating change; the criteria specifies voltage and reactive margin change from base case due to the

first contingency as well as the voltage and reactive margin that protect the voltage control area from the point of collapse for the second contingency loss of reactive resource or operating point change.

ACKNOWLEDGEMENTS: This research reported is based on Ph.D theses and research supported by Michigan State University. A special note of appreciation is expressed to Dr. Robert Barry, Mr. Chuck Arndt, Mr. Robert Ray, and Mr. Richard Popeck for their interest and support.

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Comments by

J.Zaborszky, H.Schättler and V.Venkatasubramanian
on "Dynamic/Static Voltage Security Criteria"
by R.A.Schlueter et.al.

Clarifications resulting from perspectives different from those of this monumental and interesting paper will be offered here in two areas:

1. PQ Controllability

The paper apparently uses the notion of PQ controllability as the basis for analysing voltage stability. Strong statements are made in the paper which seem to indicate that PQ controllability is a necessary and sufficient condition for voltage stability. This is questionable because *PQ controllability* is only a desirable property, but it is *neither necessary nor sufficient* for voltage stability. We start the discussions with two quotations from the paper on PQ controllability.

"It is obvious that if the system is PQ controllable for all time, which is the normal property of the voltage secure power system, loss of voltage stability is impossible because one can control the voltage at every PQ bus. It should, therefore, not be surprising that every loss of voltage stability test that gives sufficient conditions for preventing loss of voltage stability depends on the sensitivity matrix or jacobian tests that assure PQ controllability. It is also obvious that any proximity tests for loss of voltage stability should measure the proximity to violating the sufficient conditions (for) PQ controllability and necessary conditions for voltage instability should indicate (when) the sufficient conditions for PQ controllability are violated." (Section 4)

"A bifurcation of the loadflow (3.7) and transient stability model (3.3-3.5) cannot occur, if and only if, $S_{Q_L V}^a$ ($S_{Q_L V}^{**}$) defined in 3.12, exists and satisfies the conditions for assuring PQ controllability" (Section 5.1)

PQ controllability requires that the network respond in its *normal* manner. This concept is a generalisation of the classical concept of static stability, i.e. the transmission network should behave like a network of normal springs (and no balls i.e. no inertias). PQ controllability is a desirable property for the system so that the network is amenable to direct *intuitive* operator control. However, this does not mean that the system cannot be operated properly when this condition is not satisfied or that it can be so operated when this condition is satisfied. In fact, the static condition (used long time ago) which ignores the inertias (mechanical and magnetic) is an unrealistic assumption which should be replaced by local dynamic stability for realistic analysis. The system can be operated at any point that is viable (i.e. voltages etc. are within tolerances [1]) and dynamically stable without the danger of voltage collapse. Also, nonlinear control theory tells us that this operating point can be shifted around locally by *suitable* control action without loss of voltage stability. If we want to steer the system to a different operating condition, this is possible only if the desired operating condition is also dynamically stable and viable, and the system is stable all along the path. This implies that the desired operating point belongs to the same *Feasibility Region*, a concept introduced in [1]. Moving the operating point can be achieved by changing parameters such as reactive power demands and the voltage set points among other parameters. PQ controllability is a narrow notion in the sense that it does not take into account dynamic system stability or instability, and it is a desirable property only when the system is stable. *Loss of PQ controllability does not always imply a voltage collapse.*

Is it really obvious that "*the necessary conditions for voltage instability should indicate (when) the sufficient conditions for PQ controllability are violated*"? The quotations seem to say that the paper considers PQ controllability as a necessary and sufficient condition for voltage stability. If so, then both of these claims would be questionable. *For sufficiency*, local dynamic stability would need to be assumed in addition to the static PQ controllability. A simple counterexample would be the unstable section on the upper branch of the QV curve between the Hopf bifurcation and the nose which has PQ controllability. Conversely, *necessity* would be contradicted by the evidence of stable operation, (possibly at too low a voltage) on the lower branch (three industry sources cited examples of stable operation on the lower branch of the QV curve, when this came up at the Deep Creek Lake). Such stability in 'Statically unstable' and not PQ controllable area is readily confirmed theoretically [1]. More detailed counterexamples can be constructed for large systems.

2. Row dependence

In this paper, the multiplicity of row dependency "bifurcations" is offered as a guide for system analysis, but apparently without any mathematical evaluation of their relationship to the basic types of bifurcations such as the saddle node, Hopf and the singularity induced bifurcations. The singularity induced bifurcation of which "flux decay bifurcation" may be a manifestation is precisely analysed in [1]. It is shown in [1] that after taking the closure, the local bifurcations occurring on the power system fall into the above mentioned three categories (in fast, midrange etc. dynamics) but several unconnected occurrence of each is possible (e.g. a saddle node of the common PV curve variety and a saddle node connected with automatic tap changer (ULTC) dynamics). The paper unexplicably relegates Hopf only to mechanical instability and the saddle node only to static instability.

Also the paper introduces a large number of row dependency "bifurcations" without putting them into the context of the basic saddle node (not necessarily static). The fact is that while existence conditions for these can be found easily from simple matrix algebra, checking these conditions requires a very large computation load for a realistic system. Furthermore they provide no new information. It is known already that typically the row dependency "bifurcations" are part of the closure of the saddle node bifurcation manifold. The latter is quite easily computed even for large systems as a surface determined by one additional nonlinear equation, the determinant of the Jacobian set to zero to define the saddle node bifurcation, ignoring the technical transversality conditions. So, for a general system, all the excessive computation to find the row dependency cases would be redundant, finding bifurcation points already known and occurring extremely rarely (a zero measure subset of a zero measure subset of a zero measure subset... repeated a large number of times). This would be of very little practical advantage although a theoretical study of the stratification which contains these row dependency occurrences may have some intellectual appeal. Especially questionable is the proposed use (Section 5.1.1) of row dependencies is respectively the real power balance J_p and reactive power balance J_q equations to separate dominantly angle or voltage instabilities or mixed types. As shown below, this would only be an extremely restrictive (zero measure subset of a zero ...) sufficient condition typically, so its use could be quite misleading.

Analysis:

A row dependence in a flat matrix (i.e. with more columns than rows) implies a rank deficiency. If a square matrix is row dependent, then it is singular, hence it can be detected by the determinant being zero. But, when a rectangular matrix, say with dimensions $m \times n$ ($m < n$) is row dependent, then *all* the square submatrices (of size $m \times m$) of the rectangular matrix have to be singular due to the rank deficiency. This implies that the determinant of *every* $m \times m$ submatrix has to be zero. So, it can be seen that the row dependence is characterised by $n - m + 1$ determinants being zero.

Loadflow model:

The loadflow model assumes that the system is always at steady state. The system equilibrium can be described as the solution of a set of equations $f(x, p) = 0$, where p denotes a set of parameters such as real and reactive power loads, line impedances etc.. In the paper,

$$f = \begin{bmatrix} E_G(x, p) - P_G \\ E_L(x, p) - P_L \\ G_{PQ}(x, p) - Q_{PQ} \end{bmatrix} = 0$$

where $x = [\theta, \delta, V]$. So, the system Jacobian simplifies to $\frac{\partial f}{\partial x}$, denoted by $\hat{J}$ in the paper. The paper goes on to analyse the various "bifurcations" in this system. It follows from the Implicit Function Theorem that when $\hat{J}$ is nonsingular, the system has a unique solution locally when the controls are varied. So, the necessary condition for a static bifurcation is that $\hat{J}$ is singular. It is known from bifurcation theory [2] that the generic bifurcation when $\hat{J}$ becomes singular, is the static version of the saddle node bifurcation i.e. two equilibria meet and disappear at the bifurcation point. This is guaranteed by the saddle node bifurcation theorem provided certain additional transversality

conditions (inequality conditions) are satisfied alongwith $\hat{J}$ being singular [1]. Let p be the number of parameters, p . Define C_Z as the set where $\hat{J}$ becomes singular,

$$C_Z = \{(x, p) : f(x, p) = 0, \det(\hat{J}) = 0\}$$

The saddle node bifurcation theorem says that, generally, saddle node bifurcations occur at most of the points in C_Z . In other words, the closure of the saddle node bifurcation manifold, SN would be the full set C_Z . Let us look at the row dependencies as defined in the paper. The matrix $\hat{J}$ is written as

$$\hat{J} = \begin{bmatrix} J_p \\ J_q \end{bmatrix} = \begin{bmatrix} J_g \\ J_c \end{bmatrix}$$

where

$$J_p = \begin{bmatrix} A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{bmatrix},$$

$$J_q = \begin{bmatrix} A_4 & B_4 & D_4 \end{bmatrix}.$$

The matrices J_p , J_q , J_g and J_c are flat matrices i.e. they have more columns than rows. Row dependencies in the matrices J_p and J_q are defined in this paper as "Angle Bifurcation" and "Voltage Bifurcation" respectively. A row dependency in J_q (or J_p) in particular implies that $\hat{J}$ is singular. So, "Angle" and "Voltage Bifurcations" belong to the set C_Z . The dimension of the matrix J_q is $(N - M) \times ((N - M) + (N + G - 1))$. As outlined earlier, the row dependence in J_q then is characterised by $N + G$ determinants being zero. Here N denotes the number of load buses and G denotes the number of generator buses. When p parameters are considered in general, imposing $N + G$ additional equality conditions would mean that "Voltage Bifurcations" occur in a set of maximal dimension $p - (N + G)$, whereas the set C_Z is $p - 1$ dimensional. Hence, we conclude that "Angle Bifurcations" and "Voltage Bifurcations" are highly degenerate cases which are not normally encountered. Also they do not properly separate conditions where the instability originates in the angle or voltage subspaces, they are only special cases of the latter, as explained in the next paragraph. The joint row dependence of J_p and J_q , case (c) in Section 5.1, is the interesting and overwhelmingly frequent case and this typically leads to a saddle node bifurcation (This is not mentioned in the paper). Saddle node bifurcations are essentially characterised by one additional equation namely $\det(\hat{J})$ is zero. The other degenerate cases of "Voltage bifurcations" and "Angle bifurcations" are much more difficult to locate, i.e. they require excessive additional computations and moreover they are already covered in the closure of the saddle node bifurcation computation.

With the understanding that the saddle node bifurcation is the typical static bifurcation, we want to analyse whether the *instability* originates in the voltage subspace or the angle subspace. First let us consider the voltage subspace by assuming that the Jacobian J_a defined as

$$J_a = \begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}$$

is nonsingular. Then, the (Static) instability in the voltage subspace is characterised by $S_{Q_L V}^*$ becoming singular. By Schur's formula, as $\det(\hat{J}) = \det(J_a) \det(S_{Q_L V}^*)$, the singularity of $S_{Q_L V}^*$ implies the singularity of $\hat{J}$. So, this subcase also corresponds to a saddle node bifurcation in general. The row dependency condition, referred to as the "Voltage bifurcation" in the paper, is a

sufficient condition for the singularity of $S_{Q_L V}^*$. It clearly is not necessary because the singularity of $S_{Q_L V}^*$ is described by one additional equation whereas the row dependence implies $N + G$ additional equations. "Voltage bifurcation" is a very special case of such an instability. Similarly, assuming that the Jacobian D_4 is nonsingular, the static instability in the angle subspace can be detected by the singularity of the matrix

$$J_{angle} = \begin{bmatrix} A_1 & B_1 \\ A_2 & B_2 \end{bmatrix} - \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} D_4^{-1} \begin{bmatrix} A_4 & B_4 \end{bmatrix}.$$

The row dependence in J_p is a very special case of this instability.

If the voltage and angle are completely decoupled, then the matrices J_{angle} and $S_{Q_L V}^*$ simplify to J_a and D_4 respectively i.e. the Jacobian $\hat{J}$ simplifies to a block diagonal matrix with blocks J_a and $S_{Q_L V}^*$

$$\hat{J} = \begin{bmatrix} J_a & 0 \\ 0 & D_4 \end{bmatrix}.$$

Then, we can clearly separate the instability in voltage and angle subspaces. With this decoupling assumption, the singularity of the matrices J_a and D_4 become equivalent to the row dependencies in J_p and J_q because trivially all the other determinants are singular. But, in general, complete decoupling is not a realistic assumption. With the weak coupling that exists in the system, row dependencies would be still difficult to verify computationally and they provide sufficiency information on the angle-voltage decoupling only in a few extremely rare special cases.

Needless to say that angle-voltage decoupling is a very important issue in power system analysis and indeed it has a vast literature. However nothing offered in this paper on row dependency is helpful in any useful way on decoupling and it seems unlikely that any useful decoupling results could be developed along the lines of the paper.

Transient Stability Model:

The system is assumed to be described by a differential algebraic model of the form

$$\begin{aligned} \dot{x} &= f(x, y) \\ 0 &= g(x, y) \end{aligned}$$

The paper develops an extensive model for the general power system. The full system Jacobian in this case is defined as

$$J^{***} = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial y} \end{bmatrix}$$

defined in Equation 3.15. It has been shown that for such a system, instability is typically caused by one of the three types of bifurcations namely Saddle node, Hopf and Singularity Induced bifurcations [1]. Let us analyse the set C_2^* where the Jacobian J^{***} becomes singular. Once again, it follows by the saddle node bifurcation theorem that the points in C_2^* lead to a saddle node bifurcation if certain inequality conditions are also satisfied. So, generally the saddle node bifurcations occur at most of the points of C_2^* .

In section 5.2.1 the matrices J_p , J_q , J_θ and J_c are casually redefined as

$$J_q^{***} = \begin{bmatrix} A_4 & B_4 & C_4 & D_4 & E_4 \end{bmatrix}$$

and so on. It should be noted that the analysis is now different because the dynamics brings in additional equations and variables. As before, it follows that the row dependence in J_q^{***} and the row dependence in J_{PL}^{***} is highly degenerate (i.e. extremely rare in occurrence) and it is anyway already covered in the closure of the saddle node bifurcation manifold. Moreover, now the joint row dependence in J_{PL}^{***} and J_q^{***} is also not interesting because unlike in the loadflow model,

$$\begin{bmatrix} J_{PL}^{***} \\ J_q^{***} \end{bmatrix} \neq J^{***}$$

For analysing whether an instability originates in voltage or angle subspace, we have to be careful in considering the dynamics. The full calculations are not shown here because of space limitations. Yet, it is clear that the relevance of such terms in the paper as "Angle Bifurcation" are now questionable because any such distinction should be carefully extended to include the dynamics.

Conclusions:

The paper proposes security criteria based on several cases of row dependencies. As shown in the discussions, all these row dependencies are covered in the closure of the saddle node bifurcation computation for a general power system. Whereas the saddle node bifurcation involves solving for one additional equation, the row dependencies demand solving a large number of additional equations. They also do not provide much additional information, because already we know that the saddle node bifurcation leads to an instability. So, the three subcases listed under Type IIA in section 6.1 and the three subcases in Type IID(a) are unnecessary in general as they provide no additional information. A security criterion for a general system should be based on viability considerations and on the saddle node, Hopf and singularity induced bifurcation[1]. In [1], it is also shown that each of the three bifurcations can be located by solving one additional equation hence these are easily computable. In this paper, the usage of the word 'bifurcation' is not consistent with the general literature. To establish a phenomenon as a bifurcation, structural changes in the system for perturbations (known as 'unfolding'), have to be demonstrated. The paper makes no attempt at such an analysis though the term "bifurcation" is freely attached to many algebraic conditions. Considering the degeneracy of these "bifurcation"s (high codimension surfaces), the interest in these is mainly theoretical. The usage of the terms "Angle Bifurcation" and "Voltage Bifurcation" is questionable because the row dependencies only identify very special cases of the general scenario in which an instability originates in the angle or voltage subspaces.

[1] V.Venatasubramanian, H.Schättler and J.Zaborszky, *A Taxonomy of the Dynamics of a large Power system with emphasis on its Voltage Stability*, in this Proceedings.

[2] J.Guckenheimer and P.Holmes, *Nonlinear Oscillations, Dynamical instabilities and Bifurcations of Vector fields*, Springer-Verlag, New York, 1983.

Comments by John Zaborszky on
"Dynamic Static Voltage Stability Criteria"
by R.A. Schlueter et al

We engineers have our own exacting standards of precision in our research and development work. Some of our tools include mathematics, physics, chemistry, computation or computer science etc., all of which have their own methodologies. For instance in abstract mathematics--in the absence of direct physical connotations--every detail of the assumptions and every characteristic of the objects (such as functions) and processes (such as limit operations) must be defined and stated with complete precision. In engineering many of these details are implied by known physical characteristics of the object or process, so in engineering research using "mathematical precision" would often seem redundant and unnecessarily cumbersome. These differences cause normally no problem in power engineering where the needed mathematical tools are well within the grasp and competence of the engineering researcher. An exception is the area of system stability and control involving algebraically constrained differential equations and possibly discontinuous controls. Basic research for the power system here requires minimum preparation on the level of a good Ph.D. in mathematics. Of course once the basic mathematical results become available for the problem, it does not require excessive mathematical depth to use them in engineering research. But the different ambience of the two fields often results in an awkward interface. An example may be the use of the term bifurcations which is often quite casually attached to events in voltage stability and collapse in the recent literature in our field. I must make it clear that while some such instances seem to occur in this monumental paper (e.g. arbitrary restriction of meanings of saddle node bifurcation in Section 5.2.2, in conflict with its actual definition and statements like "some researchers would call Hopf bifurcation loss of voltage stability and J^{***} would not be singular when Hopf bifurcations occur" in Section 3, or terms like "voltage collapse bifurcation", etc.), it is essentially accidental that my comments are sent to this particular paper, perhaps because it is the most extensive or because I feel that the author will provide an insightful answer, perhaps starting a fruitful discussion.

Bifurcation is an important concept precisely defined in mathematics. It does play an important role in some types of voltage collapse, but the two are far from synonymous. A bifurcation point or surface in the parameter space simply means a change of topology in the state space trajectories, typically in the number or stability properties of equilibria (for local bifurcations), but also in the topology of trajectories as a whole (global bifurcation.) These changes do not need to imply any voltage collapse or anything else violent. For instance, at the singularity induced bifurcation [1] one equilibrium crosses from one state space component to another (reversing its stability) thereby respectively increasing or decreasing

the number of equilibria in the two components in the state space. This should cause no voltage collapse at an operating point not directly involved, in fact very little immediate effect would be noticeable there. So a bifurcation is not automatically a voltage collapse--neither does it imply an approaching one.

Conversely a loss of a major system component such as a line or generator is not usually a bifurcation in the mathematical sense even when the number of stable and unstable equilibria changes in the system. It is a sudden change of the system itself with a new systemwide set of equilibria, new boundaries including bifurcation and stability boundaries etc. being substituted. The operating point thus finds itself in a changed ambience. It is no longer an equilibrium but it may now be 1) inside some region of attraction so it converges to a stable equilibrium with around normal voltage or 2) inside some region of attraction where it may be trapped in a low voltage but stable equilibrium point (three examples for this were reported immediately by industry sources when this came up at Deep Creek Lake) which is of course a case of voltage collapse [1] or 3) it may be in a state space region where (the onset point of an in extremis condition) it starts on a trajectory which reaches a singular surface (the voltage collapse point). No bifurcation plays a role in this event, although it is definitely a voltage collapse. (See classification in [1].)

An apparent example of this type from the paper seems to be the "flux decay bifurcation." The onset point here seems to be the disabling of the dynamic voltage control part of the system and changing to a system with fixed excitation which may or may not be unstable. This onset point is the beginning of an in extremis event that finally reaches the point of collapse of voltage. This part of the paper presents a possibly quite useful analysis of an interesting phenomenon on the power system. But the paper leaves open the question of how and why this could be called a bifurcation overall. One special case described after eqn. 5.32 may indeed be a bifurcation of the type usually induced by singularities, but the text is too loose to tell. The paper's tendency to associate bifurcations with physical situations like flux decay or voltage is unfortunate because one type like saddle-node may occur under quite different physical connotations on the same system.

A different type of example is presented by the plethora of row dependency bifurcations presented in the paper. These may exist (existence conditions are easily established) and, if they do, they are indeed bifurcations. The problem here is that almost all would be of (very) large codimension. This means roughly that 1) they will occur with very little likelihood and 2) even if they occur they provide no news because the points of occurrence are already known to be contained in the closure of the basic saddle node bifurcation (of the P-V curve type) which can be computed with relatively little effort for even large systems. This latter situation is analyzed in a companion comment.

Another example from [1] where bifurcations are actually involved at both *onset* and *collapse* might be the chain reaction voltage descent connected with automatic tap changers. The *onset point* here is a saddle node bifurcation (different from the P-V type) where upward tap changes start causing descending voltage. The chain continues unless taps run out or are otherwise stopped) until the common P-V type saddle node is reached where then the *collapse point* is reached and the voltage collapses. Note also that in the common P-V type collapse (without taps) *onset* and *collapse* would coincide at the "nose." So this is also a possibility.

Naturally we could establish a concept called bifurcation for engineering purposes which may or may not intersect with the mathematical concept. Two such concepts however would be a continuing source of confusion so the preferred course might be to select a different terminology altogether. For instance specifically for voltage stability

1) An *onset point* or surface in the parameter space is where an in extremis condition commences which, if unchecked, will lead to

2) A *collapse point* or surface where the actual collapse of the voltage occurs. A few instances have already been given above.

Either or both of 1) and 2) may or may not be connected with bifurcations in a mathematical sense. When bifurcation is involved this fact may be mentioned, but, since this fact makes no practical difference, in application directed results, it would be better to settle for some well defined and easily understood terms such as proposed in 1) and 2). Other similar terms like feasibility, viability [1] or controllability boundaries would also be readily understood.

Naturally in the basic theoretical research which produces such results the vital role of bifurcations must be precisely analysed and understood.

[1] V. Venkatasubramanian, H. Schattler, J. Zaborszky, "A taxonomy of the Dynamics of a Large Power System, Particularly its Voltage Stability", in this volume.

[2] J. Guckenheimer and P. Holmes, "Nonlinear Oscillations, Dynamical instabilities and bifurcations of vector fields", Springer-Verlag, New York, 1983.

DISCUSSION

Adam Semlyen (University of Toronto): This study of voltage stability security criteria is very detailed, comprehensive and carefully presented. It should certainly be of great value for practical implementations and as a basis for study and further research.

My discussion pertains to the row dependence analysis of a Jacobian matrix J in section 5.1. Singularity of J implies, of course, row dependence for J , i.e.,

$$w^T J = 0 \quad (1)$$

where w is a non-zero weighting vector, subject to normalization. If J is $N \times N$, then (1) represents $N-1$ independent equations for the $N-1$ independent elements w . No problem at this point. If, however, row dependence is expected to apply to a subset of the rows of J , constituting the $n \times N$ ($n < N$) rectangular matrix J' , then row dependence for J' , i.e.,

$$w'^T J' = 0 \quad (2)$$

implies finding the $n-1$ independent components of the vector w' that would satisfy the same number of equations, $N-1$, as before. Since this would require more than one parameter to make it possible in general, it seems that strict row dependence for a rectangular submatrix of J is an unrealistically strong requirement. Instead, an *approximate row dependence*, meaning that part of the elements of w of (1), are small (but not necessarily zero) could possibly be the consequence of some particular physical conditions. Could the authors, please, comment on the above remarks, in particular, explaining what are those operational circumstances which would lead to the different types of approximate row dependence which are significant in the classifications and the mathematical analysis presented in the paper.

CLOSURE

by

R. A. Schlueter and T. Y. Guo

I appreciate the thorough review and discussions of the paper, entitled "Dynamic/Static Voltage Stability Security Criteria," by Dr. Zaborszky, Shättler, and Venkatasubramanian; Dr. Zaborszky; and Dr. Adam Semlyen. The discussion of Dr. Zaborszky, and Dr. Zaborszky with Drs. Shättler and Venkatasubramanian, based on their extremely insightful description of power system stability boundaries [1], corrects our usage of the term "bifurcation," and allows us an opportunity to clarify the relationship of the papers [1] and [2]. Dr. Zaborszky's paper [1] is clearly a landmark. This paper and these discussions have brought me to a new level of understanding of voltage instability. My closure to the two discussions reflects this new level of understanding. This closure will reply to each of these discussions separately.

Zaborszky, Shättler, and Venkatasubramanian

Each engineer at the conference brings a piece of the great puzzle called voltage collapse. Some bring definitions and concepts from the mathematical sciences, some bring understanding and concepts from the physical sciences, and some bring understanding and knowledge from the art of practice of engineering.

Dr. Zaborszky's, Shättler's and Venkatasubramanian's comments attempt to classify the types of bifurcations by the type of dynamic behavior observed at the bifurcation point. Our point of view is that although this is the practice in the mathematical sciences and the approach used by Tamura [as

discussed in our paper] and Zaborsky [1], it does not provide physical insight into why voltage instability occurs. Thus, our classification of types of voltage instability is based on whether

- (a) the system experiences sustained low voltage that causes tripping of large motor loads and possibly generators. This Type I voltage instability is a true voltage collapse;
- (b) The algebraic part of the model is row dependent so that J_C''' and J''' are row dependent. This is a Type IIA voltage instability and is called an "algebraic bifurcation" in the paper since the bifurcation in the transient stability model is solely due to the algebraic equations; and,
- (c) the differential and network equations of the transient stability model experience bifurcation. This is the Type IID voltage instability.

One may not use terms "algebraic bifurcation" or dynamic bifurcation" without causing confusion. The classification is however important because of the radical differences in observed behavior, the physical cause of that behavior, and the actions required to control or remove the behavior when bifurcations occur solely in the algebraic equations or in both the differential and algebraic equations. One could use "bifurcations in the dynamics" and bifurcations in the network equations" to avoid difficulty with the usage in the mathematical science.

Subclassification of "bifurcations in the dynamics" and subclassification of "bifurcations in the network equations" are equally important. The discussions in their paper [1] have three subclasses of "bifurcations in the dynamics"

- Hopf bifurcations
- saddle node bifurcations
- singularity induced bifurcations

and no "bifurcations in the network equations." Our paper identifies two different saddle node bifurcations, one due to loss of transient stability and the other due to instability of the flux decay dynamics. Our paper [2] identified only one Hopf bifurcation due to generator mechanical interarea oscillation although there is at least one other due to oscillation in the generator/exciter electrical dynamics as noted in [3]. The discussion of the saddle node bifurcation in flux decay dynamics and the Hopf bifurcation in mechanical dynamics is the first time such bifurcations have been discussed in terms of voltage instability. Our belief is that classifying bifurcations solely as saddle node bifurcations or Hopf bifurcations is not sufficient since it does not indicate the variables, dynamics and controls involved in the oscillations as well as what operating or control actions might reduce its effects. One could utilize terms such as

- saddle node bifurcation in generator flux decay dynamics
- saddle node bifurcation in generator mechanical dynamics
- Hopf bifurcation in generator mechanical dynamics
- Hopf bifurcation in generator electrical/exciter dynamics

Our research [2] identifies three subclassifications of "bifurcation in the network equations" (algebraic bifurcation) where [1] has no such classification since there is no classification for the case where J''' and J_C''' are both row dependent. Subclassification of "bifurcation in the network equations" into

- bifurcation in the real power balance equations (angle bifurcation)
- bifurcation in the reactive power balance equations (voltage bifurcation)
- bifurcation in the real and reactive power balance equations (voltage and angle bifurcation)

is important since the theory shows that there are two physically different subclasses of bifurcation in the network equations that cause voltage instability and three overall bifurcations of the network equations. Bifurcation of the real power balance equations has traditionally been associated with steady state stability limits. Bifurcation in the reactive power balance equations occurs when the network reactive losses within a bus group for a specific small voltage decrement exceeds the reactive power that can be imported across the boundary of the bus group for that voltage decrement. Thus, all three bifurcations of the network equations are due to physical limits that once violated cause blackout. Furthermore, these "bifurcations of the network equations" form a surface [4] which contains singular points and impasse points. The behavior of the system on this surface can't be described exactly. At impasse points, solutions may not continue for all time. This is not the case for singular points. If the initial condition is a singular point on this surface, called a frontier, then solutions exist but are not uniquely defined. A solution can switch from component to component of the regular part of the solution set via singular points. Singular points on the surface are quite rare and almost all points on the frontier are impasse points. Jump phenomena and complicated periodic solutions can occur at these points. Thus, the behavior of the system near "bifurcations of network equations" is ill behaved. Writing a partial differential equation model to replace the real and reactive power balance model would allow a more precise indication of what occurs as one approaches a bifurcation in the network equation.

Zaborsky, Shättler, and Venkatasubramanian have included "bifurcations of the network equations" as saddle node bifurcations in their discussion even though they were not considered in [1]. The network algebraic model is considered to be a singularly perturbed differential equation model to be able to consider "bifurcations of the network equations" as saddle node bifurcations. Although the algebraic equation model may not be correct, the singularly perturbed differential equation model is even less correct as noted in [1]. Furthermore, classifying "bifurcations in the dynamics" as saddle node bifurcations loses sight of the fact that they are certainly not a bifurcation of the differential and algebraic equations, but rather a "bifurcation in the network equations." Classifying bifurcation of the network equations as saddle node bifurcations also loses sight of the behavior near these bifurcations, the causes of the bifurcations, as well as the methods for preventing these bifurcations.

A quote from [1] states: "Dynamically the singularity acts as a generator of a random process. It is not possible to predict

where the continuation of a trajectory after reaching the singularity will carry the system off the load flow surface and where it will again land on or off the load flow surface, except statistically - and that only in principle."

This type of behavior seems to describe the behavior after loss of synchronism which is exactly what is likely to occur due to "bifurcation of the network equations." Treating algebraic constraints as singularly perturbed differential equations also leads to loss of synchronism but the bifurcation itself doesn't explicitly show the loss of synchronism. Replacing the network model by a partial differential equation model may give even more explicit description of what occurs at or after bifurcation of the network equations but will not give any additional insight into what causes the voltage instability to occur.

The analysis by Zaborszky, Shättler, and Venkatasubramanian in their discussion correctly points out that "bifurcation in real and reactive power balance equations is by far the most common" bifurcation, which is also noted in [2]. A more complete discussion of when and how "bifurcation in real power balance equations" and "bifurcation in the reactive power balance equations" occur is given in our closure to Dr. Semlyen's discussion.

Two important hypotheses in our work are that:

- (a) "bifurcations in the network equations" and "singularity induced bifurcations" that result due to Q-V curve tests at any bus or combination of buses are the only "voltage instability bifurcations." Hopf and saddle node bifurcations that are not Q-V curve based singularity induced bifurcations are not due to voltage instability but are due to some other type of instability; and
- (b) "bifurcations in the network equations," which is not considered as one of the typical causes of voltage instability in [1], is considered to be by far the most common cause of voltage instability. The behavior described when one encounters "bifurcation in the network equations" justifies classifying it as a bifurcation that results in voltage instability.

These hypotheses are based on industry practice of using:

- (a) load flows to assess proximity to voltage instability and
- (b) load flow indices or variables to measure proximity to voltage instability.

The above two hypotheses are the basis of our third hypothesis that PQ controllability is often sufficient to prevent any voltage instability bifurcation. PQ controllability is lost at the minimum of the Q-V curve computed at any bus or bus combination. At the Q-V curve minimum, a bifurcation of the network equations is proved to occur in [2]. A "bifurcation of the dynamics" may occur prior to reaching the Q-V curve minimum and thus the "bifurcation of the dynamics" would be the closest point of collapse. It is clear that one could experience a whole sequence of voltage instability "bifurcations in the dynamics" as one computes a Q-V curve and approaches loss of PQ controllability at the Q-V curve minima. Thus, PQ controllability is not necessary or sufficient for voltage instability. However, PQ controllability is necessary and sufficient for voltage instability in a load flow model when constant power load is assumed and "bifurcation in the real power equations" is assumed not to

occur. This is the context of the statement in our paper and is correct.

It is our view that classifying bifurcations of the transient stability model as

- bifurcation of the network equations
- bifurcation of the dynamic equations

as well as establishing subclassification of each of these two bifurcation classes is important and mathematically and physically correct. The subclassification of "bifurcation of the dynamics" are

- saddle node bifurcation in the generator flux decay dynamics
- saddle node bifurcation in the generator mechanical dynamics
- Hopf bifurcation in the generator mechanical dynamics
- Hopf bifurcation in the generator electrical/exciter dynamics

This subclassification is by no means exhaustive. The subclassification of "bifurcation in the network equations" are

- bifurcation in the real power balance equations
- bifurcation in the reactive power balance equations
- bifurcation in the real and reactive power balance equations

Classifying all of these bifurcations classes and subclasses as solely saddle node, Hopf and singularity induced bifurcations does not differentiate between voltage instability and nonvoltage instability bifurcations, makes all bifurcations appear to be "bifurcations of the dynamics," does not indicate the physical variables, dynamics, controls involved in that bifurcation, and does not indicate the method for preventing a system from experiencing them.

Zaborszky

Our experience indicates that virtually every known voltage instability has been due to a progression of loss of voltage controls due to reaching field current limits and/or due to some contingency. If one did not consider that a bifurcation could occur after contingencies or after a loss of voltage controls due to field current limits, one would experience very few true bifurcations that result in voltage instability.

The concept of an onset point and a collapse point seems similar to our closest point of collapse bifurcation concept. It was observed in [5] that a Hopf bifurcation in mechanical dynamics was the onset point. The collapse point was the saddle node bifurcation in generator flux decay dynamics. Our view is to set operating limits based on the onset point which is our closest feasible point of collapse.

The very large number of different closest feasible points of collapse is a very important concept in our paper [2]. The large numbers of different closest feasible modes of collapse are

encountered in computing a Q-V curve at every bus and every combination of buses. The closest feasible modes of collapse are thus due to possible voltage instability in different local areas of the system and are not to be confused with the several different classes and subclasses of bifurcations that can occur in each local area.

Although a plethora of row dependencies on J'''' were proposed in [2] for the various classes and subclasses of voltage instability, they would not be used to determine the set of closest feasible modes of collapse. These tests are given as an indication of the independence of each class and subclass of bifurcation and the physical variables and dynamics involved in each class and subclass of bifurcation. Tests for collapse would be based on closest point of collapse methods discussed in [2].

Semlyen

Dr. Semlyen's discussion is correct and insightful. Obtaining row dependence of a subset of the rows of a square matrix requires more than one parameter to change. For example, "bifurcation in the real power balance equations" (angle bifurcation) can be obtained by assuming all buses are PV buses. Bifurcation in the real power balance equations can be closely approximated in a system with adequate voltage control. "Bifurcation in the reactive power equations" could be achieved if real power injections at all buses were adjusted simultaneously. Bifurcation in the reactive power balance equations can be approximated in a lightly loaded network if sufficient voltage control was not available. It is true that the most common bifurcation of the network equations is a bifurcation in the real and reactive power balance equations.

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Applications of the Optimal Power Flow to Analysis of Voltage Collapse Limited Power Transfer

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Abstract

A key component to analysis of voltage-limited power transfers using power flows is the determination of the conditions at which voltage collapse is imminent. Optimal Power Flows (OPFs), by their intrinsic capability for finding extreme conditions while satisfying power flow equations, are well-suited for the direct determination of voltage collapse operating limits.

A form of the OPF using barrier functions [1] is capable of determining: (a) the transfer level at which voltage collapse is imminent and (b) the reactive power margin of a given transfer condition. Both solutions are traditionally carried out using a series of power flows; the voltage collapse transfer level is obtained from P-V curves, and the reactive power margin from Q-V curves. In tests of west-to-east power transfers across the EHV transmission of the Allegheny Power System (APS), the OPF method provided direct solutions of the extreme points of selected P-V and Q-V curves. The method is flexible and can be expanded to examine conditions beyond the coverage of conventional P-V and Q-V curves. Information from an OPF solution is useful in identifying the combination of dispatch and transformer tap settings for maximum transfer, and optimal sites for reactive support.

Introduction

Power flow methods have been used in the industry to determine power transfer margins across interconnected EHV AC transmission systems limited by voltage instability or voltage collapse [For discussion purposes of this paper, power transfer "margins" will be defined as the difference between maximum transfer limited by steady-state constraints and a selected transfer level]. Power flow methods are typically based on a series of actual or projected power flow solutions to find the maximum transfer conditions while maintaining reliable system operation. The methods can be classified into two general types depending on the measure of margin: real power transfer margin and reactive power margin.

Optimal power flow (OPF) methods, by their intrinsic capability for finding extreme conditions while satisfying power flow equations, have a strong potential for applications in direct determination of transfers that are normally limited by incipient voltage collapse conditions. Evolving optimization technology is now mature enough to compete with conventional power flow algorithms in many applications. This paper discusses the application of an OPF implementation using the barrier function method. The implementation, developed by Burchett and Grant, improves on the computational speed and robustness of a conventional quadratic programming-based OPF

algorithm [1]. Koessler and Rosecrans, applying this algorithm to a test system, found direct solutions of the transfer level at steady-state voltage collapse and measures of reactive power margins at different operating points [2]. These solutions have traditionally been obtained through serial power flow methods. In addition, [2] includes suggestions for uses of the OPF which would expand upon the capabilities of existing methods. The availability of a fast OPF implementation and promising results on a test system provided sufficient impetus to examine applications of OPFs in large-scale network analysis for both operational and system planning environments.

Further investigations on the application of OPF to the study of voltage collapse-limited transfers are reported in this paper. The investigations were performed as part of a larger study of voltage collapse analysis methods for use on the Allegheny Power System.

Optimal Power Flow

Optimal power flows are a class of network optimization algorithms which include the conventional power flow nonlinear equations in the formulation. The OPF problem can be stated as follows:

Optimize: $f(x)$

Subject to:

Power Flow equations $e(x) = 0$

Operating constraints $g(x) \geq 0$

where x is a vector of network variables; i.e., bus voltage magnitude and angle, and transformer taps; $f(x)$ is a scalar cost function; $e(x)$ is the vector of power

flow equalities; and $g(x)$ is a vector of inequality constraints; i.e., equipment rating limits, reactive power limits, and upper and lower bounds on x . The forms of the objective $f(x)$ and constraints $g(x)$ are varied. Typical examples of $f(x)$ are: transmission losses, generation production costs and load shedding. Examples of $g(x)$ are: bus voltage limits, reactive generation limits and interface flow limits.

In the generic OPF formulation, the vectors $e(x)$ and $g(x)$ define a feasible region for x . Typically the optimal vector x is constrained by one or more of the inequality conditions. Various OPF algorithms use different techniques to find the constrained optimal solution, such as linear programming and quadratic programming. The approach presented in [1] and used for the large-scale network study reported here solves the inequality constrained problem by adding barrier terms to the objective to create a minimum inside the feasible region. The barrier functions can be chosen such that the solution can be as close to the boundary defined by an inequality constraint as desired. One advantage of this approach is that it does not have to find the correct set of binding constraints to start with since the starting solution point is always feasible with respect to the inequality constraints. The barrier function method has also been found to be quite robust in handling highly-constrained formulations typically found in large-scale network problems.

Applications to Voltage-Limited Transfer Analysis

A. Real Power Transfer Margins (P-V Curve Application)

Many utilities utilize P-V curves as an aid to analysis of transfer margins against unacceptable voltage conditions. Also referred to as "knee-of-the-

curve" analysis, calculation of P-V curves involve power flow solutions of the network at increasing transfers until the solution diverges or "blows up." The procedure of using a series of power flow solutions to construct a P-V curve can be computationally expensive especially since the accuracy is dependent on the size of transfer increment. Variations to this method involve the use of linear projections based on sensitivity of bus voltage and generator reactive power to increases in transfer [3] or linear projections based on the network Jacobian [4]. The variations offer speed improvement but essentially provide the same information as conventional P-V curves. Examples of P-V curves are shown in Figure 1. A plot of voltage measured at a selected node versus transfer (real power flow across a system) would typically be characterized by a curve with two distinct sections. The first section, representing bus voltage at lower transfers, reflects the gradual drop in voltage as transfers increase. The second section, representing bus voltage at higher transfers close to the blow-up point, shows a sharp drop in voltage with respect to transfer. The transition between the two sections forms a "knee." The knee is the result of reactive power sources being exhausted in the vicinity of the monitored bus. Transfer margins have been calculated from transfer either at the last converged power flow or a point in the region of the knee. Using the knee for specifying margin is subject to the difficulties of ascertaining where the knee begins; the conditions for knee formation are not the same for each transfer study. The transfer condition at the last converged power flow, on the other hand, represents the network as it nears singularity; i.e., the point of steady-state voltage collapse. A robust power flow algorithm can bring the P-V curve consistently close to this point [Reactive power margins from Q-V curves converted into a real power transfer limit (discussed in the next section) would also find the

same extreme transfer level]. Margins measured from the extreme point are reliable.

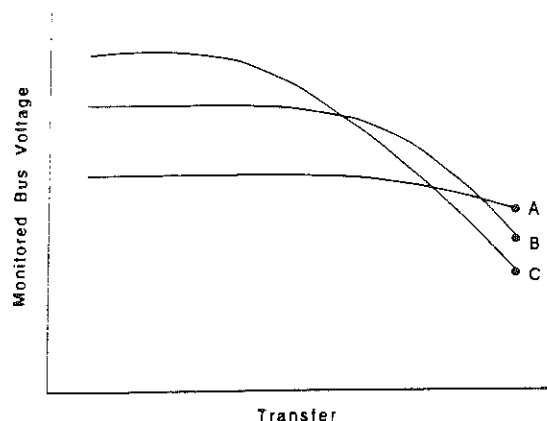


Figure 1. Examples of P-V Curves For Voltage Monitored at 3 Different buses. A, B, C

The load model is critical in this analysis. Dynamically, real and reactive loads will change as auto tap changing, local controls and load diversity are affected by low voltage. Typically a constant real and reactive power load is assumed. This provides a steady-state transfer margin which can be adjusted or compensated for dynamic response.

The operating point at the extreme of a P-V curve represents the transfer condition at which all reactive reserves and voltage control (from generators, transformer taps and switched shunts) in the local area of voltage instability have reached limits and voltage collapse is incipient.

The OPF algorithm using the barrier method can also find extreme points of the type corresponding to the last converged power flow of a P-V curve. A formulation of the OPF to solve this type of problem is as follows:

Minimize: Generation Cost
 Subject to: Generator MW, MVAR and voltage limits ($0 < V < V_{pre-contingency}$)
 Satisfying Power flow equations

By specifying generation costs which reflect the advantage of increasing transfers (i.e., cheaper sending end than receiving end costs), this formulation determines a power flow whose transfer is limited by voltage collapse. In the optimization, the algorithm assumes global control over dispatch and such other variables as transformer taps and phase-shifts. When applied to a particular contingency, the procedure determines the dispatch (and tap and phase-shift settings, if chosen) which maximizes transfer limits. The OPF provides information on the sensitivity of the cost function to operating constraints, indicating which constraints have more effect in limiting transfers.

In tests on a representation of the Allegheny Power System (APS), a conventional P-V curve was obtained by measuring West-to-East transfer across APS. The transfer is incremented 20 MW per power flow solution by reducing generation at selected Eastern units and picking up at selected Western units, where dispatch changes at individual units are defined by participation factors. Transmission losses are picked up by the power flow swing bus which is

Table 1: Transfer Limits for APS Contingency A.

1. Conventional P-V Curve using participation factors for dispatch swing bus picks up losses	4440 MW
2. OPF with global control on dispatch sending end buses pick up losses	4362 MW
3. OPF same as (2) plus open generator real power limits	4749 MW
4. OPF same as (3) plus global control on transformer taps	4888 MW

located in the West. The post-contingency P-V curve

for contingency A, measured at selected buses along the transfer path, is shown in Figure 2. The transfer is measured by the amount of generation reduced in the receiving Eastern units. The last converged power flow solution occurs when the incremental transfer is 4440 MW. This transfer limit was verified by the method of projection of reactive power margins from Q-V curves.

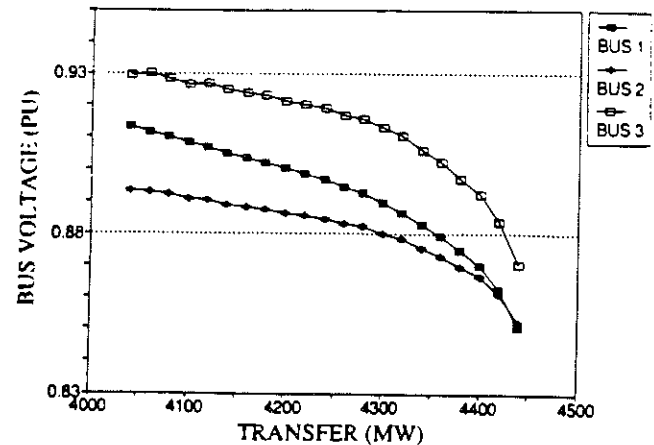


Fig. 2: P-V Curves after APS Contingency A

In the OPF solution, the same Western and Eastern units are placed under global control for dispatch. Whereas the P-V curve method requires explicit handling of transmission losses, the OPF formulation implicitly accounts for losses in the dispatch. Excluding the power flow swing bus from participating in the dispatch results in the sending end generators picking up for transmission losses. With fewer units on dispatch, the OPF solution is constrained by generator MW limits and as shown in Case 2 of Table 1, finds an extreme operating point with a transfer less than that obtained from a P-V curve solution. The dispatch vector from the OPF solution indicates the manner in which to bias dispatch to increase the transfer limit for this contingency. This dispatch can be convolved with other dispatch vectors from other contingencies to produce a security-constrained dispatch [5,6].

Opening the MW generation limits in the OPF solution results in an extreme operating point which is constrained by voltage-related control variables. The sensitivity of the cost function to constrained variables indicates which constraints have the most effect in limiting transfer. For instance, in examining the results for case 3 in Table 1, it is observed that the following have highest values for sensitivity:

- Generator voltage upper limits for three units; one is a Western unit, and the other two are Eastern units. All are on the periphery of the area where voltage problems have been observed for West-to-East transfers.
- Generator reactive power limits for all APS units and several units just East of APS.
- Transformer taps for several off-load tap-changing transformers, the most significant of which are two 500/138 kV transformers just north and south of the main West-to-East transfer path.

System and operations planners can use this information to design for increased transfer capabilities or to determine remedial action for contingencies to maximize transfer margin.

Because of the OPF's global control over dispatch, opening MW output limits result in a higher transfer limit than the P-V curve solution. For Case 3 of Table 1, an increase of 309 MW is shown over the transfer limit of Case 1. The OPF achieved the increase by increasing flow via dispatch on parallel paths not yet constrained by voltage collapse. The increase, however, is finite and, moreover, is only a small

fraction of the P-V curve transfer limit indicating only so much can be gained using alternate paths.

Including transformer taps under global control results in an even higher transfer limit as shown by Case 4 of Table 1. The information on the tap settings available from the power flow produced by this OPF case can be used to plan corrective actions in case of contingency.

Determination of Reactive Power Margins (Q-V Curve)

Another common application of power flows to voltage-limited transfer studies is the use of Q-V curves to determine reactive power margin (RPM). RPM is a measure of the reactive power that can be taken out of a bus for a given operating state without causing steady-state voltage instability. It is useful as a gauge of transfer margin to voltage collapse. It has also been used to specify the amount of shunt reactive support needed to increase transfer capability.

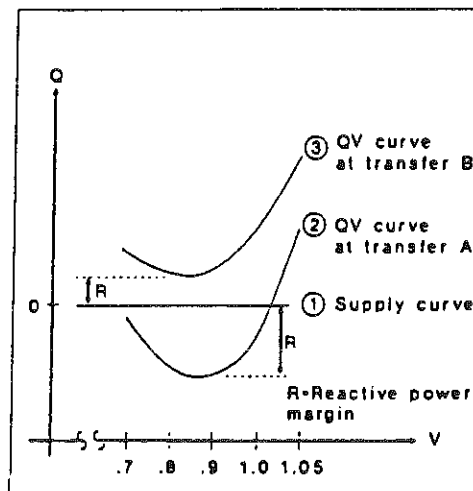


Fig. 3: The reactive power margin determines if system is voltage stable or unstable.

The RPM at a bus for a given transfer condition can be determined by construction of a Q-V curve. This has conventionally been done with a series of

power flows, but sensitivity methods have also been proposed [3]. Reactive power injection at a bus is monitored as voltage setpoint at the bus is changed. Reactive power is supplied at the monitored bus by an assumed reactive source such as a synchronous condenser model. The reactive source acts to eliminate convergence problems in the vicinity of the Q-V curve minimum such that the method works even for cases where transfer is beyond the voltage collapse limit. Figure 3 shows two Q-V curves, corresponding to two transfer conditions, A and B, and a supply curve which is the amount of shunt reactance at the monitored bus (exclusive of the reactive source) as a function of voltage. The supply curve lies at $Q=0$ in this example. RPM is the difference in reactive power between the lowest point of the Q-V curve and the supply curve. If the Q-V curve intersects the line where $Q=0$ as in the case of transfer A, the reactive power margin is positive and the operating state is voltage stable. If the Q-V curve does not intersect $Q=0$ as in transfer B, reactive power margin is negative and the operating state is voltage unstable. The transfer condition at which reactive power margin is zero (the Q-V curve just touches $Q=0$) is a voltage collapse-constrained transfer limit.

The OPF provides a direct means for finding the reactive power margin for a transfer condition. [Van Cutsem [7] developed a decoupled optimization procedure which estimates reactive margin to voltage collapse]. Using the barrier method, the OPF formulation for finding reactive margin is:

Minimize:

Shunt Reactive additions

Subject to:

Generator MW, MVar and voltage limits

Satisfying power flow equations

The OPF using the barrier function approach observes generator voltage limits by means of penalty functions. The cost of each of these penalty functions increases exponentially as one approaches the specified limit. Since the total cost being minimized by the OPF includes these penalty costs along with the costs of the explicit control variables, there will be a trade off between the goal of minimizing shunt capacity additions and keeping the generator voltages close to specified limits. Thus, if the cost per MVar of shunt capacitors is increased, generator voltages will be pushed slightly closer to their upper limits, lowering the need for additional shunts. Similarly, if the cost of shunt capacitors is reduced, the amount of shunt capacitors computed by the program will increase. Because of these effects, the reactive reserve requirements as computed by the OPF is not as well defined as those computed by the Q-V curve technique.

Table 2 compares the RPM for APS contingency

Table 2: Reactive Power Margins at Various Transfer Levels for APS Contingency A.

Transfer (MW)	RPM by Q-V Method (MVAR)	RPM by OPF (MVAR)
4000	457	N.A.
4500	-102	-110
5000	-780	-842

A as determined by conventional Q-V method and OPF at three transfer levels. The OPF does not calculate an RPM when the case is voltage stable; i.e., at the 4000 MW transfer level unless compensation methods are used, for example, a shunt reactance at the monitored bus.

In tests on the APS data base, RPM is observed to behave linearly at transfers close to voltage collapse.

Figure 4, showing the RPM measured at four different locations and for various transfers, illustrates this phenomenon. The transfer limit, or the transfer when $RPM=0$, is 4440 MW for all four buses. The authors propose that RPM measurements at two transfer levels can be converted into real power transfer limits by linear projection. For example, for contingency A results given in Table 2, the RPMs at the 4500 and 5000 MW transfer levels project zero RPM at a transfer limit of 4425 MW using either the Q-V curve or OPF results. (This is a slightly lower value than the 4440 MW previously obtained because in these cases transformer taps are fixed.) As shown in Figure 4, the technique is not sensitive to the location of the monitored bus, as long as the selected bus is within the local area of voltage instability.

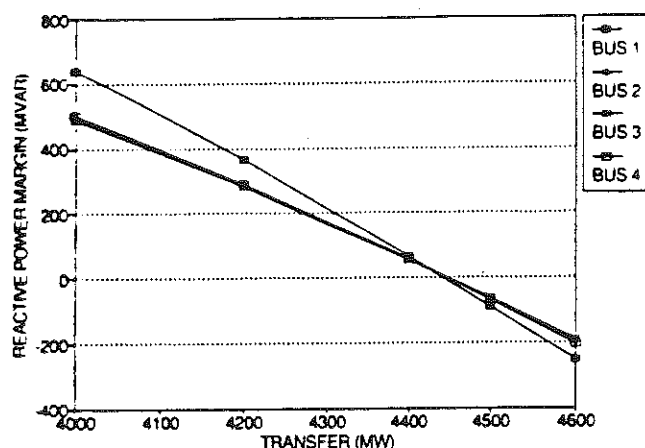


Fig. 4: Reactive Power Margin (RPM) measured at 4 buses as a function of transfer. The transfer limit occurs when $RPM=0$ and is about the same value at all 4 buses.

The OPF formulation allows for several sites to be specified for potential shunt reactive injections, making it possible to define an area RPM. In a further test on the APS contingency A case, several sites scattered along the West-to-East transfer path were specified as candidates for shunt additions in an OPF formulation. The optimal solution distributed shunt additions among the sites, allocating larger components

for sites which have been previously observed to experience the worst voltage depressions as West-to-East transfers increase. The distribution of shunt additions identify the most effective sites for shunt compensation to improve transfer capability.

Computational Speed Comparisons

All power flow and OPF solutions were performed on a VAX 3300 machine using an 800-bus

Table 3: CPU Usage for Various Methods reported in this paper.

A. Transfer Limit Calculation

1. P-V Curve generated by a sequence of Power Flow solutions, 25 power flow solutions, a total of 100 full Newton-Raphson iterations 18:33 min
2. OPF to determine transfer limit average 18:46 min
Maximum 25:23 min
Minimum 8:20 min

B. Reactive Power Margin Calculation

3. Q-V Curve generated by a sequence of Power Flow solutions, 18 power flow solutions, 96 full Newton-Raphson iterations 17:36 min
4. OPF to determine RPM (average) 16:41 min

representation. Comparison of CPU usage for the various methods is shown in Table 3.

Both items (1) and (3) are generated by a sequence of power flow solutions. Considerable savings on the times reported for power flow solutions are possible with search methods for finding the extreme points, or use of computation saving techniques described in [3] and [4]. Item (2) timing is an average value. As shown in Table 3, variations to this time can be significant. The OPF solution is quicker if fewer constraints are binding.

The timing figures show that the OPF is comparable with conventional power flow-based transfer limit analysis methods.

Acknowledgement

The authors would like to thank Allegheny Power System and Power Technologies, Inc. for allowing the use of network data base and software reported in this paper. The software include Power System Simulator for Engineering (PSS/E) and TPLAN Automatic Contingency Analysis for power flow tests, and PSS/OPF for the OPF cases. The P-V and Q-V curves using a sequence of power flows were generated with PSS/E's user programming language IPLAN.

Conclusions

Recent development of an OPF algorithm using the barrier method has produced an important tool for voltage collapse-constrained transfer analysis.

In tests on the Allegheny Power System network model the OPF provided information on the generation dispatch and transformer taps that can maximize transfer margin to voltage collapse and on the operating constraints that limit transfer for a particular contingency. The information is useful for long term planning, for operations planning in determining transformer taps and setpoints and biasing dispatch to increase transfer limits, and for planning emergency remedial action for limiting contingencies.

The OPF also provides a direct method for determining reactive power margins which can be used to identify the voltage collapse-limited transfer limit by projection. The OPF can calculate reactive margins at several sites in the same solution, eliminating the need

to pre-select a site and identifying, through the distribution of reactive additions, the most effective sites for shunt compensation to improve transfer capability.

In all of the tests, the OPF computational speed, measured in CPU time units, was comparable to conventional solution methods using a sequence of power flows.

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Voltage Stability Margin Enhancement Via Optimal Power Flow

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Abstract

Voltage stability analysis is approached from the standpoint of sensitivity analysis of generated reactive powers. The load model of electric power system is represented in reduced parameter space, and voltage collapse proximity indication investigated with three different approaches. The improvement is then monitored of the stability margins following application of the robust optimal power flow formulation, which is designed to remove violations of operating constraints with minimal control action. The results of simulations on a 24-bus test system are presented.

1 Introduction

Voltage stability in power systems has been long recognized as a very important problem. Almost every major utility has experienced some kind of voltage-related disturbance. Although a clear distinction between various types of disturbances cannot be made, many of the observed cases of voltage collapse are a consequence of the static bifurcation of dynamic system model undergoing slow parameter changes (i.e. increase of loading). Another possibility for occurrence of instability is sudden change of power system topology, which may be caused by a contingency, and may produce post-contingency state without an equilibrium, resulting in almost instantaneous voltage collapse, often precipitated by an avalanche of activity of protective equipment due to the abnormal transient state. In the operating practice, voltage collapse has been accelerated by malfunctions of protective devices. Several such cases have been documented in the literature. Unfortunately, protection models are usually excluded from the analyses of stability, because of the unwanted added complexity, and high degree of uncertainty in predicting their dynamics. Very often, voltage collapse develops in a system where no abnormal operating condition precede the

failure [11]. Typically, voltage decline slowly develops as a consequence of system loading, and its strongest effects may be concentrated in a very localized region of the power system. Contrary to the popular belief that large amounts of reactive power support effectively prevent voltage instabilities, such cases have been observed in practice [4][11], and recreated by simulation [19].

Numerous attempts to quantify stability margins in the literature have produced a variety of results, many of whom have provided interesting insights into different physical aspects of the problem. The early analyses [4] were directed toward understanding the nature of instability in power systems with predominantly radial topology. The role of under-load-tap-changing (ULTC) transformers in evolution of instability was recognized, and analyzed on simple, practical examples, but results could not be extended on power systems of arbitrary topology. The concept of ULTC coordination (or lack thereof) was refined later, and put on a solid mathematical foundation in [22], producing an explanation for evolution of instability, but failing to show that the instability could be voltage collapse. The important role of other factors, like reactive energy balance was investigated in [9], inspiring a large number of subsequent related results. The relationship between loss of steady-state stability and singularity of the load flow Jacobian was observed in [23]. As a consequence, appropriate indicators of proximity to voltage collapse were proposed in the literature: determinant [13], and minimum singular value of the load flow Jacobian [10], both lacking reliability when monitoring voltage stability in systems where generators make transitions from PV into PQ mode. Another possibility to assess the proneness to instability was found in the property of stressed power systems to have multiple close solutions of power flow equations [13][14], which lead to a number of new indicators, whose calculation required various degrees of computation. The important contribution was pre-

sented in [3], where loss of steady state stability was put in the framework of overall system dynamics and identified as a static bifurcation of power system equilibria, which was later extended, and verified [19][5]. Several methods for testing the voltage stability based on sensitivity analysis were proposed [9,17,18,7] and tested by simulation.

The objective of this paper is to combine some of the findings in voltage stability analysis with newly developed methodology for remedial action calculations in power systems. The application of remedial actions on stressed power network always leads toward improvement of voltage stability margin, even though voltage stability improvement may not be the explicit objective of the algorithm. Remedial actions are calculated from a robust optimal power flow algorithm [1] based on linear programming. The objective is to remove the violations of operating constraints from system while using the minimal action of control variables. The efficient formulation of the algorithm makes it suitable for application in large scale systems, with modest computational effort. The linearization imposed by the LP approach requires to limit the region of control action in every iteration, in order not to invalidate the applicability of the results on nonlinear power system model. This constraint is particularly pronounced in the neighborhood of the stability boundary, where it was successfully tested on a number of cases. Remedial actions can also be applied to improve the stability margin of post-contingency steady states of power system, which may be of crucial importance because of degraded stability margins and system vulnerability associated with that type of operation. In all simulations, remedial action algorithm was found to be beneficial in reducing the imminent risk of voltage collapse and eliminating part, or all violations of the operating constraints in the system.

2 Description of the Model

The equilibria of the dynamic model of power system

$$\begin{aligned}\dot{x} &= f(x, y, \lambda) \\ 0 &= g(x, y, \lambda)\end{aligned}\quad (1)$$

are given in the form of load flow equations

$$\begin{bmatrix} f(x, y, \lambda) \\ g(x, y, \lambda) \end{bmatrix} = 0 \quad (2)$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$, $\lambda \in \mathbb{R}^k$, and $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^k \mapsto \mathbb{R}^n$, and $g : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^k \mapsto \mathbb{R}^m$

are the active and reactive power balance equations. Vectors x and y represent the state of the system, while λ is a parameter vector. Normally, (2) has multiple solutions, whose number depends on the parameters (loading level) of the system. As the loading increases, the number of solutions decreases, until the point (corresponding to the parameter value λ_0) is reached, where only one solution to (2) could be found. Further increase of loading yields no solutions. That point is characterized by

$$\begin{aligned}\begin{bmatrix} f \\ g \end{bmatrix} &= 0 \\ \det \begin{bmatrix} \left(\frac{\partial f}{\partial x} \right) & \left(\frac{\partial f}{\partial y} \right) \\ \left(\frac{\partial g}{\partial x} \right) & \left(\frac{\partial g}{\partial y} \right) \end{bmatrix} &= 0\end{aligned}\quad (3)$$

where operators $\partial/\partial x$ and $\partial/\partial y$ are Jacobians. It represents the static bifurcation of the system (1). It was shown [3,5,13,14,19] that voltage collapse is a consequence of static bifurcation. The slow variation of the parameter vector λ , which encompasses the loading requirements of the system, is the cause of the occurrence of static bifurcation, and system's subsequent descent into voltage collapse. Another possibility for development of voltage collapse may be a structural change in the system, which would be reflected in the model by a change of f and g , such that the post-disturbance system model does not have any equilibria. This scenario corresponds to the line, or generator outage, which triggers the development of instability. It should be noted that, in practice, even those contingencies which result in stable equilibria, may cause overloads in some transmission lines, starting the protective action which may lead to voltage collapse. Both described types of voltage instabilities have been observed in actual system operation, and obtained by simulations. Other than having quite different time intervals corresponding to the evolution of disturbance (the later type is much faster), both types of disturbances share some similarities: declining voltages in parts of the system, and big increase of reactive power transfers, and generation, quite disproportionate with the increase of reactive load requirements. The reactive power injected at bus i of the network is

$$Q_i = V_i \sum_{j=1}^n V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (4)$$

where $\theta_{ij} = \theta_i - \theta_j$ are the angles between phasors at buses i and j , and $Y_{ij} = G_{ij} + j B_{ij}$ are the elements of the admittance matrix. Equation (4) may be linearized with respect to the state vector x

$$J_{gi} = \left(\frac{\partial Q_i}{\partial x} \right) \quad (5)$$

Sensitivity of the reactive power injected at bus i with respect to the loading parameters $\xi = [P_1 \dots P_{m+n-1} Q_1 \dots Q_m]^T$ is

$$\begin{aligned} \left(\frac{dQ_i}{d\xi} \right) &= \left(\frac{\partial Q_i}{\partial \xi} \right) - \left(\frac{\partial Q_i}{\partial x} \right) \left(\frac{\partial F}{\partial x} \right)^{-1} \left(\frac{\partial F}{\partial \xi} \right) \\ &= \left(\frac{\partial Q_i}{\partial \xi} \right) - \hat{x} \left(\frac{\partial F}{\partial \xi} \right) \end{aligned} \quad (6)$$

where $F = [f \ g]^T$ is a set of load flow equations, and $\hat{x}$ is a co-state vector [1], defined by the equation

$$J^T \hat{x} = \left(\frac{\partial Q_i}{\partial x} \right) \quad (7)$$

where J is a load flow Jacobian matrix. The equation (6) may then be represented as

$$\left(\frac{dQ_i}{d\xi} \right) = \left(\frac{\partial Q_i}{\partial \xi} \right) - J_{gi} J^{-1} \left(\frac{\partial F}{\partial \xi} \right) \quad (8)$$

The expression (8) is the sensitivity of the injected reactive power at bus i with respect to loading parameters of the system. It grows infinitely at the bifurcation point, which complies with the observations about heavy reactive power transfers, and generation in near voltage-unstable situations. One should note that the injected reactive power at any node in the system does not become infinite at the bifurcation: according to (3), it remains finite in every point (x_0, y_0, λ) which satisfies (4), but (8) suggests that its *rate of change* becomes infinite for an infinitesimal change of the parameter vector. Therefore, (8) offers one possibility to assess the proximity to voltage collapse. Some practical problems need to be resolved before it could be used reliably. Size of the parameter vector is one such problem. The number of parameters in power system which define the loading is relatively high, but some of them are strongly correlated with each other. That property could be used to effectively reduce the number of relevant sensitivities which need to be monitored for assessment of voltage stability. Reference [2] proposes the following load model

$$\begin{aligned} F_1(B)z(t_k) &= F_2(B)\eta(t_k) \\ F_3(B)z(t_k) &= x(t_k) \\ P(t_k) &= L x(t_k) \\ S_i(t_k) &= P_i(t_k) + j\alpha_i Q_i(t_k) \end{aligned} \quad (9)$$

where

$\eta(t_k)$ is an m -vector of independent white noise processes

$z(t_k)$ is an m -vector of stationary stochastic processes

$x(t_k)$ is an m -vector of nonstationary stochastic processes

$P(t_k)$ is an n -vector of bus real power electric loads

$S_i(t_k)$ is the complex power injection at bus i

$F_i(*)$ are vectors of the polynomials of the argument

B is a backward operator

L is an $n \times m$ matrix, such that $m \ll n$

t_k are discrete time instants

Using the above procedure, with an appropriate choice of $F_i(*)$ and L , one can generate the large set of n complex power injections $S_i(t_k)$ from a small number m of independent white noise processes. This model has been known to accurately model the periodicities as well as nonperiodic properties of the electric load [2]. From the standpoint of steady state analysis, we are primarily interested in the structure of matrix L , which defines the relationship between individual electric loads and independent parameters, representing the elements of the reduced parameter vector λ_r . The analysis of voltage stability in the reduced parameter space λ_r is accomplished through the linear model L , since

$$\xi = L \lambda_r \quad (10)$$

and the sensitivity of the injected reactive power at node i with respect to the loading parameters becomes

$$\left(\frac{dQ_i}{d\lambda_r} \right) = \left[\left(\frac{\partial Q_i}{\partial \xi} \right) - J_{gi} J^{-1} \left(\frac{\partial F}{\partial \xi} \right) \right] L \quad (11)$$

It should be noted that the Jacobian J is not being inverted during calculation of (11), since it would be very demanding task. The calculation is accomplished by obtaining the costate vector (7), and using it in the equation (6), then postmultiplying by L . Calculation of costate requires just one forward and backward substitution, which is computationally much more efficient than the inversion of the Jacobian matrix. Both expressions (8) and (11) represent an indication of the proximity to stability boundary. The closer look at the individual elements of $[dQ_i/d\lambda]$ in (8) yields the following interpretation: if $dQ_i/d\lambda_j = \beta_{ij}$ at some operating point (x, y, λ) , the linearized model would require the increase of generated reactive power at node i by β_{ij} MVAR per unit

increase of the loading parameter λ_j (active, or reactive load at some node in the system). For all reactive injections in the network,

$$S_j = \sum_{i=1}^n \frac{dQ_i}{d\lambda_j} \quad (12)$$

represents the total increase of reactive injection with respect to an individual (active, or reactive part of) load. Whenever λ_j is a reactive load, S_j is expected to be larger than 1, to accommodate the load increase, and increase of reactive power exchange between generators and the transmission network. Near bifurcation, however, S_j tends to grow infinitely, indicating that disproportionate amount of newly generated reactive power is directed toward covering reactive losses in the transmission network. Similar conclusions can be drawn from (12), where sensitivities reflect the effect of increase of loading factors λ_{r_i} rather than individual loads. In both cases, the norm of sensitivity vector (8), or (12) may be considered as an indicator of proximity to voltage instability, for they both grow infinitely in the bifurcation

$$\lim_{\lambda \rightarrow \lambda_0} \left\| \frac{dQ_i}{d\lambda} \right\| = \lim_{\lambda \rightarrow \lambda_0} \left\| \frac{dQ_i}{d\lambda_r} \right\| = \infty \quad (13)$$

By appropriately choosing a threshold for values of the norms (13), an effective monitoring scheme may be accomplished for tracking of voltage stability. Figure 1 shows the generated reactive power of a power system consisting of two buses interconnected by a transmission line. The active and reactive power of the constant complex power load connected at the receiving end of the transmission line is a parameter vector, in terms of which the generated reactive power is represented. The projection of the bifurcation points onto (P, Q) plane is also shown. It is quite obvious from Figure 1 how relation (13) can be visualized. Another advantage of this approach to quantifying proximity to voltage instability is that it is not sensitive to the transitions of generators into constant reactive generation mode. In fact, whenever any of the generators switches from voltage control into constant reactive power mode, the sensitivities of the remaining generators with respect to loading parameters increase even faster with further increase of the loading. Among alternative indices for voltage stability monitoring, which are multiple, two others will be used: the minimum singular value of the load flow Jacobian matrix $\sigma_{\min}\{J\}$, and a heuristic indicator of line overloading [23][24] defined as

$$J_p = \sum_{i \in \ell} \omega_i \left[\frac{Q_i}{T_{i\max}} \right]^{2n} \quad (14)$$

where

ℓ is the set of lines in power system
 Q_i is the reactive power flow through the line i
 $T_{i\max}$ is the rating of the line i
 ω_i are weight factors
 n is a constant

The choice of the alarm (14) is based on observation that line flows are large in heavily loaded systems which operate close to stability boundary. If n is chosen large enough, J_p will rise sharply when the circuits start operating near, or above their respective ratings, thus giving a warning about possibility of voltage instability. Since this indicator is based on purely heuristic considerations, its performance will be assessed herewith along with the two which have been shown to correspond closely to voltage collapse (sensitivity of generated reactive power with respect to loading parameters (11), and $\sigma_{\min}\{J\}$).

3 Remedial Action – Optimal Power Flow

The operation of the stressed power system, especially the one following contingencies, or large disturbances, is jeopardized by violation of many operating constraints: voltage magnitudes, line flows, etc. This is particularly true for near voltage-unstable systems, which operate under very stressed conditions. The importance of determination of appropriate remedial actions to alleviate violations of operating constraints cannot be emphasized enough. In addition to that, proposed methodology needs to be efficient and lend itself well to applications in large scale systems, producing usable output in short interval of time. A desirable property is also the flexibility, which would allow the system operator to consider inclusion (or exclusion) of all available controls in order to accommodate various system conditions.

The method described herein uses linearized model which associates the undervoltages, overloads, and other violated constraints with available controls. After running a load flow, violated operating constraints are identified by comparison of calculated data with specified maxima and minima. The considered constraints are:

- bus voltage magnitudes
- line MW flows
- line MVA flows
- reactive power generation
- area MW interchanges

The block diagram of the algorithm is shown in Figure 2. It is characterized by the following features:

- after detection of the violations, reduction of the model is attempted by determination of coherent constraints (which increase, or decrease with pre-specified degree of coherency with respect to control actions); an example of such constraints are line overloads in cascaded radial links; the worst violations are singled out as active constraints for subsequent optimization.

- active constraints are then linearized with respect to available controls (which may be prioritized); technique used for linearization is based on adjoint network analysis [1], analog to the equation (6): if $\psi(x, u)$ is the operating constraint, expressed as a function of the state vector x and control variables u , and $F(x, u) = 0$ are load flow equations, the sensitivity of ψ with respect to a particular control variable u_i is

$$\frac{d\psi}{du_i} = \frac{\partial \psi}{\partial u_i} - \left(\frac{\partial \psi}{\partial x} \right) \left(\frac{\partial F}{\partial x} \right)^{-1} \frac{\partial F}{\partial u_i} \quad (15)$$

- subsequent model reduction is attempted by excluding the control actions which have been identified as having small sensitivities with respect to all active constraints; such controls are declared inactive and left out of the linear program, thus reducing its size without affecting the accuracy; the complete set of controls includes:

- active generation
- reactive generation
- generator voltage
- transformer phase shift
- transformer tap
- switchable capacitors
- switchable reactors
- load transfer
- interruptible load curtailment
- firm load shedding
- critical load shedding

- the LP algorithm is formulated as minimization of the objective function (the weighted sum of absolute values of control actions) subject to

- power balance equation
- line loading constraints
- voltage constraints
- reactive power constraints
- operating limits of remedial actions

The algorithm takes advantage of the sparsity techniques to increase speed and reduce memory requirements, usually producing solution in a couple of iterations.

Without going into detailed description of the algorithm, which is known from the literature [1], we shall emphasize some aspects of its application in near voltage-unstable power systems. The issue of linearization requires closer scrutiny, for it is well known that the models with near-singular load flow Jacobian matrix have pronounced nonlinearity. Indeed, it was found out in simulations that the linearization bounds which are deemed acceptable and yield accurate results in not-far-from-normal system conditions, cannot be used when power system operates near stability boundary. The heuristic formulae

$$\begin{aligned} \left| \frac{\Delta Q_i}{\sum Y_i} \right| &\leq \epsilon_1 \\ \left| \frac{\Delta P_i}{\sum Y_i} \right| &\leq \epsilon_2 \end{aligned} \quad (16)$$

are normally used (ΔP_i , ΔQ_i , are changes of active and reactive injections at bus i , $\sum Y_i$ is the sum of all admittances connected to node i , and ϵ_1 , ϵ_2 , are user determined thresholds, which are the functions of errors of calculation). For calculations near steady state stability boundary, (16) needs to be investigated in stressed conditions, which results in reduced acceptable thresholds. Also, not all control variables should be considered for application of the remedial action algorithm. Arguably, the controls based on reduction of the load (interruptible load curtailment, firm, and critical load shedding) need not be considered, because they belong to the group of emergency controls whose application should be initiated as a result of real-time monitoring [7]. By assuming that application of remedial actions should insure the highest reliability of supply to the customers in situations which are close to voltage instability, one should concentrate on the controls which do not affect supply: it is quite clear that reduction of load results in the increase of stability margins, but the real strength of remedial actions is that the algorithm is capable of improving the system condition (performance index, stability margin), without radical measures such as load shedding. Even when there is no feasible solution which eliminates all violations of operating constraints, the algorithm will find the partial solution which is characterized by smaller violations of the constraints.

4 Test System and Simulation Results

The test system for evaluation of the four voltage instability proximity indicators was the IEEE 24-bus system [22] often used for reliability analysis. It consists of 10 power plants with a total of 32 generators, and 17 loads. The following modifications were made on the original system data [22]:

- lines between buses 11 and 13, and 12 and 13 were assumed to be twice as long without change in their respective ratings; this was done to avoid severe overloads of those lines when loading is being increased in order to reach the bifurcation point.

- two new voltage control devices were added at buses 8, and 9: static VAR compensators with 200 MVAR capacity each; they were assumed to be continuously adjustable in the range (-200,200) MVAR.

The available controls in the test system are:

- 10 active power generation variables
- 10 voltage set point variables for the power plants
- 1 phase shifter variable
- 5 transformer tap control variables
- 3 switched capacitor control variables
- 3 switched inductor control variables
- 17 firm load control variables (for load shedding)

The firm load control variables were disabled for the simulations presented in this paper. Whenever load shedding is available in the system, remedial actions can use it to reduce the load level to the point where no violations of operating constraints occur in the system. Such control is rather drastic, and should be reserved for emergency purposes only. Therefore, only the remaining 32 variables were considered active for simulations.

The load model is given by

$$\begin{bmatrix} P_1 \\ P_2 \\ \dots \\ P_n \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ \dots \\ b_{n1} \end{bmatrix} V_1 + \begin{bmatrix} b_{12} \\ b_{22} \\ \dots \\ b_{n2} \end{bmatrix} V_2 \quad (17)$$

$$Q_i = \alpha_i P_i \quad (18)$$

where a_i , b_{1i} , b_{2i} , and α_i are constants, and V_1 , and V_2 are variables called load factors. In terms of the equation (10), we are representing 17 loads (with 34 parameters) by means of only two independent parameters, V_1 and V_2 . The base case loading corresponds to the values $V_1 = V_2 = 1.0$. For these values of V_1 , V_2

the bus loads of the system are equal to the peak load defined for the IEEE/RTS-24 bus system [22]. The coefficients a_i , b_{ij} , and α_i are given in Table 1. In addition to being justified by the records of load coherency in actual power systems, the two-parameter representation of the load allows to visualize the bifurcation surface in the (V_1, V_2) plane. Figures 3,4,5, and 6 represent the values of the following indicators for various loading levels determined by points in (V_1, V_2) plane, obtained as load flow solutions for various loading levels:

- norm of the vector of sensitivities of generated reactive powers with respect to the loading factors V_1 and V_2 ; it grows infinitely at bifurcation point and could be used very reliably to assess the imminent danger of voltage instability through increasing loading; while Figure 1 and equation (8) are explaining the qualitative performance of the indicators, Figures 3, and 4 are showing its performance on the test system. It starts growing sharply for the loading well below the bifurcation level, giving an early indication of instability (it approximately triples during the first 80% of the loading between base case and bifurcation, but it grows six fold for 90% load increase toward bifurcation, and starts growing even faster thereafter).

- minimum singular value of the load flow Jacobian ($\sigma_{\min}\{J\}$, Figure 5) is zero in the bifurcation point, but its change with respect to loading is nonuniform – 40 percent of the change from the value at base case loading occurs in the last few percent of the loading toward bifurcation. This situation is aggravated by the fact that generators which switch from PV into constant reactive generation modes change the size of Jacobian and make the minimum singular value a discontinuous function of the loading; although accurate, $\sigma_{\min}\{J\}$ is not very reliable indicator of voltage collapse.

- The indicator based on formula (14) is purely heuristic, but it grows continuously as the loading increases towards bifurcation. It is the most uniform of the indicators tested, but has one serious drawback: the value of the indicator in the bifurcation point is not known, and depends on the chosen system, and its parameters. Technically, the indicator (14) is not an indicator of proximity to voltage collapse, but rather an indicator of the reactive transmission network loading, but it is well known that those two conditions are closely related, which explains the well-behaved changes of (14) in Figure 6.

For several operating points defined by loading levels between base case and stability boundary, the remedial action algorithm was applied in order to assess its

performance with respect to voltage stability. Figures 7, 8, 9, and 10 represent the same four indicators in the system which underwent the remedial action following initial loading. It may be noted that the values of the indicators have been cut in half for many operating conditions which are characterized by heavy loading. Predictably, remedial actions affected very little the low loading levels around the base case loading, because those contain no violations of the operating constraints to be removed. But the algorithm produced the best results where it was needed most – close to stability boundary, lowering the values of all indicators in almost all tested operating conditions. It should be noted that in many instances very close to stability boundary the algorithm did not actually produce a feasible solution – it was only able to apply available controls to reduce the existing violations of bus voltages and line overloads, without removing them completely. The additional control by load shedding, which was intentionally disabled from the list of remedial actions, would complete the task. The application of remedial action algorithm on the heavily loaded system not only reduces the values of indicators, but it also extends the stability boundary beyond the originally calculated one, obtained from load flow solutions. This effect was not illustrated in the paper, because the initial load flow calculation for any loading level (V_1, V_2) beyond bifurcation would result in a divergent load flow, although the subsequent application of the remedial action could produce a feasible solution.

A very useful application of the remedial action algorithm is to rectify the consequences of various contingencies in the system. Contingencies destroy the optimality of system operation, and often create stressed conditions with a number of violations of operating constraints, and degraded stability margins. By attempting to bring the system back into the secure region, remedial actions algorithm also improves the stability margin. Table 2 shows the changes of indicators used in previous simulations following the application of remedial actions for all single line outages in the test system which resulted by some violations of operating constraints. In all cases, changes of the values of the indicators suggest that the stability margin was also improved as a result of remedial action. That improvement was especially notable in the cases where contingencies create very stressed system (like the outage of the line 6-10). As a result of extensive simulations on the test system, it was concluded that the application of remedial action algorithm never degrades, and almost always improves the voltage stability of the power system, although voltage stability is not explicitly part of the objective function, nor

the operating constraints.

5 Conclusions

The analysis of the power system performance with respect to voltage stability is proposed to be done in the reduced parameter space, which is justified by the coherent load behavior in the actual power systems, and allows simpler interpretation of the results.

Three indicators of voltage stability were tested on a 24-bus system: sensitivity of the generated reactive power with respect to the elements of the reduced parameter space (loading factors), minimum singular value of the load flow Jacobian matrix, and the index of reactive transmission network loading. The first indicator produced the best results, while the third, although purely heuristic, displayed surprisingly uniform pattern of increase toward voltage stability boundary.

The remedial action algorithm was tested for performance with respect to voltage stability. It is formulated as a robust optimal power flow with the objective to remove all the violations of operating constraints from the system with minimum action of control variables. As a result of remedial actions, almost all tested situations resulted in net improvement of the voltage stability margins, judged by all previously introduced indicators. In no case the remedial action algorithm produced the deterioration of voltage stability margin, although voltage stability was not formulated as an operating constraint in the present form of the algorithm. Future research may encompass explicit inclusion of such considerations in the formulation of remedial action algorithm.

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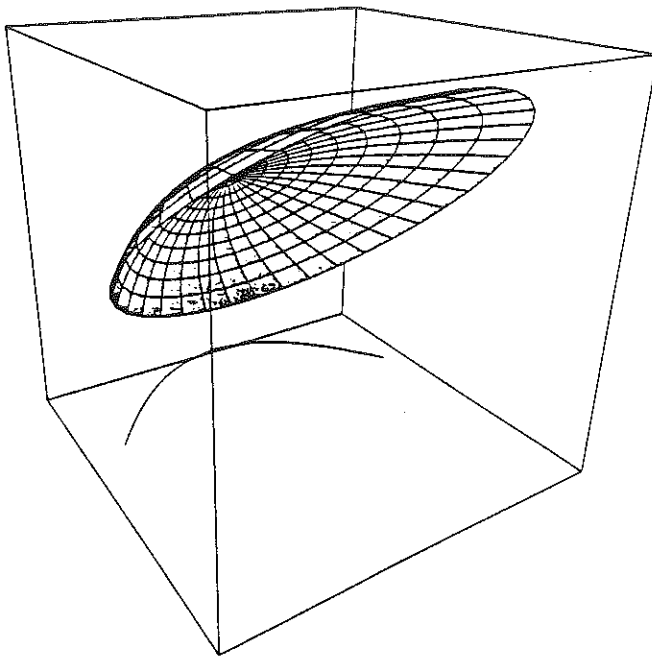


Figure 1. Generated reactive power of a single machine connected to a constant PQ load, represented in the (P,Q) parameter space. Projection of the bifurcation points in (P,Q) plane is also shown.

BUS #	a	b1	b2	alpha
1	0.37	0.40	0.31	0.2
2	0.33	0.34	0.30	0.2
3	0.61	0.60	0.59	0.2
4	0.25	0.20	0.29	0.2
5	0.24	0.20	0.27	0.2
6	0.46	0.50	0.40	0.2
7	0.42	0.33	0.50	0.2
8	0.58	0.63	0.50	0.2
9	0.59	1.00	0.16	0.2
10	0.66	1.10	0.19	0.2
13	0.90	1.00	0.75	0.2
14	0.66	0.28	1.00	0.2
15	1.07	0.59	1.50	0.2
16	0.34	0.60	0.06	0.2
18	1.13	1.50	0.70	0.2
19	0.61	0.20	1.00	0.2
20	0.43	0.35	0.50	0.2

Table 1. Parameters of the load model.

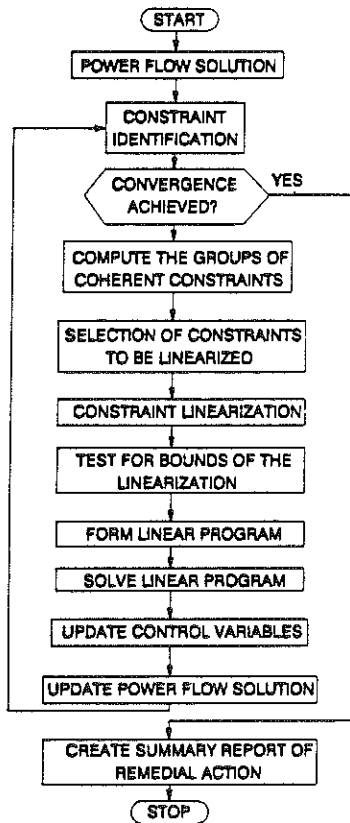


Figure 2. Block diagram of the remedial actions algorithm.

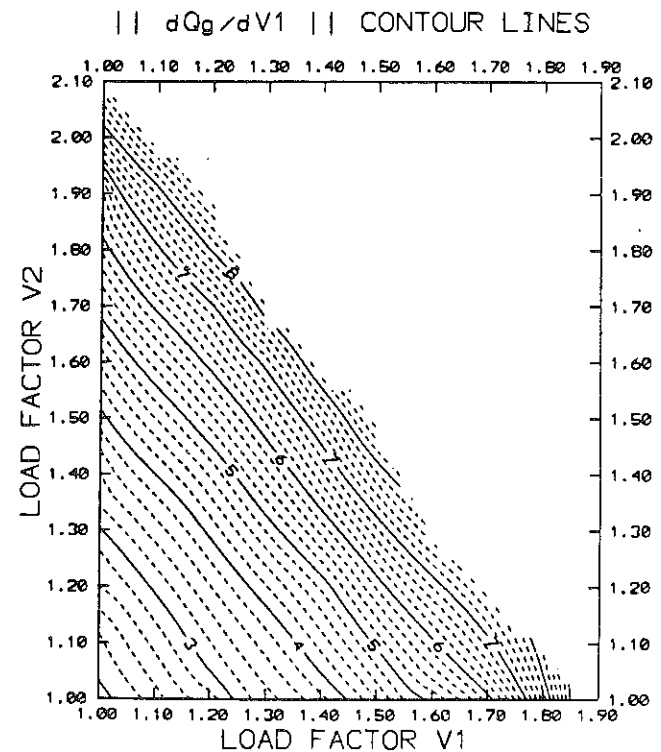
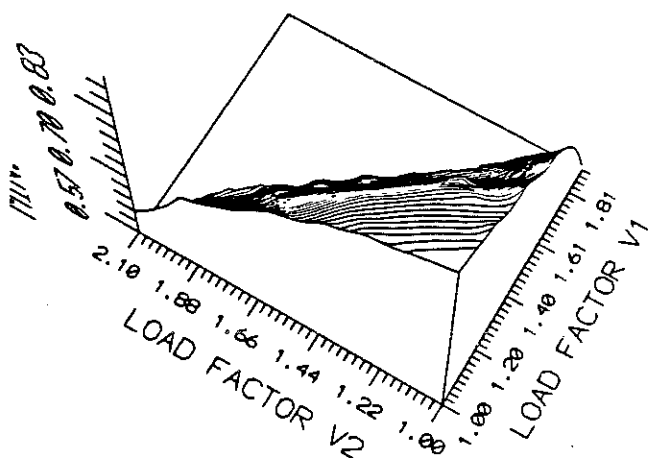
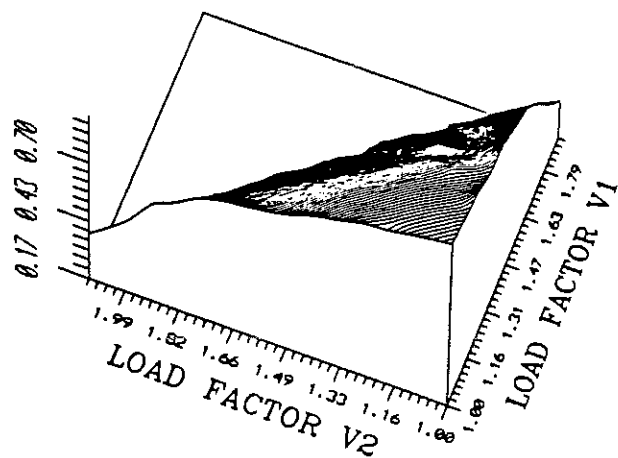
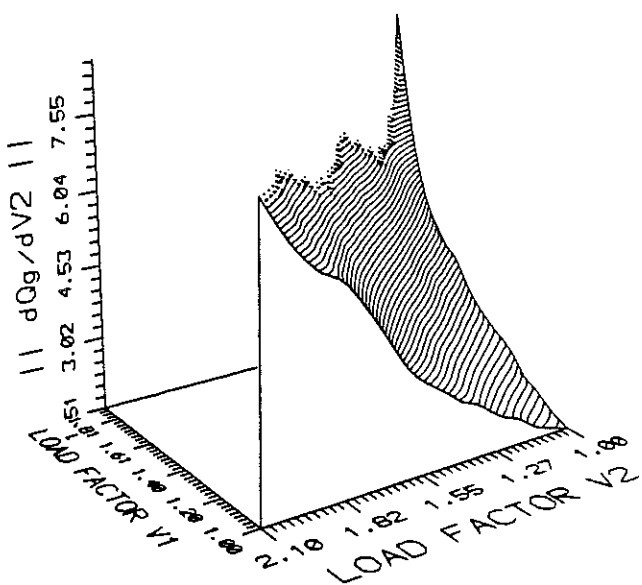
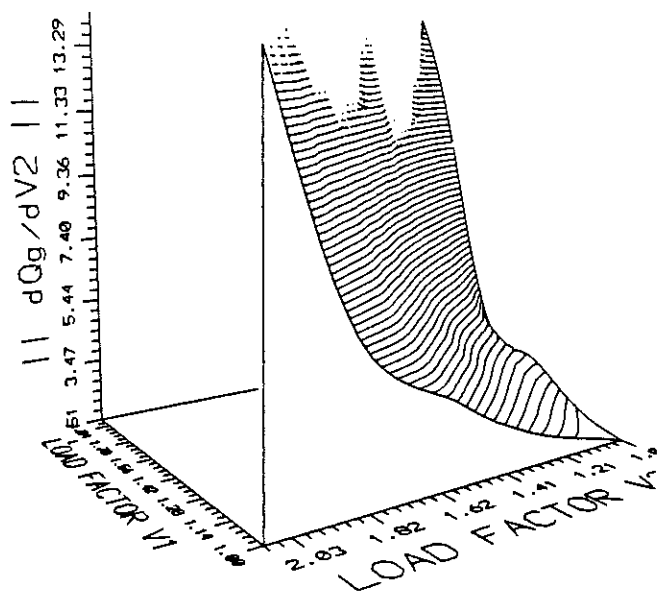
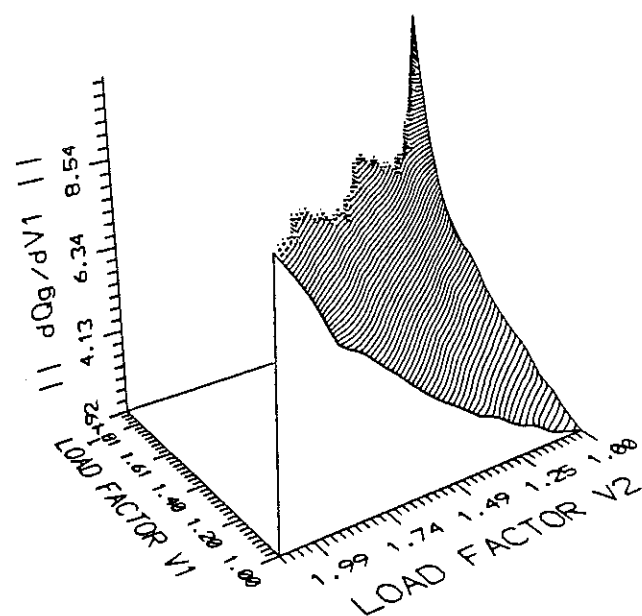
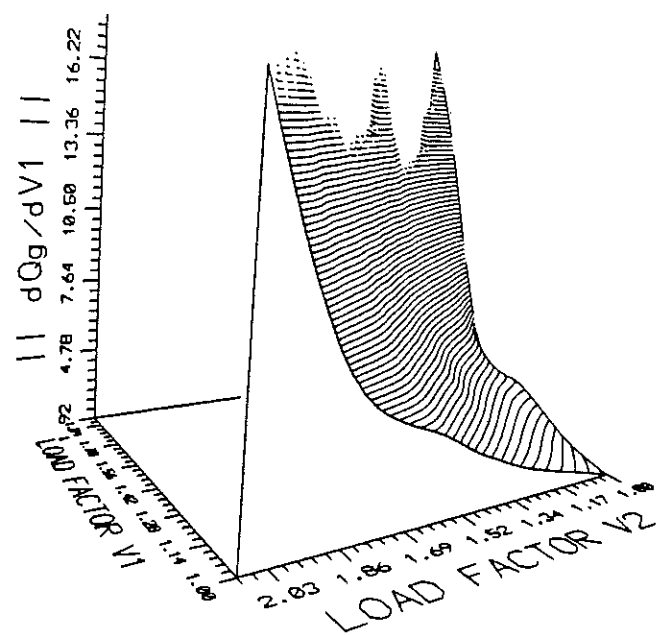


Figure 11. Contour lines of $\|dQ_g/dV_1\|$ in the region of stability of the test system in (V_1, V_2) space.



Figures 3,4,5 Sensitivities of the generated reactive power with respect to the load factors V_1 , and V_2 , and $\sigma_{\min}\{J\}$ for various loading conditions before application of remedial actions.

Figures 7,8,9 Sensitivities of the generated reactive power with respect to the load factors V_1 , and V_2 , and $\sigma_{\min}\{J\}$ for various loading conditions after application of remedial actions.

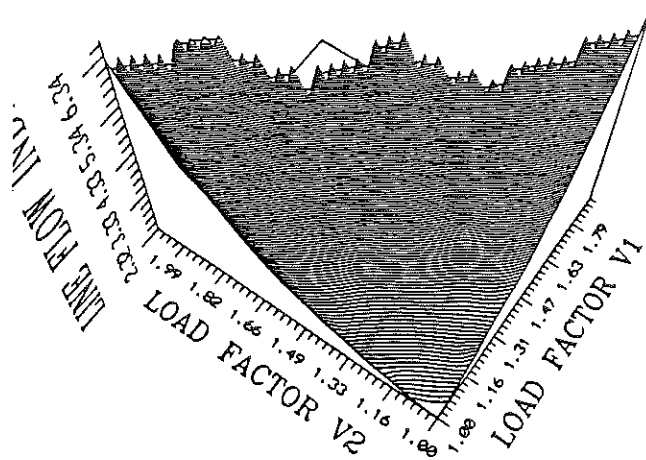


Figure 6. Reactive network overload indicator for various loading conditions before application of remedial actions.

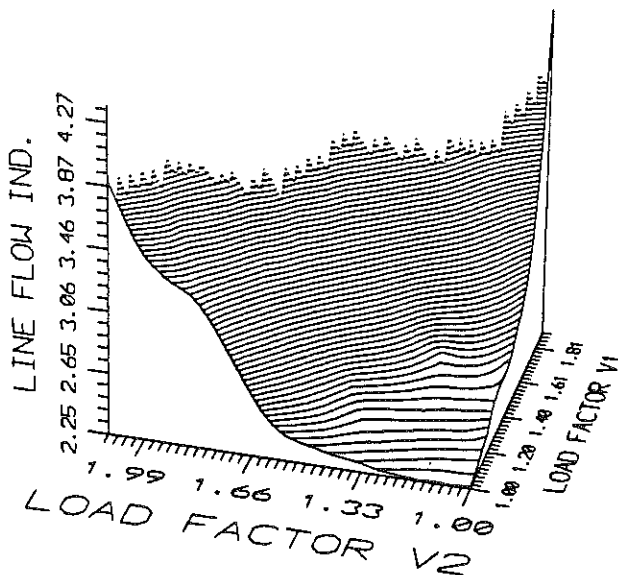


Figure 10. Reactive network overload indicator for various loading conditions after application of remedial actions.

	pre-RA	post-RA	pre-RA	post-RA	pre-RA	post-RA	pre-RA	post-RA
	IND. #1	IND. #1	IND. #2	IND. #2	IND. #3	IND. #3	IND. #4	IND. #4
1-3	2.149	2.033	1.680	1.594	0.890	0.920	2.639	2.218
1-5	2.198	2.163	1.706	1.681	0.884	0.893	2.567	2.439
2-4	2.165	2.070	1.698	1.627	0.887	0.914	2.579	2.292
3-6	2.064	1.639	1.634	1.335	0.889	0.963	2.367	4.374
4-6	2.227	2.195	1.762	1.737	0.837	0.846	2.691	2.429
6-10	4.949	3.298	3.898	2.593	0.466	0.769	5.980	4.436
8-10	2.312	2.258	1.865	1.820	0.835	0.851	2.675	2.495
9-11	2.456	2.327	1.866	1.770	0.847	0.876	2.639	2.160
10-11	2.823	2.736	2.066	2.006	0.799	0.818	2.786	2.459
11-13	3.259	3.179	2.685	2.622	0.555	0.565	2.602	2.316
11-14	2.755	2.575	1.847	1.721	0.807	0.838	3.377	2.638
12-13	2.965	2.892	2.379	2.324	0.618	0.629	2.661	2.327
12-23	2.641	2.569	1.836	1.784	0.861	0.878	2.791	2.512
13-23	1.564	1.139	1.098	0.995	0.638	0.650	2.515	2.449
16-17	2.068	2.032	1.559	1.531	0.887	0.899	2.903	2.758

INDICATOR 1 - SENSITIVITY OF GENERATED REACTIVE POWER W.R.T. LOADING INDICATOR V1
INDICATOR 2 - SENSITIVITY OF GENERATED REACTIVE POWER W.R.T. LOADING INDICATOR V2
INDICATOR 3 - MINIMUM SINGULAR VALUE OF THE JACOBIAN MATRIX
INDICATOR 4 - BASED ON REACTIVE LINE LOADING

Table 2. Effect of remedial actions on post-contingency states.

AN APPROACH TO STUDY STEADY-STATE VOLTAGE STABILITY OF LARGE SCALE POWER SYSTEMS

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1 Introduction:

When transmission systems are highly interconnected and heavily loaded, systems become more stressed and vulnerable. A close look at the problem reveals that the causes of today's stressed power systems are many and often involved various issues. For example, the high cost of upgrading existing transmission lines to meet increasing energy consumption; the difficulty of acquiring right-of-way from land owners; the delay of obtaining license to build new transmission lines from state agency; the increasing concern of the electromagnetic field (EMF) to human health; the shift of generation pattern due to environmental constraints. One of the consequences of this stressed power network is related to voltage security. Voltage collapse has been deemed responsible for several major disturbances [1] and significant research efforts are underway in an effort to further understand voltage phenomena [2]. Numerous articles have been published around the world to discuss and find ways to tackle this pressing problem [3-21]. A particular difficulty being encountered in such research is that the Jacobian of Newton-Raphson power flow becomes singular at the steady state voltage limit. Infact this stability limit, also called the critical point, is often defined as the point where the power flow Jacobian is singular. As a consequence, attempts at power flow solutions near the critical point are prone to divergence and error. For this reason, double precision computation and anti-divergence algorithms such as the one found in [22] have been used in an attempt to overcome the numerical instability.

In general, non-linearity gives rise to complex and unexpected behavior in many physical, chemical and biological systems. Power systems are no exception for this complicated behavior. A particular feature seen in many non-linear systems is the abrupt change in steady-state behavior that may occur as a parameter changes smoothly. The parameter value at which the abrupt change occur generally correspond to singular points in the governing equations. At a singular point (or limit point), the Jacobian matrix of the linearized system is singular and one real eigenvalue for the linearized system crosses the imaginary axis at such point. Hence it is accompanied by a qualitative change in behavior. In terms of physical significance, limit points characterize ignition and extinction phenomena in chemical reactors [23], structural collapse in civil engineering [24] and in power systems they can be related to voltage collapse, which is the subject of this paper.

As mentioned earlier the Conventional Newton-Raphson method of finding solution to the above non-linear equations fail at the singular points. The authors proposed in [25] a locally parameterized continuation technique which overcomes the singularity in the Jacobian. They named

the resulting algorithm, the "Continuation Power Flow". The purpose of the continuation power flow was to find a continuum of power flow solutions for a given load change scenario. It has the ability to find a set of solutions from a base case upto the critical point in a single program run. It also provides intermediate results which provide valuable insight into the voltage stability of the system and the areas prone to voltage collapse. The problem is formulated in a bifurcation theory framework, which has become a major focus of research in the analysis of many nonlinear dynamical systems.

In the original continuation power flow, only constant power load models were considered. This paper extends the continuation power flow to include nonlinear load models, where the load model includes voltage dependent terms. The paper is organized as follows. Section 1 briefly explains the bifurcation theory. Section 2 outlines basic steps involved in the continuation power flow. Section 3 describes the modifications needed to include nonlinear load models in the continuation power flow and section 4 demonstrates through a test system, the effect of nonlinear load model on the steady-state voltage stability.

2 Brief introduction to Bifurcation

Consider a power system dynamical model described by autonomous differential equations of the vector form in n -dimensional space

$$\dot{X} = F(X, \lambda) \quad \dots\dots (1)$$

$$X = (x_1, x_2, \dots, x_n)^T$$

$$F = (f_1, f_2, \dots, f_n)^T$$

Here x_i denotes the state variables (generator angle, generator angular velocity, load voltage magnitude or angle etc.), λ is a time invariant scalar parameter (real or reactive power etc.) and $\dot{X}$ denotes $\frac{dX}{dt}$ where t is time.

At an equilibrium point, the left hand term $\dot{X}$ of equation (1) becomes zero, i.e. the steady state solution of (1) satisfies the set of nonlinear algebraic equations

$$F(X, \lambda) = 0$$

Let λ_0 be the critical value of the parameter for which the real part of one or more eigenvalues of the Jacobian matrix $J = \frac{\partial F}{\partial x_j}$ becomes zero.

At (x^*, λ_0) , if the eigenvalue becomes zero, then according to implicit function theorem the equilibria of (1) for a value of λ different from λ_0 can be expressed as the smooth function $x = x(\lambda)$. The function $x(\lambda)$ is called a branch of equilibria. If at (x^*, λ_0) , several branches of equilibria come together, then the point (x^*, λ_0) is said to be a bifurcation point.

We will discuss the instances in which (i) a real eigenvalue becomes zero or (ii) a pair of complex conjugate eigenvalues become pure imaginary. In the former case, new branches of stationary solutions usually arise and are called static bifurcations. The later instance may lead to the birth of a branch of periodic solutions called dynamic bifurcation.

This paper deals mainly with static bifurcations. For dynamic bifurcation refer to [26]. There are three simplest static bifurcations corresponding to a zero real eigenvalue [27].

1. Fold or saddle node
2. Transcritical
3. Pitchfork

Steady state voltage instability is closely related to fold or saddle node type of bifurcation [16]. By observing certain transversality conditions at the critical point one can separate those three static bifurcations.

For example, if we assume the system is one dimensional, then at the critical point (x^*, λ_0) , the saddle node bifurcation satisfies the following transversality conditions

1. $\frac{\partial f}{\partial \lambda} \neq 0$
2. $\frac{\partial^2 f}{\partial x^2} \neq 0$

Fig.1 illustrates the saddle-node bifurcation for the prototype equation $\dot{x} = \lambda - x^2$; $(x^*, \lambda_0) = (0, 0)$

for
 $\lambda < \lambda_0$ no equilibrium exists
 $\lambda > \lambda_0$ two branches of equilibrium emerge one being stable the other being unstable
 For multidimensional systems the above conditions become [27]

- $\psi F_\lambda \neq 0$
- $\psi F_{xx} \phi \neq 0$

where ψ and ϕ are the left and right eigenvectors corresponding to zero eigenvalue respectively. F_{xx} is the Hessian matrix. Similar conditions can be derived for transcritical and pitchfork bifurcations (Appendix)

Identification of these bifurcation points needs robust numerical techniques. The next section briefly explains a locally parametrized continuation process which results in the continuation power flow. This power flow is capable of tracing solution paths that can go beyond the critical point for various load change scenarios.

3 The Continuation Power Flow:

The continuation power flow employs a predictor-corrector scheme to find the solution path of a set of power flow equations that have been reformulated to include a load parameter. As shown in Fig.2, it starts from a known solution and uses a tangent predictor to estimate a subsequent solution corresponding to different value of the load parameter. This estimate is then corrected using the same Newton-Raphson technique employed by a conventional power flow. The local parametrization provides a means of identifying each point along the solution path and plays an important part in avoiding the singularity of Jacobian.

Basic algorithm

If F is used to denote the whole set of steady state power flow equations, then

$$F(\delta, y, \lambda) = 0$$

where

δ is vector of bus voltage angles
 y is vector of bus voltage magnitudes
 λ is load parameter

To solve the problem, the continuation algorithm starts from a known solution and uses a predictor-corrector scheme to find subsequent solutions at different load levels.

Predictor:

Once a base solution has been found ($\lambda=0$), a predictor of the next solution can be made by taking appropriately sized step in a direction tangent to the solution path. This tangent is derived by taking the derivative of both sides of the power flow equations.

$$d[F(\delta, y, \lambda)] = F_\delta d\delta + F_y dy + F_\lambda d\lambda = 0$$

which results in

$$\begin{bmatrix} F_\delta & F_y & F_\lambda \end{bmatrix} \begin{bmatrix} d\delta \\ dy \\ d\lambda \end{bmatrix} = 0$$

This problem can be solved by choosing a non-zero magnitude (say one) for one of the components of the tangent vector. If $\underline{t}$ is used to denote the tangent vector, then

$$\begin{bmatrix} F_\delta & F_y & F_\lambda \\ & \epsilon_k & \end{bmatrix} \begin{bmatrix} \underline{t} \end{bmatrix} = \begin{bmatrix} 0 \\ \pm 1 \end{bmatrix}$$

where

$$\underline{t} = \begin{bmatrix} d\delta \\ dy \\ d\lambda \end{bmatrix}$$

$$t_k = \pm 1$$

where ϵ_k is an appropriately dimensioned row vector with all elements equals to zero except the k^{th} , which equals one. If the index k is chosen correctly, letting $t_k = \pm 1$ imposes a non-zero norm on the tangent vector and guarantees that the augmented Jacobian will be nonsingular at the critical point [28]. Once the tangent vector has been found by solving (3), the prediction can be made as follows

$$\begin{bmatrix} \delta^* \\ y^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} \delta \\ y \\ \lambda \end{bmatrix} + \sigma \begin{bmatrix} d\delta \\ dy \\ d\lambda \end{bmatrix}$$

where $\star$ denotes the predicted solution for a subsequent value of λ (loading) and σ is a scalar that designates the step size.

Corrector:

The predicted solution is corrected by a new set of equations

$$\begin{bmatrix} F(\underline{x}) \\ x_k - \eta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\underline{x} = \begin{bmatrix} \delta \\ y \\ \lambda \end{bmatrix}$$

This is nothing but augmentation of original set of equations by one equation that specifies the value of one of

the state variables. In the case of reformulated power flow equations, this means specifying either a bus voltage magnitude, a bus voltage angle, or the load parameter λ . Once a suitable index k and value of η are chosen, a slightly modified Newton-Raphson power flow method can be used to solve the set of equations. This provides the corrector needed to modify the predicted solution found in the previous section. This is basic continuation power flow algorithm. Details can be found in [25] and [29].

4 Load Modelling

4.1 Constant power load model:

In this model the load and generation at each bus are independent of voltage. Thus, both are made to vary in direct proportion to any change in λ , with $\lambda = 0$ corresponding to the base load. This is written as

$$P_{Li} = P_{Lio} + \lambda(K_{Li}S_{\Delta BASE}\cos\psi_i) \quad \dots(2)$$

$$Q_{Li} = Q_{Lio} + \lambda(K_{Li}S_{\Delta BASE}\sin\psi_i) \quad \dots(3)$$

where the following definitions are made

P_{Lio}, Q_{Lio} : original load at bus i , active and reactive respectively.

K_{Li} : multiplier to designate the rate of load change at bus i

ψ_i : power factor angle of load change at bus i

$S_{\Delta BASE}$: a given quantity of apparent power which is chosen to provide appropriate scaling of λ

With this load model and any given value of λ , the change in active power load from the base load can be calculated and the load change is distributed among the generators.

$$\Delta P_{TOTAL} = \lambda PMULT$$

where, for a given set of K_{Li} 's and ψ_i 's, $PMULT$ is a constant defined as:

$$PMULT = S_{\Delta BASE} \sum_{i=1}^n K_{Li}\cos\psi_i$$

Then, if $GFRAC_i$ is the fraction of the load change to be served by the i^{th} generator, the general expression for the generation at bus i becomes

$$P_{Gi} = P_{Gio} + \lambda(GFRAC_i)(PMULT)$$

where P_{Gio} is the generation at bus i in the base case.

When the newly formulated load and generation terms are inserted into the general form of power flow equations, the result is

$$\Delta P_i = P_{Gio} + \lambda(GFRAC_i)(PMULT) - P_{Lio}$$

$$-\lambda(K_{Li}S_{\Delta BASE}\cos\psi_i) - P_{Ti}$$

$$\Delta Q_i = Q_{Gio} - Q_{Lio} - \lambda(K_{Li}S_{\Delta BASE}\sin\psi_i) - Q_{Ti}$$

The set of equations that describe the entire system are made up of a combination of these two general equations. The details of calculating the augmented Jacobian can be found in [29].

Using the formulation just described, the values of K_{Li} , ψ_i , and $GFRAC_i$ can be uniquely specified for every bus in the system (with the exception of the swing bus, since no equations are written for it). As a result, almost any load change scenario can be simulated by the appropriate selection of these control parameters. However, while

choosing the $GFRAC_i$'s and ψ_i 's is fairly straightforward, the way in which the K_{Li} 's are chosen and scaled deserves further explanation. In choosing K_{Li} 's for a simulation of load increase, one can choose $K_{Li} = 1$ to designate some standard load multiplier and then choose the others relative to this standard.

Once the load change multipliers are chosen to have the proper size with respect to one another, it may be necessary to scale them in order that the change in λ during continuation has an acceptable order of magnitude. The general object here is to assure that the change in λ has about the same order of magnitude as the change in the state variables. This keeps the tangent vector components on a comparable scale so that the correct continuation parameter can be chosen.

4.2 Nonlinear load model:

In order to obtain a more accurate model of the load in a power system, it is often desired to model the response of load to a change in voltage magnitudes. However, developing such a model is far from trivial since the characteristics of a wide variety of motors, appliances, etc. must be considered and an estimate must be made of the composition of load at each bus. Fortunately, very sophisticated load modelling software, called LOADSYN, has been recently developed by the General Electric company under contract with the Electric Power Research Institute [30]. This software makes it possible to obtain nonlinear load models by inputting the geographic area that the load is in and the percent of each load class that makes up the load at each bus. This program uses the user input information and a built in data base to calculate the coefficients for the steady-state load model. Although the full model includes terms that account for frequency deviation, only the voltage dependant terms are used here. This results in the model

$$P = P_0 P_{a1} \left(\frac{V}{V_0}\right)^{KPV1} + P_0(1 - P_{a1}) \left(\frac{V}{V_0}\right)^{KPV2}$$

$$Q = P_0 Q_{a1} \left(\frac{V}{V_0}\right)^{KQV1} + (Q_0 - P_0 Q_{a1}) \left(\frac{V}{V_0}\right)^{KQV2}$$

where the following definitions are made:

P_0 : initial active power consumed by the load (from base case)

V_0 : initial voltage at the bus (from base case)

P_{a1} : frequency dependent fraction of active power load

$KPV1$: voltage exponent for frequency-dependent active power

$KPV2$: voltage exponent for non-frequency-dependent active power load

Q_0 : initial reactive power consumed by the load (from base case)

Q_{a1} : ratio of uncompensated reactive power load to active power load

$KQV1$: voltage exponent for the uncompensated reactive power load

$KQV2$: voltage exponent for the reactive power compensation

As these definitions show, the reactive portion of this model is broken into both load and compensation terms. In order to use this load model in the continuation power flow, the load parameter and load increase multiplier must be somehow added so that various load changes can be simulated. This can be done as follows:

$$P_{Li} = (1 + K_{Li}\lambda)P_{Lio} \left[P_{a1} \left(\frac{V_i}{V_{oi}}\right)^{KPV1} + (1 - P_{a1}) \left(\frac{V_i}{V_{oi}}\right)^{KPV2} \right]$$

$$Q_{Li} = (1 + K_{Li}\lambda)[P_{Lio}Q_{a1}(\frac{V_i}{V_{oi}})^{K_{QV1}} + (Q_{Lio} - P_{Lio}Q_{a1})(\frac{V_i}{V_{oi}})^{K_{QV2}}]$$

where the Li subscript is used to denote load bus i. Now instead of λ corresponding directly to actual load in terms of MW and MVAR as it did in the constant power formulation, λ in this formulation corresponds to the quantity of connected motors, appliances, etc., that has a given characteristic. As a consequence, in this model it is probably more accurate to refer to λ as a connection parameter rather than a load parameter.

Now the problem of increasing generation must be dealt with. If the active power generation is to match the active power load, an expression for the total load change is needed. Once obtained, this expression can be used to divide generation increases among the generators available. The load change can be expressed as

$$\Delta P_{TOTAL} = \sum_{i=1}^n P_{Li} - P_{TOTALo}$$

where P_{TOTAL} is the total active power load at any given instant and P_{TOTALo} is the total active power load in the base case. If each generator is made to take up a fraction $GFRAC_i$ of the load change, generation at a given bus i is given by

$$P_{Gi} = P_{Gio} + (GFRAC_i)(\Delta P_{TOTAL})$$

5 Application to a test case

In order to demonstrate the continuation power flow with different load models, a scenario from the 30 bus New England test system was simulated. A base case load that accompanies the system and the system data was taken from [31].

5.1 Constant power load model:

The scenario to be simulated using this model is slightly modified version of one found in [32]. Here, the load increase is such that at any point in the continuation, the active and reactive power at any bus is a given multiple (say "l") times the active and reactive power that was specified in the base case. Similarly, the active power generation will be increased the same way by choosing $GFRAC_i$ of each bus i to be equal to the fraction of the total generation that bus i provided in the base case. That is

$$GFRAC_i = \frac{P_{Gio}}{\sum_{i=1}^n P_{Gio}}$$

Reactive generation will vary within the specified limits so as to regulate the voltage at a given generator bus. In terms of the control parameters used in the model, this load increase is set up by letting

$$K_{Li} = \frac{S_{Lio}}{ADJ}$$

where ADJ is some scaling factor common to each load multiplier and where $S_{Lio} = (P_{Lio}^2 + Q_{Lio}^2)^{\frac{1}{2}}$ is the apparent power at bus i in the base case. In addition, the power factor angle of the load change at each bus is chosen to be equal to the original power factor angle at the bus.

$$\psi_i = \arccos \frac{P_{Lio}}{S_{Lio}}$$

Then, from equation (2), the active load at a given bus will be

$$P_{Li} = [1 + \lambda(\frac{1}{ADJ})S_{\Delta BASE}]P_{Lio} = lP_{Lio}$$

and from equation (3), the reactive load is

$$Q_{Li} = [1 + \lambda(\frac{1}{ADJ})S_{\Delta BASE}]Q_{Lio} = lQ_{Lio}$$

The test case just described was run using both a nominal system configuration and a contingent configuration that involved the removal of the branch joining buses 9 and 30. The results are given in the form of P-V curves for two buses that lie in opposite corners of the system. The two P-V curves for bus 7 are shown in Figure (3) and the two for bus 24 are shown in Figure (4)

The most significant aspect of these results is the successful completion of the solution path around the peak loading condition. This is a feat that could not be completed with a conventional Newton-Raphson power flow. In fact, during continuation, the non-augmented Jacobian was verified to pass through a singular point at the peak power condition. In addition to this basic result, the P-V curves for both buses show the additional power transfer limitation imposed on the system by the removal of a branch. This is ofcourse an expected result. It should also be noticed that the P-V curve for bus 24 tends to drop off rather sharply just before peak power transfer occurred. This sharp drop off actually occurs after the generators in the area near bus 24 reach their maximum reactive power output limits.

5.2 Nonlinear load model:

For the purpose of comparison, the performance of the nonlinear load model will be demonstrated on a case that is basically identical to that used for the constant power load model. That is, the amount of load connected at each bus will be increased at the same rate. The prime difference between this demonstration and that for the constant power load model is that doubling the amount of load connected will not necessarily double the load in terms of MWs and MVARs. This is ofcourse due to the fact that voltage is now allowed to impact the load. The generator multipliers used in this case are same as the one used in constant power load model. But, the load increase multipliers are chosen differently than for the constant power load model. These parameters are set by ignoring the voltage dependence of the load as shown below. Let $\frac{V_i}{V_{oi}} = 1$, then

$$P_{Li} = (1 + K_{Li}\lambda)P_{Lio}$$

similarly

$$Q_{Li} = lQ_{Lio}$$

Thus, by setting the K_{Li} multiplier of each bus equal, the active and reactive load connected increase at the same rate. The above described nonlinear load model was used on the same two system configurations used for the constant power model. The constants for the load model were obtained from the LOADSYN program. Each bus was assigned constants corresponding to load in the Northwest United States with a composition of 35% heavy industrial, 35% commercial, and 30% residential load. The resulting set of constants are shown in Table 1.

Again P-V curves were plotted for buses 7 and 24 and the same basic results were obtained (Figure.5 and Figure.6)

However, two distinct results were noticed. First, for both system configurations the maximum power transfer increased by about 3.0 per unit over that obtained with the constant power load. Thus, the constant power load model is shown to be the more conservative. The second distinct result is that the non-augmented Jacobian, which is simply the power flow Jacobian, did not become singular until

after the peak power transfer had been encountered. For the normal configuration the active load peaked at about 110.6 per unit, but the non-augmented Jacobian does not pass through a singular point until the load had decreased to between about 96.0 to 98.0 per unit. Similarly, for the contingent configuration, the active power load peaked at about 99.4 per unit, but the singular point does not occur until the load has decreased to between 79.5 to 81.7 per unit.

The fact that the non-augmented Jacobian passes through a singularity after peak power transfer is significant in that the singular point has been shown to divide the stable and unstable parts of the solution path when a constant power load model is assumed.

Further comparison between load models

In addition to the difference in results discussed above, it is interesting to compare the load variation with respect to λ for the two different load models. Figure (7) show this variation for the constant power load model under normal condition and Figure (8) show the same thing for the nonlinear load model.

As expected, load changes linearly with a change in λ for the constant power load model, but hysteresis is apparent in the load change for the nonlinear model. Namely, for this particular set of load model parameters, power is shown to peak before a peak in the load connection parameter λ occurs. Physically, this means that after the peak power transfer is encountered, more load is being connected but power consumption actually decreases due to the dominating effect of the voltage dependence of the load.

6 Conclusions:

Bifurcation theory has become a major focus in the analysis of nonlinear dynamical systems. Central to this topic is the question of whether the qualitative properties of a dynamical system change with the variation of certain of its parameters. Generally, if certain of the parameter of any physical or natural system continue to vary, then a critical stage may be reached in which the system makes a sudden jump from one state to another. Continuation methods are valuable tools to study such types of bifurcation phenomena. Voltage collapse can be related to saddle-node type of bifurcation. To analyse the steady state voltage instability, a continuation power flow was developed [25] and is extended to include nonlinear load models. Continuation power flow has been tested on the systems as large as 2300 buses and worked well. Future developments include a strategy for sharing increase in load among generators and using interchange as a parameter.

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Appendix

Transcritical Bifurcation: (Fig. 9)
Prototype equation is

$$\dot{x} = \lambda x - x^2$$

Transversality conditions are

$$\frac{\partial^2 f}{\partial \lambda \partial x} \neq 0 \quad ; \quad \frac{\partial^2 f}{\partial x^2} \neq 0$$

Pitchfork Bifurcation: (Fig. 10)
Prototype equation is

$$\dot{x} = \lambda x - x^3$$

Transversality conditions are

$$\frac{\partial^2 f}{\partial \lambda \partial x} \neq 0 \quad ; \quad \frac{\partial^3 f}{\partial x^3} \neq 0$$

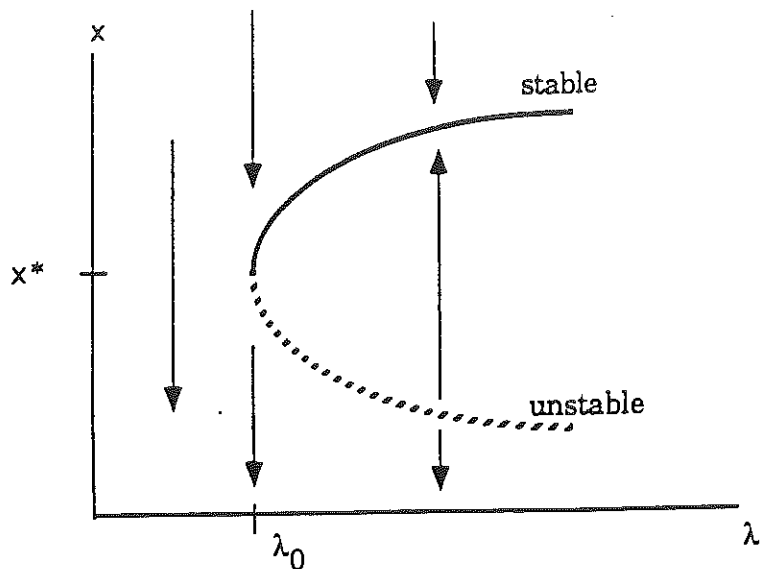


Fig. 1. Fold bifurcation

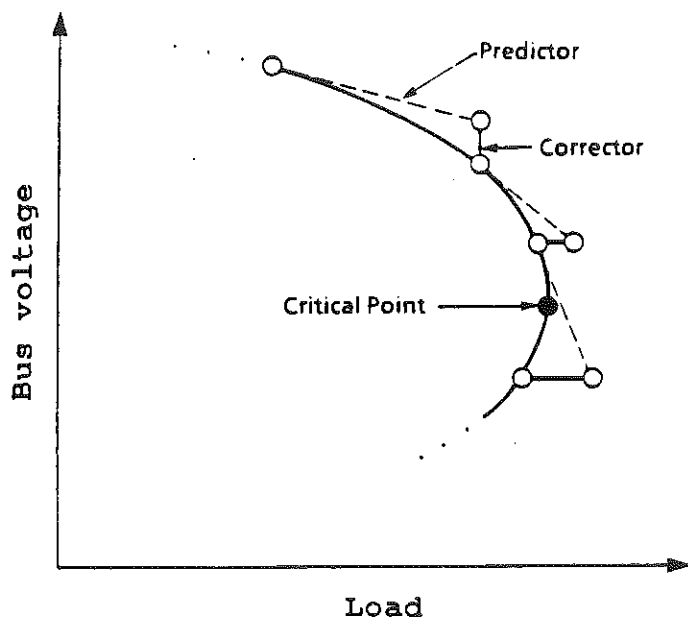


Fig. 1. An illustration of the predictor-corrector scheme used in the continuation power flow

Table 1. Constants used in nonlinear load model

CONSTANT	VALUE
P_{a1}	0.70
$KPV1$	0.38
$KPV2$	1.78
Q_{a1}	0.44
$KQV1$	1.64
$KQV2$	2.0

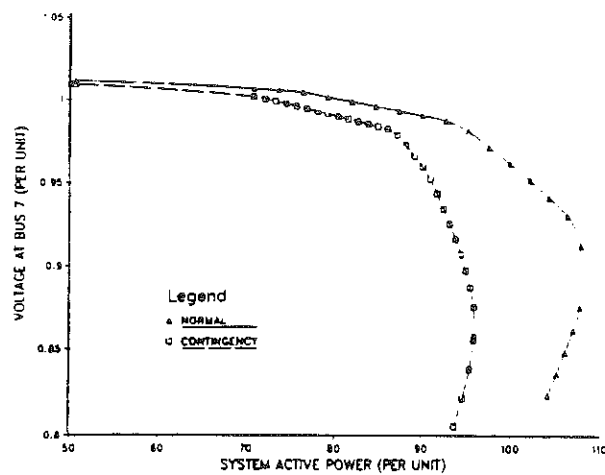


Fig. 3. P-V curves (bus 7) with constant power load model

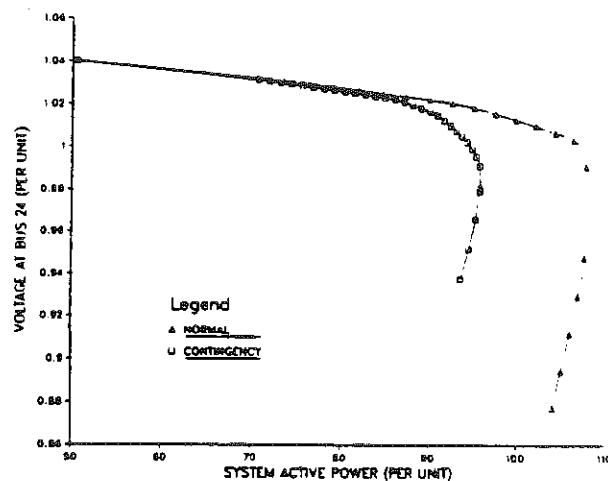


Fig. 4. P-V curves (bus 24) with constant power load model

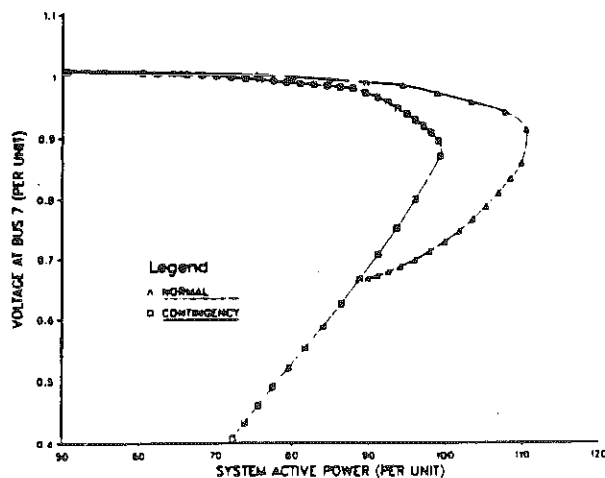


Fig. 5. P-V curves (bus 7) with nonlinear load model

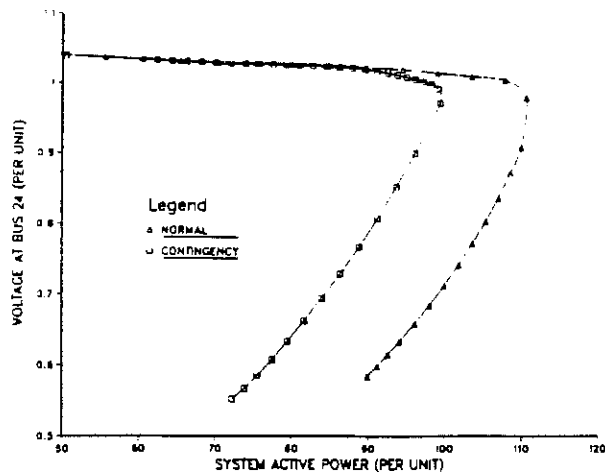


Fig. 6. P-V curves (bus 24) with nonlinear load model

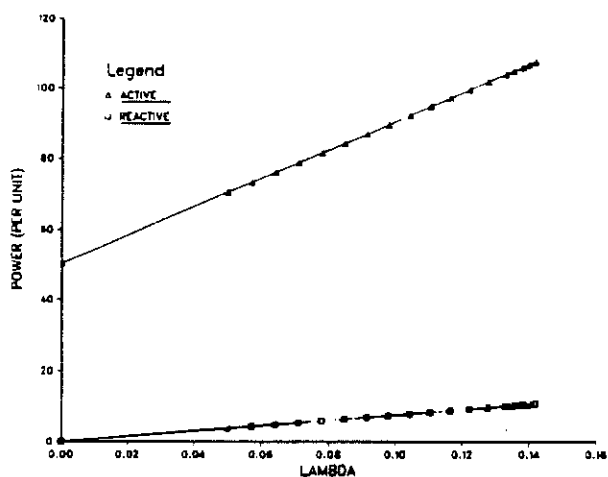


Fig. 7. Lambda Vs. P & Q load with normal system configuration for constant power load model

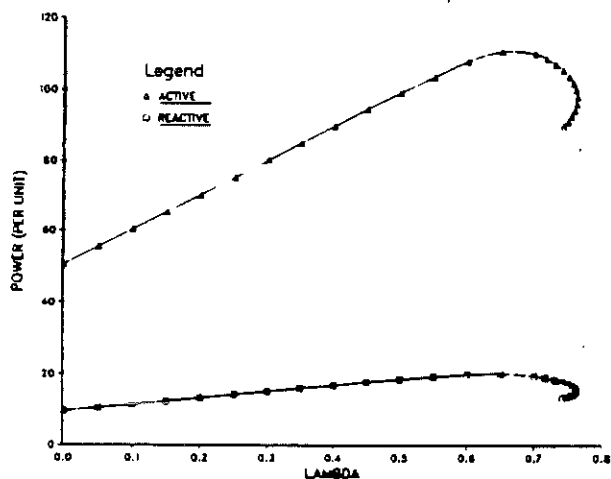


Fig. 8. Lambda Vs. P & Q load with normal system configuration for nonlinear load model

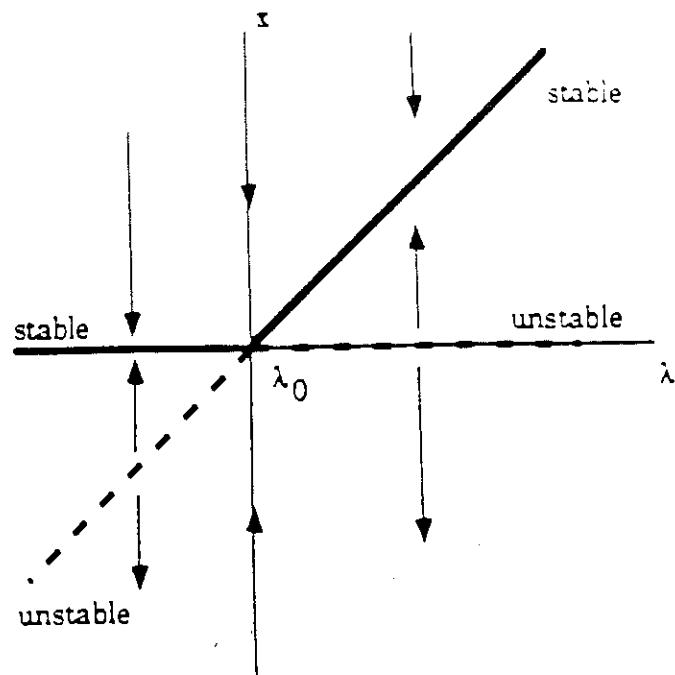


Fig. 9. Transcritical Bifurcation

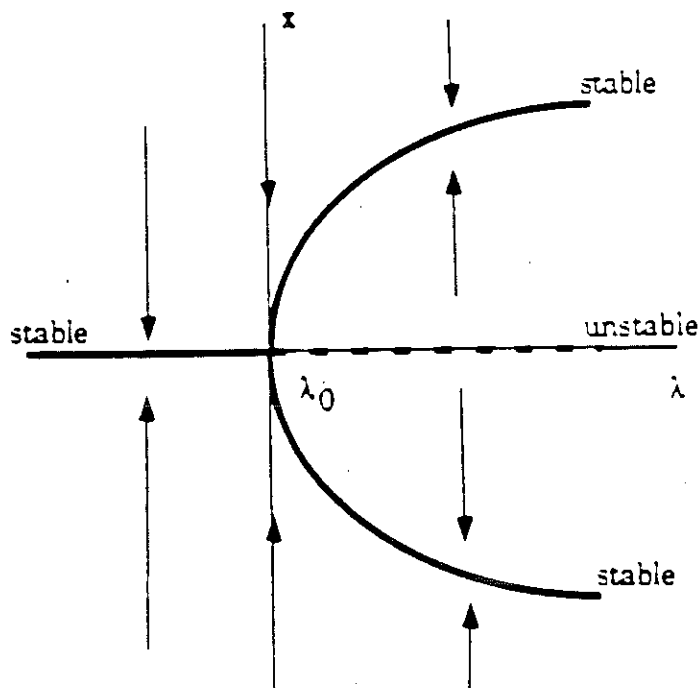


Fig. 10. Pitchfork Bifurcation

A NEWTON-RAPHSON METHOD FOR STEADY-STATE VOLTAGE STABILITY ASSESSMENT

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ABSTRACT

This paper deals with the definition and the development of a numerical algorithm for a fast steady-state voltage collapse analysis, based on the maximization of the total load.

A particular representation of voltage dependent loads is used to set up the first-order conditions in a very convenient form. The equations are then solved with a Newton-Raphson approach, using a modified Jacobian matrix which appears to be well-conditioned under the various loading conditions, even in proximity of the voltage stability critical conditions.

The proposed algorithm is so structured as to become practically a variant of the conventional Newton-Raphson load-flow procedure, with a few modifications. It is possible to retain the convenient features that characterize the load-flow programs, from the use of optimal ordering to the possibility of implementing dispatching strategies and verifying the inequality constraints.

With respect to the conventional load-flow, the computational overhead is really reduced and is of the order of two or three times the normal load-flow burden.

Tests have been performed on several systems. Some results on the standard IEEE 57-bus test system are presented.

1. INTRODUCTION

The voltage stability problem has become one of the most significant items in the power system area in the last few years. The increasing electric power demand makes the power networks work close to their limit conditions: high line currents, low voltage profile, relatively high angle differences indicate a system operating under heavy loading conditions. In these situations some harmful effects, like loss of system stability, islanding or voltage collapse, may occur. Voltage collapse starts with a slow voltage decay, which progressively accelerates and in some unfortunate circumstances culminates in the complete system blackout.

The voltage stability problem involves both static and dynamic aspects. The dynamic formulation of the problem requires the definition of appropriate models for the system components and the construction of a comprehensive mathematical model for the aggregated system. The resulting models are complex and become very cumbersome for large networks, making it difficult to obtain an effective practical application.

From the operator point of view, what is really wanted is a reliable and easily computable global index, which allows a relative comparison of the operating points with respect to the voltage collapse phenomenon. An index of this type can be obtained via the static approach.

Several static indexes have been proposed and computed by using various techniques. Basically, these indexes can be divided into two main categories, according as to whether the index is computed using some information embedded in the operating point (*a priori* indexes), or whether it is computed directly by evaluating the critical conditions (*a posteriori* indexes).

To the former category belong indicators derived from the analysis of the load-flow Jacobian determinant [2] or minimum singular value [9], multiple load-flow solutions [14,15], feasibility of load-flow solutions [7], sensitivities, etc..

As the critical conditions are not exactly computed, some *a priori* indexes are by definition less accurate than the *a posteriori* counterparts. This lack of accuracy may result in an overly optimistic or conservative index, which may undermine the user's confidence in them.

Recently there has been a move towards the direct computation of the critical point, which means that the distance between the operating point and the critical point can be better evaluated.

Some authors formulate the steady-state voltage stability problem as a corresponding maximization of the suppliable load. The dispatching strategy and the inequality constraints do affect the critical condition. Their treatment at present is inevitably heuristic.

The solution is approached via non-linear programming [6], or applying the gradient method [12], or by means of the Lagrangian multiplier method [8,17].

Within this maximization framework, this paper presents a new procedure for a very fast computation of the critical point. The procedure is based on a particular transformation of the first-order conditions.

2. PROBLEM DESCRIPTION

Given an operating point, a load increase law and a dispatching strategy, it is well known that a normal load-flow approach cannot be used to explore the entire path to the voltage collapse phenomenon.

In fact, as the load-flow program is executed with increasing load patterns and the critical point is approached, the load-flow problem becomes more and more ill-conditioned and the Jacobian matrix approaches singularity. If we want to find directly the critical point, it is then necessary to circumvent the Jacobian singularity. Some keen methods have been proposed in the literature, with different degrees of complexity. This paper presents a contribution which appears particularly promising in reducing the complexity of the direct computation of the critical point.

We assume that, given a load increase path and a dispatching strategy, the critical point corresponds to the maximum allowable load increase (i.e. to the maximum load for which the static equations can still be satisfied, but from which originates the voltage collapse).

Let us consider a $(N+1)$ -bus system, in which the first N_Q buses are of the PQ type, buses from N_Q+1 to N are of the PV type, and the last is the slack bus.

Let $M = N + N_Q$. We define as usual the vectors:

(state vector) $x = [\delta_1 \dots \delta_N, V_1 \dots V_N]^T$

(control vector) $u = [P_{G1} \dots P_{GN}, Q_{G1} \dots Q_{GN}, V_{Nq+1} \dots V_N]^T$

(load vector) $p = [P_{d1} \dots P_{dN}, Q_{d1} \dots Q_{dN}]^T$

Given an initial point (x_0, u_0, p_0) , a load increase path:

$$p = p_0 + z p_r \quad (1)$$

and a dispatching strategy $u = u(p, x)$, we shall indicate as (x_c, u_c, p_c) the corresponding critical point, i.e. the point in which the voltage collapse occurs.

The scalar z quantifies the load evolution along the given path p_r , z_c being the critical value.

We will make the assumption of a 'coherent' load increase, in which p_r is obtained from p_0 by simply zeroing those components corresponding to fixed nodal loads.

Several coherent load increases can be considered:

active only: only the active loads are allowed to change in selected buses. Then p_r is obtained from p_0 by setting to zero all the reactive components and the fixed active components;

reactive only: only the reactive loads are allowed to change in selected buses. Then p_r is obtained from p_0 by setting to zero all the active components and the fixed reactive components;

global: changes are allowed in selected buses, but at a constant power factor. Correspondingly, p_r is obtained from p_0 by setting to zero all the fixed active and reactive components.

Any load increase calls for a dispatching strategy, which is reflected on the control vector u .

We transform the search for the critical point into the constrained maximization problem:

$$\max z \quad (2)$$

subjected to the load-flow equality constraints:

$$f(x, u) + p_0 + z p_r = 0 \quad (3)$$

The correspondent Lagrangian function is:

$$L(x, z, \lambda) = z + \lambda^T (f(x, u) + p_0 + z p_r) \quad (4)$$

the first-order conditions (5) are easily derived:

$$\partial L / \partial x = 0 = J^T \lambda \quad (5a)$$

$$\partial L / \partial z = 0 = 1 + p_r^T \lambda \quad (5b)$$

$$\partial L / \partial \lambda = 0 = f(x, u) + p_0 + z p_r \quad (5c)$$

In (5a), $J = \partial f(x, u) / \partial x$ is the usual $(M \times M)$ Jacobian matrix of the load-flow equations. It can be noted (from (5b)) that λ cannot be a null vector. Hence (from (5a)) it is confirmed that the Jacobian matrix must be singular in the critical conditions.

In what follows we shall endeavour to shape these first-order conditions into a form suitable for a convenient numerical treatment.

The use of these first-order conditions for finding the critical point is not new. A formulation can be found in [8]. The same approach is basically used in [17] for the reactive case.

The direct method proposed in [10] can be related to the same approach, as (5b) and (5a) are practically substituted with the equivalent ones:

$$v^T v = 1 \quad (\text{condition for a non-zero } v) \quad (6a)$$

$$J v = 0 \quad (\text{singularity of the Jacobian matrix}) \quad (6b)$$

The vector v , while differently introduced, plays in (6) the same role as the Lagrange multiplier λ in (5). As a matter of fact, λ and v are respectively the left and right eigenvectors associated to the zero eigenvalue of the critical Jacobian matrix.

With a Newton-Raphson algorithm, the iterative process to solve (5) takes the form:

$$x^{k+1} = x^k + \Delta x^k \quad (7a)$$

$$\lambda^{k+1} = \lambda^k + \Delta \lambda^k \quad (7b)$$

$$z^{k+1} = z^k + \Delta z^k \quad (7c)$$

where:

$$\begin{bmatrix} H & J^T & 0 \\ J & 0 & p_r \\ 0 & p_r^T & 0 \end{bmatrix}^k \begin{bmatrix} \Delta x \\ \Delta \lambda \\ \Delta z \end{bmatrix}^k = - \begin{bmatrix} J^T \lambda \\ f(x, u) + p_0 + z p_r \\ 1 + p_r^T \lambda \end{bmatrix}^k \quad (8)$$

Each element of the submatrix H contains a linear combination between the load-flow Hessian terms and the Lagrange multipliers. The generic term is in fact:

$$H_{ij} = \sum_{k=1}^M \lambda_k \frac{\partial^2 f_k}{\partial x_i \partial x_j} \quad (9)$$

This approach requires at each iteration the calculation of the load-flow Hessian components and the solution of a $(2M+1)$ linear system.

However, with some transformations, it is possible to avoid any Hessian calculation and still to obtain a well-conditioned set of equations. This approach is described in the next sections.

3. THE PROPOSED METHOD

In a heuristic search for a fast and reliable numeric method, we sought a compromise between the expediency of modelling the load power/voltage dependency and the need to respect the relative comparison between the various loads.

Let us assume that the load increase takes the form:

$$p = p_0 + y h(V_{ref}) p_r \quad (10)$$

The load factor z in (3) is now obtained by multiplying a scale factor y with the corrective function $h(V_{ref})$. To preserve the physical coherence of the load variations, the corrective function has been given the form:

$$h(V_{ref}) = (V_{ref}/V_{ref,0})^\alpha \quad (11)$$

which means that each nodal power is made dependent not on the corresponding nodal voltage, but on the voltage magnitude V_{ref} in a selected reference bus. This reference bus can be chosen in various ways. The simplest choice is to use as reference bus the one with the minimum voltage. The positive exponent α modulates the power/voltage dependence and for constant admittance loads is $\alpha = 2$.

The maximization problem assumes the aspect:

$$\max z = y h(V_{ref}) \quad (12)$$

$$\text{s.t. } f(x, u) + p_0 + y h(V_{ref}) p_r = 0 \quad (13)$$

The corresponding Lagrangian function (14) and first-order conditions (15) are then:

$$L(x, z, \lambda) = y h(V_{ref}) + \lambda^T (f(x, u) + p_0 + y h(V_{ref}) p_r) \quad (14)$$

$$\partial L / \partial \lambda = 0 = f(x, u) + p_0 + y h(V_{ref}) p_r \quad (15a)$$

$$\partial L / \partial y = 0 = 1 + p_r^T \lambda \quad (15b)$$

$$\partial L / \partial x = 0 = J^T \lambda + y \partial h(V_{ref}) / \partial x (1 + p_r^T \lambda) \quad (15c)$$

Using the notation $h_x = \partial h(V_{ref}) / \partial x$, equation (15c) becomes:

$$(J^T + y h_x p_r^T) \lambda + y h_x = 0 \quad (16)$$

We may now define a very useful 'modified' Jacobian matrix:

$$J_{\text{mod}} = J + y p_r h_x^T \quad (17)$$

and rewrite (15c) as

$$J_{\text{mod}}^T \lambda + y h_x = 0 \quad (18)$$

It is very interesting to note that the modified Jacobian matrix is obtained from the original one adjusting the terms in one column only (corresponding to the derivatives $(\partial P/\partial V_{\text{ref}}, \partial Q/\partial V_{\text{ref}})$ of the powers w.r.t. the voltage magnitude of the selected reference bus) and only for the rows corresponding to the buses where the load is allowed to change.

From (18) we find formally the Lagrange multiplier vector:

$$\lambda = -(J_{\text{mod}}^{-1})^T y h_x \quad (19)$$

Substituting in (15b) and transposing:

$$1 - y h_x^T (J_{\text{mod}}^{-1}) p_r = 0 \quad (20)$$

Let us now assume:

$$\mu = J_{\text{mod}}^{-1} p_r \quad (21)$$

The equation (20) becomes:

$$1 - y h_x^T \mu = 0 \quad (22)$$

After all these changes, we obtain finally the new form for the original set of equations:

$$f(x, u) + p_o + y h(V_{\text{ref}}) p_r = 0 \quad (23a)$$

$$J_{\text{mod}} \mu - p_r = 0 \quad (23b)$$

$$1 - y h_x^T \mu = 0 \quad (23c)$$

The old set of variables (x, λ, z) as here been modified to (x, μ, y) and the solution is sought in the transformed space, using the typical Newton-Raphson approach for the load-flow equations (23a).

We may use different schemes in performing the computations. After extensive testing, the following organization has proven to be the most efficient:

$$J_{\text{mod}}^k \Delta x^{k+1} = -(f(x^k, u^k) + p_o + y^k h(V_{\text{ref}}^k) p_r) \quad (24a)$$

$$J_{\text{mod}}^k \mu^{k+1} = p_r \quad (24b)$$

$$y^{k+1} = 1/(h_x^T \mu^{k+1}) \quad (24c)$$

Starting from a base point (x_o, u_o, p_o) and $y = 0$, eq. (24a) is used to update the current value of x , while the use of eqs. (24b) and (24c) give directly the updated values of μ and y .

We may stress some very distinguishing features of the above equations.

The two linear systems (24a) and (24b) share the same matrix of coefficients J_{mod} , which means that at each iteration the two systems can be solved simultaneously, as they differ only for the right-hand side terms.

The third equation is trivial, as it gives directly y , using the updated values of h_x .

As already noted, J_{mod} differs only slightly from the original J : the differences are confined to one column (corresponding to the reference bus) and only for the rows corresponding to the loads where changes are allowed ($P_{ri} \neq 0, Q_{ri} \neq 0$):

$$(\partial P_i/\partial V_{\text{ref}})_{\text{mod}} = (\partial P_i/\partial V_{\text{ref}}) + y \alpha V_{\text{ref}}^{\alpha-1} P_{ri}/V_{\text{ref},o}^{\alpha} \quad (25a)$$

$$(\partial Q_i/\partial V_{\text{ref}})_{\text{mod}} = (\partial Q_i/\partial V_{\text{ref}}) + y \alpha V_{\text{ref}}^{\alpha-1} Q_{ri}/V_{\text{ref},o}^{\alpha} \quad (25b)$$

All the other terms remain unchanged.

There is, however, a key remark to be made: in the critical point, J is singular, while J_{mod} is not. This can be seen clearly from eqs. (23b) and (23c), as μ and p_r are non-zero vectors. J_{mod} is well-

conditioned even in the critical point, and the linear systems (24a) and (24b) may be solved with the usual optimal ordering schemes and sparsity techniques. Pivoting is not required. In case of load variations in more than a few buses, it is convenient to keep the reference bus in the last position.

4. COMPUTATIONAL CONCERNS

We can now summarize the main aspects of the procedure.

The iterative process is described from eqs. (24). The real complexity of these equations is not very much different from the usual load-flow equations.

Practically the program to find the critical point is a slight modification of the normal load flow procedure. The basic steps are:

- Select the type of load evolution (only active, only reactive or global). Note that by proper selection of p_r , we can restrict the load variations to any combinations of system buses.
- Select the type of control strategy. The simplest control strategy is to keep constant the control vector u , which means that the slack bus supplies all the load changes. More realistically, the total load can be redistributed among the generation buses, either proportionally to the base point, or applying some other law.
- Select the type of corrective function $h(V_{\text{ref}})$. This selection is really double. First, we have to select the reference bus. The criterion used was based on the minimum voltage, but other criteria can be tested. The second selection concerns the form of the corrective function (i.e. the exponent α , for which a default value of 2 was used). As a result (11):

$$h(V_{\text{ref}}) = (V_{\text{ref}}/V_{\text{ref},o})^{\alpha}$$

The corrective function may be considered as an extension of the constant admittance load:

$$P_i = P_{io} (V_i/V_{io})^{\alpha} \quad (26a)$$

$$Q_i = Q_{io} (V_i/V_{io})^{\alpha} \quad (26b)$$

Every load depends on the same reference voltage, in order to obtain the same load increase path as given in (1).

- Apply the updating formulas (24), until the convergence criteria are satisfied. The convergence criteria are based on the power mismatches for the load-flow equations and on the total load.

Part d) can be heuristically modified, to accommodate for the dispatching strategy and for the treatment of the inequality constraints. At each iteration, we may in fact modify the control vector u and verify the inequality constraints on x , just applying the same corrective strategy used in the normal load flow program (i.e. switching violating PV buses to PQ and viceversa).

The main very practical result is that the conventional load-flow program can be easily modified to find the critical point. Computationally, what we have to do is just to modify a few terms of the usual Jacobian matrix and apply the updating formulas (24).

The convergence speed is very fast. It emerges that the entire updating process numerically is somewhat equivalent to running two-three load flows. Moreover, the modifications on the available load-flow program are very low.

5. THE VOLTAGE STABILITY INDEX

Of course the objective of any static method is to find a readily assessable index that in some way quantifies the position of the operating point in relation to the critical conditions.

The critical point is determined by maximizing the total coherent load in the buses where changes are assumed to take place. It is then quite obvious to characterize the closeness of any operating condition to the limit conditions by considering the ratio between the critical total power and the base total power, limiting the computation of the total to the buses where the loads are allowed to change.

With a small modification, we will use as the voltage stability index VSI:

$$VSI = z_c = y_c h(V_{\text{ref},c}) \quad (27)$$

A VSI = 0.65 will mean that the changing loads may rise up to 65% before the static voltage collapse occurs.

Instead of using a deterministic framework, we may also define a probabilistic voltage stability risk index VSRI. As in [8], we may interpret the load vector p as a Gaussian distributed probabilistic vector, with mean p_0 and variance Σ_p .

Working in the reduced subspace of the changing loads, we can define:

$$VSRI = \text{Prob}(p_i > p_{ic}) \quad (28)$$

i.e. as the probability that the nodal load reaches the critical value. As all the changing loads vary proportionally (correlation factor = 1), we may use any non-zero component of p_i to perform the calculation.

Once the critical values are found, computation of these indexes is a trivial matter.

6. NUMERICAL RESULTS

The setting up of procedures of this kind necessarily entails an approach which has much of a heuristic flavor about it and certainly more thorough mathematical investigation is called for.

To validate the approach, a quite intensive set of tests have been carried out on several systems (including the CIGRE 10-bus, IEEE 30-bus and IEEE 57-bus standard test systems).

All the tests have given consistent results and the algorithm has shown very good properties of speed and reliability.

In what follows, we will pinpoint some of these results, using as example the IEEE 57-bus network, assuming a base case as described in [1].

EXAMPLE 1:

All the loads in the PQ buses have been increased, and the following combination of cases has been considered:

(inequality constraints)

- a) no inequality constraints
- b) with constraints on reactive generations in the PV buses (Q_{Emin} , Q_{Emax})

(dispatching strategies)

- 1) additional generation only from the slack bus
- 2) additional generation redispatched proportionally to the initial generations among the generation buses

For each of the above combinations (a1, a2, b1, b2), the critical point was found using:

- a global maximization (coherent load increase in all the buses, with constant power factor (PF))
- an active maximization only (increasing all P in the PQ buses)
- a reactive maximization only (increasing all Q in the PQ buses)

The reference bus selection is determined by the condition of the minimal voltage. While the reference bus is identified at each iteration, it can be said that changes do occur rarely during the iterative process. The exponent α is set to 2.

Table 1 reports the results for the VSI. The same results were also obtained using other previously published algorithms [8],[10].

It can be noted that switching from strategy 1 to 2 the VSI increases. The negative effect of the constraints is also evident.

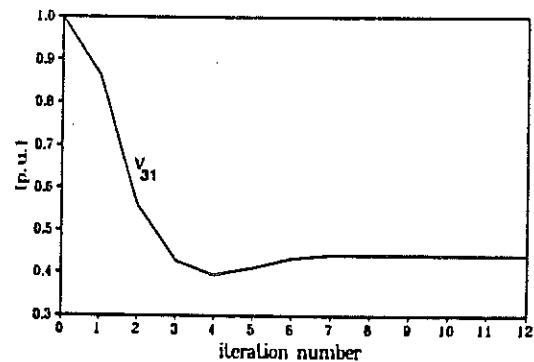
Some details on the iterative process are reported in Fig.1, for the case (a1) using the global load maximization criterion.

The interesting point is that VSI converges after seven or eight iterations. More iterations are required if one wishes to obtain an accurate evaluation of the voltages, but this aspect seems to be of less interest than the load power limit evaluation.

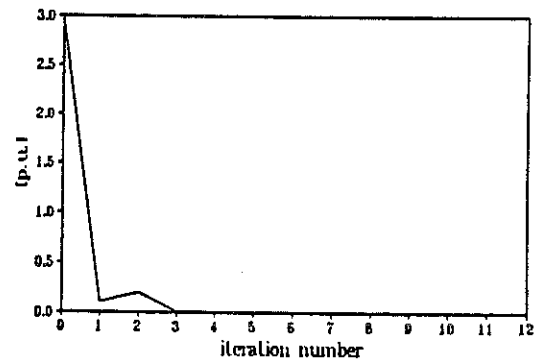
load maximization criterion	reactive constraints off (a)		reactive constraints on (b)	
	slack only (a1)	PV-buses + slack (a2)	slack only (b1)	PV-buses + slack (b2)
global	0.6592	0.6670	0.5866	0.6323
active only	1.0527	1.0779	0.9069	1.0334
reactive only	1.4768	-	1.3780	-

Table 1
Voltage stability indexes (VSI) for load variations at all the PQ buses

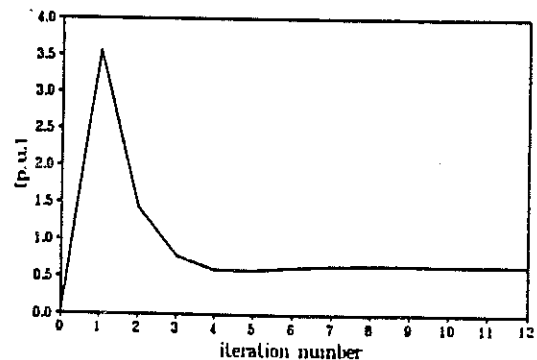
Recall that one iteration of the procedure is computationally similar to a single iteration of a conventional Newton-Raphson load-flow. Therefore, as already noted, the computational burden of the proposed method can be indicated as comparable with the execution of a low number (two or three) of load-flows.



a) V_{ref} : voltage magnitude in the reference bus



b) Maximum power mismatch



c) VSI: Voltage Stability Index

Fig. 1 - Case (a1) with global load maximization criterion

In Fig.2 the effect on VSI of different dispatching strategies and of the reactive constraints is shown.

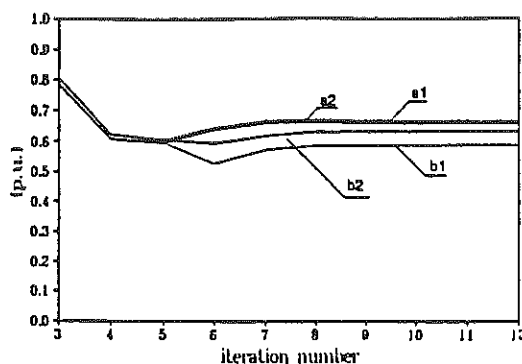


Fig. 2 - Effect on VSI of various cases

EXAMPLE 2:

Here we allow load variations at one bus only (bus 31, which has proven to be the reference bus in all the cases considered in Example 1). Of course, the reference bus remains coincident with bus 31 itself.

In this case, the proposed method with $\alpha = 2$ is equivalent to consider a constant admittance load in the bus subjected to the load variation. The results are shown in Table 2.

load maximization criterion	reactive constraints off (a)		reactive constraints on (b)	
	active power redispatched by		active power redispatched by	
	slack only (a1)	PV-buses + slack (a2)	slack only (b1)	PV-buses + slack (b2)
global	2.1749	2.1755	2.1729	2.1735
active only	3.1675	3.1688	3.1646	3.1659
reactive only	4.9794	-	4.9765	-

Table 2
Voltage stability indexes for load variation at bus 31

CONCLUSIONS

A very simple and fast method for finding the voltage collapse critical point of a power system from a steady-state viewpoint has been presented.

The method is based on the maximization of the total load. Using a heuristic power/voltage model for the loads, the first order conditions are transformed into a very convenient set of equations, whose solution can be obtained with a program that is practically a minor modification of the conventional load-flow procedure.

A modified Jacobian matrix is introduced, which is obtained from the original load-flow Jacobian by changing just a few terms, but which exhibits the important property of being non-singular in the critical collapse point.

The numerical experimentations have shown that the convergence properties of the algorithm are very remarkable and that the critical point can be found with a computational burden that is two to three times the burden associated with the normal load-flow program. The method can take into account both the inequality constraints and the dispatching strategy to attune the generation to the increasing loads.

The fast detection of the voltage stability limits makes it easy to calculate an index used to represent the closeness of the operating point to the steady-state voltage collapse conditions.

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An Artificial Neural-Net Based Approach to Monitoring Power System Voltage Stability

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Abstract - This paper presents a new method for monitoring voltage stability margin in power systems. The proposed method is based on the Kohonen neural network that can cope with pattern classification of input variables. The artificial neural net (ANN) is used to estimate an index of the voltage stability obtained from a pair of multiple load flow solutions. The estimated index is determined by firing an output neuron which corresponds to the input vector and recalling the index that is assigned to the output neuron. The advantages of the ANN are both fast estimation of the voltage stability index through pattern recognition and the self-organization feature mapping of the index that allows us to understand the transition of the voltage stability margin on the two-dimensional output neuron grid. The proposed method has been successfully applied to a sample system.

Keywords: voltage stability, artificial neural net, competitive learning, self-organization feature mapping

I. INTRODUCTION

Voltage instability problems are one of current key issues in power systems. Traditionally, the relationship between active power and angle has been discussed in dynamic stability, transient stability, and long-term dynamics studies. Voltage instability is involved in another category that often deals with the limitation of reactive power supply. As power systems become heavy-loaded, voltage instability or voltage collapse is inclined to occur for the lack of reactive power sources. Since the problem is related to load characteristics, it is essential to take into them in the problem. So far, several

approaches are studied for the following problems:

- (i) qualitative analysis [2], [3],[5],[20]
- (ii) voltage instability Indices [3], [6],[8],[9],[12],[14]
- (iii) evaluation of bifurcation or critical points[15], [16],[21],[22]
- (iv) dynamic models [11],[13],[17],[18]

This paper deals with the voltage instability indices to measure margin among the above problems. Evaluation of the indices is important in understanding how close the power system to be studied is voltage instability. The indices serve as the operator's guideline for monitoring voltage instability. So far, several indices have developed. Galiana proposed an index that makes use of the eigenvalue analysis [3]. Carpentier developed an indicator that is obtained by the optimal power flow calculation. Tamaru and Mori proposed an index called VIPI that is based on the multiple load flow solutions [6], [7], [9]. Kessel and Glavitsch contributed to developing an indicator called L which utilizes the results of the load flow calculation [12]. Titanuchit and Thomas proposed a security index using the minimum singular value of Jacobian [14]. In this paper, the idea of VIPI is employed because the index is more understandable in a sense that power system conditions can be expressed in terms of the angle of a quantitative measure.

In recent years, artificial neural networks (ANNs) is one of the promising technologies in all the engineering fields. The technique is helpful for solving the so-called complex problems. So far, a lot of ANNs have been proposed. Typical ANNs used are as follows:

- 1) multi-layer perceptron
- 2) Hopfield network
- 3) Kohonen network

The multi-layer perceptron is effective in system

identification, estimation, prediction, and classification problems. The Hopfield network can solve combinatorial and associative memory problems. The Kohonen network serves as a classifier of input patterns. Application of ANNs to power systems have been studied in several areas since Sobjic and Pao [23]. Multi-layer perceptrons are applied to many areas: security assessment [23],[25],[35], alarm processing [24], harmonics identification and detection [26],[30], fault detection[27],[31],[39], voltage control [29], harmonics prediction [32],[36], load forecasting [34],etc. Hopfield type networks have been used in topological observability analysis [28],[33], etc. Recently, the Kohonen neural network is employed in security assessment [37],[40].

In this paper, the Kohonen neural network is proposed to estimate the voltage instability index so that the computational effort is reduced and the feature extraction of power system conditions is made on two-dimensional grid. The index requires a set of the load flow calculation to find the closest pair of the multiple load flow solutions. One of the drawback in the index is that evaluating the index is quite time-consuming in power systems. It is expected that the Kohonen network speeds up evaluation of the index through the pattern recognition or the pattern classification function. Unlike other ANNs, the Kohonen network has the informative function that input pattern is mapped on the two-dimensional output grid so that similar patterns are allocated to output neurons near to each other in a topological sense. This function is quite useful in monitoring power system condition from a standpoint of voltage instability. The effectiveness of the proposed approach is illustrated in a sample system.

II. REVIEW OF CONVENTIONAL METHODS FOR FINDING MULTIPLE LOAD FLOW SOLUTIONS

Before dealing with the voltage instability index, some methods for finding the closest multiple load flow solutions are reviewed. The closest pair of the multiple load flow solutions is required to calculate the index.

2.1 Exhaustive Search Method

This paragraph describes an exhaustive search method for finding multiple load flow solutions [4]. The method is based on combining initial conditions that are obtained from two kinds of load flow solutions in the quadratic power flow equation.

Now, let us consider a nonlinear load flow equation such as

$$y_s = f(x) \quad (1)$$

where

y_s : specified value vector

x : bus voltage vector

$f(\cdot)$: nonlinear load flow equation

The load flow equation may be written as a quadratic form in rectangular coordinates [1]. Thus, Eqn. (1) may be rewritten as

$$y_s = \frac{1}{2} x^T H x \quad (2)$$

where, H : cubic matrix

It should be noted that Eqn. (1) is the exact load flow equation. Suppose that load flow solution x_1 is obtained with the flat start. The elements of x_1 can be expressed as follows:

$$\begin{aligned} x_1 &= (e_1, f_1, e_2, f_2, \dots, e_k, f_k, \dots, e_n, f_n)^T \\ &= (E_1, E_2, \dots, E_k, \dots, E_n)^T \end{aligned} \quad (3)$$

where

e_k : real part of bus voltage at bus k obtained with the flat start

f_k : imaginary part of bus voltage at bus k obtained with the flat start

E_k : bus voltage vector at bus k so that

$$E_k := (e_k, f_k)^T \quad (4)$$

Noting that the load flow equation is quadratic, the other voltage vector corresponding to E_k can be evaluated by solving the load flow equation at bus k [4]. Defining the voltage as

$$E_k' := (e_k', f_k')^T \quad (5)$$

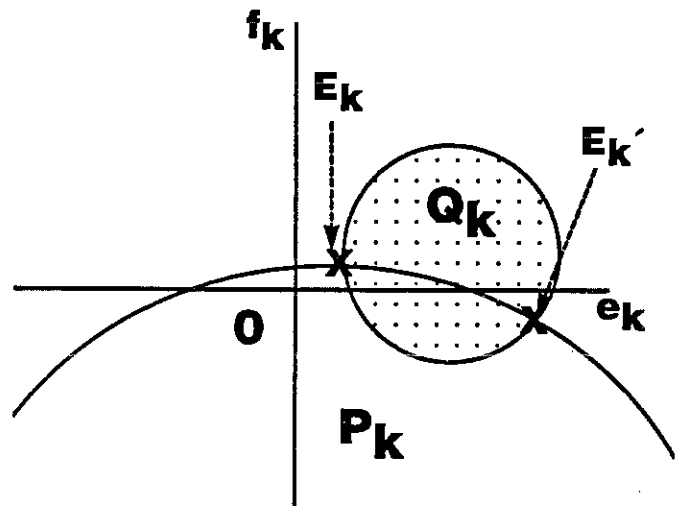


Fig. 1 A Pair of Multiple Load Flow Solutions at Bus k

where

- e_k' : real part of bus voltage at bus k obtained by solving the load flow equation at bus k
- f_k' : imaginary part of bus voltage at bus k obtained by solving the load flow equation at bus k

Using Eqn. (5), the other solution corresponding to x_1 can be represented as

$$\begin{aligned} x_1' &= (e_1', f_1', e_2', f_2', \dots, e_n', f_n')^T \\ &= (E_1', E_2', \dots, E_n')^T \end{aligned} \quad (6)$$

The multiple load flow solutions are found by combining the elements of x_1 with those of x_1' . For an n-bus system, the number of potential initial values is equal to $2^{(n-1)}$ excluding the reference bus.

2.2 Simplified Exhaustive Search Method

In this paragraph, a simplified exhaustive search method is outlined. The exhaustive search method mentioned in 2.1 seems reliable but it is extremely time-consuming for real-sized power systems. Compared with the load flow calculation, it requires at most $2^{(n-1)}$ times computational time. As a result, a simplified exhaustive search method was proposed to reduce the computational effort [4]. The idea is to select (n-2) bus voltages from x_1 and one bus voltage from x_1' . For an n-bus system, (n-1) kinds of initial condition candidates are prepared to find the multiple load flow solutions. For example, the j-th initial conditions are given by

$$\begin{aligned} x_{1cj} &= (e_1, f_1, e_2, f_2, \dots, e_j', f_j', \dots, e_n, f_n)^T \\ &= (E_1, E_2, \dots, E_{j-1}, E_j', E_{j+1}, \dots, E_n)^T \end{aligned} \quad (7)$$

where, bus 1 is assumed to be the slack node and $j=2,3,\dots,(n-1)$.

Also, E_k' is not involved in the initial condition candidates in a case where bus k is a floating bus or PV-specified bus. Thus, the number of the initial condition candidates is less than (n-1). Although the computation effort is reduced in this method, it seems that the method needs enhance the computational efficiency. Later on, this method is used to create the learning and test data of the Kohonen neural network for reducing the computational effort.

2.3 Method with Optimal Multiplier μ_3

This paragraph outlines a method of Iba, et al. that makes use of an optimal multiplier μ_3 [19]. The method is based on minimizing a cost function that represents the error between the specified value and the load flow equation.

Eqn. (2) can be written with a Taylor's series expansion

around x_e as follows [5]:

$$\begin{aligned} y_s &= f(x) \\ &= f(x_e + \Delta x) \\ &= f(x_e) + J(x_e)\Delta x + y(\Delta x) \end{aligned} \quad (8)$$

where

- x_e : estimate of x_e
- Δx : error(correction) vector
- $J(\cdot)$: Jacobian matrix

Now, introducing an adjusting parameter μ in updating bus voltage vector x , the updated voltage vector may be expressed as

$$x := x + \mu \Delta x \quad (\mu: \text{scalar}) \quad (9)$$

Using Eqn. (8), the load flow calculation can be formulated by minimizing the following cost function with respect to μ :

$$\begin{aligned} g &= \|y_s - y(x)\|^2 \\ &= \|y_s - y(x_e) - \mu J(x_e)\Delta x - y(\Delta x)\|^2 \end{aligned} \quad (10)$$

Parameter μ is evaluated by solving a scalar cubic equation with respect to μ . There exist three kinds of parameter μ , i.e., μ_1 , μ_2 , and μ_3 . Since μ_3 corresponds to the adjusting parameter for finding the counterpart of a pair of the multiple load flow solutions, the multiple load flow solution can be calculated by updating the following equation:

$$x := x + \mu_3 \Delta x \quad (11)$$

The method is quite effective in a case where there exists only a pair of the multiple load flow solutions. However, the method has a problem that it might provide misleading results in a case where the number of the solutions is more

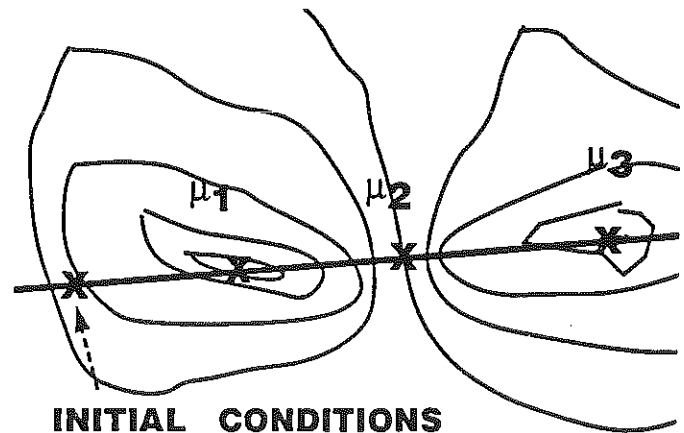


Fig. 2 Three Kinds of Optimal Multiplier, μ_1 , μ_2 and μ_3

than three. That is because the method makes an assumption that the counterpart lies on the extrapolated line connecting the initial condition with the load flow solution obtained using μ_1 as shown in Fig. 2. In practice, it is quite difficult to evaluate how many load flow solutions the power systems to be studied have.

III. INDECIES OF STATIC VOLTAGE INSTABILITY USING A PAIR OF MULTIPLE LOAD FLOW SOLUTIONS

This section deals with an index of static voltage instability using a pair of the multiple load flow solutions. The angle between two voltage vectors is utilized as a margin to the singularity of the load flow equation.

Now, consider a pair of the multiple load flow solutions, x_1 and x_2 . According to reference [5], the pair can be expressed as

$$x_1 := a + b \quad (12)$$

$$x_2 := a - b \quad (13)$$

where, vector a is a singular vector in x-space.

Since the pair satisfies the same specified value y_s , we have

$$\begin{aligned} y_s &= f(a \pm b) \\ &= y(a) \pm J(a)b + y(b) \\ &= y(a) + y(b) \\ &= y_a + y_b \end{aligned} \quad (14)$$

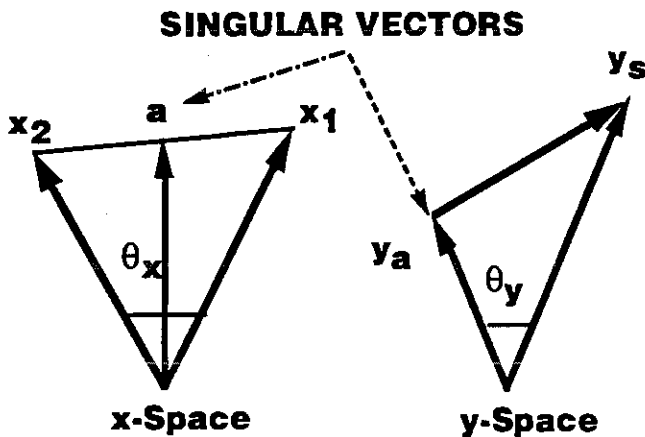


Fig. 3 Singular Vector in x-Space and y-Space

where

$$y_a := y(a)$$

$$y_b := y(b)$$

y_a : a singular vector in y-space.

Fig. 3 depicts the singular vectors in both x- and y-space. As power system conditions become heavy-loaded, x_1 and x_2 approach singular vector a in x-space while power system conditions y_s does singular vector y_s in y-space. Thus, an index for static voltage instability can be written as

$$\theta_x := \cos^{-1} \frac{x_1^T x_2}{\|x_1\| \cdot \|x_2\|} \quad [\text{deg.}] \quad (15)$$

or

$$\theta_y := \cos^{-1} \frac{y_s^T y_a}{\|y_s\| \cdot \|y_a\|} \quad [\text{deg.}] \quad (16)$$

The index given by Eqn. (16) is called VIPI [12]. The indecies have an advantage that they are easily understood in a sense that they are expressed in degree. If the counterpart of the load flow solution obtained with the flat start is calculated by a method for finding multiple load flow solutions, the above indices are obtained.

IV. KOHONEN ARTIFICIAL NEURAL NETWORK

This section outlines the Kohonen neural network for pattern classification.

4.1 Competitive Learning

The Kohonen neural network makes use of unsupervised learning based on competitive learning. The process of the competitive learning is carried out as follows: Before the learning, several output neurons are fired by a given input pattern. During the learning, output neurons in some neighbourhood are trained to be fired for the corresponding input patterns. Eventually, neighbor output neurons around the fired output neurons are trained not to be fired. In other words, after the learning, only one neuron is fired for the corresponding input neurons. Kohonen introduced the Mexican hat function so that output neurons near the fired neuron are strongly excited while those not so far are inhibited from being excited. The function is shown in Fig. 4, where the horizontal axis denotes the distance from the most excited neuron and the vertical axis implies the interaction between output neurons.

4.2 Characteristics of SOFM

Kohonen developed a simplified approach called self-organization feature mapping (SOFM). The conventional competitive learning approaches have difficulties in handling nonlinearity of the process. Thus, it is not easy to apply the

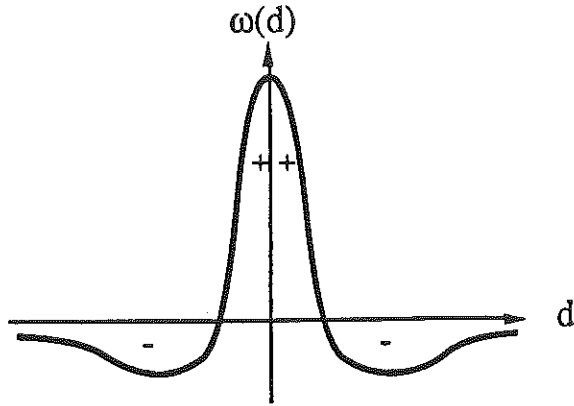


Fig. 4 Mexican Hat Function

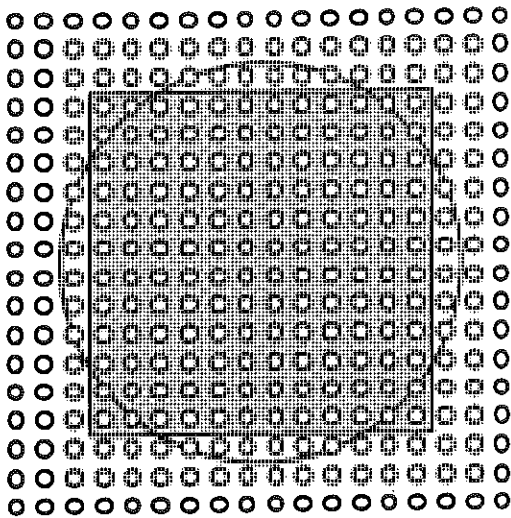


Fig. 5 Allocation of Output Neurons

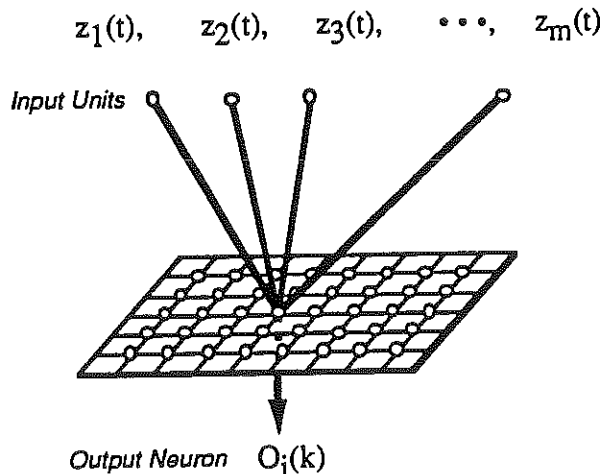


Fig. 6 Self-Organization Feature Mapping

conventional approach to larger systems. The main characteristics of the Kohonen network are given as follows:

(a) The network has function that high dimensional patterns are transformed into two or more dimensional output patterns. For example, Fig. 5 illustrates two-dimensional rectangular grid that represents output neurons. The mapping from input to output patterns with the self-organization can reflect the ordering of input patterns. In other words, input patterns with similar features are allocated to output neurons close to each other.

(b) The weighted vector between input neurons and a fired output neuron is trained so that the weight is close to input patterns. In other words, the fired output neuron has the weight vector which is most similar to the input pattern. Fig. 6 shows the Kohonen neural network where a input pattern fires the output neuron. To be specific, the concept of "bubble" was proposed to determine which neurons should be activated during the learning process. At the beginning, a set of neurons close to the most excited neurons are activated. The set is called bubble. As the learning steps proceed, the area of the bubble was shrinking into one neurons. Fig. 7 shows the transition of the bubble. It should be noted that other neurons outside of the bubble are weakened not to be activated. The topological neighborhood at learning step k is defined as the area in which the activated neurons satisfy

$$\|n_i - n^*(k)\| \leq \gamma(k) \quad (17)$$

where, $\|\cdot\|$: Euclidean norm

n_i : output neuron i within bubble

$n^*(k)$: most excited output neuron

$\gamma(k)$: radius from $n^*(k)$ which is a decreasing function with respect to learning step k

According to Eqn. (17), it is hard to determine the bubble in terms of the circle with radius $\gamma(k)$. So, the rectangular is utilized to determine the bubble in the two-dimensional grid like Fig. 5.

(c) Input vectors $x(t)$ and weight vector $w_i(k)$ are normalized as follows:

$$\|z(t)\| = 1 \quad (18)$$

$$\|w_i(k)\| = 1 \quad (19)$$

where, $\|\cdot\|$: magnitude of vector

$$z(t) = (z_1(t), z_2(t), \dots, z_m(t))^T \quad (20)$$

t : input pattern number ($t = 1, 2, 3, \dots, t_{\max}$)

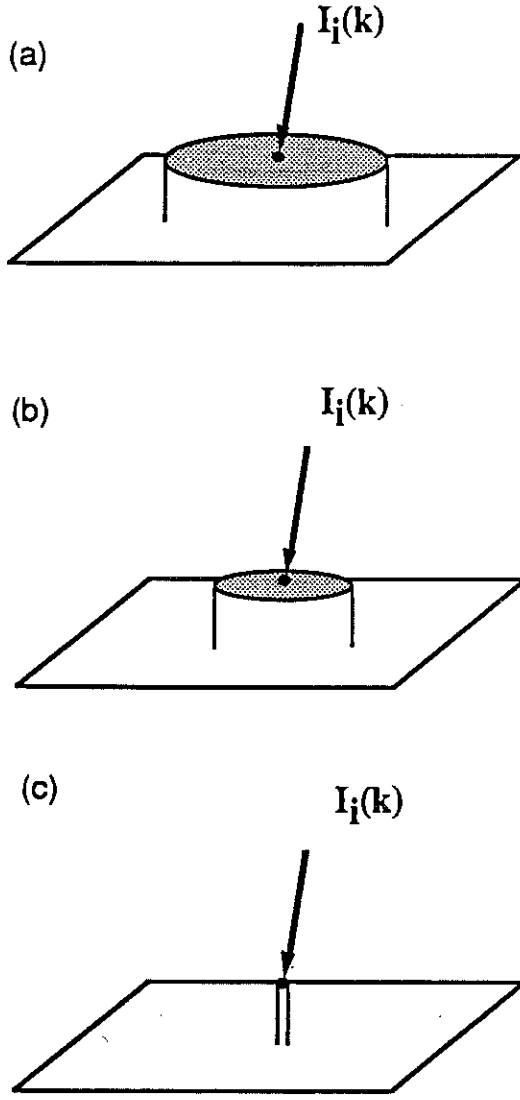


Fig. 7 Transition of Bubble

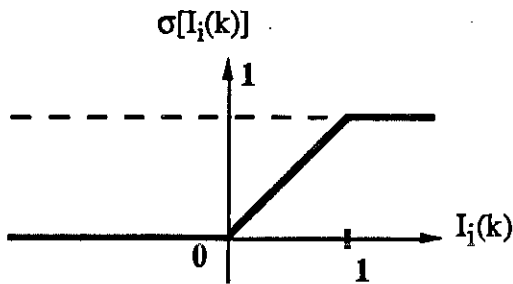


Fig. 8 Output Function

($l_{\max}$: number of input patterns)

$w_i(k)$: weight vector of neuron i such as

$$w_i(k) = (w_{i1}(k), w_{i2}(k), \dots, w_{im}(k))^T \quad (21)$$

$w_{ij}(k)$: weight between neurons i and j

(d) Input of the Kohonen neural network is given by

$$I_i(k) = \sum_j^m z_j(t) \cdot w_{ij}(k) \quad (22)$$

The relationship between input and output neuron can be written as the following squashing function:

$$O_i(k) = \sigma(I_i(k)) \quad (23)$$

where, $\sigma(\cdot)$: nonlinear function as shown in Fig. 8

$$\begin{aligned} \sigma(I_i(k)) &= 1 & \text{if } I_i(k) > 1 \\ \sigma(I_i(k)) &= I_i(k) & \text{if } 0 \leq I_i(k) \leq 1 \\ \sigma(I_i(k)) &= 0 & \text{if } I_i(k) < 0 \end{aligned} \quad (24)$$

In general, the weights are updated by

$$w_{ij}(k+1) = w_{ij}(k) + \Delta w_{ij}(k) \quad (25)$$

The correction of the weights for the neurons within the bubble can be expressed as

$$\Delta w_{ij}(k) = \alpha(k) (x_j(t) - w_{ij}(k)) \quad (26)$$

where, $\alpha(k)$: decreasing function with respect to learning step k

$$0 < \alpha(k) < 1 \quad (27)$$

On the other hand, the correction for the neurons outside the bubble can be written

$$\Delta w_{ij}(k) = 0 \quad (28)$$

4.3 Algorithm

The algorithm of SOFM can be written as follows:

Step1: Set learning step $k=0$ and inner loop count $t=0$.
Then prepare the initial value of weights $w_{ij}(0)$.

Step 2: Set $k = k + 1$.

Step 3: Set $t = t + 1$ and give input signal $x(t)$.

Step 4: Calculate neuron $n^*(k)$ that have the closest distance, $d^*(k)$, to input pattern $x(t)$, where $d^*(k)$ is given

$$d^*(k) = \min_i \{d_i(k)\} \\ = \min_i \left\{ \sum_j (z_j(t) - w_{ij}(k))^2 \right\} \quad (29)$$

Step 5: Find topological neighbourhood around $n^*(k)$.

Step 6: Update the weights for $n^*(k)$ and topological neighborhood.

Step 7: If $k = k_{\max}$, then stop. Otherwise go to Step 8.

Step 8: If $t = t_{\max}$, then set $t = 0$ and go to Step 2. Otherwise, go to Step 2.

V. ESTIMATION OF A VOLTAGE INSTABILITY INDEX USING SOFM

In this section, a neural network based approach is proposed to estimate voltage instability index θ_x with SOFM mentioned in the previous section. The advantage of the proposed method is twofold; pattern recognition and feature extraction. The idea of the proposed method is shown in Fig. 9.

Firstly, SOFM is utilized to carry out pattern recognition to reduce the computational effort. In the figure, the conventional method finds the multiple load flow solutions using the simplified exhaustive search method, where the conventional method means the simplified exhaustive search method mentioned in 2.2. In an n -bus system, at most $(n-1)$ pairs of multiple load flow solutions are calculated to find the closest counterpart. Among them, the pair with the smallest index was selected as the solution pair that expresses most critical power system conditions.

Secondly, the proposed method can provides an insight into power system conditions from a standpoint of feature extraction of input patterns. This is quite useful in power system monitoring because the different power system conditions from the normal ones are easily identified. Let us consider the situation in Fig. 10, where the daily load patterns are expressed in shaded area S on the feature extraction mapping. In the figure, the curve via A, B, C, and D shows the typical power system conditions that represents the typical daily load curve. The power system conditions are continuously monitored by tracing the transition of the power system conditions on the mapping. If the power system conditions moves from the shaded area and head for point E in a lower index value, power system operators can understand that the power system encounters the different

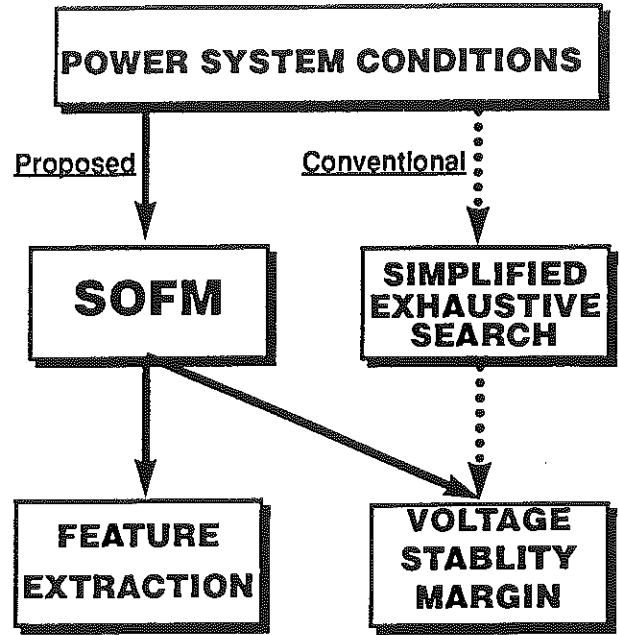


Fig. 9 Comparison between Proposed and Conventional Methods

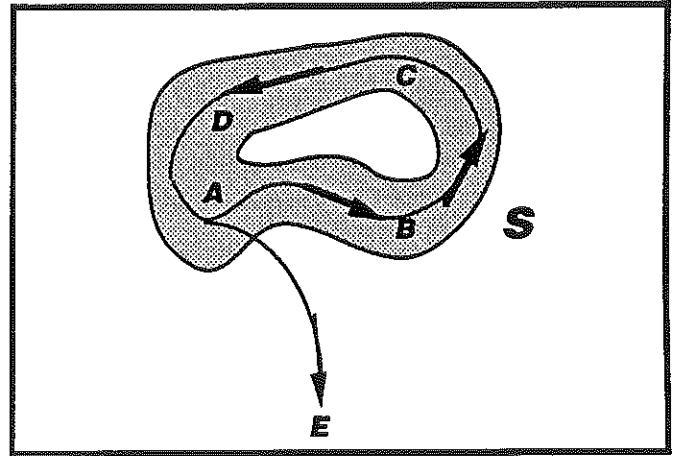


Fig. 10 Monitoring Voltage Instability Index

power system conditions from the scheduled pattern, which implies insecure power system conditions with respect to voltage instability.

For a given input pattern, SOFM recalls the output neuron which is most similar to the input pattern. The voltage instability index is allocated to each neuron. The index is calculated by Eqn. (15) which makes use of the angle of a pair of the multiple load flow solutions.

The input vector is given by the specified vector. For example,

$$z(t) = (V_1(t), P_2(t), Q_2(t), \dots, P_n(t), Q_n(t))^T \quad (30)$$

where, P_p : active power at node p

Q_p : reactive power at node p

V_p : magnitude of voltage at node p

n: number of nodes

If input vector is linearly dependent on another vector, i.e.,

$$z(t_1) = c z(t_2) \quad (c: \text{constant}, t_1 \neq t_2) \quad (31)$$

It is impossible to distinguish $z(t_1)$ with $z(t_2)$ due to the normalization. The unity is added to the original input vector to cope with the problem. As a result, Eqn. (31) can be rewritten as

$$z(k) = (V_1(k), P_2(k), Q_2(k), \dots, P_m(k), Q_m(k), 1)^T \quad (32)$$

In this paper, output neurons are expressed in two-dimensional grid like Fig. 5.

Two methods are utilized to estimate voltage instability index θ_x at each output neuron after finishing the learning process of the weights. Method A makes use of the average index that is obtained from a set of power system conditions classified into the same output neurons. It is assumed that the index is allocated to each power condition in advance. On the other hand, Method B recalculate the index with the weights that implies the power system conditions.

The algorithm of Method A can be written as

<Method A>

Step 1: Prepare for learning patterns of SOFM and index θ_x for each learning pattern.

Step 2: Carry out SOFM.

Step 3: Calculate θ_{xi}^e at output neuron i on the grid using the following equation and store it at each output neuron:

$$\theta_{xi}^e = \frac{1}{N_i} \sum_k \theta_{xik} \quad (33)$$

where,

θ_{xi}^e : estimated index at output neuron i

θ_{xik} : index of input pattern k classified into output neuron i

N_i : number of input patterns classified into output neuron i

Step 4: Give an unknown input pattern to the neural

network. Then, find output neuron i^* that is closest to the input pattern.

Step 5: Recall θ_{xi}^e at output neuron i^* .

On the other hand, the algorithm of Method B can be written as

<Method B>

Step 1: Prepare for learning patterns of SOFM.

Step 2: Carry out SOFM.

Step 3: Calculate θ_{xi}^e with the weight vector at each output neuron i.

Step 4: Give an unknown input pattern to the neural network. Then, evaluate output neuron i^* that is closest to the input pattern.

Step 5: Recall θ_{xi}^e at output neuron i^* .

VI. SIMULATION

6.1 Simulation Conditions

The proposed method with SOFM was applied to a five bus system in Fig. 11. The following conditions are used:

(a) The input vector representing power system conditions is given by

$$z(k) = (e_1(k), P_2(k), Q_2(k), \dots, P_5(k), Q_5(k), 1)^T \quad (33)$$

Since the input vector has ten elements, there are ten input neurons. In this paper, 6000 power system conditions were created by the uniform random number generator in which 5000 power system conditions were the learning data and 1000 system conditions were the test data. The upper and lower bounds of the specified value y_s were set as follows:

$$y_s^U = 1.25 y_s \quad (34)$$

$$y_s^L = 0.75 y_s \quad (35)$$

where, $U(L)$ denotes the upper (lower) bound.

Also, the slack voltage $e_1(k)$ has the following range:

$$e_1(k)^U = 1.02 e_1 \quad (36)$$

$$e_1(k)^L = 0.98 e_1 \quad (37)$$

For 6000 power system conditions, voltage instability

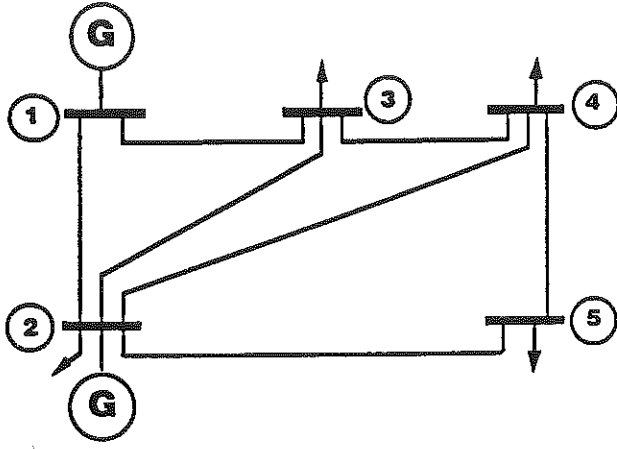


Fig. 11 Five Node System

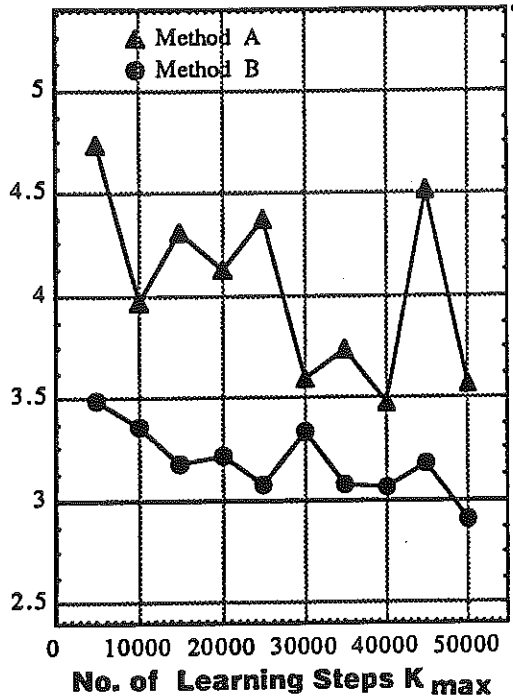


Fig. 12 Average Error vs. No. of Learning Steps

index θ_x was calculated by the simplified exhaustive search method and Eqn. (15).

- (b) Output neurons were arrayed on the two-dimensional grid consists of 30×30 output neurons. The grid is expressed as a rectangular lattice.
- (c) The decreasing functions $\alpha(k)$ in Eqn. (26) and $\gamma(k)$ in Eqn. (17) are expressed in the following nonlinear functions:

$$\alpha(k) = 0.7 \exp(-3 \times k / k_{max}) \quad (38)$$

$$\gamma(k) = 10 \exp(-3 \times k / k_{max}) \quad (39)$$

where, k_{max} is the number of learning steps that is employed for the learning process. The following ten kinds of k_{max} is used:

$$k_{max} = t_{max} \times n_k = 5000 \times n_k \quad (40)$$

(where, $n_k = 1, 2, 3, \dots, 10$)

For each k_{max} , the neural networks are constructed to obtain SOFM with Methods A and B. Totally, this paper evaluates twenty kinds of the neural networks.

- (d) The accuracy of the networks was examined by the average error such as:

$$\epsilon = \frac{1}{M} \sum_k \frac{|\theta_{xk}^e - \theta_{xk}^t|}{\theta_{xk}^t} \times 100[\%] \quad (41)$$

where

θ_{xk}^e : estimate of the index for input pattern k

θ_{xk}^t : true value of the index for input pattern k

M : number of test patterns

5.2 Results

Fig. 12 shows the relationship between the number of the learning steps and the average accuracy. As the number of learning steps increases, the accuracy is apt to be enhanced in Methods A and B. However, the accuracy of Method A is changeable with the number of the learning steps. It can be observed that Method B has higher accuracy than Method A. In fact, the average error at $k_{max} = 50000$ is about 2.9[%] and 3.5[%] in Methods A and B, respectively. Since Method B calculates the index with the weight vector after finishing learning, the accuracy of the index using the weights that the fired output neuron has is better than that of Method A which is based on the average value. It seems that the average error of 2.9 [%] for Method B is acceptable in tracing the transition of power system conditions.

The voltage instability index at each output neuron in is depicted in Fig. 13, where Method B is employed with $k_{max} = 50000$. The areas where the voltage instability index is lower correspond to power system conditions that have fewer margin with respect to voltage instability.

Fig. 14 illustrates the self-organization feature mapping of the voltage instability index on two-dimensional grid which consists of 30×30 output neurons. A mapping representation is easily to understand how close the power system is to the critical conditions. In the figure, the voltage instability index lacks margin as the shaded area becomes whiter. It can be seen in Fig. 13 that there exist two area with less

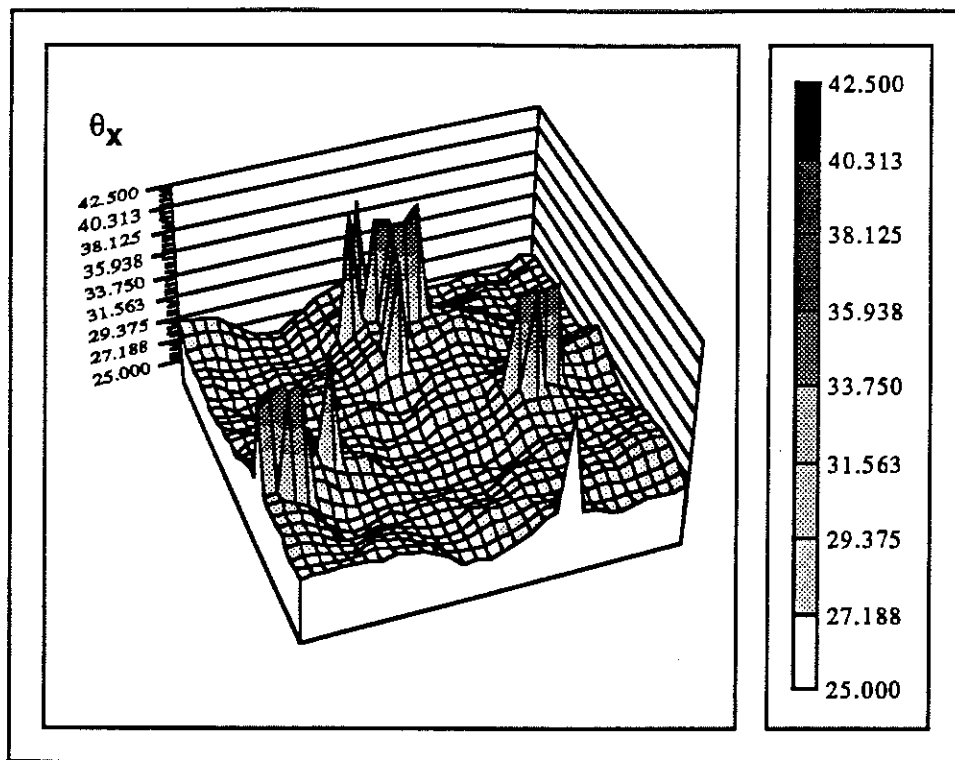


Fig. 13 Voltage Instability Index on Two-dimensional Output Neuron Grid with Method B ($k_{\max}=50000$)

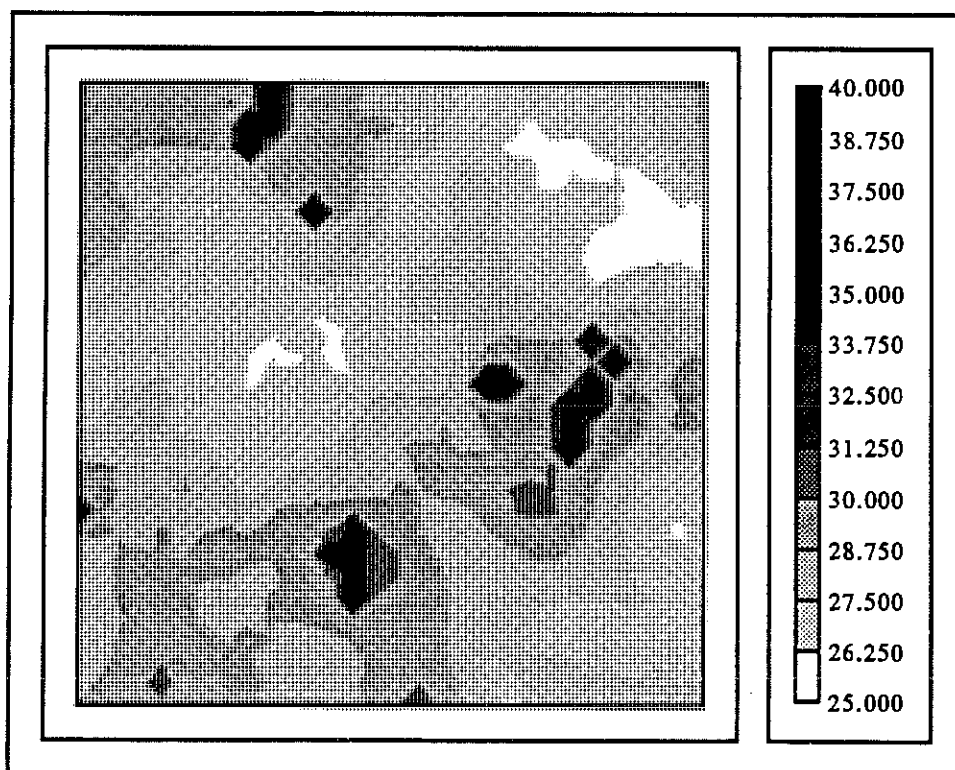


Fig. 14 Self-Organization Feature Mapping with Method B ($k_{\max}=50000$)

margin at almost the center and the right upper side. This mapping can advise power system operators if power system conditions approach the critical conditions. Thus, it is expected that the representation is displayed on the CRT as an operator's guideline for voltage stability monitoring and control.

VI. CONCLUSIONS

In this paper, a neural network-based approach has been developed to help power system operators monitor the voltage instability margin. The proposed approach with the Kohonen neural network has two advantages; pattern recognition based on the classifier and the feature extraction of power system conditions. The former is useful for reducing the computational effort while the latter is quite informative in understanding characteristics of power system conditions.

What this paper demonstrated is easily applied to larger systems although the proposed approach has been applied to a five bus system in this paper. The subject will be discussed in a forthcoming paper.

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OVERVOLTAGE CONTROL DURING RESTORATION

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Abstract: Restoration of bulk power supply system presents the operator with unique challenges not normally encountered in daily operations. The initial, and even intermediate transmission system topologies are a far cry from the well integrated system of normal operation. There are a number of issues of common concern to the operators which need to be addressed [1]. One of these, the problem of overvoltages, is the subject of this paper. For those studying the domain of voltage phenomena, this paper may provide examples of extraordinary system conditions that may occasion unusual manifestations.

Introduction

During early stages of restoring high voltage overhead and underground transmission lines the concerns are with the following three related overvoltage areas:

- sustained power frequency overvoltages,
- switching transients, and
- harmonic resonances.

The sustained power frequency overvoltages are caused by charging currents of lightly loaded transmission lines. If excessive these currents may cause underexcitation, or even self excitation and instability. The sustained overvoltages also overexcite transformers, generate harmonic distortions, and cause overheating.

The transient voltages or switching surges are caused by energizing large segments of transmission system or by switching capacitive elements. The switching transients which are usually highly damped and of short duration, in conjunction with sustained overvoltages may result in arrester failures [2].

Transient overvoltages are not usually a significant factor at transmission voltages below 100 kV. At higher transmission voltages, overvoltages caused by switching may become a significant factor, because arrester operating voltages are relatively close to normal system voltages and lines are usually long so that energy stored on the lines may be large. In most cases, though, surge arresters have sufficient energy absorbing capability to clamp harmful overvoltages to safe levels without sustaining damage. The likelihood and the effect of transient overvoltages are determined by the study of special system conditions [3]. Computer aided analysis

has proven to be a valuable tool in understanding switching surge overvoltages [4,5].

Harmonic resonance voltages are oscillatory undamped or only weakly damped temporary overvoltages (TOVs) of long duration. They originate from switching operations and equipment non-linearities [2]. During restoration several factors may combine to cause harmonic resonance: first, natural frequency of the series resonant circuit formed by the source inductance and line charging capacitance may, under normal operating conditions, be a low multiple of 60 Hz; second, "magnetizing inrush" caused by energizing a transformer produces many harmonics; and finally, during early stages of restoration the lines are lightly loaded, resonance is therefore lightly damped, which in turn means the resulting resonance voltages may be very high [6]. If transformers become overexcited, due to power frequency overvoltage, harmonic resonance voltage will be sustained or may even escalate.

Transmission system equipment has only a limited ability to withstand overvoltages, as discussed in the next section.

Transmission System Equipment Limitations

Power transformers, surge arresters and circuit breakers will be the equipment earliest affected by overvoltages. Figure 1 shows power frequency voltage withstand vs. time for typical power transformers and surge arresters as would be applied on a solidly grounded system. As can be seen, a power transformer can only survive a seemingly small overvoltage of 1.2 p.u. for about one minute! Both the surge arrester and the transformer can withstand 1.4 p.u. voltage for 10 seconds; above 1.4 p.u. voltage, surge arrester failure will precede transformer damage.

Any voltage above 1.1 p.u. will put the transformer into saturation causing core heating and copious harmonic current generation. Note that the per unit base for the transformer is the particular voltage rating on the tap being energized.

Circuit breakers if called upon to operate during periods of high voltage will have reduced interrupting capability (or none at all). At some voltage, even the ability to interrupt line charging current will be lost. This point will vary among specific breaker designs. Although standards require circuit breakers to interrupt against larger than normal recovery

voltages with reduced fault currents, there are no overvoltage requirements for line charging current interruption. The authors believe that most HV or EHV circuit breakers could interrupt line charging current at least up to 1.2 p.u. of maximum rating.

Slow clearing of phase-to-ground faults could aggravate an otherwise acceptable overvoltage to the point of damaging surge arresters. Even on well grounded systems the voltage on the un-faulted phases during phase-to-ground faults can be on the order of 1.3p.u. The longer the fault persists, the higher the likelihood of equipment damage.

In light of the foregoing equipment voltage limitations and fault contingency, it is recommended that no line be energized if by doing so voltage at the remote end would raise to more than: 1.2 p.u. of normal or $1.1 \times$ (transformer tap or circuit breaker rated maximum voltage) whichever is lower.

In developing a power system restoration plan the above concerns and constraints can be addressed by analysis and simulation as illustrated by the three sample studies described below.

Sustained Voltage Study

In the following example a load flow program was used to study sustained voltages of a system shown on Figure 2. The system has been reduced to three generating buses, five load buses and nine 220 kV lines. Figure 3 shows the voltage profile for the 220 kV System. The upper profile shows voltages to be at unacceptably high level, i.e., about 130% when keeping the Generators 1 & 2 terminal voltages at 13.8 kV or 1.0 per unit. The total charging reactive power for the system at this voltage level is about 90 MVAR. The lower profile shows that to maintain the sustained voltages within the acceptable limits, Generators 1 & 2 terminal voltages should be kept at about 11.0 kV or 80%, with the resulting reduction in charging reactive power to about 55 MVAR, which is within the reactive absorbing capability of the generators. It is important to note, however, that the extent of the generator's voltage reduction is usually constrained by underexcitation of generators brought about by a number of limiting factors including; generator terminal low voltage limit (as limited by power plant auxiliary equipment), under reactive ampere limit relay, minimum excitation limit relay, and rotor core end heating limit [7].

Control of Sustained Overvoltages

The sustained overvoltages can be controlled by absorbing the large charging reactive power of the lightly loaded transmission lines. This can be accomplished by:

- having sufficient under-excitation capability on the generators,
- connecting reactive loads (lagging power factors) to the underlying system including shunt reactors,
- removing all sources of reactive power and switching off shunt capacitors,
- running generators at maximum possible reactive power output to allow margin to adjust for large charging reactive power when lines are switched on,
- operating parallel transformers on different taps to increase circulating currents and generate reactive power,
- energizing only those transmission lines which carry significant load and avoiding the energization of extra lines which will generate unwanted reactive power, and
- maintaining a low voltage profile on the transmission lines, since the charging currents are proportional to the square of the voltage.

Inability to perform the above tasks would cause a reactive power imbalance resulting in, for example, generator self-excitation and a runaway voltage rise.

Considering the last item, e.g., a 100 miles of 500 kV line with charging reactive power of 1.8 mvar per mile, generates about 180 MVAR. A 5% voltage reduction will reduce this charging by about 10%, or about 18 MVAR!

Switching Transients Study

Severe overvoltages resulting from switching surges may cause flash-over and serious damage to equipment. Switching transients are fast transients that occur in the process of energizing transmission line and bus load capacitances right after a power source is connected to the network. Inductances of the transmission lines and the power sources interact with the capacitance to cause very fast oscillations in the process. The main cause of overvoltages is ill-timed closure of the breaker; if it occurs at the point when the instantaneous voltage of the source

goes through its maximum a large overvoltage can result. In order to illustrate this phenomenon results of the 220 kV System study are shown in Figures 4 and 5. The 220 kV System of Figure 2 was energized using Generators 1 and 2 at Bus 1, with no surge arresters modeled. When the generators are running at 105% of the rated bus voltage, it can be seen from Figure 4 that the maximum instantaneous line-to-line voltage reaches about 715 kV (2.3 p.u.) on buses 6 and 7. Typical surge arresters would limit this voltage to 2.1 p.u. or less while using only 5% of their energy absorbing capability. Figure 5 shows the minimum instantaneous voltage of about -439 kV (1.41 p.u.) for Bus 8. By reducing terminal voltage on the two generators to 80% of their nominal values, the switching transients are kept well within the arresters ratings.

An RLC model of the network is used for this study. Generators are represented as sinusoidal sources; lines and transformers are represented as RLC circuits, with the charging capacitance concentrated at the line ends. Bus loads are converted to equivalent resistances and inductances (for absorbing VAR loads), or capacitances (for generating VAR loads). A model relating unknown (state) variables of the system and known parameters and the input is obtained by observing basic physical laws of balance of current at each node and balance of voltage in each loop. The state variables consist of currents through each inductor, and voltages across each capacitor. A result is a mix of algebraic and ordinary differential equations, as given in the Appendix. This set of equations is solved by a method that utilizes automatic integration step size adjustment [8].

Control of Switching Transients

Energizing transmission lines or switching capacitive elements causes switching surges of fast front, low energy or slow front high energy transients. Switching transient are usually not the limiting factor in re-energizing a system. Generally, if the steady state voltages (power frequency) are less than 1.2 per unit of their nominal values, the switching transients can be managed by typical arresters with ease. A notable exception is energizing transformer terminated lines which may result in harmonic resonance and damaging overvoltage.

Harmonic Resonances

During the reintegration phase, with a "fledgling" system, the capacitive voltage rise due to charging currents will be sufficient to overexcite transformers and generate significant harmonics. If the combination of the system impedance

and the line capacitance is adverse, then a harmonic resonance will result. Harmonic distortions produced by transformer saturation will excite these resonances which can result in damaging overvoltages. Even if transformers are not continuously (due to sustained overvoltage) overexcited, harmonics generated by magnetizing inrush on energization is sufficient to excite the resonance. To assure that resonance is avoided, sufficient load should be connected to the underlying system at both ends of line to damp the resonance. Analysis for a 500 kV line has shown that a load of about three mega-watts per mile is adequate.

Harmonic Resonance Study

A major concern with energizing transmission system is the increase in harmonic overvoltages due to the different line and transformer configuration from the normal operation. The overall objective of the harmonic analysis study is to determine if the line-transformer configuration resulting from energizing of the line would be prone to excessive harmonic levels. In the following example, EMTP was used to model a simple system consisting of a 500/230 kV transformer at the sending end, a variable length 500 kV line, and a 500/230 kV transformer at the remote end. The underlying loads were modeled on both transformers. The line length was varied to attain resonance at 3rd, 4th, 5th, 6th and 7th harmonics. Pre-switch voltage was varied from 0.9 to 1.1 p.u. At these resonances, switching angles were varied to produce the maximum sustained overvoltage possible. Lower order resonances produce higher overvoltages. Load levels on both the sending-end and receiving-end transformers were gradually increased until the resonance was controlled by sufficient damping. Approximately, 3 MW load on the underlying 230 kV system for every mile of 500 kV line connected will damp resonance voltages. Figure 6 shows the one-line diagram of the model and Figures 7 and 8 show sample wave forms.

In the case shown, a 650MVA transformer is being energized at the end of an 88 mile 500 kV line. B phase has been tuned to 180Hz (3rd harmonic). The pre-switch voltage is 500 kV and the transformer being energized is on its 550 kV tap. Switching takes place at 0.1 seconds near a voltage zero with no remnant magnetism in the core. The voltage shown is the sending end bus voltage. The receiving end bus voltage is about 8 kV higher. In Figure 7 there is no load. The voltage escalates to 2.0 p.u. and shows no sign of decay during the simulation. Such a sustained resonance voltage would certainly cause surge arrester failures, and create system faults. Figure 8 shows the effect of placing 200 MW load on the underlying 230 kV system at the source end. The resulting voltage is damped to a harmless 1.05 p.u.

Control of Harmonic Resonance

The following table lists factors aggravating overvoltage and remedies:

- Sustained harmonic overvoltages caused by over excitation of transformers can be controlled by selecting a transformer tap which equals or exceeds the power frequency voltage applied (or lowering system voltage to at or below the tap) before energizing.
- The harmonic resonances can be damped by connecting sufficient underlying loads at both ends of line or by connecting dead load on the transformer to be energized.
- The high source impedance can be reduced by starting more generators and connecting underlying loads.
- The reactive power of a lightly loaded system can be reduced by minimizing the number of unloaded lines to be energized and setting the sending-end transformers at the lowest tap position.

CONCLUSION

Power system equipment are protected against the effect of overvoltages resulting from normal operation by surge arresters which provide a relatively low-impedance path for transient overvoltages. During restoration however, certain line-transformer configurations when energized generate a combination of sustained, transient and harmonic overvoltages which may cause surge arrester failures and thus system faults, or the subsequent flash-over and damage to equipment.

The switching surge overvoltages have become increasingly more important with the design of compact transmission lines and substation facilities. The energy stored in long high voltage lines is large causing significant transient or resonant overvoltages which must carefully be considered during restoration. With the help of analytical tool such as EMTP and those described in this paper it should be possible to establish general procedure and restoration guidelines to avoid overvoltages.

Appendix

The RLC model of the 220 kV System of Figure 2.

Notation:

Subscripts:

B - bus

L - line (with terminals i and j)

G - generator; GH - high voltage generator bus

v - instantaneous bus voltage,

V - voltage magnitude

i_L - current through line L,

i_G - current through generator G

Model

$$di_G/dt = 1/L_G [V \sin(2\pi ft) - R_G i_G - v_L]$$

$$di_L/dt = 1/L_L (v_i R_L - i_L v_j),$$

$$\sum_L c_{BL} i_L + \sum_G c_{BG} i_{GL} = i_B,$$

$$c_{BL} = (1 \text{ if } B = i; -1 \text{ if } B = j; 0 \text{ otherwise})$$

$$c_{BG} = (1 \text{ if } G \text{ connected to bus } B; 0 \text{ otherwise})$$

$$dv_B/dt = 1/C_B (i_B - v_B/r_B), \quad r_B \neq 0$$

$$dv_B/dt = 1/C_B i_B, \quad r_B = 0$$

$$B = 1, 2, \dots, 8;$$

$$L = 1, 2, \dots, 9;$$

$$G = 1.$$

Line Data

<u>Line L</u>	<u>From (i) To (j)</u>	<u>Inductance L_{ij}</u>	<u>Resistance R_{ij}</u>
1	1 - 2	0.051457	2.6620
2	1 - 4	0.084888	4.0656
3	1 - 5	0.14436	7.5020
4	2 - 3	0.011709	0.3872
5	3 - 8	0.19289	6.3888
6	4 - 5	0.050532	2.4200
7	5 - 6	0.010076	0.50336
8	5 - 8	0.082269	4.2592
9	6 - 7	0.0020182	0.09922

Note: Charging capacitance is lumped at the terminal buses

Source Data

<u>Bus G</u>	<u>Inductance L_G</u>	<u>Resistance R_G</u>
1	8.0737e-4	2.4757e-3

Bus Data

<u>Bus B</u>	<u>Capacitance C_B</u>	<u>Resistance R_B</u>
1	7.2673e-7	0
2	1.584e-7	0
3	5.332e-7	0
4	3.7596e-7	0
5	9.3047e-7	0
6	1.4717e-6	0
7	1.5155e-7	0
8	8.1567e-7	0

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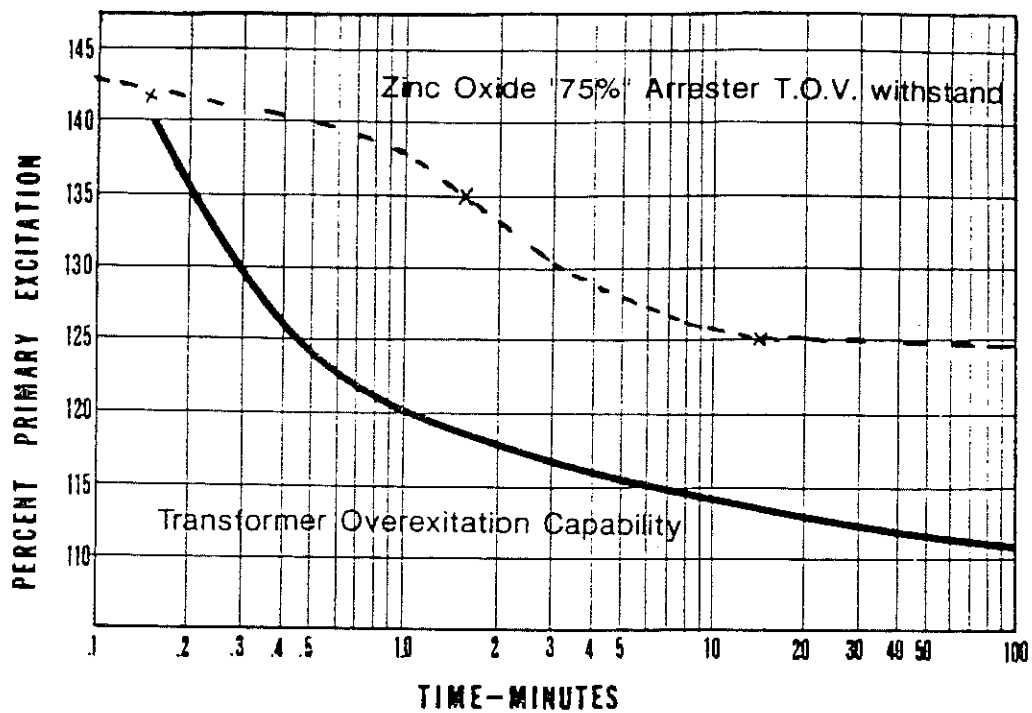


Figure 1 - Temporary Overvoltage (TOV) Capability of Power Transformer and Surge Arrester

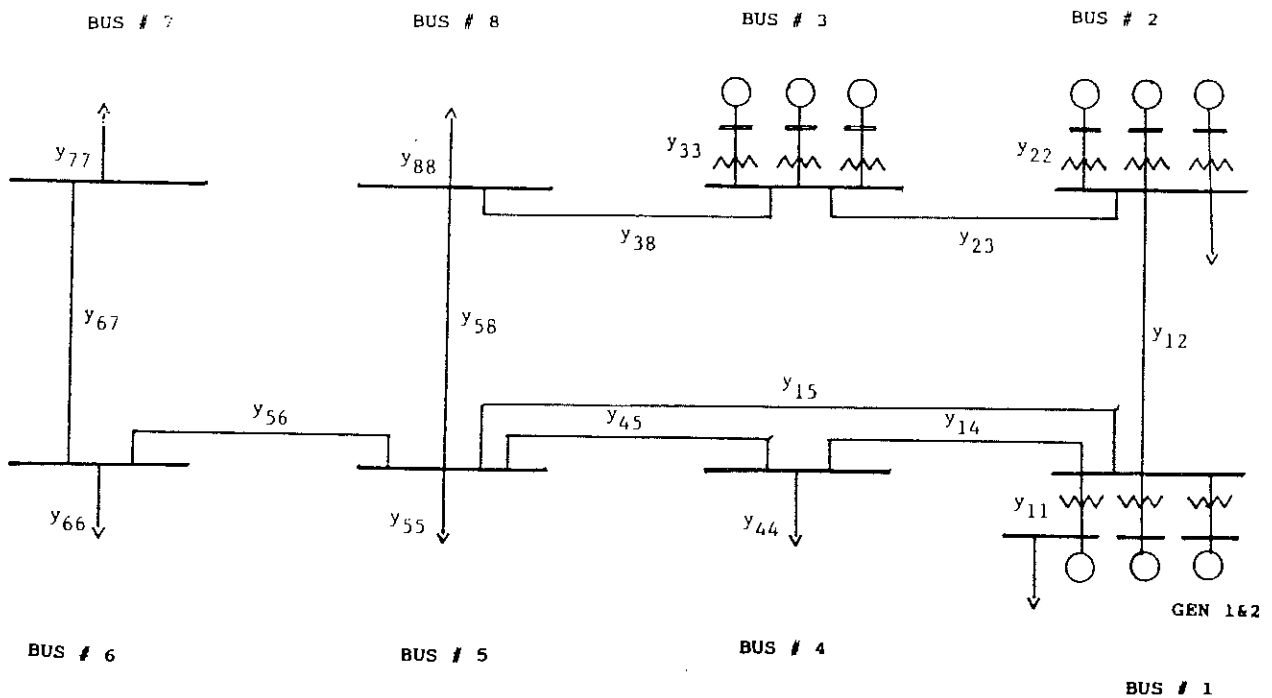


Figure 2 - The 220kV System

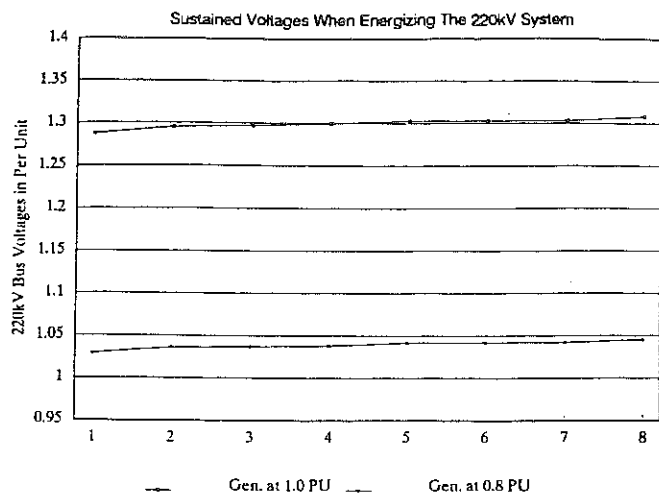


Figure 3 - The 220kV System Voltage Profile

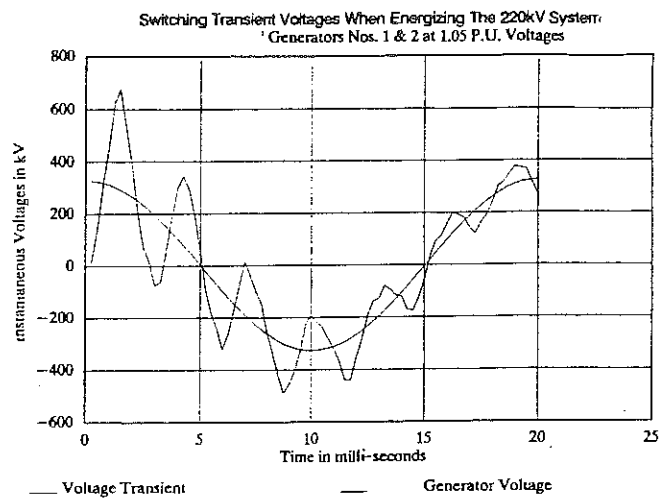


Figure 4 - Switching Transient Voltages When Energizing The 220 KV System - Maximum

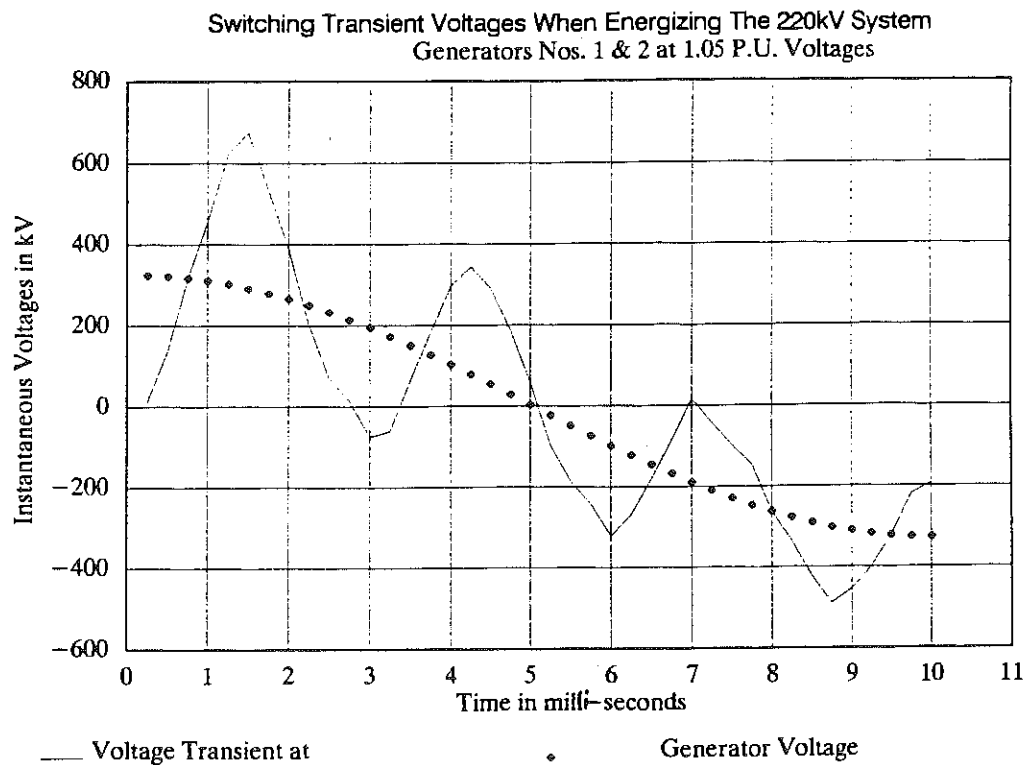


Figure 5 - Switching Transient Voltages When Energizing The 220 KV System - Minimum

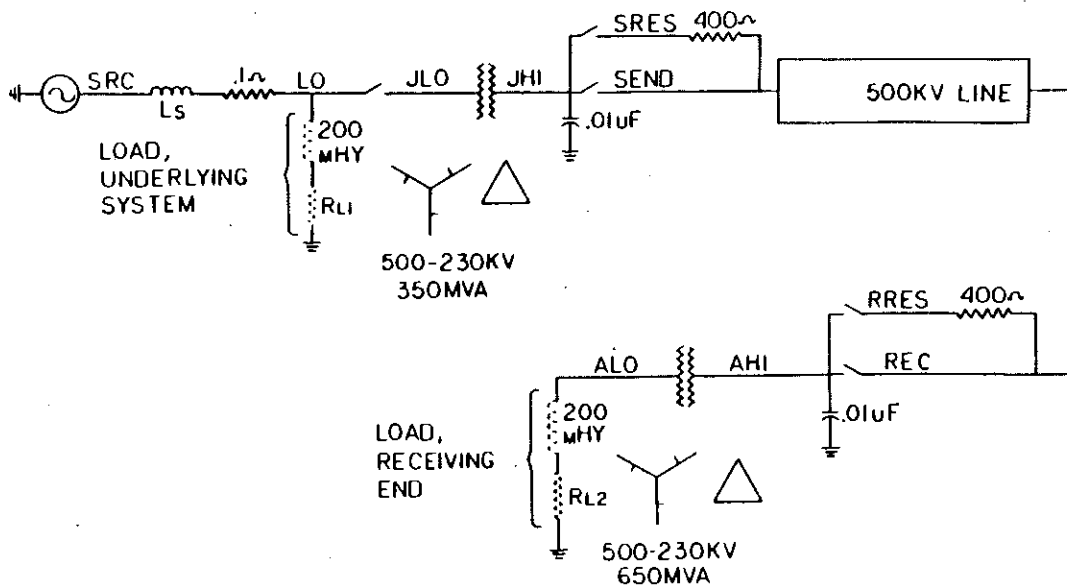


Figure 6 - EMT Model for Harmonic Resonance Study

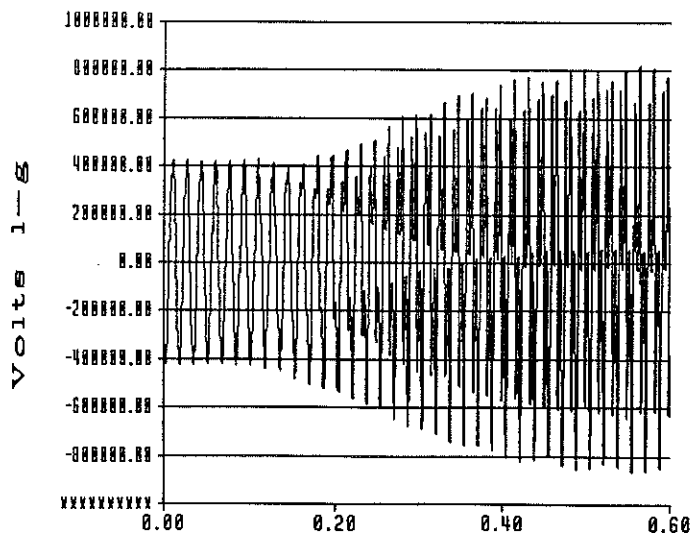


Figure 7 - Energizing Transformer Terminated Lines
No Load

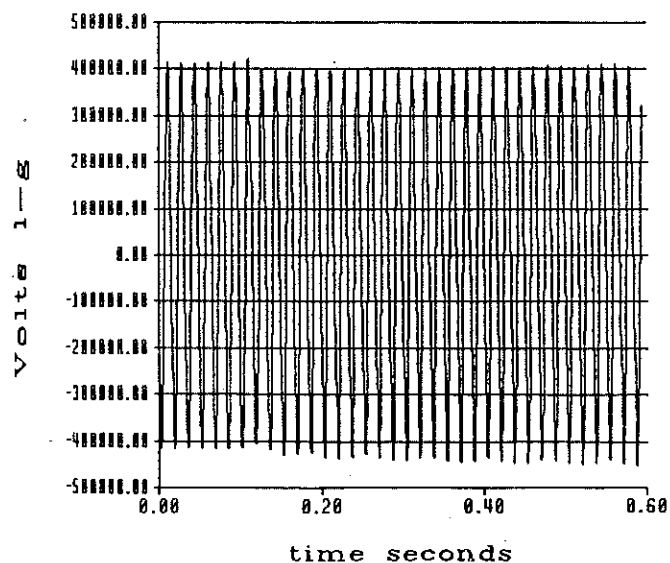


Figure 8 - Energizing Transformer Terminated Lines
200 MW Load on Underlying 230 KV

DISCUSSION - SESSION FOUR

Edited by Lester H. Fink, ECC, Inc.

Thomas J. Overbye (Univ. of Illinois):

On Bob's [Schlueter] paper, I would like to agree with him the need to calculate multiple voltage collapse modes or multiple indicators of voltage security of the system. I think this is a point that's been somewhat neglected in looking at the voltage security of a system. That is that we cannot come up with one index for the system security. There's a need to calculate a number of indices for the voltage security of the system. This comes because of the localized nature of the voltage problem because of the high reactive losses on the transmission lines. You can experience voltage prompts in number of areas in your system somewhat independently so that to really calculate from an operational point of view the voltage security, you would have to calculate a number of these indices and this is a point that I think Bob made well in his paper with the example he showed of the Pacific northwest system where there are a number of different areas that had independent indices associated with them. This is something that Chris DeMarco and I have also seen using the energy function technique in that we would have a number of different energy measures where each measure would be associated with the voltage security of a particular portion of the system so that I liked to thank Bob again for emphasizing this point in his presentation.

The second question is for Mr. Reppen's paper. That is in formulating his OPF problem it looked like the idea was to find the point which was the nose point of the PV curve and use this point to determine the maximum power transfer. My question is really (and I think he answered this partially right at the end of his talk) the intention isn't to operate at this point, is it, since that would be right at the point where the system would lose its voltage stability? If it's not the intent to operate at that point, is there a way using the OPF formulation to back off some distance from that point and get the OPF to solve some distance away from this nose point, so that you're somewhat maximizing your power transfer but yet you also have some amount of security margin. That's the question.

Christopher L. DeMarco (Univ. of Wisconsin):

This may actually be an attempt to get a dialogue going between two of the authors. With regard to Colin's paper, it seems to me the continuation power flow that he's

developed allows you to trace all the way back and work backwards to a parameter value that corresponds to the original loading you had, but gives you, if you will, the alternate low voltage power flow solution - in some sense a companion solution to the normal operating point which you started from. I hope I've interpreted that right. If that's so, then the dialogue I would propose is with Dr. Mori, where for your VIPI security index you're in essence looking to search for what roughly I would label the closest low voltage power flow solution on which this angle calculation could be based, and I would be curious if the continuation power flow is actually producing that same closest low voltage power flow solution. That might be a hypothesis worth investigating.

Claudio A. Canizares (Univ. of Wisconsin):

I've got a couple of questions regarding one slide that Professor Schlueter presented here regarding Hopf bifurcations. It was on the fly, I might have picked up wrong one of the points, but anyway I understood from the slide that he mentioned that Hopf bifurcations were detectable in the full transient Jacobian for the dynamic equations he presented. He seemed to suggest that a couple of rows in the Jacobian were linearly dependent; however, as he mentioned in his paper, I don't believe that Hopf bifurcations can be detected by looking at singularities of the Jacobian. The other question was with regard again to Hopf bifurcations, when he highlighted the fact, or mentioned the fact that Hopf bifurcations seem to be related to mechanical dynamics. I'm not sure about what he means by that, but my experience with that phenomena seems to point toward problems with voltage regulators in generators, rather a voltage phenomena than an angle phenomena; and in DC systems you can also observe it in a sort of closed loop voltage control called the voltage dependence or normal limit. If he might clarify those points, I'd appreciate it.

Marija Ilic (MIT):

I have basically two questions, for Dag Reppen and Begovic, on the OPF. It's been our experience with the method I discussed this morning that when you have serious voltage problems then the typical set of control actions includes load shedding, and my question is: how do

the methods you propose deal with that? Second question: I'm not sure I understand directly the relationship between the choice of the performance criteria, of which there was more than one, and the bifurcation point.

Lester H. Fink (ECC, Inc.):

With respect to Dag Reppen's presentation, I was a little curious. If I understood him he's using the optimal power flow with a performance index of minimum production cost. Is that right? I was wondering whether you took a look at how the situation would improve, how your transfer limit would be relaxed, if you looked at minimum reactive loss rather than minimum production cost.

Robert A. Schlueter (Michigan State Univ.):

In regard to the first comment, it turns out that, in the areas I showed in BC Hydro and BPA, there are several clusters of what I would call reactive modes of collapse in each one of those areas. The first thing is that when the people at BC Hydro have been looking at QV curves, they have not detected the most severe points of collapse, the ones that are most sensitive to contingencies. The second thing is that each one of the modes of collapse within a cluster is affected by different contingencies. Not only that; you can have contingencies that are very electrically close to one of those clusters causing collapse in clusters that are very far away. The point is that in a system you've got to know all the modes of collapse in order to understand what the stability is, and you have to assess which contingencies are affecting each one of them. It turns out that you are fortunate that there are within each cluster what we call super critical ones. There are critical ones that are very affected by contingencies far more than any others, and those are the ones you want to detect.

The second is a response to Mr. Canizares. His comment on Hopf bifurcations is obviously true, that you can't detect Hopf bifurcations by singularities in the Jacobian, but the reason I included it as a label associated with the Jacobian is that if you use a similar approach to what we use to flux decay bifurcations, and that is that you set all of the dynamic's derivatives to zero except the flux decay dynamics and back-solve everything in an appropriate sequence, you can show that the network sensitivity matrices, those that are associated with load flow problems in the network, are actually causing loss of damping in the mechanical dynamics, as we did for the flux decay dynamics in this paper. So now the question is whether the oscillations are mechanical or electrical in large systems. I think from what my understanding and experience are that there are a large

number of interarea oscillations that develop as one approaches collapse. Now there could also be electrical oscillations. They may well be not interarea but local, and we have a proposal to a large utility that experiences oscillations prior to collapse, to apply our methods and technique on a very large system to detect which kinds of modes are actually happening, if they happen first, and what actual modes are happening throughout the system. So that question perhaps is going to be resolved with time.

N. Dag Reppen (PTI, Inc.)

The first couple of questions related to using this for determining limits. Our intent was that, because the focus of the study was to look at the number of different methods and decide which one to use for determining operating limits. As far as the OPF was concerned, we decided it was not really all that good for determining operating limits. We apply this for what we know are the limiting contingencies, and we have to find them by doing screening studies. So you apply the OPF for each contingency. Now if you are maximizing the transfer for a particular contingency by adjusting the generation, you are indeed finding the best possible solution for this purpose for that particular contingency. That is not the way the system will be operated. You're not going to immediately reshuffle the power system or generation after you lose a line. So it's not really good for that purpose, but it is good for the planner and the operator looking at what are the options for preparing for a particular contingency, or if the contingency occurs, for the remedial actions. If you should use this for an operating limit you would need to have some sort of margin which covers a number of uncertainties. Some of these would relate to what we have neglected, maybe uncertainties because of dynamics. Some might be there because there are all these things happening in the system, particularly when it's heavily stressed. Some generator could fall off line, which could be quite likely; you might want to protect yourself against that. In the latter case you might want to use a megaVAr margin rather than a megaWatt margin. You can mix these two.

In response to Maria Ilic: load shedding is no problem. Load shedding is just an injection, like generator powers, so you can manipulate load shedding just as well as the generation. How does this thing relate to the bifurcation point? It does so in the same way as these PV curves and QV curves relate to it. All that we're doing here is asking the program to find the maximum feasible point, and they find that point. Similarly here, in the algebraic solution, the equality constraints we have describes this particular curve, and we're asking to find that minimum point, the minimum reactive margin.

Marija Ilic:

How do I know that minimizing fuel costs corresponds to that point?

N. Dag Reppen:

Consider a system with western and eastern portions, and make the cost of the generation in one area less expensive than in the other; quite arbitrarily. Our objective was not to come up with an economic dispatch, but to maximize the transfer. So calling the cost in one area one per unit and that in the other two per unit, we get our maximization. We could equally well maximize the transfer per se, it was just that this was as easy or easier to do. And that leads right into Les's question. Finding the maximum transfer will I believe take into account the appropriate level of reactor losses to attain the maximum transfer. The only purpose is to maximize the transfer, and if you do it this way, by saying that the sending end has uniformly lower cost than the receiving end, you will obtain that effect.

Miroslav M. Begovic (Georgia Inst. of Tech.):

I would like to make a couple of comments. First to Mr. Overbye, regarding his question about formulation of the optimal power flow which would back off the bifurcation point. The concept of remedial action that was presented in our paper did not include any of the voltage security nor stability criteria that would be taken as operating constraints. If that were done, then by extending calculations a step forward, and calculating a Hessian in both those sensitivities, we would be effectively able to guarantee backing off that point even though the system might not violate any of the other operating constraints. So such a formulation is quite a natural next step to extend on what was presented today.

Regarding Maria's (Ilic) question about load shedding: in all of the simulations that we did, we intentionally suppressed load shedding to preserve correspondence between results obtained with and without a remedial action algorithm. In all cases we've been able to reduce the sensitivities of generated reactive powers with respect to loading substantially. In cases of heavy loading when regular controls that we were applying (meaning generator set points, tap changers, and shunt capacitors) were exhausted, the algorithm would stop at that point; if we allowed load shedding in addition to that (which we consider some sort of an emergency control) then the system would end up not violating any of the constraints. Most of the time we would have line loading limit violations

or undervoltages resulting as a consequence of that approach, but load shedding becomes a problem when we near the bifurcation.

Ian Dobson (Univ. of Wisconsin):

Chris DeMarco raised a question concerning the continuation power flow, and the relation between finding the closest UEP with neural net methods and also finding it by continuation by going around the end of the curve. I wondered if any of the authors want to address that.

Colin Christy (Commonwealth Edison Co.):

The only thing I can say is that the real purpose of the continuation algorithm is a path following mechanism, which is different in that respect than homotopy, where you're after a particular point. So for this algorithm, we're just interested in following the path, and it's up to the discretion of the user to stop it. So I would think that you'd get your relevant UEP as long as you didn't have a branch that you'd jump off on to. There are algorithms for testing to see if you've got branches and that sort of thing; we haven't gotten that fancy. The other thing is to find the bottom solution corresponding to a high voltage solution; you'd also have to use a target point algorithm to hone in on the exact value of λ on the lower voltage solution. We again don't do that but in the math work on this type of algorithm, there're those algorithms too. So, it's all there for the taking. It seems like it would give you the right answer as far as what I know about the manifold. I don't know whether it's more complex than that.

Hirofumi Mori (Meiji Univ.):

My understanding is that the existing approach to find the critical point is quite time consuming. We should apply neural networks to find the critical point and thus take a short cut to find it.

Ian A. Hiskens (Queensland Electricity Commission):

We actually use a method that is based on the Euler method which can follow one manifold every way, instead of just finding the UEP. We've in fact, in a little three bus system found something like seventeen equilibrium points using this method, so it really does work for following the curve the whole way around and gives a lot more than just the two equilibrium points.

Claudio A. Canizares:

Just one little comment regarding finding a specific point using the continuation method. We were playing with that quite a while. Doing that is rather simple by just by cutting the size of the step you're taking. We've been quite successful in finding specific points in the UEP region.

R. Navarro-Perez (Westinghouse Systems Ltd.):

Just a comment on Mr. Reppen's presentation. I'm not quite sure whether the maximum MW transfer will correspond to the bifurcation point. If we assume that the OPF active power transfer was found due to some active power constraints, then that won't be bifurcation or voltage collapse.

N. Dag Reppen:

You are quite right. In working with these things, there are a number of constraints that could be active at the same time. For example, one of the things that we ran into was limitations on just the generation of dispatch; we ran into limits there to the point where that limited the dispatch rather than the voltage collapse. Also you may be seeing limits and constraints of voltage per se, in which case we would not be talking about the bifurcation point.

What I believe I've learned over the last couple of days here, if we have a constant NVA model, which we do have in our load model, and get to the extreme point, then you are indeed talking about the steady state bifurcation point. Have I learned the right thing here?

Marija Ilic:

A quick question for Dag [Reppen] and Begovic. On the convergence, what has been your experience when you are close to the bifurcation point? Is load flow worse or OPF.

N. Dag Reppen:

The OPF that you're using when you find these extreme points does not seem to have any kind of convergence problems whatsoever. Our experience has been, and I guess it's contrary to some other people's experience, is that if you use a Newton-Raphson algorithm and do approach the singular points in small steps, you don't seem to have much of a problem; that is, we can get to within a few megawatts of the actual extreme value on this 4,000-5,000

megawatts transfer. I'm sure that's system dependent; there may be some reason why on some other system it wouldn't work.

Miroslav M. Begovic:

Regarding convergence properties of the algorithm, our formulation was linear programming, based on linearization. We had to be very cautious about step size chosen. There are heuristic indicators that we proposed and that we followed and they dynamically changed depending on the distance that the system has from the bifurcation point, but we did not experience any problems with it, and compared to the regular load flow, the OPF is a little more expensive computationally, but not that much not to be used for our purpose. An additional comment regarding load shedding is that we have provision for an additional control called load transfer. Whenever a certain amount of load is possible to be connected to a number of buses by topology, then we investigate the possibility by sensitivity analysis to change the location of the connection of that load. When those opportunities are exhausted, then load shedding might be the consideration.

Peter W. Sauer (Univ. of Illinois):

I'd like to ask Professor RN [Napoli] about his NR method. We use to have a lot of trouble. You finished by saying "Try it, it works." We have tried the older methods of the enlarged standard optimization type problems and found, depending on the initial guess for the Lagrange multipliers, we could get many, many different solutions. Do you encounter any problems like that of converging to local minima or local maxima or whatever?

Roberto Napoli (Politecnico di Torino):

No actually, in our experience we did not have this kind of effect.

Robert A. Schlueter:

I wanted to make a comment in terms of running PV curves and QV curves again, but let me start with the following. If you look at one of the ways that you use a PV curve, it's a normal loading of the system in some pattern, and if you looked at an analogy to a building, what you're doing is putting some kind of weighting pattern on the roof and waiting for the thing to collapse. Well, the fact is that you will get a collapse, but it will generally be

one mode only; if you look at a different loading pattern, perhaps you could get another mode, and you could continue to do that. Generally the way you could get a mode is to go locally in real power. However, in our methods for loading and reactive power locally, what we do is pick out those areas that are theoretically most susceptible to collapse, so we don't have to do too many tests. The problem is that if you don't have some kind of structure to your search procedure, which using QV curves gives you, you could find out those that are most vulnerable to collapse. What you're doing in that case is hitting each node point in the building and finding out which ones potentially are close to collapse in terms of just adding stress, and then running contingencies against those to find out, if something happened - you took out a beam or something, a member of the network - what would happen to that particular mode. That is a far more effective search procedure. Ultimately, when you come back after you've understood that, you'd again want to go back and add transfers up to find which mode of collapse is going to hurt you for a particular transfer, for a particular combination of transfers for a particular loading level that you want to look at. You want to see how far you can go, and find out which group of generators are going to exhaust and ultimately collapse the system.

Nizam U. Ahmad (Oklahoma Gas & Electric):

I have a comment. You guys are going to write all of these operating procedures in about twenty-five pages, and the operators is going to read it in seventy seconds. Right? Do all of the switching, load shedding, switching, transfers, and whatever. I think we need to be slightly real, and look at the operator and see what he's doing and what we're giving him to do, whatever the window is, just before and after the collapse.

Marija Ilic:

I'll respond as related to the similar corrective action we were talking about this morning. You really end up not having that many steps to take. If you have one contingency then, if you've done your job right, there are only so many generators whose voltages you have to reset, and the operator sees that on the screen. There are no seventy pages of manuals there. This happens whenever you have another contingency or change in operation conditions. The matter generates again and says "You change three generator settings, and you may have to shed three loads and add two capacitors." That's a very complicated contingency where you've already had four generators out and the fifth goes out. That was a fairly realistic scenario

in the New England system. That is why I like the second criteria that Dag had there, that you minimize the number of control actions that you take. That's the one that's really going to do the job. If you're just looking for control criteria which are going to minimize losses, I think the typical answer is going to be changed a little bit everywhere, and that's going to kill the operator. If you've formulated your objective right, then it should not be too bad.

U. George Knight (Imperial College):

A suggestion connected with the discussion earlier on what should be the objective function: it seems to me under normal conditions it almost certainly's going to be minimum cost as you are proposing. Under emergency conditions or related conditions, it could well be to maximize transfers, and so I think you need a method to decide such basic things.

N. Dag Reppen:

This is another operator's friend. I think there are two issues. Number one is what you can do with mathematical methods. You can compute the sensitivities of the actions that you're proposing; you can look at how much range you have for the control actions; you can try to evaluate how effective they are. So you can do something within the methods and mathematical range. But then I think the operator has a lot of other constraints on him, a lot that are not easily formulated in a mathematical procedure, and maybe this is where an expert system can come in and do something.

U. George Knight:

In a real emergency, one of the things the operator may well wish to do is to re-switch the network. If you've got a real overload on the system that may well be the quickest way of getting out of it. Equally, some utilities do tend to do a lot more switching than others, and we haven't touched on that area at all. I think that's something that shouldn't be neglected in the operator's range of actions.

Louis G. Leffler (Public Service Electric & Gas):

I'm an operator. I've been keeping my head down, partly because of that and partly because I'm giving a talk tomorrow on the same subject. It's true that the operators have been handed an enormous amount of complexity in the

power system in the last ten years, from pretty much going by rote instruction and seat-of-the-pants to now literally tons of various remedial action schemes to solve a potpourri of different kinds of problems. I'm very sympathetic to our operators because of that. However, I think that the last two days pretty much convinced me (if I didn't already know it) that the complexities associated with the problems we're dealing with are so great that there's going to have to be a reaching between academia and us operators, so that we can solve our problems mutually. I don't think that we can expect simplicity because it's just not a simple situation, if we're going to continue to stress our power systems for economy, as Dag pointed out, the way we are and the way we're going to do it because there's just no other way to satisfy our customers demands economically. It's a complex situation and there's going to have to be a stretching between the two of us.

Miroslav M. Begovic:

I would like to make a comment on Maria's comment regarding the number of controls. In our formulation of the optimum power flow, we consider several aspects of that particular program. First, in the identification process of the violated operating constraints, we take the least number of constraints into consideration. For instance, if several constraints are identified, such as line overloads in the section of several cascaded transmission lines, then only the worst violation of one of those lines is taken and the other ones are assumed to be fairly coherent with that one. On the other hand, when we consider the choice of control actions and calculate sensitivities with respect to violated constraints, then those control actions that have sensitivities lower than a pre-specified threshold are automatically discarded. That way we minimize the number of constraints one way. The other way is by definition of the objective function itself which is supposed to minimize the range of action of operating constraints.

Ian A. Hiskens:

Just a comment regarding operator overload. Many of you know Walter Lachs. He has proposed an expert system for how to reschedule items of plant, etc., to overcome voltage collapse situations. QEC will be working with Walter however long it takes to see how this works. There are people who are looking at expert systems for this problem.

Carson W. Taylor (Bonneville Power Administration):

A comment on the operator overload problem: as a planner

or a control engineer, I would not expect an operator to do anything in the first two, three, or four minutes following a disturbance. We're putting in undervoltage load shedding, which is completely local control that operators have nothing to do with, that would shed up to 15% of our load, about 1800 megawatts in the Puget Sound area. That's one possible sort back up to reduce the concern about blackout; that's one possibility and there are other lots of other local automatic controls that can be independent of operator actions.

Mohammed J. Beshir (LA Dept. of Water & Power):

I was going to wait until tomorrow to make a comment before I leave, but I'm not going to be here until the end of the discussion so I'll make my comments now. Just a general comment. I'm from the planning division and a lot of the things you've been talking about are over my head, maybe that is understandable. Where I come from, I think we have voltage instability problems. We have some approaches. Maybe the first approach should be the QV curve; it is one of the things we've been using and it has been very helpful in trying to get a margin to stay away from causing any impacts to the system. Another thing we are looking forward to is some kind of dynamic approach, long time dynamics; people in my area are looking forward to getting a good handle on that approach and it seems to be logical from a transient analysis point of view; it seems to be an extension of what we're doing right now. Another thing I would like to get a good handle on is how load modeling affects voltage instability analysis; a lot of things have been said here and I think I've gotten a few things as they go, but that would be very helpful.

Session 5: Wednesday Morning

SESSION 5:

Utility Concerns

L. G. Leffler (Public Service Electric & Gas)

Voltage Control

*A Survey of the Present State-of-the-Art Voltage Control
with Emphasis on Potential Role of FACTS Devices*

M. D. Ilıc (Massachusetts Institute of Technology)

A Novel Method to Improve Voltage Stability Using Shunt Capacitors

C. W. Taylor (Bonneville Power Administration)

Load (and Induction Motor) Modeling

First-Order Induction Machine Models Near Voltage Collapse

S. Ahmed-Zaid, M. Taleb (Clarkson University), W. W. Price (General Electric Co.)

Imminent Voltage Instability with Large Induction Motor Loads

A. Semlyen (University of Toronto)

A Study of Dynamic Load Models for Voltage Collapse Analysis

K. Jimma, A. Tomac (Puget Sound Power & Light), K. Vu, C. C. Liu (University of Washington)

Distribution System and Load Modelling for Voltage Collapse Studies

U. G. Knight, C. Roa-Sepulveda (Imperial College)

Discussion

UTILITY CONCERNS

Louis G. Leffler
Public Service Electric & Gas Company

WHAT HAPPENS IN A VOLTAGE COLLAPSE?

The following scenario, as viewed from a System Operator's perspective, embodies some of the concepts of voltage instability.

You're sitting at your console (in an unnamed system operations center) on what appears to be a nice, calm, routine day. Loads are not spectacular, transmission flows are normal and well within limits. Everything looks just fine, the logs are prepared, and the boss even seems happy. You just begin to get up for a cup of coffee when... an alarm goes off. You check the bulk board and the computer, which indicate that a key EHV line has tripped. You begin seeking the reason and need to make a decision regarding reclosure. Data begins coming in on the computer as well as verbally from the field locations. Your interconnected partner to the west calls asking "what happened?" You request a contingency evaluation from the system security program. After a few minutes you have learned apparently that the line has tripped due to a terminal fault, so reclosure is inhibited. The security analysis completes in another minute or two and very disturbingly informs you "AC load flow non-convergent for contingent loss of generation in S.E. region." Further analysis indicates that, while there will be no particular thermal problems if a southeast region unit trips, voltages over a well-defined area will be extremely low.

So, the first contingency has been weathered and the system is in good shape. However, for that subsequent contingency the system is likely to be in desperate condition. In fact, you're not really sure what would happen in the event of the subsequent tripout. But, it's a nice day and you're not really expecting the second contingency to occur. However, you do begin restoration of system conditions to be prepared for a possible subsequent tripping. Spinning reserve is called for in the eastern part of the system with a corresponding generation reduction in the west. Since you are really concerned about the consequences of the second contingency, you order preparation for manual load shedding, just in case. Generation begins to come up. Everything looks like it is going to be OK. The boss has come out of his office and is hovering all over the place. You even consider the possibility of going for that cup of coffee when... you get a call from the control room operator in that southeastern region generating station.

With trepidation, you pick up the phone and he informs you that his 800-MW unit has a very serious tube leak. He expects that he will be unable to maintain water to the boiler and that he will have to trip it momentarily. You tell him that the system will be in a state of extreme stress and you implore him to hold off for at least another 15 minutes. He tells you he sure will try.

You now begin thinking about all the possibilities. You know you're ok right now, but if that unit trips prior to establishing the generation which you've already ordered, you may indeed be entering the realm of -- VOLTAGE COLLAPSE! You put an "all-call" message to the stations requesting them to continue loading as fast as they can and to start-up the gas turbines.

The lights DIM. You immediately look toward the generator chart panel and, with a sinking feeling, you see that the unit has indeed tripped. Your eyes next scan the total gen chart and you see that although an 800-MW dig is there, there IS response from all the other units (due in large measure to the well-maintained governors).

You know from your experience and from the EPRI "Tutorial on Interconnected System Dynamics" that:

1. The voltage is low now due to the tripping and in a short period of time the voltage will have a tendency to go lower yet due to:
 - the movement of distribution voltage regulators,
 - the reduction of field forcing of generator voltage regulators after control time-out;
2. Under a situation of this kind, there is a heightened possibility that a unit will trip due to field overcurrent;
3. There is an inherent possibility that lines will trip due to relay operation if the voltage is so low that restraint is removed and distance relays sense what appears to be a fault.
4. You probably have less than two minutes for effective action.

The lights come back to normal brilliance as the Operation Center's emergency generators cut in. You look at the critical bus voltage and see that it is at 87%.

You realize, from the above, your own experience, and the fact that this case has been studied in the past, that your best course of action is probably to do some manual load shedding. You know that the frequency is still at just under 60 Hz and that the system does not embody under voltage load shedding. The studies that have been done on this contingency indicate the need to shed approximately 500-MW of load in certain areas -- basically the eastern part of the system.

You call up the load shedding program on your computer screen. You arm the load shedding and select the regions and the number of megawatts to be shed. You know that the automatic controls have been tested frequently and you have confidence that the load shedding will be prompt and complete. All that is necessary is to press "EXECUTE."

You look up at the supervisor on-shift and he looks back at you. His face is grim. You say, "I'm going to shed 500 megawatts now." He nods his head; you press EXECUTE. You get acknowledge alarms from the computer indicating that 500 megawatts of load was shed. You breathe a sigh of relief as the system voltages pop back. In fact, your estimate of load shedding and location thereof has been so good that voltages at the critical buses are just about returned to their pre-disturbance values.

You realize the need to return balance to the system because the area control error chart, due to the magnificent response from the spinning reserve and gas turbines, the load shedding, and those governors, is actually significantly in the over-gen position, and you also know that you'll have to restore the 500 megawatts of lost load and you'll want to do this in the next half hour. But you have effected a good save! You look over at the supervisor and he has a partial smile on his face; in fact, he says "You've done a damn good job." You say, "Thanks." And all those folks in the field deserve a lot of credit too, as well as the planners who put together the scenario so that we knew exactly what to do.

Evidently, this was an occasion where, while we approached the "nose of the curve," the system was managed in such a way that it be retreated from the abyss.

I've got a few questions. Has this sequence of the events ever occurred? Yes, almost like that. Could this sequence of events occur on your system? I think there's a non-zero probability that it could. Are you prepared? I think there's a non-zero probability that we aren't. And finally, what are we going to do about it?

NERC CONCERNS AND ACTIVITIES

The topics of the remainder of my brief discussion with you this morning are:

The NERC Interconnection Dynamics Task Force (IDTF) - what it is;

The Interconnected System Dynamics Tutorial;

A voltage collapse questionnaire, which we put out to the systems about a year ago;

A survey of the voltage collapse phenomenon - a paper that you will receive after it is published by NERC;

Some insights - a few of many that we have gained from the discussions that we've had with a number of investigators;

Proposed action; and

Some next IDTF activities.

What in the world is IDTF? It's a group formed in 1985 by the NERC Operating Committee. It now reports to the Operating and Engineering Committees of NERC to look at our large dynamic complex systems. Its objectives are:

Identify areas for further study, including dynamic operation and reliability;

Examine the need to promote, urge, focus, persuade development of analysis techniques and operating procedures;

Recommend operating guides;

Recommend modeling practices and planning criteria for the Engineering Committee.

We do this through a variety of techniques including: questionnaires; roundtable exchanges; very importantly, discussions with experts; identification of R&D needs and urging their study; industry advisors; promotion or development of tutorial material for planners and operators.

The "Tutorial on Interconnected System Dynamics" that was put together and was published by EPRI in 1989 contains these headings:

Preface

Tutorial Scope/Organization

- Review of Power System Dynamics
- Overview of Power System Dynamics
- Frequency Deviations
- Voltage Deviations
- Power System Oscillations
- Effect of Power Plants and Their Control/Protection Equipment
- Additional Topics on System Dynamics
- Case Histories
- Glossary of Terms
- Equipment/Device Descriptions

It contains about 300 pages, and a hundred or so descriptive illustrations with graphic presentations. It is intended for operators of power systems - operators in control centers, and operators in power plants. It was done under the auspices of EPRI and IDTF industry advisors, and the actual preparation and development of the manual was done by our friends from PTI. This is available from EPRI. It is EL6360L.

The questionnaire which we sent out to the systems contained only two questions:

Do you believe voltage stability/collapse is an existing or impending concern for your system (or for systems in your NERC region)?

Please give us your view on monitoring and mitigating voltage stability/collapse problems.

It is a very simple questionnaire. I'm going to dwell for a moment or two on the first of those two questions: "Do you believe that voltage collapse is a problem for you?" We got responses from twenty systems representing eight of the nine NERC regions, and the response was an overwhelming "Yes." There were one of two "No"s, and one person said, "I have no problem but my neighbor does." This began an effort by the Task Force, with the approval of the parent committees of NERC, to study the voltage collapse phenomenon. This was to be not from the technical theoretical point of view, but to focus attention and to bring back information. A wealth of information will be brought back from this meeting and it will be of tremendous value, to the operators and planners of our power systems.

With their suggestion, we took on the subject of voltage instability and voltage collapse and we've been working on it for a couple of years. The suggestion was made that perhaps we could put all that learning together in one place, some sort of a little survey, if you will, and publish it by NERC. Well, that survey is now ready for publication; the draft copy is one hundred pages long. You will each get a copy of it. It consists of these segments:

- Introduction
- Recommendations
- Incidents
- Voltage Stability Analysis and Visualization
- Technical Insights
- References
- Appendices
 - Voltage Stability/Collapse Questionnaire Summary
 - Understanding Voltage Collapse in Bulk Transmission Systems
 - Voltage Stability, Parts I, II, III
 - Power Flow Simulation Methodology for Analysis of Classical Voltage Stability
 - A Conceptual Analysis of Voltage Stability as Related to Load Characteristics

The recommendations are all practical from the point of view of what power systems can do now. We will report on three incidents. There are several papers in the report. There are a number of technical insights which we have gleaned from the eleven or so investigators with whom we've had discussions, and a summary of the questionnaire. From the discussions with some of the investigators we came up with a potential definition of what we're talking about. It's a definition; it may not necessarily be subscribed to by everyone, but it's one that we think has some merit, and it's one that we can think about in physical terms.

A DEFINITION

SYSTEM NORMAL >>

- Increasing power flow
- System angle not challenged
- Decreasing area bus voltages
- Increasing reactive requirement
- Reactive exhausted

>>> VOLTAGE INSTABILITY

- Motor stall
- Increased reactive requirement
- Lower voltage
- System voltage below usable level
- Probable relay actions
- System separations

>>> VOLTAGE COLLAPSE

Some of the technical insights that were mentioned are:

Complex regions of attractions and "chaos."

One person said, "I have never seen anything as complex as voltage collapse;" and the greatest comment of all: "The whole state of the art of this today is that of comparing an inaccurate measurement with an unknown limit."

A few other things that we've learned, picked pretty much at random: The load commutating inverters are reported to have been a contributing factor to the problems in Tokyo a couple of years ago.

The time for collapse can be long. Many conditions can change. It's a non-trivial analysis.

Load tends to be constant P and nearly constant Q, and as time goes on more and more so.

Modeling of unexpected events: contactors and manual adjustments may not be modeled.

The self defeating tap changing that was reported on in one of their very first papers on Monday.

A solar magnetic disturbance: we have had ample evidence in this North America. An SMD can change system reactive requirement by thousands of megavars, and you don't know when they're going to happen. I have spoken with the people at NOA and they claim that forecasting solar magnetic disturbances is like forecasting the weather was a hundred years ago.

Voltage collapse is associated with areas, not with single buses.

We believe in monitoring reactor reserves. One of the recommendations is that we have to know them extremely well, together with the limits.

The importance of generator regulators: regulators, together with turbine governors are vital.

It's a misconception that was around for a while, that load shedding on the underside of the P-V curve can do more harm than good.

Undervoltage relays: there's a lot of things that can be done manually, but more importantly (I think as an operator) automatically, which include blocking TCs, increasing taps in the right places, and ultimately load shedding.

The problem, though it is complex, is solvable, understandable, and not mysterious.

So again, the same questions are posed: Has it occurred yet? Yes. Could it occur again? Are you prepared? Maybe yes, maybe not. What do we do about it?

In our discussions we have, not a well formulated plan, but at least some ideas for a proposal on which we encourage activity:

1. Develop, test, and validate a basic methodology for detecting potential voltage instability conditions and for providing operating guidelines to steer the system away from voltage collapse. After the basic methodology has been implemented, research could be continued to refine the methodology based on operating experience and to expand the methodology to encompass a wider range of voltage instability scenarios.
2. Develop a simple, yet accurate technique for on-line determinations of area or system reactive balance. Such a technique would be helpful in scheduling reactive power and, in conjunction with the methodology developed in Recommendation 1, would aid in maintaining adequate reactive reserve margin to minimize possible voltage instability.
3. Develop or expand system operator simulation tools to incorporate the concepts and techniques described in Recommendations 1 and 2.
4. Develop the format for and arrange a forum between planners, operators, and academics to exchange information relative to system operations and planning reality and the technical aspects of voltage instability and collapse analysis.

A SURVEY OF THE PRESENT STATE-OF-THE-ART VOLTAGE CONTROL WITH EMPHASIS ON POTENTIAL ROLE OF FACTS DEVICES

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I. Introduction

This paper is written with an emphasis on control coordination in voltage related problems. Hard engineering problems of reactive power supply and control are viewed more as a system-wide transmission, rather than simple local load supply problems. For example energy transfer over far electrical distances require coordinated reactive power support; historically generators have supplied reactive power. They are not sufficient in present operation, and many voltage controls are being supplemented. In addition to the slow, switching controls, many fast voltage controls are being implemented. This paper looks at the system theoretic aspects of coordinating all of the existing controls so that they do not cause undesirable closed-loop dynamics. Many concepts discussed in this paper could be found in the Proceedings of the First Workshop on Voltage Phenomena in Potosi, MO, in 1989 [49].

A primary objective of controls distributed throughout large electric power systems is to maintain frequency and voltages within allowable constraints as operating conditions on the system change. Under normal operating conditions, controls are expected to regulate system performance so that energy cost is minimized. When the system is subject to severe disturbances in its inputs and structure, controls are considered to be remedial tools that should act in unison to preserve system integrity and remedy the effects of a disturbance.

Most utilities employ a two-level approach to power control: a secondary level, at which systemwide changes are monitored in Energy Management Systems (EMS), and a primary (local) level, where information from the EMS is used by a human operator to make decisions about setting reference points at particular control locations or about switching in more power support. Specific control tools are activated at the primary level to maintain desired local set points. These could be activated manually or automatically—for instance, governors, under load tap changing transformers (ULTCs), capacitor banks, and automatic voltage regulators (AVRs). If the tools are automated, they are often time-set to begin

functioning at a time when load changes are expected, rather than being triggered by changes in the state of the system. At present, many power systems throughout the world are undergoing changes, for various reasons. This calls for exploring the potential of control tools that would respond more flexibly than they do now to changes in system state.

It is the premise here that many systems possess a certain amount of power reserves, which could be summoned in proper amounts and in response to system state changes to help achieve better system performance. It is believed that more adaptive control design could improve overall performance and slow down a rather expensive trend toward implementing excessive amounts of new power support on systems. (Even now, operational power resources could be equipped with more flexible microprocessor and electronics-based controls.) However, theoretical work to support such efforts is not simple and could be harmful if the theoretical assumptions do not adequately model the operating power system.

A detailed survey of the existing power control literature offers ideas ranging from those that view control problems as exclusively systemwide to those that would apply controls only at locations where a state violation is present. The latter thinking is based on the assumption that power changes do not have systemwide effects, primarily because of large power losses. This paper seeks to put earlier work in perspective and to suggest open questions in this area. A system theoretic formulation is emphasized.

Large-scale electric power systems are extremely complex, so up to now they have been designed very conservatively, to operate well within the region where dynamic problems would be unlikely to occur. The inputs to such systems have traditionally been viewed as quasi-stationary, that is, as long as first-generation inputs are adjusted for anticipated load demands, no dynamic problems are likely to occur. The main emphasis in such normal system operation is on the most economical scheduling of events; cost is measured in terms of total generation cost of a single utility.

A typical way to ensure that the system does not experience serious problems when subject to sudden disturbances (contingencies) has been to perform off-line

simulations of system responses to the most critical (frequent or serious) single outages and then, by trial and error combined with operator insight, to devise countermeasures to be taken in case such an outage occurs. Often, during the fault specification, system variable limits are relaxed. This approach known as preventive control approach, rather than on-line control, is very much ingrained in the current philosophy of operating interconnected systems.

Under such quasi-static conditions, not much challenge would appear to be left in operating a large-scale system. Computer-based software is viewed as useful for off-line studies, in preparation for actual events. The only time critical actions are called for in this kind of operation is when a sequence of unplanned changes takes place, as happened for instance, during the 1968 New York blackout. As such events unfold, operator actions or preventive type computer-assisted software do not suffice to establish control. Usually, such events are further complicated by human errors or equipment malfunctioning, and it is almost impossible to predict the proper constellation of automatic controls for such situations. In such extreme scenarios, modern control techniques often have not been very helpful because the underlying theoretical assumptions have not been valid. For instance, while many theoretical solutions do not take into account limits on controls and states, implemented on the actual system through a variety of protection devices, many out-of-control situations may have arisen because of the system's exceeding the limits of the controlling devices.

The power systems of today require more dynamic operating conditions than in the past, and the trend has been in this direction, beginning with the Public Utilities Regulatory Policy Act (PURPA) [1], which allows privately owned generators to sell electricity into the electric power system and requires that utilities buy it. Two changes apparent in modern power system operation are the presence in the single utility setting of non-utility-owned generators (NUGs), and the increase in the multiutility environment of unusual modes of energy trades.

The NUGs are smaller and more dispersed without having firm obligations to supply minimum energy continuously, and their price-driven load management could move what used to be a nominal operating point closer to steady-state stability margins [2]. This means that if an unplanned change (including an outage) occurs, system response will not be identical to the response anticipated by off-line simulations that did not include effects of NUG's. There is a great deal of concern about the impact of NUGs on the dynamics of power systems, particularly if their numbers continue to increase. Clearly, under such conditions, controls must become more responsive to system state changes.

Real Power/Frequency Control Versus Reactive Power/Voltage Control

The purpose of power system control is to regulate both frequency and voltages. While frequency is strongly affected by real power changes, voltages are affected primarily by changes in reactive power support and demand. The original design of power systems provided sufficient reactive power support to maintain voltage magnitudes very close to 1 p.u. In this sense, only frequency monitoring and control were of prime concern, because changes in frequency are very sensitive to changes in real power, and the use of real power is directly related to present-day energy tariffs. It is very common not to charge for reactive power demand but only for kilowatt-hours [3]. Only very recently are utilities charging large customers for excessive reactive power associated with their presence on the system.

In describing system operation, a comparison of controls primarily intended for reactive power/voltage regulation with the existing controls intended for real power/frequency regulation will be useful in describing qualitative differences between the two phenomena and outlining the respective roles of their related controls. One should recognize, however, that the voltage dynamics are not fully separable from the frequency dynamics and that each control affects both, i.e., complete system dynamics of the interconnected system. This is particularly true of high gain, fast controls, which are intended for voltage regulation but which change frequency dynamics at the same time. In this sense, separation between voltage controls and frequency controls may seem artificial in many instances. A fairly detailed review of the two level real power/frequency control can be found in [4].

Two-Level Approach to Reactive Power/Voltage Control

Most operating power systems are currently equipped with a large variety of reactive power/voltage controllers. Depending on the voltage level at which it is used, a particular type of control tool may be preferred over others. This often depends on the economic and reliability criteria associated with the implementation and operation of these tools.

The activation of voltage controllers may be manual or automatic, depending on the voltage level of the controlled system. Typically, operation of voltage controllers is more likely to be automated for distribution networks than for transmission systems, although this may not hold in every case.

The most frequently used tools for voltage control are shown in Table I [5]. Most of them are relatively slow and thus are geared toward controlling mid- and long-range system dynamics.

Table I. Reactive Power/Voltage Control Tools

CONTROL BY GENERATING REACTIVE INJECTION						
	<i>Continuously adjustable</i>	<i>Capacitive MVar</i>	<i>Inductive MVar</i>	<i>Fast response</i>	<i>Set point for local control centrally adjustable</i>	
Synchronous generators	Yes	Yes; but more limited.	Yes; thermally limited.	Yes, depending on excitation system may be less	Yes	
Synchronous condensers	Yes	Yes	Yes	Yes	Generally yes	
Static var control	Yes	Yes	Yes	Yes; under 1 second.	Maybe.	
Capacitor banks	No; one, two, or three steps	Yes	No	Depends on operating arrangements	Setpoint for switching maybe	Series capacitors mostly for stability
Reactors	No; usually just single units per line	No	Yes	Depends on operating arrangements	-	
Line dropping	No	Yes	No	Depends on operating arrangements	Usually operator decision	
Load dropping	Depending on operating arrangement potentially in relatively small taps	No	Yes, but connected with real power	Depends on operating arrangements	Depends on operating arrangements	Emergency only
Voltage reduction	Yes	Yes	No	Depends on operating arrangements	Depends on operating arrangements	
REROUTING BUT NOT GENERATING REACTIVE POWER DIRECTLY						
Transformer taps in transmission network	Usually in up to a dozen steps of 1% each	-	-	No 1 tap per control step	Depends on location and operating arrangements	Rerouting flow does not directly generate MVAs
Transformer taps between transmission and individual subtransmission networks	-	-	-	-	Usually, if located in main substation	Keeping subtransmission on voltage up source of problem in emergencies
Transformer taps in radial subtransmission and distribution	-	-	-	-	Usually not	Keeping distribution voltage up source of problem in emergencies

Although very little attention has been given to exploring the potential of fast controls for improving the systemwide voltage profile, this could be done. The impact of electronics-based control equipment has made these ideas much more feasible than in the past [6-8]. As is the case with real power control philosophy, the two-level approach is fully ingrained in reactive power/voltage control design. In normal system operation, the trend by U.S. utilities has been to separate the control of steady-state set points (optimal load flow control) from the control design for voltage dynamics, governed by the natural system response to changes. This clearly conforms to the two-level approach [4]. The information structure for advanced voltage control is currently available on most U.S. utilities (system control and data acquisition (SCADA) systems), leading to the on-line monitoring of the systemwide voltage profile. Most control actions are still taken by a human operator, based on insight about the best operation of the system. Some utilities are experimenting with tools like optimal power flow [9]. The process of updating voltages as a result of optimal power flow calculation has not been automated on any utility.

The philosophy of voltage control for more severe system changes has been based on preventive control approaches similar to those utilized in controlling real power flow. Most abnormal voltage conditions are anticipated off-line by contingency screening algorithms, and control planning is supposed to prepare for worst-case contingencies. Extensive load flow simulations are currently performed by most utilities to anticipate worst-case voltage problems (see, for instance, [5] for a report on the Pennsylvania-Jersey-Maryland (PJM) system experience). It appears at this stage that most strategies for voltage control under hard operating conditions are a combination of preventive actions planned off-line and operator decisions. The French voltage control strategy [10, 11] and in part the Italian are the only exceptions; they look toward more automated ways of controlling voltage at the secondary level.

All control strategies discussed in this paper are assumed to be of a two-level (decentralized) nature unless stated otherwise. It should be noted that any two-level control scheme is based on the assumption that the power system dynamic response (primary control-driven) and steady-state responses (secondary control-driven) could be controlled independently of each other. This property, although not much studied in the systems theory literature, is one of the most striking differences between centralized and decentralized control design. We give some examples under which this assumption is not satisfied and discuss problems that evolve with the present state-of-the-art control logic in this case. For example, in discussing the coordination of underload tap changing transformers (ULTCs) a point is made that

conditions may arise under which currently applied decentralized controls cannot stabilize the system whereas centralized control can, for the same operating conditions. An interesting example is also given in [12] of a class of controls (proposed in the context of excitation control improvements on generators) for which the steady state voltage control design could be viewed completely separately from the control design for the primary controllers. The potential of this class of controls for improving voltage-related dynamics appears to be significant.

In order to understand dynamics related to voltage problems, and their dependence on related controllers, we review primary control design and some state-of-the-art results by introducing control-driven dynamics concepts. Viewing slowly evolving mid- and long-range voltage dynamics in this context could prove useful, since under mild conditions, most of the evolving voltage dynamics are control-driven.

Secondary Real and Reactive Power/Controls

Both real and reactive power control at the secondary level are based on the principle of monitoring systemwide steady-state data, using SCADA systems and adequate software to assist the operator in his decision making. The main decisions are concerned with resetting reference points on all local controllers of real power/frequency (such as governors, HV-DC links, and phase-shifting transformers) as well as on all local controllers of reactive power/voltage (such as excitation systems, static var compensators, synchronous condensers, and ULTCs). Decisions about when and how much new real power and reactive power support (shunt capacitor and reactor switching problem) should be connected on the system are also made at the EMS level.

While in the past, much more work has been done on real power secondary control methods reference [4] provides a survey of available reactive power secondary controls.

Primary reactive power/voltage controls

Although natural system response is such that frequency dynamics are fastest and need to be stabilized quite rapidly in case of transient instability, the stabilization resources are quite scarce. Governor-based frequency regulation is much slower in character and cannot be used for transient stabilization. On the shortest time scale available resources are HV-DC links, fast valving braking resistors, transmissionline switching [63] and PSSs. More recently, fast-modulated phase-shifting transformers are considered as a new option [13].

Studies of system dynamics under the assumption that no transient instability occurs, i.e., that the fastest dynamics of δ and v are asymptotically stable, are relevant for understanding mid-term voltage dynamics and the effects of AVR/PSS on

their stabilization; the time frame of interest is shorter than the time constant of governor control. Reduced-order modeling needed to extract the dynamics of the voltage behind transient reactance E'_q in particular, while taking into account the effect of fast variables and a fast AVR, is more involved. It requires systematic procedures for model reduction based on singular perturbation techniques [14-16, 18] and the selective modal analysis [17]. It is not surprising to find that at least one variable associated with the AVR must be preserved in such a reduced-order model [12, 18-19]. This is so, partly because high gain controls [20-22] are often incorporated into modern AVRs, and lead to qualitative changes in the time scale separation process of the open-loop (natural) system dynamics. A theoretical question not well understood at present is the effect of high gain AVR/PSS on the transient stabilization of system frequency. While an AVR was originally implemented to directly regulate the terminal voltage of the machine (it is therefore recognized as a primary voltage controller), the addition of a PSS has led to recognizing the AVR/PSS as a means for transient stabilization of frequency.

On the slowest time scale, the governor controller regulates frequency by controlling mechanical power output T_m . This control is only effective provided faster variables are asymptotically stable. Standard model reduction techniques are directly applicable for deriving a simpler model for studying the effect of controls that regulate T_m based on that frequency output; they are often employed in analyzing the effects of governor controls.

Given a two-level control philosophy, system dynamics are only relevant to reactive power/voltage control at the primary level. This is one of the sources of confusion in trying to separate dynamics-related issues from those of steady-state operation. Since most of the effort is currently focused on designing new control centers on the secondary level, voltage-related dynamics are typically not discussed. Two-level control, as applied to power systems, relies on an implicit assumption that the transition from one steady state to a new steady state is always well defined, that is, asymptotically stable. If this assumption is not satisfied, the scheme may have problems. In power systems terminology, this means that viabilization and stabilization problems [23] are treated independently. In this context, it is assumed that reference set points of primary controllers are assigned from the secondary control level and known to primary controllers. Possible consequences of centralized control design, when this approach is not taken, are discussed later in this paper.

A look at Table I shows two qualitatively different types of reactive power/voltage control tools, the first being continuous and fast (like AVRs on generators) and the second being discrete in space and time and much slower (like ULTCs and a variety of capacitor banks). These two groups

of controls have evolved historically. It was easier to control the entire system under the assumption that its natural dynamics would reach a new equilibrium point between two discrete control actions. Natural system dynamics subject to fast continuous control result in fast closed-loop dynamics, on which slow control-driven dynamics are superimposed. We state that under the mild assumptions on system response, slow voltage dynamics are primarily driven by a variety of controls. In formulating voltage dynamics, because of the fact that they are primarily control-driven, it is important to formulate clearly the dynamics of interest and the conditions under which such a model holds. Otherwise, this could be a cause of misunderstanding when trying to distinguish dynamic from steady-state phenomena. For example, in studying the effects of slower discrete controls, it is typically assumed that generator dynamics are such that sufficient fast control support exists to maintain terminal voltage at the set point assigned at the secondary level. In this case, the generator model is treated as a static PV bus, with the terminal voltage taken as a parameter. Then the only state variables are voltages and tap changer positions, a type of dynamics reported in [24-26] and briefly reviewed here as well. Similarly, when the effects of AVRs are studied on a very short time scale, tap changer positions are assumed to be given parameters, not states, implying that the dynamics of generators reach a new equilibrium point before the transformer tap changes its position.

With the impact of power electronics-regulated voltage control tools such as static var compensators [27], it is possible to visualize a scenario where fast and slow dynamics start interacting when a delay time associated with discrete control tools is allowed to be shorter. Serious study is needed to assess the potential impact of such a change in control philosophy on the existing equipment in order to properly explore the potential of faster controls for the voltage profile. In this chapter, both fast and slower (discrete) dynamics are modeled. An example of a fast voltage controller is the conventional AVR/PSS system. Slow control-driven voltage dynamics are briefly reviewed later in the paper. Further understanding of these model properties would lead to answering more practical questions. Mathematical conditions under which control-driven dynamics due to fast and slow control tools could be separated need further studies.

Electric Power Systems of the Future—Flexible AC Transmission Systems (FACTS)

Some new efforts have been made toward implementing more flexible control devices on existing transmission networks, which could bring system performance closer to meeting modern energy needs [28-29]. Several studies are attempting

to demonstrate a gain due to implementing such new devices [30-32]. The recent tendency to operate systems closer to their steady-state stability margins requires the availability of such tools for system regulation. In thinking about an existing power system as a flexible AC transmission system (FACTS), the role of the control designer may become more critical than in the past. Here we briefly review the main concepts related to the FACTS idea; more details can be found in [28, 32]. Our main emphasis is on recognizing the complexity of coordination required by FACTS devices. Open control theoretical questions are discussed.

First, the steady-state concepts related to the FACTS idea are quite simple. To illustrate the most representative ones, we refer to a two bus power system. Recall that its main purpose is to deliver energy from generator bus #1 to load bus #2. This energy transfer is based on the simple formula that the real power delivered to load P_2 is

$$P_2 = \frac{E_1 E_2}{X_{12}} \sin \delta_{12}$$

Clearly, P_2 could be increased by reducing the inductance of transmission line X_{12} , or by increasing the voltage phase-angle difference δ_{12} , for example. If a capacitor C_{12} or a phase-shifting transformer were inserted in series on the transmission line, all other conditions being the same, the real power delivered to the load could be increased.

The basic role of many other tools recognized as FACTS tools can be explained similarly to the role of C_{12} and a phase-shifting transformer in their steady-state operation. However, very little is understood about the potential effects of FACTS devices on the system dynamics if they are controlled in very fast mode. For example, the question of whether the system would gain anything if a given C_{12} were modulated fast via power electronics and microprocessor-based tools has not been studied from the viewpoint of the effects on system closed-loop dynamics.

Modeling and control questions due to primarily new modes of energy management, taking into consideration that FACTS tools would play a major role in implementing more sophisticated control strategies are assessed in detail in [4].

One of the main conclusions is that at present good controls (i.e., their hardware) can be designed at the primary level but that the systemwide effects of these controls are not clear. Also, it is challenging to design controls with least losses on control devices. Some work in this direction has begun [33, 35]. It has been uniformly accepted that two-level, or decentralized, control design is the preferred mode for fast stabilization and regulation of power system dynamics. This paper therefore, accepts the deviation from this approach since

it allows that slower switching-type devices could be subject to centralized control, if absolutely necessary.

Further, recognizing the fact that many advanced nonlinear control techniques for stabilization on the primary level have been proposed in the recent past [36, 40], it becomes important to define and evaluate each type of controller in order to choose the most efficient controls.

Relevant modeling problems as related to more flexible control design are assessed [4]. On the control side, the basic premise of separating steady-state (secondary) system monitoring and control from system dynamics control on the local (primary) level is revisited. Possible scenarios of condition violations are discussed in [4]; the need for and the role of partly centralized control in such situations is emphasized.

The consequences of violating the separation principle on changes in secondary and primary controls are critical. The role of more advanced control design is viewed in this light. For the first time in the literature, the critical role of FACTS devices as basic tools for structural systemwide stabilization is emphasized for improving system performance under unusual conditions.

Modeling issues relevant for adequate voltage control design

It is intriguing to stop in the middle of talking about changing operating modes on the interconnected power system and, while being tempted by more sophisticated FACTS hardware, to assess the adequacy of current modeling and control practices on this system. As has been rightly pointed out many times, modeling often composes 80 percent of a control design process. Modeling of specific systems involves experts on the way these operate and experts familiar with a rigorous system theoretic approach. Modeling the dynamics of electric power systems has had a long history and is well understood.

The complexity of an interconnected power system can be appreciated by examining its general structure, [4]. Depending on the phenomena studied, a different degree of model detail is required on each device level as well as on the interconnected system level. For this reason, modeling has become an art in its own right in the power systems community. Excellent references are available on systematic model reduction, for example [14-16, 41].

It is widely recognized that models applicable for short-term modeling are qualitatively different from the models applicable for mid- and long-term dynamics. Historically, only very short-term dynamics have been studied to assess if the system maintains its synchronism. Since voltage (magnitudes) did not change much because of a conservative design and the provision of sufficient reactive power support,

a typical modeling assumption led to employing the so-called classical model for machines. A further direct consequence of this assumption is that the model of the interconnected system could be represented in the standard state-space form [42], defined by a set of coupled ordinary differential equations. Much research has been done on analysis of such models.

However, with the more prominent role of voltage dynamics in system operation, higher-order models of specific components, electrical machines in particular, which capture electromagnetic as well as electromechanical phenomena, are needed. Although a higher-order model on the device level should not critically increase the interconnected system model complexity, in this case a qualitative change of model structure takes place because the interconnected model cannot be expressed as a set of ordinary differential equations. Coupling algebraic equations are introduced, and the typical dynamic model is defined as a set of differential algebraic equations (DAEs) [43]. The complexity of defining a consistent set of initial conditions for such models, as well as the sensitivity of the same response if the conditions are not consistent, is just beginning to be studied [44, 45].

A new question is how realistic are present state-of-the-art models for analyzing the effects of FACTS tools. This is an area where much new modeling effort is needed. Modeling methods employed in power electronics [46] need to be combined, most likely, employing averaging techniques [47] to capture the effect of fast power electronics-modulated dynamics of FACTS devices on frequency and voltage dynamics. Time scale separation techniques, carefully nurtured in power systems modeling, are bound to play a major role in generating models for power electronics-modulated closed-loop power system dynamics.

The presence of some of these new devices on the system is expected to violate some of traditional assumptions about transmission system properties. In particular, symmetry of transmission lines cannot be automatically assumed when phase-shifting transformers are present on the system. This will have a direct effect on the need to modify existing programs for eigenvalue analysis, for example.

In modeling mid- and long-term system dynamics, the effect of slowly switching control devices, such as ULTCs and capacitor banks, could be analyzed by investigating closed-loop, control-driven mid- and long-term system dynamics [24, 26]. Instead of the common practice of simulating this process as a sequence of equilibria, each corresponding to the new tap position or the new number of capacitor banks, a formulation of slow dynamic models, primarily control-driven, sets a basis for investigating closed-loop system stability design of new stabilizing controls for unusual operating modes [48]. A simple example of the averaging process for incorporating

faster, in this case natural system, response, can be found in [24].

Open Research Questions Regarding the Two-Level Control Concept

To emphasize the importance of assumptions under which monitoring and control of large-scale electric power systems are currently developed, a specific term for one of the assumptions was recently proposed in [49]. It is referred to as the separation principle, and its definition is reviewed here.

Definition 1. *The separation principle of control design for power systems is an assumption that the systemwide control design can be performed in two independent, separate steps. The first step is to control the systemwide equilibrium, (steady state performance) under the assumption that the system dynamics are stable in going from one equilibrium to the next. The second step is to control the system dynamics, knowing that the unique equilibrium exists.*

This definition implies that secondary monitoring and control methods are only steady-state in nature, while primary controls only deal with stabilization problems, assuming that the unique postfault equilibrium exists in the region of interest. Obviously, the separation principle holds if primary control does not destabilize the system, or if transition from one equilibrium to the next is asymptotically stable.

In questioning the validity of the separation principle one should point out that under abnormal operating conditions two-level control may not work [24]. On the other hand, given all operating conditions identical and the option to control the power system using full (centralized) information about its dynamics one may succeed in establishing an acceptable equilibrium. Examples of this, related specifically to slow voltage dynamics, can be found in [24], where it is shown that centralized control of ULTC dynamics leads to a viable equilibrium whereas, two-level control does not manage to coordinate systemwide voltage dynamics. In this case voltage collapse [49] is prevented by coordinating slowly evolving mid-term voltage dynamics, provided the validity of the separation principle is not assumed a priori. Of course, the centralized solution proposed in [24] requires much more communication than exists under present two-level control.

A second way to improve system performance under a separated two-level decentralized design is to make local controllers more dynamic [50]. This is probably the most promising path, since fast reliable communication cannot be assumed to be readily available. Using decentralized dynamic compensation will require more local measurements than are currently used. Robustness issues to model and parameter uncertainties may be critical in determining the actual success of the implemented controls. Recent voltage problems due to slow switching controls are related to some known sufficient

conditions for the separation principle to hold, at least when the system is subject to slow controls. An illustration is given on a simple two-bus system of the controls and their dependence on these conditions.

Some Sufficient Conditions for Slow Controls to Guarantee Systemwide Stabilization. In attempting to understand the operating and mathematical conditions under which the separation principle may be violated, we revisit cases of slow voltage controls "malfunctioning" [24]. In particular, many instances have been reported of ULTC transformers equipped with the latest control logic reducing directly controlled voltages, instead of increasing them as desired. The system is said to be experiencing voltage collapse, since the primary voltage controls do not manage to bring voltage within the threshold acceptable values.

A simple engineering explanation of inadequate ULTC actions is as follows. Parameters of the steady-state π -equivalent model of a ULTC transformer change as a function of tap position a . It should therefore be clear that a ULTC does not generate power; it only distributes it from bus i to bus j and effectively manages to increase voltage at bus j at the expense of reducing voltage at i . (In the case when i is a generator bus, it requires the generator to produce more reactive power in order to maintain voltage.) At extremely low voltages the ULTC tries to maintain voltage at j , E_j , close to E_j^{ref} , although there is a severe reactive power deficiency at bus i , lowering voltage at bus i even further. As a result, the sending-end voltage could drop below acceptable limits.

The most common automated control on ULTC transformers at present is of the form [51]

$$a_{i,j}(k+1) = a_{i,j}(k) + d_j f(E_j - E_j^{ref}) \quad (1)$$

where subscripts i and j indicate that the ULTC located in the line between buses i and j monitors and controls voltage at bus j ; d_j denotes the step size in the change of the tap position during one operating cycle; and $f(E_j - E_j^{ref})$ is the control function governing operation of the ULTC and is given by

$$f(E_j - E_j^{ref}) = \begin{cases} 1, & E_j - E_j^{ref} > \Delta E_j \\ 0, & |E_j - E_j^{ref}| \leq \Delta E_j \\ -1, & E_j - E_j^{ref} < -\Delta E_j \end{cases} \quad (2)$$

At each tap position the transformer should also satisfy the load flow equations. While the conventional approach of treating tap position a as a parameter in the load flow equations and recomputing new voltages each time a changes is useful for simulating mid-term voltage changes due to transformer actions, it cannot be used for predicting if the transformer will bring the system within the desired threshold of E_j^{ref} , given the initial voltage and reactive power

demand. Predicting the effect of ULTC transformers was for the first time formulated in [24], in which a sequence of slowly changing directly controlled load voltages via ULTC transformers is viewed as slow voltage dynamics. Approximate equations for such dynamics are derived in [24] by combining the ULTC control function given by Eqs. (1)-(2) and the load flow equations linearized around the initial steady state voltages. Without reviewing details of this model, we state it here because it is important for further conclusions. The model takes on the form

$$\underline{E}(k+1) = \underline{E}(k) - ADE(\underline{E} - \underline{E}^{ref}) \quad (3)$$

where $\underline{E}(k+1)$ and $\underline{E}(k)$ represent load voltages directly controlled via transformers; matrix A is typically a low-order matrix of dimension $m \times m$ (m being the number of ULTCs) and is defined as

$$A = C \left[\frac{\partial Q}{\partial E} \right]^{-1} \left[\frac{\partial Q}{\partial a} \right] \quad (4)$$

and C is a matrix of zeros and ones extracting the controlled load voltages from the vector of all load voltages. Matrices $\left[\frac{\partial Q}{\partial E} \right]$

and $\left[\frac{\partial Q}{\partial a} \right]$ are computed once around the initial operating point. In the same work [24], network conditions are derived that are sufficient for directly controlled load voltages $\underline{E}$, to converge to within threshold $\Delta \underline{E}$ of a desired $\underline{E}^{ref}$. They can be summarized simply by requiring that matrix A given in Eq. (4) be a positive definite matrix. From a systems theory viewpoint, one could talk about sufficient conditions for the slow dynamics of Eq. (3) to be mid-term stable. Therefore, the condition on matrix A can be used as a test for predicting the qualitative behavior of transformers instead of performing a sequence of load flow computations each time a tap position changes. Further extensions of this result to more realistic scenarios when transformers do not react equally fast to a voltage violation (different duty cycles) or when the taps change in different step sizes can be found in [25].

In the simple two-bus example, matrix A becomes a scalar number of the form

$$A = 1 \cdot \left(\frac{\partial Q_2}{\partial E_2} \right)^{-1} \left(\frac{\partial Q_2}{\partial a_{1,2}} \right) = \frac{\partial E_2}{\partial a_{1,2}} \quad (5)$$

For the ULTC to "malfunction" in this case, $\left(\frac{\partial E_2}{\partial a_{1,2}} \right)$ should become negative, meaning that an increase in $a_{1,2}$ causes a decrease in the controlled voltage E_2 , which is contrary to the basic reason for designing a control function of the form of Eq. (2) to increase controlled voltage. This means that under certain unusual conditions it could happen that, because of the network conditions, the basic control function of Eq. (2) does not have the desired systemwide effect of increasing load voltage E_2 by increasing the tap position $a_{1,2}$. Further analysis of a sign change in Eq. (4) shows that the sign change could occur if either $\left(\frac{\partial Q_2}{\partial E_2} \right)^{-1}$ or $\left(\frac{\partial Q_2}{\partial a_{1,2}} \right)$ change their signs.

A very interesting connection with conventional voltage collapse analysis [49] is apparent here. Recall that on the upper part of a P-V curve [49] $\left(\frac{\partial Q_2}{\partial E_2} \right)$ is always negative, i.e., an increase in load demand causes a decrease in load voltage. This is the normal expected system behavior. However, on the lower part of the P-V curve, which is declared a voltage collapse area, $\left(\frac{\partial Q_2}{\partial E_2} \right)$ becomes positive, and a decrease in load demand causes a further decrease in load voltage. Note that for larger power systems (more than one load) cancelation of terms in Eq. (4) is not valid, and therefore a simpler criterion (5) for $\left[\frac{\partial E}{\partial a} \right]$ is not applicable. Moreover, the entire matrix A needs analyzing. In this case, matrix $\left[\frac{\partial Q}{\partial E} \right]$ could change its nature, i.e., become a negative definite matrix. Understanding the preceding phenomenon would require new technologies for detecting specific causes for changing the qualitative nature of $\left[\frac{\partial Q}{\partial E} \right]$. In the case of one ULTC, the sensitivity $\left(\frac{\partial Q_2}{\partial a_{1,2}} \right)$ is always positive and therefore does not affect the sign of matrix A in Equation (4). In systems which have many transformers, the effect of matrix $\left[\frac{\partial Q}{\partial a} \right]$ on the nature of matrix A needs careful studies. Numerical results indicating the change of behavior in matrix A for some

voltage collapse scenarios in the six-bus Ward-Hale example can be found in [24].

To summarize, it is straightforward to conclude from the preceding analysis that if modified control strategy for ULTC transformers were proposed that took into account network and loading conditions on the system instead of just the regulation of voltage values, many present problems could be avoided. Examples of such new controls can be found in [48].

An open question here is whether designing systemwide voltage controls so that the closed-loop steady-state equations satisfy the M -matrix property for the entire range of viable operating conditions is the best way of making the reactive power network behave similarly to the real power network of the same system. All indications point in this direction, but it is not easy to prove this thinking rigorously, more research is needed to support the idea. Some other analytical results and newly introduced definitions relevant for understanding the voltage transfer problem [49] seem to indicate the same results. However, to achieve a similar behavior for voltage propagation as for real power propagation, it is clear that reactive power support will be needed to compensate directly for large reactive power losses and that it would have to be electrically close to the causes of most reactive power losses, like very long transmission lines. Equipment allocation to achieve such performance could form a challenging synthesis problem. This process is definitely not unique and different criteria (like implementation and maintenance costs, and rights of way) would influence the choice of the most attractive voltage control support for a given system. FACTS-based tools could play a major role in enabling the transmission system to preserve the M -matrix property, for example, even in unusual operating conditions.

A direct consequence of preserving the diagonal dominance property of equations defining real power steady-state and the M -matrix property of equations defining steady-state dependence of reactive power/voltage in the region of interest for steady-state voltages and phase-angle differences is that the unique equilibrium will exist. This means that if these conditions are satisfied, the secondary control level will provide for equilibria around which the transmission network will behave as expected. The most important implication of this is that present state-of-the-art switching type primary controls, such as ULTCs could guarantee local stabilization in such operating regions. Further, the separation principle assumption is met, leading to the following:

Result . *The unique equilibrium always exists in the operating region in which the real power steady-state problem and the reactive power/voltage steady state problem can be characterized as nonlinear monotone network problems [52].*

Thus, the equilibrium monitored and controlled from the secondary level (EMS) will be uniquely defined.

Note that the result is stated under the real/reactive power decoupling assumption; eliminating this assumption is not straightforward. At present, there are no theoretical results that generalize [53, 54] - [52, 55] to the fully coupled power network. More work is needed here, since it is known that newly evolving modes of energy transfer take place under a strong real/reactive power coupling. Some theoretical results on decoupling error measurement are available and could be used as tests of validity of the decoupling assumption for specific analysis.

Prior to addressing the topic of very fast voltage controls on the power system, we examine the potential for improving the performance of slow switching-type controls even in operating regions that do not meet the diagonal dominance property (in the case of real power studies) or M -matrix property (in the case of reactive power/voltage studies). Specifically, we look at the question of ULTC transformer controls that work in the voltage collapse area. In principle, one way of getting around the problem of system matrix A in Eq. (4) losing its positive definiteness property is to have a new control law that accounts for qualitatively different system response to controls when matrix A changes to negative definite. In the slow voltage dynamics model of Eq. (3) the term determining how load voltage $\underline{E}(k+1)$ due to the $(k+1)$ st transformer action will be related to the load voltage $\underline{E}(k)$ due to the previous transformer action is $ADF(\underline{E} - \underline{E}^n)$. In the operating range where A is positive definite, a decrease in reactive power load causes voltage increase. Since, as discussed earlier, in more complex situations initial load voltage could cause matrix A to become negative definite, it is important to take this change into account when formulating a new control law.

One such control law that accounts for qualitatively different network response to input changes when matrix A becomes negative definite can be found in [48] and [4], in which theoretical results relating the proposed control to the well-known sliding mode control design are also presented. However, an immediate practical problem in implementing the new control law arises because this is not a decentralized control scheme. More work is underway toward developing sufficient conditions under which this control law could be implemented as a two-level control. Another, not fully explored, direction is to design more dynamic controllers, following some of the general system theoretic results [50, 56], to preserve decentralization of a ULTC control law that would work in the area of voltage collapse.

For the time being, however, the concept of monotone systems [57, 58] has not been applied rigorously to studies of power system dynamics. While analyzing effects of ULTC

actions, only slow voltage dynamics due to these controls are modeled, under the assumption that between two, slow control actions the natural system dynamics stabilize. Thus it was possible to interpret potential problems due to "malfunctioning" of such slow controls by employing steady-state concepts of monotone networks. A similar attempt to understand unstable situations of fast primary controls (such as AVRs, PSSs, and even governors in some cases) in the light of violation of the separation principle remains an open problem at this time because an interpretation of dynamic monotone systems and their analogous role is missing at this time. It is probably only a matter of time before serious investigation in this direction takes place. For the time being, we review new developments at the primary control level without linking them fully to the separation principle.

Open Research Questions Regarding Fast Primary Controls

All fast primary controls currently used on electric power systems can be recognized as fully decentralized, i.e., local with respect to measurements on which their control design is based. The fastest controls directly regulate voltages and are an essential part of the electrical machine's AVR/PSS system. Present controls are constant gain, at best, of a proportional-integral-differential (PID) type [42]. Under the most frequent operating conditions, they regulate local dynamics, and, at the same time, no systemwide instabilities occur. In the past, AVRs have been enhanced by PSSs since high gain voltage regulation could have destabilizing effects on frequency. Much work has been done on improving the AVR/PSS systems. They are successfully implemented throughout the world.

Research is still continuing to develop more adaptive AVR/PSS-type controls that could make possible wider ranges of stable operation. Some theoretical developments are decentralized high gain controls for nonlinear systems [20, 21, 22], variable structure controls [50], and feedback linearizing controls [59, 36-40], etc. Little work has been done on assessing systemwide coordination problems of AVR/PSS-type controls. Some recent studies propose centralized solutions to the problem [59, 60].

Taking the very specific structure of nonlinear power systems into account, sufficient conditions can be found in [19] under which decentralized full-state feedback linearizing controls are guaranteed to stabilize systemwide dynamics.

Potentially, there are two system theoretic issues. The first is the question of sufficient conditions under which the decentralized dynamics will be stabilized, provided unlimited control is available. The second is related to comparing ranges of equilibria of the closed-loop dynamics due to a control based on a linearized model with the limits on control resources. An important practical problem associated with

these theoretical issues is how to design controllers, with specified bounds on the control vector, that would, after a significant system change, bring the system to a new equilibrium within physically acceptable constraints on output. In trying to achieve such maximum performance when the system is under stress, one must design controllers that would achieve this task for the largest set of input or structural changes.

Results unique to the two-level control design, with the primary controls that use the feedback linearizing technique, are stated in [4, 19].

Measures of Most Effective Controls of Available System Reserves

Under new modes of electric power systems operation, it appears attractive to define meaningful measures of power system potential to reach a new viable and stable operation following any major change. It is well known that such measures are often expressed in terms of system Lyapunov functions. However, keeping in mind the general philosophy of decentralized control design for regulating electric power systems as well as the general structure of their nonlinear dynamic models, it seems reasonable to associate such measures with each subsystem. The transmission system needs to have certain properties under which optimal control design of each subsystem would lead to optimal operation of the interconnected system.

In dealing with large-scale interconnected systems two types of conceptual problems arise. One is defining nonconservative stability measures for the interconnected system, given stability measures of each subsystem. A typical solution is to take the weighted algebraic sum of all subsystem functions as the scalar function characterizing the entire system. Such an approach often leads to very conservative estimates of stability regions. The second is defining an adequate stability measure for each subsystem. Efficiency of the proposed control design critically depends on both of these processes. An interesting conjecture is proposed in [35]; here a dynamic reactive power reserve, closely related to the stored energy in each subsystem, is suggested as a meaningful measure for comparing performance of specific decentralized controllers. To briefly review this concept, recall the following.

Definition 2. *Steady-state reactive power reserve of an electric machine operating at the reactive power output Q_G is*

$$Q_G^{SR} = Q_G^{\max} - Q_G \quad (6)$$

where SR stands for static reserve.

A real difficulty in monitoring and controlling electric machines in their transition from one steady state to the next

following a major disturbance comes from not having measures of a dynamic reserve, similar to the steady state reserve defined in Eq. (6). One such possible definition was recently proposed in [35]:

Definition 3. *Dynamic reactive power reserve of an electric machine over a fixed time interval $[0, T]$ is given by*

$$(Q_G^{DR})^2 = \frac{1}{T} \int_0^T (Q_G^{\max})^2 - (E_d' i_d(t) - E_d'(t) i_d(t)^2) dt \quad (7)$$

where $Q_G^{\max}$ is the same reactive power limit specified for the steady state operation as in Definition 2.

Note that T is unrelated to the period of 60 Hz frequency; it just defines a specified time interval of interest after a disturbance. The value of Q_G^{DR} depends on the time interval T . For practical purposes, if the time allowed for system stabilization is 10 seconds, $T = 10$ seconds. The measure Eq. (7) is useful for evaluating the effects of different control designs on improving the stability region of an electric power system in its closed-loop operation, given the same hard limits on controls. For example, recall that there exists a field voltage control E_{fd}^* that assures operation of an electric machine with the largest possible stability region for a specified control limit $E_{fd}^{\max}$. An unproven conjecture in [35] is that this control law is equivalent to the control law that maximizes the dynamic reactive power reserve defined in Eq. (7) with respect to both time T and field voltage E_{fd} . This control law ensures the largest stability region of the machine in the same time interval. An illustration of the dynamic reactive power reserve measure for two different field voltage controls, the constant gain controller of the IEEE type and the feedback linearizing exciter control discussed above can be found in [35].

The ideas proposed in [35] need to be extended to develop less conservative measures of the interconnected system while using a measure of the type of Eq. (7) on the subsystem level.

Conclusions - Potential Role of FACTS Devices in Implementing Most Effective Controls

Many of the candidate FACTS devices are capable of changing the dynamic properties of the transmission system. Therefore, series capacitors, phase-shifting transformers, shunt capacitors, and transmission line switching could be employed to help systemwide stabilization, particularly in more stressful operating regions.

Two distinct directions could be taken to better understand the opportunities offered by these new devices. If the FACTS devices are controlled approximately at the rate at

which ULTCs and shunt capacitor banks are at presently controlled, i.e., fairly slow relative to the natural system dynamics, they could play a role in structural stabilization [61]. By switching such devices in and out, they could move the equilibrium from an operating region in which the transmission network does not respond to input changes as expected to a qualitatively different operating region that is well understood by system operators. In this way the existence of the steady-state solution within acceptable operating limits would be ensured. It is useful to recall the earlier example of controlling a ULTC transformer in two qualitatively different operating ranges. Therefore, the main role of these slowly varying FACTS devices would be at the secondary EMSs control level.

A more challenging application of FACTS devices would be to modulate them very fast by power electronics and microprocessor-based implementations. It appears that to fully understand theoretical aspects of their presence on the closed-loop system dynamics a deeper understanding of the concept of dynamic monotone systems as applied to electric power systems must be pursued. It is reasonable to still expect FACTS devices to ensure fast structural stabilization, in the sense that they would guide the transmission network to behave as a dynamic monotone system, while currently employed primary controls would play the same role of stabilizing their subsystems locally. General theoretical results in the area of vector Lyapunov functions and nonlinear decentralized control design by variable structure control should be revisited as a starting point in developing nonconservative stability measures of interconnected power systems. Understanding the effects of such high gain nonlinear controls on the natural time scale separation within the electric power system dynamics requires further study employing integral manifold tools as in [62]. Robust control design both with respect to uncertain parameters and modeling, will have to be pursued in operating electric power systems in more dynamic modes. One should, of course, recognize that if many new operating modes, such as NUG's and wheeling, become common, the temptation to employ FACTS devices will be enormous. Their systemwide effects on these newly evolving dynamics form a serious challenge to the community of power systems researchers.

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M. M. Adibi:

The reason is circuit breakers. Circuit breakers do not operate very nicely when you are switching static capacitors. In fact, Mr. O'Grady of Potomac Electric mentioned that they use this for voltage regulation, but they actually take turns. They have a complete logic that you never operate these breakers a lot for static capacitors, so shorting them is much easier than opening them.

Michael J. O'Grady:

He's right. In fact, that was one of the heuristic rules I was talking about. That's part of the logic you build into part of a distribution automation. If you're going to switch capacitors, part of the heuristic rules may be you're only going to switch so many times a day. You're only going to switch certain size banks, if you have more than one bank, depending on its size, for what you need to use it for, etc. You enter into all those considerations.

David Bertagnolli (New England Power Exchange):

I would like to address this comment to Lou Leffler's presentation this morning of a voltage collapse incident. The incident he described was a low voltage problem where the ultimate solution was to shed load. For most of our low voltage problems that is the fail-safe solution which will almost always solve our problems. Let me suggest the inverse problem with the high voltage situation, which is a much more difficult problem to solve on the part of the operator because you don't have a last resort control action to reduce voltages.

The second question I have addresses George Knight's presentation on load modeling. All the references he cited in his paper were at least five years old, and some were over fifteen years old. What we found, at least in New England, is that the nature of our load has changed in the last ten years dramatically towards the constant power load model. We see this when we invoke a voltage reduction: whereas fifteen or twenty years ago we would get about 4% reduction in our megawatt load, recently we only get about a 1° - 2% reduction. Various explanations have been offered for this change. One is the prevalence of personal computers and switched power supplies (constant regulated power supplies), and the same kind of regulation applies to induction motors. So whatever assumptions you make about load modeling, I expect only marginal benefits to power transfers, and with the kind of reliability standards we operate to in the northeast, that doesn't seem to be

worth it. We will continue, unless someone convinces us otherwise, to use constant power load models.

Isao Iyoda (Mitsubishi Electric Corporation):

There is a central control for active power: ELD in EMS, and for the reactive power there is distributed control on AVR. On the other hand, the time constants of the reactive power are shorter than that of real power. My question is, which is preferred in the future, central control for reactive power, or to improve on the distributive control of reactive power?

Louis G. Leffler (Public Service Electric & Gas Co.):

To respond to David's question about high voltages, yes, we have seen them too, particularly in heavily cabled systems such as we have. One solution, which is not a very appealing one, of course, is to remove EHV lines in service after carefully studying them in advance, and you do it with a great deal of concern. But you take them out for maintenance purposes, too. Otherwise, you just sit there and watch the system high voltages climb. One closing shot, talking about brittleness and robustness and some of the work that Mr. Dag Reppen did with the people from APS: we did go through a contingency, back in (I think) it was April, when tornadoes struck that part of the country taking out, over a period of time, not simultaneously, two major east-west 500 KV lines. The system did survive with a lot of hectic activity on the part of a whole bunch of people in gas turbines and one thing or another.

Washington came along with this dynamic load model concept that was presented this morning. Essentially, our concern is the under voltage load shedding that was proposed by Carson (Taylor), which we are in fact purchasing and will be in service later this year. The question is now, how are we going to set it? All you can do with undervoltage load shedding is that there are two primary controls you can set, the time and the threshold voltage. What Dr. Vu's presentation showed this morning is the chance of saving the system is really 50-50 in some cases, and you could overshoot or undershoot depending on the rate of change of voltage. The next step, as far as this project goes, is to test that model on a large scale computer program. We have a PSSE program in house; now we are going to try to model this load modeling.

Having said that, I would like to make one general observation about this meeting. I came with a set of questions, hoping to have them answered. I have to admit, I told some of you this morning I'm going to be leaving with a whole bunch of questions, so the challenge for the academics is in the next couple of years to come up with solutions in three general areas with which we have a bit of concern. One is study methodologies. Second, of course, is margin, what is a reasonable margin - we have been really struggling with this kind of problem for quite a while. The last is criteria. So helpfully in the next meeting like this we will have some answers to these questions.

Lester H. Fink (ECC, Inc.):

I want to throw in a comment on behalf of Charlie Concor dia, who is not here. If Charlie had heard that, I'm sure he would have said, "you have a third parameter that is very important: not only the amount and the time but also where on the system you shed the load." This can be critically important, also.

Said Ahmed-Zaid (Clarkson University):

Concerning the model validation, I'm going to draw the first order slip model I have shown in my presentation. This model has been used to estimate the accelerating time of both small and large induction machines with reasonable accuracy, but for large induction machines you may even want to go one step further and model the double cage induction machines with an additional circuit like this. What was pointed out in the paper was the fact that this kind of representation is erroneous for the kind of voltage studies that we are interested in. In order to perform experiments, we would probably need to do these experiments on-line to verify the analytical results that we have on these machines.

I think this is a well known, well founded model that has been used for several years for estimating the accelerating time, but we're talking about a totally different problem, that of performing simulation on-line with large induction machines. I will relay your comments to Mr. Bill Price and I will let you know what he thinks.

Carson W. Taylor:

In response to Pete's questions, if I understood them right, I think one of them was, why not just use conventional mechanically switched capacitors with 115 or 230 KV circuit breakers or 500 KV circuit breakers. The reason is mainly economics; the medium voltage switch gear is very much less expensive. I think for the Tacoma, WA, prototype at 230 KV we're talking about a total of \$50,000 for the three phase vacuum breaker, the current limiting reactors, the control closing device, and when you put in the controls, the installation costs and so on, it is \$400,000. High voltage circuit breakers are quite expensive; I think our planning numbers for 500 KV circuit breakers are one and a half million dollars now, and 230 KV circuit breakers are something like \$300K, so the whole project is driven by economics. The other possibility is, if you're putting in a 230 KV capacitor bank, you could just make it somewhat bigger, but then you would lose the control of it. You may have too much capacitance in during normal conditions, and you don't have the switching capability.

I can respond to one of the other questions on other control devices. I think we are going to have voltage stability problems with us for many years. One of the major solutions I see involves distribution automation where you have the capability to do very nice things, the most obvious being demand side management where you can turn off air conditioners or electric heaters or water heaters during emergencies, which is really a painless way to shed load. There is a lot of future in that area, particularly with the new fiberoptic systems and so on. I have a paper on undervoltage load shedding at the winter meeting that talks about that a little bit. Other things that can be done with distribution automation is capacitor switching, voltage reduction and automated testing of load characteristics, which can be done inexpensively from a central site, perhaps.

Peter W. Sauer:

My question was, in shorting one out, would you really have to remove it? Why not put it in parallel with the other ones?

M. M. Adibi (IRD Corporation):

I have a comment or question, particularly for those authors who are presenting induction motors models. Yesterday, I presented a diagram showing how you would start up a very large induction motor with a relatively small generator. The significant thing about that was not the fact that we had response of the frequency and voltage. In my opinion, the significance of that was that an operating company such as Illinois Power accepted Clark University's modeling and then went through the expenses of isolating a 138 KV line and starting up a synchronous generator and starting an induction motor and providing instrumentation to see whether the models that they had, which Mr. Schultz had created, were correct. So the significance of that was now we have confidence in that model and we can use it elsewhere. Now there've been three second or third order induction motor models presented, but none of these were tested. I believe that utilities are very willing to test these things these days. Of course, you couldn't hear about these things before, particularly Dr. Ahmed-Zaid who is at Clarkson and in fact co-authored with Mr. Price of General Electric. They have got so many induction motors there, it would be very nice to see whether their models can be validated against tests.

Leonard J. Panzica (New York Power Authority):

This isn't a question as much as a response to Dr. Knight's earlier request for some additional information on past near voltage collapses and specifically the one in the northeast U.S. in June 1984. To the best of my recollection from reviewing operator logs and reports from that day, which was June 11, 1984, as experienced in the New York power pool, the slow voltage decay was precipitated by the near simultaneous loss of a large 345 KV transmission line and a 600 MW generating facility. Initially, the corrective actions that were taken by operators were to reduce transfer across the critical interface in the system into which the power was heading, or the critical bus which was starting to decay, which was in the Albany, NY, area. In addition to that action, we had a unique situation in that we have a pumped storage power project in that area, a 1000 megawatt pumped storage plant consisting of four 250 megawatt units at the time of the problem. Three of the four units were on-line generating their capacity, one was off-line. We were able to start up the fourth unit as a synchronous condenser and spin it in the air, and over the course of the next 30-45 minutes we were able to take two units off-line as generators and convert them into synchronous condensers. Over the period of an hour we were able to restore the system. Since that time, in that part of the New York power pool system, we have installed

a major double circuit 345 KV transmission line, and associated with it two large static VAR compensators and some 2000 MVAR's of switched shunt capacitors. So now our problem has converted into high voltage, especially with the incidence of reduced power transfers, or lower power transfers than we had anticipated at the time we built the project. So I hope that provides a little more information as I can recall of some seven plus years ago. I think the unique feature we had at the time was not only rapidly reducing transfers but converting those pump storage generators to synchronous condensers, which was pretty much our ace in the hole at the time.

Peter W. Sauer (University of Illinois, Urbana-Champaign):

I want to ask Carson Taylor about his choice of putting the capacitors in series and then shorting them out, as opposed to just buying more and switching them in parallel which, apparently seems to me, would work also. And then on top of that, why short them out? Why not change them from series to parallel so that you are using them all the time? It seems if you short them out then you don't get the advantage of them.

Nizam U. Ahmad (Oklahoma Gas & Electric Co.):

I was wondering, besides LTC tap changing and the voltage load shedding, are there any other schemes that any other utility is using at this point, or if any kind of a monitoring system is showing up in other utilities? That's the question to which just about anybody can respond.

Kebede M. Jimma (Puget Sound Power & Light Co.):

Perhaps if I give you some background on practical application of the paper presented this morning by Dr. Vu, I may give you a little bit of appreciation of what we are doing. Back when we recognized that we had a potential for voltage instability in the Puget Sound region, we went to what we call the established methodology as far as studying the problem. That was the QV curve methodology, which served the purpose for planning and comparing the relative merits of plan A versus plan B. The problem we ran into was when we started looking at the short term response of the system following a major outage in the Puget Sound region. That is when we recognized the significance of dynamic load modeling. What essentially happened was we did some sensitivity studies and it showed that we really needed to address that issue a little more. That's when the University of

formers would be complete and prevent operation of the lower voltage tap changing, and in the process support the lower voltage shunt capacitors and reduce reactive losses all the way down into the distribution system. The bad part of that is that the loads get restored faster than they normally would. Blocking of tap changers has been used at a few places, Ontario-Hydro, for instance, has a scheme in the Ottawa area where they monitor the unregulated or high side voltage and for certain voltage depressions of the high side voltage they block tap changing and release the blocking once voltage recovers and stabilizes - hysteresis characteristics on the voltage setting. I guess it's been proposed to actually reverse control of the tap changing, but I'm not aware of anyone that's done that: in other words, support the high side voltage and actually reduce the load intentionally. It's been used in voltage reduction schemes, but not elsewhere.

Marija Ilic (Massachusetts Institute of Technology):

With regards to Mike O'Grady's comments, I'm really glad the comments were made. We talked yesterday and it's nice to hear. I didn't know there's been an automated scheme on the distribution level for 15 years. It is nice to know when one does theoretical work on this, so I do appreciate the link that you made and I hope that we can work together in the future on understanding the practical issues and the relationships to the theoretical. I don't think that there was any specific question except on the terminology. I have to go again over the practical terminology that they are using. It appears that maybe the direct control is what we call primary and indirect is EMS level secondary control. Then Harry Kwatny came and he mixed them both a couple of days ago, so I think we have to work on the terminology and communications, but that was one of the points I was trying to make with my block diagram. Historically, there has been a clear cut separation between these two levels and it may be disappearing under some unusual operating conditions, so it's important to keep that in mind.

The second question was on modeling of FACTS elements. All the switching devices (power electronics based controllers) are known to generate very high harmonics so this is going to be a problem in the future. We need more solutions on filtering, and the protection has to be revisited also, so it's really an open area as far as the harmonics go. The modeling, I think, of the steady state models for practical devices are well understood and have been well understood for a while. We have been working with these devices, it's just when one wants to take advantage of them in the real dynamic modes, I think we have to do the averaging I mentioned, of the effects of very fast dynamics on the time

scale that we are interested in, so there is a lot of research type problems on the modeling side.

Regarding Mr. Davis's question on tap changers, I would like to illustrate that, if I can, with just one transparency. The questions of blocking tap changers versus letting them work, it really depends. If you have a situation now related to all these PV curves, assuming that the fast dynamics are stable, the generator dynamics don't have problems. If you're on the upper part of the PV curve and the tap changers are trying to bring the voltage higher, then the present controller is going to do its job. If you can with the same controller, the way things are implemented now, you can start from here on the lower part of the PV curve. You then can go down and down and this is the dynamical action that we are talking about. But one can change the controller so that you recognize the gradient that you are on the lower part of the PV curve. The mathematics of this has actually been developed in several theoretical papers. But you can actually go from the voltage which is point H_2 to the voltage where you want to drive it, if you reverse the action and you start from the lower part of the PV curve. I didn't have time to address that issue in my presentation but there is a possibility to take advantage if you change the control logic of the under load tap changing transformers. So by locking, you are not exploiting, possibly, the fully reactive reserve that you have. This is one of the points I was going to make.

Khoi Vu (University of Washington):

I have some more comments regarding tap changing locking, but I don't know if I'm going to clarify it or give you more confusing issues about it. As you can see from the simulation, if I have fast tap changing then I can save the system but if I have no tap changing then I have to shed load under the same system. Let me put back the so called Leaf theory and try to explain that. This is the academic view of tap changing, tap changer locking. The Leaf refers to the voltage recovery region, so if we have no tap changing, in other words we lock the tap changers, we have very slow tap changing and we are in this situation then we miss a Leaf, so we won't have voltage recovery. If we have fast tap changing, then we can get into a Leaf then we can have voltage recovery. The final comment regarding tap changer locking, when we lock the tap changers we also have to worry about what is connected down the line. For example, if you have a very hot summer then you have a lot of air conditioning, if we have a very cold winter, we have a lot of electric heaters. Then if you lock a tap changer and the voltage is still low then the thermostatic control will try to add more equipment into the system and that is making the situation worse.

DISCUSSION - SESSION FIVE

Edited by Thomas J. Overbye, University of Illinois, Urbana-Champaign

Michael J. O'Grady (Potomac Electric Power):

In reference to Marija's (Ilic) paper on voltage control, this is the first paper I've seen so far, at least in this session, that starts to address, or attempts to address, the problems of actually applying the controls calculated by the models and techniques. We've heard a lot about the techniques themselves and how you can go from state A, where you're at right now, to state B, where you would like to be, to protect yourself against collapse or potential voltage problems; but after you collect the set of controls that you've come up with, generator set points, capacitor switching, transformer taps, how do you order or group these controls, and then issue them to the power system in such a way that you don't adversely affect the power system along the way in going from state A to state B? We've done a lot of work in that area because we're trying to bring our next control system on line where we have some closed-loop voltage control programs for our system. We've found it to be quite a task to incorporate this into the scheme of things that you're actually one day going to apply. A lot of what we found also is that it is system dependent. We've had to apply heuristic rules depending on the types of equipment you have installed in your system and everything in this process in order to get the control out reasonably. Of course, I'm talking about automated controls here more than the dispatcher manually issuing them, but the dispatcher still has to be concerned with these things even if he manually issues these controls -- this set of controls he has in front of him.

The second part of my question or point is in regard to the paper also. We talked about types of controls and I know Marija was very sensitive the other day about pointing out the difference between primary and secondary controls and the terminology. I am trying to relate our terminology to hers. For us, primary controls are controls that will most affect the problem. For example, for voltage in a certain area our primary controls will be generators or capacitors, with secondary controls maybe of different types of transformers. On the other hand, we have direct controls. Direct controls are controls that are issued by the application program directly to the piece of equipment, such as a transformer tap movement. We also have indirect controls where the control is a target that's calculated for the local controllers of a piece of equipment in the field. So the indirect control sends a target off to the other program or local control, which takes care of it from there or incorporates it into its little scheme of things.

The other thing I want to mention is that we have a closed loop voltage control program that works on each of our distribution substations. It's been in service for about fifteen years and it has some of these concerns built into it. It seems to work pretty well. We're just adding some new applications on top of it, nesting the old application into the newer application like a hierarchy of programs, and one of the things we are adding is a voltage dispatch to go along with our megawatt dispatch. We're quite interested ourselves to see what happens.

Miroslav M. Begovic (Georgia Tech):

I have a question regarding modeling of FACTS elements and the power system itself in the light of their harmonic interaction. I would appreciate comments on that.

Edward J. Davis (Entergy Services, Inc.):

I'm more of an applications kind of person, and one of the things I'm having trouble dealing with here is what are you trying to tell me? Is it best in regards to tap changing to change taps or to block them, and are we talking about the 500 KV system or the distribution system, if there is a difference in the way you handle taps. If somebody would address that, I'd appreciate it.

Carson W. Taylor (Bonneville Power Administration):

I'll address the last question on blocking of taps. In some cases, for the normal control, where you regulate the low side voltage, the regulation can be beneficial, other times detrimental. It tends to be detrimental on the high power factor of the residential loads where the tap changing restores the load. In some cases it can be beneficial if there are a lot of capacitors downstream like in industrial loads, induction motors, which is sort of a constant power load anyway, but a lot shunt power factor compensation, and then the tap changing can be beneficial in supporting the output of the shunt capacitors. In some cases you have several tap changers in series, say one at the distribution level, maybe one has automatic control of a large LTC auto transformer. Then the strategy would be to have the tap changing on the auto transformer be fast in comparison with the distribution tap changing, and for all system disturbances the tap changing at the auto trans-

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SOLUTION METHODS

From equations (6) and (7) the mathematical model has a similarity with the transient stability problem. However, when considered further it can be seen that the size and complexity of the problem is significantly greater. Moreover, the computation required to perform the simulation of one contingency for a specific operational condition in a voltage problem can be much larger than those for transient stability.

Solution of Differential Equations

In solving this set of equations, the problem appears when the system is close to the voltage collapse point because the Jacobian of the whole system becomes very stiff pushing the numerical solution to its numerical stability boundary. To avoid this problem, a stiff differential equation solver is being used based on Gear's method which manipulates the integration step according to the stiffness of the system.

Network Solution

Equation (7) is the system representation and corresponds to a load flow problem. To avoid extensive calculations and to allow an easy implementation of the on load tap changers and any network compensation, Norton models for all components and a special network solution based on [28] are being used. This formulation allows modifications of the network parameters and machine-load characteristics without producing excessive solution time.

Actual procedure

The procedure being used initially is to take a past operating or planned state of the system, to weaken the network transmission capability by taking one or more circuits out, and then to increase power transfers over critical paths by adjusting the generation and demands appropriately. The full dynamic analysis will be started near the point of voltage collapse.

FINDING THE DYNAMIC OPERATIONAL BOUNDARY

It is intended to study the boundary between stable and unstable operating states as voltage collapse is approached through analysis of the rates of change of the eigenvalues of the state of the system. This should enable both the efficiency of the various control actions and times available for actions to be analysed.

CONCLUSIONS

An attempt has been made in this paper to summarise some of the data available on the characteristics of utility load as affected by system voltages, to suggest those characteristics important in voltage disturbance studies and to consider the availability of such data. Much data exists, but with the exception mainly of the work supported by EPRI, it tends to be localised and not always in a form or detail which can be easily used.

Although there is a measure of agreement between the values quoted for the coefficients n and m , particularly for individual components of load, it seems advisable that studies on actual or typical systems should

- i) include assessment, as far as possible, of the load mix and composition likely to be found;
- ii) include wide ranging sensitivity studies.

It would seem worthwhile for one of the international or large national engineering institutions to prepare guidelines for the testing and reporting of load characteristics.

ACKNOWLEDGEMENTS

Dr Knight wishes to acknowledge financial support for this project from the National Grid Company, UK.

Mr Roa-Sepulveda wishes to thank the following institutions for their support, University of Concepcion-Chile, University of London (Edmund Davis Scholarship), Committee of Vice-Chancellors and Principal of the Universities of UK (ORS award) and the Rotary Foundation.

Thanks are also due to Dr B.J. Cory and Dr M.J. Short, Imperial College for helpful discussions on this subject.

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If the effect of control actions on the development of voltage disturbances is to be analysed, the various control mechanisms should be modelled, including those on the distribution networks.

The main mechanisms at these voltage levels will be tap changers and switched capacitors. Taking the tap changers, their control strategy needs to be integrated with that of the tap changers in the transmission network and the joint strategy may for instance be

	Step size (%)	Time delay (secs)
tap change on transmission network	2.5-4	50-90
tap change on distribution networks	1.5-2	120

A large step size at transmission voltages will both minimise the number of tap changes and reduce the number of tap changes on the distribution system, eliminating any need for consequential adjustments and hunting between transmission and distribution tap changers. Time delays are chosen so as to restore transmission voltage levels before the restoration of load.

The above settings are well defined for normal operation and will help the system to recover after a transient situation. However, incipient voltage collapse usually produces a 'slow' decay of voltage in one area which would produce, in some circumstances, LV tap changer operation first - thereby producing an 'extra' perturbation to the system which could be seen as a 'worsening situation'.

There are thus, contradictory goals to be fulfilled. On the one hand, the steady state and transient analyses dictate the above settings. On the other, voltage collapse analysis indicates that using those settings the system recovery situation could be worsened. A compromise solution needs to be found.

Noting also the interaction of the distribution network and load characteristics it would seem worthwhile to develop a model which includes these effects. A proposal which we are investigating is shown in Figure 1. It falls between the full model of [6] and the very reduced one of [7]. The configuration would be adapted to the number and values of voltage levels and to the data available on the switching voltages of load. The total transformer capacity between each voltage level is summated and an equivalent transformer, including tap change, used with a reactance equal to the paralleled reactance of the actual transformers. The transfers over the equivalent transformers are estimated or judged and the losses assessed. Hopefully, some estimate will be possible of circuit impedances (possibly based on switchgear fault level ratings) and losses. Not least, this proposal should allow tap change characteristics at the various voltage levels to be modelled without difficulty.

PROJECT DESCRIPTION

The objective of the Imperial College research is to study the mechanism of voltage collapse with particular reference to the impact of demand characteristics and the dynamics of the process. An additional objective will be to study operational measures, including the time available, to prevent a voltage disturbance degenerating into a voltage collapse.

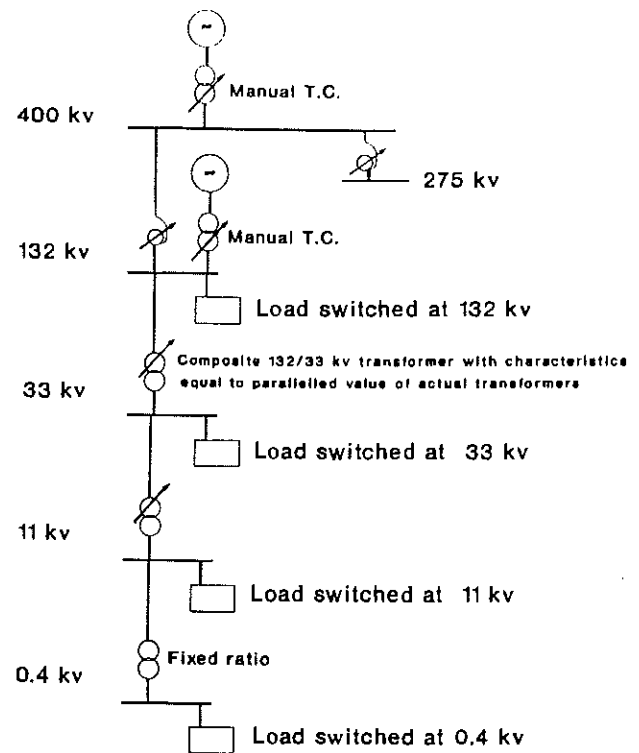


Figure 1. Equivalent Distribution Network

MATHEMATICAL MODEL

The voltage instability problem can be studied using the generalised model as follows:

$$\dot{x}(t) = F(x(t); \lambda(t)) \quad (6)$$

$$0 = G(x(t); \lambda(t)) \quad (7)$$

where

$x(t)$ = is a vector which contains the chosen state variables of the system such as internal and field voltages of generators, tap positions, etc.

$\lambda(t)$ = is a slowly changing vector which contains those variables possibly responsible for voltage collapse and often causing changes in the appropriate variables in $x(t)$.

The voltage instability critical point is found when the Jacobean formed by F and G has a singularity for a given value of λ .

THE SYSTEM MODELLED

A detailed model of the system is being used, paying particular attention to the various control mechanisms (excitation systems, tap changers, var sources) and allowing a flexible load model.

Organisation - Turkish Electricity Authority [9], Rome University and CESI
Date - 1970

These tests were done on the 154 kV network in Turkey and concentrated on the transient response. They were unique in that very large voltage drops (up to 60% or so) were applied for short periods to residential/commercial and industrial loads. In one test at a residential/commercial supply point (domestic and public lighting = 70%, asynchronous motors = 30%), the active power load dropped by about 90%, and the reactive power (leading) by about 70% with a voltage drop of 60%, appearing to stabilise at these values after some 0.2 secs. In another test on an industrial area (motors = 79%, electrochemical = 3.5%, lighting = 5%, domestic = 12.5%, magnitude at time of test 27 MW), the active and reactive power dropped by about 85% and 90% with a voltage depression of some 50%, appearing to stabilise after some 0.5 secs (the voltage was restored after 0.7 secs.)

Organisation - EDF
Date - 1987

In simulations of the voltage collapse incident which occurred in 1987 (see Table 1), EDF used a composite value of $n = 1.4$ based on measurements and analysis from the time of the disturbance. A value of 3.0 was used for m , this taking into account capacitors included with the load. The time response of tap changers was modelled.

Organisation - Various Utilities
Dates - Mainly 1950's

Reference [17] summarises the tests made in several countries, generally at distribution voltage levels (6 kV- 27 kV) and for voltage drops up to 10%. Values for n range between 0.6 and 1.3 (most in the range 0.8 and 1.1) and for m between 1.5 and 4 (most in the range 1.7 to 2.6). Concordia [12] has listed values of n and m published mainly between the late 1950's and early 1970's, n varying between 0.5 and 1.6 (the commonest values in the range 1 to 1.5) and m between 0 and 4 (the commonest in the range 1 to 2). The voltage variations were variously down to -10% and up to +15%.

ASSESSMENT OF THE VOLTAGE SENSITIVITY OF LOAD BY AGGREGATION

The most exhaustive studies of techniques to assess the static and dynamic characteristics of load as functions of voltage and frequency has been that supported by EPRI [6, 7]. In the later method [7], the load at each node to be modelled (e.g. infedding point) is synthesised from data on the load mix (industrial, commercial, domestic), the composition of each component of the mix (amount of heating, lighting, motor loads, etc.) and the characteristic of each of these components. Using mix and composition, the amount of each load type is calculated (a load type being the amalgam of loads with similar characteristics, e.g. residential direct space heaters with $n = 2, m = 0$). The load types are then amalgamated into composite static and dynamic loads (a similar method is given in [18]). The distribution networks are represented as summated reactive power gains or losses.

Although the data requirement is considerably less in content and detail from that of the earlier proposal, it is much more consumer orientated than that usually needed for system technical and economic studies and may be difficult to obtain. Essentially one needs for voltage studies the summated totals of each type of load for the area of the system equivalent to the nodes to be modelled in the system analysis study. Accepting that this is unlikely to be available in this detail, the mix and composition approach proposed in [7], tailored to meet the available knowledge on the characteristics of each load type, may well be the best approach.

Considering mainly published information out, so far, limited experience is that utility-wide figures are much easier to obtain than breakdowns. Thus proportions of different classes of load (say domestic, commercial, agriculture, industrial, public lighting) are readily obtainable in terms of energy consumptions. Proportions in terms of power may be found in papers dealing with tariffs, load management or forecasting but again the information is likely to be in global terms [e.g. 19, 20]. If the assumption is made that the daily load curves for all types of load are similar in shape, the ratios of energy will also give the power ratios. With this assumption then, billing data could provide a breakdown of the load by type and location. The size of such a task is evident.

Obtaining the composition of load within each class is also difficult. As suggested earlier, transmission voltage problems are likely to involve zones of the system and hence aggregated compositions over such areas are needed. It is judged that the compositions of domestic and commercial load classes across a utility will be broadly the same, at least over areas of similar social/economic status. Published information is often in energy terms, but sometimes data is given in power terms.

The composition of the industrial class is likely to be more variable obviously depending on the types of industry. Useful information has been found in large scale studies of energy requirements [21] or public enquiries on system developments, although again the data is often in energy rather than power terms [22].

Quite a lot of information has been published on the characteristics of different components of load, some of which has been abstracted in Tables 2-4. Default data for 26 domestic, commercial and industrial components is given in [7].

A potential problem in obtaining data for longer term studies is that of the change of load characteristics with development of equipment, typically to obtain higher operating efficiencies. The particular requirements here are to assess whether the composition of the load will change and how the characteristics of individual components will be affected.

A forecast of the growth of energy use in the USA made by EPRI projected minimum, mean and maximum estimates for the year 2000, incorporating views on changes in efficiency of use. Using these estimates, a brief analysis in terms of mix and composition of loads suggests that the domestic/commercial/industrial mix will be broadly the same (each class has some 30-35% of the total and the maximum changes are some 4 percentage points from the 1987 values). In the domestic and commercial spheres, the proportion of the main components (heating, lighting, mixed) remain broadly the same, with the proportion of mixed loads increasing slightly, and domestic/commercial falling.

A long term influence could be the evolution of the whole economy, for example away from heavy manufacturing towards light engineering and service industries.

THE MODELLING OF DISTRIBUTION NETWORKS

Measurement based techniques automatically include the effect of distribution networks. In the aggregation methods, the network effect can be determined accurately (for the given data) by power flow studies or approximately by summing circuit active and reactive losses and gains, including reactive compensation. The assumptions made concerning the load response to voltage may have a significant effect on the characteristics of the network itself [7, 17].

Table 4 Voltage response characteristics of motor loads

Size (kVA)	P.F.	n_1	m_1	R_s	X_s	X_m	R_r	X_r	Load level	Source ref.
110	0.88	-0.1	1.1						0.85	5
Small	0.83	0.1	0.8	.031	.10	3.2	.018	.18	0.6	7
Large	0.89	.05	0.5	.031	.067	3.8	.009	.17	0.8	7
General		=0								12
Static aux (2850)	0.7	0	1.8							13
4		0.26	1.5							11

THE P/Q/V CHARACTERISTICS OF NETWORKS

Some of the results available from tests on networks are summarised below. The voltage range is often restricted to avoid disturbance to consumers.

Organisation - CEEB Date - mid-1970's

A number of tests were made by the CEEB in conjunction with Distribution Boards. The tests were made at the 33 kV busbars of 5 132/33 kV points and at the 132 or 66 kV busbars of 4 supergrid (400 or 275 kV to 132 kV or 66 kV) points. These covered industrial, mixed and domestic areas of supply. The tests were repeated for different days and times of day and included measurement of transient response. The voltage range covered was from say -6% to +4% from nominal [13, 14]. The overall responses are shown in Table 5.

The authors note that when translated into the forms

$$\frac{\Delta P}{\Delta V} = n \frac{P_o}{V_o} \quad \text{and} \quad \frac{\Delta Q}{\Delta V} = k \frac{P_o}{V_o}$$

all the active power responses and most of the reactive power responses lie within the bounds

$$1 \leq n \leq 2 \quad \text{and} \quad 1 \leq k \leq 2.$$

The analysis was extended to derive equivalent networks and the static and dynamic proportions of the load.

Table 5 Demand characteristics at CEEB substations (tests - 1970's)

Substation and voltage level (kV)	n $\frac{\Delta P}{\Delta V}$ (MW/V)	m $\frac{\Delta Q}{\Delta V}$ (MVA/V)	load level (min load) MW	n $\frac{\Delta P}{\Delta V}$ (MW/V)	m $\frac{\Delta Q}{\Delta V}$ (MVA/V)	load level (max load) MW
1, 33	.09	0.1	4	.33	0.31	21
2, 33	.27	0.5	28	.30	0.59	31
3, 33	.38	0.25	22	0.60	0.64	48
4, 33	.45	0.41	25	.45	0.61	40
5, 33	.43	0.43	27	.6	0.41	36
1 (132 or 66)	1.1	0.7	90	-	-	-
2 (132 or 66)	1.3	1.0	105	1.9	1.0	115
3 (132 or 66)	2.4	1.1	145	2.4	2.1	155
4 (132 or 66)	1.0	3.0	130	3.0	4.8	220

Organisation - EPRI and USA utilities Date - Late 1970's

The EPRI projects [6, 7] included tests at several substations to assess the validity of the load synthesis models. The tests were done over a range of about 0.95 to 1.10 pu volts. Using formulations

$$P = P_o (1 + n \Delta V)$$

and

$$Q = Q_o (1 + m_1 \Delta V + m_2 \Delta V^2)$$

the results obtained were:

Domestic type substation with load of about 3 MW, over the day n varied between about 1.0 and 1.5, m usually about 7. P variation with voltage was linear, Q variation non linear in any one test.

Mixed type substation with load of about 20 MW, over the day n varied between 0.63 and 0.88, m between 3.1 and 8.26. P variation with voltage was linear, Q non linear.

Organisation - Tokyo Electric Power Company (TEPCO) Date - 1985

The loads at several substations in Tokyo were monitored for about one year [23]. The load types were industrial, residential/commercial and building. The data for various seasons of the year was fitted to equations as indicated in Table 6.

A seasonal sensitivity analysis showed that the values of n and m could vary by up to $\pm 50\%$ of a mean value over the day and year.

Table 6 Demand characteristics in Tokyo

Type of load	n	m	Season
Industrial	0.2	0.6	September night and day
Residential/commercial	1.9	1.6	January and April night
Building	1.5	1.5	January night

Equations used:

$$P = K V^n$$

$$Q = K_o + K_1 V^m$$

(K_o assumed equal to apparent power at nominal voltage.)

Organisation - USSR Power System [16]

The characteristics at 15 substations with demands above 10 MW on 110 kV networks were measured and fitted to polynomial equations, giving for example

$$P = P_o \left(-0.9 + 1.2 \left(\frac{V}{V_o} \right) \right)$$

Complex equations were used for Q_o .

The voltage variation was from -15% to +5% of nominal.

A first order relationship is often used

$$P = P_o \left(\frac{V}{V_o} \right)^{n_1} \quad Q = Q_o \left(\frac{V}{V_o} \right)^{m_1}$$

EPRI has proposed that the nominal power rather than nominal reactive power should be used for Q to avoid difficulties with near unity power factor loads, i.e.

$$\frac{Q}{P_o} = \left(\frac{V}{V_o} \right)^{k_1}$$

Most of the dynamic load will be induction motors. The P and Q transients following a sudden voltage depression as calculated by models of various complexities converge to the solution given by the simple equivalent circuit in well under a second [8, 10]. Hence for the timescales involved in voltage collapse phenomena, it would seem quite adequate to use this model, with parameters

R_s, X_s (stator resistance and reactance)
 R_r, X_r (rotor resistance and reactance)
 X_m (magnetising reactance)
 s (slip)

Some of the published information on static loads (mostly domestic) is given in Table 2. Combined static and dynamic loads, usually small domestic appliances, are listed in Table 3 and typical induction motor information in Table 4. In some cases, the original references show the transient characteristics, the information having been obtained for transient stability analysis. In these cases, the final steady state response has been quoted.

Table 2 Voltage response characteristics of static loads

Load type	Power factor	Coefficients n_1	m_1	Comment(1)	Source ref (2)
Resistive - (consumers) Fire Radiator Water heater Cooker	1.0	2.0	0		7
Incandescent lamp	1.0 1.0	1.54 1.6 1.5-1.6	0 0	several refs given n varies with voltage change	7 5 9
Fluorescent lamp	0.90	1.0 0.95	3.0 3.1		7 5
TV (colour)	0.77	2.0	5.2		7
TV (monochrome)		2.0	5.2		5
Mercury Vapour lamp		2.0	2.3		9
Computer		2.0	5.2		
Electrochemical plant	-	0.8	-	(as measured on d.c. side)	9
Electrolysis plant	0.9	1.8	2.2		7
Arc furnace	0.72	2.3	1.61		7

Note (1): In the EPRI sponsored work, most tests on individual components were over the range 0.7-1.2 p.u.

(2): More detail on the EPRI results will often be given in [6].

Table 3 Voltage response characteristics of combined loads

Typical Load	Overall p.f.	Overall static coeff.		Dynamic load as % of total	Static Coeff. (non-motor)				Dynamic coeff.					Comment	Source
		n ₁	m ₁		p.f.	n ₂	m ₂	k ₂	R _s	X _s	X _m	R _r	X _r		
Refrigerator & freezer	0.84	0.8	2.5	0.80	1.0	2.0	-	0	.056	.087	2.4	.053	.082	Domestic	7
	-	0.44	2.13											Domestic	11
	0.79	0.8												Domestic	9
Washer	0.65	.08	1.6	1.0	-	-	-	-	.11	.12	2.0	.11	.13	Domestic	7
Air conditioner (central)	0.75	0.1	2.5	1.0	-	-	-	-	.053	.083	1.9	.036	.068	Commercial	7
Air conditioner (room)	0.75	0.5	2.5	1.0	-	-	-	-	.10	.10	1.8	.09	.06	Domestic/ commercial	7

Having obtained a broad breakdown of the load into essentially heating, lighting and various forms of motive power, one should have some knowledge of the mix of the load on a geographical basis, for instance what are the proportions of domestic, commercial and industrial loads plus, broadly, what types of industry at each of the interfaces between the transmission and lower voltage networks.

Nothing has been said so far about the range of voltage over which the characteristics of the load and network should be determined. It has been suggested that there is cause for worry when the transmission voltage drops to 10% below nominal. On the other hand, the authors know of large, but plant deficient, systems in which voltages down to about 85% nominal are occasionally accepted. It would seem reasonable to suggest that the PQ/V characteristics (i.e. the response of load P and Q to voltage changes) down to at least 80% nominal and preferably 60% nominal should be sought.

THE TECHNIQUES AVAILABLE

An attempt has been made in the previous section to outline the reasonable set of data required for voltage studies (reasonable is used in the sense of necessary and sufficient in the context of the difficulties of obtaining such data). It is against this requirement that the two methods outlined below to obtain such data should be judged. The methods are direct measurement and synthesis by aggregating the elementary components of the different types of load, with known characteristics, into composite totals at the various nodes or for each area.

Direct Measurement

In this method, the changes in power and reactive power taken by the load at selected points following a sudden change in voltage on the busbar supplying the load is measured. The voltage change may be caused by setting paralleled transformers on different taps and switching one out, switching out an unloaded line, etc. Tap changers on the networks subjected to the changed voltage may be locked, although the load response and measurement times will often be less than the tap change time.

The problems of this method are:

- 1) it is difficult to obtain voltage depressions greater than say 10% because of consumer reaction.
- 2) it requires quite extensive planning, instrumentation and manpower and because of this is only likely to be used on a small scale.
- 3) it will only give data for the times of the tests; these would probably include several over a day, perhaps weekday and weekend, but repeating for other seasons and years could be difficult.

On the other hand it does, within its limitations, give directly the information required.

Synthesis of Load Characteristics

In this method, as touched on above, the PQ/V characteristics are synthesised from knowledge of the characteristics of the elementary components of the load (lights, freezers/refrigerators, different motor loads) at each of the nodes represented. These techniques have been studied in detail by EPRI. The first proposal was to aggregate the load at nodes on the distribution network,

carry out load flows on the network for various values of voltage at the supply node from the transmission network and hence obtain the relationship between the value of this voltage and the P , Q supplied [6]. This technique was apparently not taken up by utilities and a simpler proposal was put forward. In this, the PQ/V characteristics are estimated at the bulk supply points by aggregating the elementary components of load (as just described), at these points and adding to this an overall approximation for the distribution networks themselves [7]. This is considered in more detail below.

A REVIEW OF SOME PUBLISHED DATA ON THE PQ/V CHARACTERISTICS OF LOAD

An extensive review of the literature on this subject has been made and some of the information found has been used in the review below (as a matter of interest one often finds the same basic sources quoted). The results may be given in different forms, e.g.

$$\frac{\Delta P}{P_o} = n \frac{\Delta V}{V_o} \quad (1)$$

or

$$\frac{P}{P_o} = \left(\frac{V}{V_o} \right)^n \quad (2)$$

It is easy to show that these are identical if the first term only of the expression

$$\text{for } \left(1 + \frac{\Delta V}{V_o} \right)^n \text{ is taken.}$$

The corresponding equations for Q are

$$\frac{\Delta Q}{Q_o} = m \frac{\Delta V}{V_o} \quad (3)$$

or

$$\frac{Q}{Q_o} = \left(\frac{V}{V_o} \right)^m \quad (4)$$

Not least to avoid difficulties for near unity power factor loads ($Q_o \rightarrow 0$), equation (4) is sometimes modified to

$$\frac{Q}{P_o} = \left(\frac{V}{V_o} \right)^k \quad (5)$$

THE PQ/V CHARACTERISTICS OF INDIVIDUAL COMPONENTS

Numerous tests have been reported on various types of equipment. For the static characteristics, it is usual to assume polynomial relationships between the values of P , Q at some voltage V and those P_o , Q_o at nominal voltage V_o , i.e.

$$P = P_o \left\{ a_1 \left(\frac{V}{V_o} \right)^{n_1} + a_2 \left(\frac{V}{V_o} \right)^{n_2} \right\}$$

$$Q = Q_o \left\{ b_1 \left(\frac{V}{V_o} \right)^{m_1} + b_2 \left(\frac{V}{V_o} \right)^{m_2} \right\}$$

Table 1a Examples of Voltage Collapse

ACTUAL

Date, system & structure	Initiating cause	Area Affected (sq km)	Demand (GW)	Demand lost (GW)	Approx. time init. event to start of decay	Approx. time of decay of voltage	Mechanism
Tokyo Electric Power July, 1987. meshed 500 kV	High demand, very high rate of increase	West and central Tokyo	39	8	12 mins	4 mins	No reactive reserve; voltage fell to ~ 75% nominal at three major substations which were shut down by protection [15].
B.C. Hydro July, 1979. radial 287 kV	Sudden increase in power transfers caused by (small) demand trip near sending end.		0.2	-	1 min	1 sec	Increased power flow as result of demand loss led to decreased voltage at an intermediate smelter (constant Z) load. As a result the load decreased further, the voltage at the line mid point fell towards zero. Some generation on manual excitation[3].
EDF January, 1987. meshed 400 kV	3 generators tripped over 45 mins at a station in Western France. 4th generator tripped on field protection	Western France, some 150,000	58		Main changes occurred over some 45 mins.	8 mins	Voltage decayed to about 180 kV at one point. 9000 MW of generation lost during voltage fall. Situation contained by load shed and GT start [24].
Sweden December, 1983. meshed 400 kV	Loss of 2 circuits on busbar fault, decreased transmission capacity to Southern Sweden.	Southern Sweden	18	-11	Demand restored to normal by tap change in 1 min		Voltage fell rapidly and as result more lines tripped within 1 min after first fault. Collapse followed. Generators on excitation limit. [11, 25]
EDF December, 1978. meshed 400 kV	Very high demand; high power flows led to circuit tripping	Western, central and northern France	38.5	29	20 mins to first circuit trip. Voltages fell by some 6% in this time	Other circuits and generators tripped in 1 to 1 1/2 mins	Large voltage and current oscillations developed which led to operation of system separation protection [25].
Belgium August, 1982. meshed 380 & 150 kV	Tripping of generation led to voltage drop		5.5	2.4	2 1/2 mins. Further trips over 2 mins		Voltage drop led to 5 generators tripping on excitation protection. Excessive power flows then led to oscillations and tripping of lines to north and centre of country [25].

Table 1b Examples of system voltage problems

Date and system	Conditions	Area affected	Comment
USA June 1984. Interconnected ehv	High load due to unseasonal hot weather plus planned maintenance led to low voltages.	North east United States	No loss of load but voltage reductions implemented in NYPP and PJM. Transfers and generation adjusted to improve Q position [26].
CEGB May, 1986. meshed 400 kV	Six 400 kV circuits were lost due to thunderstorm in 1 minute.	South-East England	Voltage decayed to 325 kV at one point within 5 minutes. Situation contained by GT start [27].

will require the load and distribution system characteristics to be modelled for such areas. Detailed representation of the distribution networks will be impossible but the power and reactive power characteristics of the load and distribution system seen at the interfaces between transmission and lower voltage systems and as affected by the interface voltages, would seem to be an essential requirement for detailed analysis.

As regards the mix of load, the essential requirement will be to assess the proportions of the types of load with different voltage responses for instance:

In the domestic sphere

The resistive heating load: electric fires and radiators, irons, dryers, cookers, water heating

Other forms of space heating involving motors.

Air conditioning.

Lighting (incandescent)

Lighting (fluorescent)

TV and computers

Freezers and refrigerators

Utensils involving motors - e.g. dish-washers, vacuum cleaners.

In commerce

The types of load will be similar to those listed above, with the addition of public lighting.

In industry

There is a wide range of industries (one classification for instance lists over 100) each with its own broad use of electricity for heating, motive power, etc. Common classifications of types of load used for other purposes but which would seem to be appropriate for voltage studies are:

Motive power

Process heating

Space and water heating

Lighting

Electrolytic processes

Distribution System and Load Modelling for Voltage Collapse Studies

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ABSTRACT

This report considers first the evolution and features of major voltage disturbances and the impact of these on the characteristics of distribution system and load required for voltage disturbance studies. A few of the large scale disturbances of recent years are described.

The techniques available to estimate the changes with voltage of power and reactive powers of a distribution network and load are outlined and published results available from staged tests on substations and systems reviewed. The data requirements for the aggregation method, in which composite characteristics are synthesised from estimates of the mix of types of components and (mainly) measured characteristics of each type are also considered.

The paper concludes with a short description of the research currently underway at Imperial College, London into the dynamics and time scale of incipient voltage collapse, including measures and time available to prevent voltage collapse occurring.

INTRODUCTION

It is difficult to keep abreast of the state of knowledge of the phenomenon of voltage collapse in power systems. Recent work seems to have concentrated either on the derivation of approximate, readily computable and perhaps somewhat empirical measures for use in system planning, operation and control [1, 2, 3,] or on fundamental research into the mechanism of collapse. One of the issues has been whether steady state techniques and criteria, as currently used in practice, are adequate or whether dynamic analysis should be used.

An area which seems not to have received the attention it deserves in this context is that of the effect of voltage changes on the characteristics of load and on the distribution network. Many studies and tests have been done, but are more usually related to transient stability and load control; the effect of frequency changes is usually studied as well.

One reason for this relative lack of attention may be the use of tap changers. These will act to keep the voltages at consumer terminals constant at or near the nominal values in spite of changes of 10% or more on the higher voltage networks. Nevertheless, there are several reasons why one needs to consider further the effect of voltage changes at the distribution/ load level:

- 1) typically preventive measures in the event of insufficient generation leading to low system frequency on an isolated system or to high power transfers leading to the risk of thermal overloads or electromechanical or voltage instabilities on an interconnected system will include reduction of load. This will be done by altering tap positions and/or 'locking taps' thereby exposing the load to a reduced voltage and reducing its magnitude [4]. It is accepted that the voltage dependence of load is usually a stabilising factor. In more severe cases, load may be disconnected.
- 2) tap changers will take several seconds to operate; during this period, the load will be exposed to uncorrected values of voltage.
- 3) the system voltage drops may be greater than the correction available from tap changers and hence the voltage at the load will fall.

As noted by Concordia [5], load modelling is qualitatively different from, for example, generator modelling. There will be a vast number of components on the consumers' premises making up the load, the relevant characteristics of most of which will be obtainable from test or basic theory. The difficulty is in determining the mix of these components which aggregate to the load placed on the utility at any time. Furthermore, it will often be power and reactive power estimates which will be required rather than energy demands which on the whole are more easily obtainable.

The way in which a voltage problem is assumed to develop may also be important in analysis. Most of the severe disturbances seem to have occurred following increasing demands, increasing transfers, or decreasing generation or transmission capacities. Unlike transient instability in which the instability develops immediately from the system state during the disturbance, in voltage problems there seems usually to be a period of a few seconds to many minutes between any identifiable initiating event and the start of a significant fall in voltage; voltage collapses seem often to be more examples of 'graceful degradation' rather than very sudden collapse. Some aspect of magnitude, timing and mechanism of actual voltage disturbances have been summarised in Table 1.

Any transmission system voltage changes not corrected at the input to the subtransmission/ distribution networks will affect, in particular, the reactive gain. The importance of the characteristics of the distribution networks is identified in EPRI work.

THE MEASUREMENT AND ESTIMATION OF THE VOLTAGE SENSITIVITIES OF LOAD

THE DATA NEEDED

Voltage collapse in practice seems frequently to be a semi-global problem, i.e. large areas of a system are effected, as illustrated in Table 1. Voltage problems will tend to develop as a result of the aggregate power and reactive power balance in areas of the system rather than the conditions at one particular node. Hence analysis

ACKNOWLEDGEMENT

This study is supported by Puget Sound Power & Light Company.

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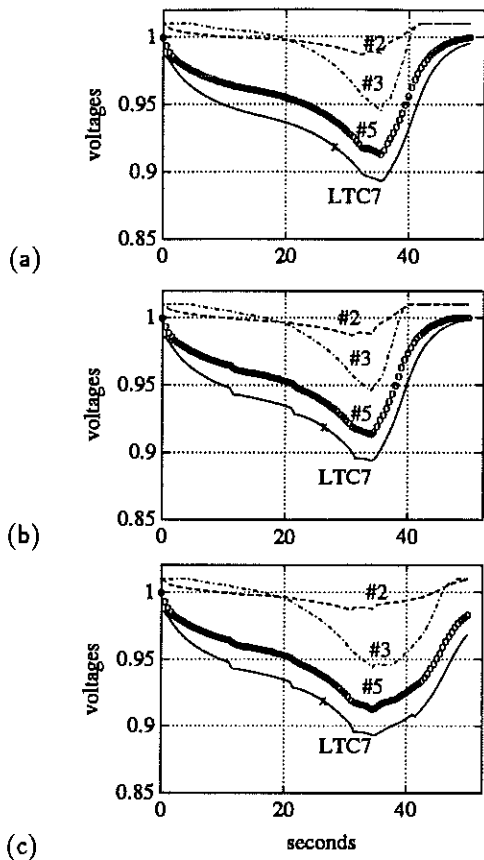


Figure 8: Load shedding as a remedial control. Load model: $\dot{V}$.

- (a) Shed 15% of load. LTC blocked.
- (b) Shed 15% of load. Tap-changing with 10-sec delay.
- (c) Shed 10% of load. Tap-changing with 10-sec delay.

analysis therefore might point to the conclusion that the system stabilizes following the disturbance. Note that such analysis only makes use of the availability of tap-changing at bus 7, but not the dynamic of this tap-changer. With tap-changing of 10-sec delay, the system stabilizes, hence the static prediction is correct. However, the prediction is wrong with slower tap-changing, such as a 15-sec or longer delay!

The results for the $\dot{V}$ model are not the same as those for the 3rd-order model. This is in part due to the fact that $k(V)$ assumes a constant value throughout the simulation. However, control actions are still possible. For example, a shedding of 15% load can save the system. This is shown in Fig. 8(a) where LTC is blocked, and in Fig. 8(b) where tap-changing with a 10-sec delay is used. Similar to the case of the 3rd-order model, tap-changing can also help avoid overshedding. Fig. 8(c) demonstrates that with tap-changing of 10-sec delay, only 10% of load needs to be shed (i.e., 5% less when compared to Figs. 8(a,b)).

3-rd order model; this action automatically reduces both real and reactive power demands (not necessarily by the same proportion). For the $\dot{V}$ model, the same strategy is used, but more calculations are involved. This is because the new static demands (dashed curves in Fig. 4) are computed for the new load torque. To simplify the computation, k assumes a constant value ($k = 5$ for bus 5, $k = 7.5$ for bus 7) throughout the simulation.

Unless otherwise stated, the following 3-step scheme of staggering load shedding is used:

1. Shed 5%, 0.90 voltage setpoint, 3.5-sec delay.
2. Shed 5%, 0.92 voltage setpoint, 5.0-sec delay.
3. Shed 5%, 0.92 voltage setpoint, 8.0-sec delay.

Note that the listed steps might occur in any order depending on which clock expires first. The relay monitors the primary side of tap-changer 7 (no load tripping at bus 5).

The results for the 3rd-order and $\dot{V}$ models are shown in Figs. 7 and 8, respectively. In Figs. 7–8, LTC7 denotes the primary-side voltage of bus 7. On the curve: an ‘x’ marks the instant the setpoint of 0.92 voltage is reached (i.e., the clocks of the 2nd and 3rd steps of the shedding scheme start); the equidistant corners, if any, correspond to the tap-changing at bus 7; other corners correspond to the shedding of one or more blocks of load.

For the 3rd-order model, the collapse situation seen in Fig. 6(b) can be avoided by any of the following actions:

- shed 15% of load, with LTC blocked. This is shown in Fig. 7(b). The system cannot be saved if less than 15% is shed as shown in Fig. 7(a) where 10% load is shed, i.e., only the first two steps of the load shedding scheme are implemented.
- shed 10% of load (i.e., block the last step of the shedding scheme) with tap-changing of 15-sec delay. Comparing Figs. 7(b) and (c), it is seen that tap-changing helps saving one block of load.
- Tap-changing with 10-sec delay.

REMARKS:

1. It is sometimes believed that to save the system, tap-changing should be disabled. However, the simulation here provides a counterexample: voltage collapses in the case of blocked LTC, but not in the case of fast tap-changing—Fig. 6(b) vs. Fig. 7(d)! Therefore, tap-changers should be locked/unlocked only when certain conditions are met. A (sufficient) condition which guarantees the positive effect of locking LTC is given in [9].
2. Fig. 7(d) clearly indicates that there is a post-disturbance equilibrium which is stable. The static

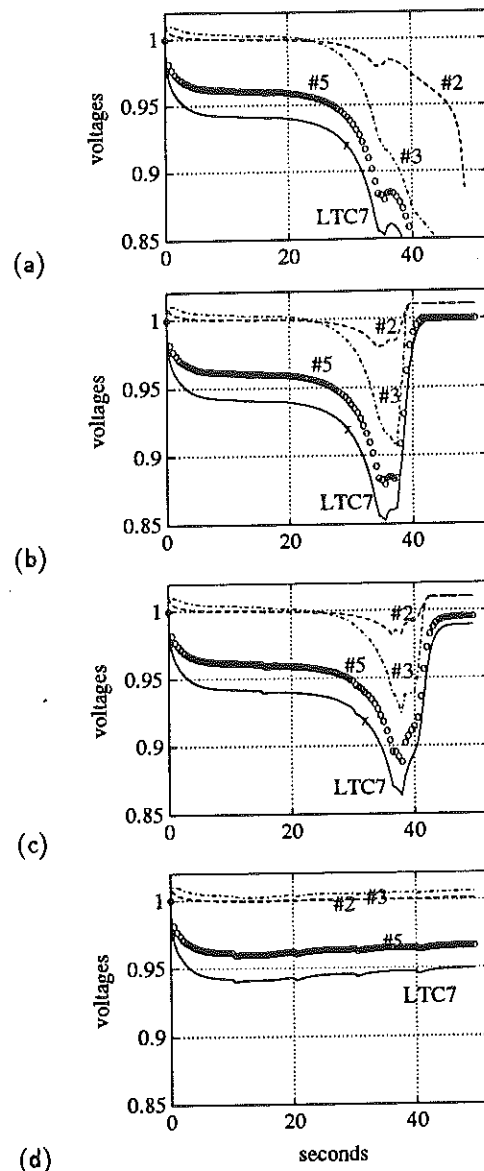


Figure 7: Load shedding as a remedial control. Load model: 3rd-order induction motor.

- (a) Shed 10% of load. LTC blocked.
- (b) Shed 15% of load. LTC blocked.
- (c) Shed 10% of load. Tap-changing with 15-sec delay.
- (d) Tap-changing with 10-sec delay.

In the simulation cases, we assume that the loads are dynamic. Three different cases are used for comparison: 1st-order, 3rd-order, and $\dot{V}$ models. Simple models are assumed for generators: G1 is a slack bus, whereas G2 and G3 have current limits.

The system is in steady-state operation prior to time $t = 0$ when a double-line outage occurs. This disturbance is simulated by replacing X_{24} by $X_{24} \times 5/3$. The result is a voltage collapse as depicted in Fig. 6, but the time duration (time delay) is different for each load modeling. If loads are 1st-order induction motor then the collapse is almost immediate (5 seconds), Fig. 6(a). A longer time delay (about 25 seconds more) is obtained if the 3rd-order model is used for the loads, Fig. 6(b). If the $\dot{V}$ model is used, with $k = 5$ for bus 5 and $k = 7.5$ for bus 7 (a larger value for bus 7 because of heavier loading), then the time duration is nearly the same as that in the case of the 3rd-order model. It is therefore seen that there is more time for control actions in the last two cases. We will concentrate on the controls later on.

A remark is made here about the $\dot{V}$ model. Since a constant value for k is used in the simulation, the system behavior is not strictly the same as that of the 3rd-order model. A comparison of the voltage curves (load buses 5 and 7) of Figs. 6(b) and (c) reveals the followings:

- From $t = 0$ to $t \approx 5$, the voltages V_5 and V_7 in Fig. 6(b) are falling at a faster rate; this means that a small value for k should be used for the $\dot{V}$ model.
- From $t \approx 5$ to $t \approx 25$, V_5 and V_7 appear to stabilize in Fig. 6(b), but decay progressively in Fig. 6(c); this means that a very large value for k should be used.
- From $t \approx 25$ on, the voltage curves in Fig. 6(b) fall more rapidly; thus a small value of k should be used in Fig. 6(c).

A method to "tune" k is yet to be developed.

CONTROL ACTIONS

We shall investigate the effect of tap-changing and undervoltage load shedding. Only the cases of the 3rd-order and $\dot{V}$ models are considered since the resulting time delays appear significant (Fig. 6).

The data are the same as those used to obtain Fig. 6. In addition, the following assumptions are made:

- Automatic on-load tap-changing (LTC) takes place at the end of the radial link, i.e., bus 7. Three cases of tap-changing are considered: 10-sec delay, 15-sec delay, and blocked.
- Load shedding is placed at bus 7. A load shedding refers to a partial removal of the load torque in the

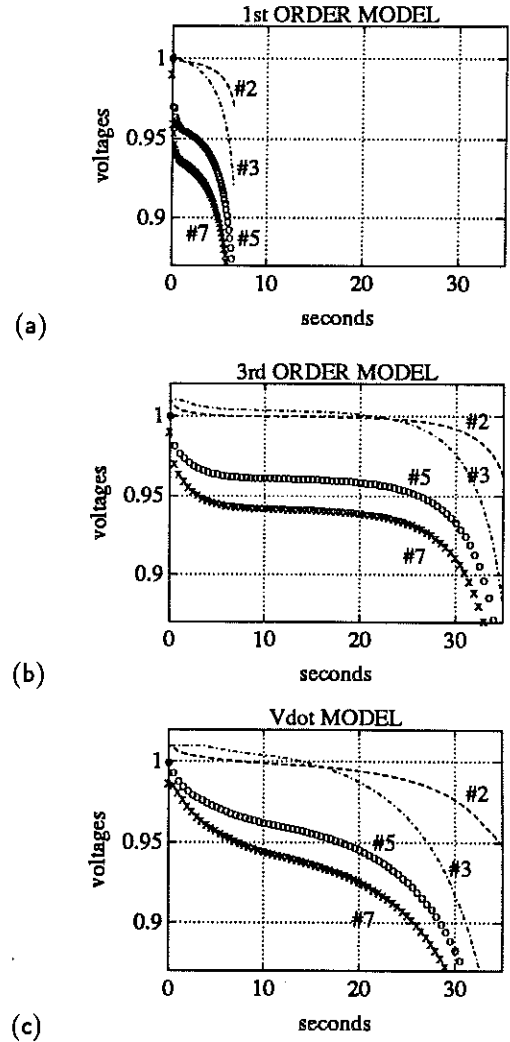


Figure 6: Voltage collapse due to double-line tripping:

(a) loads: 1st-order induction motor.

(b) loads: 3rd-order induction motor.

(c) loads: $\dot{V}$ model.

In view of the dependence of Q on dV/dt , we can express the external behavior of the 3rd-order model in a crude way as follows:²

$$\begin{aligned} P &= P_{stat.}(V) \\ Q &= Q_{stat.}(V) + k(V) \frac{dV}{dt} \end{aligned} \quad (7)$$

where $k(V) > 0$. Note that the term kdV/dt makes the demand less in case of voltage decay (curves D1 and D2), and more in case of voltage increase (curve D3). The static terms $P_{stat.}$ and $Q_{stat.}$ can be represented in any traditional way, e.g., the ZIP model.

The inclusion of dV/dt term in the power demand is first proposed in [1]. It is believed that the dV/dt -dependence is generic [8]. We shall refer to this type of load model as the $\dot{V}$ model ($\dot{V}$ means dV/dt).

Denote by Q_{in} the reactive flow into the device. We must have $Q_{in} = Q$ where Q is the reactive power being drawn. Eq. (7) implies that

$$\frac{dV}{dt} = \frac{\overbrace{Q_{in} - Q_{stat.}(V)}^{\text{"Q-mismatch"}}}{k} \quad (8)$$

which allows the following interpretation, since $k > 0$:

- reactive deficit ($Q_{in} < Q_{stat.}$): voltage decay.
- reactive surplus ($Q_{in} > Q_{stat.}$): voltage increase.
- reactive balance ($Q_{in} = Q_{stat.}$): steady state.

A remark is made here regarding the characteristic of power system stability (angle and voltage) where the modeled dynamics are governed by "mismatches" (P -mismatch for angle problem, and Q -mismatch of voltage problem). In the angle stability problem, the dynamic of generator rotor is described by the "swing equation" which is of 2nd order. This results in a possible oscillation behavior. In the study of voltage problem where the modeling makes use of the $\dot{V}$ model, each load is described by a 1st order equation, Eq. (8), and the system behavior is quite different. In [3], it is shown that such behavior is monotonic.

It should be kept in mind that the behavior of the $\dot{V}$ model might not always be identical to that of the 3rd-order model. This is due to the fact that we use one differential equation to mimic the external behavior of a 3rd-order dynamical system. As a remark, the simple representation of the $\dot{V}$ model has proved to be useful to the mathematical analysis of voltage collapse, e.g., [2, 9, 3]. Note also that only one differential equation is used to describe the behavior of the 1st-order model. However, according to this model, the demand-voltage relationship is

²As a note, the 1st-order model does not show the Q - dV/dt relationship. This is due to the fact that e_r' is the same as $e_{r,EQ}$, thus $Q = Q_{stat.}$.

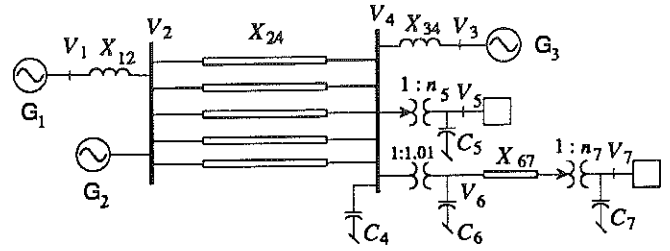


Figure 5: A five-line system, [10].

basically static, i.e., $k(V) = 0$ in Eq. (7). More comparison of the three models is made in the next section.

The use of the $\dot{V}$ model implies that V is a state; thus, V cannot change instantaneously. It may appear that the model is invalid on the ground that in practice the bus voltage drops precipitously due to a fault nearby. Actually, from Eq. (8), the magnitude of k affects the rate at which V can change: a smaller k means a faster change; and in the limit, when k approaches 0, an instantaneous change is allowed.

LOAD MODELS AND CONTROL ACTIONS

In the study of voltage collapse, the time duration over which the event takes place is important since it determines whether the control actions are feasible. Note that the static approach does not provide such information because it is concerned with the existence of an equilibrium.³ In this section, we use dynamical system simulations to illustrate that the load modeling plays an important role in the time delay. The time delay predicted by the 1st-order model is very short, but those by the 3rd-order and $\dot{V}$ models might be significant enough so that control actions such as tap-changing and load shedding can save the system.

TIME DELAYS IN A VOLTAGE COLLAPSE

The power system shown in Fig. 5 is chosen. Its topology is based on [10], and resembles the system in the Puget Sound area of the Pacific Northwest. This system has limited local generation, represented by G_3 . The load centers, buses 5 and 7, have to rely on import over the five long transmission lines. The low-voltage problem is a concern when the load level is high and the 5-line transmission is weakened by a single- or double-line outage.

³The existence of an equilibrium does not always guarantee stabilization, especially when the disturbance is severe enough so that the initial operating point is outside the post-disturbance stability region. Furthermore, the disappearance of equilibria, though a very serious situation, does not readily imply a voltage collapse in the modeled system; as a matter of fact, one should carefully investigate the dynamics of the post-disturbance system in this case, [2].

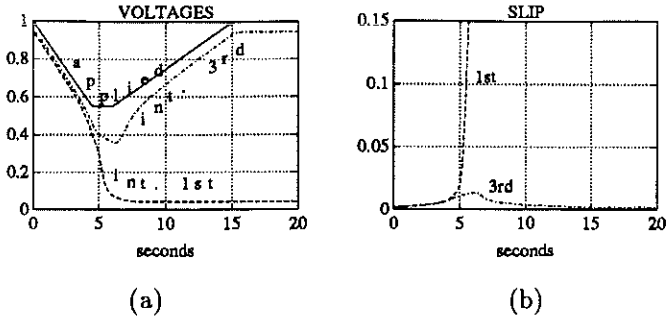


Figure 3: Behavior of the 1st- and 3rd-order models due to a dip in applied voltage.

- (a) applied and induced (internal) voltages.
(b) slip.

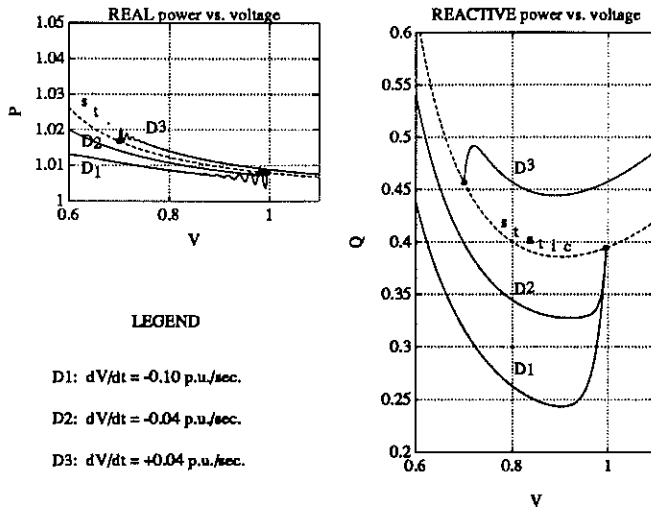


Figure 4: Dependence of power demand on dV/dt .

P); the exponent model where the demand is proportional to V^α . In this section, we address an external representation for dynamic load.

Assume that we are given a device, and are to study its behavior as the applied voltage changes. Our interest is in its external behavior, i.e., power demand vs. voltage— P - V and Q - V . Two types of power demand need to be distinguished: static and dynamic. The difference between the two depends on whether the device is in a steady state. For a resistor, both types are always identical since $P = V^2/R$ and $Q = 0$. For a device represented by a set of dynamical equations such as an induction motor, the static demand corresponds to its equilibrium, and the dynamic demand to its actual state. If the variations in the applied voltage (magnitude) are slow enough so that the state can closely track the equilibrium, then the static and dynamic demands are essentially the same. However, in case of large disturbance the two might be significantly different.

Fig. 4 shows the powers drawn by a 3rd-order induction

machine for various disturbance cases. The dashed curves marked 'static' is obtained by solving for the equilibrium for each value of the applied voltage (magnitude) V . Only values of $V > 0.6$ p.u. are of interest; smaller values result in a stalled machine since the torque induced is less than the load torque. The dynamic demands are different from the static one, and they are dependent on how fast the voltage is changing: curve D1 corresponds to a voltage changing at a constant rate of -0.10 p.u. per second, D2 to -0.04 , and D3 to $+0.04$. Note that a steady-state condition is assumed prior to the variations in voltage (D1 and D2 with 1 p.u. initial voltage, D3 with 0.7 p.u. voltage); this is why the dynamic curves start at a point on the static curves.

Observe that the dynamic and static *real* demands are essentially the same; this is due to the fact that the (stable equilibrium) slip s changes very little over the wide voltage range, and therefore the required real power is practically unchanged. The static and dynamic reactive demand curves are quite different. It is interesting to observe that as the voltage falls, the dynamic reactive demand is less than the static one. An implication of this is that the use of static P - V and Q - V models in a voltage collapse study would yield conservative results. Also note that as the voltage increases, the dynamic demand is larger than the static one. (Naturally, when the voltage ceases to change, the dynamic demand will approach a point on the static curve.)

An explanation of why the dynamic reactive demand is smaller when the voltage falls and larger when the voltage increases is given here. Since $R_s \ll X'$, it can be shown that the reactive power drawn is approximately

$$Q = \frac{V(V - e'_r)}{X'}$$

where e'_r is the component of e' in phase with V . When V changes, the equilibrium¹ shifts; we denote by $e'_{r,EQ}$ its e'_r -coordinate. If V falls as in cases D1 and D2, then $e'_{r,EQ}$ and e'_r also fall, but $e'_r > e'_{r,EQ}$ (in other words, e'_r is tracking a decaying $e'_{r,EQ}$). When V increases, the opposite is true, i.e., both e'_r and $e'_{r,EQ}$ increase but $e'_r < e'_{r,EQ}$. Therefore,

$$\dot{V} < 0:$$

$$Q = \frac{V(V - e'_r)}{X'} < \frac{V(V - e'_{r,EQ})}{X'} = Q_{stat.}$$

$$\dot{V} > 0:$$

$$Q = \frac{V(V - e'_r)}{X'} > \frac{V(V - e'_{r,EQ})}{X'} = Q_{stat.}$$

It is also true that the faster V changes, the faster $e'_{r,EQ}$ drifts. This implies that $e'_{r,EQ}$ and e'_r are further apart; thus the difference between Q and $Q_{stat.}$ in the above inequalities is more. This is why the static curve differs from curve D1 more than it does from curve D2.

¹For the 3rd-order model, the state space is 3-dimensional, (e'_r, e'_i, s) .

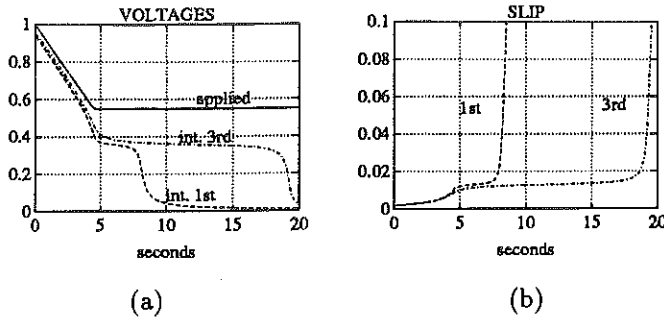


Figure 1: Response of 1st- and 3rd-order models due to a decaying applied voltage.

- (a) applied and induced (internal) voltages.
(b) slip.

$$0 = -\frac{1}{T_{do}} [e'_{EQ} - j(X - X')i] - je'_{EQ}2\pi fs \quad (4)$$

$$\frac{ds}{dt} = \frac{1}{2H} [T_L(s) - \Re\{e'_{EQ}i^*\}] \quad (5)$$

$$i = \frac{V - e'_{EQ}}{R_s + jX'} \quad (6)$$

In fact, Eqs. (4, 6) can be represented by the standard equivalent circuit, [5].

The dynamic of the 1st-order model has been discussed in [6] in conjunction with voltage instability. It is proved that the motor stalls when the slip exceeds a certain value (the unstable equilibrium). When the flux dynamic is included as in the 3rd-order model, the slip evolves more slowly than it does in the 1st-order model. This is due to the fact that the induced torque is sustained by the flux. In this section, two differences in dynamic behavior of the models are addressed: time delay before stalling, and stability under severe disturbance.

Fig. 1 shows the behavior of the two models when the applied voltage is dropped from an initial value of 1 p.u. to 0.55 p.u. An applied voltage of $V = 0.55$ will result in a stalled motor since the induced torque is less than the load torque. Even though both models predict the same consequence, the time duration before which the motor stalls is different in each model. The 3rd-order model takes more time before stalling since its internal voltage e' decays more slowly.

Besides having a longer time duration in the case of stalling, the 3rd-order model may have a totally different response from that of the 1st-order model, especially when a large excursion in voltage is encountered. Let's consider the qualitative behavior of the two models when there is a dip in the applied voltage such as the one shown in Fig. 3. An example of a voltage dip is the 1987 event in France [7] which starts with a successive loss of reactive sources; the voltage stabilizes to as low as 50% of the nominal level, and recovers following load shedding. Response of loads dur-

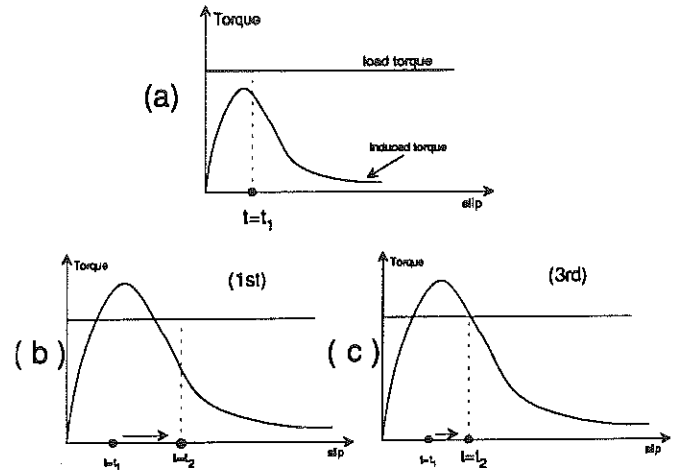


Figure 2: The 1st- and 3rd-order models predict different consequences in a voltage dip.

- (a) prior to voltage recovery.
(b) and (c) at full voltage recovery: the unstable slip is exceeded in the 1st-order model (b), but not in the 3rd-order model (c).

ing this process is worth considering. We can use Fig. 2 to explain the qualitative behavior of each model. Assume that the dip is severe enough so that the induced torque is less than the load torque (a constant) as in Fig. 2(a). This means that the slip continues to increase. Note that the slip increases faster in the 1st-order model than it does in the 3rd-order model. At time t_2 when the voltage fully recovers, the slip in the 1st-order model might be beyond the unstable slip as in Fig. 2(b); the unstable slip being the rightmost intersection between the load and induced torque curves. Thus, according to this model the motor will eventually stall. For the 3rd-order model, the slip increases at a slower rate, and at time t_2 it might still be less than the unstable equilibrium, as in Fig. 2(c). Consequently, the slip would converge to the stable point; that is, the motor resumes its normal operation.

To illustrate that the two models can predict different behavior in a voltage dip, the result of a numerical simulation is shown in Fig. 3.

LOAD MODEL

The above two models for induction motor are concerned with the internal behavior of the device (s and e'). Such representations may make analytical (qualitative) studies of voltage collapse difficult since they offer no explicit relationship between the applied voltage and power demands. Usually the load models involving P , Q and V explicitly are static (we refer to this type of representation as external). Examples are the ZIP model (quadratic model) which views the load as consisting of an impedance, a constant current, and a constant power (hence the name Z-I-

A STUDY OF DYNAMIC LOAD MODELS FOR VOLTAGE COLLAPSE ANALYSIS

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ABSTRACT

In the study of voltage collapse, the qualitative behavior of load over a large range of voltage variations should be modeled, especially when a severe disturbance is considered. Several dynamic load models are studied in this paper, where the external behavior of the load (demand vs. voltage) is the main concern. When the applied voltage falls, a dynamic model may suggest that the reactive demand is significantly lower than the one from the static model. As a consequence, in case of system collapse, an extra time delay is predicted by dynamic models; such a time delay might be long enough to allow control actions against system breakdown.

INTRODUCTION

Load modeling has been viewed as an important component in the studies of power system dynamics, but the behavior of load is, in general, poorly understood. It is well-known from studied cases of voltage collapse that the familiar constant PQ-load model yields very conservative results since the associated load demand is unchanged as the voltage decays. Other models which allow load demand to vary with the applied voltage have been used, such as the quadratic model. However, these static models do not account for the dynamic behavior of the load. The dynamic aspect is usually believed to be associated with induction motor load. There are several mathematical models for induction motors, the complexity varying from very simple to very detailed. However, all these models involve many parameters and computational burden. To avoid these problems, an equivalent model which is concerned with the external behavior of the machines has been proposed [1]. This model consists of two parts: a voltage-dependent static part, and a dynamic part which is dependent on the rate of change of voltage, dV/dt . The simplicity of this model has proved to be very useful in the development of voltage collapse theories [2, 3].

This paper is concerned with the dynamic load modeling for the study of voltage collapse. The discussion begins with a comparison of two different induction motor models (one with flux dynamics and one without). These two models have different stability properties when subject to a large disturbance such as a severe voltage dip. The dependence of load demand on dV/dt is verified for the 3rd-order induction motor model, and a physical explanation is given. Time delays in a voltage collapse event are

dependent on load modeling, and may or may not allow room for control actions. These points are illustrated by a numerical simulation of a system whose structure is similar to that in the Puget Sound region.

INDUCTION MOTOR MODELS

Two induction motor models are considered in this paper: a model with flux dynamic (3rd-order system), and a model without flux dynamic (1st-order).

3rd-order model (with flux dynamics) [4]: In this model, the flux dynamic is represented by Eq. (1), and the slip by Eq. (2).

$$\frac{de'}{dt} = -\frac{1}{T_{do}} [e' - j(X - X')i] - je'2\pi fs \quad (1)$$

$$\frac{ds}{dt} = \frac{1}{2H} [T_L(s) - \Re\{e'i^*\}] \quad (2)$$

$$i = \frac{V - e'}{R_s + jX'} \quad (3)$$

where

- e' : $e'_r + je'_i$ = emf induced in rotor winding
- V : applied voltage (angle chosen as reference)
- T_{do} : time constant
- X : reactance at synchronous speed
- X' : reactance with blocked rotor
- R_s : stator winding resistance
- T_L : load torque, function of speed
- H : inertia (of motor plus driven machine)
- s : slip

There are a total of 3 differential equations since Eq. (1) has real and imaginary parts (e'_r and e'_i). The 3rd-order model can be viewed as an internal voltage e' behind an impedance, Eq. (3), and the dynamic of the internal voltage is governed by Eqs. (1-2).

1st-order model (without flux dynamics): In this model, only the mechanical dynamic (i.e., slip dynamic) is considered; the flux is assumed to be able to change instantaneously. For this model, Eqs. (1-3) can be used, except that Eq. (1) is replaced by an equilibrium condition. That is,

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a dynamic one if reduced to the form (45a), and a static one if reduced to form (48). Naturally, the two views are not unrelated and it is clear that the static analysis will feature only part of the more complex phenomena that could be obtained from the dynamic analysis.

For the examination of the local behavior of the dynamic system (46), we linearize it around the fixed point given by Eqn.(47). With $\frac{d}{dt} = \lambda$, we obtain

$$\frac{\partial \phi}{\partial x} \Delta x + \frac{\partial \phi}{\partial y} \Delta y = \lambda \Delta x \quad (49a)$$

$$\frac{\partial \psi}{\partial x} \Delta x + \frac{\partial \psi}{\partial y} \Delta y = 0 \quad (49b)$$

or

$$\begin{bmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \\ \frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \lambda \begin{bmatrix} \Delta x \\ 0 \end{bmatrix} \quad (50a)$$

This identifies the system matrix

$$J_{sys} = \begin{bmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \\ \frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y} \end{bmatrix} \quad (51)$$

The alternate form

$$\begin{bmatrix} \left(\frac{\partial \phi}{\partial x} - \lambda I\right) & \frac{\partial \phi}{\partial y} \\ \frac{\partial \psi}{\partial x} & \frac{\partial \psi}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (50b)$$

of Eqn.(50a) shows that the dynamic problem associated with J_{sys} of Eqn.(51) is not of conventional type, since λ does not take all positions in the diagonal.

We note that this hybrid problem description is analogous to the approach used for the small signal stability analysis of large power systems, where it is preferred because it preserves sparsity [12]. Eqns.(50) can, however, be reduced to the standard form

$$(J_{dyn} - \lambda I) \Delta x = 0 \quad (52a)$$

where

$$J_{dyn} = \frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial y} \left(\frac{\partial \psi}{\partial y} \right)^{-1} \frac{\partial \psi}{\partial x} \quad (52b)$$

is a function of μ , and λ is an eigenvalue, Δx an eigenvector of J_{dyn} .

Dynamical bifurcation occurs for those values of μ which result in a λ being either purely imaginary or zero. In the former case, which leads to Hopf bifurcation, $\det(J_{sys}) \neq 0$ (see Eqn.(51)), while in the

latter case, $\det(J_{sys}) = 0$ and also $\det(J_{dyn}) = 0$, and Eqns.(50) can be reduced to the form

$$J \Delta y = 0 \quad (53a)$$

where

$$J = \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial x} \left(\frac{\partial \phi}{\partial x} \right)^{-1} \frac{\partial \phi}{\partial y} \quad (53b)$$

Eqn.(53a) indicates that $\det(J) = 0$ and the Jacobian is singular. Thus J and J_{dyn} become singular together, as the consequence of J_{sys} becoming singular for some μ_{cr} . However, J_{dyn} can display Hopf bifurcation without this fact being reflected in J .

CONCLUSIONS

The paper has focused on the effect of load characteristics on the static loading limits of a power system. While in the standard load flow formulation, P and Q at the load buses are specified, it is seldom justified to consider as critical the extreme loading condition where $P, Q \rightarrow \max$. Criticality is in general the result of a bifurcation for the combined system and load equations and it may appear after or before the nose point of power-voltage characteristics representing the supply system. Consequently, the assessment of critical loading, based on the singularity of the load flow Jacobian, may not be satisfactory, and more realistic results could be achieved if the Jacobian is adjusted to reflect the load characteristics.

A particular and interesting example, where the load characteristics adversely affect the voltage stability of the system, is in the case of induction loads. These are shown to produce extemporaneous and possibly unsuspected collapse conditions due to their drooping Q - V characteristics.

The discussion on the significance of load characteristics concludes with a proposal for a voltage collapse Proximity Index based on the singularity of the adjusted Jacobian. The index is easier to calculate than the determinant or the smallest eigenvalue of the Jacobian but is essentially equivalent to these. It may serve to guide a sequence of load flows to obtaining the critical loading condition.

Finally, a discussion on static and dynamic bifurcations ends with the conclusion that the static bifurcations reflect essentially particular aspects of dynamic bifurcations, which are related to the general problem of dynamic stability of the power system.

ACKNOWLEDGEMENTS

Financial support from the Natural Sciences and Engineering Research Council of Canada is gratefully acknowledged.

$$\begin{aligned}
P_1 &= P_1(V_1, s_1, \mu) \\
Q_1 &= Q_1(V_1, s_1, \mu) \\
P_2 &= P_2(V_2, s_2, \mu) \\
Q_2 &= Q_2(V_2, s_2, \mu)
\end{aligned} \tag{39c}$$

We note that these are functions of the state variables $x = \{s_1, s_2\}$. (In general, state variables could be internal angles θ , speeds ω or slips s , fluxes, control variables, etc.) Correspondingly, the interface (network) variables y of Eqn.(40a) will now be functions of s_1, s_2

$$\{V_1, V_2, \delta_1, \delta_2, P_1, P_2, Q_1, Q_2\} = F(s_1, s_2, \mu) \tag{40b}$$

while instead of Eqns.(41a) we have

$$f_i(V_1, V_2, \delta_1, \delta_2, s_1, s_2, \mu) = 0, \quad (i = 1, 2, 3, 4) \tag{41b}$$

which, of course, also give the solution (in conjunction with (39a)) of Eqn.(40b).

We note that in steady state (when the state variables s_1, s_2 assume constant values), the dynamic equations reduce to the static equations: (40b)→(40a) and (41b)→(41a). We also note that the adjusted load flow equations (41b) have the general form

$$\psi(x, y, \mu) = 0 \tag{42}$$

The differential equations for the induction motors describe their acceleration and are of the form

$$M\dot{s} = P_{mech} - P \tag{43}$$

where the electrical power is a function of V, s and μ . Thus the dynamic equation is of the general form

$$\dot{x} = \phi(x, y, \mu) \tag{44}$$

Relation Between Static and Dynamic Bifurcation

Generally, the term bifurcation is used both in the context of static and dynamic stability analysis of power systems. In the static analysis of voltage stability, it is often connected to the classical load flow problem where P and Q are assumed to be specified and functions of a loading parameter μ . Bifurcation in this description corresponds to the critical transition point from two classical load flow solutions to none, appearing at $P, Q = \max$. If, however, the load characteristics are taken into account, then loading is not simply equivalent to increasing P and Q but rather to increasing the connected nominal load (related to the rated voltage). This leads to the condition of critical loading, again related to the transition point from two

to zero load flow solutions, this time, however, with P_n rather than P, Q as inputs to the load flow problem.

The relation between static and dynamic bifurcation is not straightforward in the case of power systems, because the dynamic equations do not have the simple form

$$\dot{x} = f(x, \mu) \tag{45a}$$

in terms of state variables only, as usually presented in the literature [3],[4].

The above differential equation has fixed points x , function of μ , given by the algebraic equation

$$f(x, \mu) = 0 \tag{45b}$$

which represents the steady state condition of the system. The local, dynamic behavior at a fixed point is characterized by the Jacobian matrix $\partial f / \partial x$ at that point, and dynamic bifurcation appears whenever an eigenvalue of the Jacobian matrix has zero real part. Static bifurcation requires a zero eigenvalue, i.e., $\det(J) = 0$, while dynamic bifurcation can occur even if $\det(J) \neq 0$, but with purely imaginary eigenvalues that lead to Hopf bifurcation.

As we have seen in the previous example (Eqns. (44) and (42)), the general dynamic equations of a power system are of the form

$$\phi(x, y, \mu) = \dot{x} \tag{46a}$$

$$\psi(x, y, \mu) = 0 \tag{46b}$$

This formulation is valid not just in relation to load buses (as in the case of the induction load discussed above) but, quite generally, for the whole power system, including generators and their controls.

If we could substitute y from (46b) into (46a), we would obtain the classical form (45a). If we set $\dot{x} = 0$, then Eqns.(46) give the fixed point (or steady state) equations

$$\phi(x, y, \mu) = 0 \tag{47a}$$

$$\psi(x, y, \mu) = 0 \tag{47b}$$

which yield either Eqn.(45b) or the adjusted load flow equations (that include the load characteristics)

$$g(y, \mu) = 0 \tag{48}$$

if x is substituted from (47a) into (47b). Clearly, Eqn.(48) is equivalent to (45b), but it is given in terms of the interface variables y , rather than the state variables x .

We see thus that the dynamic system (46)—due to the fact that it includes both state variables x and interface (or load flow) variables y —offers two aspects,

smallest eigenvalue of the matrix under consideration.

We note that during the simple process of calculating the proximity index PI of Eqn.(33), the eigenvector e , corresponding to the smallest eigenvalue λ_{min} , has also been obtained. This could be of significance since the eigenvector provides information regarding the participation of different system components in the mechanism of loss of stability. The importance of this fact has been pointed out and documented in the recent reference [11]. It is shown there how the eigenspace corresponding to several of the smallest eigenvalues can be identified, if desired. To achieve this objective, the method of Eqns.(32) can be generalized, as shown in the discussion to reference [11] (to be published alongside with the paper), for the case of several eigenvalues approaching zero.

Calculation of Critical Loading by Sequential Load Flows

In order to obtain the critical loading point, we increment the loading parameter μ until the Proximity Index becomes zero. Let $\mu_1 < \mu_2$ be two consecutive values of μ and PI_1, PI_2 the corresponding proximity indices, obtained as shown above, after complete load flow solutions. Then the secant method gives the new value

$$\mu_3 = \frac{\mu_2 PI_1 - \mu_1 PI_2}{PI_1 - PI_2} \quad (38)$$

This process has superlinear convergence.

Illustrative Examples

Static Induction Load

In the following we illustrate the use of the Proximity Index on the system of Figure 10 with induction loads at each of the two load buses.

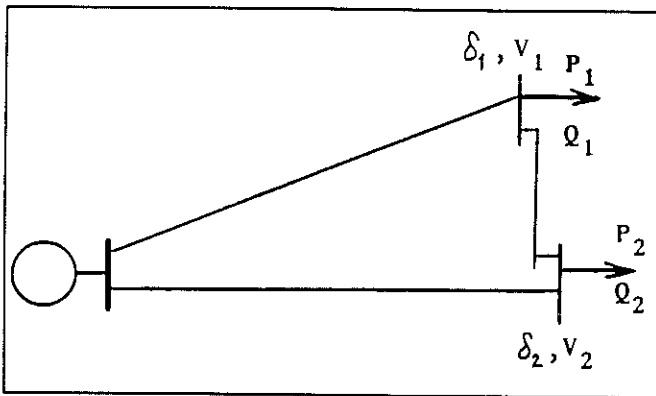


Fig. 10 Two load bus system
The load flow equations will have the form

$$\begin{aligned} P_i &= P_i(V_1, V_2, \delta_1, \delta_2) \\ Q_i &= Q_i(V_1, V_2, \delta_1, \delta_2) \end{aligned} \quad (i = 1, 2) \quad (39a)$$

and the induction load equations will be, based on the results of Eqns.(22),

$$\begin{aligned} P_1 &= P_{n1} = \alpha_1 \mu \\ \frac{Q_1}{P_{n1}} + \frac{P_{n1}}{Q_1} &= 4 \left(\frac{V_1}{V_n} \right)^2 \\ P_2 &= P_{n2} = \alpha_2 \mu \\ \frac{Q_2}{P_{n2}} + \frac{P_{n2}}{Q_2} &= 4 \left(\frac{V_2}{V_n} \right)^2 \end{aligned} \quad (39b)$$

The number of Eqns.(39) is eight and they contain eight unknowns, for which they could conceptually been solved and which, therefore, can be viewed as functions of μ :

$$\{V_1, V_2, \delta_1, \delta_2, P_1, P_2, Q_1, Q_2\} = F(\mu) \quad (40a)$$

On the other hand, we can eliminate pairwise the interface variables P_1, P_2, Q_1, Q_2 from Eqns.(39) so that we end up with four equations, in terms of voltages only. These are the adjusted load flow equations

$$f_i(V_1, V_2, \delta_1, \delta_2, \mu) = 0, \quad (i = 1, 2, 3, 4) \quad (41a)$$

Equations (41a) contain μ as parameter. If solved, they yield, together with (39a), the values listed in (40a). They also yield the 4×4 Jacobian J , which we use, as indicated in (32) and (33) to obtaining the Proximity Index PI, for any specified μ . The secant equation (38) will then lead us to μ_{cr} as $PI \rightarrow 0$. For all values of μ , Eqn.(40a) gives the corresponding information for the state of the system (except at μ_{cr} itself, when the calculations become ill-conditioned and, therefore, approximations from neighboring values may be appropriate.)

Dynamic Induction Load

This will be a slight digression from the static point of view of the paper: its justification is the discussion of dynamic bifurcations in the next section. We generalize the discussion by introducing the dynamics of the load, which consists of induction motors in this particular case. Clearly, the state variables are the slips s of the induction machines and there is no dynamics in the network, so that Eqns.(39a) remain valid. However, instead of (39b) we will have equations of the form

characteristics, is the fact that all loads, at all system buses, become critical at the same time. This fact does not appear immediately if the load characteristics and the nose curves are examined independently at the different buses. However, the voltages of the different load buses are interrelated, and when the system Jacobian (adjusted to incorporate the load characteristics) becomes singular, then all voltage and angle increments, ΔV and $\Delta \delta$, will vary proportionally, in the null-space of the Jacobian, limited only by second order effects [10]. In other words, voltage collapse is a system-wide problem and criticality does not appear at individual buses only. This point of view is further strengthened if one takes into consideration the fact that system loading is essentially a one parameter problem: each P_n , the loading parameter of an individual bus, is a function of a single loading parameter $\mu = P_D$ representing the total demand on the system. Consequently, the Jacobian J is function of the single parameter μ .

Finally, we note that the load flow Jacobian J_{LF} is generally not the same as the adjusted Jacobian J , which also reflects the load characteristics. Indeed, in a load flow program, P and Q are specified, which is equivalent to voltage-independent (vertical) load characteristics. It is J_{LF} that is relevant for the load flow while the action of the tap changers keeps the loads constant, but it is J that determines whether the critical point has been reached. Therefore, the critical loading point can, in general, be obtained by standard load flow solutions even though J tends to zero at the solution point.

Proximity Index for Voltage Collapse

As mentioned above, steady state voltage collapse occurs when the system Jacobian J , adjusted to include the load characteristics, becomes singular. Different measures for closeness to singularity exist: $\det(J)$, the smallest eigenvalue, or the smallest singular value [9] of J , among others. They can serve as Proximity Indices to control the sequence of load flows (with parameter μ) to the critical point. Here we describe a PI that is equivalent to those mentioned but easier to calculate.

If J is singular, then

$$Je = 0 \quad (32a)$$

where e is the eigenvector corresponding to the zero eigenvalue. We normalize it so that its last element (assumed to be non-zero) is unity. Then, the above equation can be written in the partitioned form

$$\begin{bmatrix} J' & a \\ b^T & \alpha \end{bmatrix} \begin{bmatrix} e' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (32b)$$

From the first equation, where we assume that the submatrix J' is non-singular, we calculate e' and then we evaluate the residue of the second equation: this will be the Proximity Index

$$PI = b^T e' + \alpha \quad (33)$$

Clearly, when $PI = 0$, then Eqns.(32) are satisfied and the Jacobian is singular.

In the following we demonstrate, using a simple example, that the proposed PI is proportional, in the proximity of a singularity of a matrix A , to $\det(A)$ and to λ_{min} (the smallest eigenvalue of A). We choose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 + \varepsilon \end{bmatrix} \quad (34)$$

where $\varepsilon \ll 1$.

Clearly, with $\varepsilon = 0$, matrix A is singular: as in (32), we have

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (35)$$

If $\varepsilon \neq 0$, then neglecting higher powers of ε , we obtain from (34)

$$\det(A) = -3\varepsilon, \quad \lambda_{min} = -\frac{\varepsilon}{10} \quad (36a)$$

while the procedure of Eqn.(32b), i.e.,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 + \varepsilon \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (37a)$$

yields, for the residue (33) of the third equation,

$$PI = \varepsilon \quad (36b)$$

Similarly, if the first element of the eigenvector had been normalized to unity, i.e.,

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 + \varepsilon \end{bmatrix} \begin{bmatrix} 1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (37b)$$

then we would have obtained for the residue of the first equation

$$PI = -9\varepsilon \quad (36c)$$

We see from the three results in Eqns.(36) that the proximity index depends on the choice of normalization and is equivalent to the determinant and to the

V_s and V_l . Their ratio, in per unit, due to the tap changer, is $\theta = V_l/V_s$. As long as the LTC is in its active range, $V_l = V_n$, so that $P = P_n$ and $Q = Q_n$. In order to use a single voltage variable V , let $V_s = V$ and $V_l = \theta V$.

We illustrate the calculation of the incremental load characteristic on hand of equations (14). From (14a), we have

$$\frac{dP}{dV} = \frac{dP}{dV_s} = \theta \frac{dP}{dV_l} = \theta p P_n \frac{V_l^{p-1}}{V_n^p} = \theta p \frac{P_n}{V_n} \quad (28a)$$

and, similarly, from (14b),

$$\frac{dQ}{dV} = \theta q \frac{Q_n}{V_n} \quad (28b)$$

Clearly, the derivatives of the load characteristics are slightly increased if $\theta > 1$.

We note here that the incremental load characteristic of an induction load, given by Eqn.(22b), can be calculated taking into account that in this equation Q appears in an implicit form. Consequently,

$$\left(\frac{1}{P_n} - \frac{P_n}{Q^2} \right) \frac{dQ}{dV} = \frac{8V}{V_n^2} \quad (29)$$

Example for Calculation of Critical Loading Point

We consider a supply system as in Figure 2, with $E = 1$ and $X_0 = 0.2$. The load is assumed to be given by

$$Q = \mu V^2 + 1 \quad (30a)$$

where μ is the loading parameter. The supply power is

$$Q = V \frac{1-V}{0.2} = 5V - 5V^2 \quad (30b)$$

According to the general methodology outlined above, we first substitute the load power Q from (30a) into (30b). This yields

$$f(V, \mu) = (\mu + 5)V^2 - 5V + 1 = 0 \quad (31a)$$

This equation corresponds to Eqn.(18b). It has (see Figure 9) two real roots for loadings μ below $\mu_{cr} = 1.25$ which gives $V_{cr} = 0.4$ and $Q_{cr} = 1.2$. Note, that the critical solution point is below the nose point where $\mu = 1$, $V = 0.5$ and $Q = Q_{MAX} = 1.25$. In fact the load characteristic (30a) for $\mu = 1$ intersects the supply characteristic (30b) also at $V = 0.3333$ and $Q = 1.1111$. The first of the last two "load flow solutions" is often perceived as leading to voltage collapse while the second one is viewed as unstable. (There are, however, no dynamic elements in our model to substantiate such judgments).

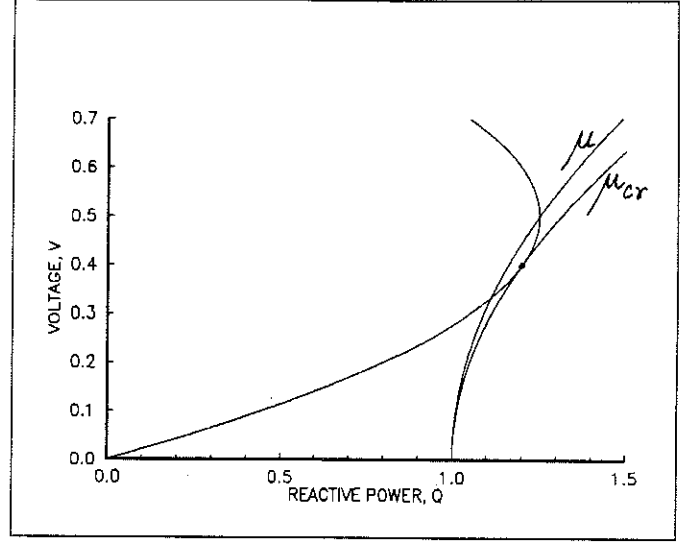


Fig.9 Simple example for nose curve and load characteristics

By the procedure of Eqn.(18c), we would have

$$\frac{\partial f}{\partial V} = 2(\mu + 5)V - 5 = 0 \quad (31b)$$

Simultaneous solution of Eqns.(31) yields, of course, the same critical values, μ_{cr} , V_{cr} , and Q_{cr} we have found above by examining the roots of the quadratic equation (31a). We note that Eqn.(31b) expresses the fact that at the critical point the supply and the load characteristics have equal slope so that the two curves become tangent. Therefore, voltage collapse could occur before P_{MAX} , Q_{MAX} , when the load characteristic has a negative slope, due for example to the influence of induction loads.

COMPUTATION OF CRITICAL LOADING

Voltage Dependent Load Models

The purpose of having discussed in the previous section different load characteristics—with special attention to the induction load—was to emphasize that the critical loading of the system is in general not at the extreme power limit for the system itself and, in particular with induction loads, it could be at smaller powers and higher voltages, since the induction load has a negative slope of its Q - V characteristic. This is, of course, a purely static point of view and it is quite possible that the dynamic stability limit could appear even before. However, in this latter case, the situation can be mitigated by successful control actions.

Another important remark, related to the interaction between load and supply system

or, with $\varphi_n = 15^\circ$,

$$Q + \frac{P^2}{Q} = 4P_n \left(\frac{V}{V_n} \right)^2 \quad (21c)$$

Eqns.(21) represent a surface which includes the point P_n, Q_n, V_n . A point on the surface is not expected to be far away from this nominal operating point since increased loading is the result of more motors being added to the load. The purpose of the flexibility afforded by Eqns.(21) is only to explore the stability of the system in the neighborhood of the operating point. In fact, however, the active power P cannot 'slip', being fixed by the mechanical load. Therefore, it is reasonable to assume that

$$P = P_n \quad (22a)$$

and thus Eqn.(21c) becomes

$$\frac{Q}{P_n} + \frac{P_n}{Q} = 4 \left(\frac{V}{V_n} \right)^2 \quad (22b)$$

Corresponding to the constraint (22a), the characteristic of the supply system of Eqn.(17a) becomes

$$P_n^2 + \left(Q + \frac{V^2}{X_0} \right)^2 = \left(\frac{EV}{X_0} \right)^2 \quad (17c)$$

This equation gives an unconventional nose curve which corresponds to the cross-section of the supply surface with the plane $P = P_n$, rather than with a plane at a constant power factor, $P = \rho Q$. Figure 8 represents both the supply characteristic and a family of induction load characteristics. The load characteristics have minima for V when $Q = P_n$. Clearly, stable operation requires $Q < P_n$, when the reactive power has a negative slope. We can also see that the critical reactive loading Q_{max} is less than the extreme value Q_{MAX} on the nose curve.

Composite Load

We have found that even in the case of an induction load the expressions of P and Q are of the general form (12a) and (12c), respectively. Since on any given bus we may have several load components, we will simply have to add up the relevant equations. For instance, for two loads, P_{n1} and P_{n2} , Eqns.(12a) will be

$$P_1 = f_{1,1}(V, P_{n1}) \quad (23a)$$

$$P_2 = f_{1,2}(V, P_{n2}) \quad (23b)$$

If we consider a single loading parameter, P_n , so that

$$P_{n1} = \alpha_1 P_n \quad (24a)$$

$$P_{n2} = \alpha_2 P_n \quad (24b)$$

then Eqns.(23) take the form

$$P_1 = f_{1,1}(V, P_n) \quad (25a)$$

$$P_2 = f_{1,2}(V, P_n) \quad (25b)$$

Therefore

$$P = P_1 + P_2 = f_1(V, P_n) \quad (26a)$$

and, similarly,

$$Q = Q_1 + Q_2 = f_2(V, P_n) \quad (26b)$$

We note that all derivations shown above refer to a single load bus. However, in the general situation of a multi-bus system, we still may consider that each (active and reactive) load is proportional to the total demand in the system. If we keep the same notation, P_n , for this loading parameter, then Eqns. (12) or (26) apply to any bus k ,

$$P_k = f_{k1}(V_k, P_n) \quad (27a)$$

$$Q_k = f_{k2}(V_k, P_n) \quad (27b)$$

Clearly, now, the load characteristics are functions of the local bus voltage. We will return to this problem shortly, when we shall deal with the general computational problem for determining the critical loading of the entire system.

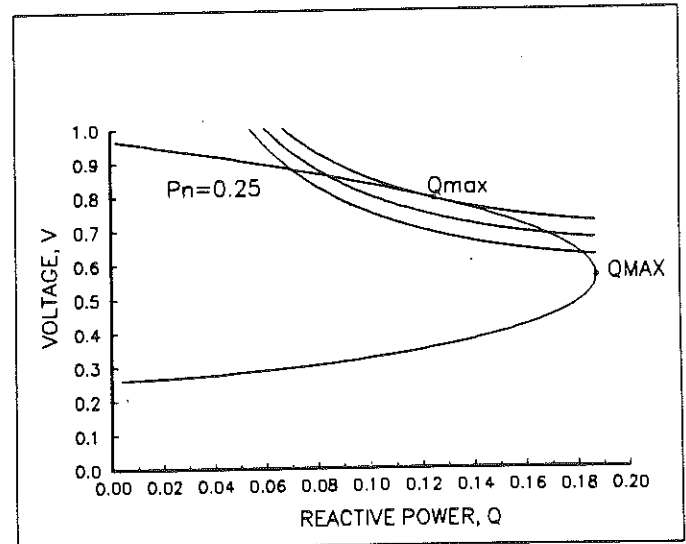


Fig. 8 Reactive load characteristics for an induction load

Tap Changers and Incremental Load Characteristics

The effect of tap changers is to keep the secondary voltage close to a constant value. For simplicity, we will assume that this value is V_n . This keeps the load nearly constant, but not its derivative, since, as mentioned, the action of the tap changer is intermittent.

Because of the presence of the LTC, we have to distinguish between the supply and load side voltages,

Supply System

The process of loading in the case of the nonlinear loads given by Eqns. (12) or (14) consists in increasing the rated powers P_n and Q_n . Even for $P_n, Q_n = \text{const}$, the power factor remains constant only if the exponents p and q are equal, which is rarely the case. Therefore, the concept of nose curves is too simplistic for a more realistic representation of the system-load interaction.

A more general understanding is obtained if the supply system is represented by the equation

$$F(P, Q, V) = 0 \quad (15)$$

In the simple case of Figure 2, the balance of voltage drops is expressed by

$$\left(V + X_0 \frac{Q}{V}\right)^2 + \left(X_0 \frac{P}{V}\right)^2 = E^2 \quad (16)$$

This yields

$$P^2 + \left(Q + \frac{V^2}{X_0}\right)^2 = \left(\frac{EV}{X_0}\right)^2 \quad (17a)$$

or, in per unit,

$$P^2 + (Q + V^2)^2 = V^2 \quad (17b)$$

Eqns.(17) are of the general form (15). They represent circles for any $V = \text{const}$. Figure 7 represents the surface corresponding to Eqn.(17b) for the supply system of Figure 2. Any radial cross-section of this surface is a nose curve for $\rho = \text{const}$.

Critical Loading

We note that the general load model (12) can be represented by a curve in the three-dimensional space of Eqn.(15). This load curve, which may or may not intersect the supply surface, is function of the two load parameters, P_n and Q_n . Thus, substitution of (12) into (15) yields

$$f(V, P_n, Q_n) = 0 \quad (18a)$$

However, since in the process of loading, Q_n is a function of P_n , say,

$$Q_n = \frac{1}{\rho} P_n \quad (19)$$

so that Eqn.(12b) becomes

$$Q = f_2(V, P_n) \quad (12c)$$

therefore, Eqn.(18a) will contain the single loading parameter P_n ,

$$f(V, P_n) = 0 \quad (18b)$$

If the load curve intersects the supply surface (see

Figure 7), then Eqn.(18b) will have generally two real solutions in V . With increased loading P_n , the two solution points may coalesce. This corresponds to the critical load $P_{n,cr}$, at V_{cr} and P_{cr}, Q_{cr} . Beyond this point, (18b) has no real solution in V .

The critical loading corresponds to the additional condition

$$\frac{\partial f}{\partial V}(V, P_n) = 0 \quad (18c)$$

Thus, we have two equations, (18b) and (18c), for the unknowns V and P_n .

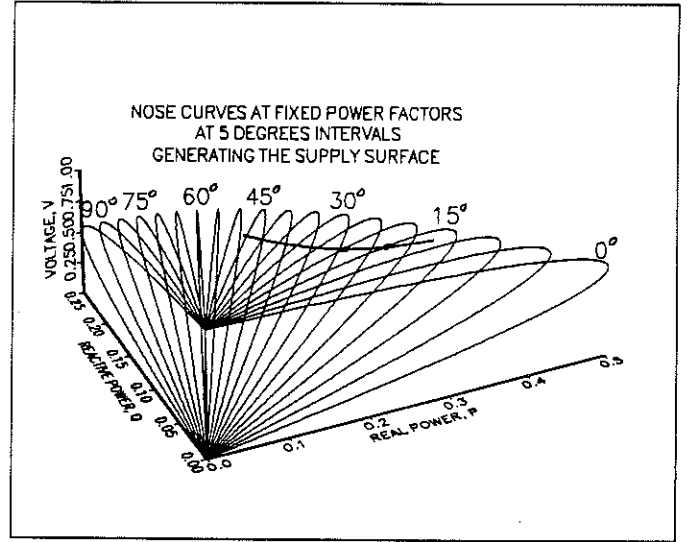


Fig. 7 Three-dimensional representation of supply system

Induction Load

An induction load can be represented by the simple series R, X model of Figure 2, plus a parallel magnetizing branch. In our analysis, the latter is assumed to be lumped with the rest of the load (Eqns.(14)). Therefore the power factor for the series branches alone is quite high; typically $\varphi_n = 15^\circ$. The rating P_n of the induction load defines also $Q_n (=P_n \tan \varphi_n)$. For the reactive power we write

$$Q = XI^2 = \frac{X}{R^2 + X^2} V^2 = \frac{V^2/X}{1 + \frac{P^2}{Q^2}} \quad (20)$$

This yields

$$Q + \frac{P^2}{Q} = \frac{V^2}{X} \quad (21a)$$

We denote $Q = Q_n$ when $P = P_n$ and $V = V_n$. Then the above equation becomes, taking into account the proportionality between P_n and Q_n ,

$$Q + \frac{P^2}{Q} = \left(Q_n + \frac{P_n^2}{Q_n}\right) \left(\frac{V}{V_n}\right)^2 = \frac{2P_n}{\sin 2\varphi_n} \left(\frac{V}{V_n}\right)^2 \quad (21b)$$

large value of P_{max} (about twice the load power P), and the corresponding critical slip s_{cr} will be much larger than the actual slip s . On the other hand, a person who looks at the *aggregate* load, will have to consider curve 1 with $E = \text{const}$. Compared to this voltage, the local bus voltage V would indicate the effect of the voltage drop on the system, the actual power P would be closer to the maximal power P_{max} and also the critical slip s_{cr} would result small, fairly close to the actual slip s . This latter perspective, based on the aggregate induction loading, is the one which is more realistic in reflecting the true danger of stalling of the motors, while the perspective of the individual user is unduly optimistic. It reflects an appearance of normalcy in a situation which may in fact be on the verge of collapse.

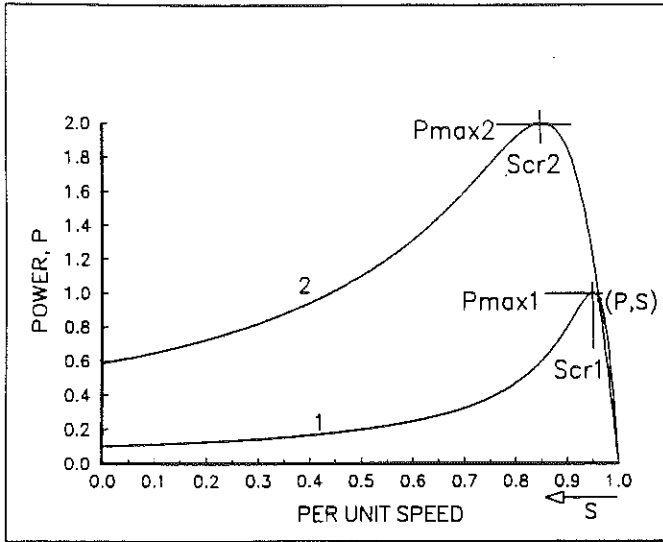


Fig. 5 Loading curves of the same induction load from two different perspectives

Phasor Diagram

Figure 6 represents a phasor diagram corresponding to the circuit arrangement of Figure 2. We see from this diagram that φ is always less than 45° , so that voltage collapse is not apparent at normal power factor; also, if E is assumed to be well beyond 1.1 p.u. (rather than 1 p.u. as shown in the figure) and V_{cr} appropriately prorated (from the shown value of 0.8), then V_{cr} could result greater than 0.9 p.u., so that an operating voltage close to normal could in fact be close to critical.

Representation of the System and Load Characteristics

A nose curve is essentially a simplified representation of the power system, as seen from a

particular load bus and, usually, for a fixed power factor. In practical power system analysis it may be viewed as a graphical image of the results obtained by load flow calculations. Its significance is therefore fully realized only when it is interfaced with the characteristic of the load. The latter is much simpler in a practical sense since it involves only a small number of local variables.

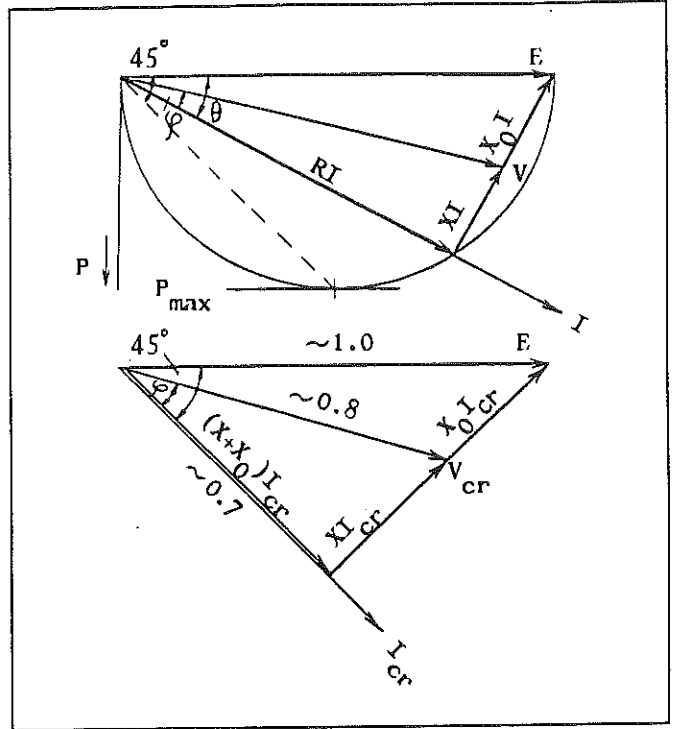


Fig. 6 Phasor diagram for normal and critical induction loading

General Load

In general, the load can be described by the nonlinear relations

$$P = f_1(V, P_n) \quad (12a)$$

$$Q = f_2(V, Q_n) \quad (12b)$$

which may reflect the presence of active sources (as for instance in the case of SVCs) and which satisfy the constraints

$$P_n = f_1(V_n, P_n) \quad (13a)$$

$$Q_n = f_2(V_n, Q_n) \quad (13b)$$

For example, in a particular case, Eqns.(12) could be

$$P = P_n \left(\frac{V}{V_n} \right)^p \quad (14a)$$

$$Q = Q_n \left(\frac{V}{V_n} \right)^q \quad (14b)$$

rating) of the motor, and $R = r/s$ is a variable resistance expressed in terms of the slip s . Thus the load is a function of a single parameter and we cannot expect having the loading limits as wide as in the previous case. For a given power factor, i.e., $\rho = \text{const}$, as we decrease X and move outward radially in Figure 3, at any given X appearing in Eqn.(3) we have therefore to check whether the motor load is still stable.

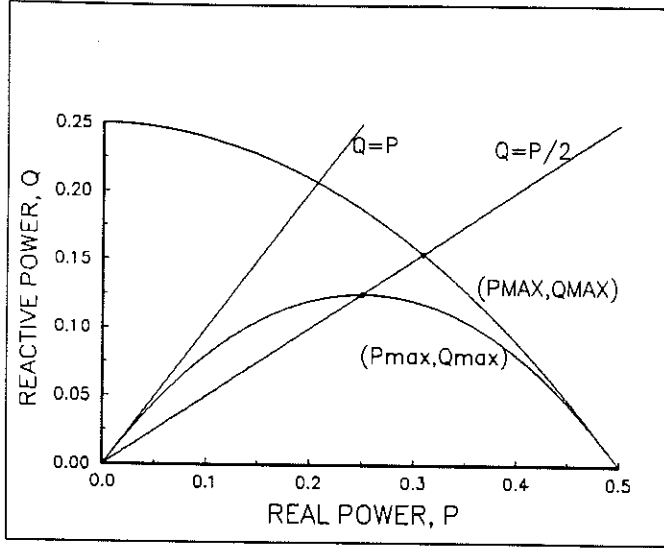


Fig. 3 Loading limits for the system and for an induction load

This requires that

$$R > R_{cr} = X + X_0 \quad (8)$$

or

$$\rho X > X + X_0 \quad (8a)$$

and, therefore,

$$X > X_{cr} = \frac{X_0}{\rho - 1} \quad (8b)$$

This means that we can not proceed on the $\rho = \text{const}$ line beyond the value of X_{cr} given by Eqn.(8b). If we substitute this value into (3) and take into account that

$$Q_{max} = \frac{P_{max}}{\rho} \quad (9)$$

we obtain

$$Q_{max} = \left(1 - \frac{2P_{max}}{E^2/X_0}\right)P_{max} \quad (10a)$$

or, in per unit,

$$Q_{max} = (1 - 2P_{max})P_{max} \quad (10b)$$

Figure 3 represents the parabola of Eqn.(10b) in addition to that of Eqn.(7b). It can be seen that the critical power P_{max} of induction motor loading can be much smaller than the critical loading P_{MAX} (which

may be obtained with a general load), especially at lower power factors. To get a precise value at a given power factor, we substitute $P = \rho Q$ in Eqns.(7b) and (10b). The resultant quadratic equations yield P_{MAX} and P_{max} as functions of ρ , and then the ratios

$$\theta = \frac{P_{max}}{P_{MAX}} = \frac{Q_{max}}{Q_{MAX}} = \frac{\rho - 1}{-1 + \sqrt{1 + \rho^2}} \quad (11)$$

Taking (2) into account, we obtain for $\varphi = 15^\circ$ the ratio $\theta = 0.954$, and for $\varphi = 30^\circ$ the ratio $\theta = 0.732$. At $\varphi = 45^\circ$ we obtain, of course, $\theta = 0$, as it can be seen from Figure 3.

Apparent Normalcy at the Verge of Collapse

Nose Curves with Induction Loading

Figure 3 suggests that the critical induction loading P_{max}, Q_{max} , if shown on a nose curve for some fixed power factor ($\rho = \text{const}$), appears before the nose point on that curve. In Figure 4 a third dimension is used for the V axis. The P - Q diagram of the induction load (an ellipse, since here $E = \text{const}$, not V) shows that the collapse point P_{max} is at $P = \text{max}$. We can see that the critical value of V can be fairly close to normal and $P_{max} \ll P_{MAX}$ in the case of weak systems, as expected.

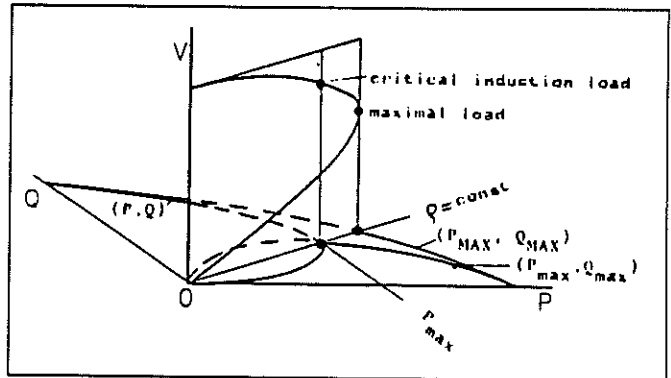


Fig. 4 Nose curve showing the critical induction loading

Voltage and Slip

Figure 5 represents two P - ω characteristics of the same large induction motor load ($X = \text{fixed}$). The point (P, s) is the operating point at a given mechanical loading P and slip s . Curve 1 corresponds to E (at the far bus) assumed constant, while curve 2 corresponds to V (the local bus voltage) being assumed constant and close to the rated value.

The operator of an *individual* (small) motor will make this latter assumption: he will see, on curve 2, a

The emphasis in this paper is thus on the effect of load characteristics on voltage stability. Special attention is given to the influence of induction motor loads. In this respect it is pointed out that distinction has to be made between the analysis of the operation of the *individual* motor and the *aggregate* induction (motor) load. Indeed, the induction load is increased (when overall augmentation of the load is contemplated) not by increasing the mechanical power of the already connected motors but by adding additional motors (with their mechanical loads). This recognition crucially affects the induction load characteristic. Although the local bus may be considered as having $V = \text{const}$ for the individual motor but for the aggregate induction load the local bus voltage can not be viewed as constant, with the effect that the motors may be closer to stalling than it would appear for the individual motor, as the voltage may be close to normal.

The proximity to voltage collapse [8] is thus strongly dependent on the load characteristics. A Proximity Index (PI) to voltage collapse [9] is formulated to reflect this fact. It is based on a Jacobian similar to the one used in the Newton-Raphson load flow but appropriately modified to account for the voltage dependence characteristics of the loads. The proposed PI is equivalent to the determinant of that Jacobian but easier to compute. An immediate application of the PI is in the calculation of the critical loading condition in a system via a sequence of standard load flow solutions.

CRITICAL LOADING

We consider the system represented by its Thevenin equivalent E, X_0 ; see Figure 2. The network losses are neglected in this representation. The load is given by R, X . First we assume that P and Q of the load can be increased independently and next we consider an induction (motor) load where this assumption does not hold any more.

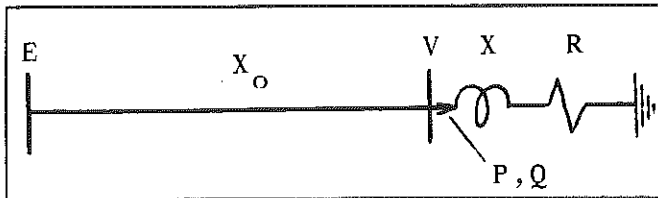


Fig. 2 Lossless line with simple load model

Critical Loading of the System

The load power is

$$P = RI^2 = \frac{R}{R^2 + (X + X_0)^2} E^2 \quad (1)$$

In order to find the critical loading of the system, we explore the loading along lines with fixed power factors, $\cos\phi$, i.e.,

$$R = \rho X, \quad \rho = \frac{1}{\tan\phi} = \frac{P}{Q} \quad (2)$$

Substitution of (2) into (1) gives

$$P = \frac{\rho}{(1 + \rho^2)X + \frac{X_0^2}{X} + 2X_0} E^2 \quad (3)$$

Clearly, we have P as a function of X . It can be viewed as a nose curve in terms of X rather than V . However, since V is a monotonously increasing function of X (which approaches E asymptotically), given by

$$\begin{aligned} V^2 &= \frac{R^2 + X^2}{R^2 + (X + X_0)^2} E^2 \\ &= \frac{1 + \rho^2}{1 + \rho^2 + \left(\frac{X_0}{X}\right)^2 + 2\left(\frac{X_0}{X}\right)} E^2 \end{aligned} \quad (4)$$

we can regard P as function of V , representing a nose curve for a given power factor (obtained by eliminating X between (3) and (4)). The maximum of P will result the same, no matter whether X or V is the independent variable. It can be obtained from (3) for

$$X_{CR} = \frac{X_0}{\sqrt{1 + \rho^2}} \quad (5)$$

Its magnitude is

$$P_{MAX} = \frac{\rho/2}{1 + \sqrt{1 + \rho^2}} \left(\frac{E^2}{X_0}\right) \quad (6a)$$

Correspondingly,

$$Q_{MAX} = \frac{P_{MAX}}{\rho} \quad (6b)$$

Eliminating ρ from Eqns.(6), we obtain

$$Q_{MAX} = \frac{1}{4} \left(\frac{E^2}{X_0}\right) - \left(\frac{X_0}{E^2}\right) P_{MAX}^2 \quad (7a)$$

or, in per unit, with E and X_0 as base values,

$$Q_{MAX} = \frac{1}{4} - P_{MAX}^2 \quad (7b)$$

Figure 3 represents the parabola corresponding to Eqn.(7b).

Critical Induction Loading

If the load of Figure 2 is an induction motor, then $X = \text{const}$ and its inverse characterizes the size (or

IMMINENT VOLTAGE INSTABILITY WITH LARGE INDUCTION MOTOR LOADS

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Abstract – This paper shows that induction motor loads bring the voltage instability limit significantly closer than $P, Q = \text{const}$ loads of the same magnitude. This fact remains essentially unnoticed in the operation of the system because both voltage and slip are close to normal. The phenomenon is explained by taking into account the particular voltage dependence characteristics of induction motor loads.

Computationally, the load characteristics do not have to be directly accounted for in load flow programs if the loads are connected through tap changing transformers, but can be used for obtaining a Proximity Index for voltage collapse based on an appropriate Jacobian matrix. Guided by this index, load flows can be performed sequentially for obtaining the critical loading condition.

INTRODUCTION

It is often convenient to examine the static voltage stability of a system by hand of P - V or Q - V diagrams. These, so called nose curves [1], clearly indicate the maximal value of P or Q by the "nose point", often called bifurcation point because the nature of the solution of a standard load flow program, having P and Q as inputs for load buses, would change at this point (from two real solutions to none or a complex pair) as the load increases. In a Newton-Raphson load flow the Jacobian would become singular at the bifurcation point [2].

On occasion, the algebraic expressions of the load flow equations (representing the nose curve) are given the dynamic interpretation of a vector field [3], [4], in a nonlinear differential equation formulation. This interpretation of the bifurcation point as being dynamic is, however, hard to justify in its concrete details [5].

Often, the point of maximal P and Q is considered to be the critical point for system loadability (cf. for example figures 3.30 and 3.42 of reference [6]. For P, Q specified at this value, the slope of the nose curve is infinite and the voltage solution by any standard approach is not unique, so that the voltage is deemed unstable. Reference [6] contains fairly comprehensive

bibliographic listings relevant to the topics of both static and dynamic voltage stability.

The above point of view, with the emphasis on the maximal values of P and Q [7], assumed to be specified, is in general not realistic, as the validity of $P, Q = \text{const}$ is based on the action of tap changing transformers (LTCs) which, in reality, is only intermittent. The concept is, of course, adequate for steady state computations, such as load flows. However, at any fixed tap position of the LTC, the load, by its physical nature, is in general voltage dependent to a certain degree. Consequently, while the nose curve correctly reflects the characteristic of the network, the load characteristics have also to be represented (by other than a perfectly vertical line, valid for constant P, Q) and interfaced with the nose curve in order to obtain the correct critical loading and bifurcation point [6]. This point may be before or after the $P, Q = \text{max}$ point, as exemplified in Figure 1. It is the point of voltage collapse, of bifurcation in both the nature of the algebraic solution and of the stability type of the equilibrium point in a dynamic problem formulation.

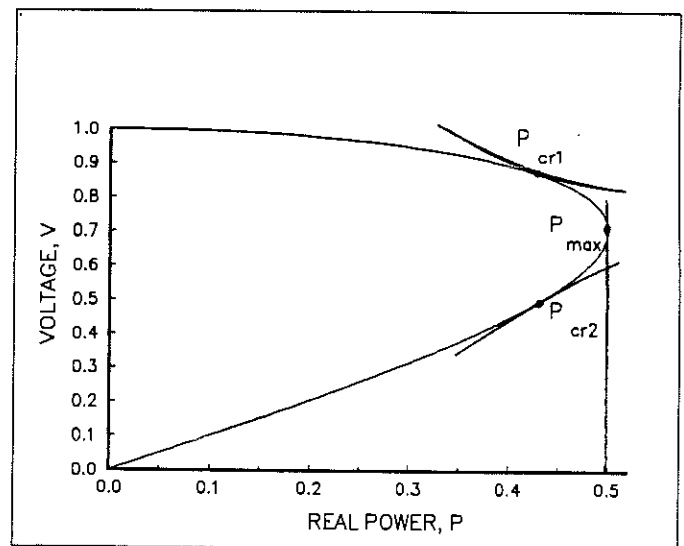


Fig. 1 Effect of load characteristic on the point of voltage collapse

Dr. Ahmed-Zaid is a member of IEEE/PES. In 1990, he was a recipient of an NSF Presidential Young Investigator (PYI) Award.

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William W. Price, Jr. received the B.S. degree in Electrical Engineering from Clarkson University in 1968 and the M. Eng. degree from Rensselaer Polytechnic Institute in 1971. He joined GE in 1968, where he is presently a Consulting Engineer in the Power Systems Engineering Department. Mr. Price is a licensed Professional Engineer in New York State. He is a member of the IEEE/PES and its subcommittees on system Dynamic performance and Computer and Analytical Methods, and is Chairman of the Task Force on Load Representation for Dynamic Performance Analysis.

8. Conclusions

A steady-state stability criterion for induction machines similar to the classical stability criterion of synchronous machines has been derived. This stability criterion can be regarded as an index indicating proximity to voltage collapse. A heuristic first-order linear model of a relatively large induction machine has been derived. This model improves upon the first-order voltage model derived in a previous work and is valid near voltage collapse. An eigenvalue analysis validates this model throughout the entire region from zero speed to synchronous speed. The linear model obtained in this paper is significant for voltage studies near voltage collapse.

Acknowledgements

This work was supported in part by the General Electric Company and in part by the National Science Foundation under Grant NSF ECS-9058174.

Nomenclature

T_o'	Transient open-circuit time constant
E_q', E_d'	Quadrature- and direct-axes rotor voltages
X, X'	Rotor steady-state and transient reactances
V	Infinite bus magnitude voltage
ω, ω_s	Shaft and synchronous speeds
H	Inertia constant
M	Angular momentum constant ($= \frac{2H}{\omega_s}$)
T_e, T_m	Electromagnetic and mechanical torques
T'	Modified transient short-circuit time constant
$T_{e,max}$	Maximum electromagnetic torque
E'	Magnitude of voltage behind transient reactance
δ	Angle of voltage behind transient reactance
K_1, K_2, K_3	Induction Machine constants
K_4, K_3', K_4', K_5'	constants
Ω	Modified speed variable
Δ	Synchronizing-power coefficient

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Biographies

Said Ahmed-Zaid was born in Algiers, Algeria in 1956. He received his B.S., M.S., and Ph.D. degrees in Electrical Engineering from the University of Illinois at Urbana in 1979, 1981 and 1984 respectively.

He held positions with the University of Illinois at Urbana-Champaign as Research Associate in 1984 and Visiting Assistant Professor in the Spring of 1989. From 1985 to 1988, he was a principal research engineer with SONEGAS, the Algerian Electric and Gas Company. He joined Clarkson University in 1989 where he is currently an Associate Professor of Electrical Engineering. His research interests include power system modeling and simulation, computational methods, optimization and advanced control applications in power systems.

The difference between the two models appears in the denominator of (25). This denominator is practically equal to 1 from synchronous speed up to the voltage collapse point then starts to increase for lower speed values because of the effects of the ratio $(K_4 K'_5 / K_1)$. This makes the real eigenvalue of the modified voltage model remain close to the real eigenvalue of the exact third-order model. This observation is not seen in the original voltage model. Figure 5(c) displays the real eigenvalues of the first order linearized models and compares them to the exact real eigenvalue obtained from the third order model. A reasonable eigenvalue agreement is obtained using the modified voltage model for the full range of speeds from standstill to synchronous speed.

7. Heuristic First-Order Model

Due to the strange behaviors observed in the participations factors in the previous section, a heuristic transformation of state variables is suggested. This transformation consists of defining a new variable Ω such that

$$\Omega = \omega - \left(\frac{X - X'}{X'} \right) \frac{V \sin \delta}{T'_0 E'} \quad (26)$$

Replacing this expression in (12)-(14), it can be checked that the new modified third-order nonlinear model is

$$T'_0 \frac{dE'}{dt} = -\frac{X}{X'} E' + \left(\frac{X - X'}{X'} \right) V \cos \delta \quad (27)$$

$$\frac{d\delta}{dt} = \Omega - \omega_s \quad (28)$$

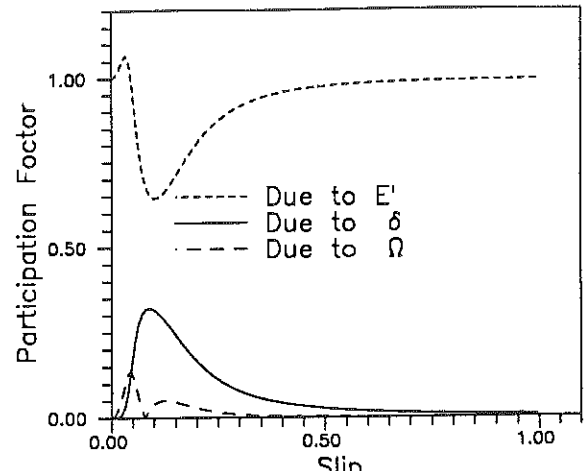
$$\begin{aligned} M \frac{d\Omega}{dt} = & -\frac{V E' \sin \delta}{X'} - T_m \\ & - \frac{M(X - X') V \cos \delta}{T'_0 X' E'} (\Omega - \omega_s) \\ & + \frac{M(X - X') V \sin \delta}{T'_0 X' E'^2} \frac{dE'}{dt} \end{aligned} \quad (29)$$

This model has a steady-state equilibrium similar to that of a synchronous machine.

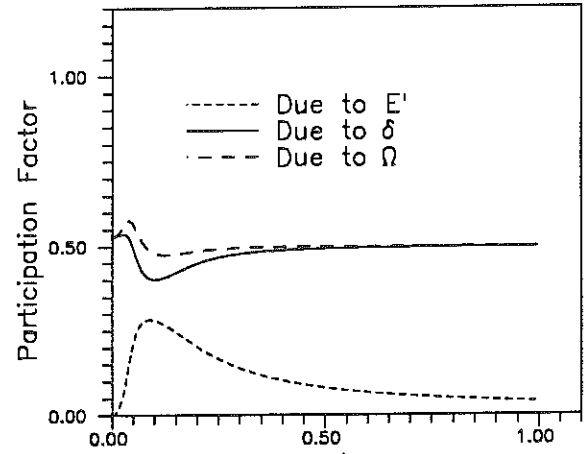
Linearizing this new model and performing a sensitivity analysis yields the results displayed in Figure 6. An improved decoupling between the state variables is observed from standstill to synchronous, and once again, the slow variable is still the voltage E' and is the one characterizing the real eigenvalue. This observation leads to an other version of the linearized first-order voltage model for large machines which can be derived as

$$\Delta \dot{E}' = -\frac{1}{T'_0 K_1 K_3} \frac{(K_1 - K_2 K_3 K_4)}{\left(1 - \frac{M}{T'_0} \frac{K_4 K'_1}{K_1}\right)} \Delta E' \quad (30)$$

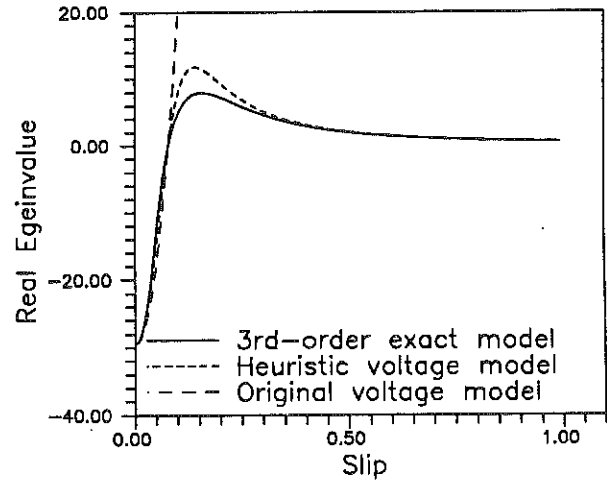
Figure 6(c) displays the real eigenvalues of the first order linearized models and compares them to the exact real eigenvalue obtained from the third-order model.



(a)



(b)



(c)

Figure 6: Sensitivity analysis and Real eigenvalue for the 500-hp induction machine. a) Real eigenvalue. b) Complex eigenvalue. c) Real eigenvalue comparisons.

machine occur and the voltage model becomes erroneous. One of the interesting characteristics of the voltage model is the appearance of the voltage collapse index in the linearization of the voltage model. The linearized voltage model in terms of K-coefficients is

$$\Delta \dot{E}' = -\frac{1}{T_o' K_1 K_3} (K_1 - K_2 K_3 K_4) \Delta E' \quad (19)$$

Motivated by the strong coupling between the voltage E' and the speed ω near voltage (see Figure 3), a new transformation of state variables is suggested. This transformation consists in defining a new variable Ω such that

$$\Omega = \omega_s + \frac{X T_m}{T_o' \sqrt{(V E')^2 - (X' T_m)^2}} \quad (20)$$

This transformation is a consequence of the higher-order manifold expressions derived in [7]. Plugging this expression in (12)-(14), the modified third-order nonlinear model has the following form

$$T_o' \frac{dE'}{dt} = -\frac{X}{X'} E' + \left(\frac{X - X'}{X'} \right) V \cos \delta \quad (21)$$

$$\begin{aligned} \frac{d\delta}{dt} &= \Omega - \omega_s - \frac{X T_m}{T_o' \sqrt{(V E')^2 - (X' T_m)^2}} \\ &\quad - \frac{X - X'}{X'} \frac{V \sin \delta}{T_o' E'} \end{aligned} \quad (22)$$

$$\begin{aligned} M \frac{d\Omega}{dt} &= -\frac{V E' \sin \delta}{X'} - T_m \\ &\quad - \frac{M X T_m V^2 E'}{T_o' [(V E')^2 - (X' T_m)^2]^{\frac{3}{2}}} \frac{dE'}{dt} \end{aligned} \quad (23)$$

$$(24)$$

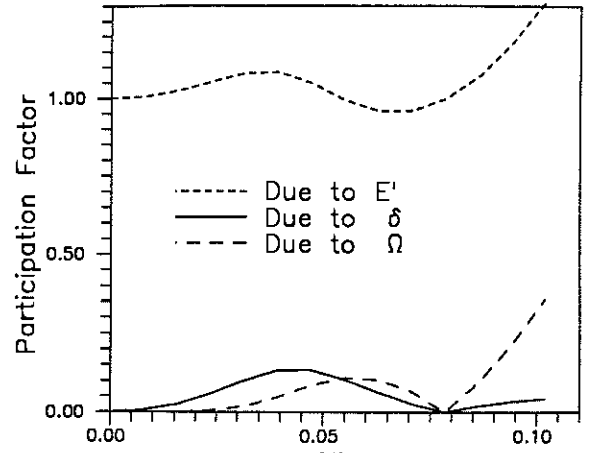
Linearizing this new model and performing a sensitivity analysis, yields the results displayed in Figure 5. An improved decoupling between the state variables is observed and it can be seen that the slow variable is the voltage E' and it is the one that characterizes the real eigenvalue up to voltage collapse. Beyond voltage collapse, we find that the participation factors are increasing in a rapid manner and become meaningless for our analysis. Since, a good decoupling between the state variables E' and Ω is obtained for the whole region before the voltage collapse, a linearized modified first-order voltage model is

$$\Delta \dot{E}' = -\frac{1}{T_o' K_1 K_3} \frac{(K_1 - K_2 K_3 K_4)}{(1 - \frac{K_4 K_4'}{K_1})} \Delta E' \quad (25)$$

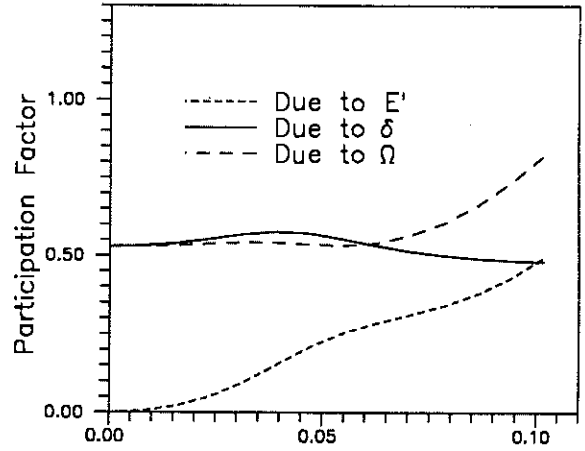
where

$$K_5' = \frac{M X T_m V^2 E_o'}{T_o'^2 [(V E_o')^2 - (X' T_m)^2]^{\frac{3}{2}}}$$

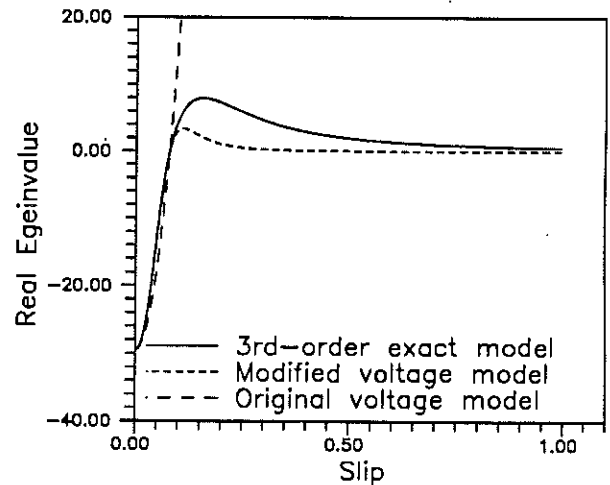
This expression is different from the linearized first-order voltage model of the the previous research (19).



(a)



(b)



(c)

Figure 5: Sensitivity analysis and real eigenvalue for the 500-hp induction machine. a) Real eigenvalue. b) Complex eigenvalue. c) Real eigenvalue comparisons.

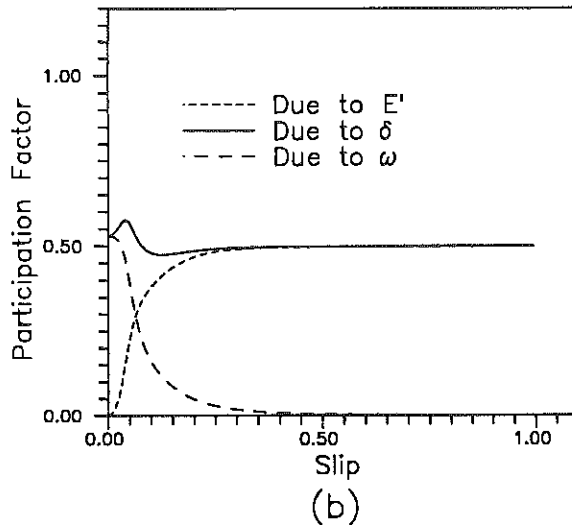
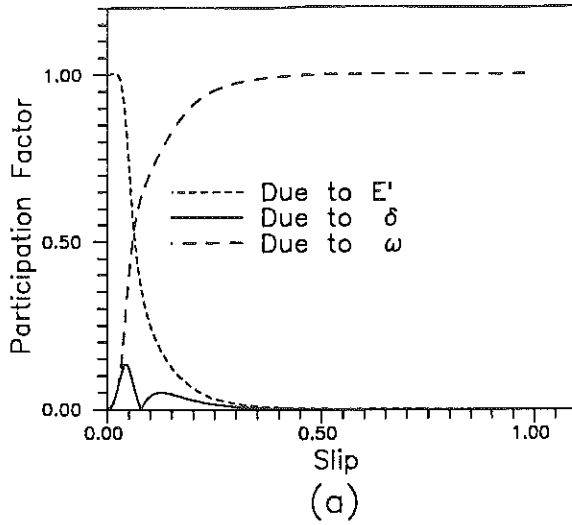


Figure 3: Sensitivity analysis for the 500-hp induction machine. a) Real eigenvalue. b) Complex eigenvalue.

near rated speed to synchronous speed (see Figure 3). However, near voltage collapse, both the speed and the voltage variables contribute significantly to the real eigenvalue and far beyond the point of voltage collapse the speed is the dominant variable in characterizing this real eigenvalue.

From these results, one can conclude that for further reduction in the order of the nonlinear system, the speed should be considered as the slow variable in describing the small induction machine whereas, for large induction machine, the voltage variable is not always the dominant variable in this model (12)-(14).

5. First-Order Model of Small Machines

The differences in the interactions between various state variables in small and large induction machines

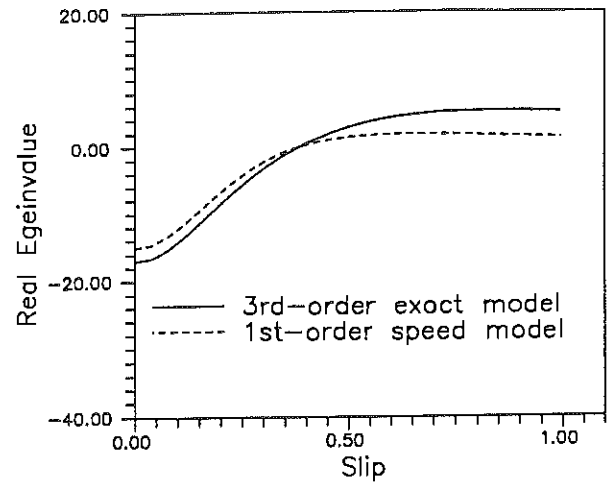


Figure 4: Real eigenvalue of the 50-hp induction machine.

are attributed to certain design parameters which endow these machines with different dynamical structures. Depending on the order of magnitude of these key parameters, a different decomposition of slow and fast variables will result [7].

The slow model governing the dominant dynamics of small induction machines is derived in [7] and has the standard expression

$$M \frac{d\omega}{dt} = -V^2 \left(\frac{X - X'}{X X'} \right) \frac{T'(\omega - \omega_s)}{1 + T'^2(\omega - \omega_s)^2} - T_m \quad (17)$$

This model is not only valid at rated conditions but also at other points of interest including near voltage collapse.

The linearized first-order speed model in terms of the K-coefficients can be easily derived as

$$\Delta \dot{\omega} = - \frac{T'_o K_3}{M(1 - K'_3 K_4 K_3^2)} (K_1 - K_2 K_3 K_4) \Delta \omega \quad (18)$$

Figure 4 displays the real eigenvalue of the linearized third-order system and the linearized model of this first-order speed model. A close agreement is obtained between the two curves and this will in fact endows the validity of the speed model in the small induction machines case. This model is exact for maximum electromagnetic torque since the synchronizing-power coefficient is zero ($\Delta = 0$).

6. First-Order Model of Large Machines

In a previous work [7], a voltage model was derived for large induction machines and was shown to perform adequately near rated conditions. However away from rated conditions, changes in the dynamic structure of the

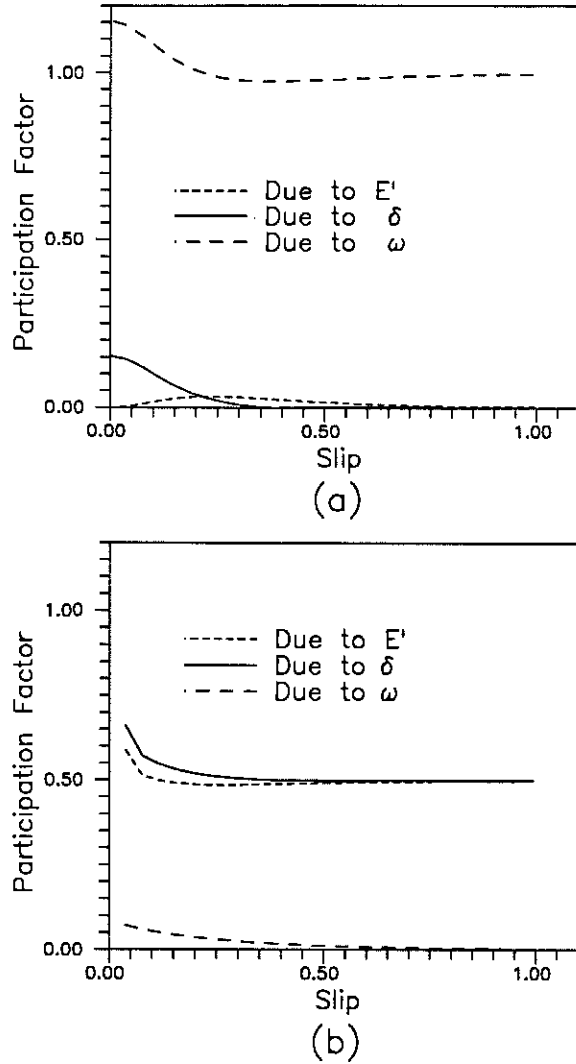


Figure 2: Sensitivity Analysis for the 50-hp induction machine. a) Real eigenvalue. b) Complex eigenvalue.

$$K_4 = \left(\frac{X - X'}{X'} \right) V \sin \delta_o$$

$$K'_3 = - \left(\frac{X - X'}{X'} \right) \frac{V \sin \delta_o}{(E'_o)^2}$$

$$K'_4 = \left(\frac{X - X'}{X'} \right) \frac{V \cos \delta_o}{E'_o}$$

$$K_2 = \frac{V \sin \delta_o}{X'}$$

$$K_1 = \frac{V E'_o \cos \delta_o}{X'}$$

K_1 - K_4 are defined in the same way as the standard deMello-Concordia constants. It can also be shown that

$$K'_4 = \frac{1}{K_3}$$

Using the Routh-Hurwitz stability criterion, it can be shown that the stability of the equilibrium point depends on the synchronizing-power coefficient (16)

$$\Delta = K_1 - K_2 K_3 K_4 \quad (16)$$

just as in the synchronous machine case. That is:

- The machine is stable if $(K_1 - K_2 K_3 K_4) > 0$.
- The machine is unstable if $(K_1 - K_2 K_3 K_4) < 0$.
- When $(K_1 - K_2 K_3 K_4) = 0$, a zero real eigenvalue is obtained and the electromagnetic torque (T_e) has reached its maximum value ($T_{e,max}$). Indeed, this condition is exactly similar to the stability criterion of synchronous machines and, therefore, can be considered as a stability criterion for induction machines.

4. Sensitivity Analysis

A simple approach to determine how the state variables interact near an equilibrium point is to perform a sensitivity analysis using the well-known participation matrix whose entries, the participation factors, indicate how much each state variable contributes to an eigenvalue and vice-versa. Such a sensitivity analysis is explained in [9] and it is used in this paper to determine the contribution of the state variables for a full range of possible speed operation (from standstill to synchronous speed ω_s) and for two sizes of induction machines (50-hp and 500-hp machines). This analysis shows the remarkable difference between the structures of the small and large induction machines. In the case of the small machine, the speed is the dominant variable in characterizing the real eigenvalue for the whole range of slip (see Figure 2). In the case of the large machine, the voltage variable E' is the dominant variable in characterizing the real eigenvalue

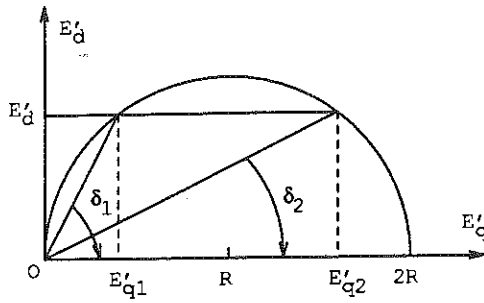


Figure 1: Equilibrium points in (E'_q, E'_d) -plane.

where $M = 2H/\omega_s$ (See nomenclature). To find the equilibrium points of this dynamical system, we set the time derivatives on the left-hand side of (1)-(3) to zero and solve the resulting algebraic equations. After some manipulations, these equations become

$$(E'_q - \frac{(X - X')V}{2X})^2 + E'^2_d = \frac{(X - X')^2 V^2}{4X^2} \quad (4)$$

$$E'_d = \frac{X'T_m}{V} \quad (5)$$

$$\omega - \omega_s = -\frac{1}{T'} \frac{E'_d}{E'_q} \quad (6)$$

where $T' = T'_o X'/X$ is a modified transient short-circuit time constant accounting for any external series reactance. Equation (4) represents a circle in (E'_q, E'_d) -plane centered at $((X - X')V/2X, 0)$ and of radius $R = (X - X')V/2X$ (see Figure 1). From (5), E'_d is single-valued and positive for motor operation. Substitution of (5) into (4) yields a quadratic equation in E'_q . Thus, for each positive value of E'_d (or T_m) correspond two values of E'_q provided the discriminant of the quadratic equation is nonnegative. From (6) there corresponds a value of speed ω for each value of E'_q . From classical arguments, it is known that the equilibrium point with the higher speed ω and the higher voltage variable E'_q is stable whereas the other equilibrium point is unstable.

The maximum steady-state torque developed by the machine is

$$T_{e,max} = \frac{(X - X')V^2}{2XX'} \quad (7)$$

and occurs when

$$E'_d = E'_q = \frac{(X - X')V}{2X} \quad (8)$$

$$\omega - \omega_s = -\frac{1}{T'} \quad (9)$$

The point of voltage collapse occurs at the maximum electromagnetic torque. A value of T_m greater than $T_{e,max}$ leads to a quadratic equation with no solutions for E'_q .

A model which presents some striking similarities with a synchronous machine model can be obtained by converting the rectangular variables E'_d and E'_q to polar variables E' and δ by means of the transformation

$$E' = \sqrt{E'^2_d + E'^2_q} \quad (10)$$

$$\delta = \tan^{-1}(-\frac{E'_d}{E'_q}) \quad (11)$$

The new model in polar form is

$$T'_o \frac{dE'}{dt} = -\frac{X}{X'} E' + (\frac{X - X'}{X'}) V \cos \delta \quad (12)$$

$$\frac{d\delta}{dt} = \omega - \omega_s - (\frac{X - X'}{X'}) \frac{V \sin \delta}{T'_o E'} \quad (13)$$

$$M \frac{d\omega}{dt} = -\frac{V E' \sin \delta}{X'} - T_m \quad (14)$$

This model shows the following differences with the classical one-axis model of a synchronous machine:

- Even though E' is a voltage behind a transient reactance, it does not arise from an external excitation current.
- The angle δ is not a stroboscopic angle of rotation of the shaft but the angle of the rotor flux magnitude (equivalently, of an induced rotor voltage magnitude) with respect to the synchronously-rotating reference frame.
- There is no excitation voltage in (12) as the rotor circuits are short-circuited.
- There is an extra term in (13) when comparing this equation with the angle equation of a synchronous machine due to the variable speed operation of the induction machine.
- The maximum steady-state torque occurs at an angle $\delta = -45^\circ$ as opposed to -90° for a synchronous motor with constant field excitation (see Figure 1). Stable operation ranges between 0° and -45° and unstable operation between -45° and -90° .

3. Linearized Third-Order Model

The linearized version of (12)-(14) around an equilibrium point $(E'_o, \delta_o, \text{ and } \omega_o)$ is

$$\begin{bmatrix} T'_o \Delta \dot{E}' \\ T'_o \Delta \dot{\delta} \\ M \Delta \dot{\omega} \end{bmatrix} = \begin{bmatrix} -\frac{1}{K_3} & -K_4 & 0 \\ -K'_3 & -K'_4 & T'_o \\ -K_2 & -K_1 & 0 \end{bmatrix} \begin{bmatrix} \Delta E' \\ \Delta \delta \\ \Delta \omega \end{bmatrix} \quad (15)$$

where

$$\frac{1}{K_3} = \frac{X}{X'}$$

First-Order Induction Machine Models Near Voltage Collapse

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Abstract — The purpose of this paper is two-fold: First, a steady-state stability criterion similar to the classical steady-state stability limit of synchronous machines is derived for induction machines. Second, the reduced-order modeling of large induction machines is investigated for operating conditions near voltage collapse. It is shown that dynamic structural changes occur in the vicinity of voltage collapse, leading to a different interaction between speed and voltage variables. A first-order linearized model characterizing the dominant behavior of large induction machines is derived.

1. Introduction

A review of the present literature reveals that more attention has been paid to the modeling of the generator side rather than the load side. Power system operation, however, depends a great deal on the nature and dynamic behavior of loads. It has long been recognized that load representation can have a significant impact on the analysis and results of system simulations. This impact is due primarily to the dynamics exhibited by rotating machines, in particular by induction machines which make up more than 60 % of the load in most power systems.

Various simplified models of induction machines [1]-[2] are available in the literature including the quasi-static first-order speed model where both stator and rotor electrical transients are neglected. This latter model has been used in numerous studies involving certain mechanisms of voltage stability in general [3] and voltage collapse associated with the interaction of induction machines and load-tap-changing (LTC) transformers [4]-[5]. Recent research, however, indicates that this overly simplified speed model may be erroneous for relatively large induction machines and does not accurately predict their dynamic performance around rated conditions as reported in [1],[6].

In a previous reference [6], the authors have investigated time-scale properties of induction machines ranging in size from 3 hp to 6000 hp. It is shown that the accuracy of the first-order speed model deteriorates substantially

for relatively large induction machines. This large discrepancy is clarified in a second reference [7] where the authors show that the structural dynamics of small and large induction machines are quite different near rated conditions. Using integral manifold theory, they show that the dominant behavior of large induction machines is characterized by a first-order voltage model rather than by a first-order speed model. These results have been confirmed independently in another reference [8] where the authors investigate dynamic structural changes with machine rating and operating conditions using sensitivity analysis and participation factors.

This paper, which is an extension of a previous work [7], pursues the modeling of small and large induction machines near voltage collapse. The issue of adequacy of both the traditional slip model and voltage model is addressed here. A modified first-order voltage model is derived for large induction machines. This model not only performs adequately near voltage collapse but also improves upon either the first-order speed or first-order voltage model in the vicinity of voltage collapse.

2. Third-Order Induction Machine Models

The third-order model of a single-cage induction machine is obtained in general by neglecting stator transients, a process which can be rigorously justified using integral manifold theory [7]. Assuming a zero stator resistance, an infinite bus with voltage magnitude V and phase angle 0° , and a constant positive machine load torque T_m , a simplified third-order nonlinear model is [7]

$$T_o' \frac{dE_q'}{dt} = -\frac{X}{X'} E_q' + \left(\frac{X - X'}{X'} \right) V + T_o' (\omega - \omega_s) E_d' \quad (1)$$

$$T_o' \frac{dE_d'}{dt} = -\frac{X}{X'} E_d' - T_o' (\omega - \omega_s) E_q' \quad (2)$$

$$M \frac{d\omega}{dt} = \frac{V E_d'}{X'} - T_m \quad (3)$$

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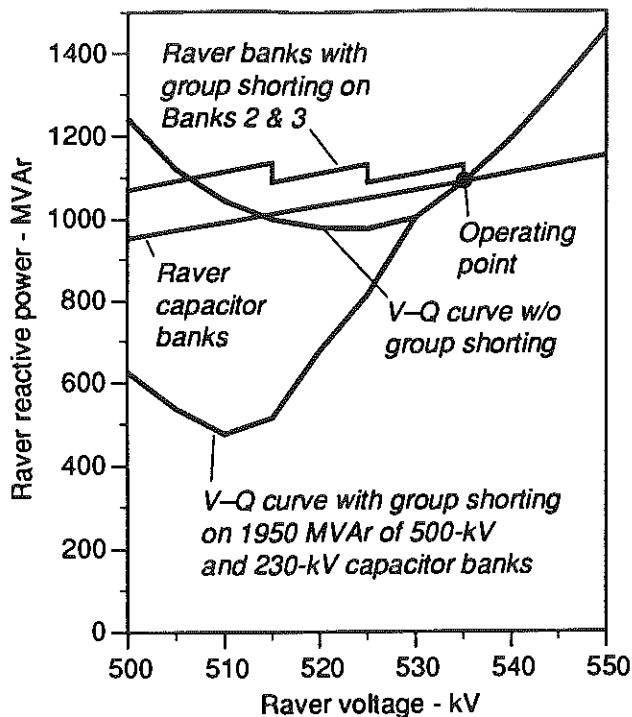


Figure 5. V-Q curve for Raver with and without capacitor bank group shorting. Outage of Chief Joseph-Monroe 500-kV line for January 1993 heavy load conditions.

Relation to Automatic Control of Autotransformer Tap Changers

Most of BPA's EHV autotransformers are equipped with on-load tap changers. The tap changers are, however, controlled by system operators. We have investigated voltage stability benefits of *automatic* control of 500/230-kV LTC autotransformers [8]. Close regulation of 230-kV voltage supports 230-kV and lower voltage shunt capacitor reactive power output and minimizes reactive losses at 230-kV and lower voltages. Autotransformer tap changing should be much faster than distribution system tap changing. A problem is that regulating 230-kV voltage sags the 500-kV voltage and reduces 500-kV shunt capacitor bank output. Shorting of series groups on 500-kV capacitor banks located at Raver and Monroe would compensate for this. Selected 230-kV capacitor bank groups could also be shorted based on the unregulated 500-kV side voltage.

Koishikawa et al. [10] describe a microprocessor-based control to integrate capacitor switching and network transformer tap changing.

BPA R&D Project to Prove Concept

BPA is starting a demonstration project. Initial work will include EMTP studies and laboratory tests of capacitor time-overvoltage capability. Full-

scale implementation on a 168 MVAR, 241.5-kV capacitor bank at Olympia Substation is planned. The bank has fourteen series groups; up to three groups will be shorted to increase the reactive output up to a factor of 14/11 or 127%.

Summary

For voltage stability, a problem and a solution are both present in the equation for shunt capacitor reactive output:

$$Q_{\text{capacitor}} = \frac{V^2}{X_c}$$

The voltage-squared dependance makes the power system brittle and prone to instability. Reducing X_c helps solve the voltage stability challenge.

Concluding Remarks

Although we have described binary progression switching with two switches, we will investigate use of a single switch. This improves economics and makes control much simpler. For the Raver banks, five or six groups could be shorted at 515-520 kV.

The microprocessor-based controller may be integrated with standard capacitor bank control and protection functions. The controller may be expanded for overall substation voltage/reactive control.

For switched capacitor banks held in reserve for major disturbances, the capacitor bank groups could be initially shorted. The group shorting switches would then be opened after voltage recovery.

The method may also be used to improve transient stability.

Transient or voltage instability is a problem at some HVDC inverter stations [11,12]. In conjunction with inverter controls to regulate ac voltage, capacitor bank group shorting could be used on converter power factor correction capacitors.

The method, using mechanical or thyristor switches, improves the "flexibility" of power systems.

Acknowledgement. R. S. Gens suggested that BPA reconsider the feasibility of dynamic capacitor group shorting. Many BPA engineers are currently contributing to the R&D project for capacitor group shorting at Olympia Substation.

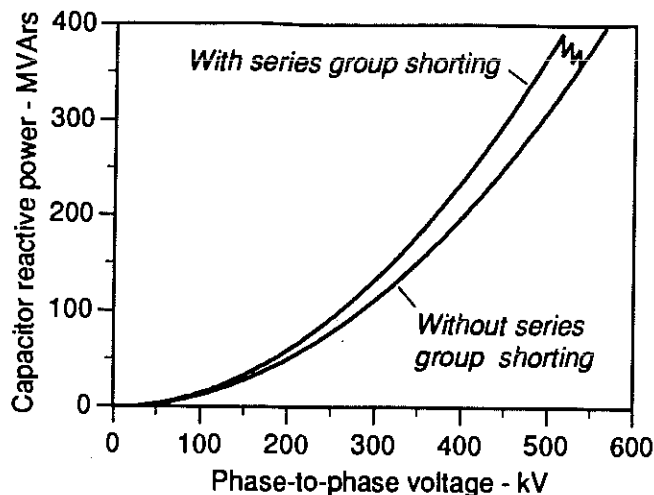


Figure 3. Raver capacitor bank reactive power characteristics with and without group shorting.

From Table 1, we note that 58 MVAR per bank are gained at 500 kV. We estimate the cost of a conventional 230-kV breaker switched capacitor bank of this size to be \$612,600 (\$300,000 for circuit breaker and \$5.39/kVAR for the capacitor bank). If a circuit switcher rather than a circuit breaker is used, the cost estimate is \$437,000.

Puget Sound Voltage Stability Improvement

Figure 4 is a schematic of the 500-kV transmission system in the Puget Sound and surrounding areas.

Experience has shown that Puget Sound area voltage stability can be judged by $V-Q$ curves computed for the Raver 500-kV bus. The fictitious synchronous condenser (fictitious PV bus) method is used to efficiently compute the curves. ($V-Q$ rather than $Q-V$ terminology emphasizes that PV bus voltage is the independent variable.)

Figure 5 shows, by $V-Q$ curves, the voltage stability improvement by extensive use of capacitor bank group shorting. The disturbance is outage of the Chief Joseph-Monroe 500-kV line (Figure 4). Group shorting is represented on 1950 MVAR of 500-kV and 230-kV capacitor banks in the Puget Sound area (exclusive of the Raver capacitor banks). The group shorting is represented by switching in twenty percent additional capacitance at 96% to 98% of the pre-disturbance voltage. At each location, the switching was done in two equal steps.

The characteristics of the three 500-kV banks at Raver are shown explicitly on Figure 5. Characteristics are shown with and without group shorting on capacitor banks 2 and 3. The characteristics

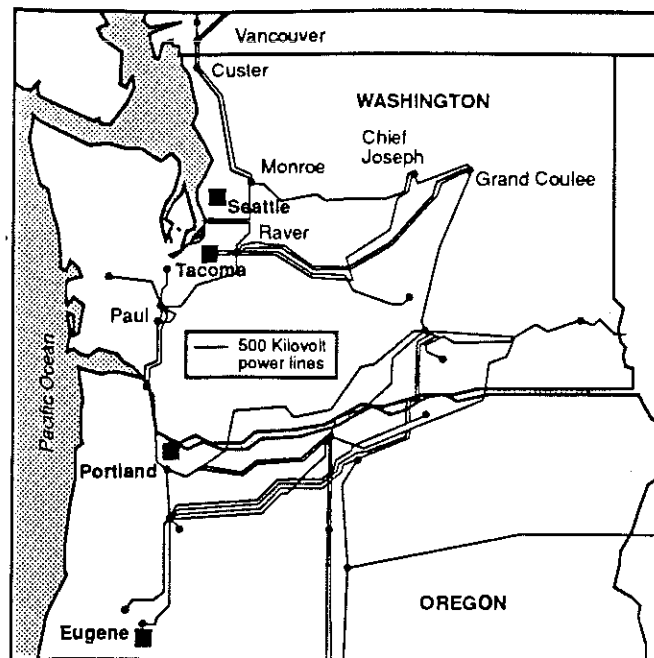


Figure 4. Puget Sound area 500-kV transmission schematic.

of banks 2 and 3 are the same as shown on Figure 3.

The operating point is the intersection of the system $V-Q$ curve and the characteristic of the Raver capacitor banks. The operating point is at a voltage high enough that capacitor group shorting is not used (Raver is at borderline of switching).

Dramatic improvement in distance from voltage instability is seen, however, with capacitor bank group shorting. Without group shorting, the reactive margin (distance from the operating point to the bottom of a $V-Q$ curve) is about 120 MVAR. With group shorting, the reactive margin is about 600 MVAR. Capacitor bank group shorting at Raver (not included in the reactive margin) further improves system reliability.

The $V-Q$ curves are computed using a large scale (5000 bus) representation of the western interconnection. The Puget Sound area representation is very detailed. For Figure 5, however, we assumed constant power loads. Modeling some voltage dependence for the loads may be more accurate. Voltage sensitive loads greatly improve voltage stability [8,9] and may reduce the benefits of capacitor group shorting.

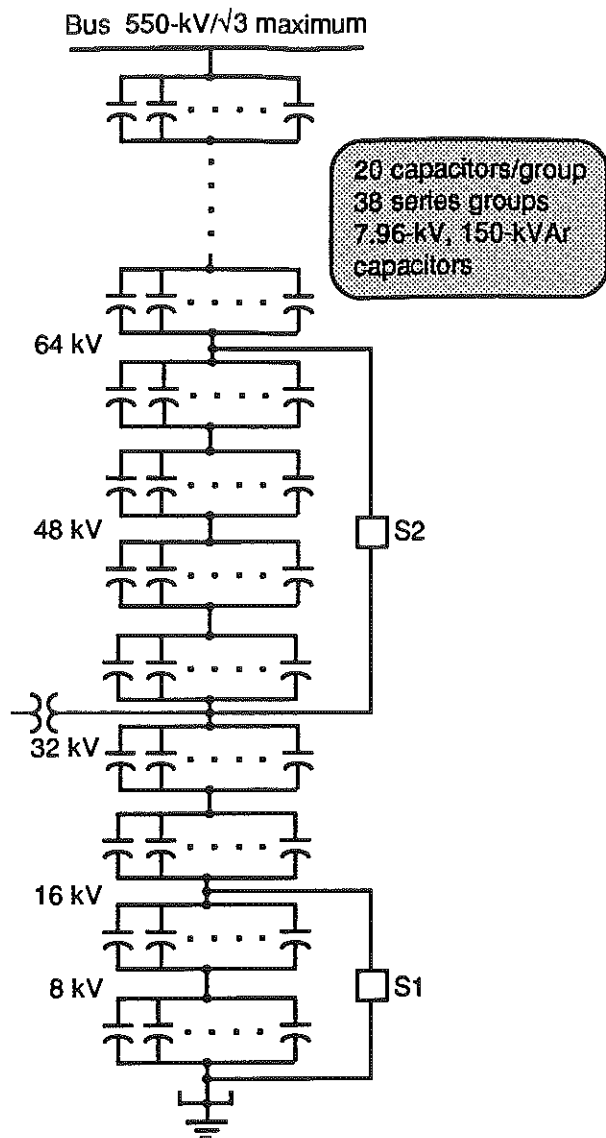


Figure 2. Preliminary design for Raver banks 2 and 3.

With a more sophisticated controller, and with additional switching, the following schedule could be followed:

- Above 535 kV, S1 and S2 open
- 525–535 kV, S1 closed
- 515–525 kV, S2 closed
- Below 515 kV, S1 and S2 closed

A phase-plane controller could be used where S1 and S2 switching is based on both voltage change and rate of voltage change. Appropriate filtering must be provided for the rate of voltage change computation. It's important to short groups at as high a voltage as possible to "keep under the voltage."

The controller requires additional logic for switch failures, excessive switch close time, hunting prevention, etc.

Table 1 shows voltages and capacitor reactive power for various combinations of voltages and shorted groups. For the above switching schedule, the highest voltage stress is 117% (at 515 kV with both switches closed); more aggressive control might be possible.

V - kV	V - pu of 523.9 kV	# Groups	V - pu/group	Q - MVAR
550.0	1.05	38	1.05	376.92
545.0	1.04	38	1.04	370.10
540.0	1.03	38	1.03	363.34
535.0	1.02	38	1.02	356.65
535.0	1.02	36	1.08	376.46
530.0	1.01	38	1.01	350.01
530.0	1.01	36	1.07	369.46
530.0	1.01	34	1.13	391.19
525.0	1.00	38	1.00	343.44
525.0	1.00	36	1.06	362.52
525.0	1.00	34	1.12	383.84
523.9	1.00	38	1.00	342.00
520.0	0.99	38	0.99	336.93
520.0	0.99	36	1.05	355.65
520.0	0.99	34	1.11	376.57
515.0	0.98	38	0.98	330.48
515.0	0.98	36	1.04	348.84
515.0	0.98	34	1.10	369.36
515.0	0.98	32	1.17	392.44
505.0	0.96	38	0.96	317.77
505.0	0.96	34	1.08	355.15
505.0	0.96	32	1.14	377.35
500.0	0.95	38	0.95	311.51
500.0	0.95	36	1.01	328.81
500.0	0.95	34	1.07	348.16
500.0	0.95	32	1.13	369.92

Figure 3 shows the capacitor reactive power versus voltage characteristics for the above switching schedule. The group shorting provides a static characteristic similar to addition of a small SVC (thyristor switched capacitor) with about a 5% droop.

Cost Estimates

The rough cost estimate for the Raver installation is \$162,000. The breakdown is \$117,000 for high voltage equipment including six vacuum switches and six current limiting reactors. The remaining \$45,000 is for control and protection.

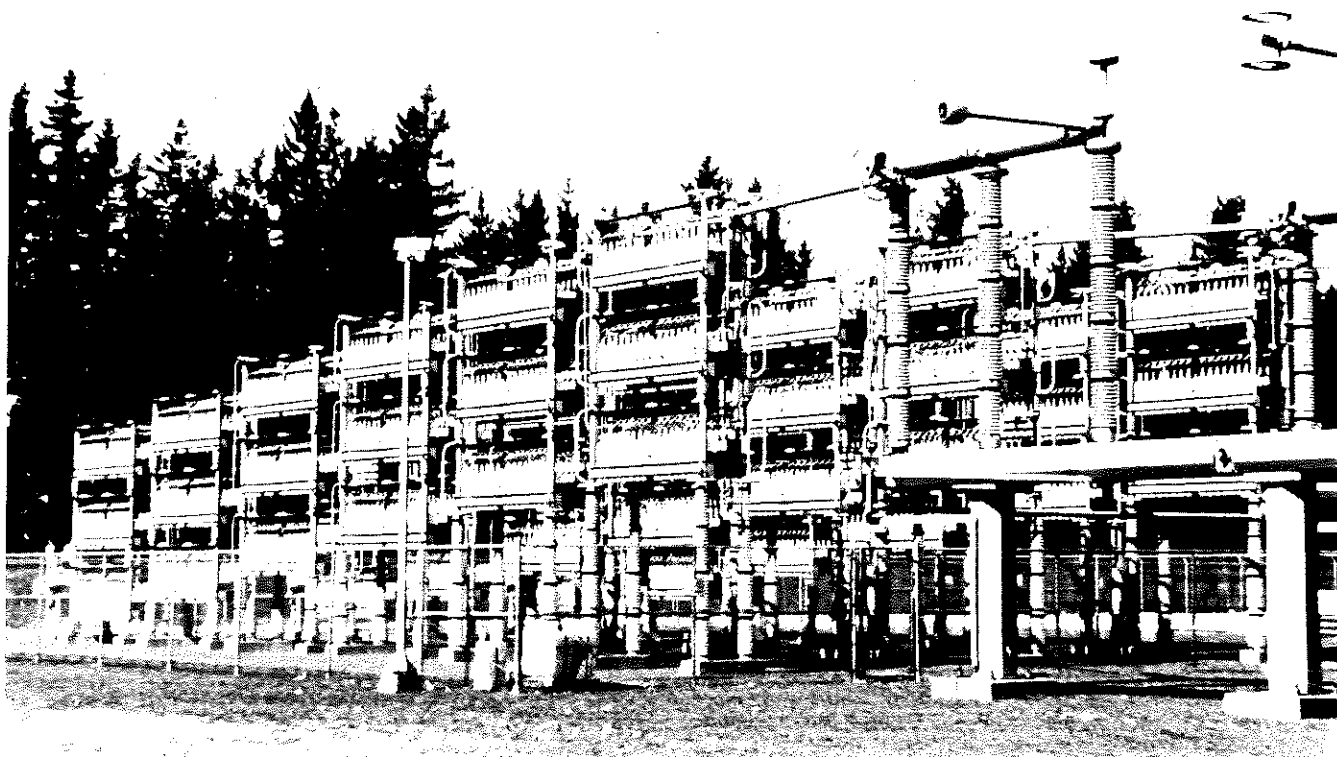


Figure 1. Raver 500-kV capacitor bank.

Actual use of the control would be very infrequent; the joint probability of very heavy loads and a major disturbance threatening voltage stability is quite low. The low probability of voltage instability favors low cost solutions, rather than expensive solutions such as static var compensators.

Preliminary Design for a 500-kV Bank

Calculations were made for two 500-kV banks recently commissioned at the Raver Switchyard east of Tacoma (banks 2 and 3). The capacitor banks are important for Puget Sound area voltage stability and include SEL voltage differential relays that possibly could be reprogrammed for adaptive setting and capacitor group shorting. The wye-grounded, externally-fused banks are rated 342-MVAR, 523.9-kV, and consist of 7.96-kV, 150-kVAR capacitors. There are 20 parallel capacitors per group and 38 groups in series. The tap point for the voltage differential relay potential transformer is four groups above ground.

Figure 1 is a photo of one of the banks.

Figure 2 shows a preliminary design with two medium-voltage switches. A binary progression is

used, allowing shorting of 0, 2, 4, or 6 groups. Shorting six groups will increase the reactive output by a factor of 38/32 or 119%. Following any switching, the microprocessor could compute the ΔV for a ΔQ to determine the correct number of groups to be shorted to maximize voltage and reactive power output without excessive voltage over-stressing.

The normal condition short circuit capacity at Raver is about 21,000 MVA, meaning a 100 MVAR shunt insertion will change voltage about 2.5 kV. Raver voltage schedule is usually 540 kV.

Possible settings for simple relay logic are:

535 kV	Open S1 (overvoltage relay).
530 kV	Close S1 (undervoltage relay), increase reactive output by about 20 MVAR/bank.
525 kV	Open S2 (overvoltage relay).
515 kV	Close S2 (undervoltage relay), increase reactive output by an additional 46 MVAR/bank.

closer to the 550 kV maximum voltage has led us, however, to higher capacitor bank voltage ratings on new banks.

The idea of dynamically shorting capacitor bank groups, as described in this paper, has been considered by BPA in the past. One factor which prevented its use was BPA's capacitor bank protection for blown fuses. New voltage stability challenges and new technology has led to "an idea whose time has come."

Detection of Blown Fuses

Although there are some advantages to internal capacitor fuses [2], BPA's practice has been to use external fuses (a single fuse for each capacitor).

An IEEE guide [3] describes design and protection of shunt capacitor banks. To detect blown fuses for capacitor overvoltage protection, BPA has used, the MTY voltage differential relay for many years [4] (the relay is named after its inventor, Moon Tom Yee, my first supervisor at BPA). For each phase, this relay compares bus voltage with the scaled voltage at a tap point near the bottom group of capacitor cans.*

In the past, high speed automatic switching of capacitor groups was not possible because of the voltage differential relay. This problem can now be overcome with microprocessor-based relays employing adaptive settings based on the number of groups in service.

A microprocessor version of the relay has been developed, with BPA sponsorship, by Schweitzer Engineering Laboratories (SEL) [5]. The relay is more sensitive in detecting blown fuses than the MTY relay. The accuracy is ± 0.25 volts based on a 67-volt PT source. For 500-kV, this corresponds to about ± 2 kV phase-phase primary voltage. The SEL relay is also a voltage-based controller for capacitor switching or other control.

An alternative to differential protection, used by many utilities, is time-delay phase unbalance protection [3,5]. This requires one rather than three relays and does not need adaptive settings for capacitor group shorting. The main disadvantage is reduced sensitivity, since the relay setting must cater for some system unbalance during normal operation.

* The voltage differential relay for each phase computes $\Delta V = V_{\text{bus}} - K(V_{\text{tap}})$, K is a scaling factor that changes with the number of series rows on either side of the tap. ΔV is zero with no blown fuses.

Pacific Northwest Voltage Stability Challenges

In the Pacific Northwest, interrelated voltage stability problems exist from Vancouver, Canada to the Puget Sound (Seattle-Tacoma) area to the Portland, Oregon area. Voltage instability is likeliest during cold winter weather. During peak winter load periods, several thousand megawatt of 230-kV and 500-kV capacitor banks are in service in the Puget Sound area.

Voltage stability problems also exist on the 500-kV Pacific AC Intertie [6]. For both transient stability and voltage stability improvement, the 500-kV shunt capacitor banks described by Furumasa and Hasibar [1] were installed at Malin Substation near the Oregon-California border. At present, the banks are held in reserve and switched on for outages of the parallel Pacific HVDC Intertie. Mainly for voltage stability, additional 500-kV banks are being installed in Northern California.

This paper will concentrate on Puget Sound area voltage stability.

Basic Concept

The idea is to short series groups based on detection of low voltage. Because of the low voltage, the capacitor stress is reduced with all groups in service. Standards [7] allow 110% continuous voltage. The continuous overvoltage capability is necessary for blown fuses and other abnormal conditions. Standards [7, section 8.3.2] also suggest capacitors can tolerate many applications of temporary overvoltage of time-voltage durations of 125% for thirty minutes, 130% for one minute, 140% for fifteen seconds, etc.

At a particular voltage, the reactive output will be increased by a factor $n/(n - n_{\text{short}})$ where n is the total number of groups and n_{short} is the number of groups shorted.

Using the thirty minute overvoltage capability can improve voltage stability. Following a major disturbance, it can provide ten to fifteen minutes for operator-directed actions such as starting gas turbines. If gas turbines are already running, it can allow time for generation rescheduling or manual load shedding.

One or two medium-voltage switches (vacuum or SF₆ switches) are required per phase for each capacitor bank. Current limiting reactors are also required. Conservative application could limit capacitor voltage to, say, 120% of rated for a maximum of fifteen minutes. Deadbands would be wide enough to prevent hunting.

A Novel Method to Improve Voltage Stability Using Shunt Capacitors

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Abstract - The "brittle" (reactive output proportional to voltage squared) characteristic of shunt capacitor compensated networks is a major factor in power system voltage instability. This paper describes how transmission system shunt capacitor banks can be controlled to increase reactive output during emergencies. The method involves dynamically shorting several series groups of a shunt capacitor bank to reduce the total capacitive reactance and thus increase reactive power output. Emphasis in this paper is on the general concept and on system benefits. BPA is currently evaluating the concept, and is planning a prototype installation on a 168-MVAr, 241.5-kV capacitor bank.

Keywords - Voltage Stability, Voltage Collapse, Capacitor Banks

Introduction

Voltage instability is associated with high utilization of transmission systems. The high reactive losses during heavy load and heavy power import into load areas require reactive support that is most economically provided by shunt capacitors. The very wide use of shunt capacitors contributes to the "brittleness" of load areas (reactive power output falls as the square of the voltage). The brittleness can be overcome, at considerable expense, by additional mechanically- or thyristor- switched shunt compensation.

Most utilities have traditionally applied shunt capacitors at subtransmission and distribution voltages. As transmission loadings increase above surge impedance loading levels, however, large capacitor banks are being applied at high voltage and extra-high voltage. This is in accord with principles of producing reactive power near points of consumption and of avoiding transmission of reactive power between voltage levels.

Transmission system shunt capacitors may be switched during the daily load cycle. In some cases, however, transmission system mechanically switched capacitor banks (MSCs) are held in reserve for emergencies, and then energized by circuit breaker closing.

Shunt capacitor banks consist of series-parallel combinations of capacitors.

This paper describes a novel and low-cost method to improve the effectiveness of shunt capacitor compensation. The method uses medium-voltage switches to short series groups (layers) of shunt capacitor banks. This reduces the capacitor bank reactance, increasing the reactive power output at a given voltage. The capacitor groups are shorted during depressed system voltage. The method takes advantage of the time-overvoltage capability of capacitors (standards suggest 125% of rated voltage for thirty minutes). Often only about ten minutes of increased output is valuable.

The method is being investigated by Bonneville Power Administration (BPA).

BPA Application of Shunt Capacitors

BPA is a transmission company with over 14,000 circuit miles of transmission lines between 115-kV and 500-kV. We have no distribution systems. Contrasted with other utilities, we have, for many years, applied shunt capacitor banks at 230-kV. Since 1976, capacitors have been applied at 500-kV.

Banks as large as 150 MVAr are used at 230-kV, and banks over 300 MVAr are used at 500-kV. The banks are wye grounded. Some new banks are designed for fast insertion, with back-to-back switching capability [1]. BPA is currently replacing smaller, PCB-contaminated 230-kV capacitor banks with larger 230-kV banks.

For many years, BPA increased the output of shunt capacitor banks during the winter season by reducing the number of series groups. One group was shorted on many 230-kV banks to increase output. This was effective on older capacitors which have much higher losses than modern plastic film capacitors.

BPA has also taken partial advantage of the 110% continuous overvoltage capability of capacitors. Two 500-kV banks were designed and built with 496.3-kV, 324-MVAr rating. Operating voltage was typically 530–535 kV. Experience over many years has been satisfactory. Problems with detection of blown fuses and recent practice to operate

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