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MASTER'S REPORT.

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An Energy Based Security Measure for Assessing Vulnerability to Voltage Collapse

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1. Background and Motivation

Voltage collapse in electrical power systems is a phenomenon characterized by declining or 'sagging' voltages throughout a large portion of the system. Conceptually, a main cause of voltage collapse can be interpreted as loss of voltage controllability due to a lack of reactive compensation. This normally results from some type of contingency on an otherwise heavily loaded system. Voltage collapse may occur either in a small subsection of a power system, or throughout the entire system (such as in the French system in December of 1978 [1]). However, care should be taken to distinguish between the relatively minor situation of having a few buses in a system close to their regulatorily defined low voltage limits, and the situation of voltage collapse. A voltage collapse scenario is characterized by the power system approaching its limit of voltage stability, beyond which no solution to the powerflow equations is possible. A typical voltage collapse situation would be a heavily loaded system which experiences a contingency, such as loss of a large generator. As the output of outaged generator is transferred to other generators, the voltage throughout a large portion of the system declines. Although the LTC transformers may maintain constant customer voltage, the sagging transmission system voltage results in an increase in system real and reactive losses, along with a decrease in capacitive voltage support, further aggravating the problem. Nearby generators reach their var limits and eventually have to be taken off-line. Therefore, in the span of a short time an otherwise stable system is rapidly moving towards its point of voltage instability. Since voltage collapse can occur quite abruptly, some method of identifying how close a system is to voltage collapse is needed, along with a method of optimally moving the system to a more secure operating point. This report addresses the first area.

A large portion of the recent literature devoted to voltage collapse has had as its goal developing a security measure to quantify how "close" a particular operating point is to voltage collapse. The crucial point in judging the effectiveness of a measure of determining proximity to voltage collapse is whether or not it provides planners and operators with an indication of when corrective control actions are necessary. The goal of this report is to evaluate an energy based measure of proximity to voltage collapse which is both physically reasonable and provides information not captured by methods which only look at a linearization of powerflow or system dynamics about a single operating point. The energy based method will be examined by first looking at a single line example in both a static setting and in a dynamic setting. Then, the energy function will be applied to more general systems in order to show how it can be used to provide a measure of proximity to voltage collapse. Lastly, the solution of the powerflow equations at "low voltage" operating points will be examined. This data is needed by the energy function method to determine proximity to voltage collapse.

2. Static Approach to Voltage Collapse

The energy based method can be motivated by examining the static powerflow in a single line example. For example, consider a system with a single lossless line connecting two buses, number 1 and 2. Bus 1 is treated as the slack bus with its voltage magnitude fixed at 1.0 pu. Since the line is lossless, the real power injection at bus 1 must equal the real power consumed at bus 2. We will assume the load at bus 2 is represented as a constant P-Q demand. The following analysis can easily be extended into the case of P and Q specified as functions of bus voltage. The resulting power balance equations at bus 2 are:

$$P_L - B_{12}V\sin(\alpha) = 0$$

$$Q_L - B_{22}V^2 - B_{12}V\cos(\alpha) = 0$$

where

V := bus voltage magnitude at bus 2

$\alpha := \delta_1 - \delta_2$ = phase angle voltage difference from bus 1 to bus2.

For $B_{12} = -B_{22} = 10.0$, the locus of the points in the α-V space satisfying these two constraints for a range of P and Q values is shown in figure 2-1. Alternatively, the same locus of solution points could be plotted as in figure 2-2. In this case the solution is plotted in the V-P space for varying values of load power factor. A radial line with a fixed sending voltage typically has two solutions; this is due simply to the quadratic nature of the reactive power constraint. This is represented in figure 2-1 by the two intersections of the P and Q constraint curves, and in figure 2-2 by the dual voltage solutions for any given P value and power factor. These solution values will be referred to throughout this report as the "high voltage solution" and the "low voltage solution". The two will be distinguished by their relative values of voltage magnitude. Section 5 examines the pattern of high and low voltage solutions for more complex networks. It should be noted that a standard Newton-Raphson powerflow was used in the solution of all the examples contained in this report. As shown in the figures, for certain critical values of P and Q the constraints have only one solution. If either P or Q is increased further, the powerflow has no solution. At this bifurcation point (i.e., the point where the two solutions coalesce into one), the Jacobian of the two power balance equations is singular. This observation has been used by some authors as a method of predicting proximity to voltage collapse. In particular, [2] recommends the use of the smallest singular value of the Jacobian of the powerflow equations, evaluated at the normal operating point of the system. This corresponds to the high voltage operating point in figures 2-1 and 2-2.

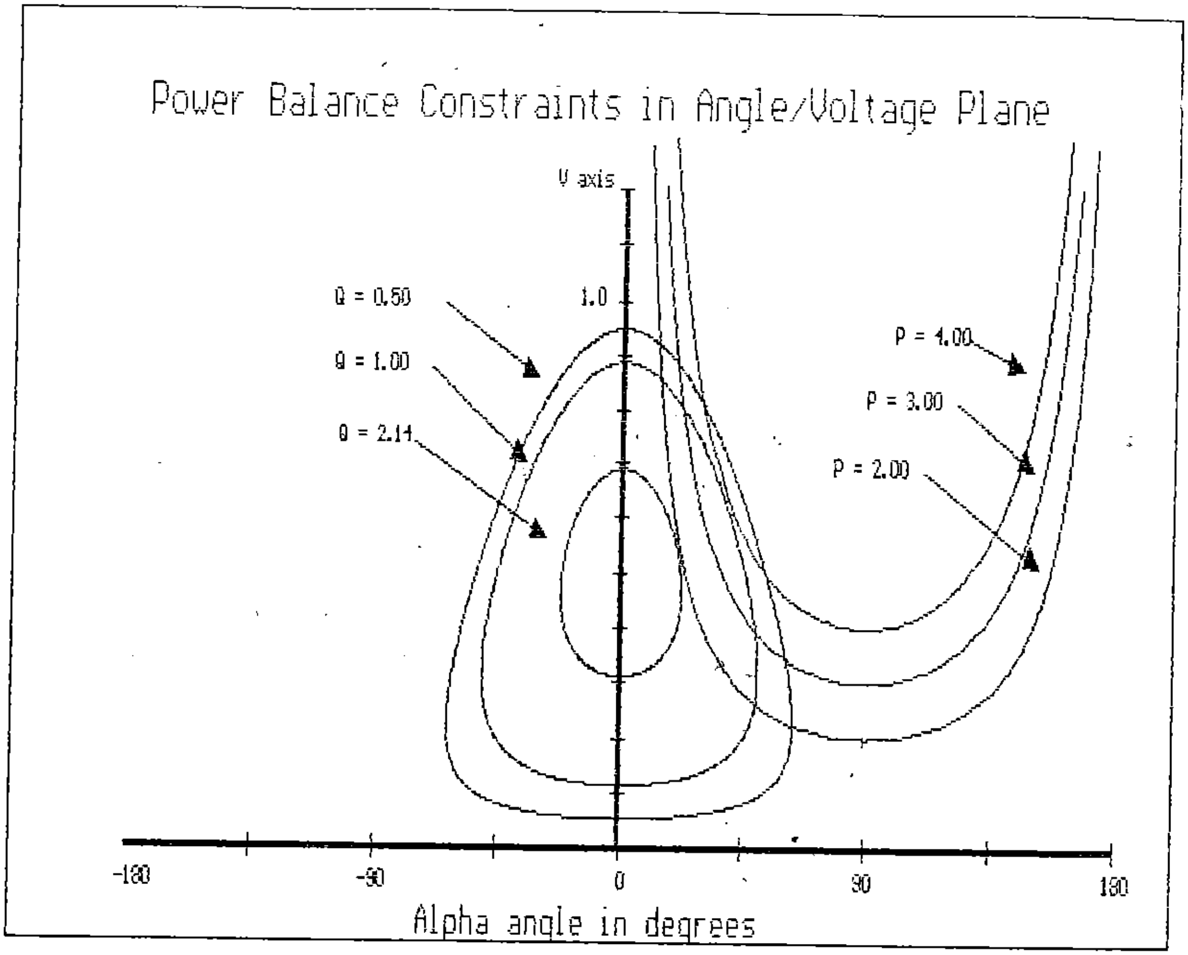

Figure 2-1

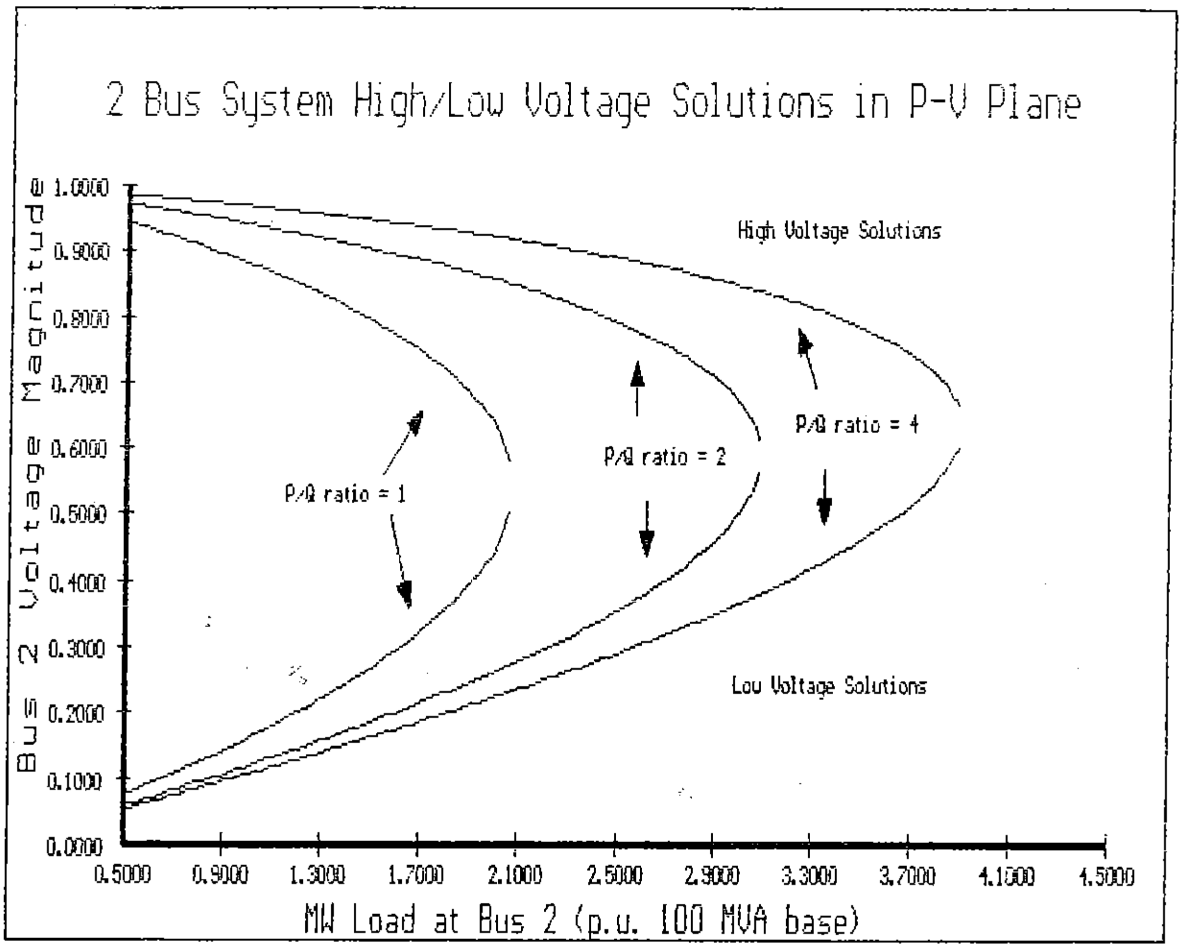

Figure 2-2

3. Dynamic Approach to Voltage Collapse with Lyapunov Energy Functions

Although a large contingency may cause a system to be in an operating state susceptible to voltage collapse, it is most often the aggregate of small, random events (such as the gradual increase in load) which actually push the system to the brink of voltage collapse. Because of this, it appears that the voltage collapse phenomenon can not be treated as strictly a static problem. Load dynamics and voltage control dynamics need to be considered. In addition, the small random variations that are typically present in customer load demand may also prove important. These small changes may be insignificant at normal operating points. However, near the point of voltage collapse where the voltage is very sensitive to changes in load, they may be able to push the system "over the edge".

As an aid to analyzing the system dynamically, a Lyapunov or "energy" function[1] is constructed [3]. For the simple two bus case from section 2, where the voltage at the generator is assumed to be fixed, the proposed Lyapunov function is given by:

$$v(w,\alpha,V) := \tfrac{1}{2}M_g w^2 - B_{12}V\cos(\alpha) + B_{12}V^{\circ}\cos(\alpha)$$

$$- \tfrac{1}{2}B_{22}V^2 + \tfrac{1}{2}B_{12}(V^{\circ})^2 - P_L\,(\alpha-\alpha^{\circ}) + Q_L\ln(V/V^{\circ})$$

The generalization of this Lyapunov function to a multimachine case is found in [4].

A Lyapunov function has the property that along any trajectory originating from a point in the region of attraction of a stable equilibrium , the "energy" of the system decays asymptotically until the equilibrium point is reached. A traditional analogy is to the dynamics of a rolling ball contained in a valley. When the ball is displaced from its equilibrium point at the bottom of the valley, one expects that it will eventually return to that point with its total energy (i.e. kinetic and potential) decreasing asympotically to zero. This is true unless the ball is given a strong enough 'kick' so that it is able to escape out over a "pass" bounding the valley. Therefore one way to determine whether the ball will return or escape is to compare its energy with the potential energy value associated with the lowest "pass" (or more formally, saddlepoint) bounding the valley. If the ball's energy is below that value, then the ball is guaranteed to return to its equilibrium position. The lowest saddle point bounding the valley is determined by the lowest energy unstable equilibrium point on the boundary. If the boundary of the valley is not uniform in height then it is possible that the ball may return to its equilibrium point even if it has sufficient energy to clear the lowest saddle; this may occur if its initial trajectory does not take it in the neighborhood of the lowest saddle point.

Figure 3-1 is a plot of the energy in a small system of two generators (modeled as constant voltage behind transient impedance) , with load at each, connected by a single lossless transmission line. As time t=0 load is suddenly shifted between the 2 buses. Note the energy's asymptotic decay to zero, with energy nonincreasing as a function of time. Figure 3-2 shows that as resistance is introduced

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[1]The terminology Lyapunov function will be used to refer to a scalar function of state that has the properties of being positive semi-definite about a stable equilibrium and nonincreasing along trajectories. For system models in which the second property does not hold, the scalar function of state will be referred to as an energy function.

into the system, the system energy is no longer nonincreasing as it decays asymptotically to zero, and thus the energy function for the system is no longer strictly a Lyapunov function. In power systems, where the R/X ratio on lines is normally substantially below 1, an assumption of lossless lines in the derivation of the energy function appears to be justifiable. For a discussion of the effects of transmission losses and the existence of Lyapunov functions for the lossy case, the reader is referred to [5] and [6].

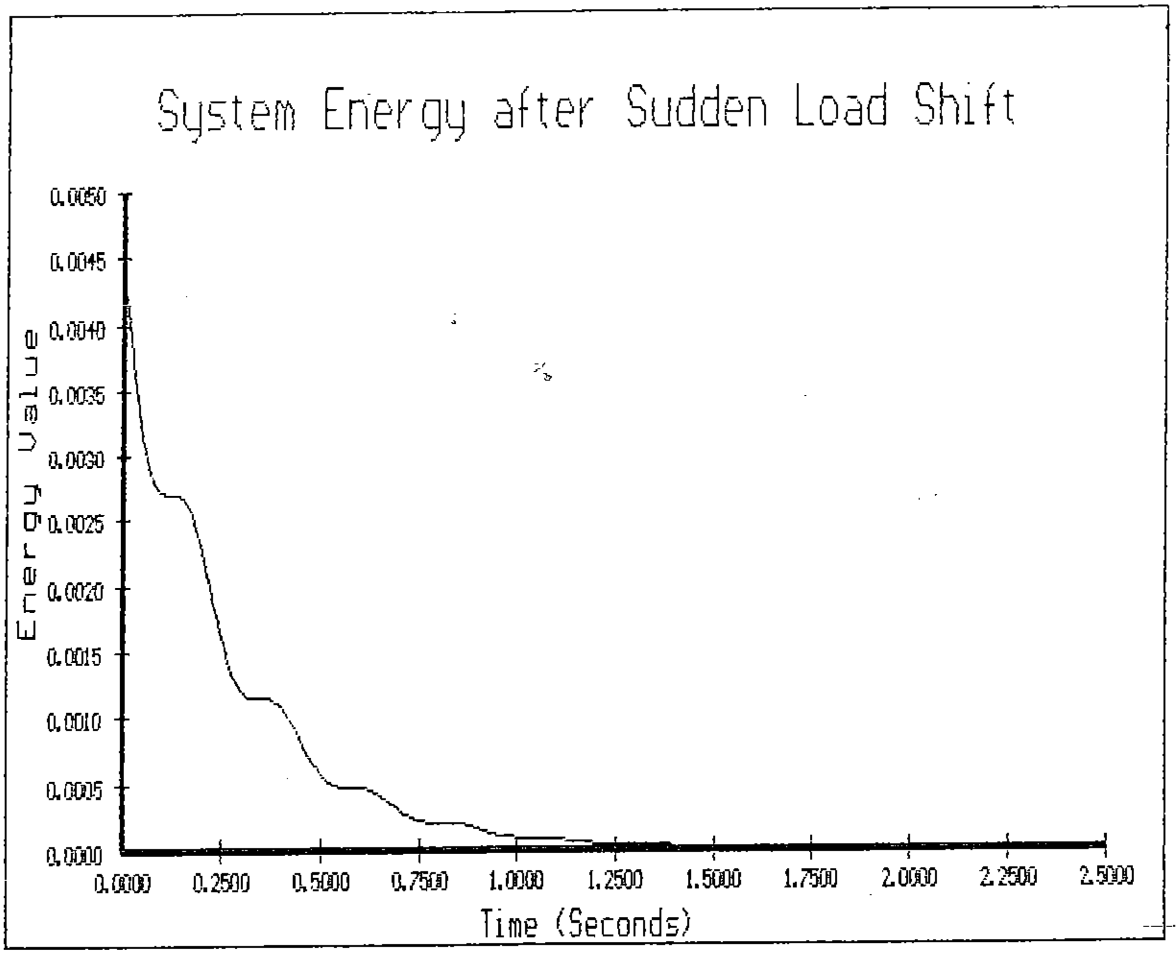


Figure 3-1

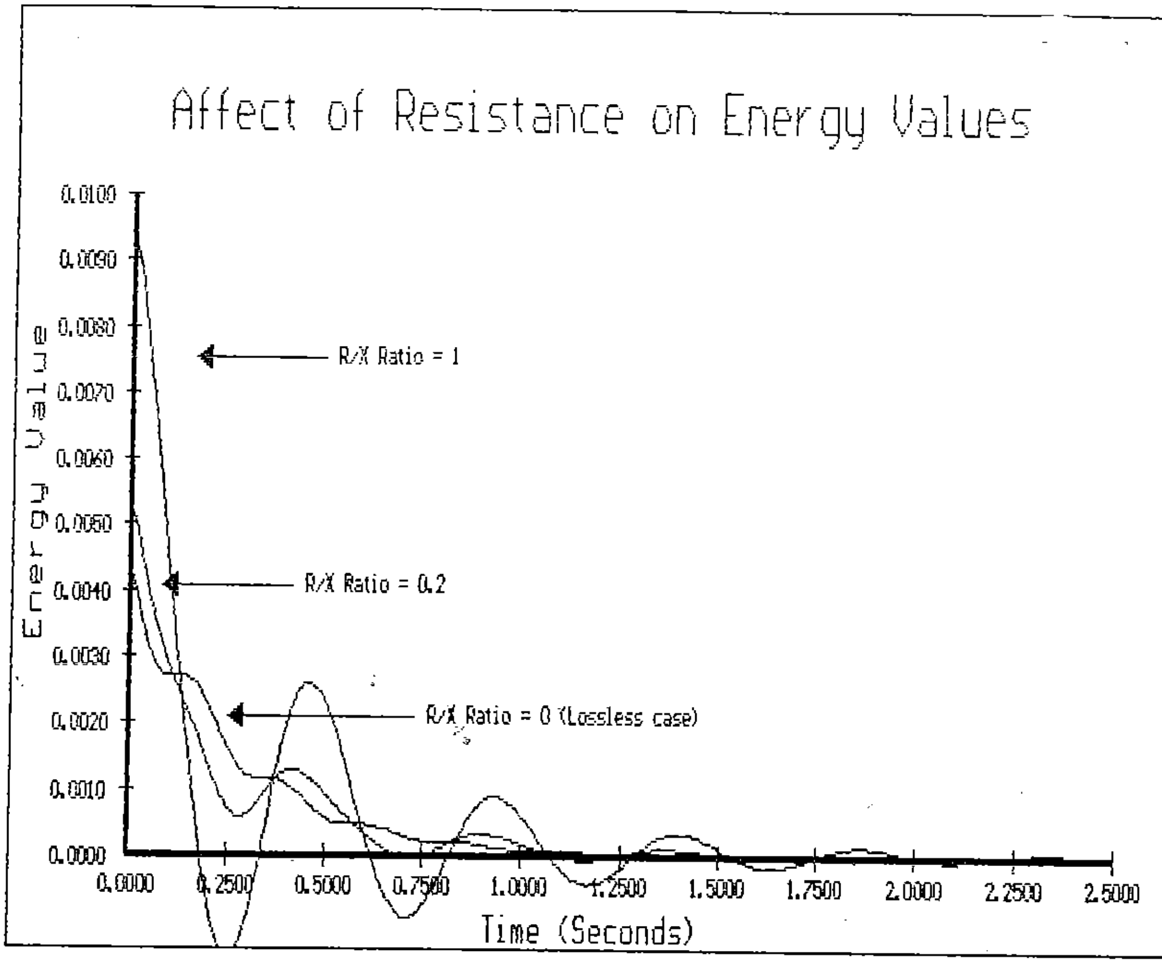


Figure 3-2

In power system models which include load bus voltage variations, the low voltage solutions correspond to the nearest unstable equilibrium points. Therefore the method proposed in this report of determining how close a given system is to voltage collapse is to measure the difference in energy between the high voltage solution and the low voltage solutions. This energy difference decreases as the system is moving closer to voltage instability. The constant energy contours of the energy function can be viewed as "potential energy surfaces" expending outward from the stable equilibrium point. The closest unstable equilibrium point (corresponding to a low voltage solution) represents the nearest saddle point by which trajectories may escape the potential well surrounding the stable equilibrium point.

An actual power system is never exactly at its equilibrium point since there are always fluctuations in the P and the Q load terms (which can be approximated as white noise) at each bus which are continually "kicking" the system away from that point. In the normal case, where the energy potential well is deep, these minor disturbances have no large scale affect. However, as the system moves closer to voltage collapse the depth of the potential well decreases. As this happens, the affects of these white noise load fluctuations can become very important. If enough of the "kicks" occur in the right direction, it is possible that the system might escape its present well. Thus the state of the system becomes a random process, and one can formally define the expected time required to exit the potential well. The calculation of expected exit times for randomly perturbed power system models was examined using the theory of large deviations in [4]. In the limit of noise much smaller than the average load term, the expected exit time is proportional to:

$$\exp\left[v(w^u, \alpha^u, V^u)/\epsilon^2\right].$$

The expected exit time provides a way in which different operating points may be ranked in terms of their vulnerability to voltage collapse. However, the expected exit time is not able to take into account more long term variations in the system state (such as load or generation ramping up). In the next section a more heuristic application of the energy function will be examined, which combines the energy calculation used to determine expected exit time with a family of energy curves based upon anticipated long term changes to the system state.

4. Use of Energy Function to Estimate Proximity to Voltage Collapse

In this section, the energy method will be used to develop a security measure which estimates distance to voltage collapse in terms of the MW load increase required to drive the system to voltage collapse. This approach will be motivated by first looking at a simple three bus system, and then by applying the security measure to larger systems. It will be shown through examples that the energy method can take into account the affects of generator var limits and static tap changing (LTC) transformer models.

4.1. Simple three bus system

The first test system consists of two strongly coupled generator buses, numbered 1 and 2, with a weakly coupled load bus (number 3) attached to the second generator bus. Bus 1, the slack bus, is attached to bus 2 by line 1, while line 2 links buses 2 and 3. This system was chosen for initial study because it is a simple equivalent representation of the type of system which is often prone to voltage collapse. Consider the following system parameters:

$$R1 = 0.005 \quad X1 = 0.05 \quad R2 = 0.02 \quad X2 = 0.06.$$

Initially assume that the generator at bus 2 is offline, and that the only load is a constant P/Q load at bus 3. In this case the energy function can be calculated for any P/Q load at bus three. Figure 4-1 plots the difference in energy between the high and the low voltage powerflow solutions as the load at bus 3 is increased. Beginning with a load of 50 MW and 25 MVAR, the load is increased with a constant P/Q ratio of 2 until the critical voltage collapse point is reached. This occurs at approximately P = 242 MW and Q = 121 MVAR.

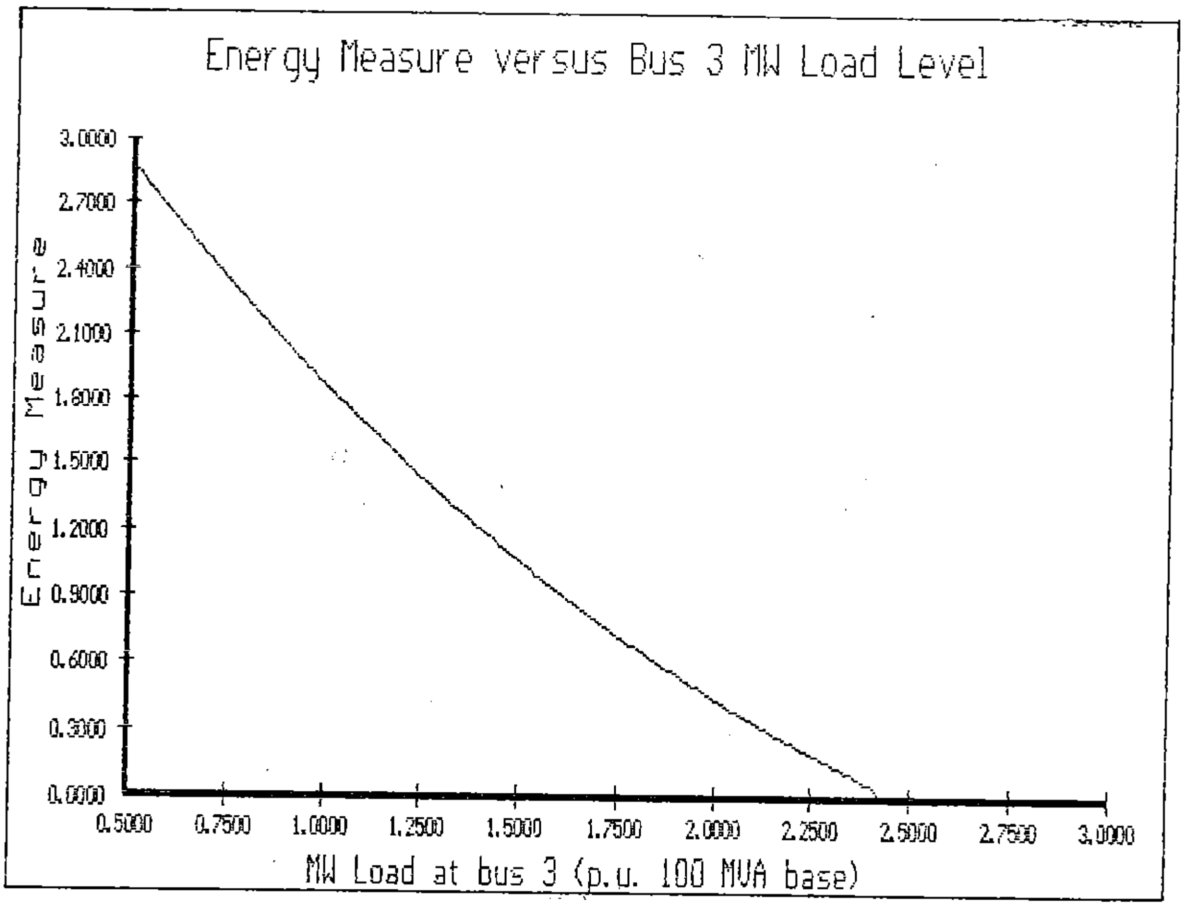


Figure 4-1

One possible application of the energy values shown in figure 4-1 is calculation of the 'expected exit time' of [4] mentioned in the previous section. Thus the energy value provides a relative measure of the system security that can be used to compare different operating points. However, the figure also suggests a more intuitive measure of system security. By shifting the x axis to align the critical collapse point (the bifurcation point) with zero MW, each energy value can be related to a "distance" in MW to voltage collapse. For example for an energy value of 0.50 (which occurs when the load at bus 3 is 192 MW and 96 MVAR) the distance to voltage collapse would be about 50 MW; i.e., when the energy value is 0.50, voltage collapse will occur when the load at bus 3 is increased by 50 MW and 25 MVAR. This suggests the following heuristic test for determining the distance to voltage collapse under general load conditions:

1) Calculate the energy value for a given load at bus 3.

2) Use a table containing the figure 4-1 data (pre-calculated off-line) to determine the 'distance' to voltage collapse.

In order for this method to be useful, the shape of the curve in figure 4-1 must be relatively insensitive to changes in the operating point of the system. Ideally, one would like to approximate distance to voltage collapse for a wide variety of operating points on-line, using only information from the off-line calculation of a single energy curve. Note that the 'energy curve' is simply the difference in energy between the high and low voltage powerflow solutions over a one parameter family of operating points (MW load at bus 3 being the free parameter used in figure 4-1). The following examples examine the feasibility of this approach.

The operating point of the three bus system is first varied by changing the load at bus 2 (with the generator at that bus again assumed to be off line). Figure 4-2 shows how the energy function varies with the load at bus 3 for different values of load at bus 2. The topmost curve corresponds to the load at bus 2 = 0, and is thus simply a repeat of the curve in figure 4-1. The remaining curves show the energy values plotted versus bus 3 load as before, but also as the load at bus 2 is increased in increments of 50 MW and 25 MVAR. As would be expected, the energy value for any given value of load at bus 3 decreases as the load at bus 2 is increased, since the system is becoming more heavily loaded and therefore less secure. Note that the five curves in figure 4-2 are nearly identical up to an x-axis shift. This is shown in figure 4-3 which contains the 5 curves aligned with their respective critical collapse point at 0 MW. Thus, for any bus 2 load in the range described, the distance to voltage collapse (expressed in tolerable MW increase in load at bus 3) can be approximated by using only the energy calculated at the current operating point and any one of the five curves.

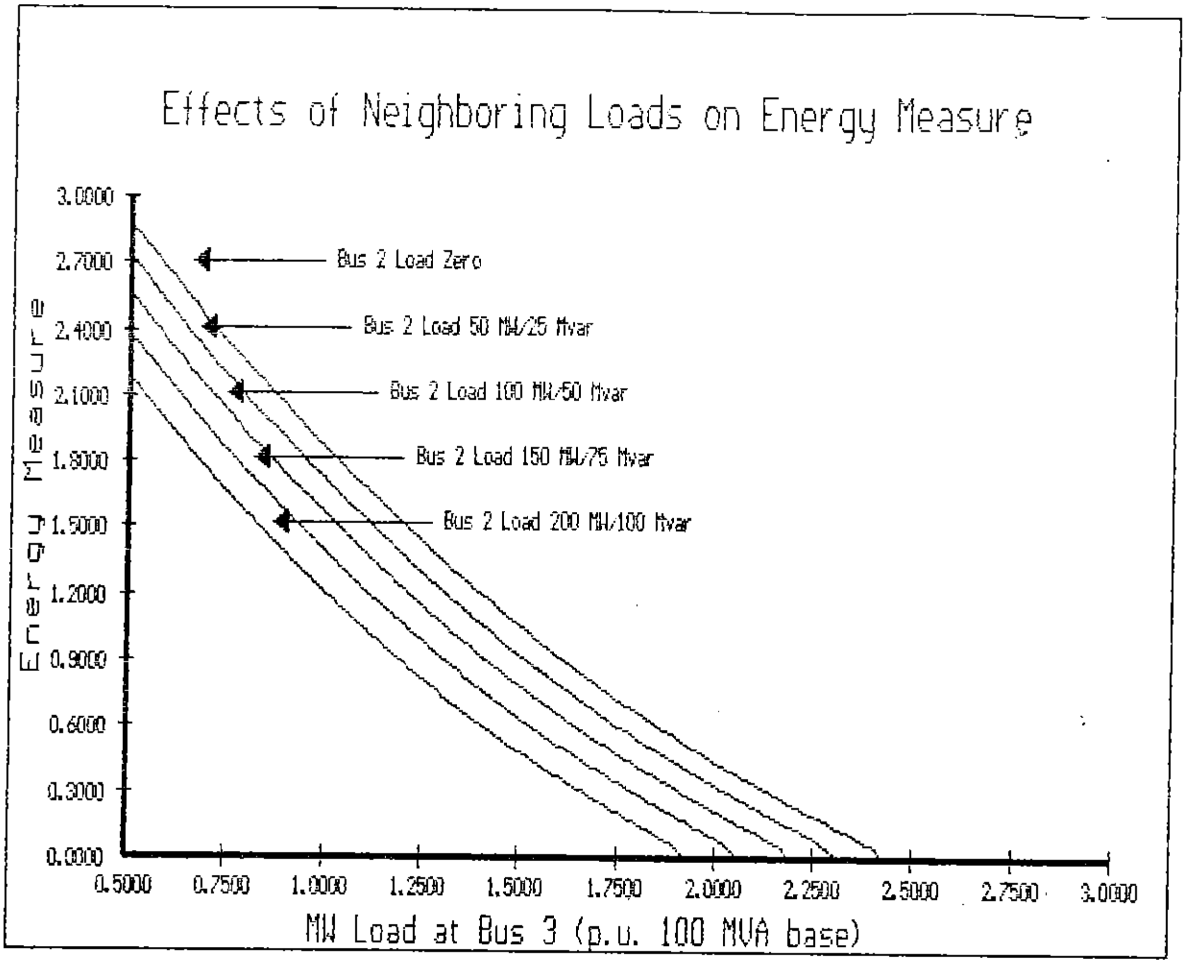

Effects of Neighboring Loads on Energy Measure
Bus 2 Load Zero
Bus 2 Load 50 MW/25 Mvar
Bus 2 Load 100 MW/50 Mvar
Bus 2 Load 150 MW/75 Mvar
Bus 2 Load 200 MW/100 Mvar
Energy Measure
3.0000
2.7000
2.4000
2.1000
1.8000
1.5000
1.2000
0.9000
0.6000
0.3000
0.0000
0.5000
0.7500
1.0000
1.2500
1.5000
1.7500
2.0000
2.2500
2.5000
2.7500
3.0000
MW Load at Bus 3 (p.u. 100 MVA base)

Figure 4-2

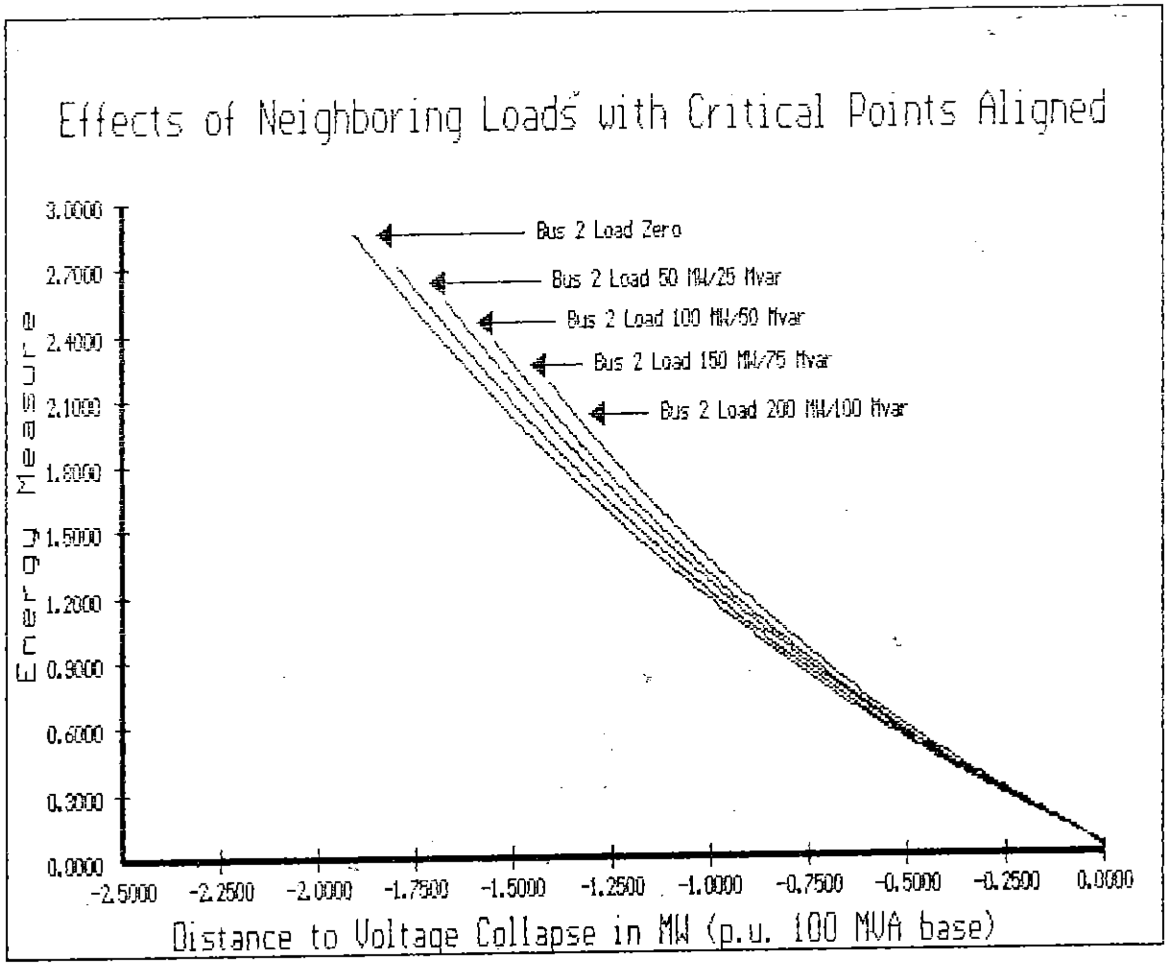


Figure 4-3

Before moving on to study the affects of generator var limits and tap changing transformers, it is important to clarify one point. In relating the energy function to distance to voltage collapse, the load and generator participation factors must be taken into account. In the preceding two examples, the increase in real and reactive power of the system has been parameterized by one independent variable (MW load at bus 3), with assumptions made on how the other free values depend on this single parameter. For example, in determining the distance in MW bus 3 is away from voltage collapse, the assumptions made were that the power factor at bus 3 remained constant and that the load at bus 2 also remained constant (incremental load participation of 0). For illustration, figure 4-4 shows the behavior of the energy function as the system loads are increased in three different ways: (i) the load at bus 2 is held constant at 100 MW and 50 MVAR while the load at bus 3 is increased with a constant P/Q ratio of 2, (ii) the load at bus 2 is again held constant, but the MVAR load at bus 3 is also held constant as the MW load increases,

(iii) the load at bus 2 changes with that at bus 3, with a ratio of 2 to 1 (incremental load participation twice that of bus 2); both buses maintain a constant power factor. As would be expected, the second curve represents a more secure scenario than (i), system while the third curve shows the system less secure than (i).

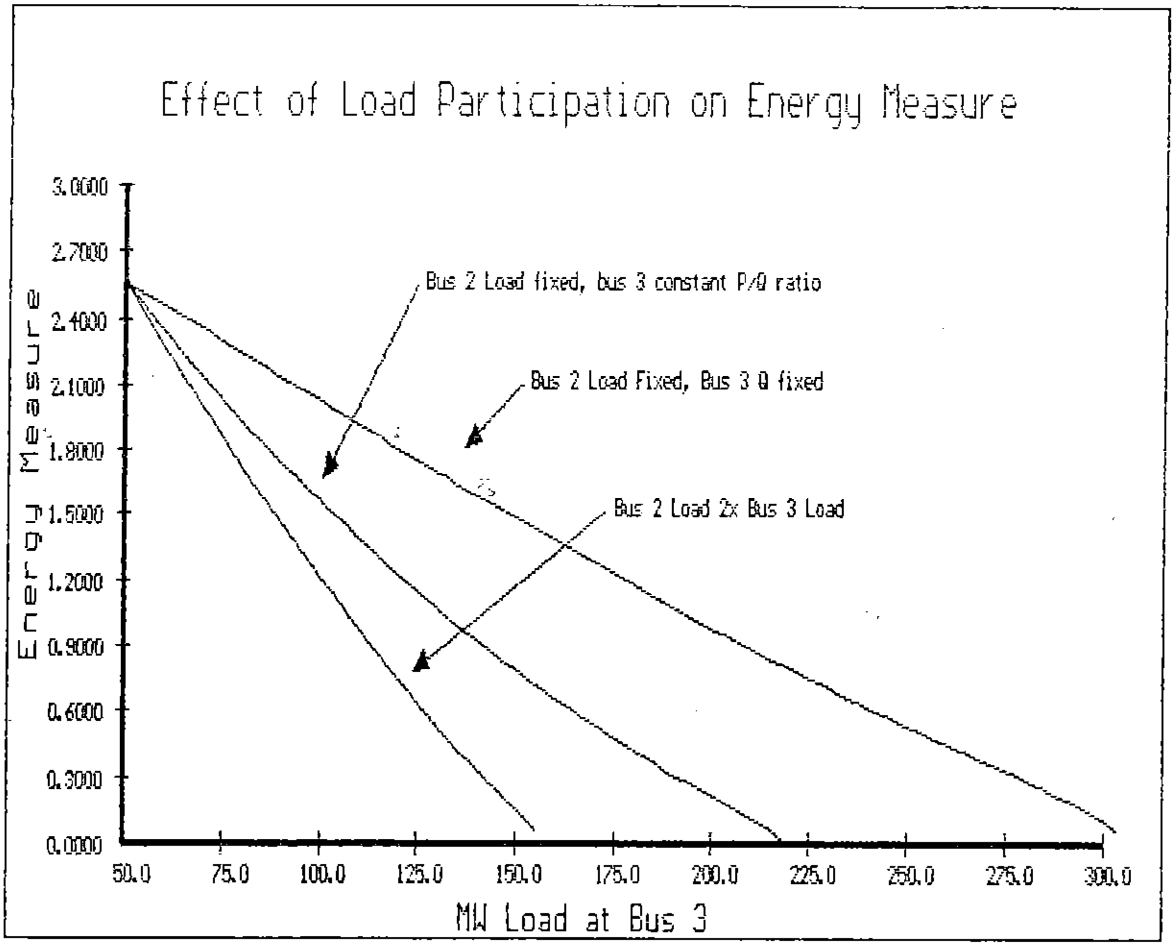


Figure 4-4

The dependence of the security measure on load participation is an inherent feature in the method described above, and reflects assumptions on expected behavior as the system evolves with time. A weak area with increasing load within its borders is clearly moving more rapidly towards voltage collapse if the net load (load-generation) neighboring the area is also increasing with time. Likewise, how the powerfactors vary as the MW loads within an area increase is also important in determining expected system behavior. This information must be taken into account when relating an energy value to distance to voltage collapse, and one would expect planners and operators to examine a variety of participation factor scenarios. However, by combining this method with the expected exit time security measure, the energy value also provides a means of ranking operating points with respect to their vulnerability to voltage collapse independent of participation factors.

4.2. Affects of Generator Var limits

The energy based method is based on a nonlinear representation of the powerflow and system dynamics, rather than working only with a linearization about a given operating point. This gives the method the very desirable property that the affects of voltage regulation controls hitting their limits can be taken into account. In this section the affects of generator var limits will be considered, while in the next section the affects of tap changing transformers will be examined. In the powerflow calculations used in this section, the var output of the generators is normally allowed to vary in order to hold its bus voltage constant; i.e., generator buses are treated as PV. However, if the var limit is reached, the exciter is considered to have saturated, and thus the generator's voltage is held constant; the bus model changes to PQ. This is the standard approach to treating var limits in powerflow calculations.

Consider again the three bus system of the previous section. This time the generator at bus 2 turned on to provide voltage support (reactive power), but initially it provides no real power output and there is no load at the bus. One would expect that the more reactive power the generator can provide, the greater the load that can be tolerated at bus 3 before voltage collapse occurs. Figure 4-5 shows that this is indeed the case. The lowest curve represents the case of generator 2 turned off and is thus just a repeat of the curve found in 4-1. The next four curves show how the energy function varies as the maximum var output of generator 2 is increased in increments of 50 MVAR. During the sequence of powerflow/energy calculations used to produce this figure, the voltage at bus 2 was held at 1.0 per unit as the load at bus 3 ramped up until generator 2 reached its var limit. Thereafter the var output was held at its maximum. Surprising, the shape of the energy function curve proves insensitive to the varying var limits of the generator; the five curves in figure 4-5 are again nearly identical up to a shift along the horizontal axis.

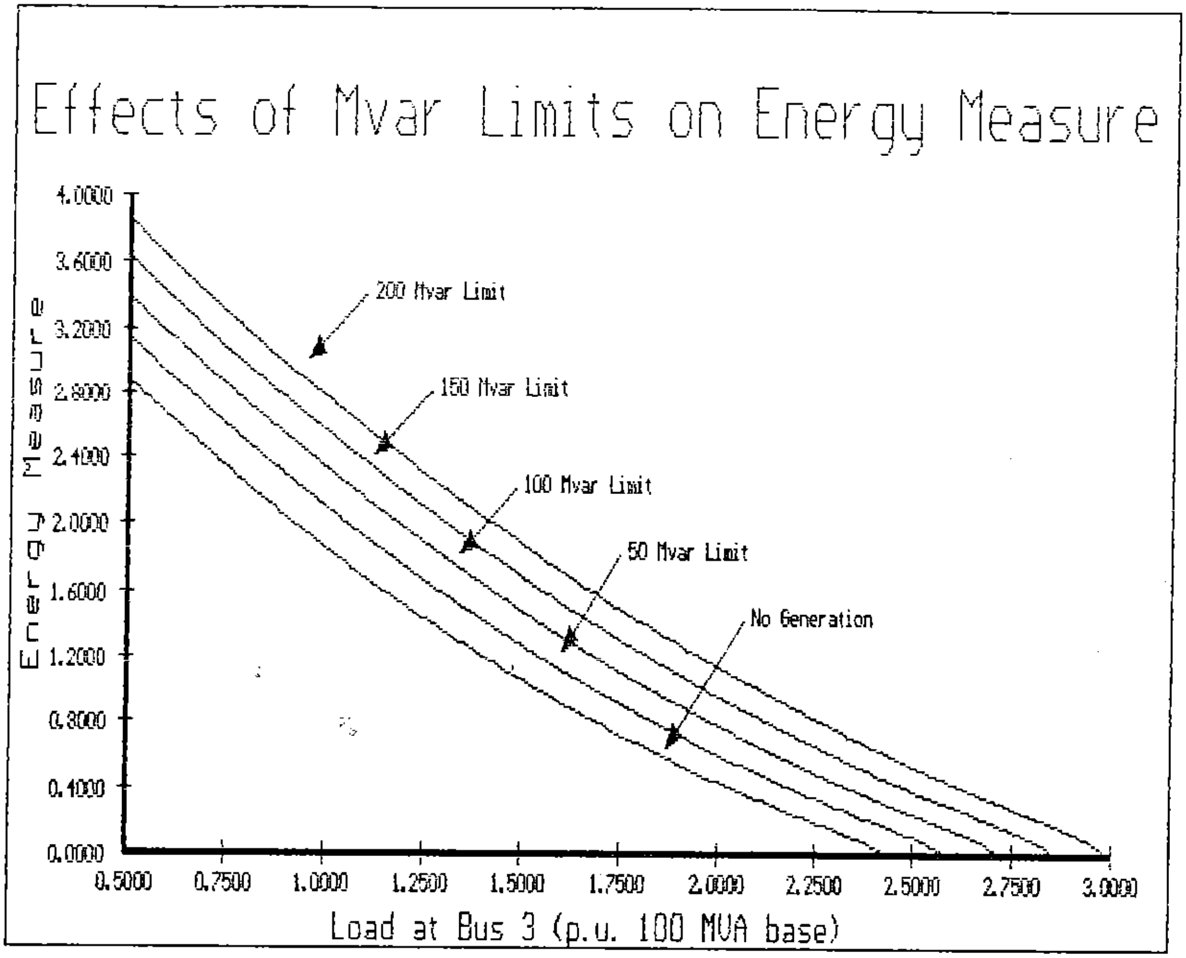


Figure 4-5

The preceding example shows that the limits on available var support are taken into account even when the current system operating point (the "high voltage solution") has not pushed the generators to these limits. Intuitively this is because the low voltage solution tends to push all neighboring var sources to their limits and thus reduce the height of the potential energy boundary that the system must cross to experience collapse. Table 1 illustrates this property.

Table 1

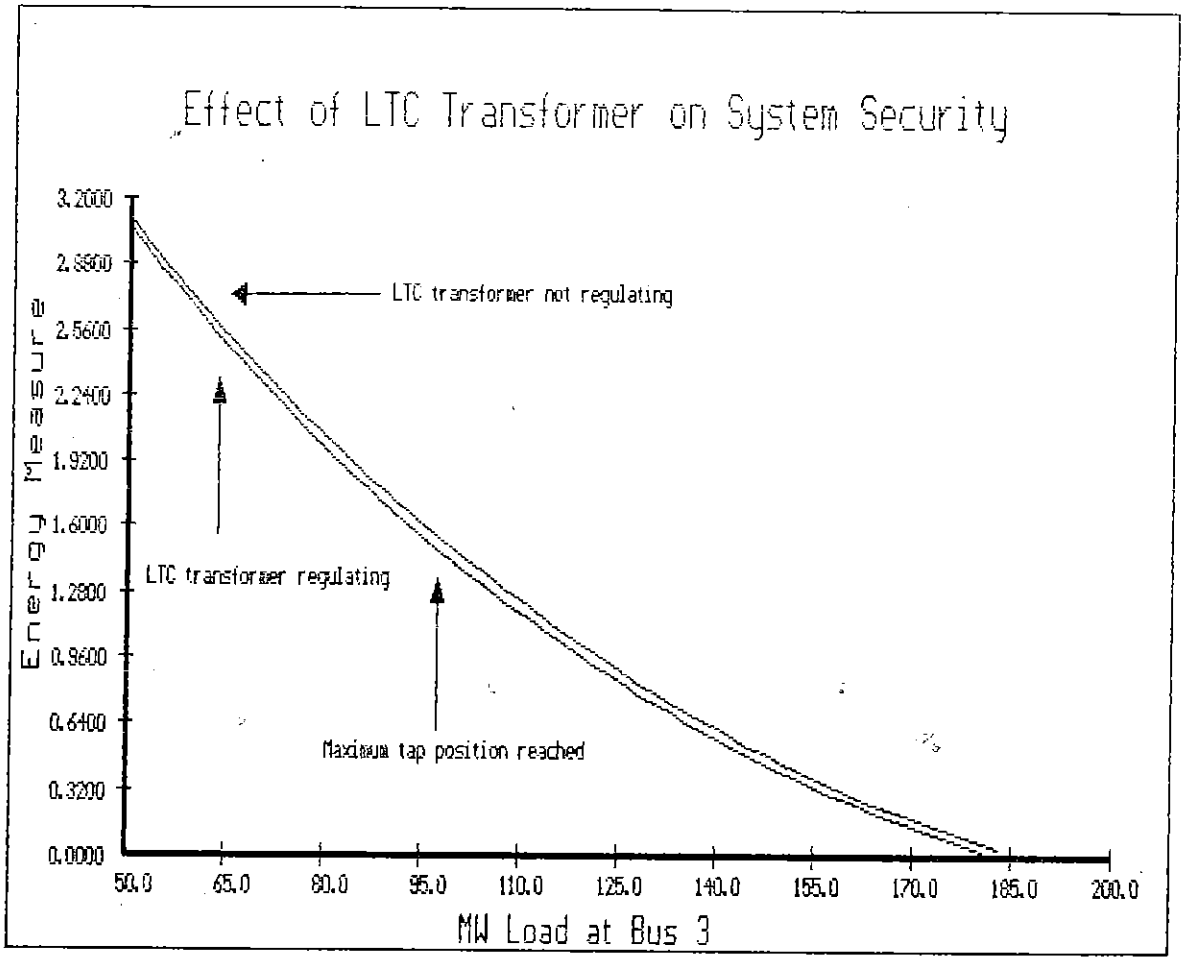
Bus 3 load:		Bus 2 Generator:			Energy
MW	MVAR	MW	MVAR	MAX MVAR Limit	
100	50	0	0	0 (gen off)	1.85
100	50	0	50	50	2.10
100	50	0	71.38	100	2.34
100	50	0	71.38	150	2.56
100	50	0	71.38	200	2.77

Notice that in the last 3 cases in the table, the generator at bus 2 has not saturated at the current operating point, but the energy function yields different values based upon the maximum var output of the generator. As expected, the larger energy values (indicating a more secure operating point) are associated with the cases where generator 2 has a greater var margin. This is a very desirable characteristic to have in a method of determining proximity to voltage collapse since a voltage collapse scenario is often characterized by nearby generators reaching their reactive power limits as the voltage in the area declines. Thus in order to gauge how far the current operating point is away from the voltage collapse point, these limits must be taken into account.

4.3. Affects of Tap Changing Transformers

In a power system, tap changing transformers (LTC transformers) are used in both the distribution system and in the transmission system. In the distribution system their primary purpose is to maintain the customer voltage within a narrow band. In the transmission system they are used both to control the voltage on the transmission system and to reduce transmission losses by decreasing the var flows on the lines. Because of their considerable ability to control voltage profile of the system, it is essential that their affects be considered in any measure of voltage security. In this section the affects of the radial load LTCs will be considered, while the affects of the transmission system transformers will be examined in the next section.

The 3 bus system used in the above examples is modified to include an LTC transformer between buses 2 and 3. This transformer is regulating bus 3's voltage to 1.0 per unit; assume the transformer has 16 steps above and below its nominal setting, with a per unit step size of 0.0125. Also the assumption of fixed MVAR load will also be relaxed by allowing the load at bus 3 to consist of inductance (Q varies as a function of V^2), along with constant MVAR load. Figure 4-6 shows the energy measure as the load at bus 3 is increased for the two cases: LTC transformer not regulating, and LTC regulating. When the LTC transformer is used for voltage regulation, the system turns out to be less secure. The LTC transformer regulation causes the load bus voltage to be higher, thus resulting in more inductive load. Conversely in the unregulated case the bus voltage is lower, resulting in less load. Figure 4-7 plots the voltage at bus 3 for both cases. Intuitively, the energy measure accounts for transformer's regulation limits because the low voltage solution pushes the transformer to its limit. This is the same phenomenon observed earlier with generator var limits.



Effect of LTC Transformer on System Security
Energy Measure
3.2000
2.8800
2.5600
2.2400
1.9200
1.6000
1.2800
0.9600
0.6400
0.3200
0.0000
LTC transformer not regulating
LTC transformer regulating
Maximum tap position reached
50.0
65.0
80.0
95.0
110.0
125.0
140.0
155.0
170.0
185.0
200.0
MW Load at Bus 3

Figure 4-6

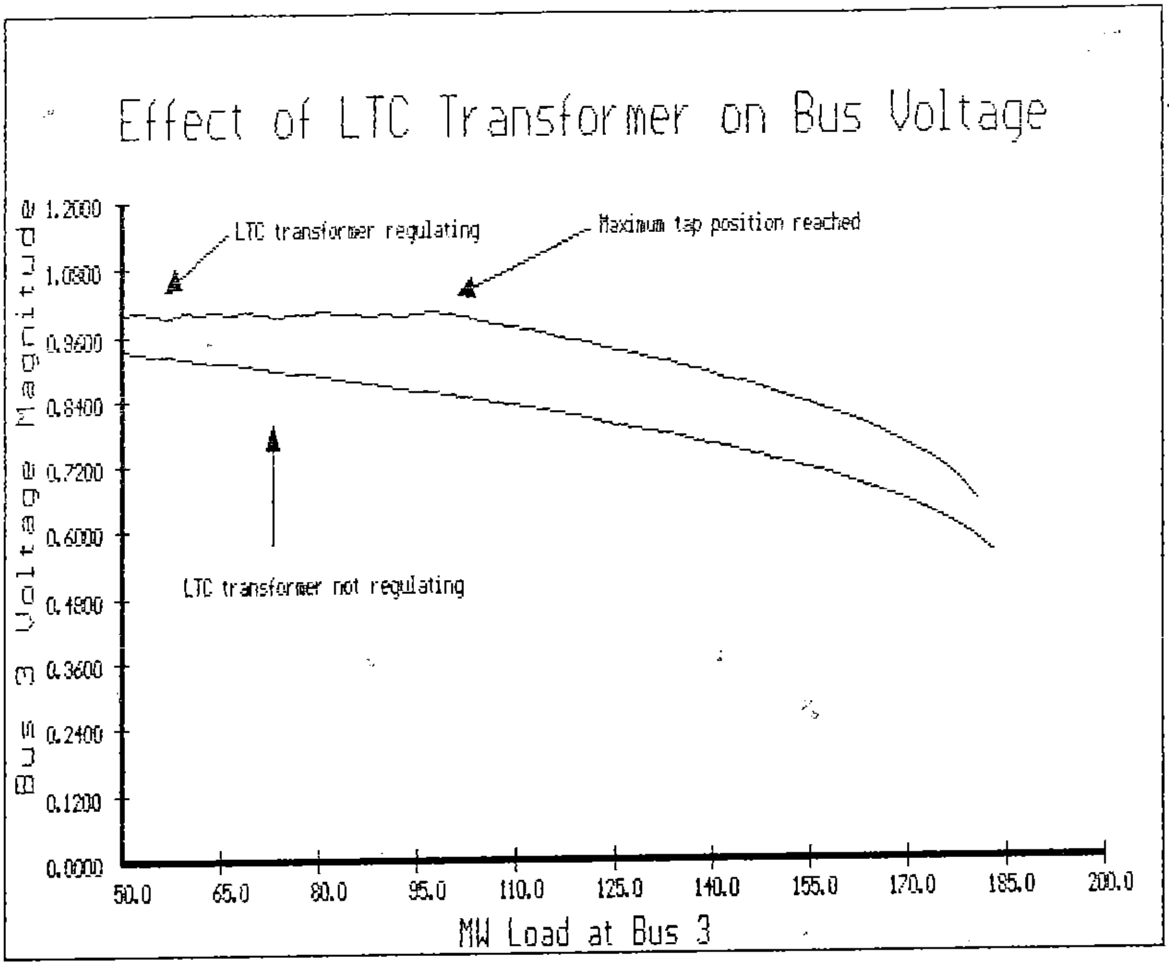


Figure 4-7

4.4. Energy method applied to a larger system.

The following set of examples illustrates the application of the energy method on a larger system. Frequently the possibility of voltage collapse exists in more than one area of the system. The low voltage areas are normally a collection of buses with very little internal reactive support, connected to the rest of the system through relatively high impedance branches. A 16 bus network was constructed to illustrate the use of the energy function method in a system having multiple weak areas. A one line diagram of the system is shown in figure 4-8 (see appendix A for a complete listing of system parameters). The system contains two areas with potential low voltage problems: area 1, which consists of buses 13, 14, 15, and 16 (generator 15 has an extremely low var limit), and area 2, which contains buses 9, 10, 11, and 12.

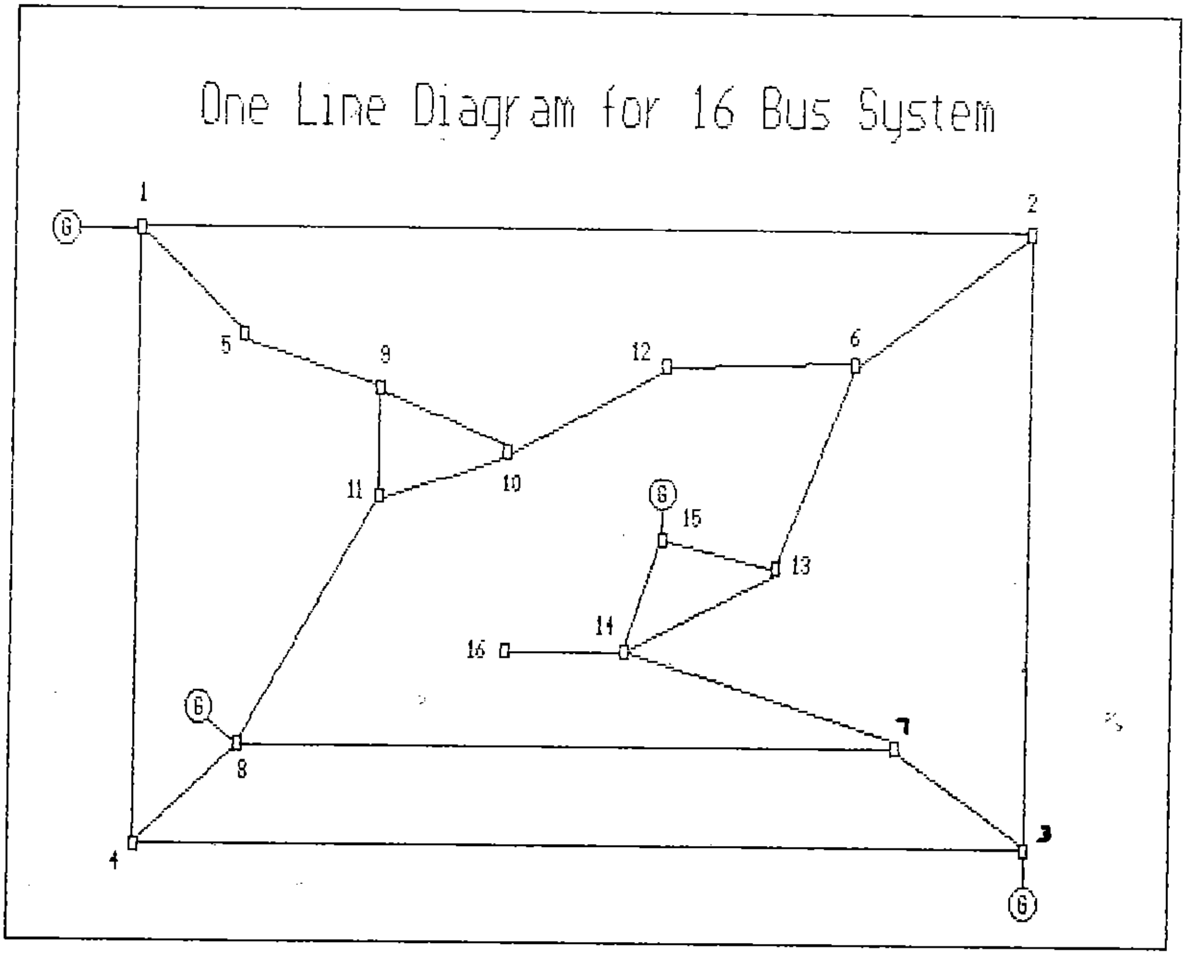


Figure 4-8

The possibility of voltage collapse in area 1 will first be examined. By gradually increasing the load in area 1, the energy difference between the high and low voltage solutions can be calculated.[2] The single free parameter in this case is the total MW load in the area. The result is shown in figure 4-9, which relates energy value to area MW load. Additional curves can be developed for any number of contingencies on the system. Figure 4-10 shows the energy curves for the following four contingencies: 1) Outage of line 3-7, 2) Outage of the generator at bus 15, 3) Outage of the generator at bus 8, and 4) Outage of line 6-13. As was seen in the examples with the three bus system, the shape of the energy function curves is nearly identical up to a shift along the horizontal axis. Table 2 provides a ranking of the contingencies by energy value (most to least severe) for an area 1 load example load of 150 MW.

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[2]See section 5 for a more detailed description of determining low voltage solutions in larger systems.

Table 2 - Area 1 Contingencies Ranked According to Severity

Contingency	Energy	Distance to collapse From Load of 150 MW
1 - Line 3-7 Out	0.41	33
4 - Line 6-13 Out	0.48	42
3 - Gen 8 Out	1.13	123
2 - Gen 15 Out	1.15	127
Base Case	1.22	134

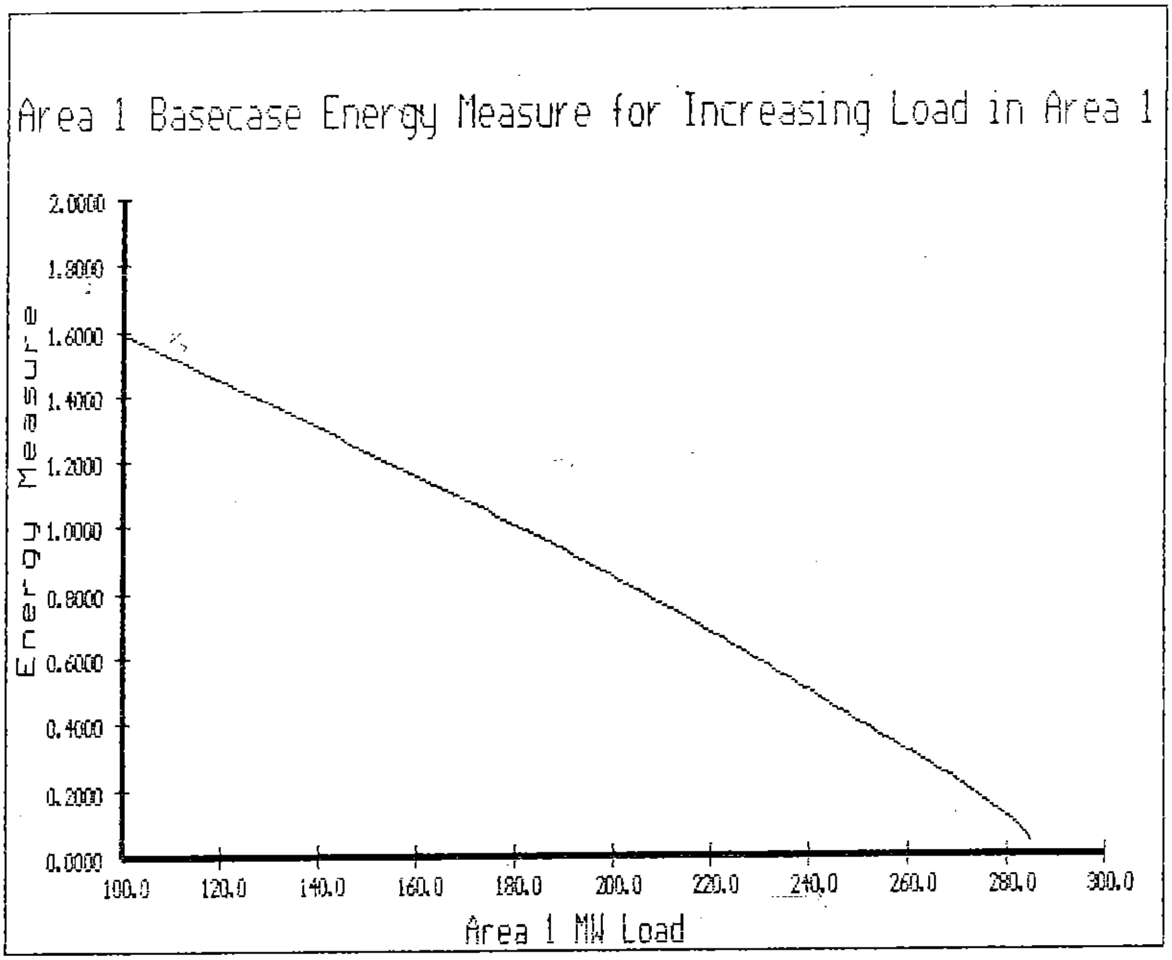


Figure 4-9

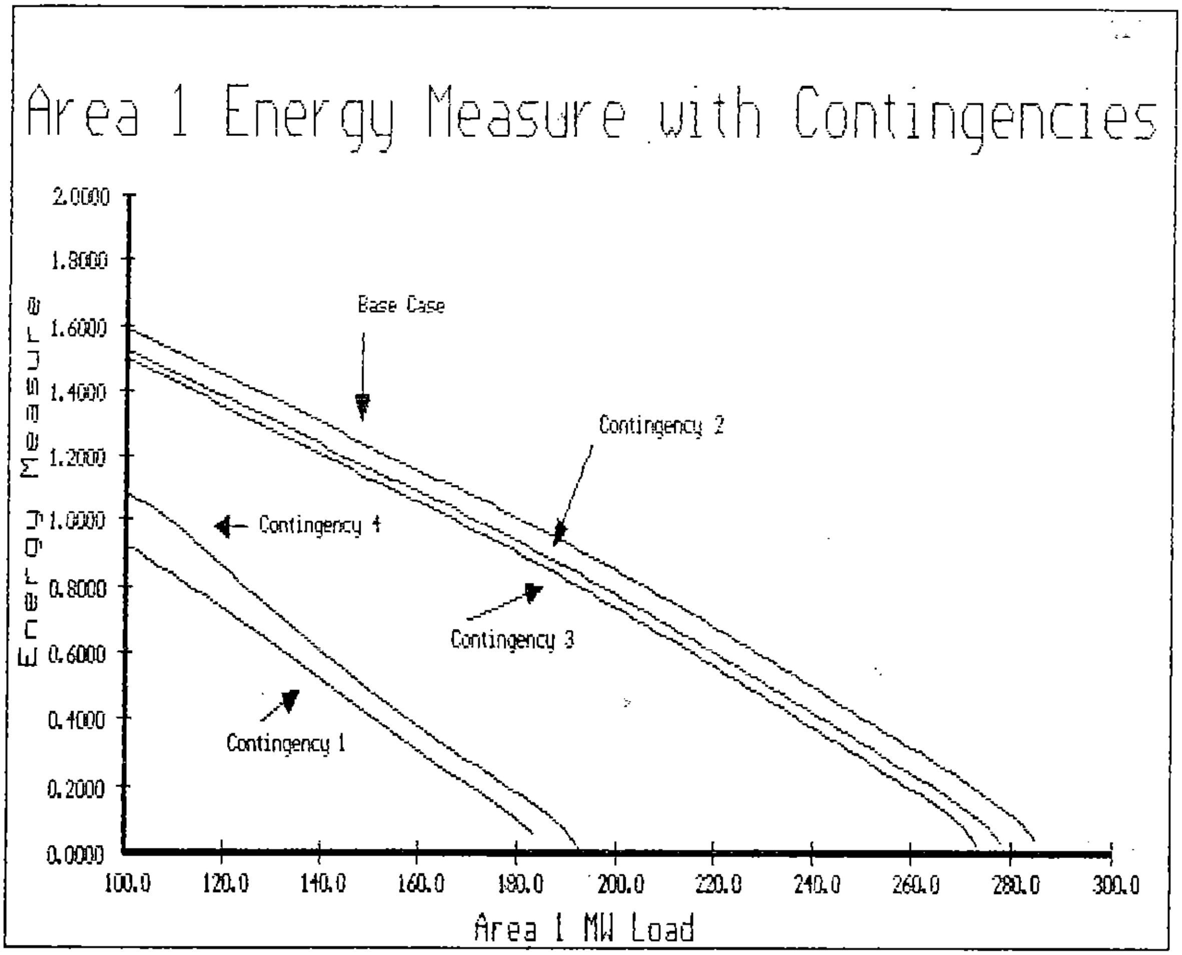


Figure 4-10

In a system with more than one weak area, it may be necessary to calculate the energy function for the low voltage solutions of each area. The energy value associated with each low voltage solution appears to correspond to the likelihood of voltage collapse being initiated by loss of solution at the bus with the lowest voltage. Table 3 shows the energy values associated with the four above contingencies when voltage collapse in is forced to occur in area 2. This is done by gradually increasing the load in area 2, rather than in area 1. Figure 4-11 displays the energy curves associated with each contingency.

Table 3 - Area 2 Contingencies Ranked According to Severity

Contingency	Energy	Distance to collapse From Load of 150 MW
1 - Line 3-7 Out	No Solution	- Collapse at bus 16
3 - Gen 8 Out	1.81	155
2 - Gen 15 Out	2.04	171
4 - Line 6-13 Out	2.08	173
Base Case	2.08	173

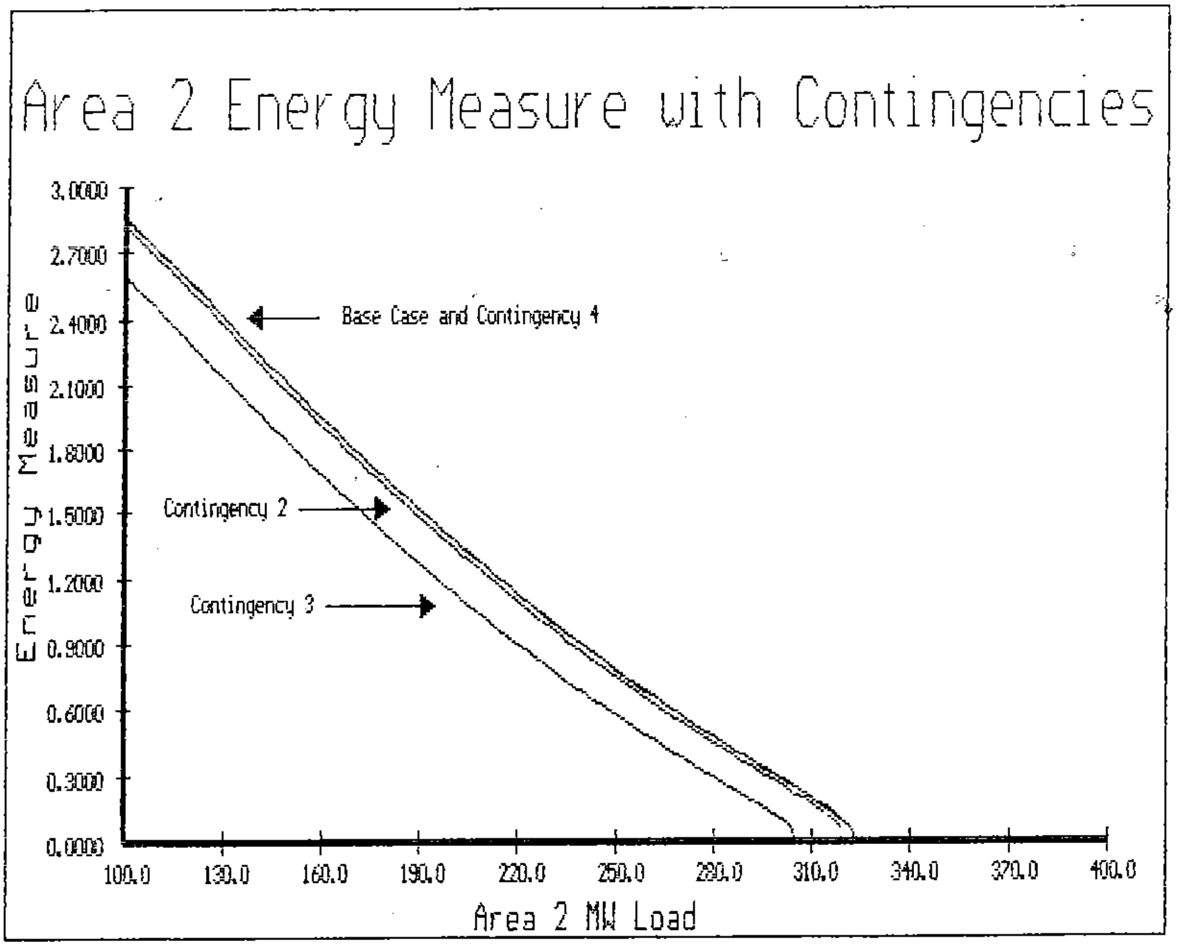


Figure 4-11

As was seen in the previous case, the energy function ranks the severity of each contingency in a manner which agrees with engineering judgement. Since the ties of area 2 to the rest of the system (and to var sources) are stronger, the associated energy values are higher. The contingency where line 3 to 7 was removed is an example where area 1 and area 2 join to form a single area of depressed voltage. In this case it appears that no low voltage solution is possible with bus 9 having the lowest voltage; in this case voltage collapse would occur at bus 16 before bus 9, even though it is the loads in area 2, not area 1, which are increasing.

Lastly, the 16 bus system is modified to include LTC transformers on the lines between 5 to 1, 6 to 2, 7 to 3, and 8 to 4. Each of the four transformers attempt to hold the voltage of the first bus of the four above pairs at 1.0 per unit. Again assume that each transformer has 16 steps above and below nominal, this time with a step size of 0.00625. Figure 4-12 compares the energy curve of figure 4-9 (with transformers not regulating) to the case where the LTC transformers are regulating the bus voltage. Note that the var loads are here assumed to be insensitive to bus voltage. Under these conditions, the presence of the LTC transformers increases the energy measure associated with any area 1 load, and hence the system is judged more secure. This is in contrast to the case examined in 4.3 where the presence of LTC's decreased security. The energy measure appears able to correctly account for LTC transformers affects regardless of whether they improve or degrade security.

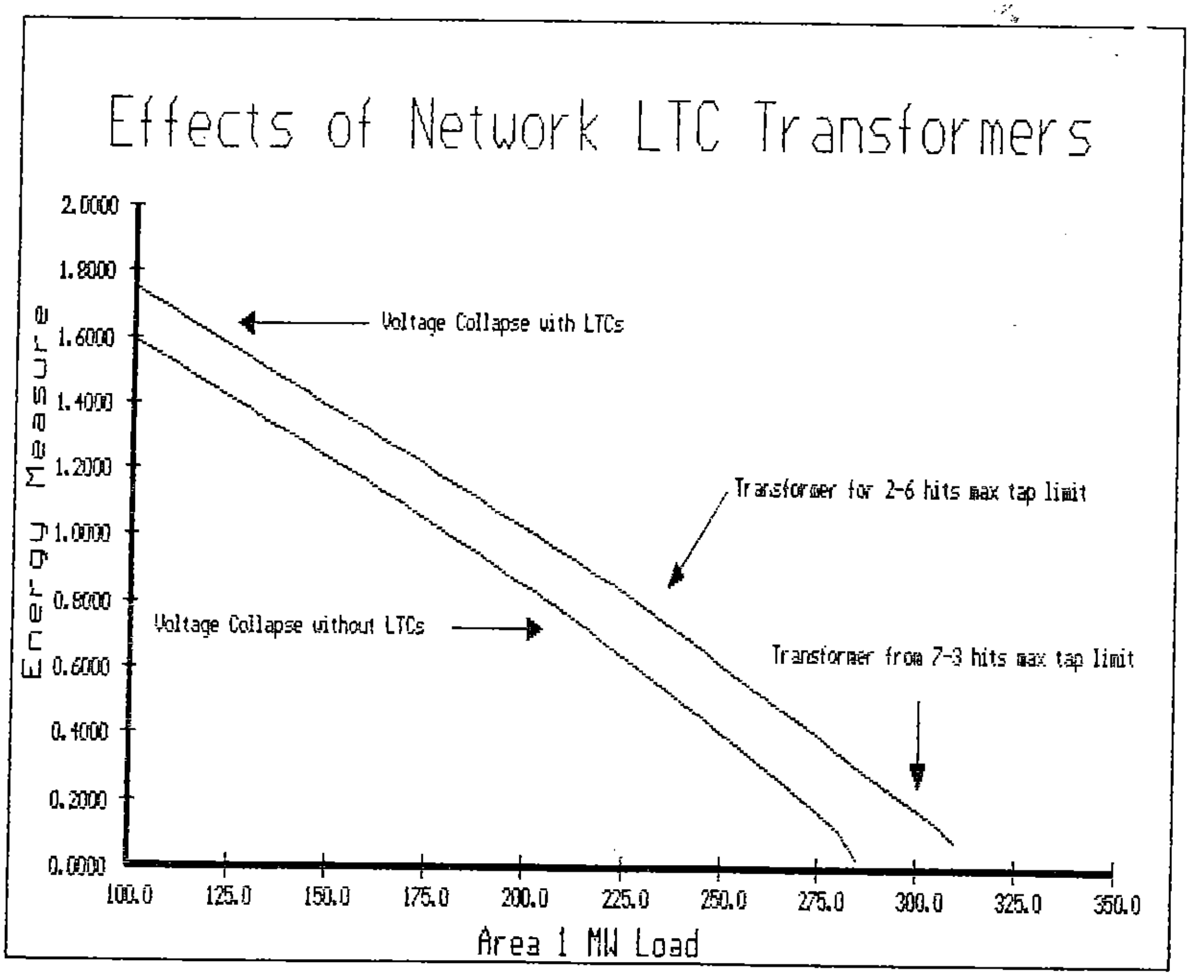


Figure 4-12

Based on the cases examined here, it appears that the energy function provides both a means of ranking the severity of each contingencies, and also provides a more intuitive relative measure of system security (in this case the tolerable increase of MW in load 1). As expected, the measures of distance to collapse require that the appropriate incremental load and generator participation factors are taken into account. Computationally, the on-line requirements of the method consist only of a few (ideally two) powerflow solutions per contingency, and associated evaluations of the scalar energy function. If one only wishes to rank various operating points with respect to their vulnerability to voltage collapse, no additional calculation is needed. If one wishes to postulate several possible patterns of expected load increase, the offline calculations require repeated powerflow solutions and energy evaluations along these "projected paths" of operating points. As shown in the preceding examples, typically only one path is needed per load pattern, not one per contingency. The energy value calculated on-line can then be used to identify where the system lines on one of the paths, and thus the tolerable load increase can be identified for either the base case or a contingent state.

5. Low Voltage Powerflow Solutions

The powerflow solutions often display more than one possible solution, as had been seen in figure 2-1. Since the energy function requires the use of the low voltage solutions, some discussion of their characteristics and methods of solution is needed.

Referring again to the two bus system, it is clear that the simultaneous solution of the P and Q constraints shown in figure 2-1 is obtained at their intersections. With low load values there are two solutions (the high and low voltage), which are relatively far apart. As the load is increased, these two solutions gradually move together until they reach the bifurcation point, where the two solutions coalesce into one. Further increase in load results a powerflow with no solution. At the bifurcation point the Jacobian of the two powerflow equations becomes singular. This is shown in figure 5-1, which is a plot of the smallest singular value of the Jacobian of the two bus system used in section two. The high and low voltage magnitude data from figure 2-2 is included for reference.

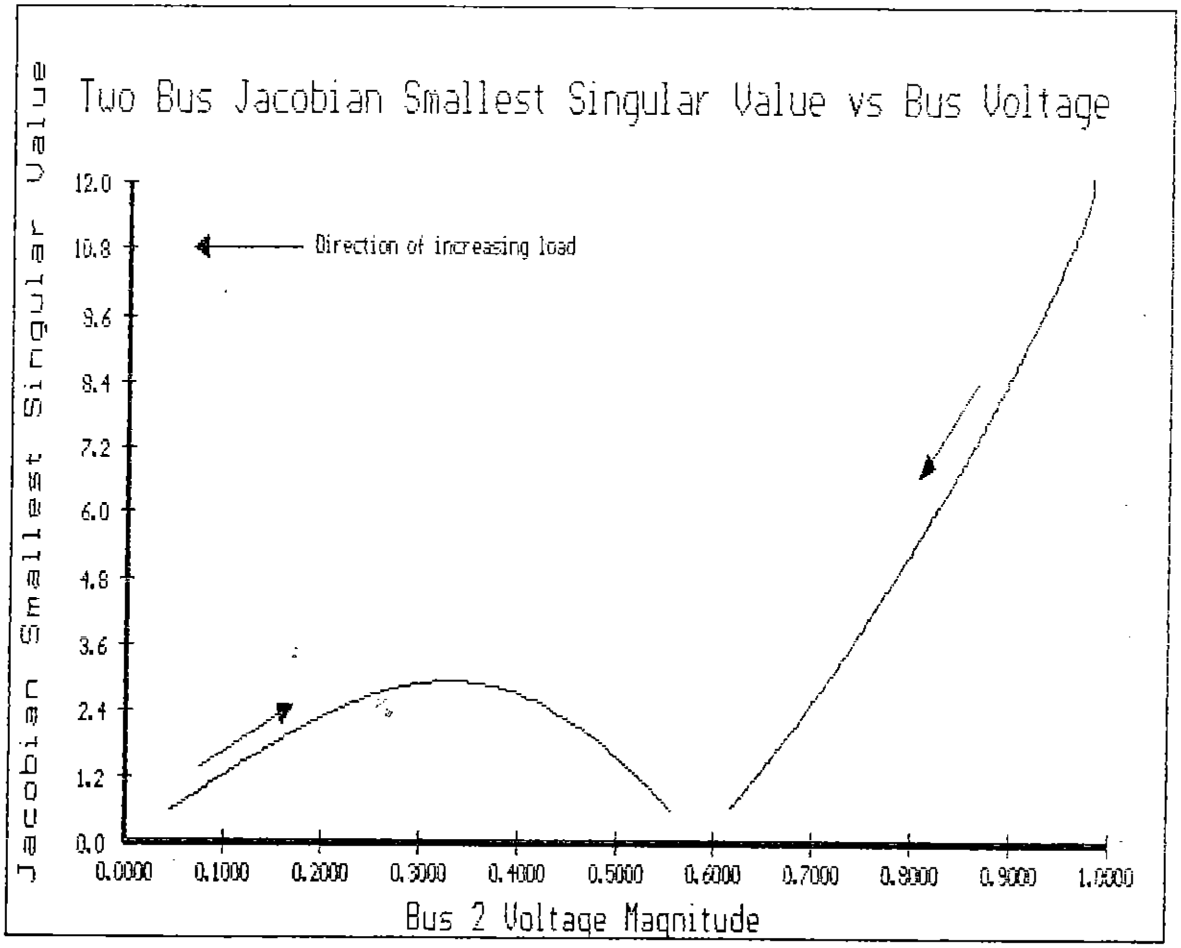


Figure 5-1

As a "next step" in complexity over the two bus system, consider the case of a three bus system with two radial lines connected to a single slack bus. Because of the slack bus between them, the two halves of the system are essentially isolated from one another. This system could be likened to a larger system with two potentially weak areas, isolated from one another by a relatively strong portion of the system. For this case the maximum number of solutions is seen by inspection to be four (both loads at their high bus solutions, both at their low voltage solution, and the two combinations of one load high and one load low), with each side reaching its bifurcation point independently as its load is increased. The next logical extension of this system is to couple the two loads by adding a third line between them. As shall be demonstrated shortly, this system results in a much more interesting set of solutions.

Consider the three bus system with bus 1 as the slack and buses 2 and 3 as load buses with constant P/Q loads. Each bus is connected to the other two with lossless lines of 0.2 per unit impedance. With an initial load of 50 MW and 25

MVAR at each of the load buses, 4 solutions are possible. Figure 5-2 shows the solution trajectories in the V_2-V_3 plane as the load at both buses is increased at the same rate, maintaining a constant powerfactor. The initial starting voltage points are labeled 1, 2, 3, and 4. Point 1 corresponds to the normal operating point of the powerflow. As the load is uniformly increased at buses 2 and 3, trajectory 1 moves downward to the left, indicating that the voltages at both buses are falling. This is the expected power system behavior. Eventually the voltage collapse point is reached (labeled point 5); at this point the Jacobian becomes singular and no further increase in load is possible. The other three points correspond to the three other initial powerflow solutions (the 'low voltage' solutions). At point 2 the voltage at bus 3 is higher than that at bus 2; at point 3 both voltages are the same; and at point 4 the voltage at bus 2 is higher than the voltage at bus 3. As the load at both buses is increased, the three trajectories converge, coalescing into a single trajectory at point 6. By the implicit function theorem [7] it is clear that at this point of coalescing, the Jacobian must also be singular. As the load is further increased, the trajectory continues to the upper right, eventually reaching the voltage collapse point 5, where the Jacobian again becomes singular. The simulations for this figure encountered few convergence problems in the vicinity of point 5. In the immediate neighborhood of this point the Newton-Raphson iteration required one or two more steps to reach a solution. This ease of solution is closely related to the choice of load step size. A larger step size meant less likelihood of load conditions extremely near to the singularity. Geometrically, the singular points comprise a set of measure zero, so the chance of landing exactly on the singularity is negligible. Figure 5-3 shows the smallest singular value of the Jacobian along paths 1 and 3 plotted versus the voltage magnitude at bus 2.

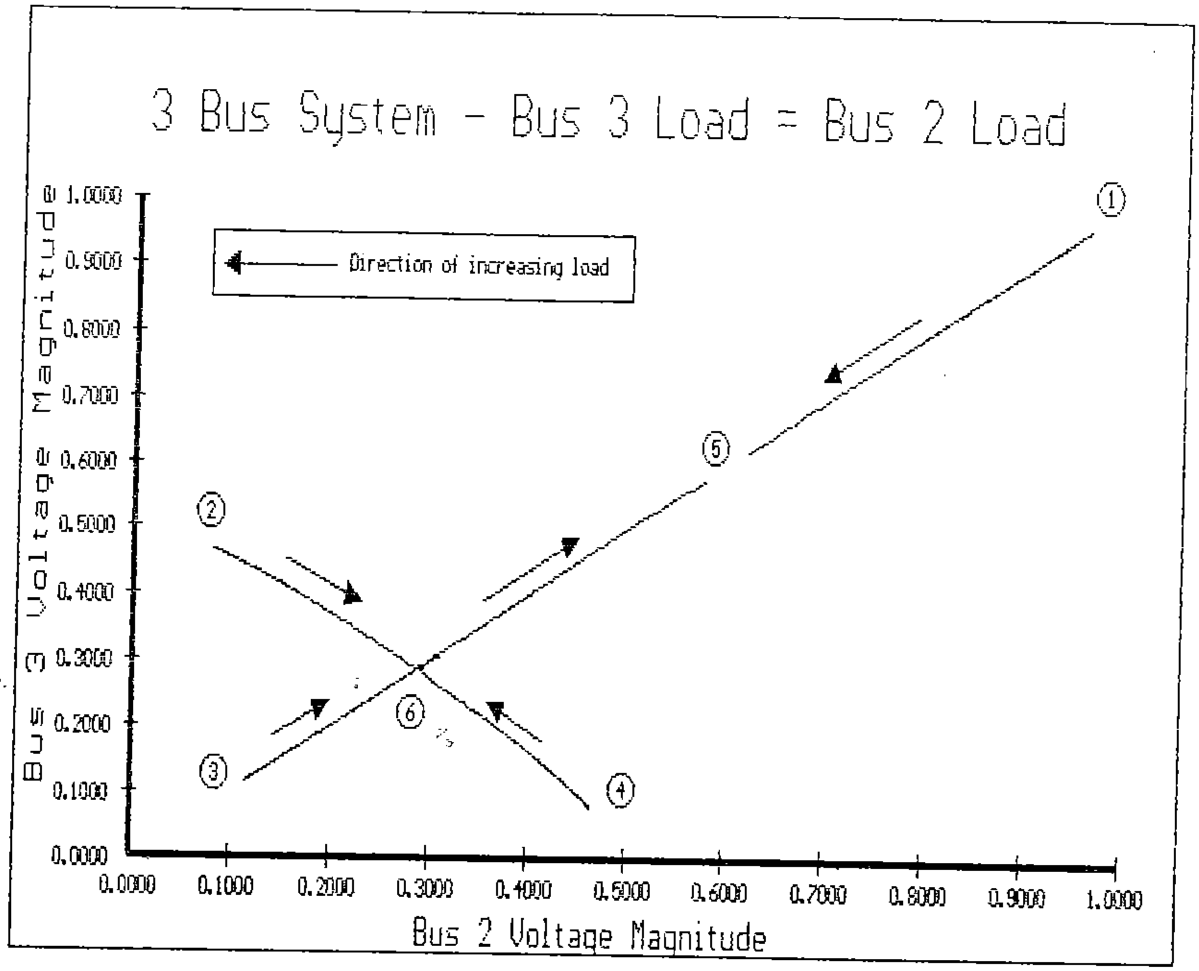

3 Bus System - Bus 3 Load = Bus 2 Load
Direction of increasing load
Bus 3 Voltage Magnitude
Bus 2 Voltage Magnitude
1.0000
0.9000
0.8000
0.7000
0.6000
0.5000
0.4000
0.3000
0.2000
0.1000
0.0000
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1.0000
1
2
3
4
5
6

Figure 5-2

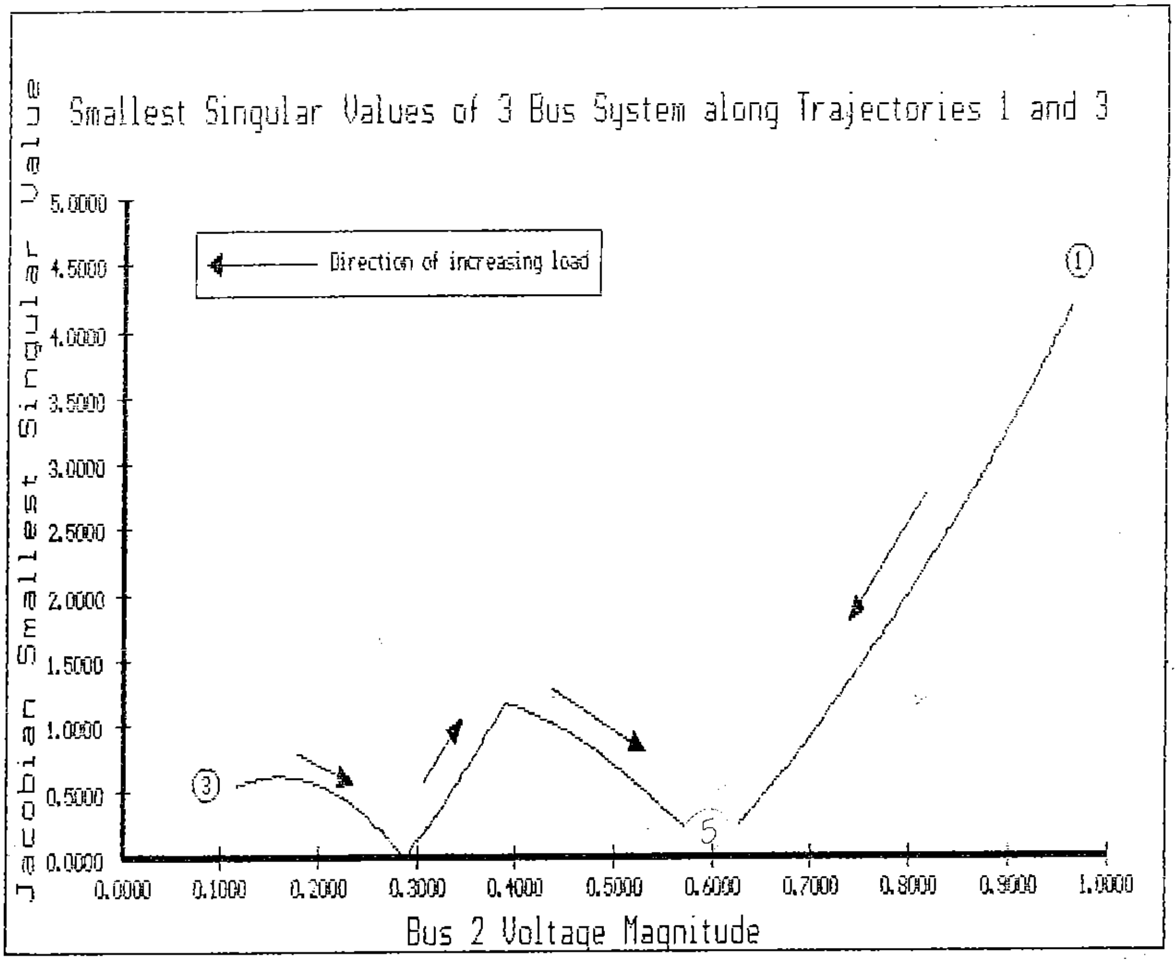


Figure 5-3

If the loads and their participation factors are changed so that the load at bus 2 is no longer equal to the load at bus 3, the three trajectories no longer join together. Figure 5-4 shows the results for the same system examined previously except that the initial loads were changed to with 50 MW and 25 MVAR at bus 2 and 45 MW and 22.5 MVAR at bus 3. As the load in the system was increased, the load participation at bus 3 was such that it remained 90% of the load at bus 2. In this example, trajectory 1 remained relatively unchanged, with the voltages dropping as the load is increased. Because of its larger load, bus 2's voltage was always slightly below that of bus 3. The low voltage trajectories changed substantially. Trajectory 2 now no longer joins those of 3 and 4, but rather moves towards an intersection with trajectory 1 at the point of voltage collapse. Trajectories 3 and 4 also move towards a point of mutual intersection, however the load value associated with this intersection point is substantially below the value associated with the point of voltage collapse for the system. As would be expected, the Jacobian of the system at the intersection point of trajectories 3 and 4 is singular. Figure 5-5 shows the results of the case where the load at bus 3 was only 60% of the load at bus 2. The bottom trajectory is continuing to move closer to the X axis. Numerically it was not possible to detect this trajectory for loads at bus 3 which were less than 50% of the initial 50 MW/25 MVAR load at bus 2.

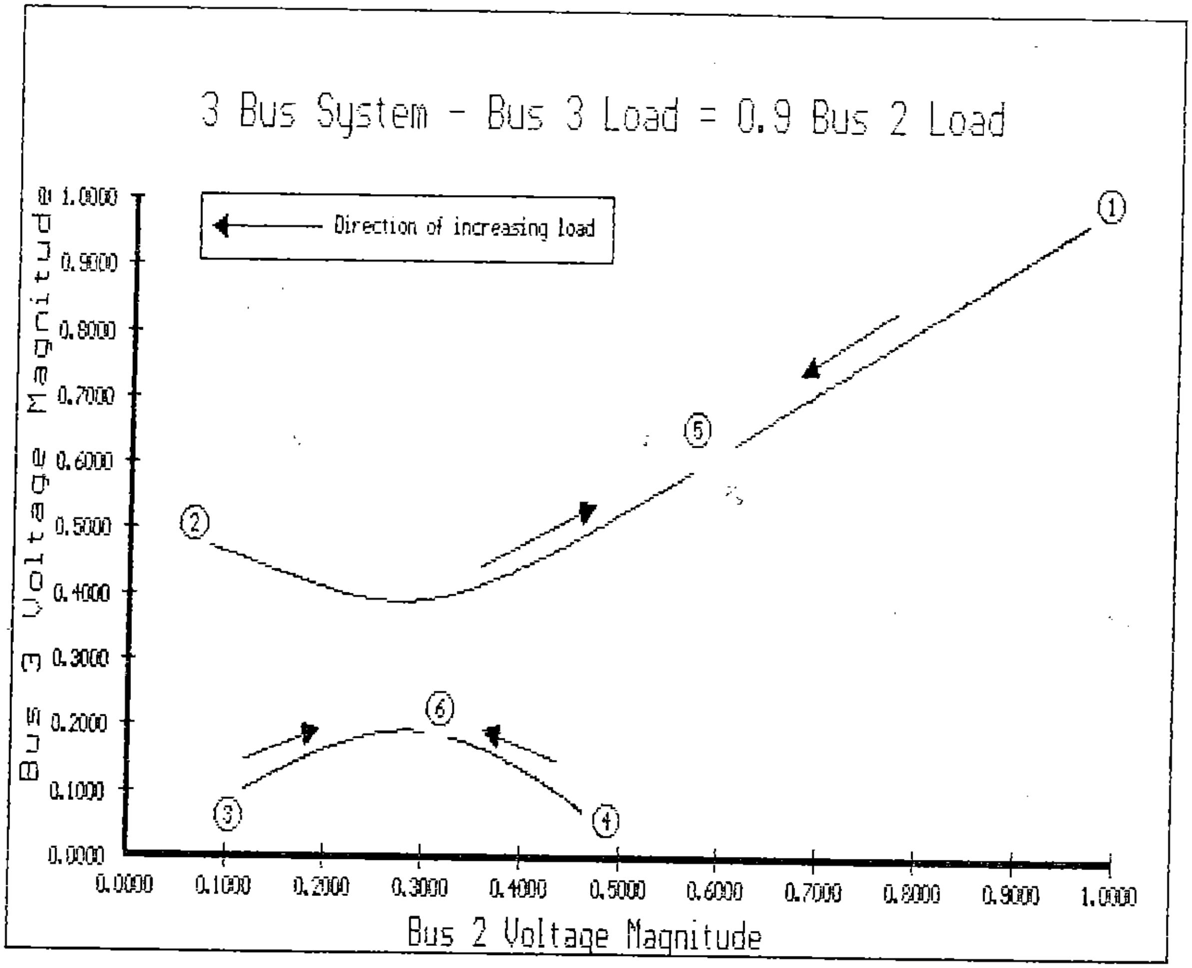


Figure 5-4

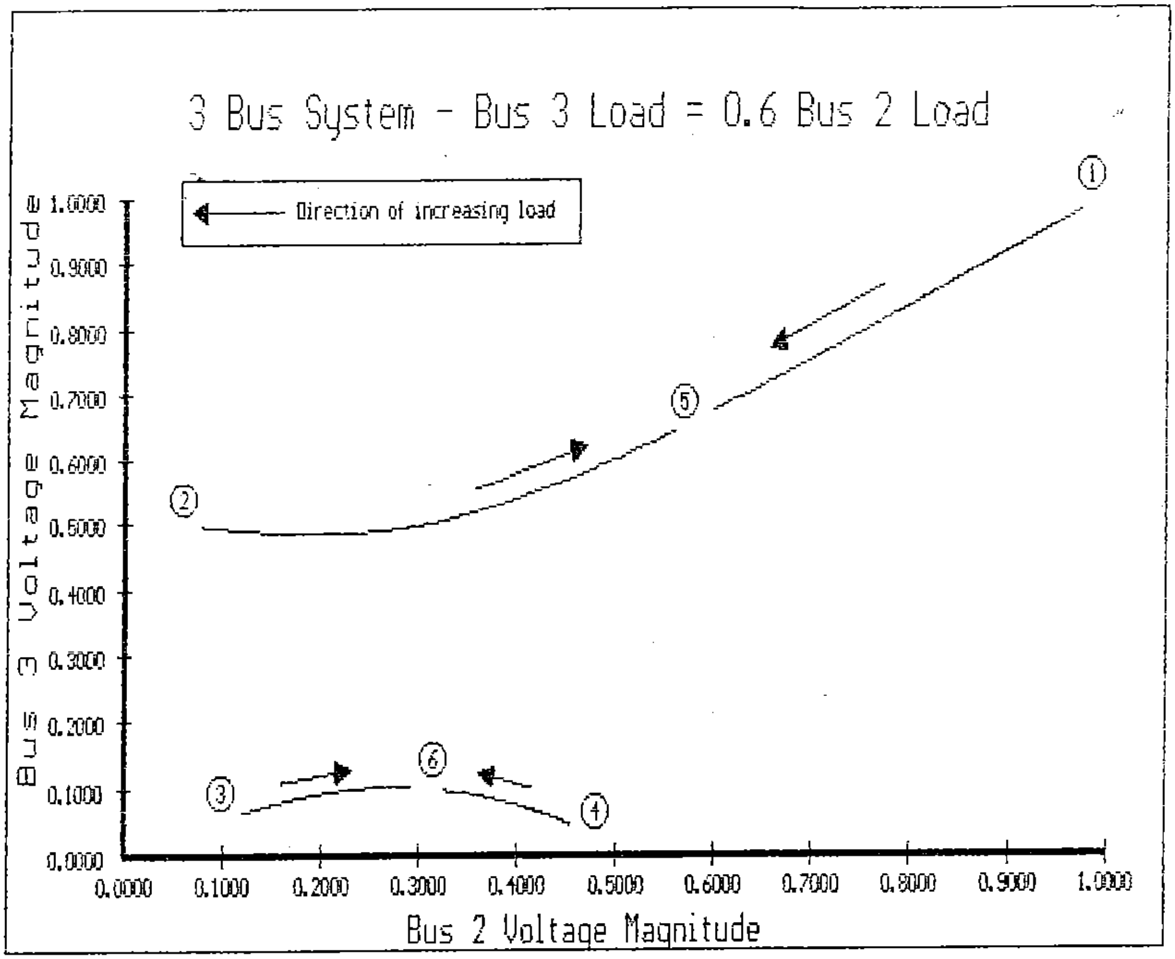


Figure 5-5

As the previous examples have shown, the power system equations often have multiple 'low voltage' solutions. In order to use the energy function method, it is necessary to determine which of this 'set' of solutions is relevant in defining the desired security measure. This remains an area for future research, but the following interpretation of the results does shed some light on the problem. Returning again to the simple three bus system consisting of two radial loads, it is fairly easy to plot the energy difference between each of the three low voltage solutions, and the high voltage solution. Figure 5-6 contains this data plotted vs the load at bus 2 (the loads at bus 2 and 3 are considered equal). As can be seen, the energy value associated with the case where the voltages at both 2 and 3 are low is significantly higher than for the cases where either 2 or 3 is high. The energy value in the former case would correspond to the expected exit time with both areas experiencing voltage collapse simultaneously (an unlikely event if they are truly independent), while in the later case the energy value would correspond to one of the buses collapsing independent of the other (a much more likely event, and hence a lower energy difference). This suggests a possible technique for determining which of the low voltage solutions to use. Since the high voltage solution corresponds to the known operating point, one may simply use the low voltage solution which yields the smallest energy difference between itself and the high voltage solution.

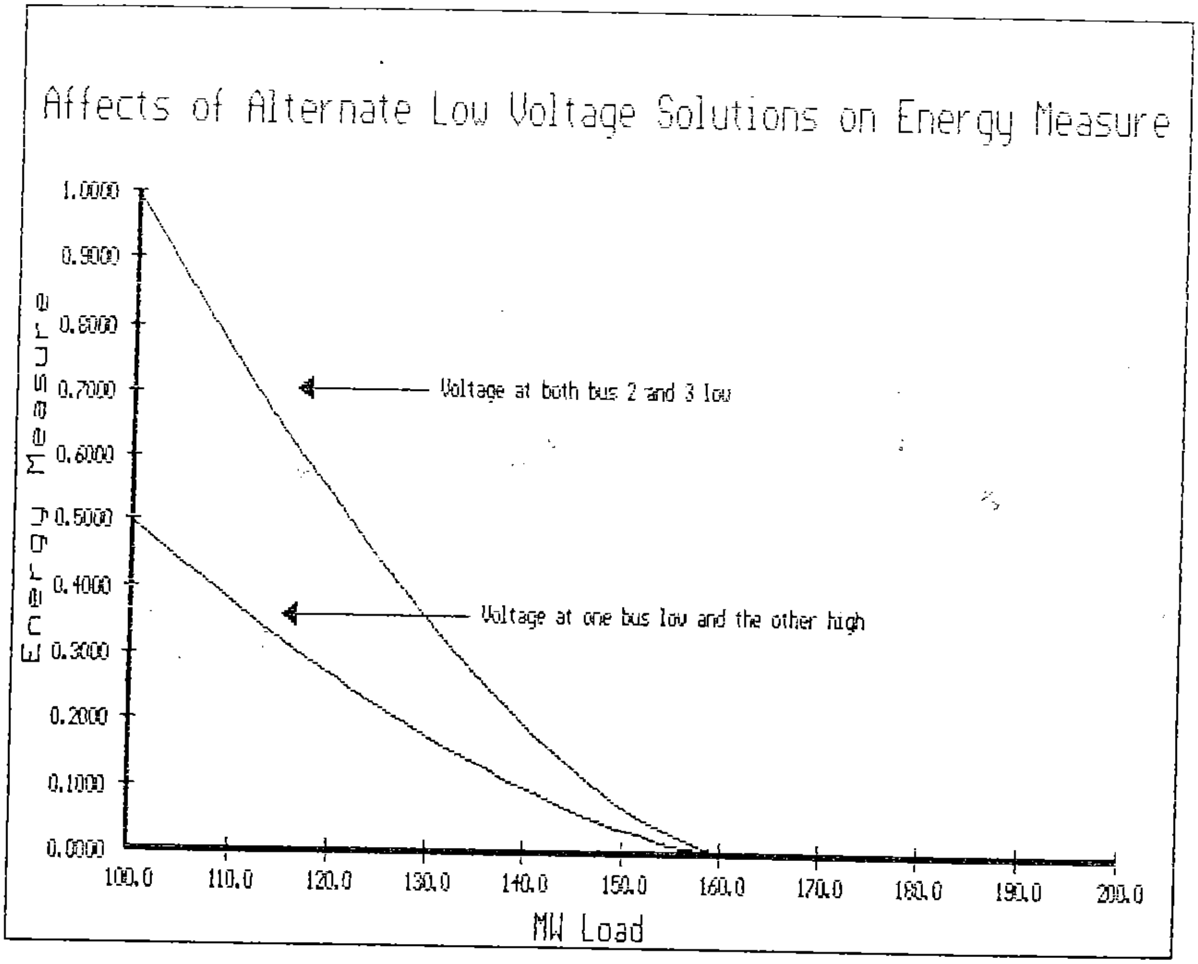


Figure 5-6

To find this solution with a Newton-Raphson iteration, a good rule of thumb is to initialize the power flow with extremely low voltage magnitude (about 0.1 pu.) at the bus which is thought to be the most vulnerable to voltage collapse.

Applying this rule to the system used for figure 5-4, one would expect voltage collapse to occur at bus 2 first since the load is greater at this bus than at bus 3. Figure 5-7 shows the plots of the energy difference between each of the three low voltage trajectories and the high voltage trajectory. As expected, the trajectory corresponding to collapse around bus 2 does indeed result in the lowest value.

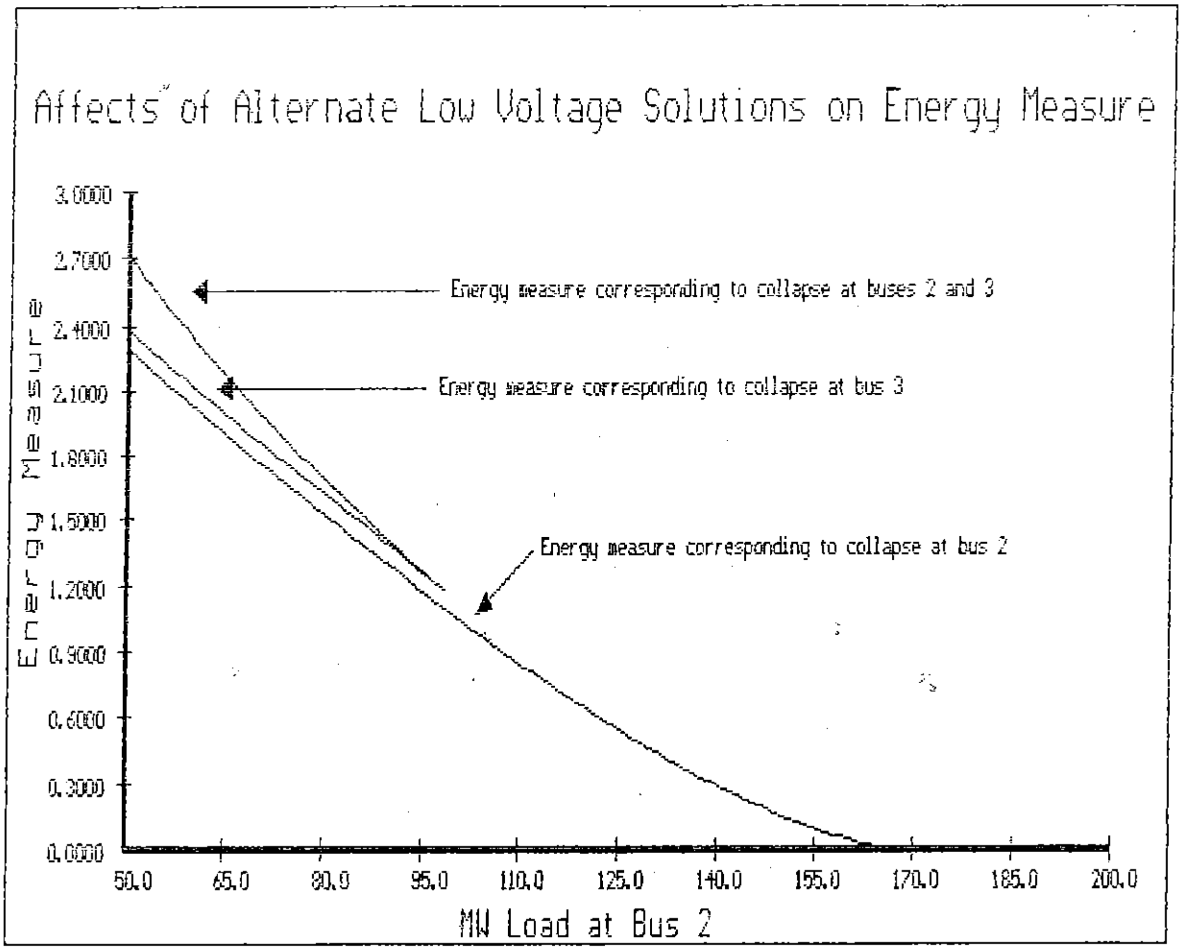


Figure 5-7

In the sample 16 bus system used in section 4 it was also found that the lowest system-wide energy margin was obtained by collapsing around the bus with the lowest energy margin in an area. This was accomplished by setting the initial guess for that voltage's bus to be 0.1 at an angle of zero, with the other voltages at 1.0. Clearly, the identification of the weakest bus is a heuristic judgement. Here it was taken to be the bus with the weakest ties to the rest of the system. The solution did not require that the initial values be extremely close to the actual values. In the case of area 1 (which contains buses 13, 14, 15, and 16) identification of bus 16 as the weakest was straightforward. Figure 5-8 shows the voltage magnitude profile for Area 1 collapse using bus 16 as the low initial guess. In the case of area 2 (which contains buses 9, 10, 11, and 12) labeling the weakest bus was more difficult. Any of the buses in the area could have been classified as weakest under the criterion established for the three bus system. The heuristic used for the larger system was to select the bus which had the lowest voltage magnitude in the high voltage solution. Figure 5-9 shows two of the low voltage magnitude profiles for area 2 collapse; one using bus 9, the other using bus 12 as the low initial guess.

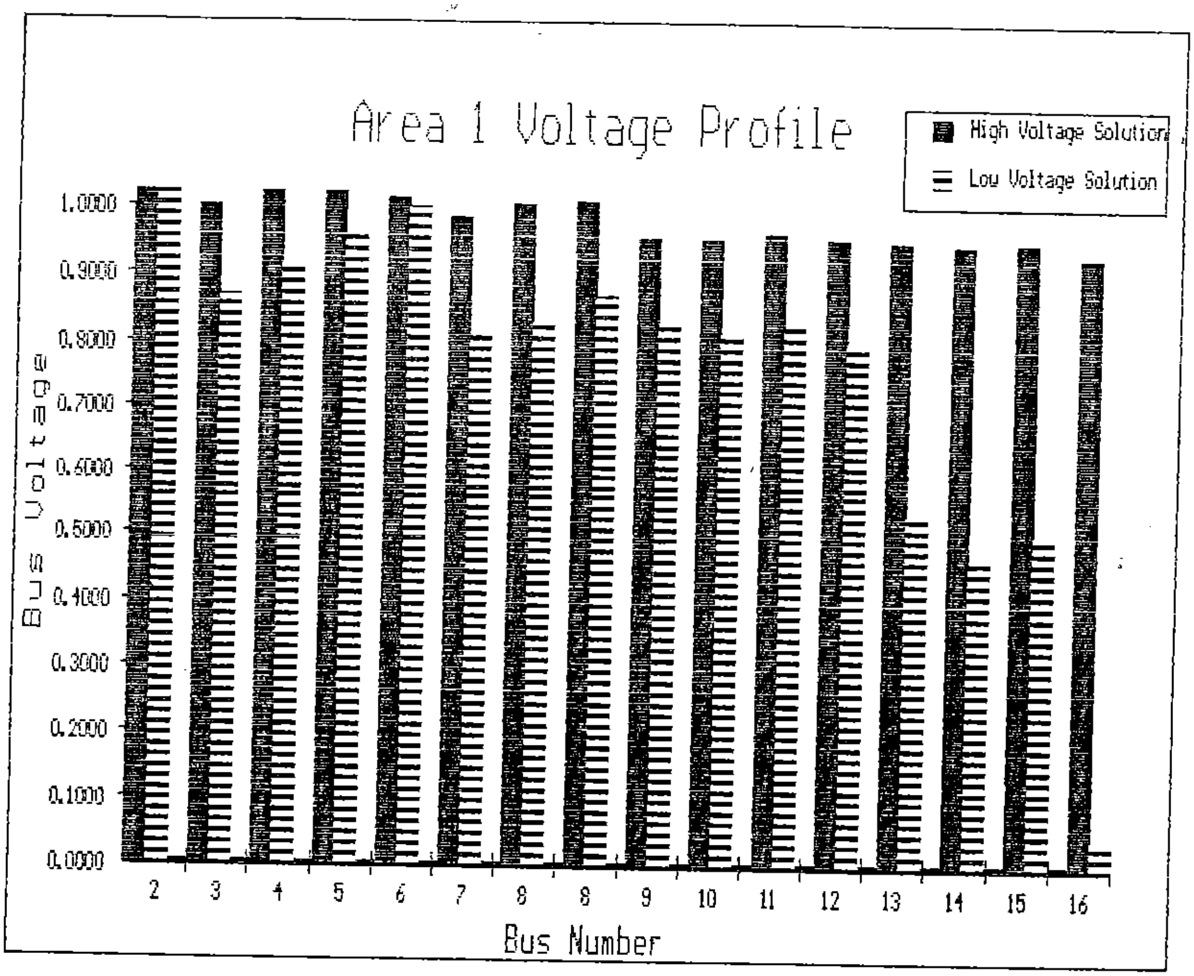


Figure 5-8

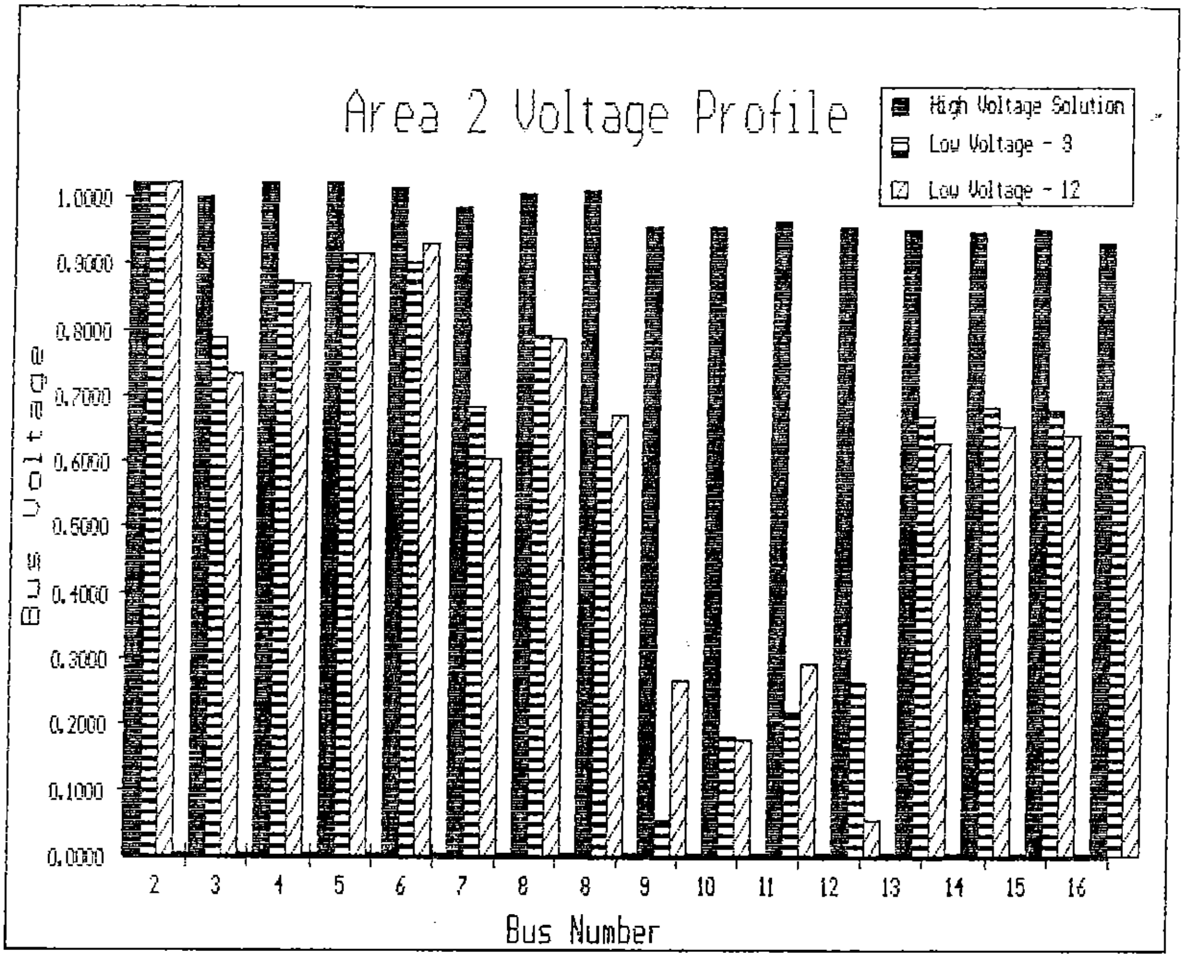


Figure 5-9

It is interesting to note that as the load in the 16 bus system was uniformly increased, areas 1 and 2 became less and less independent. Eventually it was no longer possible to obtain the low voltage solution for the low initial guess at bus 12. This was similar to what was observed in the three bus system is figures 4-2 to 4-4; eventually some solution trajectories terminate at loads below the voltage collapse point. As the load was increased further, the solution at bus 9 also terminated; when this occurred areas 1 and 2 could no longer be viewed as separate and thus it was necessary to collapse the system about the weakest bus in the combined area (which in this case was bus 16).

Identifying low voltage powerflow solutions represents a challenging application of bifurcation theory. Determining the initial guess of the voltage magnitudes and angles in order to arrive at a desired solution is not always intuitively obvious. This difficulty can be illustrated by again considering the two bus system. With a load of 200 MW and 50 MVAR at bus 2, the voltages at bus 2 corresponding to the two solutions are 1) 0.922 -12.53° and 2) 0.224 -63.43°.

Depending upon the initial voltage magnitude and angle guess at bus 2, the Newton-Raphson algorithm either converges to one of these values or does not converge. Figure 5-10 shows all of the initial voltage magnitude guesses between 0 to 2 pu. and initial angle guesses between -180° and 180° degrees which converged to the high voltage solution. Likewise, Figure 5-11 shows those which converged to the low voltage solution. As one would expect, initial guesses close to one of the two solutions converged to that solution. However the boundary between these two "regions of attraction" appears to be very complicated. Finding an initial guess which converges to a given solution does not mean that all other guesses "closer" to that solution will necessarily converge to it. The Newton-Raphson algorithm used here was forced to abort anytime the solution process caused a voltage magnitude to be negative or after 25 iterations.

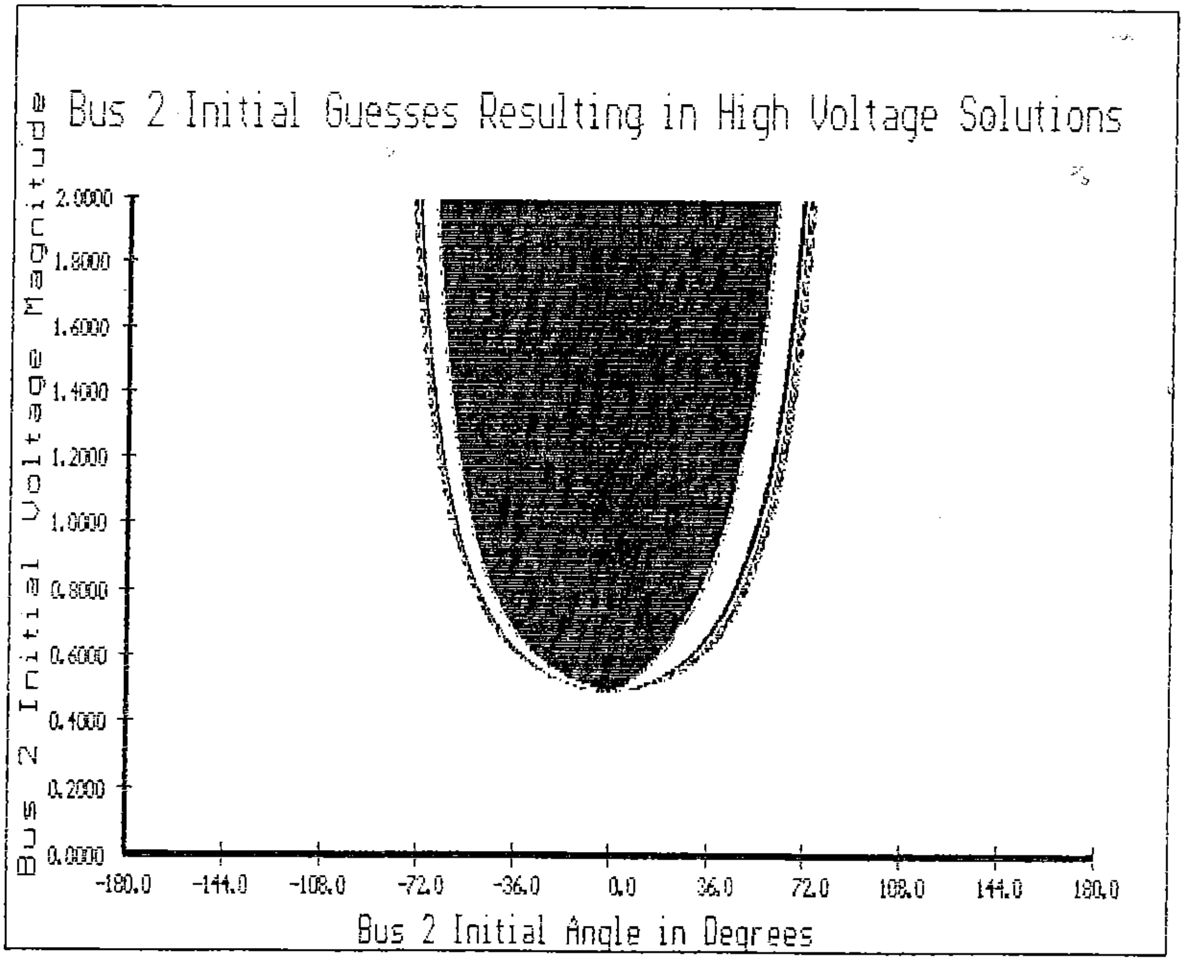


Figure 5-10

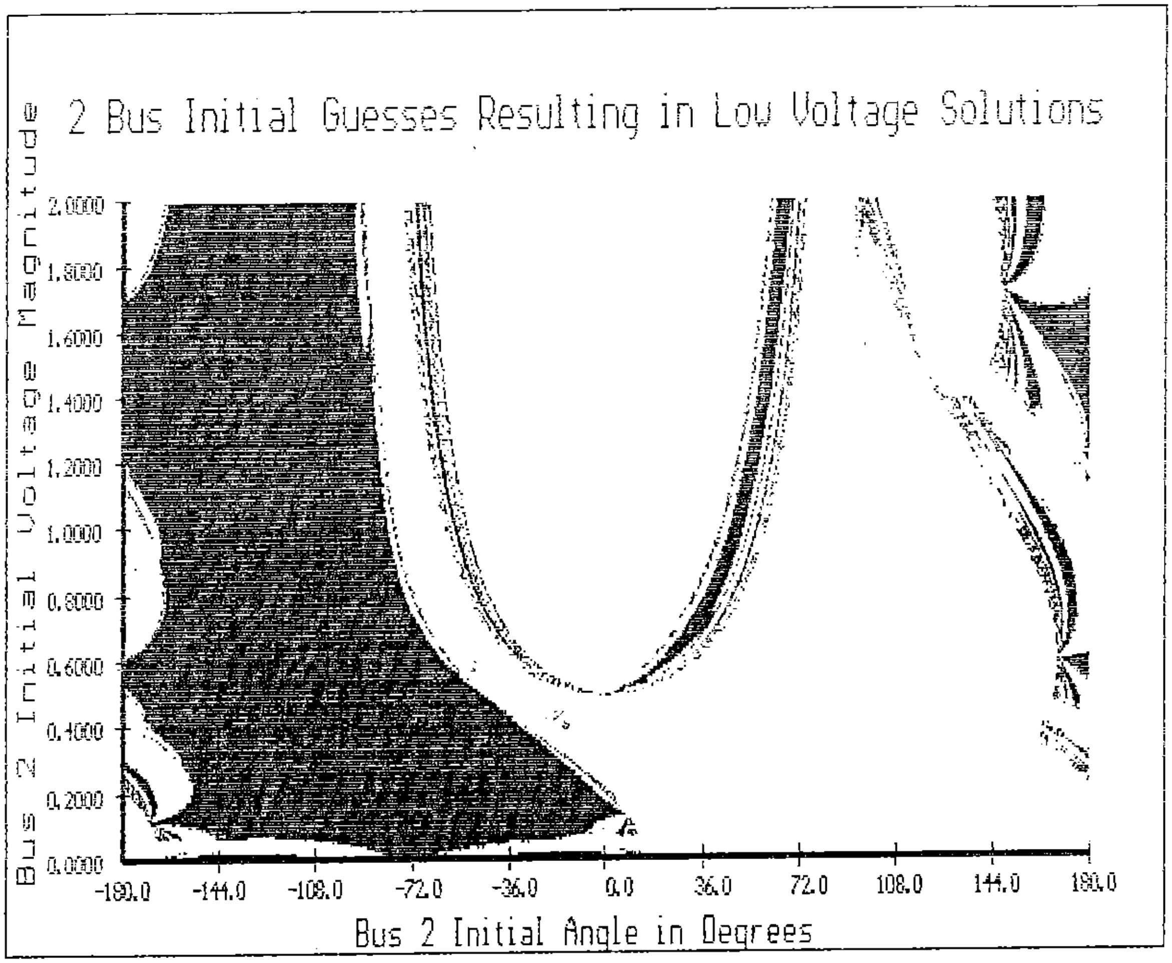


Figure 5-11

Figure 5-12 shows the "regions of attraction" for the three system used in figure 5-4 for varying initial voltage magnitude guesses at buses 2 and 3 with constant initial angle guesses of 0. In this problem, with load values of 50 MW and 25 MVAR at bus 2 and 45 MW and 25 MVAR at bus 3, four solutions are shown in Table 4.

Table 4

	|V1|	Angle 1	|V2|	Angle 2	|V3|	Angle 3
1)	1.0	0.0°	0.944	-5.88°	0.946	-5.66°
2)	1.0	0.0°	0.082	-62.33°	0.462	-14.76°
3)	1.0	0.0°	0.450	-16.03°	0.074	-63.14°
4)	1.0	0.0°	0.120	-56.54°	0.105	-58.75°

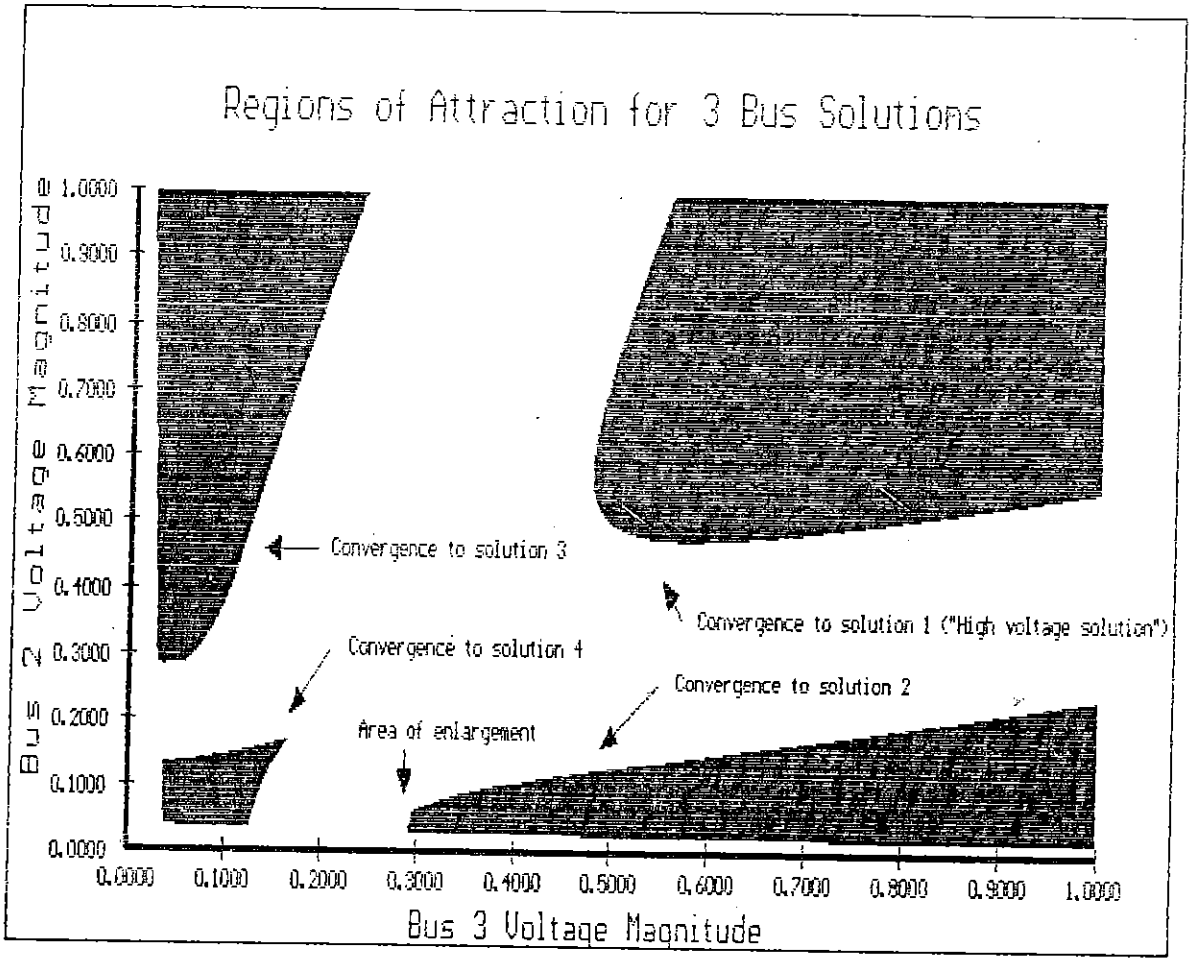


Figure 5-12

These four solutions correspond to the four "regions of attraction" in figure 5-12. The regions appear to be contiguous, however, as the enlargement of a small portion of region 2 shows in figure 5-13, their boundaries are not smooth. The white areas represent initial conditions for which a solution was not found.

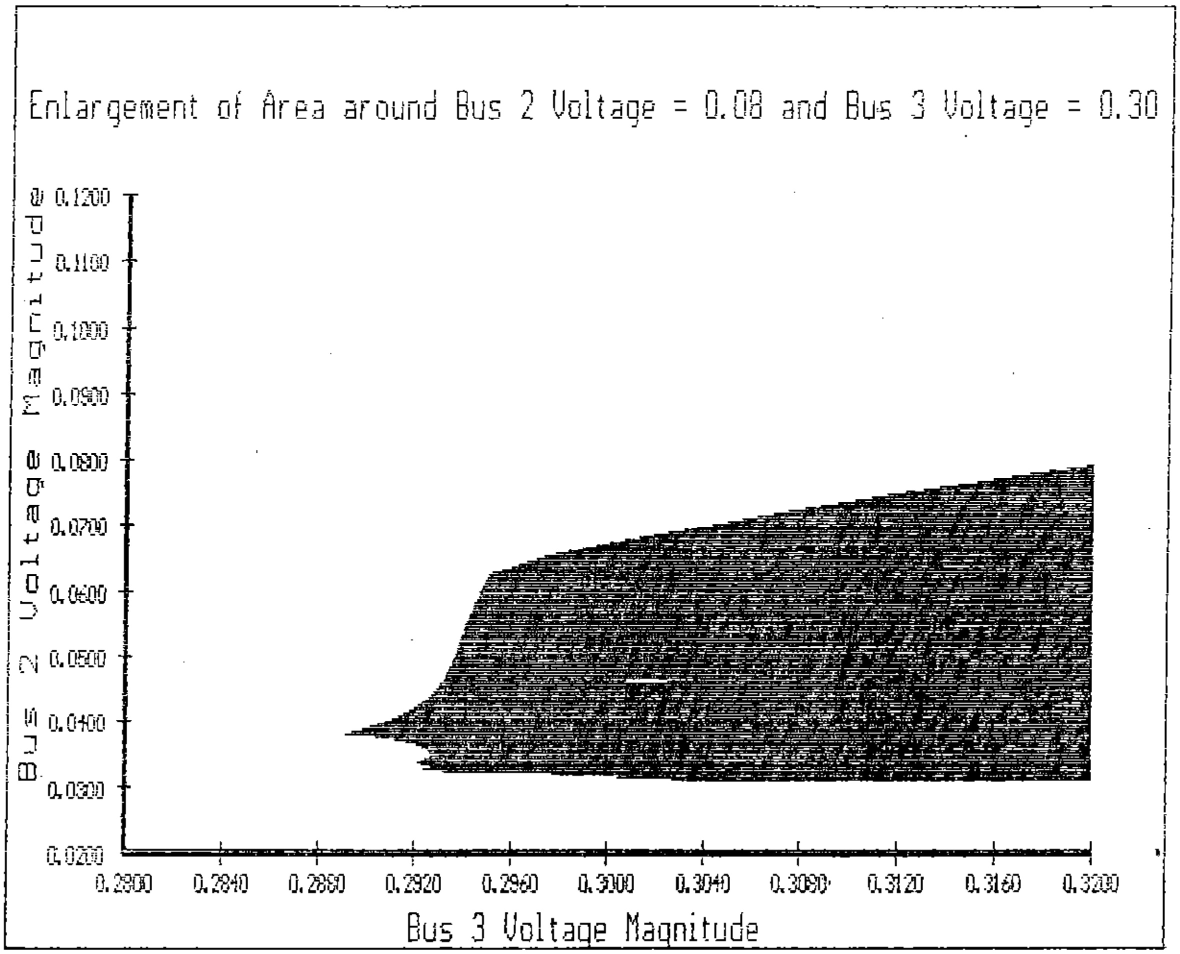
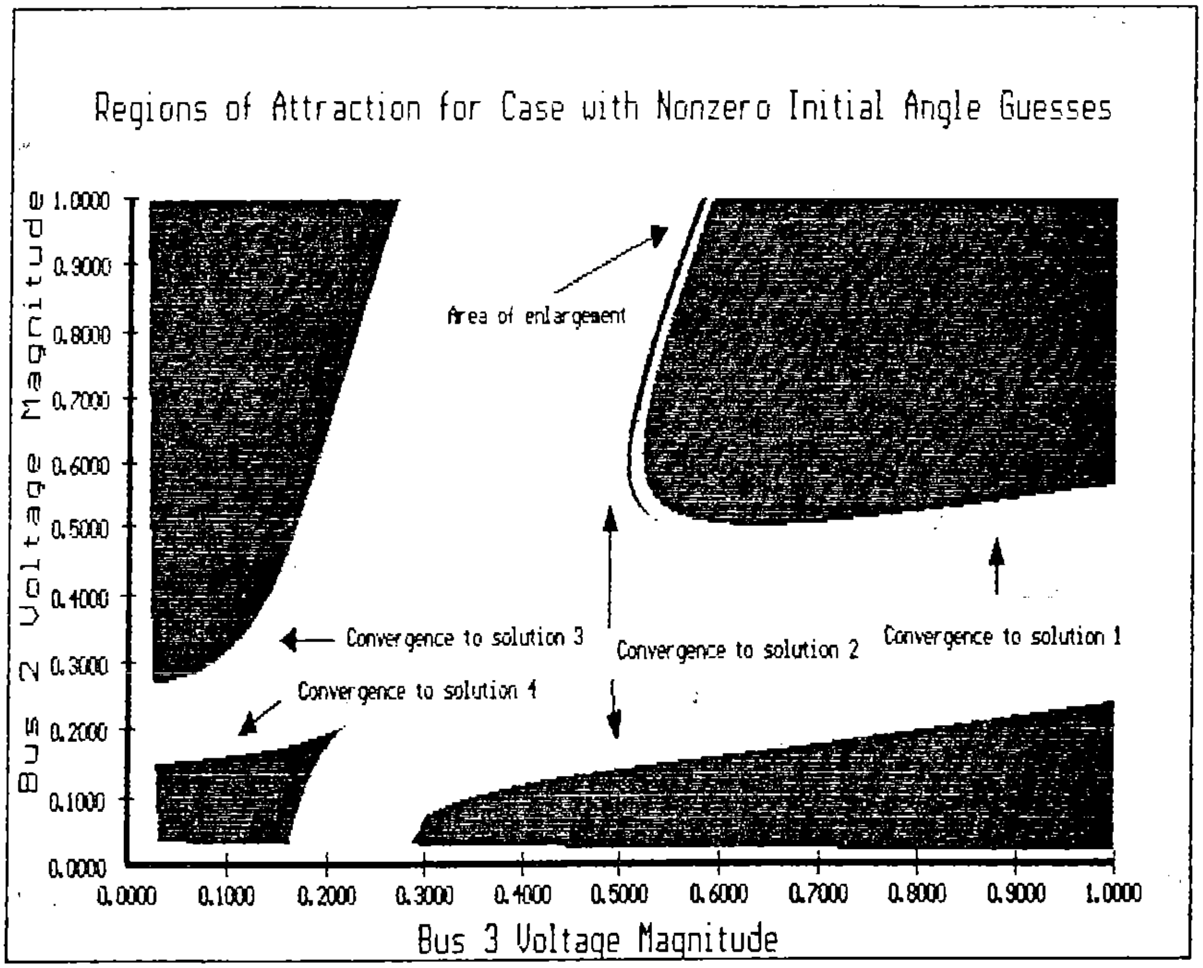


Figure 5-13

The regions of attraction become more complex in the case where the initial angle guesses were no longer zero. Figure 5-14 shows the case with initial angles at bus 2 and 3 of -5.73° -11.46° degrees respectively. The shapes of the four large regions from the previous problem appear relatively unchanged; however a new region has formed immediately to the left of area 1. Surprisingly, initial guesses in this area result in convergence to solution 2. Table 5 shows the convergence path taken for an initialize guess of $|V2| = 1.0$ and $|V3| = 0.58$. Upon enlargement, shown in figure 5-15, another small, slender area appears between the two areas. This area's attraction is to solution 3. More research is required into both the properties of these solutions, and into algorithms to locate the particular low voltage solution in an area that yields the lowest energy.



Regions of Attraction for Case with Nonzero Initial Angle Guesses
Bus 2 Voltage Magnitude
1.0000
0.9000
0.8000
0.7000
0.6000
0.5000
0.4000
0.3000
0.2000
0.1000
0.0000
Area of enlargement
Convergence to solution 3
Convergence to solution 2
Convergence to solution 1
Convergence to solution 4
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1.0000
Bus 3 Voltage Magnitude

Figure 5-14

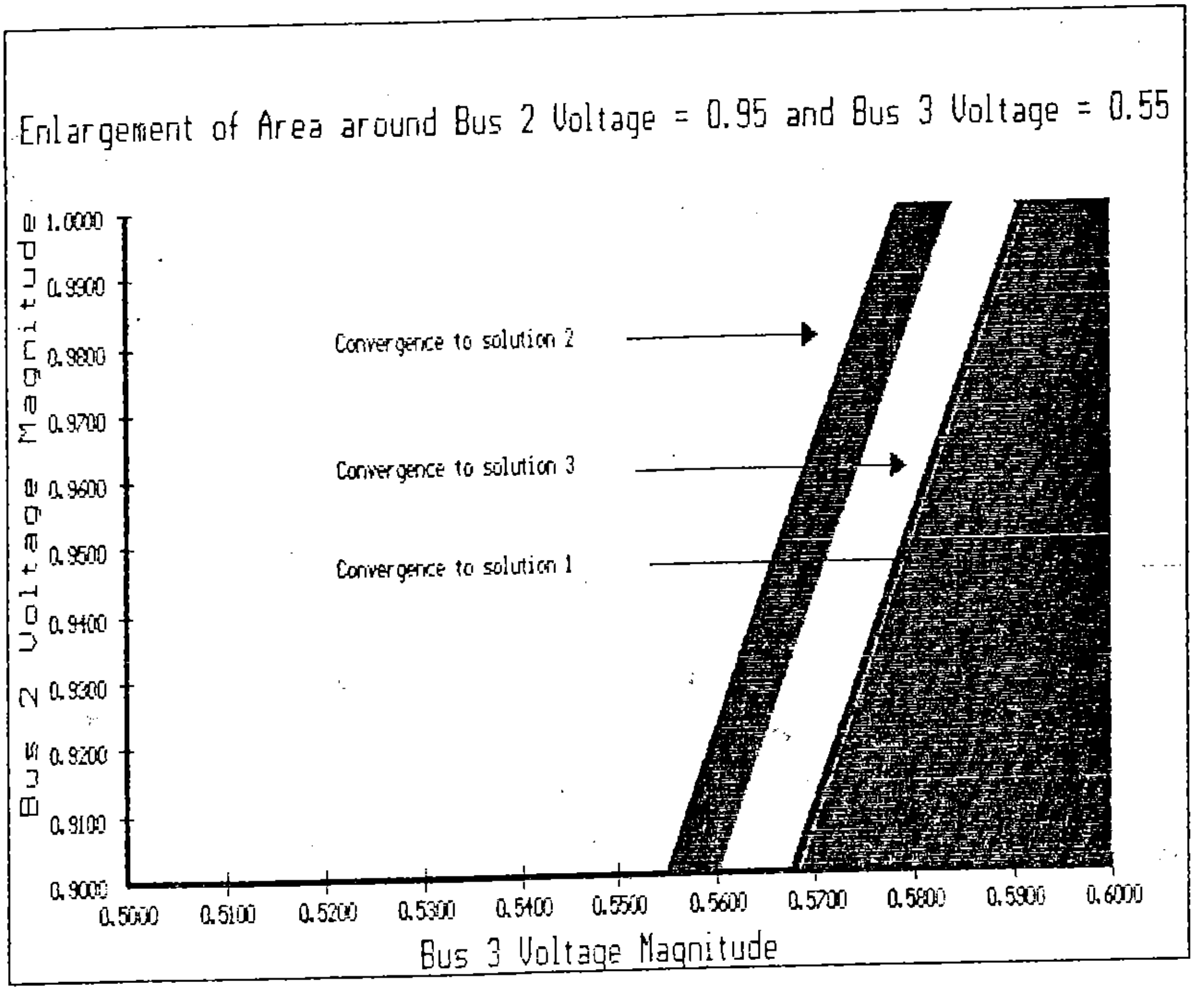


Figure 5-15

6. Conclusions and Directions for Further Research

This report has discussed a new method of assessing vulnerability of a power system operating point to voltage collapse based on an energy function defined for the system. The advantage of this approach is that it offers an improved measure of proximity to the voltage collapse point, giving a security measure that operators might use to anticipate when corrective action is necessary, before collapse occurs. From a mathematical standpoint, this advantage comes from the fact that the method treats nonlinearities in the powerflow and system dynamics, rather than working only with a linearization at the current operating point. Practically, this gives the method the ability to describe such effects as var limits on generators and load tap changing limits on transformers.

The method in its existing state of development appears to be quite promising. Further research would be valuable in several areas. First, more detailed dynamics should be included in the model and associated energy function. In particular, the effects of on load tap changing transformers, generator voltage control systems, and HVDC links would be relevant. Further work on calculation of low voltage solutions is also important. A heuristic method is needed to efficiently ascertain the critical alternate low voltage solutions.

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