ECEN 460
Power System Operation and Control
Spring 2025

Lecture 21: Load Modeling, Modal Analysis

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Special Guest Lecture Given by Sanjana Kunkolienkar

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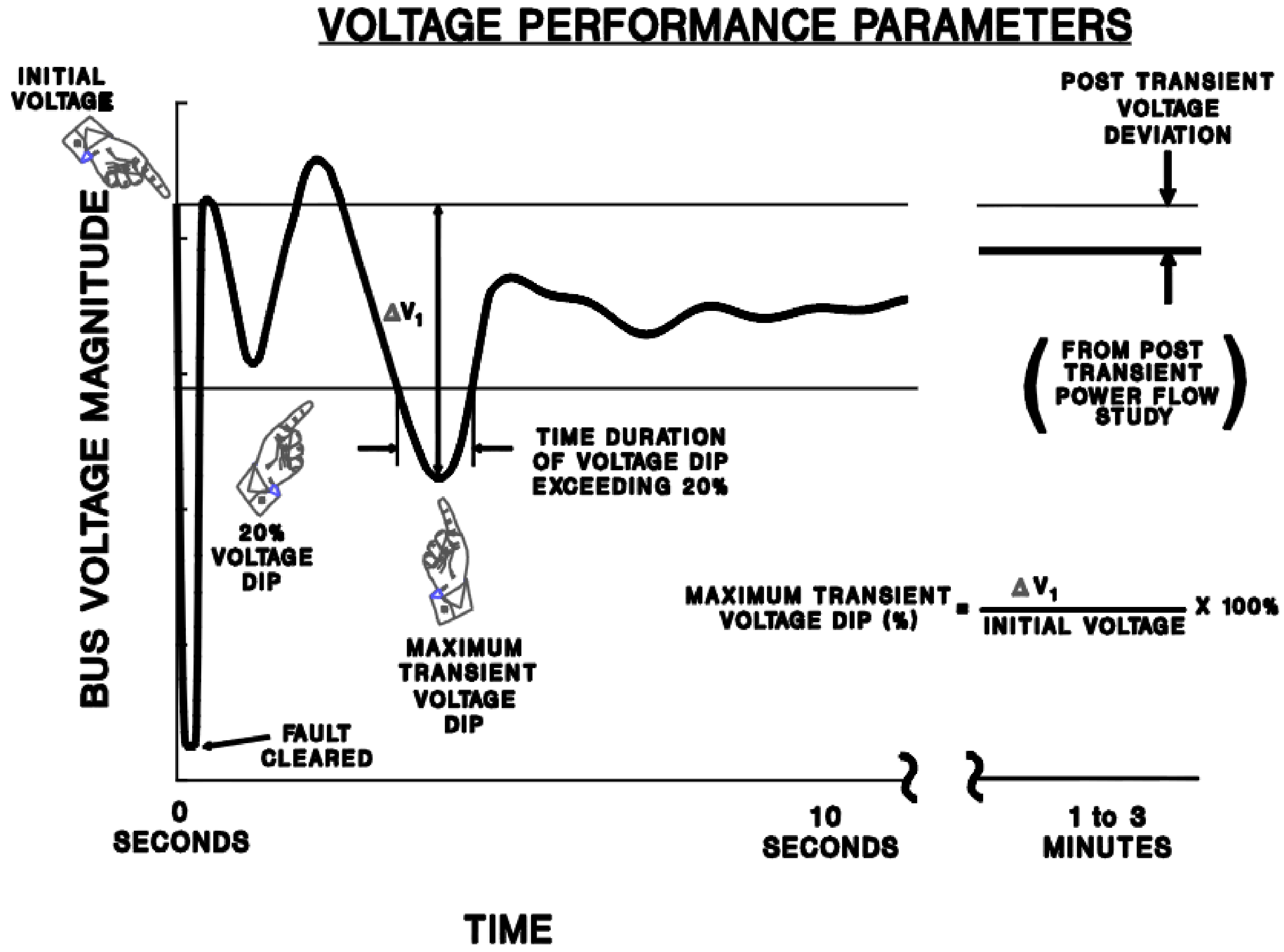
Announcements

- Please read Chapter 13
- By April 9 do Problem Set A
- Schedule for the rest of the semester is
  - Lab 10 week of April 7 consisting of time for the design groups to meet; all groups need to turn in a brief progress report in lieu of a lab report
  - Lab 3 week of April 14 (optional, ungraded machine lab)
  - Lab 11 (project presentations) by the individual teams to their TA before the end of classes (on or before April 29)
  - Exam 2 on Wednesday April 23 during class
  - Design project due at 9:30 am on May 1 (i.e., at the end of our final slot; no final)

Transient Limit Monitors

- During a transient contingency how fast the voltage recovers is a key metric



Similar performance criteria exist for frequency deviations

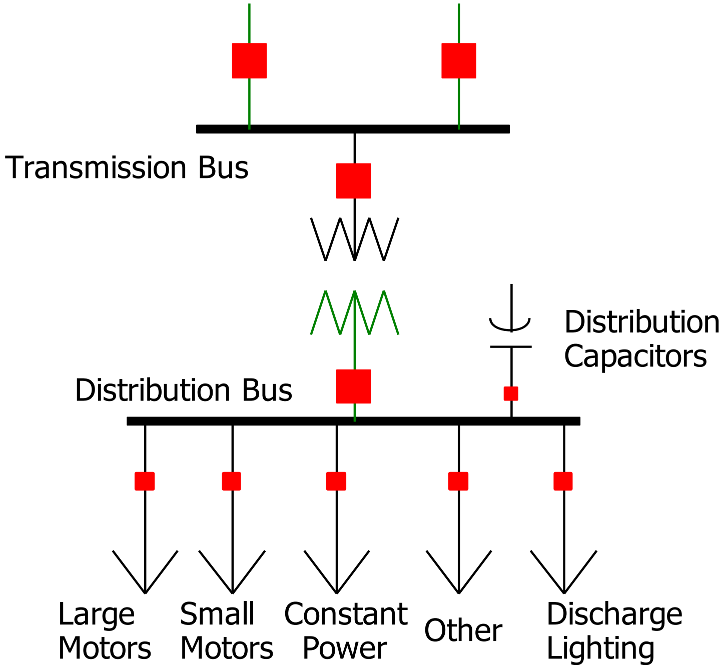
Image from WECC Planning and Operating Criteria

Composite Load Models

- Many aggregate loads are best represented by a combination of different types of load
  - Known as composite load models
  - Important to keep in mind that the actual load is continually changing, so any aggregate load is at best an approximation
  - Hard to know load behavior to extreme disturbances without actually faulting the load
- Early models included a number of loads at the transmission level buses (with the step-down transformer), with later models including a simple distribution system model

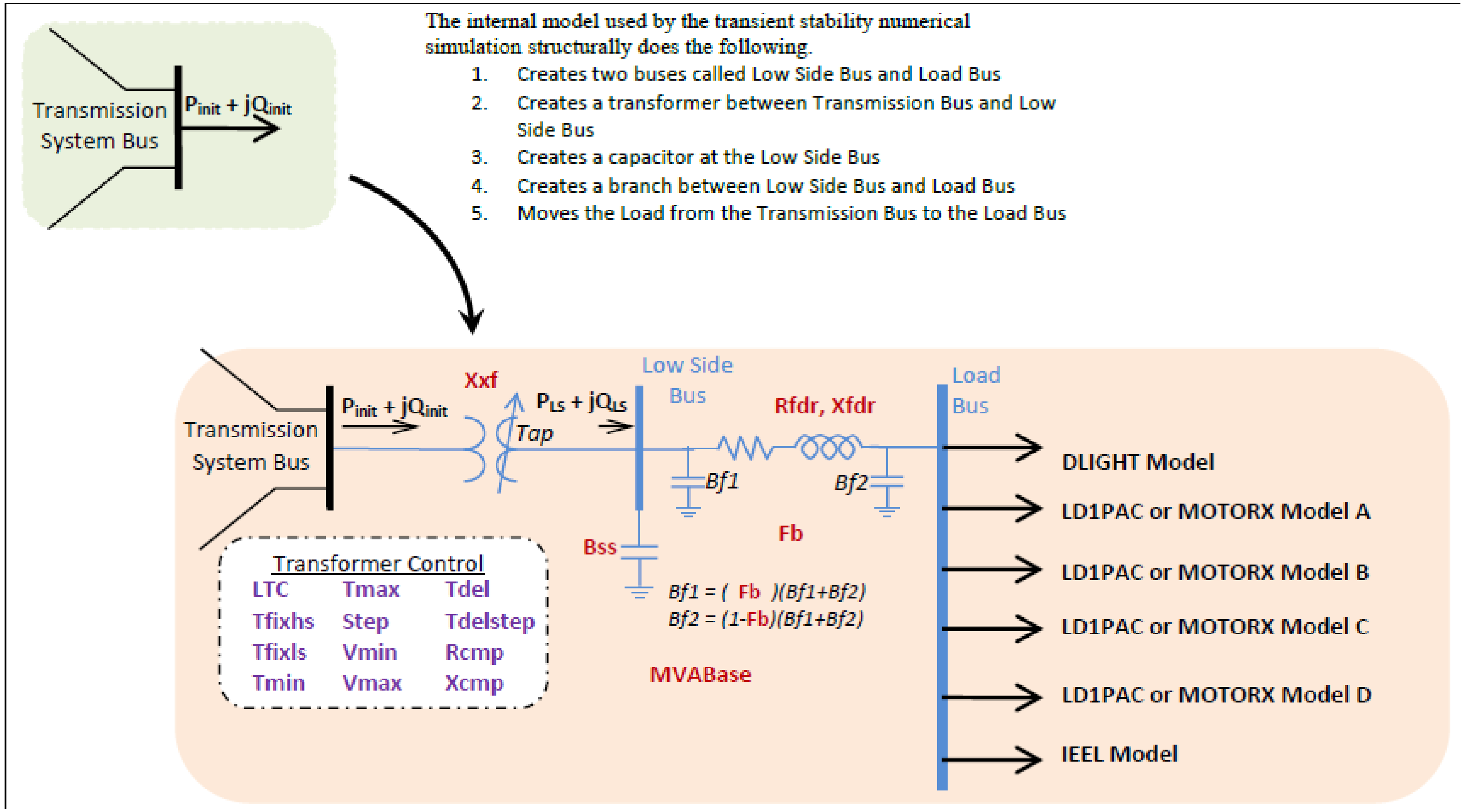
CLOD Model

- The CLOD model represents the load as a combination of large induction motors, small induction motors, constant power, discharge lighting, and other



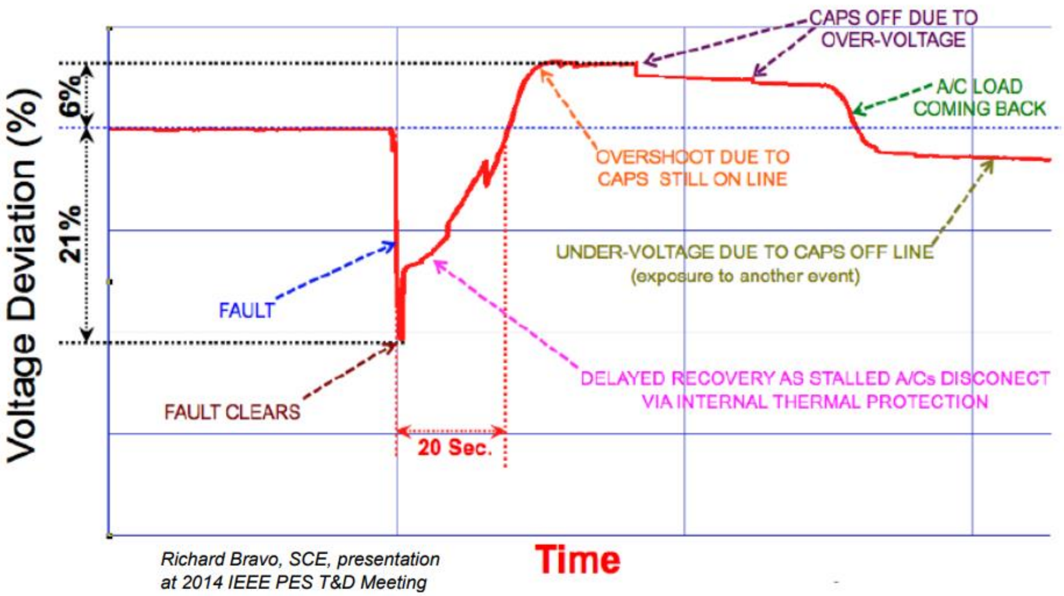
Composite Load Model

- Contains up to four motors or single phase induction motor models; also includes potential for solar PV



A Concern: Fault Induced Delayed Voltage Recovery (FIDVR)

- FIDVR is a situation in which the system voltage remains significantly reduced for at least several seconds following a fault (at either the transmission or distribution level)
  - It is most concerning in the high voltage grid, but found to be unexpectedly prevalent in the distribution system
- Stalled residential air conditioning units are a key cause of FIDVR – they can stall within the three cycles needed to clear a fault



Oscillations

- An oscillation is just a repetitive motion that can be either undamped, positively damped (decaying with time) or negatively damped (growing with time)
- If the oscillation can be written as a sinusoid then

$$e^{\alpha t}\left(a\cos(\omega t)+b\sin(\omega t)\right)=e^{\alpha t}C\cos(\omega t+\theta)$$

$$\text{where } C=\sqrt{A^2+B^2} \text{ and } \theta=\tan\left(\frac{-b}{a}\right)$$

- The damping ratio is

$$\xi=\frac{-\alpha}{\sqrt{\alpha^2+\omega^2}}$$

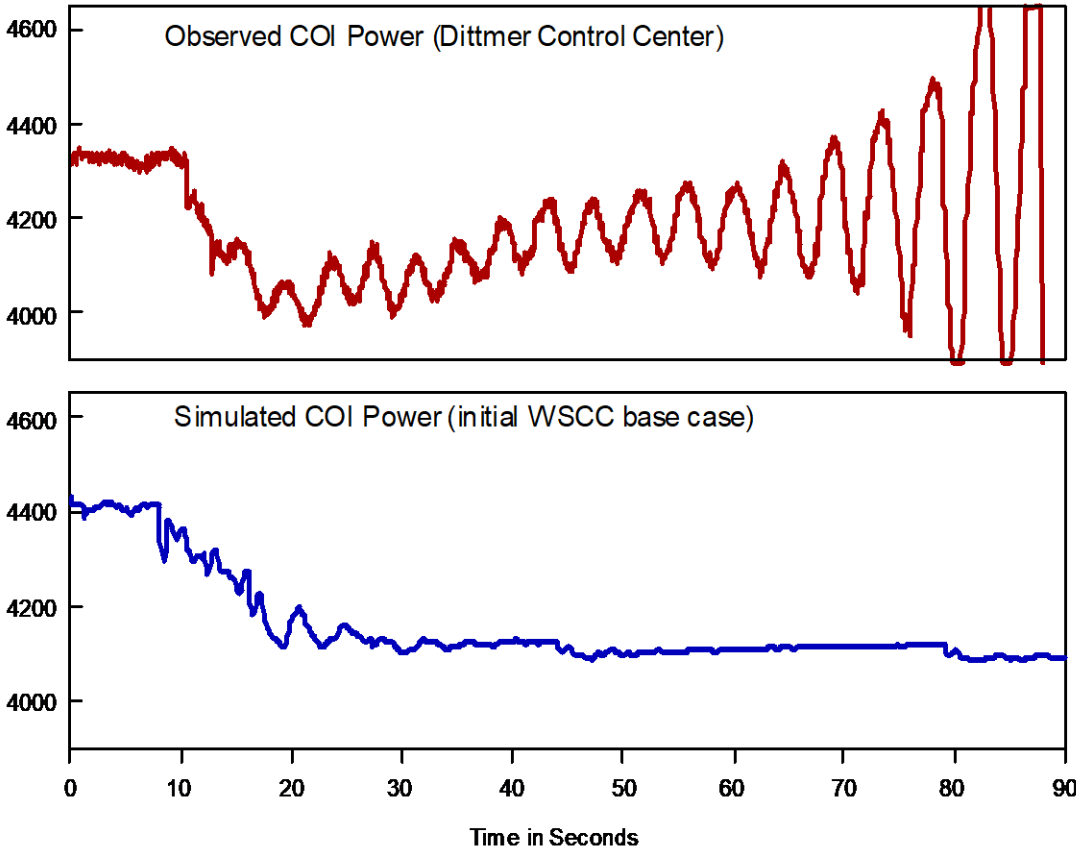
The percent damping is just the damping ratio multiplied by 100; goal is sufficiently positive damping

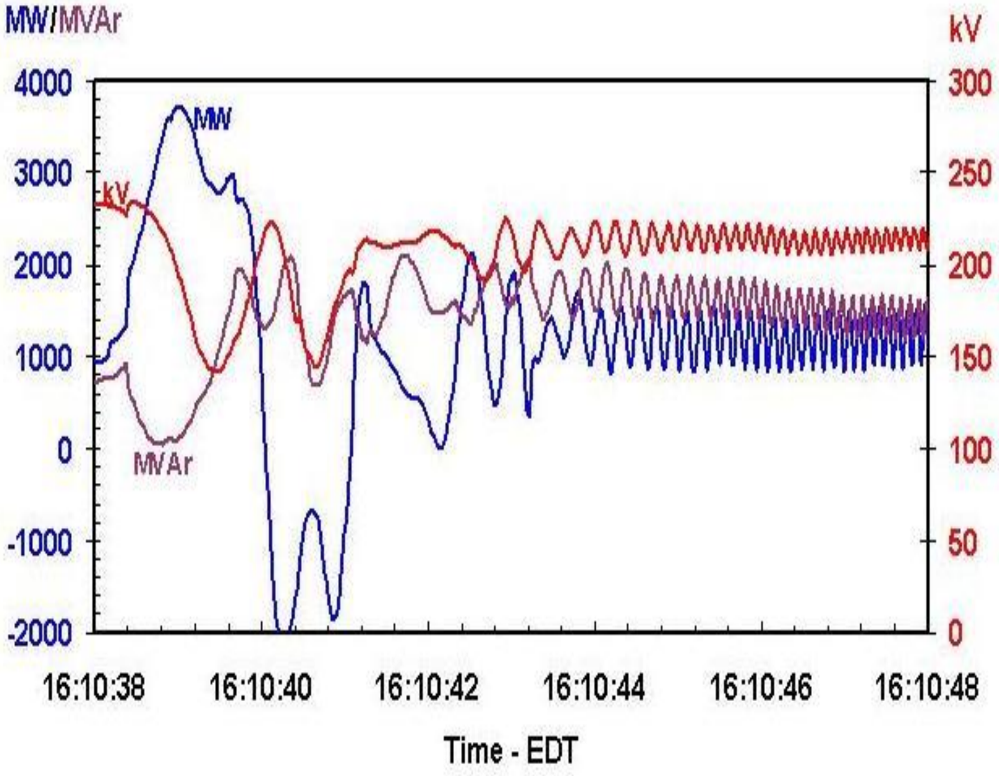
Power System Oscillations

- Power systems can experience a wide range of oscillations, ranging from highly damped and high frequency switching transients to sustained low frequency (< 1 Hz) inter-area oscillations affecting an entire interconnect
- Types of oscillations include
  - Transients: Usually high frequency and highly damped
  - Local plant: Usually from 1 to 5 Hz
  - Inter-area oscillations: From 0.15 to 1 Hz
  - Slower dynamics: Such as AGC, less than 0.15 Hz
  - Subsynchronous resonance: 10 to 50 Hz (less than synchronous)

Example Oscillations

- The left graph shows an oscillation that was observed during a 1996 WECC Blackout, the right from the 8/14/2003 blackout





Causes of Power System Oscillations

- The response of a simple system can be divided into its natural response versus its forced response
  - The natural response tells how the system will response to an initial disturbance without any additional (external) influences; this response shows the system's modes
  - A forced response is associated with an external disturbance; if the external disturbance is periodic then the system will oscillate at least partially at this frequency
  - Often forced oscillations are due to control failures
- Resonance occurs when a forced response is at a similar frequency to one of the system's modes
- An power system can experience both types of oscillations

Phasor Measurement Units (PMUs) and SynchroPhasors

- Initially a challenge with understanding power system dynamics was the lack of high speed, synchronized measurements
  - Supervisory Control and Data Acquisition (SCADA) measured the system analog values every couple of seconds
- This has gradually changed over the last several decades with the now widespread deployment of phasor measurement units (PMUs) that are able to use time synchronized measurements to accurately determine values at rates of 30 times per second

Modes

- A mode is a concept from linear system analysis
  - Electric grids certainly are not linear, but usually their response to small disturbances is approximated as linear
- A mode corresponds to one of the eigenvalues of the response or, for oscillations, a complex pair of eigenvalues
- A mode has a frequency and damping; all parts of the system oscillate with this pattern
- The mode shape tells how parts of the system participate in the mode
- There can be multiple modes in a system; power systems can have many modes

Small Signal Analysis (SSA)

- Small signal stability is the ability of the power system to maintain synchronism following a small disturbance
  - System is continually subject to small disturbances, such as changes in the load
- The operating equilibrium point (EP) obviously must be stable
- Small signal analysis (SSA) is studied to get a feel for how close the system is to losing stability and to get additional insight into the system response
  - There must be positive damping

Model-Based SSA

- Assume the power system is modeled in our standard form as

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \mathbf{y})$$

$$\mathbf{0} = \mathbf{g}(\mathbf{x}, \mathbf{y})$$

- The system can be linearized about an equilibrium point

$$\Delta\dot{\mathbf{x}} = \mathbf{A}\Delta\mathbf{x} + \mathbf{B}\Delta\mathbf{y}$$

$$\mathbf{0} = \mathbf{C}\Delta\mathbf{x} + \mathbf{D}\Delta\mathbf{y}$$

If there are just classical generator models then **D** is the power flow Jacobian; otherwise it also includes the stator algebraic equations.

- Eliminating $\Delta\mathbf{y}$ gives

$$\Delta\dot{\mathbf{x}} = \left(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\right)\Delta\mathbf{x} = \mathbf{A}_{sys}\Delta\mathbf{x}$$

We won't be covering model-based SSA in 460

Small Signal Analysis and Measurement-Based Modal Analysis

- The alternative to model-based SSA is to use measurement-based modal analysis to determine the observed dynamic properties of a system
  - Input can either be measurements from devices (such as PMUs) or dynamic simulation results
  - The same approach can be used regardless of the measurement source
- Focus in this section is on the measurement-based approach

Ring-down Modal Analysis

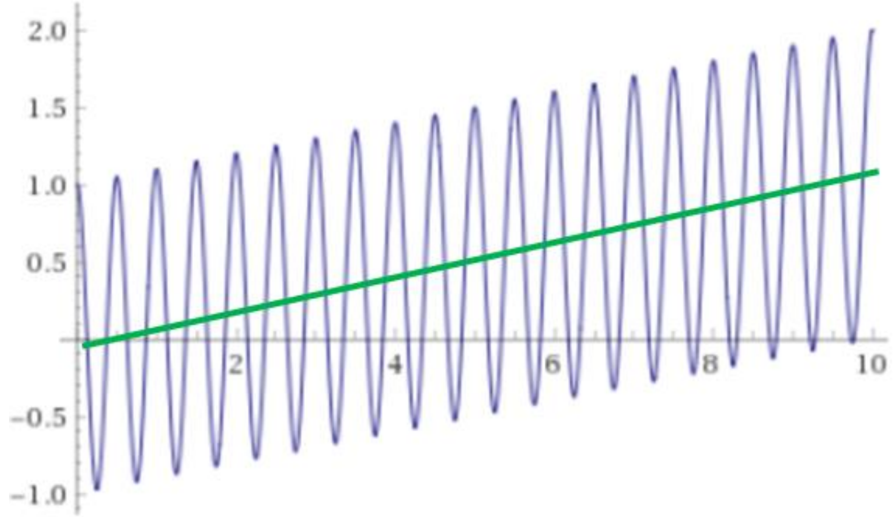
- Ring-down analysis seeks to determine the frequency and damping of key power system modes following some disturbance
- There are several different techniques, with the Prony approach the oldest (from 1795); introduced into power in 1990 by Hauer, Demeure and Scharf
- Regardless of technique, the goal is to represent the response of a sampled signal as a set of exponentially damped sinusoidals (modes)

$$y(t) = \sum_{i=1}^{q} A_i e^{\sigma_i t} \cos\left(\omega_i t + \phi_i\right) \qquad \text{Damping (\%)} = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \omega_i^2}} \times 100$$

Goal: Extracting Modes from the Signals

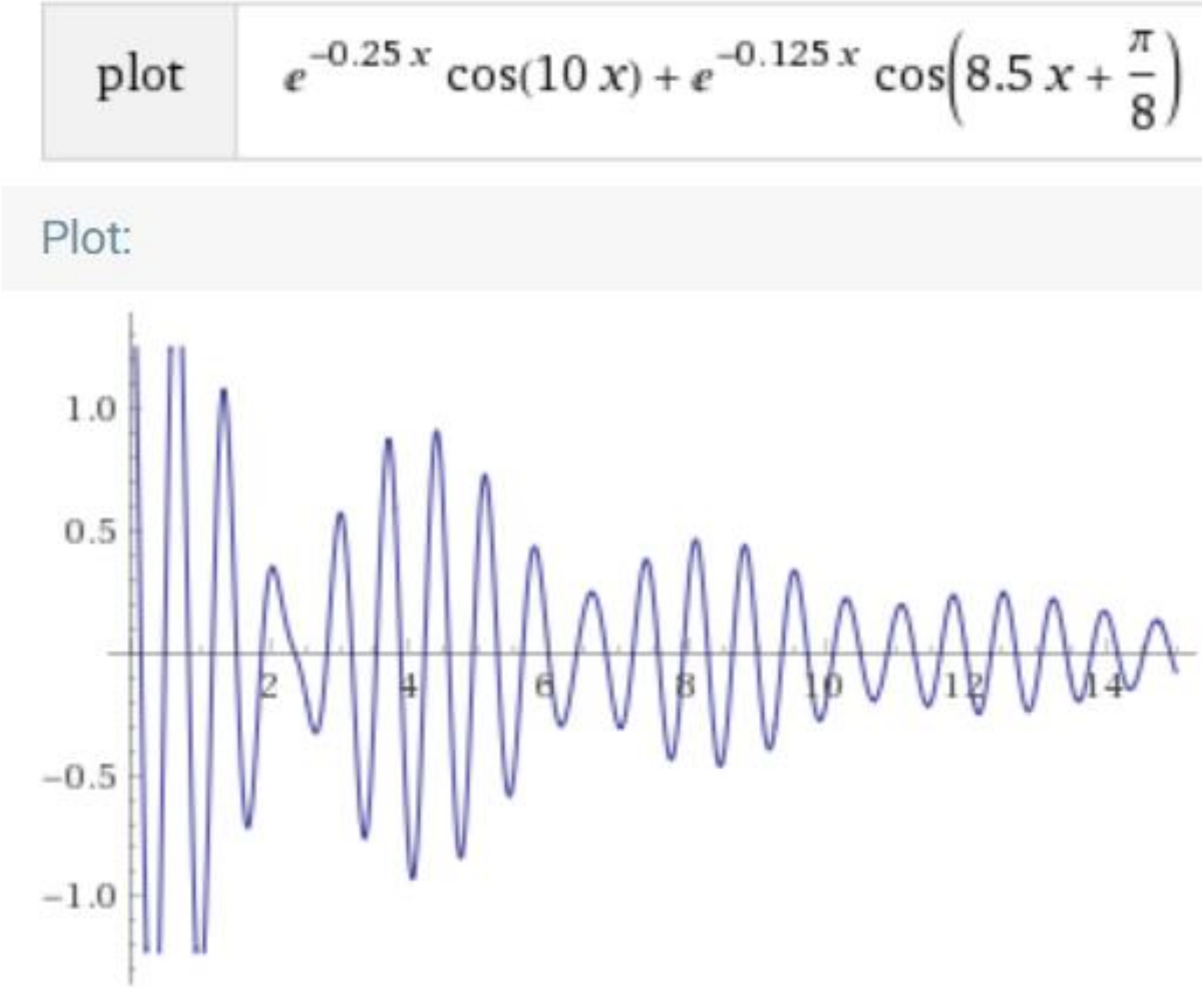
- The goal is to gain information about the electric grid by extracting modal information from its signals
  - The frequency and damping of the modes is key
- The premise is we'll be able to reproduce a complex signal, over a period of time, as a set a of sinusoidal modes
  - We'll also allow for linear detrending

$$0.1t + \cos(2\pi 2t)$$



Example: Summation of Two Damped Exponentials

- This example was created by going from the modes to a signal
- We'll be going in the opposite direction (i.e., from a measured signal to the modes)



Example: One Signal

This could be any signal; image shows the result of the original signal (blue) and the reproduced signal (red)

Start Time Used: 3.000000 | End Time Used: 10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

Statistics | Modes and Damping | Object Fields

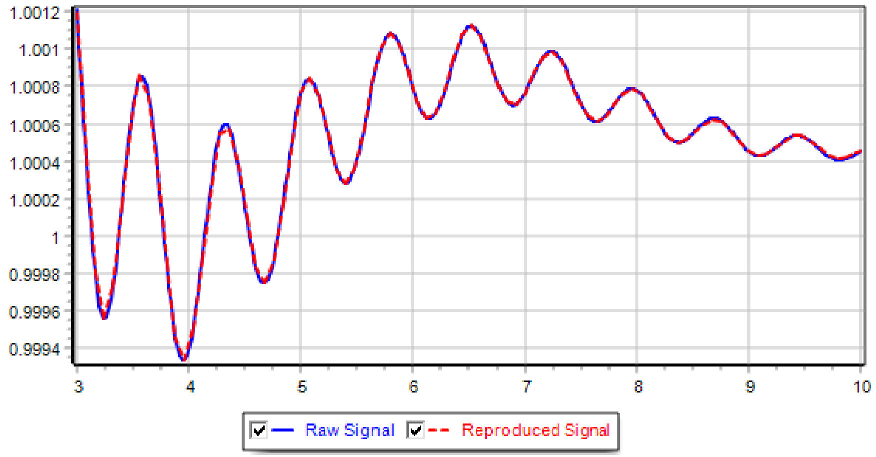
Undamped Modes: 0

Trend — A (constant): 1.00 | B (linear): 0.0000617 | C (quadratic): 0.0

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

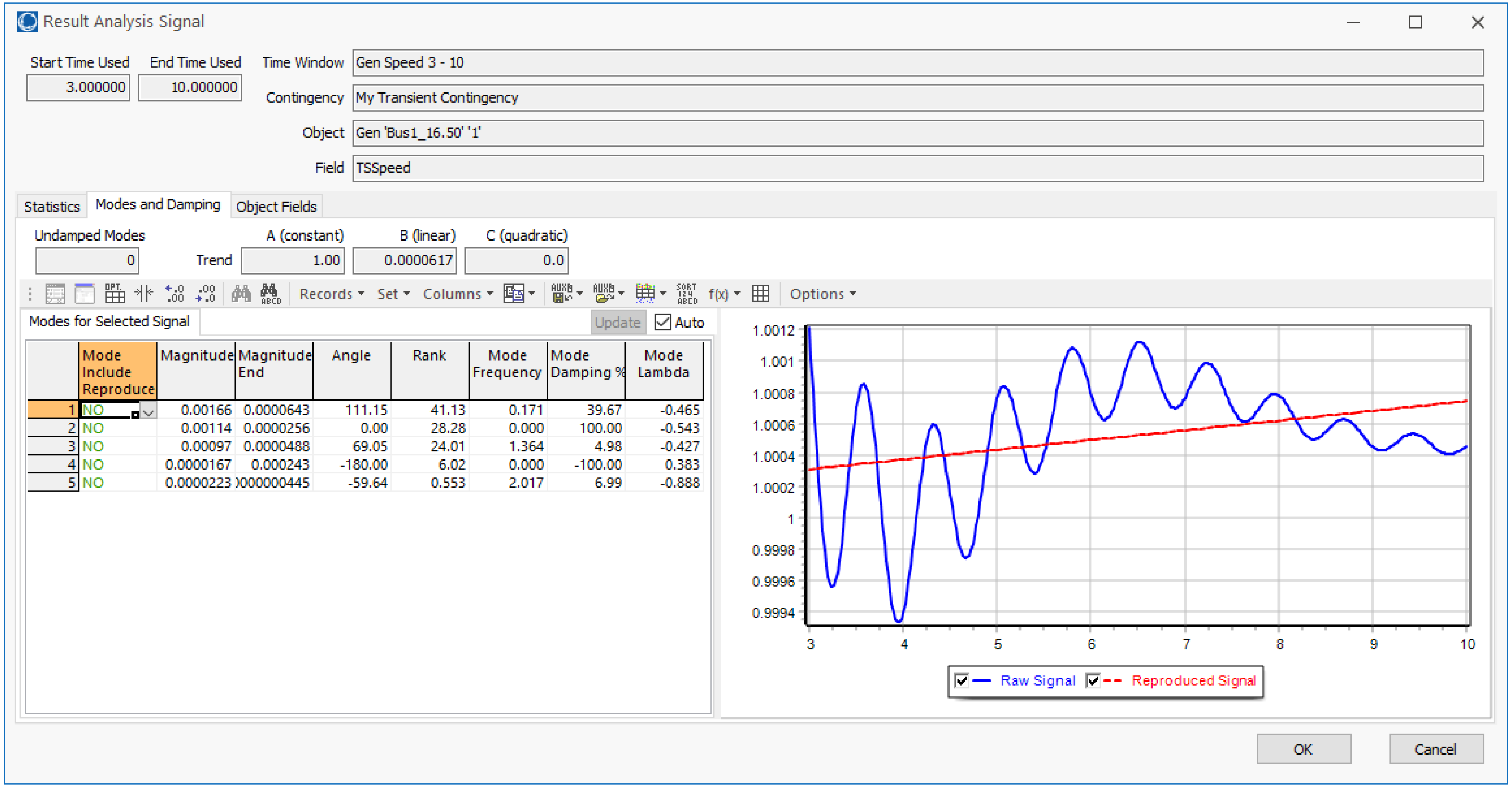
Modes for Selected Signal — Update ☑ Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	YES	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



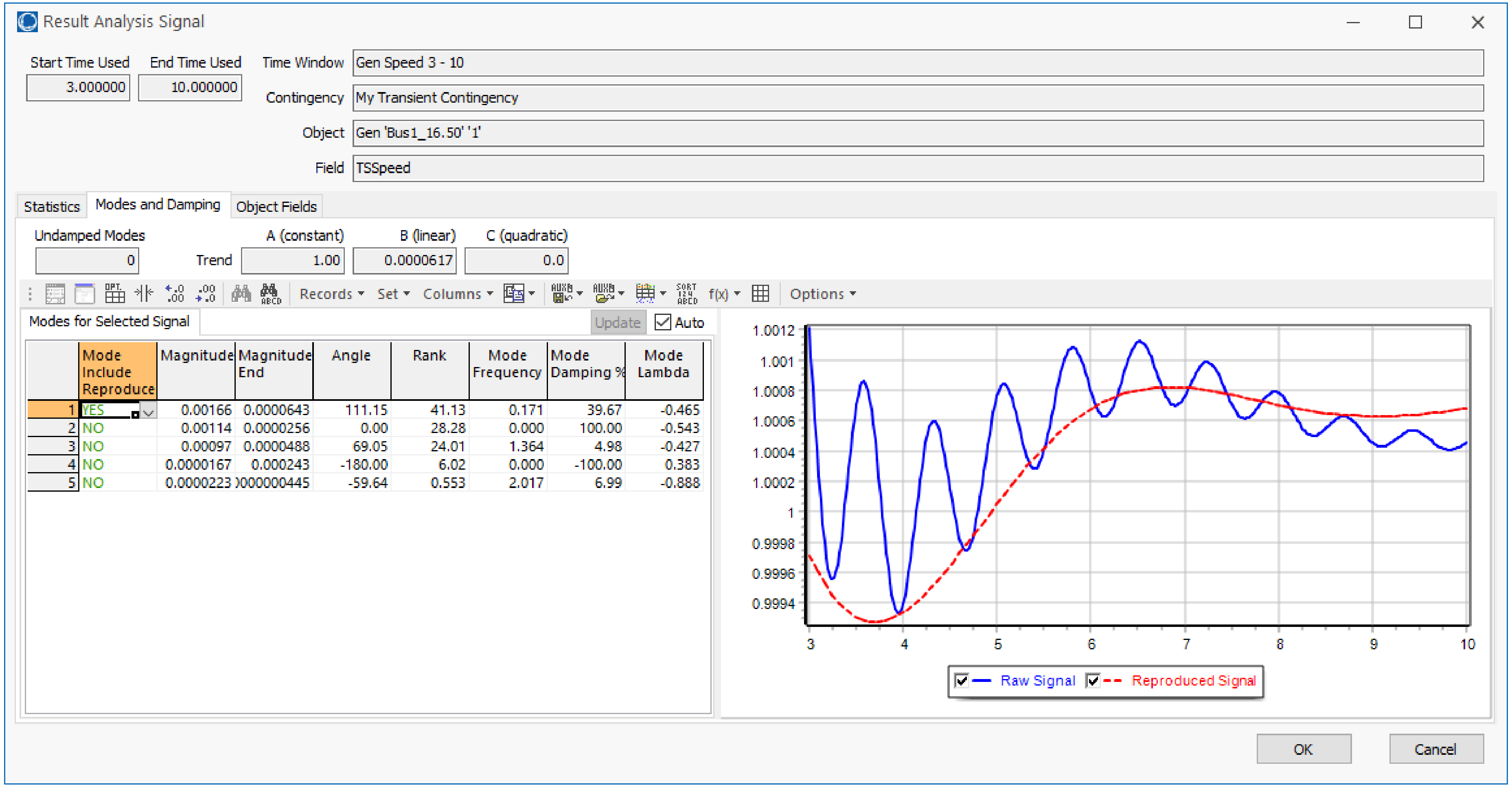
OK | Cancel

Verification: Linear Trend Line Only



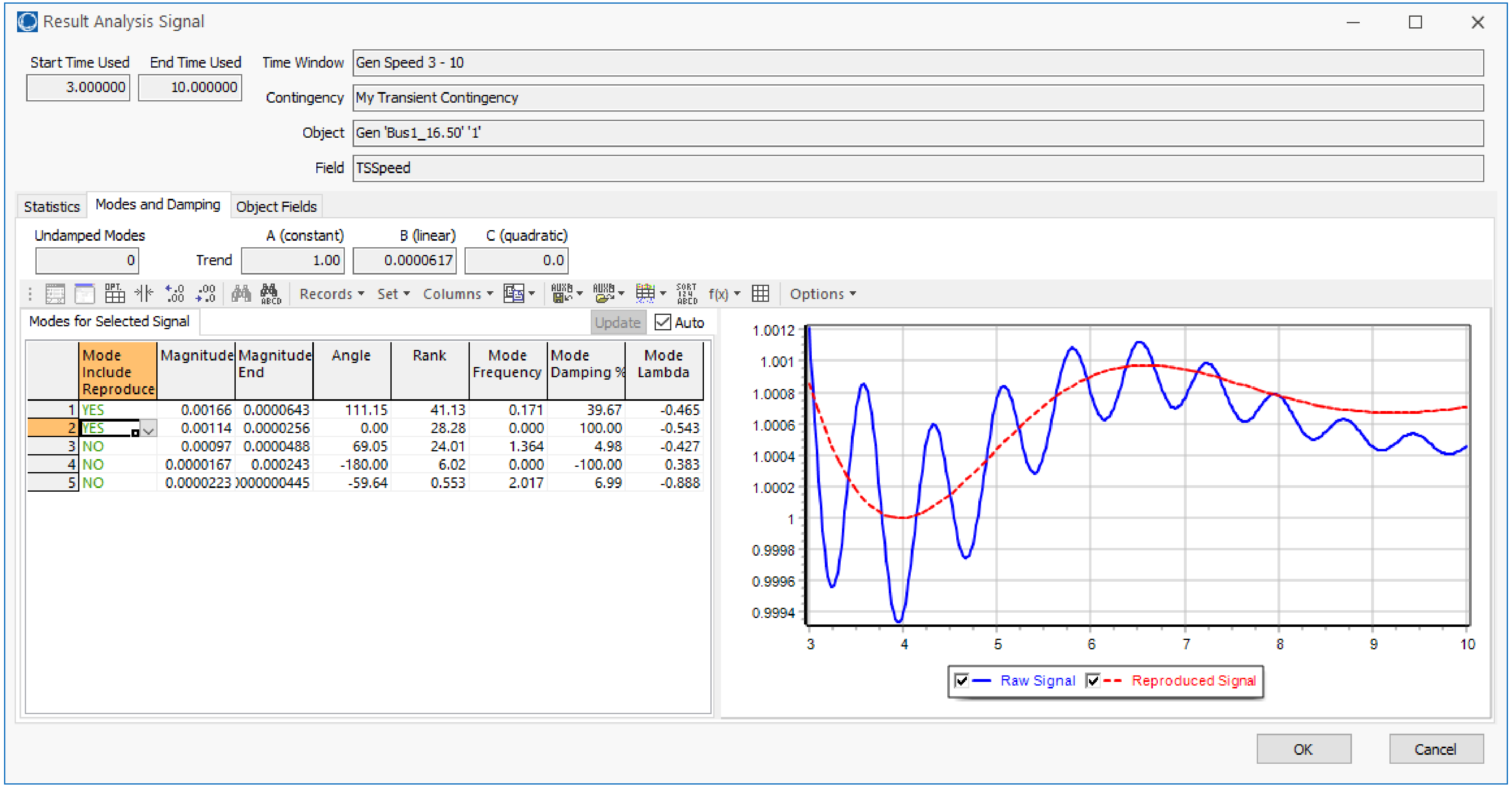
	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	NO	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	NO	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	NO	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line + One Mode



	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	NO	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	NO	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line + Two Modes



	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	NO	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888

Verification: Linear Trend Line + Three Modes

Result Analysis Signal

Start Time Used: 3.000000
End Time Used: 10.000000
Time Window: Gen Speed 3 - 10
Contingency: My Transient Contingency
Object: Gen 'Bus1_16.50' '1'
Field: TSSpeed

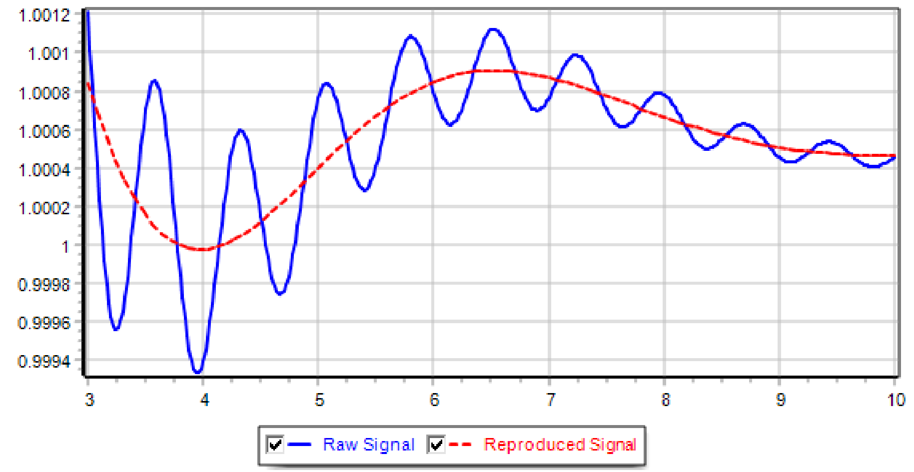
Statistics | Modes and Damping | Object Fields

Undamped Modes: 0
Trend — A (constant): 1.00; B (linear): 0.0000617; C (quadratic): 0.0

Records Set Columns f(x) Options

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	NO	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



OK Cancel

Verification: Linear Trend Line + Four Modes

Result Analysis Signal

Start Time Used	End Time Used
3.000000	10.000000

Time Window: Gen Speed 3 - 10

Contingency: My Transient Contingency

Object: Gen 'Bus1_16.50' '1'

Field: TSSpeed

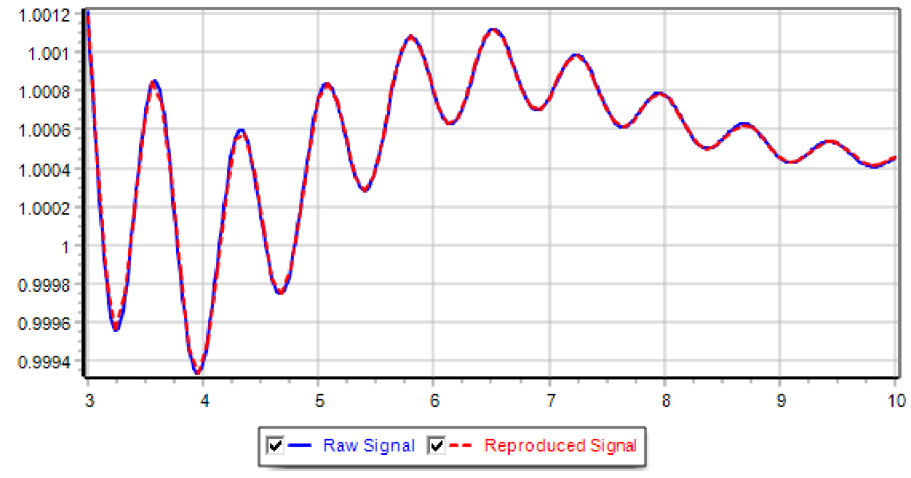
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records Set Columns f(x) Options

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	NO	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



OK Cancel

Verification: Linear Trend Line + Five Modes

Result Analysis Signal

Start Time Used	End Time Used	Time Window	Gen Speed 3 - 10
3.000000	10.000000	Contingency	My Transient Contingency
		Object	Gen 'Bus1_16.50' '1'
		Field	TSSpeed

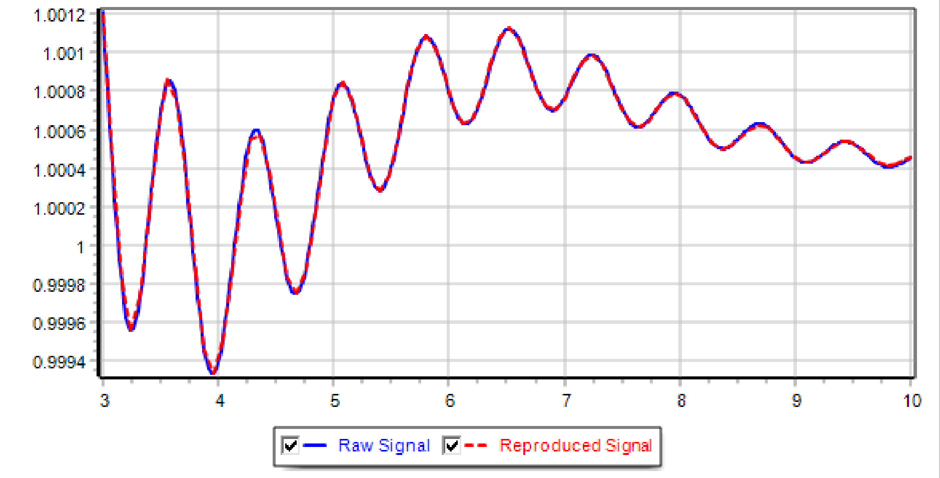
Statistics | Modes and Damping | Object Fields

Undamped Modes		A (constant)	B (linear)	C (quadratic)
0	Trend	1.00	0.0000617	0.0

Records ▾ Set ▾ Columns ▾ f(x) ▾ Options ▾

Modes for Selected Signal — Update — Auto

	Mode Include Reproduce	Magnitude	Magnitude End	Angle	Rank	Mode Frequency	Mode Damping %	Mode Lambda
1	YES	0.00166	0.0000643	111.15	41.13	0.171	39.67	-0.465
2	YES	0.00114	0.0000256	0.00	28.28	0.000	100.00	-0.543
3	YES	0.00097	0.0000488	69.05	24.01	1.364	4.98	-0.427
4	YES	0.0000167	0.000243	-180.00	6.02	0.000	-100.00	0.383
5	YES	0.0000223	)000000445	-59.64	0.553	2.017	6.99	-0.888



OK Cancel

It is hard to tell a difference on this one, illustrating that modes manifest differently in different signals

Measurement-Based Modal Analysis

- There are a number of different approaches
- The idea of all techniques is to approximate a signal, $y_{org}(t)$, by the sum of other, simpler signals (basis functions)
  - Basis functions are usually exponentials, with linear and quadratic functions used to detrend the signal
  - Properties of the original signal can be quantified from basis function properties
    - Examples are frequency and damping
  - Signal is considered over time with t=0 as the start
- Approaches sample the original signal $y_{org}(t)$

Measurement-Based Modal Analysis

- Vector **y** consists of m uniformly sampled points from $y_{org}(t)$ at a sampling value of ΔT, starting with t=0, with values y_j for j=1…m
  - Times are then $t_j = (j-1)\Delta T$
  - At each time point j, the approximation of y_j is

In 460 we won't be dwelling on the equations

$$\hat{y}_j(t_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

where **α** is a vector with the real and imaginary eigenvalue components,

with $\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}$ for α_i corresponding to a real eigenvalue, and

$$\phi_i(t_j, \boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j) \text{ and } \phi_{i+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$$

for a complex eigenvector value

Measurement-Based Modal Analysis

- Error (residual) value at each point j is

$$r_j(t_j,\boldsymbol{\alpha}) = y_j - \hat{y}_j(t_j,\boldsymbol{\alpha})$$

- The closeness of the fit can be quantified using the Euclidean norm of the residuals

$$\frac{1}{2}\sum_{j=1}^{m}(y_j - \hat{y}_j(t_j,\boldsymbol{\alpha}))^2 = \frac{1}{2}\left\|\mathbf{r}(\boldsymbol{\alpha})\right\|_2^2$$

- Hence we need to determine $\boldsymbol{\alpha}$ and **b**

$$\hat{y}_j(\mathrm{t}_j,\boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i\phi_i(t_j,\boldsymbol{\alpha})$$

In 460 we won't be dwelling on the equations; the key here is to understand the concepts

Sampling Rate and Aliasing

- The Nyquist-Shannon sampling theory requires sampling at twice the highest desired frequency
  - For example, to see a 5 Hz frequency we need to sample the signal at a rate of at least 10 Hz
- Sampling shifts the frequency spectrum by 1/T (where T is the sample time), which causes frequency overlap
- This is known as aliasing, which can cause a high frequency signal to appear to be a lower frequency signal
  - Aliasing can be reduced by fast sampling and/or low pass filters

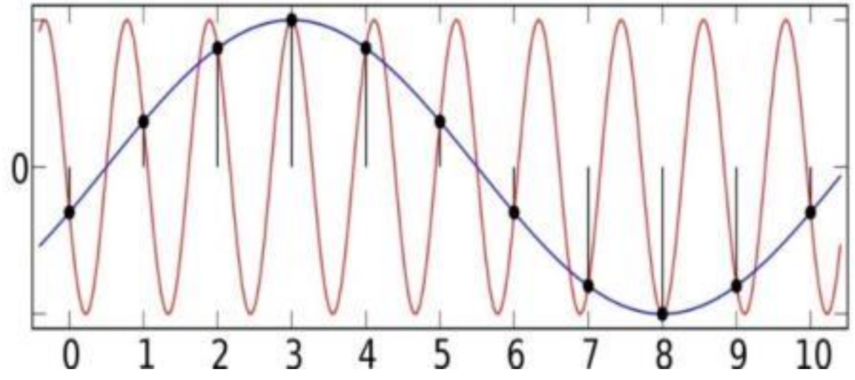


Image: upload.wikimedia.org/wikipedia/commons/thumb/2/28/AliasingSines.svg/2000px-AliasingSines.svg.png

One Solution Approach: The Matrix Pencil Method

- There are several algorithms for finding the modes. We'll use the Matrix Pencil Method
  - This is a newer technique for determining modes from noisy signals (from about 1990, introduced to power system problems in 2005); it is an alternative to the Prony Method
  - The Matrix Pencil Method is useful when there is signal noise
- Given m samples, with L=m/2, the first step is to form the Hankel Matrix, **Y** such that

$$\mathbf{Y} = \begin{bmatrix} y_1 & y_2 & \cdots & y_{L+1} \\ y_2 & y_3 & \cdots & y_{L+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L} & y_{m-L+1} & \cdots & y_m \end{bmatrix}$$

Reference: A. Singh and M. Crow, "The Matrix Pencil for Power System Modal Extraction," IEEE Transactions on Power Systems, vol. 20, no. 1, pp. 501-502, Institute of Electrical and Electronics Engineers (IEEE), Feb 2005.

Algorithm Details, cont.

- Then calculate **Y**'s singular values using an economy singular value decomposition (SVD)

$$\mathbf{Y} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}}$$

- The ratio of each singular value is then compared to the largest singular value σ_c; retain the ones with a ratio > than a threshold
  - This determines the modal order, M
  - Assuming **V** is ordered by singular values (highest to lowest), let $\mathbf{V}_p$ be then matrix with the first M columns of **V**

In 460 you should understand what an SVD is doing since the approach is widely used in many applications

The computational complexity increases with the cube of the number of measurements!

This threshold is a value that can be changed; decrease it to get more modes.

Aside: Matrix Singular Value Decomposition (SVD)

- The SVD is a factorization of a matrix that generalizes the eigendecomposition to any m by n matrix to produce

$$\mathbf{Y} = \mathbf{U\Sigma V}^{\mathbf{T}}$$

The original concept is more than 100 years old, but has found lots of recent applications

where $\mathbf{\Sigma}$ is a diagonal matrix of the singular values

- The singular values are non-negative, real numbers that can be used to indicate the major components of a matrix (the gist is they provide a way to decrease the rank of a matrix)

Aside: SVD Image Compression Example

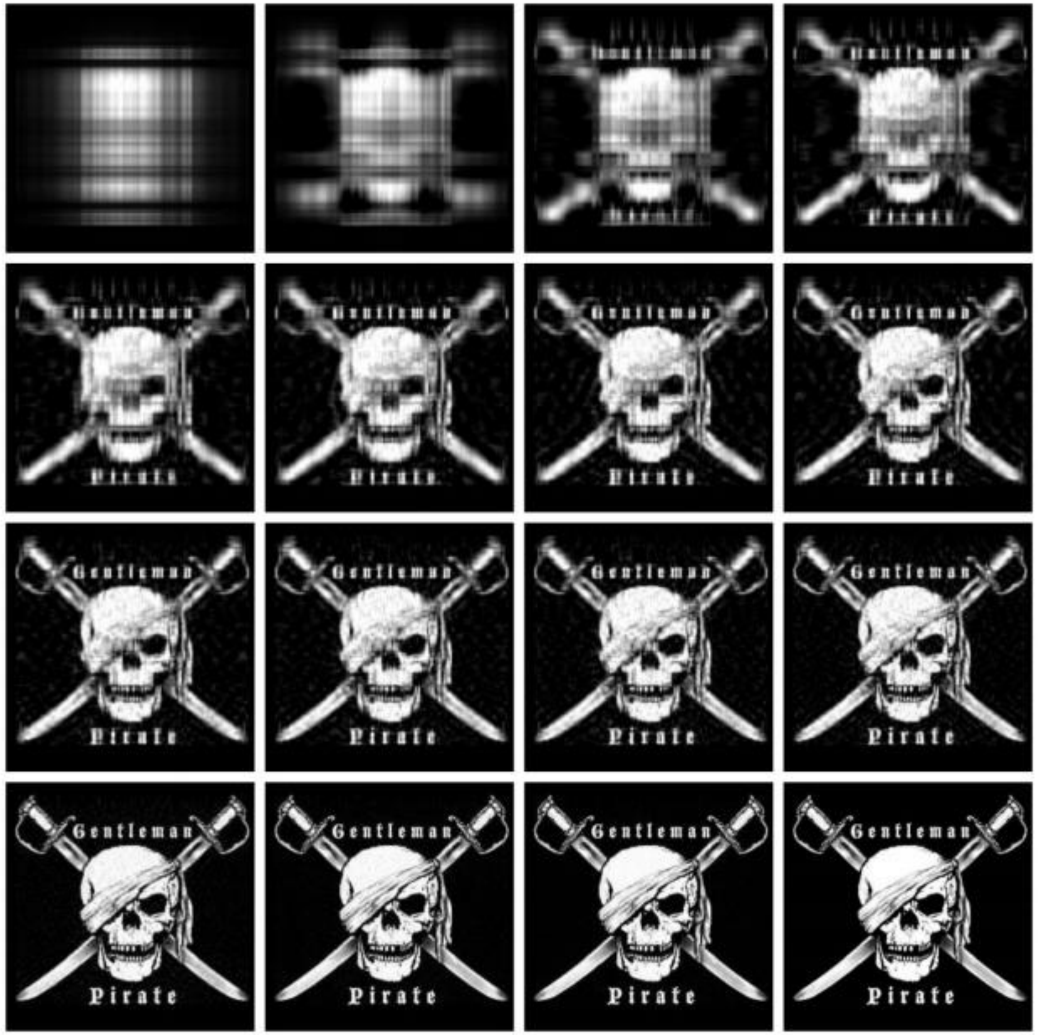


Figure 3.1: Image size 250x236 – modes used {{1,2,4,6},{8,10,12,14},{16,18,20,25},{50,75,100,original image}}

Images can be represented with matrices. When an SVD is applied and only the largest singular values are retained the image is compressed.

SVD is used in many other applications as well, including facial recognition and principal component analysis (PCA)

[Image Source: www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf](http://www.math.utah.edu/~goller/F15_M2270/BradyMathews_SVDImage.pdf)

Matrix Pencil Method with Many Signals

- The Matrix Pencil approach can be used with one signal or with multiple signals
- Multiple signals are handled by forming a $\mathbf{Y}_k$ matrix for each signal k using the measurements for that signal and then combining the matrices

$$\mathbf{Y}_k = \begin{bmatrix} y_{1,k} & y_{2,k} & \cdots & y_{L+1,k} \\ y_{2,k} & y_{3,k} & \cdots & y_{L+2,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{m-L,k} & y_{m-L+1,k} & \cdots & y_{m,k} \end{bmatrix}$$

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_N \end{bmatrix}$$

The required computation scales linearly with the number of signals

Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately we are finding

$$y_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

- The $\boldsymbol{\alpha}$ is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Quickly Determining the b Vectors

- A key insight is from an approach known as the Variable Projection Method (from Borden, 2013) that for any signal k

$$\mathbf{y}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k$$

And then the residual is minimized by selecting $\mathbf{b}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}_k$

where $\boldsymbol{\Phi}(\boldsymbol{\alpha})$ is the m by n matrix with values

$\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}$ if α_{i} corresponds to a real eigenvalue,

and $\Phi_{ji}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\cos(\alpha_{i+1} t_j)$ and $\Phi_{ji+1}(\boldsymbol{\alpha}) = e^{\alpha_i t_j}\sin(\alpha_{i+1} t_j)$

for a complex eigenvalue; $t_j = \left(j-1\right)\Delta T$

Finally, $\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

Where m is the number of measurements and n is the number of modes

A. Borden, B.C. Lesieutre, J. Gronquist, "Power System Modal Analysis Tool Developed for Industry Use," *Proc. 2013 North American Power Symposium*, Manhattan, KS, Sept. 2013

Matrix Pencil Method with Many Signals

- However, when dealing with many signals, usually the signals are somewhat correlated, so vary few of the signals are actually need to be included to determine the desired modes
- Ultimately, we are finding

$$y_j(\mathrm{t}_j, \boldsymbol{\alpha}) = \sum_{i=1}^{n} b_i \phi_i(t_j, \boldsymbol{\alpha})$$

- The $\boldsymbol{\alpha}$ is common to all the signals (i.e., the system modes) while the **b** vector is signal specific (i.e., how the modes manifest in that signal)

Aside: Pseudoinverse of a Matrix

- The pseudoinverse of a matrix generalizes concept of a matrix inverse to an m by n matrix, in which m >= n
  - Specifically this is a Moore-Penrose Matrix Inverse
- Notation for the pseudoinverse of **A** is $\mathbf{A}^+$
- Satisfies $\mathbf{A}\mathbf{A}^+\mathbf{A} = \mathbf{A}$
- If **A** is a square matrix, then $\mathbf{A}^+ = \mathbf{A}^{-1}$
- Quite useful for solving the least squares problem since the least squares solution of $\mathbf{A}\mathbf{x} = \mathbf{b}$ is $\mathbf{x} = \mathbf{A}^+\mathbf{b}$
- Can be calculated using an SVD

$$\mathbf{A} = \mathbf{U}\,\boldsymbol{\Sigma}\,\mathbf{V}^T$$

$$\mathbf{A}^+ = \mathbf{V}\,\boldsymbol{\Sigma}^+\,\mathbf{U}^T$$

Least Squares Matrix Pseudoinverse Example

- Assume we wish to fix a line (mx + b = y) to three data points: (1,1), (2,4), (6,4)
- Two unknowns, m and b; hence **x** = [m b]^T
- Setup in form of **Ax** = **b**

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} \quad \text{so } \mathbf{A} = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 6 & 1 \end{bmatrix}$$

Least Squares Matrix Pseudoinverse Example, cont.

- Doing an economy SVD

$$\mathbf{A} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T = \begin{bmatrix} -0.182 & -0.765 \\ -0.331 & -0.543 \\ -0.926 & 0.345 \end{bmatrix} \begin{bmatrix} 6.559 & 0 \\ 0 & 0.988 \end{bmatrix} \begin{bmatrix} -0.976 & -0.219 \\ 0.219 & -0.976 \end{bmatrix}$$

- Computing the pseudoinverse

$$\mathbf{A}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.976 & 0.219 \\ -0.219 & -0.976 \end{bmatrix} \begin{bmatrix} 0.152 & 0 \\ 0 & 1.012 \end{bmatrix} \begin{bmatrix} -0.182 & -0.331 & -0.926 \\ -0.765 & -0.543 & 0.345 \end{bmatrix}$$

$$\mathbf{A}^+ = \mathbf{V}\,\mathbf{\Sigma}^+\mathbf{U}^T = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix}$$

In an economy SVD the $\mathbf{\Sigma}$ matrix has dimensions of m by m if m < n or n by n if n < m

Least Squares Matrix Pseudoinverse Example, cont.

- Computing $\mathbf{x} = [m\ b]^T$ gives

$$\mathbf{A}^{+}\mathbf{b} = \begin{bmatrix} -0.143 & -0.071 & 0.214 \\ 0.762 & 0.548 & -0.310 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 0.429 \\ 1.71 \end{bmatrix}$$

- With the pseudoinverse approach we immediately see the sensitivity of the elements of **x** to the elements of **b**
  - New values of m and b can be readily calculated if **y** changes
- Computationally the SVD is order mn^2+n^3 (with $n < m$)
  - In this example it means it scales linearly with the number of points; matrices with $m >> n$ are common

Computational Considerations

- When there is just one signal, the procedure scales with the cube of the number of measurements
  - This value is usually relatively small, say 20 seconds of data sampled at 10 Hz for 200 measurements
- If multiple signals are included, it scales linearly with the number of signals
- However, a key insight is once $\boldsymbol{\alpha}$ has been determined, each $\mathbf{b}_k$ can be determined with a matrix multiply of a matrix with dimensions of the number of modes and number of measurements

$$\mathbf{y}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})\mathbf{b}_k \rightarrow \mathbf{b}_k = \boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}\mathbf{y}_k$$

$\boldsymbol{\Phi}(\boldsymbol{\alpha})^{+}$ is the pseudoinverse of $\boldsymbol{\Phi}(\boldsymbol{\alpha})$

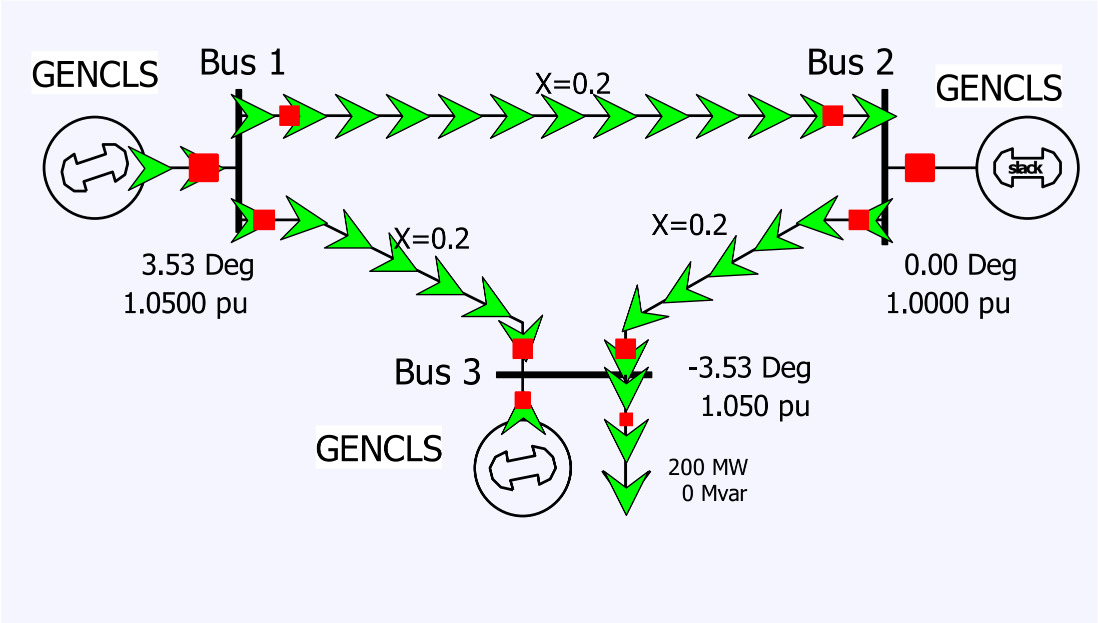
We can quickly determine how well $\boldsymbol{\alpha}$ matches each signal

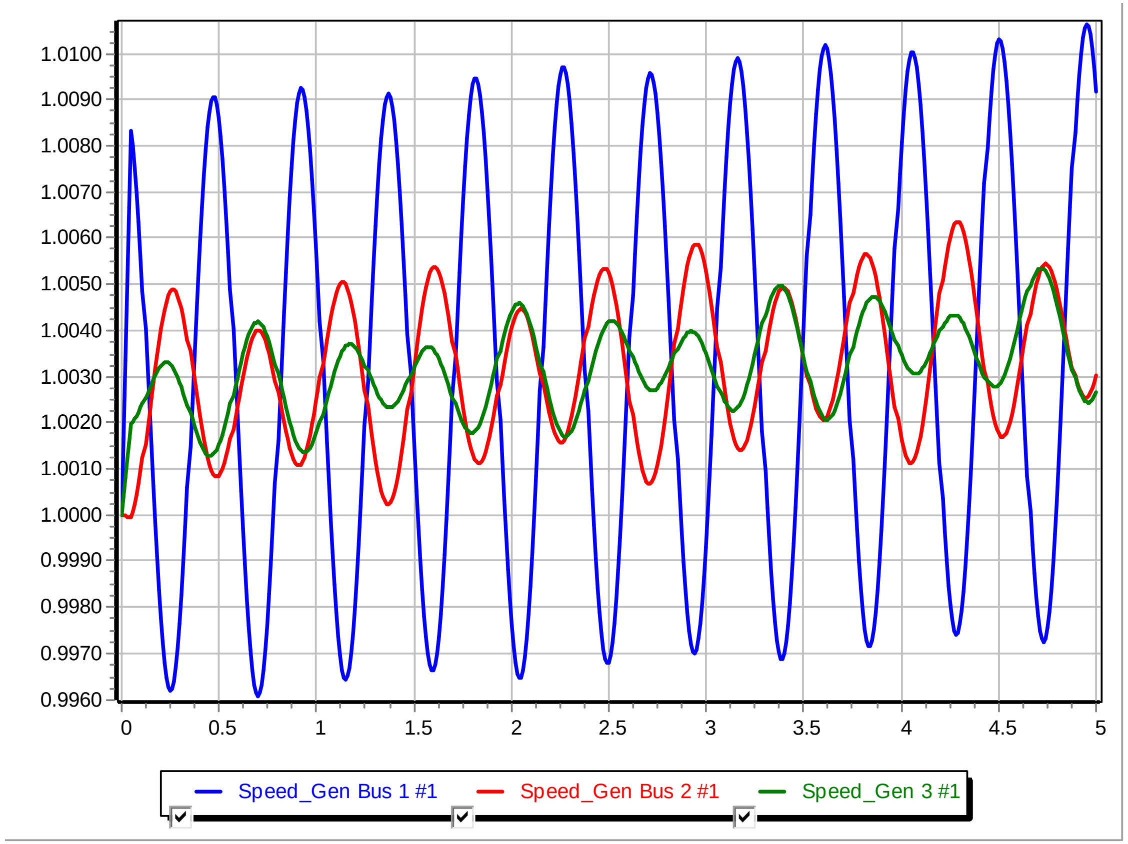
Modal Analysis in PowerWorld

- Goal is to make modal analysis easy to use, and easy to visualize the results
- Provided tool can be used with either transient stability results or actual system signals (e.g., from PMUs)
- Three ways to access in PowerWorld
  - From the Modal Analysis button (in **Add-Ons**)
  - On the Transient Stability Analysis form left menu, **Modal Analysis** (right below SMIB Eigenvalues)
  - By right-clicking on a transient stability or plot case information display, and selecting **Modal Analysis Selected Columns** or **Modal Analysis All Columns**

Modal Analysis: Three Generator Example

- A short fault at t=0 gets the below three generator case oscillating with multiple modes (mostly clearly visible for the red and the green curve)



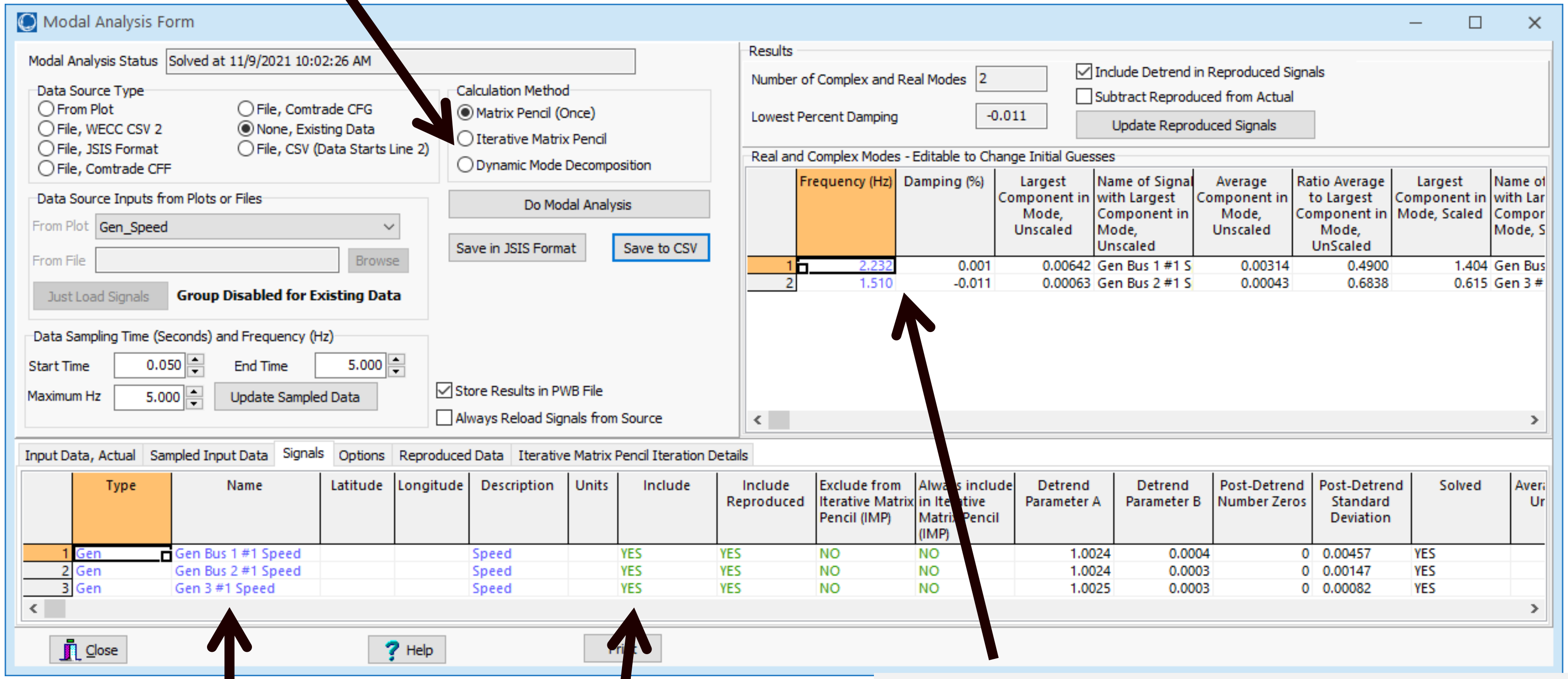


Modal Analysis: Three Generator Example

- Open the case **B3_CLS_UnDamped**
  - This system has three classical generators without damping; the default event is a self clearing fault at bus 1
- Run the transient stability for 5 seconds
- To do modal analysis, on the Transient Stability page select Results from RAM, view just the generator speed fields, right-click and select **Modal Analysis All Columns**
  - This display the Modal Analysis Form

Modal Analysis Form

First click on **Do Modal Analysis** to run the modal analysis



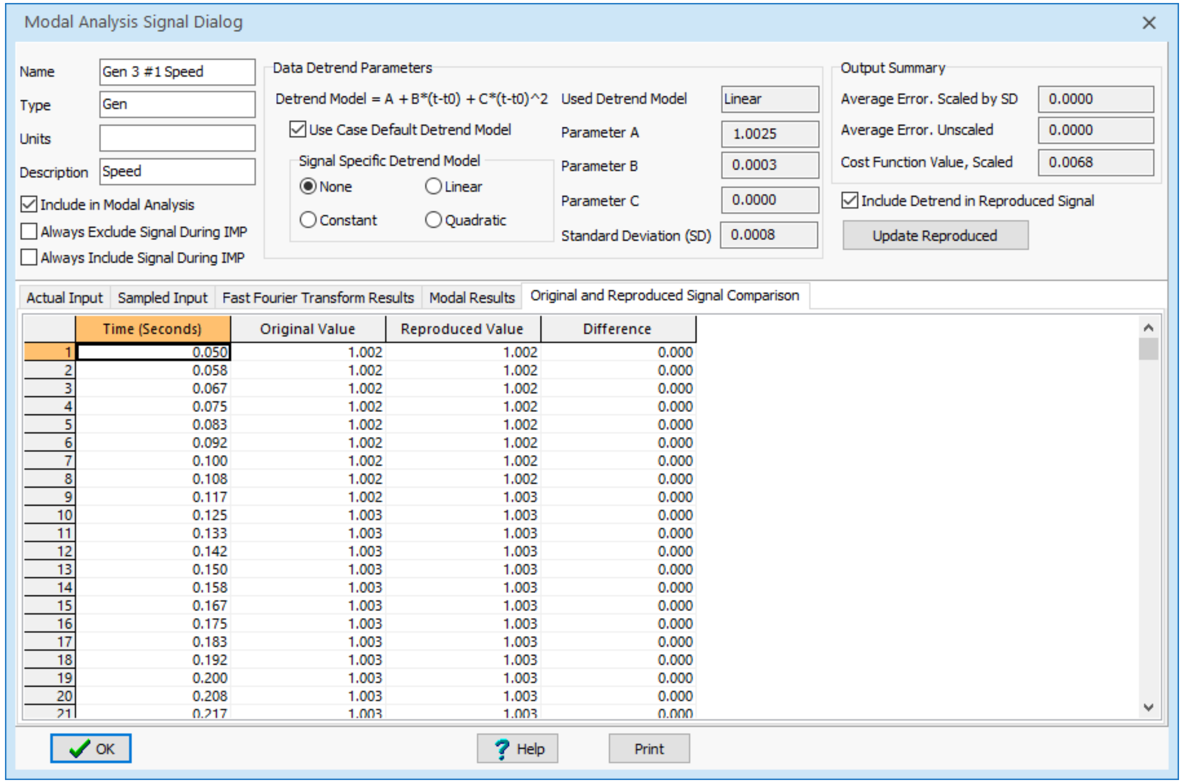
Right-click on signal to view its dialog

Signals to include

Key results are shown in the upper-right of the form. There are two main modes, one at 2.23Hz and one at 1.51; both have very little damping.

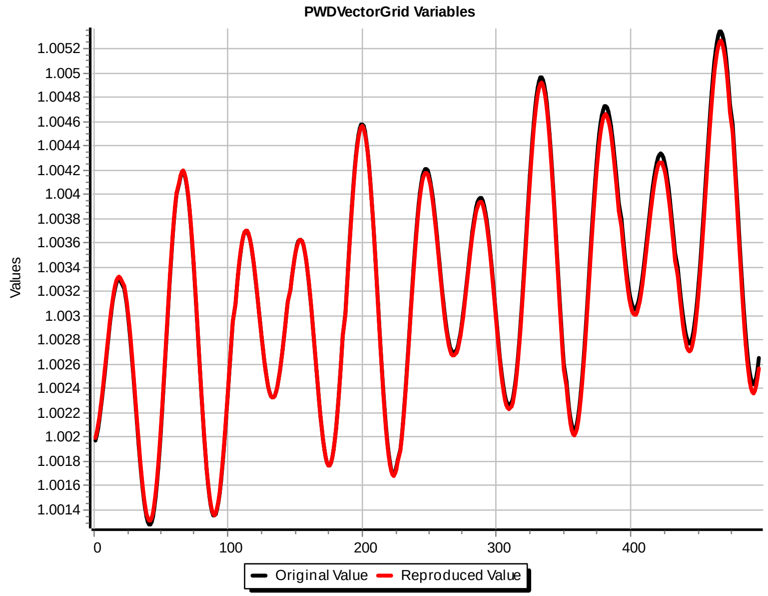
Three Generator Example: Signal Dialog

- The **Signal Dialog** provides details about each signal, including its modal components and a comparison between the original and reproduced signals (example for gen 3)



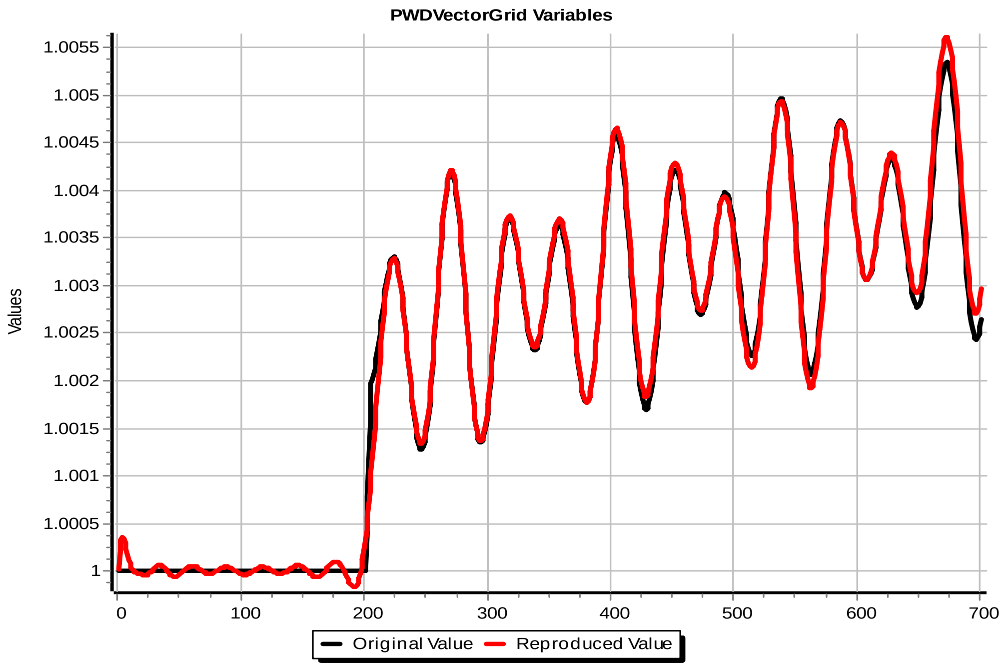
	Time (Seconds)	Original Value	Reproduced Value	Difference
1	0.050	1.002	1.002	0.000
2	0.058	1.002	1.002	0.000
3	0.067	1.002	1.002	0.000
4	0.075	1.002	1.002	0.000
5	0.083	1.002	1.002	0.000
6	0.092	1.002	1.002	0.000
7	0.100	1.002	1.002	0.000
8	0.108	1.002	1.002	0.000
9	0.117	1.002	1.003	0.000
10	0.125	1.003	1.003	0.000
11	0.133	1.003	1.003	0.000
12	0.142	1.003	1.003	0.000
13	0.150	1.003	1.003	0.000
14	0.158	1.003	1.003	0.000
15	0.167	1.003	1.003	0.000
16	0.175	1.003	1.003	0.000
17	0.183	1.003	1.003	0.000
18	0.192	1.003	1.003	0.000
19	0.200	1.003	1.003	0.000
20	0.208	1.003	1.003	0.000
21	0.217	1.003	1.003	0.000

Plotting the original and reproduced signals shows a near exact match



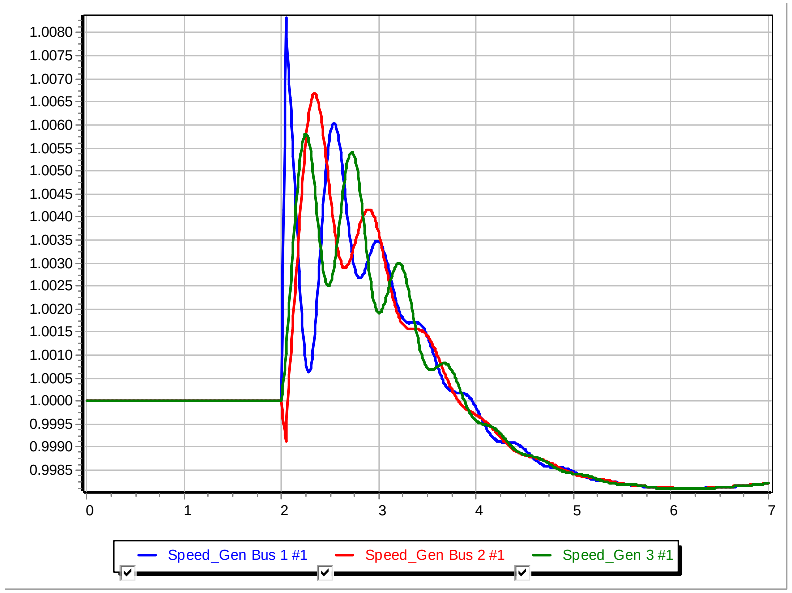
Caution: Setting Time Range Incorrectly Can Result in Unexpected Results!

- Assume the system is run with no disturbance for two seconds, and then the fault is applied and the system is run for a total of seven seconds (five seconds post-fault)
  - The incorrect approach would be to try to match the entire signal; rather just match from after the fault
  - Trying to match the full signal between 0 and 7 seconds required eleven modes!
  - By default the **Modal Analysis Form** sets thedefault start time to immediately after the last event



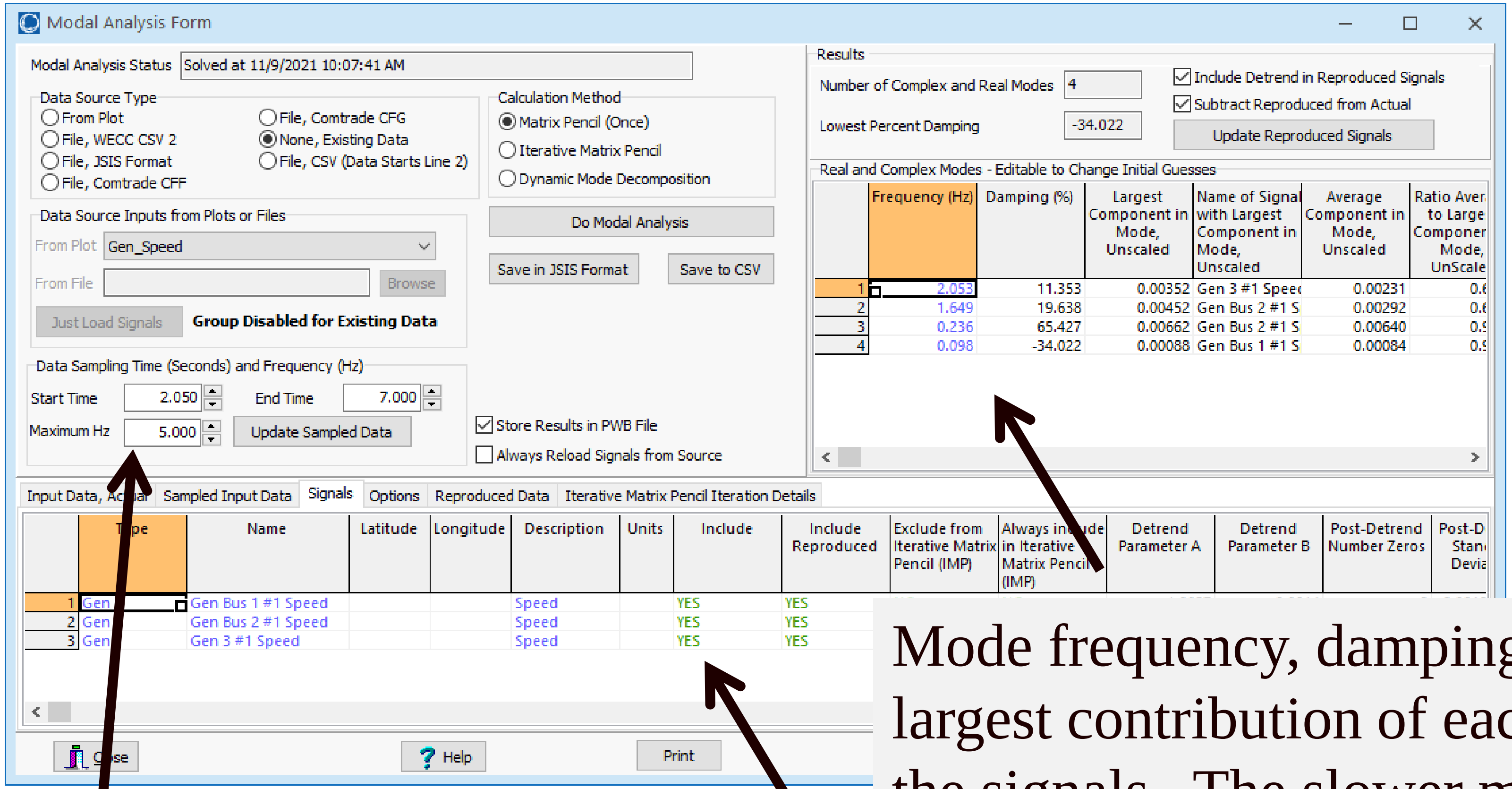
GENRUO Example with Damping

- Open the case **B3_GENROU**, which changes the GENCLS to GENROU models, adding damping
  - Also each has an EXST1 exciter and a TGOV1 governor
  - The simulation runs for seven seconds, with the fault occurring at two seconds; modal analysis is done from the time the fault is cleared until the end of the simulation.



The image shows the generator speeds. The initial rise in the speed is caused by the load dropping during the fault, causing a power mismatch; this is corrected by the governors. Note the system now has damping; modal analysis tells us how much.

GENROU Example with Damping

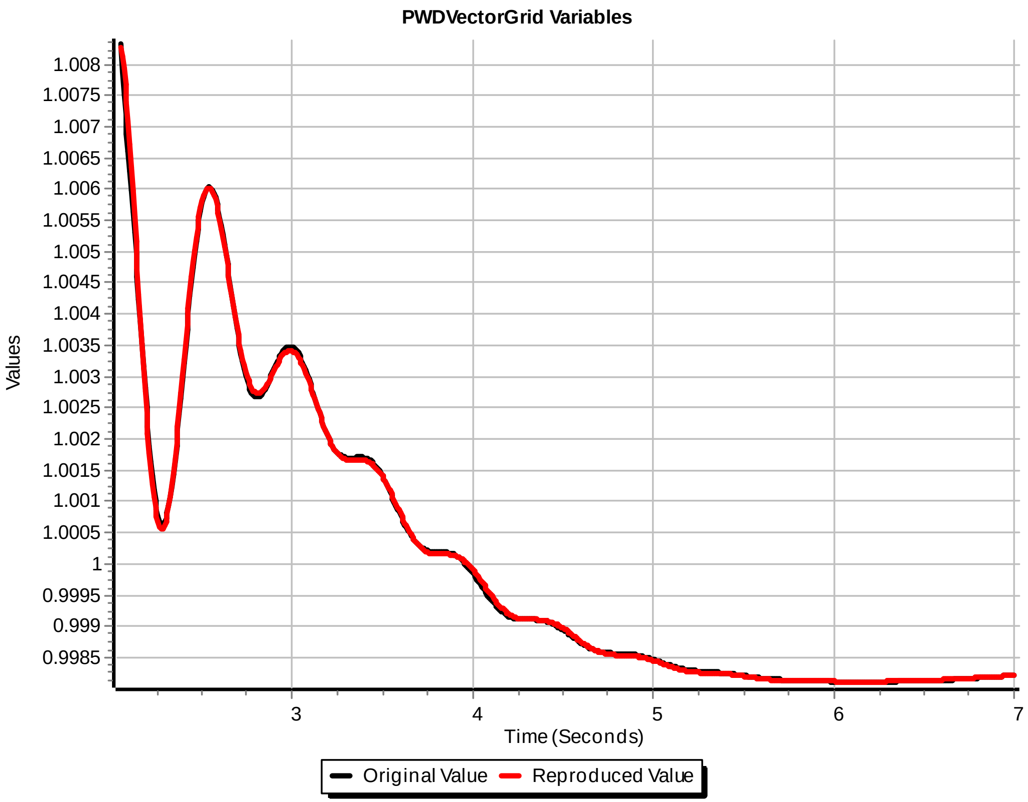


Mode frequency, damping, and largest contribution of each mode in the signals. The slower mode is associated with the governors.

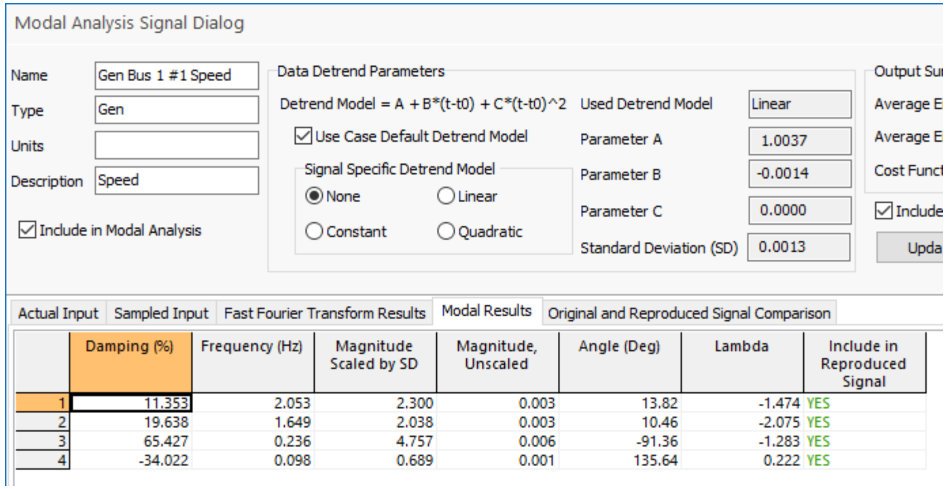
Start time default value

GENROU Example with Damping

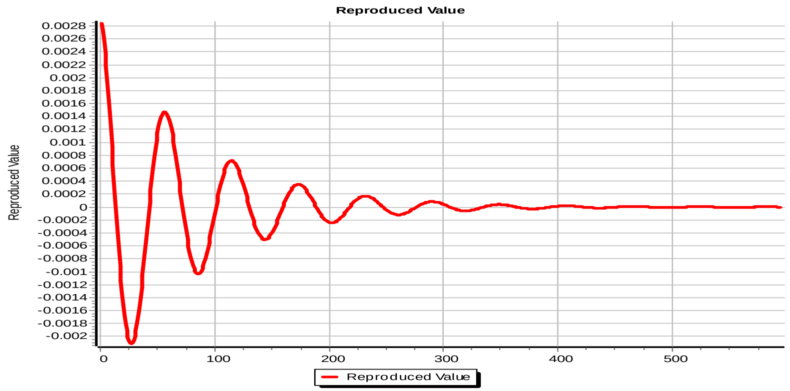
- Left image show how well the speed for generator 1 is approximated by the modes



More signal details



	Damping (%)	Frequency (Hz)	Magnitude Scaled by SD	Magnitude, Unscaled	Angle (Deg)	Lambda	Include in Reproduced Signal
1	11.353	2.053	2.300	0.003	13.82	-1.474	YES
2	19.638	1.649	2.038	0.003	10.46	-2.075	YES
3	65.427	0.236	4.757	0.006	-91.36	-1.283	YES
4	-34.022	0.098	0.689	0.001	135.64	0.222	YES



Just the 2.05 Hz mode